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# Understanding Hip Biomechanics: From Simple Equilibrium to Personalized HIPSTRESS Method

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Additional information is available at the end of the chapter

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## Abstract

It is useful to have a quantitative measure of the contact hip stress and other relevant biomechanical parameters. Parameters that correlate with clinically relevant features are sought and relations between these parameters are studied. For this purpose, two different models for the resultant hip force in the one-legged stance (the primitive model and the HIPSTRESS model) are presented with which the effect of the shape of the pelvis and proximal femora is described. Also, a special case of the primitive model—the simple balance approximation—is considered. All three descriptions are based on the equilibrium of forces of torques and differ by increasing amount of information on the shape of the particular subject. It is shown in a case of normal hip and pelvis geometry that the primitive model gives similar values of biomechanical parameters as the HIPSTRESS model that was validated by clinical studies. The primitive model (but not the simple balance approximation) merits to minimal standards to be used for understanding of the principles of the equilibrium of the forces and torques in the one-legged stance and can in certain cases (such as the one shown) also yield a valid quantitative estimation of the biomechanical parameters.

**Keywords:** hip stress, resultant hip force, hip osteoarthritis, cartilage degeneration, hip dysplasia, hip osteotomy

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## 1. Equilibrium of forces and torques

Within biomechanics the effects of mechanical forces (forces due to gravity, elasticity, and friction) on living mechanisms are considered. These forces determine the movement of human and animals which is, especially in vertebrates, enabled by a complex and interconnected network of muscles, tendons, and bones that act as a consistent kinematical chain. A living system is never static on the cellular level, however, as a whole, the body can attain certain positions which are

taken to correspond to static equilibria. The body is in static equilibrium when the sum of all external forces acting upon it equals to zero and sum of all torques subject to these forces equals to zero. The first condition is expressed by equation

$$\mathbf{F} = (-F \sin \vartheta_F, -F \cos \vartheta_F, 0) \quad (1)$$

where  $\mathbf{F}_i = (F_{x,i}, F_{y,i}, F_{z,i})$  is the  $i$ -th force and the second condition is expressed by equation

$$\begin{aligned} \mathbf{M}_F = \mathbf{r}_F \times \mathbf{F} &= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -x_F & y_F & 0 \\ -F \sin \vartheta_F & -F \cos \vartheta_F & 0 \end{bmatrix} \\ &= (0, 0, x_F F \sin \vartheta_F + y_F F \cos \vartheta_F), \end{aligned} \quad (2)$$

where  $\mathbf{M}_i = (M_{x,i}, M_{y,i}, M_{z,i})$  is the torque of the  $i$ -th external force, defined as a cross product

$$-x_{CM}(W_B - W_L) + F(x_F \cos \vartheta_F + y_F \sin \vartheta_F) = 0. \quad (3)$$

with  $\mathbf{r}_i = (x_i, y_i, z_i)$  the momentum arm of the  $i$ -th external force. Index  $i$  runs over all forces acting upon the body.

The cross product can be expressed by the matrix

$$\mathbf{M}_i = \mathbf{r}_i \times \mathbf{F}_i = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_i & y_i & z_i \\ F_{x,i} & F_{y,i} & F_{z,i} \end{bmatrix} \quad (4)$$

with the result

$$\mathbf{M}_i = ((y_i F_{z,i} - z_i F_{y,i}), (z_i F_{x,i} - x_i F_{z,i}), (x_i F_{y,i} - y_i F_{x,i})) \quad (5)$$

In the description of the static equilibrium, the image of the body is divided into segments. These segments act one upon another which is expressed by means of intersegment forces. The segments are also subjected to attraction of the Earth. As these forces and their momentum arms in general attain different directions in space, all torque components have in general nonzero values. However, in certain situations the expressions are simplified, such as in the case where the balance consists of a dimensionless rigid rod supported in a certain point, with two vertical load forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$ , each acting on a different side of the support, with momentum arms  $r_1$  in  $r_2$  (**Figure 1**). Let the positive  $x$ -axis point in the medial direction, positive  $y$ -axis in the superior direction, and positive  $z$ -axis in the anterior direction.

There are three forces acting on the balance, the two load forces  $\mathbf{F}_1$  in  $\mathbf{F}_2$  and the ground force originating in the support point. This force is called the resultant force  $\mathbf{R}$ . As the forces  $\mathbf{F}_1$  and  $\mathbf{F}_2$  act in the negative vertical direction,

$$\mathbf{F}_1 = (0, -F_1, 0), \quad (6)$$

$$\mathbf{F}_2 = (0, -F_2, 0). \quad (7)$$

The resultant force is not known; therefore, we will consider that it has three components,

$$\mathbf{R} = (R_x, R_y, R_z) \quad (8)$$

To determine the momentum arms, a choice of the origin of the coordinate system must be made. It is convenient to choose it at the origin of the resultant force  $\mathbf{R}$ . In general, the momentum arms have three components,

$$\mathbf{r}_1 = (x_1, y_1, z_1) \quad (9)$$

$$\mathbf{r}_2 = (x_2, y_2, z_2) \quad (10)$$

however, in the case presented in **Figure 1**, the rod extends in the direction of  $x$ -axis only, and therefore the components of the momenta in the directions of  $y$  and  $z$  axes are equal to zero. The momentum arm of the force  $\mathbf{F}_1$  points in the negative direction of  $x$ -axis,

$$\mathbf{r}_1 = (-x_1, 0, 0) \quad (11)$$

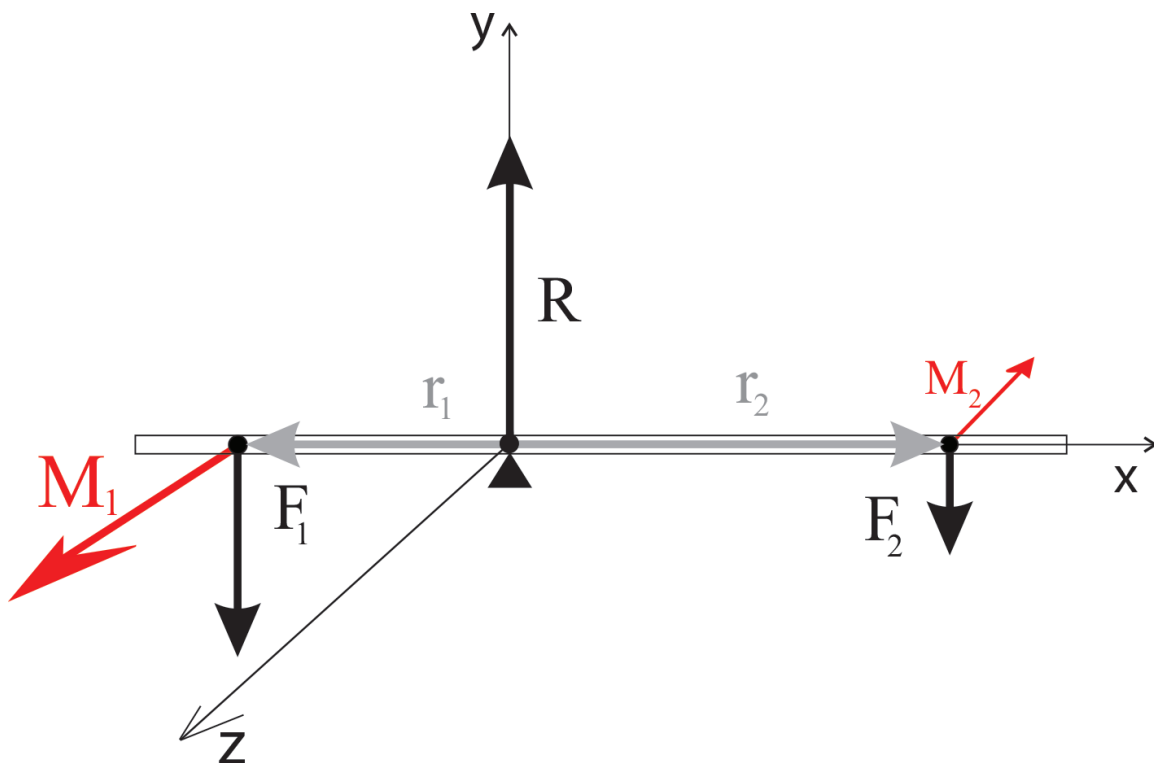
while the momentum arm of the force  $\mathbf{F}_2$  points in the positive direction of  $x$ -axis,

$$\mathbf{r}_2 = (x_2, 0, 0) \quad (12)$$

The momentum arm of the resultant force  $\mathbf{R}$  is zero, due to our particular choice of the origin,

$$\mathbf{r}_R = (0, 0, 0) \quad (13)$$

The torques of all three forces are



**Figure 1.** Scheme of a simple balance if the load forces act in the vertical direction.

$$\mathbf{M}_1 = \mathbf{r}_1 \times \mathbf{F}_1 = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -x_1 & 0 & 0 \\ 0 & -F_1 & 0 \end{bmatrix} = (0, 0, x_1 F_1) \quad (14)$$

$$\mathbf{M}_2 = \mathbf{r}_2 \times \mathbf{F}_2 = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_2 & 0 & 0 \\ 0 & -F_2 & 0 \end{bmatrix} = (0, 0, -x_2 F_2) \quad (15)$$

$$\mathbf{M}_R = \mathbf{r}_R \times \mathbf{R} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0 \\ R_x & R_y & R_z \end{bmatrix} = (0, 0, 0) \quad (16)$$

