Chapter 4

# Photon Propagation Through Dispersive Media

Patricio Robles and Francisco Pizarro

Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/66673

**Abstract**

In the present chapter, we study the propagation of photons through dispersive media, starting from a description of the dynamics of free photons using a Dirac-like equation with an analysis of the energy solutions arising from this equation. A comparison with the case of a free electron is made. We present an analysis of the interaction between photons with the medium considering both a classical and a quantum treatment of light, and also we analyse the propagation of photons along a waveguide where they behave as if they did have a finite mass. As a technological application of the theoretical frame here presented, we consider the use of the properties of metamaterials to control the propagation of waves through waveguides filled with this kind of materials.

**Keywords:** photon, Hamiltonian, wave function, antiparticle, dielectrics, coherence, metamaterials

## 1. Introduction

Up to now, the photon is understood as the quantum of electromagnetic radiation. In 1905, Einstein proposed that energy quantization was a property of electromagnetic radiation itself. Accepting the validity of Maxwell's theory, he pointed out that several experiments with results unpredictable by the classical electrodynamics theory could be explained if the energy of a light wave was localized into point-like quanta moving independently of one another. A very simple and intuitive interpretation at the level of undergraduate teaching of quantum physics is that photons are the fundamental particles of light having the property that they behave both as a particle and a wave (wave-particle duality). They also have characteristics, which make them different from other particles. One of these characteristics is that, as theorized up to now, when freely propagating, they behave as massless particles not interacting between them and carrying linear and intrinsic angular momentum.

In modern terms, a photon is considered as an elementary excitation of the quantized electromagnetic field, and it can be treated as a (quasi-) particle, roughly analogous to an electron.

It has unique properties, arising from its zero rest mass and its spin-one nature. In particular, since the early days of quantum mechanics, it has been argued that there is no position operator for a photon, leading someone to conclude that there can be no properly defined wave function, in the Schrödinger sense, which allows to know the probability of finding the particle in a given spatial region. Nevertheless, photon position operators have been postulated whose eigenvectors form bases of localized states, as in Ref. [1].

The aim of this chapter is study the propagation of photons through dispersive media. This chapter is organized as follows. In Section 2, a semiclassical description of the dynamics of free photons is presented using a Dirac-like equation. In Section 3, the positive and negative energy solutions arising from these equations are analysed. A comparison with the case of a free electron is made. Section 4 presents an analysis of the interaction of photons with the medium considering both a classical as a quantum treatment of light. Section 5 includes an analysis of the propagation of photons along a waveguide where they behave as if they did have a finite mass. In Section 6, some technological applications of the theoretical frame here presented are shown, such as the use of the properties of metamaterials to control the propagation of waves through waveguides filled with this kind of materials.

## 2. A Dirac-like equation for the photon

Maxwell's equations can be considered as a classical field theory for a single photon that can be field (or "second") quantized to obtain a quantum field theory of many photons.

In Ref. [2], it has been shown that in a region without sources Maxwell's equations can be written in the form of a Schrödinger-like equation for a single photon adding a transversality condition. Although in quantum mechanics Schrödinger's equation is valid for describing the dynamics of a nonrelativistic particle, its application for the case of a photon must be considered only within the context of classical electrodynamics and taking into account that we are dealing with an equation which has the form of Schrödinger's equation and that it is equivalent to Maxwell's equations.

Considering that one important requirement of a quantum theory for describing the dynamics of photons is Lorentz invariance, in this section, we study the application of a Dirac-like equation. In Refs. [3, 4] it has been shown that Maxwell's equations without sources can be written in a form analogous to that of Dirac's equation for a free electron. These last works also show that optical spin and light orbital angular momentum can be obtained from this Dirac-like equation. As an extension of these works we give arguments for obtaining this equation in a similar form to those used for the deduction of Dirac's equation for an electron, starting from the relativistic expression for the energy. For example, see Ref. [5]. We also study the positive and negative energy states obtained from the corresponding Hamiltonian and the form that this equation takes for the propagation of a photon in a magnetodielectric medium.

As in Ref. [6], we begin with a derivation of a Dirac-like equation for a photon starting from Dirac equation for a massless particle in free motion, so that we postulate an equation of the form:

$$i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi \tag{1}$$

Since Eq. (1) is linear in the time derivative, it seems natural to construct a Hamiltonian operator also linear in the spatial derivatives. This is compatible with the energy-momentum relation for the photon $E = c|\vec{p}|$ for the photon. Therefore, we postulate a Hamiltonian of the form:

$$\hat{H} = c\vec{\alpha}\cdot\vec{p} \tag{2}$$

where $\vec{p} = -i\hbar\nabla$ is the momentum operator and $\hat{H}$ is the Hamiltonian operator.