In general, the equilibrium of forces is given by three equations for three components,

$$F_{1,x} + F_{2,x} + R_x = 0 \quad (17)$$

$$F_{1,y} + F_{2,y} + R_y = 0 \quad (18)$$

$$F_{1,z} + F_{2,z} + R_z = 0 \quad (19)$$

Following Eqs. (17)–(19), the components of the force  $R$  are

$$R_x = 0 \quad (20)$$

$$R_y = F_{1,y} + F_{2,y} \quad (21)$$

$$R_z = 0 \quad (22)$$

and the resultant force can be given as

$$\mathbf{R} = (0, F_1 + F_2, 0) \quad (23)$$

The equilibrium of torques is given by three equations for three components,

$$M_{1,x} + M_{2,x} + M_{R,x} = 0 \quad (24)$$

$$M_{1,y} + M_{2,y} + M_{R,y} = 0 \quad (25)$$

$$M_{1,z} + M_{2,z} + M_{R,z} = 0 \quad (26)$$

As the torque of the force  $\mathbf{R}$  is equal to zero and also the components of the torques due to load forces in the  $x$  in  $y$  directions are equal to zero, there remains only one nontrivial equilibrium equation for torques,

$$M_{1,z} + M_{2,z} = 0 \quad (27)$$

Considering also the expressions (14) and (15), we obtain

$$x_1 F_1 - x_2 F_2 = 0 \quad (28)$$

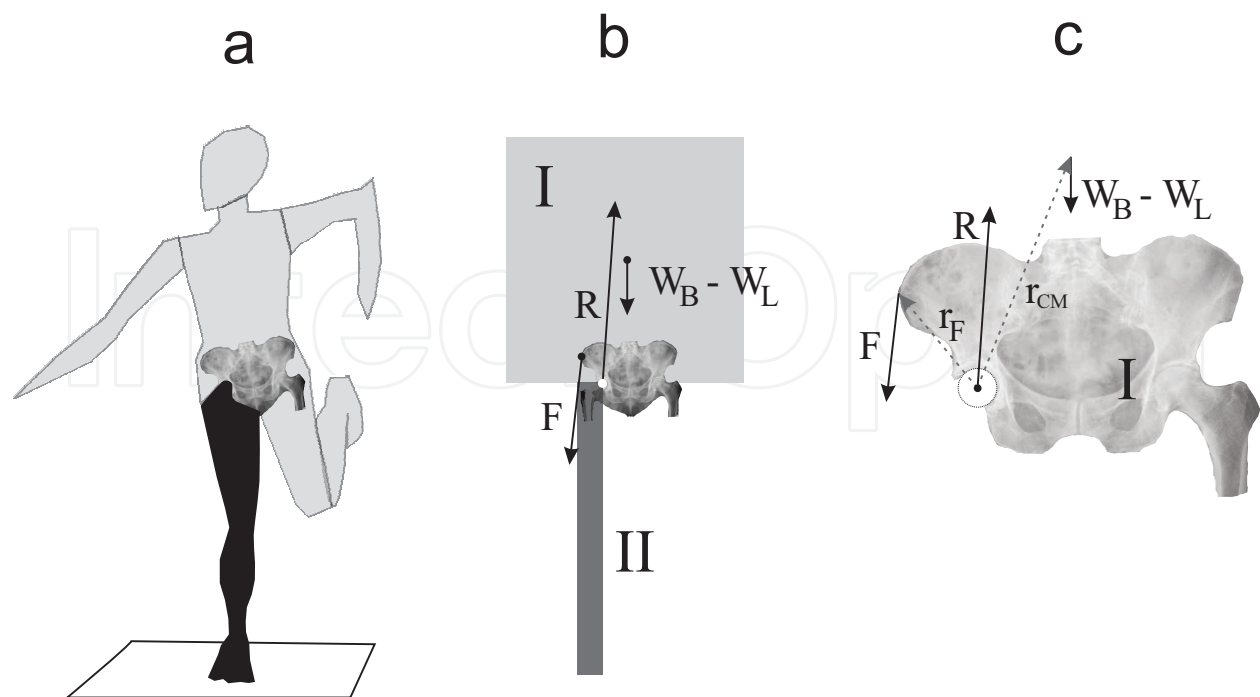
and finally

$$\frac{F_1}{F_2} = \frac{x_2}{x_1} \quad (29)$$

## 2. A two-segment model for the resultant hip force in the one-legged stance

In a simple model of a one-legged stance (**Figure 2**), the body is divided into two segments: the loaded leg and the rest of the body (**Figure 2a**). The two segments are connected by the hip joint. **Figure 2b** presents an abstraction of the two segments (labeled I and II, respectively). For simplicity, the pelvis is taken to be leveled in the model. The sizes of the boxes correspond to approximate weight proportion of the two segments. Further, it is assumed that all the forces lie in the frontal plane of the body through the centers of both femoral heads (their components in the  $z$  direction are zero). The forces and momenta arms acting on the segment I are indicated in panels b and c. The hip is loaded at the medial side by the weight of the segment I (denoted as  $\mathbf{W}_B - \mathbf{W}_L$ ), where  $\mathbf{W}_B$  is the weight of the entire body and  $\mathbf{W}_L$  is the weight of the loaded leg, and at the lateral side by a force of an effective muscle (denoted by  $\mathbf{F}$ ), which pulls the segment toward the loaded leg. There are several muscles which are active in the one-legged stance, but in this simple model all of them are represented by one effective muscle with one origin at the crista iliaca and the other at the greater trochanter (**Figure 2c**). It is taken that the muscle force acts in the direction of the line connecting both origins, expressed by the inclination angle  $\vartheta_F$ .

The model is based on equilibrium equations of forces and torques (Eqs. (1) and (2), respectively) acting on the segment I. Momentum arms of the weight of the segment I and of the



**Figure 2.** Scheme of a two-segment model of the one-legged stance. The body is divided into two segments: the loaded leg and the rest of the body (a). Abstraction of the two segments (labeled I and II, respectively) (b). Forces and their momentum arms (c).

effective muscle force can be determined from the geometry of the pelvis and proximal femur and the weight of the segment I can be determined from the body weight and an approximation that the leg weights about 1/7 of the entire body [1]. There are three unknown parameters in the model: the magnitude of the effective muscle force ( $F$ ) and the magnitude and direction (inclination with respect to vertical) of the resultant hip force ( $R$  and  $\vartheta_R$ , respectively).

### 2.1. A primitive model for resultant hip force

In the model (Figures 3 and 4), we have chosen the origin of the coordinate system at the center of the hip joint (that coincides with the center of the femoral head and the center of the acetabular shell). The loading forces are the weight of the segment I,

$$\mathbf{W}_B - \mathbf{W}_L = (0, -(W_B - W_L), 0), \quad (30)$$

with momentum arm  $\mathbf{r}_{CM}$ ,

$$\mathbf{r}_{CM} = (x_{CM}, y_{CM}, 0) \quad (31)$$

and the force of the effective muscle, which lies in the frontal plane through centers of the femoral heads,

$$\mathbf{F} = (-F \cos \vartheta_F, -F \sin \vartheta_F, 0), \quad (32)$$

with momentum arm  $\mathbf{r}_F$ ,

$$\mathbf{r}_F = (-x_F, y_F, 0). \quad (33)$$

The origin of the weight of the segment I is taken at the center of mass of the segment. It is approximated that this point lies in the sagittal plane of the body through the midline. Note that the components of the forces  $\mathbf{W}_B - \mathbf{W}_L$  and  $\mathbf{F}$  in the direction of the  $y$ -axis were taken to be negative, as these forces point downward and we have chosen that the positive direction of the  $y$ -axis is upward. Also, the component of the force  $\mathbf{F}$  in the direction of the  $x$ -axis and the momentum arm of the effective muscle force in the direction of the  $x$ -axis are negative. The resultant hip force  $\mathbf{R}$  is written as

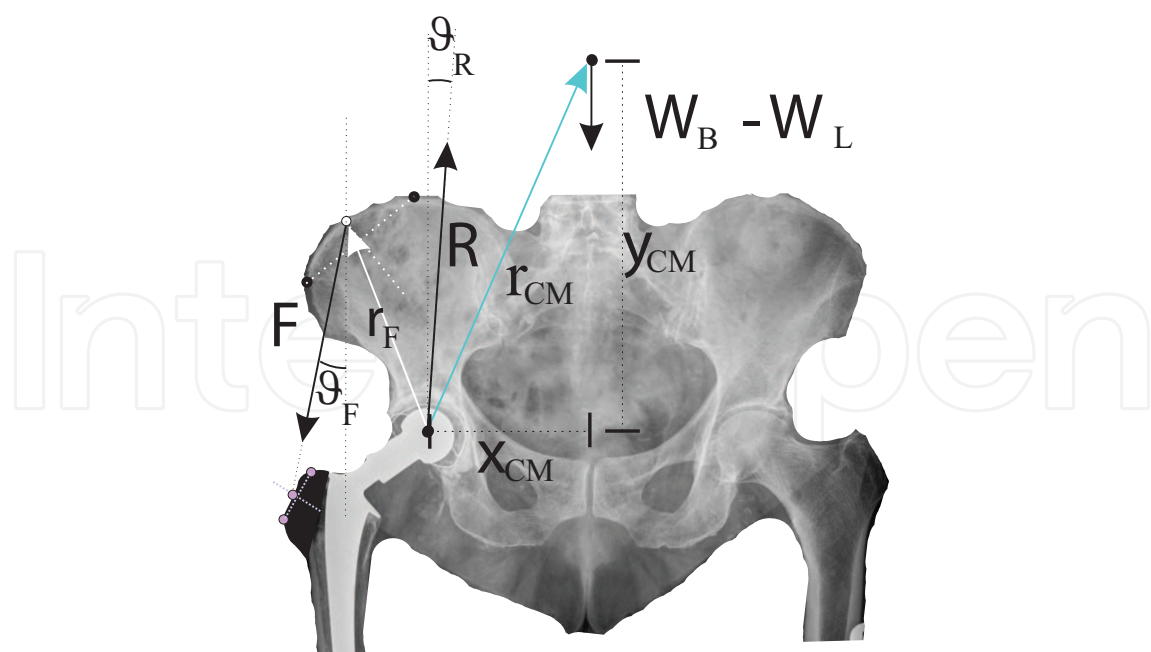
$$\mathbf{R} = (R \sin \vartheta_R, R \cos \vartheta_R, 0). \quad (34)$$

The respective torques are

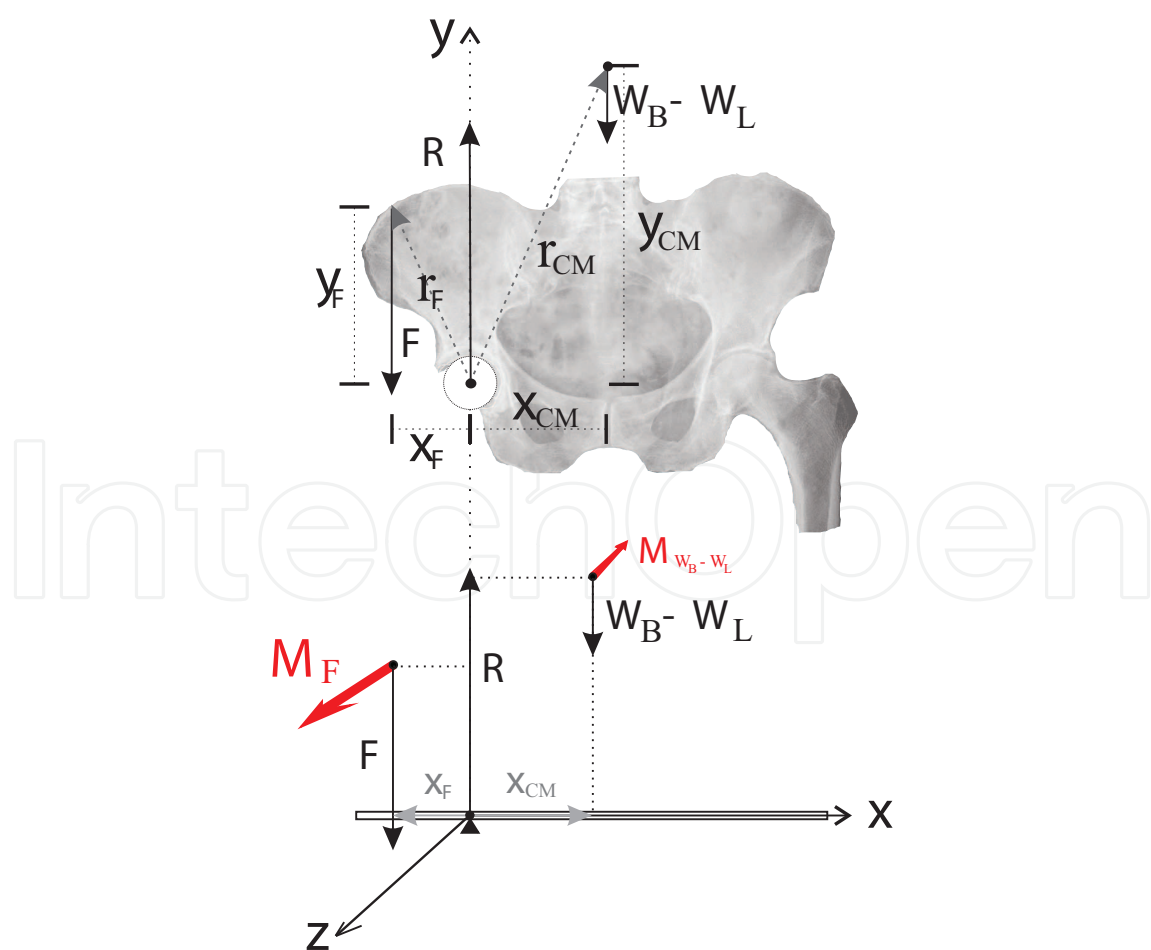
$$\mathbf{M}_{W_B - W_L} = \mathbf{r}_{CM} \times (\mathbf{W}_B - \mathbf{W}_L) = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_{CM} & y_{CM} & 0 \\ 0 & -(W_B - W_L) & 0 \end{bmatrix} = (0, 0, -x_{CM}(W_B - W_L)) \quad (35)$$

$$\mathbf{M}_F = \mathbf{r}_F \times \mathbf{F} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -x_F & y_F & 0 \\ -F \cos \vartheta_F & -F \sin \vartheta_F & 0 \end{bmatrix} = (0, 0, x_F F \sin \vartheta_F + y_F F \cos \vartheta_F) \quad (36)$$

and



**Figure 3.** Scheme of forces and momentum arms in the primitive model subject to segment I.



**Figure 4.** Scheme of a two-segment model of the one-legged stance.



$$\mathbf{M}_R = (0, 0, 0), \quad (37)$$

as the momentum arm of the resultant hip force is zero due to the choice of the origin of the coordinate system.