As in Ref. [7], a possible election for $\vec{\alpha}$ is a vector operator of the form

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\tau} \\ \vec{\tau} & 0 \end{pmatrix} \tag{3}$$

where $\vec{\tau}$ is a vector matrix whose components are the spin-1 matrices

$$\tau_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}; \tau_y = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}; \tau_z = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{4}$$

In Section 3, it is shown that the above equations lead to the relation $E = c|\ \vec{p}\ |$ for a free photon propagating in an unbounded medium, so that the Dirac-like equation given by Eq. (1) gives a solution analogous to the energy of a free fermion obtained from Dirac's equation in the limit of zero mass. Nevertheless, this must be considered only as a formal analogy since photons have spin 1 and Dirac's equation is applicable for particles of spin ½.

There exist a variety of Dirac-like formulations of Maxwell's equations and alternative ways for choosing the wave function ψ. Considering that photons have only energy and no other scalar quantities such as mass or charge, it is convenient to choose ψ so that its modulus squared correspond to energy density not of probability density for localization as is the case of a particle with mass like the electron. Therefore, we choose as wave function the following column vector of dimension $6x1$

$$\psi = \begin{pmatrix} k_1\ \vec{E} \\ ik_2\ \vec{B} \end{pmatrix} \tag{5}$$

In this last equation, the components of $\vec{E}$ and $\vec{B}$ are written as column vectors $3x1$ and taking $k_1 = \sqrt{\epsilon_0/2}$ and $k_2 = 1/\sqrt{2\mu_0}$ we obtain

$$\psi^{+}\psi = \frac{1}{2}\epsilon_0\vec{E}^2 + \frac{\vec{B}^2}{2\mu_0} = w_{\text{em}} \tag{6}$$

This last expression corresponds to the density of energy in the electromagnetic field.

For an electromagnetic wave propagating in a linear magnetodielectric and nonconducting medium, there is an induced polarization and magnetization classically represented by the polarization and magnetization vectors $\vec{P}$ and $\vec{M}$, respectively. These vectors are related with the electric and magnetic fields as $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ and $\vec{B} = \mu_0(\vec{H} + \vec{M})$. Using these relations, Maxwell's equation corresponding to Ampere's law may be rewritten as

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}_{\mathrm{ind}} \tag{7}$$

where $\vec{J}_{\mathrm{ind}} = \frac{\partial \vec{P}}{\partial t} + \nabla \times \vec{M}$ is an equivalent current density associated to the polarization and magnetization of the medium. In this case, the Dirac-like equation becomes

$$i\hbar \frac{\partial}{\partial t}\psi = c\vec{\alpha} \cdot \vec{p}\ \psi - \frac{i\hbar}{2k_1}\tilde{J} \tag{8}$$

where $\tilde{J} = \begin{pmatrix} \vec{J}_{\mathrm{ind}} \\ 0 \end{pmatrix}$ is a column matrix of dimension 6x1.

From Eq. (8), a continuity equation for the wave function $\psi$ may be derived taking the scalar product with $\psi^{+}$on both sides, obtaining

$$\frac{1}{2}\frac{\partial}{\partial t}|\psi^2| = -c\psi^{+}\ \vec{\alpha} \cdot \nabla\psi - \frac{i\hbar}{2k_1}\psi^{+} \cdot \tilde{J} \tag{9}$$

Using Eqs. (6)–(9) and the definition of the column matrix $\tilde{J}$ given previously, one obtains the classical equation for conservation of electromagnetic energy

$$\frac{\partial}{\partial t}\left\{\epsilon_0 \vec{E}^2 + \mu_0 \vec{H}^2\right\} + \vec{E} \cdot \vec{J}_{\mathrm{ind}} = -\nabla \cdot (\vec{E} \times \vec{H}) \tag{10}$$

## 3. Positive and negative energy states for a photon

The energy eigenvalues are obtained by looking for stationary state solutions of the Dirac-like equation. A plane wave solution of this equation has the form

$$\psi_k(\vec{r}, t) = A_k \exp\left[\frac{i}{\hbar}(\vec{p} \cdot \vec{r} - \hbar\omega t)\right] \tag{11}$$

for $k = 1$ to 6, where $\vec{p}$ is the momentum vector and $\omega$ is the angular frequency. Considering a momentum vector with components along two directions, for example, parallel to $x$ and $y$

axes, from Eqs. (2) and (3), it can be shown that the values of the energy for a photon can be obtained from the eigenvalues of the 3x3 matrix $(c\vec{\tau}\cdot\vec{p})^2$ for this case given by:

$$(c\,\vec{\tau}\cdot\vec{p})^2 = c^2\begin{bmatrix} p_y^2 & -p_x p_y & 0 \\ -p_x p_y & p_x^2 & 0 \\ 0 & 0 & p_x^2+p_y^2 \end{bmatrix} \tag{12}$$