Following the above procedure, in particular Eq. (26), which describes equilibrium of torques, we obtain

$$-x_{CM}(W_B - W_L) + F(x_F \sin \vartheta_F + y_F \cos \vartheta_F) = 0. \quad (38)$$

Rearranging the above equation yields for the unknown magnitude of the effective muscle force  $F$ ,

$$F = \frac{x_{CM}(W_B - W_L)}{(x_F \cos \vartheta_F + y_F \sin \vartheta_F)}. \quad (39)$$

Following Eqs. (20)–(22), we obtain for the components in the direction of the  $x$ -axis

$$R \sin \vartheta_R = F \sin \vartheta_F \quad (40)$$

and in the direction of the  $y$ -axis

$$R \cos \vartheta_R = (W_B - W_L) + F \cos \vartheta_F. \quad (41)$$

Dividing Eq. (40) by Eq. (41) eliminates the unknown magnitude of the resultant hip force  $R$  and yields the expression for the inclination of the resultant force with respect to the vertical  $\vartheta_R$ ,

$$\tan \vartheta_R = \frac{\sin \vartheta_F}{\cos \vartheta_F + (W_B - W_L)/F}. \quad (42)$$

By knowing  $F$  and  $\vartheta_R$ , the magnitude of the resultant hip force  $R$  is then expressed from Eq. (40),

$$R = F \frac{\sin \vartheta_F}{\sin \vartheta_R}. \quad (43)$$

It is often convenient to present the results with respect to the body weight  $W_B$ . We also take into account that  $W_L = W_B/7$  [2] to get the expression for the normalized effective muscle force

$$\frac{F}{W_B} = \frac{6}{7} \frac{x_{CM}}{(x_F \cos \vartheta_F + y_F \sin \vartheta_F)}, \quad (44)$$

the inclination of the resultant hip force

$$\tan \vartheta_R = \frac{\sin \vartheta_F}{\cos \vartheta_F + 6W_B/7F}, \quad (45)$$

and the normalized resultant hip force

$$\frac{R}{W_B} = \frac{6}{7} \left( 1 + \frac{x_{CM}}{x_F + y_F \tan \vartheta_F} \right). \quad (46)$$

In a special case when the effective muscle force points in the vertical direction, i.e.,  $\vartheta_F = 0$  (**Figure 4**), the expressions (44)–(46) simplify into

$$\frac{F}{W_B} = \frac{6}{7} \frac{x_{CM}}{x_F}, \quad (47)$$

$$\tan \vartheta_R = 0, \quad (48)$$

$$\frac{R}{W_B} = \frac{6}{7} \left( 1 + \frac{x_{CM}}{x_F} \right). \quad (49)$$

Note that these expressions (Eqs. (47)–(49)) are the same as if obtained for a simple balance with the two loading forces

$$\mathbf{F}_1 = (0, -F, 0) \quad (50)$$

and

$$\mathbf{F}_2 = (0, -(W_B - W_L), 0) \quad (51)$$

and respective momentum arms

$$\mathbf{r}_F = (-x_F, 0, 0) \quad (52)$$

and

$$\mathbf{r}_{CM} = (x_{CM}, 0, 0). \quad (53)$$

Following Eqs. (29), (50), and (51), we obtain

$$\frac{F}{(W_B - W_L)} = \frac{x_{CM}}{x_F} \quad (54)$$

or (by taking into account that  $W_L = W_B/7$ )

$$\frac{F}{W_B} = \frac{6}{7} \frac{x_{CM}}{x_F}. \quad (55)$$

Following Eqs. (22), (50)–(51), and  $W_L = 6W_B/7$ , we obtain

$$R = F + \frac{6}{7} W_B, \quad (56)$$

or, normalized

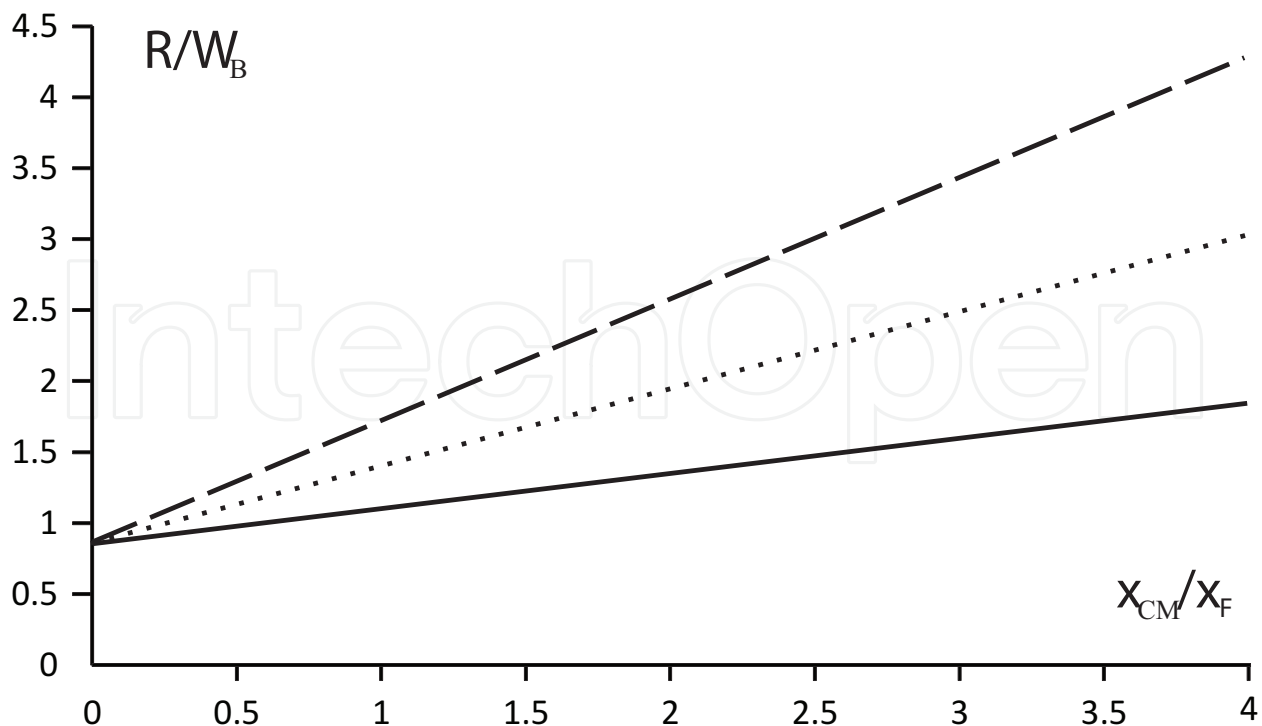
$$\frac{R}{W_B} = \frac{6}{7} + \frac{F}{W_B}. \quad (57)$$

Taking into account Eqs. (55) and (57) yields

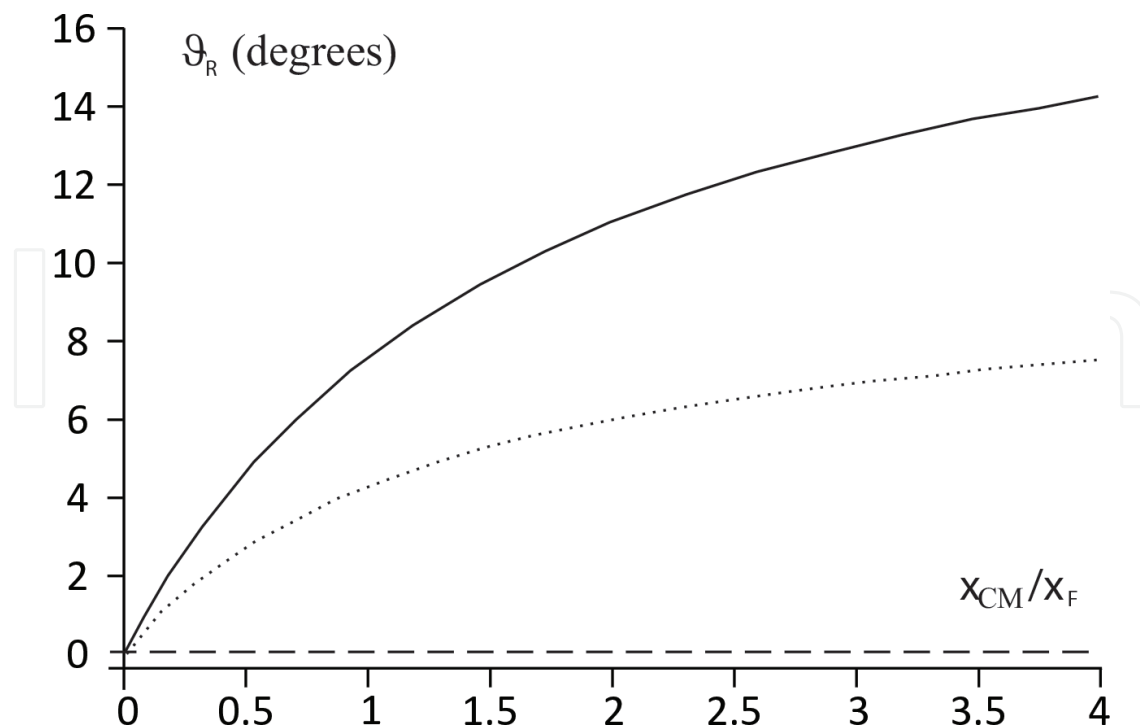
$$\frac{R}{W_B} = \frac{6}{7} \left( 1 + \frac{x_{CM}}{x_F} \right). \quad (58)$$

It can be seen that Eqs. (47) and (55) are identical. Likewise, Eqs. (49) and (58) are identical. Although the effective muscle attachment point on the iliac bone, the center of the femoral head, and the center of mass of the body segment I do not lie in the same horizontal plane, the model of simple balance derived for a weightless rigid bar with all forces originating in the same horizontal plane, gives the same solution, owing to a special case that the forces lie in the vertical direction only. It should however be kept in mind that this is a consequence of the simplifications used in the model of the one-legged stance and that in reality segment I has a characteristic shape that may impact the forces, which is not considered in the simple balance model. Some textbooks use a simple balance as an illustrative model to explain the principles of the effect of the muscle forces (the principles of different types of levers). It should be borne in mind that such approximations are valid only if all forces act in the same direction.

**Figure 5** shows the dependence of the magnitude of the resultant hip force  $R$  on the ratio between parameters  $x_{CM}$  and  $x_F$ , for the primitive model with two different inclinations of the effective muscle force ( $\vartheta_F = 20$  degrees, solid line, and  $\vartheta_F = 10$  degrees, dotted line),



**Figure 5.** Dependence of the normalized resultant hip force  $R/W_B$  on the ratio between geometrical parameters  $x_{CM}/x_F$  for the primitive model (Eq. (46)) with two different inclinations of the effective muscle force ( $\vartheta_F = 20$  degrees, solid line, and  $\vartheta_F = 10$  degrees, dotted line), and for the simple balance model (Eq. (58)) (broken line).  $y_F/x_F = 2$ .



**Figure 6.** Dependence of the inclination of the resultant hip force with respect to vertical direction  $\vartheta_R$  on the ratio between geometrical parameters  $x_{CM}/x_F$  for the primitive model (Eq. (45)) with two different inclinations of the effective muscle force ( $\vartheta_F = 20$  degrees, solid line, and  $\vartheta_F = 10$  degrees, dotted line), and for the simple balance model ( $\vartheta_R = 0$ , Eq. (48)) (broken line).  $y_F/x_F = 2$ .

and for the simple balance model (broken line). It can be seen that for larger  $x_{CM}/x_F$  and larger inclinations  $\vartheta_F$ , the difference between the models becomes substantial. **Figure 6** shows the dependence of the inclination of the resultant hip force with respect to vertical direction  $\vartheta_R$  on the ratio between parameters  $x_{CM}$  and  $x_F$ , for the primitive model with two different inclinations of the effective muscle force ( $\vartheta_F = 20$  degrees, solid line, and  $\vartheta_F = 10$  degrees, dotted line), and for the simple balance model (broken line). It can be seen that in the primitive model the inclination of the resultant hip force increases with increasing  $x_{CM}/x_F$ , the effect being more pronounced for larger inclination of the effective muscle force  $\vartheta_F$ . In the simple balance model, the resultant hip force points in the direction of the  $y$ -axis (i.e.,  $\vartheta_R = 0$ ).

## 2.2. HIPSTRESS model for resultant hip force

The primitive model and the simple balance approximation consider only one muscle acting in a hip in the one-legged stance. Measurements however indicate that there are several muscles that are active in this body position. The static equilibrium requires that the resultant of all external forces acting on each segment is zero and that the resultant of all external torques acting on each segment is zero, therefore in a more realistic model, contributions of all active muscles should be taken into account. The equilibrium equation for forces acting on segment I is

$$\mathbf{W}_B - \mathbf{W}_L + \sum_i \mathbf{F}_i + \mathbf{R} = 0, \quad (59)$$

where index  $i$  runs over all muscles that are active in the one-legged stance. The equilibrium of torques is expressed by equation

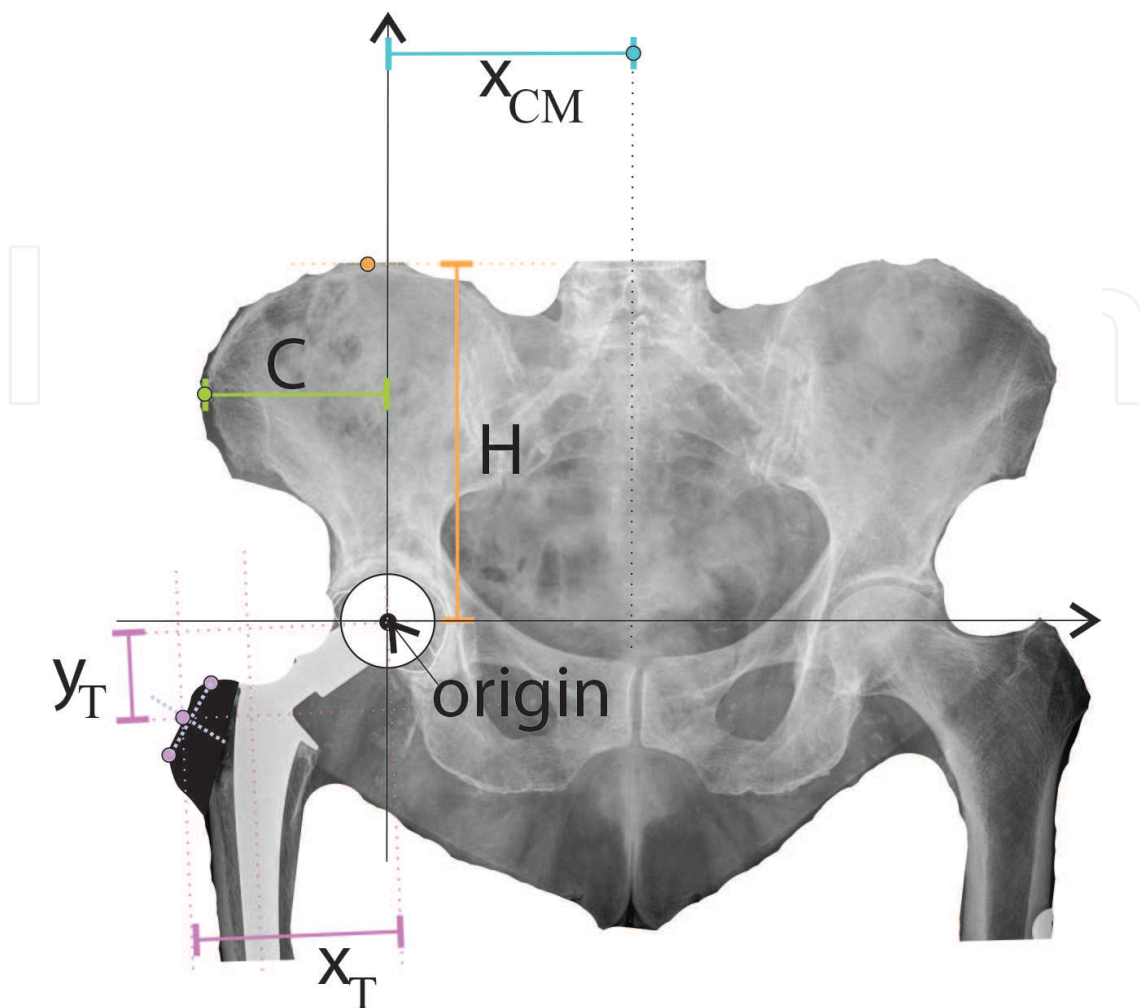
$$\mathbf{r}_{CM} \times (\mathbf{W}_B - \mathbf{W}_L) + \sum_i \mathbf{r}_i \times \mathbf{F}_i = 0, \quad (60)$$

where  $\mathbf{r}_i$  is the momentum arms of the respective muscle forces and  $i$  runs over all the forces that are active in the one-legged stance. It was taken into account that the torque of the resultant hip force is zero since we have chosen the origin of the coordinate system in the center of the femoral head, that is, the origin of the resultant hip force. The HIPSTRESS model for resultant hip force takes into account nine effective muscles: gluteus minimus anterior, gluteus minimus middle, gluteus minimus posterior, gluteus medius anterior, gluteus medius middle, gluteus medius posterior, tensor fasciae latae, piriformis, and rectus femoris [2]. The geometry of the individual subject is taken into account by rescaling the coordinates of the reference muscle attachment points according to the geometry of the pelvis and proximal femur. However, if the standard anteroposterior radiogram is used to assess the geometrical parameters, only the coordinates in the directions of the  $x$  and  $y$  axes can be taken into account. The magnitude of the force of the  $i$ -th muscle is taken to be proportional to the muscle cross section area  $A_i$  and average tension in the muscle  $\sigma_i$ . Muscle forces are considered to act in straight lines between the muscle attachment points,

$$\mathbf{F}_i = A_i \sigma_i \frac{(\mathbf{r}_i - \mathbf{r}'_j)}{|\mathbf{r}_i - \mathbf{r}'_j|}, \quad (61)$$

where  $\mathbf{r}_i$  is the coordinate of the origin of the  $i$ -th muscle on segment I and  $\mathbf{r}'_j$  is the coordinate of the origin of the  $i$ -th muscle on segment II. Both coordinates are measured with respect to the center of the articular sphere (i.e., the center of the femoral head and the acetabular shell).

The forces and the torques have three dimensions, therefore the model consists of six equations (three for equilibrium of forces and three for equilibrium of torques). For known origin and insertion points of the muscles and known cross-section areas, the unknown quantities are the muscle tensions and three components of the resultant hip force  $R$ . Since there are 9 effective muscles and 3 components of the force  $R$ , there are 12 unknowns and 6 equations. To solve this problem, a simplification was introduced by dividing the muscles into three groups (anterior, middle, and posterior) with respect to the position. It was assumed that the muscles in the same group have the same tension. This reduced the number of unknowns to six as required for solution of the complex of six equations. The muscle origin and insertion points and the muscle cross-section were taken from Refs. [3] and [4], respectively. The geometry of the individual patient was taken into account by correction of muscle attachment points according to the geometrical parameters obtained from the standard anteroposterior radiograph, the distance from the center of the femoral head to the midline  $x_{CM}$ , the height of the pelvis  $H$ , the width of the pelvis  $C$ , and the position on the greater trochanter relative to the center of the femoral head  $x_T$  and  $y_T$  (**Figure 7**). Results obtained with the HIPSTRESS model for resultant hip force showed that the force lies almost in the frontal plane of the body through both femoral heads [1]. To further simplify the calculations it was assumed in most clinical studies



**Figure 7.** Geometrical parameters needed for determination of resultant hip force within the HIPSTRESS model.

using HIPSTRESS model that the force lies in the frontal plane and is, like in the primitive model, represented by its magnitude  $R$  and its inclination with respect to the vertical  $\vartheta_R$ .

### 3. HIPSTRESS model for contact stress in the hip

Once we know what is the overall load  $\mathbf{R}$  (the magnitude of the resultant hip force  $R$  and its inclination with respect to the vertical direction  $\vartheta_R$ ) that the hip must bear in order to keep the balance in the one-legged stance, it should also be clarified how this load is distributed over the load-bearing area. Namely, it is the local load that determines the development of cells. Therefore, we are interested in stresses connected to the load. The model HIPSTRESS for contact hip stress has previously been described in detail in Ref. [5]; therefore, only brief description will be given here. The readers who wish to understand the derivation of the equations are kindly asked to refer to the pointed literature.

We neglect all other stresses but the contact hip stress acting perpendicularly to the spherical articular surface, by assuming that the joint is well lubricated. A surface is imagined that is a



part of a sphere with radius  $r$ , representing the hip joint. The contact hip stress  $p$  is connected to the resultant hip force,

$$\oint p \, d\mathbf{A} = \mathbf{R}, \quad (62)$$

where  $\mathbf{A}$  is the area element and the integration is performed over the load-bearing area of the articular surface.

It is assumed that stress is proportional to strain due to the squeezing of the cartilage between the femoral head and the acetabulum [6], which yields

$$p = p_0 \cos \gamma, \quad (63)$$

where  $p_0$  is the stress at the stress pole and  $\gamma$  is the angle between the vector pointing from the origin of the coordinate system to the pole and the vector pointing from the origin of the coordinate system and the chosen point on the articular surface. The load-bearing area is bounded on the lateral side by the acetabular roof given in the radiogram by the center-edge angle of Wiberg  $\vartheta_{CE}$  and on the medial side by the line where the cosine function (63) vanishes. Eq. (62) is represented by three equations for three components of the force and is subject to three unknown parameters of the model, that is, the position of the stress pole on the articular surface given by two angles  $\Theta$  and  $\Phi$ , and the value of stress at the pole  $p_0$ . The azimuthal angle of the pole is  $\Phi = 0$  or  $\pi$ , as the resultant hip force in the one-legged stance lies in the frontal plane of the body. In order to get the solution for  $\Theta$ , a nonlinear algebraic equation should be solved,

$$\tan(\vartheta_R + \Theta) = \frac{\cos^2(\vartheta_{CE} - \Theta)}{(\frac{\pi}{2} + \vartheta_{CE} - \Theta + \frac{1}{2} \sin(2(\vartheta_{CE} - \Theta)))} \quad (64)$$

which simplifies into

$$\tan(x + y) = \frac{\cos^2(y - x)}{(\frac{\pi}{2} + (y - x) + \frac{1}{2} \sin(2(y - x)))} \quad (65)$$

by introducing the expressions

$$x = \Theta + \frac{1}{2}(\vartheta_R - \vartheta_{CE}), \quad (66)$$

and

$$y = \frac{1}{2}(\vartheta_R + \vartheta_{CE}). \quad (67)$$

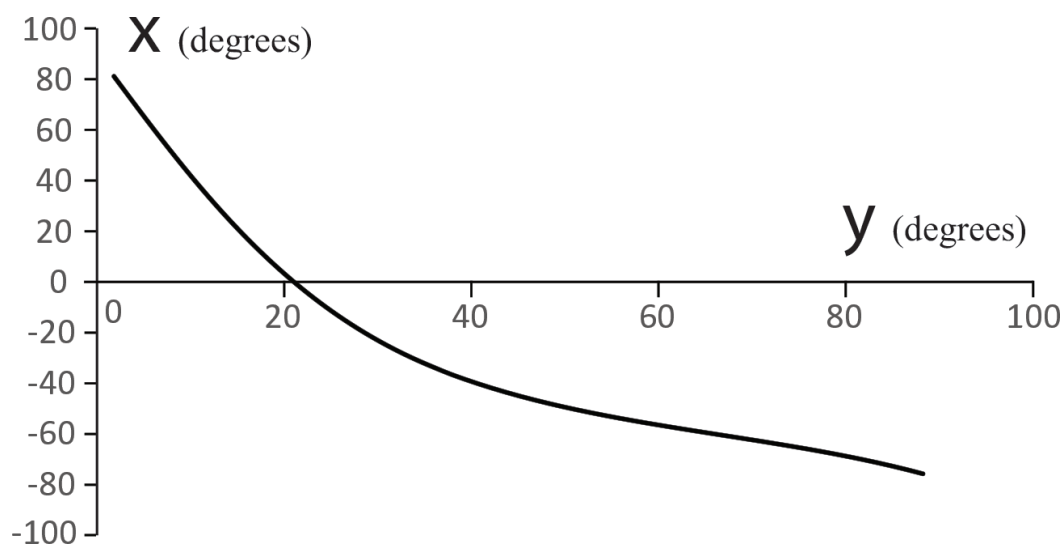
As  $\vartheta_R$  and  $\vartheta_{CE}$  are the input parameters, and the unknown parameter is  $x$ , the solution of Eq. (64) is determined solely by the parameter  $y$ . The normalized value of stress at the pole is then expressed from

$$\frac{p_0 r^2}{R} = \frac{3}{2} \frac{\sin(y+x)}{\cos^2(y-x)}, \quad (68)$$

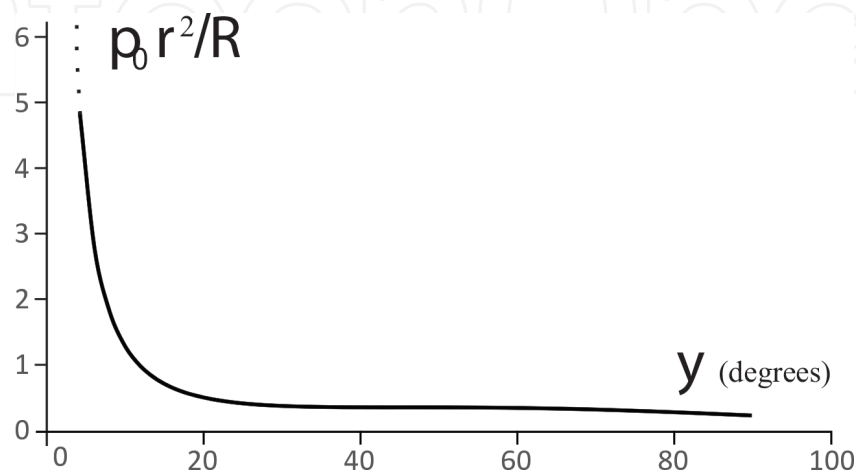
while its proper value can be calculated by multiplying the left side of Eq. (68) by  $R$  and dividing it by  $r^2$ . The polar angle is given by

$$\Theta = x - \frac{1}{2}(\vartheta_R - \vartheta_{CE}). \quad (69)$$

**Figures 8 and 9** show the dependence of the polar angle and stress at the pole (Eqs. (69) and (68), respectively), on parameter  $y$ . Clinical studies that have validated the HIPSTRESS method have used the parameter peak stress on the weight-bearing area as the relevant quantity.



**Figure 8.** Dependence of the position of the pole  $\Theta$  on parameter  $y$ .



**Figure 9.** Dependence of the value of contact stress at the pole  $p_0$  on parameter  $y$ .



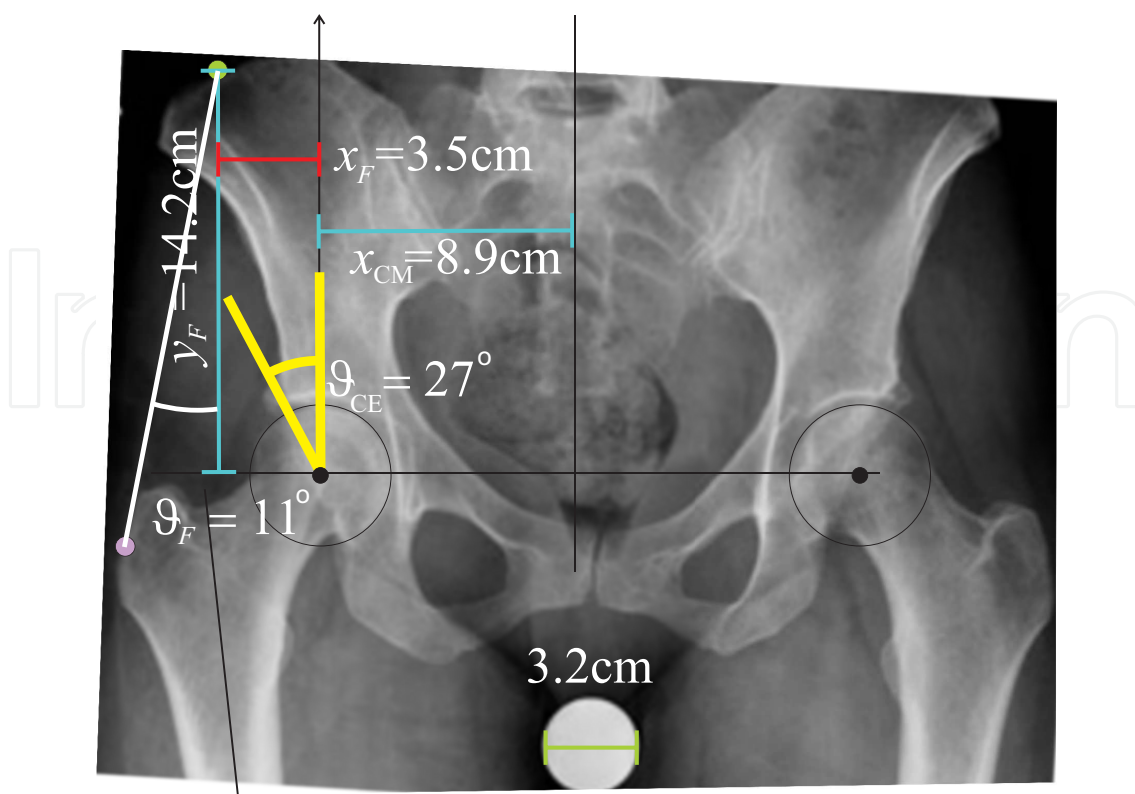
Namely, the stress pole is an abstract point in which the respective spheres outlining the femoral head and the acetabulum most closely approach each other upon loading of the joint. The pole may therefore be located within the load-bearing area of the joint or outside it. In the first case, the peak stress is identical to the value of stress at the pole  $p_{\max} = p_0$ , while in the second case, the peak stress is taken at the point on the load-bearing area that is closest to the stress pole. If this takes place at the acetabular rim, the peak stress is calculated according to the expression  $p_{\max} = p_0 \cos(\vartheta_{\text{CE}} - \Theta)$  [5]. It was shown that biomechanical parameters calculated with HIPSTRESS models for resultant hip force and contact hip stress were useful in explaining early osteoarthritis in dysplastic hips [7], hips with primary osteoarthritis, hips subject to avascular necrosis of the femoral head [5], hips that were in childhood subject to the Perthes disease [8], effect of different osteotomies [9–12], and the direction and volumetric wear of total hip endoprosthesis [13]. Evidently, the models include the relevant parameters of the individual hip to have a predictive value.

#### 4. Comparison of the primitive model and the HIPSTRESS model

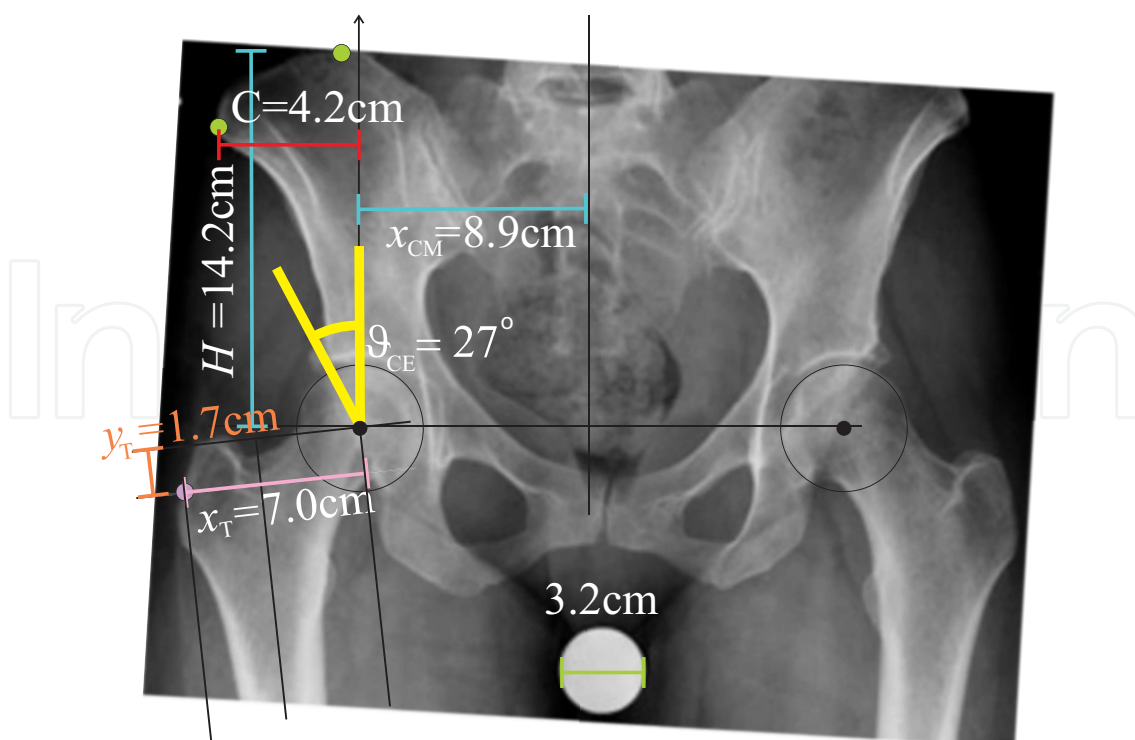
The primitive model and the HIPSTRESS model both use the same characteristic points on the iliac bone and on the greater trochanter (i.e., the highest and the most lateral points). In both models, the center-edge angle and the radius of the articular surface (i.e., the radius of the femoral head) is needed to calculate stress distribution. Both models consider the center of mass and the corresponding momentum arm. There are however differences in parameters for the resultant hip force. The HIPSTRESS model includes more parameters ( $H$ ,  $C$ ,  $x_{\text{CM}}$ ,  $x_{\text{T}}$ , and  $y_{\text{T}}$ ) than the primitive model ( $x_{\text{CM}}$ ,  $x_{\text{F}}$ , and  $\vartheta_{\text{F}}$ ) to characterize geometry of the individual hip and pelvis. The parameters of HIPSTRESS (but not the primitive model) enable consideration of the inclination of the femoral neck.

For illustration we calculate the biomechanical parameters by using both models and also the simple balance approximation. **Figure 10** shows the measured geometrical parameters for the primitive model and **Figure 11** shows the measured parameters for the HIPSTRESS model.

To determine the magnitude and the inclination of the resultant hip force ( $R$  and  $\vartheta_R$ , respectively) in the primitive model, we use the measured parameters and Eqs. (44)–(46), while in the simple balance approximation, with  $\vartheta_R = 0$ ,  $R$  is obtained by using Eq. (58). To estimate  $R$  and  $\vartheta_R$  in the HIPSTRESS model, we used the nomograms as described in [1]. The results of all three models are depicted in **Table 1**. It can be seen that for the chosen hip and pelvis, the magnitude of the resultant hip force in the primitive model and in the HIPSTRESS model differ by only 9%, while in the simple balance approximation the result deviates by about 40%. The inclination of the resultant hip force  $\vartheta_R$  is by definition zero in the simple balance approximation, but it is also small in the primitive model and in the HIPSTRESS model. By using these results we can estimate the parameter  $y$  in all three models. Knowing  $y$ , we estimate also parameter  $x$  in all three models by using **Figure 10**. Parameter  $x$  is needed to calculate the position of the pole  $\Theta$  by using Eq. (69). Finally, the value of stress at the pole is obtained by using the respective values of  $y$  and **Figure 9**. The inset of the figure with the values corresponding to all three models is shown in **Figure 12**.



**Figure 10.** Geometrical parameters needed for the determination of the resultant hip force within the primitive model.



**Figure 11.** Geometrical parameters needed for the determination of the resultant hip force within the HIPSTRESS model.

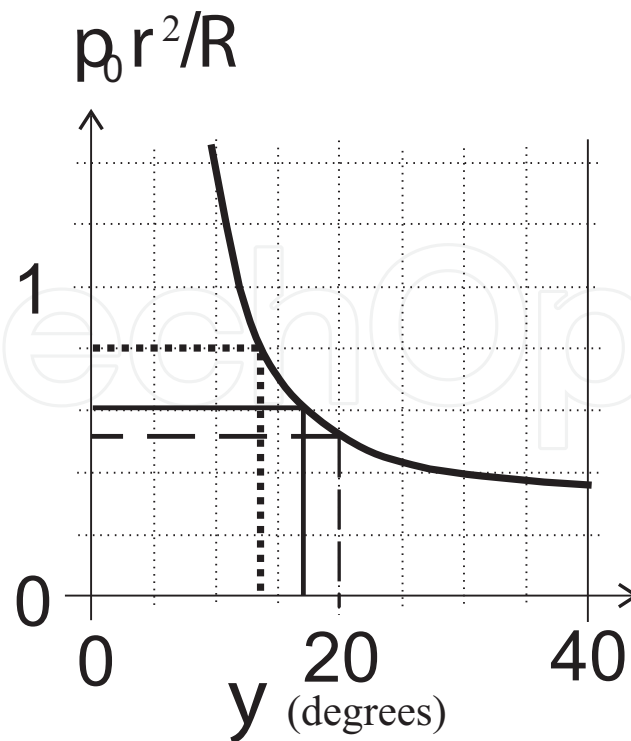
It can be seen that in the primitive model and in the HIPSTRESS model the pole lies within the load-bearing area while in the simple balance approximation it falls outside the load-bearing area (**Table 1**). The HIPSTRESS model in this case yields the lowest stress. Note that in the simple balance approximation the hip would according to the criteria of the HIPSTRESS [14, 15] be considered as dysplastic since it exhibits rapidly decreasing stress at the lateral acetabular rim. However, the center-edge angle is 27° which is considered as a healthy hip. The simple balance model overestimates hip stress and is in most cases not suitable to give quantitative result regarding biomechanical parameters of the hip and pelvis.

The example that we have shown corresponds to a normal hip geometry. Also, the values of peak stress that were obtained by the primitive model and the HIPSTRESS model are within the values corresponding to hips that would remain without clinical problems up to about 85 years of age [16]. In this case, the primitive model proved successful in estimating biomechanical parameters. However, to see whether it has a predictive value, it should be validated by clinical studies. The advantage of the primitive model is that it is simpler and does not need

Parameter	SBA	Primitive	HIPSTRESS
$r$ (cm)	2.47	2.47	2.47
$\vartheta_{CE}$ (degrees)	27	27	27
$x_{CM}$ (cm)	8.9	8.9	8.9
$x_F$ (cm)	3.5	3.5	
$y_F$ (cm)	14.2	14.2	
$\vartheta_F$ (degrees)	0	11	
$C$ (cm)			4.2
$H$ (cm)			14.6
$x_T$ (cm)			7.0
$y_T$ (cm)			1.7
$R/W_B$	3.2	2.2	2.4
$\vartheta_R$ (degrees)	0	7	12
$y$	13.3	17	20
$x$	27	12	2
$p_0/W_B$ (m <sup>-2</sup> )	4693	2693	2172
$p_{max}/W_B$ (m <sup>-2</sup> )	4572	2693	2172
$\Theta$ (degrees)	40	22	10

SBA, simple balance approximation.

**Table 1.** Geometrical and biomechanical parameters for a hip with total hip endoprosthesis as determined by simple balance approximation, primitive model and HIPSTRESS model of a one legged stance.



**Figure 12.** Estimation of the value of  $p_0$  for the primitive model (solid lines), simple balance approximation (dotted lines), and HIPSTRESS model (broken lines).

special software. Determination of the resultant hip force with the primitive model is scale independent which is an advantage over the HIPSTRESS model. Namely, the HIPSTRESS model uses three-dimensional coordinates of the muscle attachment points of a reference hip and pelvis but only the  $x$  and  $y$  coordinates are rescaled according to the hip considered, while the  $z$  coordinates of the reference hip remain in the model. Therefore, the HIPSTRESS model for the resultant hip force is biased by the artifact that it depends on the size of the hip.

We have used standard anteroposterior radiograms to measure geometrical parameters. Imaging with magnetic resonance has recently improved to enable determination of three-dimensional positions of muscle attachment points for the needs of the HIPSTRESS method, but has not yet been used for the determination of biomechanical parameters by this method. This would be a major improvement over using radiograms, as the direct data on the muscle attachment points could be used and there would be no need for rescaling of the reference geometry. In considering the three-dimensional data the primitive model could not do justice to the system as its assumptions are bounded to the simplification to two dimensions. However, the primitive model (but not the simple balance approximation) merits to minimal standards to be used for understanding of the principles of the equilibrium of forces and torques in the one-legged stance, and can in certain cases (such as the one shown here) also yield a valid quantitative estimation of the biomechanical parameters.

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