The corresponding solutions for the energy are $E = +cp$, $E = -cp$ and $E = 0$ where $p = |\vec{p}|$. The negative energy solution may be interpreted using Feynman concept of antiparticles associating to photon states going backward in time and postulating that the photon is its own antiparticle. For example, see Refs. [8, 9]. The solution $E = 0$ has no physical meaning since it would imply that a photon in vacuum could be at rest (zero momentum) and from the point of view of classical electrodynamics, it would result in an electromagnetic wave propagating in an unbounded medium with a field component parallel to the direction of propagation, what is not compatible with Maxwell's equations. That can be seen if one considers, for example, motion along the $z$ direction with momentum $p_z = p$. In that case, the calculation of the eigenvalues and eigenvectors of the Hamiltonian given by Eq. (2) shows that the positive and negative energy solutions are valid because they are compatible with an electromagnetic wave propagating in vacuum and satisfying the condition that the electric and magnetic field vectors must have only transversal components (that is perpendicular to the $z$ axis in this case). On the other hand, the solutions with zero energy are not valid since they imply that the electric or magnetic field have longitudinal components (parallel to the direction of propagation).

It is important to consider that Dirac's equation for a particle like an electron only gives positive and negative energy solutions.

## 4. Interaction between photons and the medium

### 4.1. Introduction

When light passes through a material, there is an electromagnetic interaction with the particles of the medium. This interaction is macroscopically manifested by two main effects: absorption of energy from the incident beam and scattering.

Considering that every particle has electric charge that acquires a motion due to the electric field associated to the incident electromagnetic wave, the absorption of energy may be understood using a phenomenological model of electric dipoles with negative charges whose positions oscillate with respect to the centre of positive charges, with a frequency corresponding to that of the incident light. This oscillatory motion has a damping associated to the dielectric losses.

The scattering of light may be thought of as the redirection that takes place when an electromagnetic wave encounters an obstacle or non-homogeneity. The accelerated motion of the charges gives rise to radiation of electromagnetic energy in all directions producing secondary waves, process known as scattering.

### 4.2. Classical model for the interaction between light and matter

A classical model for representing the optical response of a polarizable medium through which travels a monochromatic electromagnetic wave of frequency $\omega$ is the Drude model where it is imagined that due to the electric field vector associated to this wave, each electron bound to the nucleus of an atom performs harmonic oscillations. Therefore, oscillating electric dipoles are formed, and considering that the electric dipolar moment is related to the electric field through the atomic polarizability $\alpha(\omega)$, we get

$$\alpha(\omega) = \frac{e^2/m}{\omega_{\mathrm{oe}}^2 - \omega^2 - i\Gamma_{\mathrm{d}}} \tag{13}$$

In this last equation, $\omega_{\mathrm{oe}} = \sqrt{k/m}$ is the natural frequency of oscillation, $\Gamma_{\mathrm{d}} = \gamma/m$ is the absorption parameter and $m$ is the mass of the electron.

At a macroscopic scale, the formation of electric dipoles in a dielectric material subjected to an applied electric field is described by means of the polarization vector $\vec{P}$ defined as the electric dipolar moment per unit of volume and related with the electric field and displacement vectors as

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 \vec{E} + \chi\epsilon_0 \vec{E} = \epsilon\epsilon_0 \vec{E} \tag{14}$$

In this last equation, $\chi$ is the dielectric susceptibility and $\epsilon = 1 + \chi$ is the dielectric function which in general depends on the frequency $\omega$.

If $n$ is the volumetric density of molecules dipoles each one with $Ze$ electrons, the polarization vector is rewritten as

$$\vec{P} = \frac{nZ_{\mathrm{e}}^2/m}{\omega_{\mathrm{oe}}^2 - \omega^2 - i\Gamma_{\mathrm{d}}} \vec{E} \tag{15}$$

Therefore, we obtain the Drude model for the dielectric function:

$$\epsilon(\omega) = 1 + \frac{\omega_{\mathrm{pe}}^2}{\omega_{\mathrm{oe}}^2 - \omega^2 - i\Gamma_{\mathrm{d}}} \tag{16}$$

where $\omega_{\mathrm{pe}}$ is the plasma frequency given by $\omega_{\mathrm{pe}}^2 = nZ_{\mathrm{e}}^2/m\epsilon_0$.

From Eq. (16), it can be seen that a lossy dielectric medium has a complex refraction index $n_{\mathrm{r}} = \sqrt{\epsilon(\omega)}$ whose imaginary part is associated to the attenuation of the intensity of an electromagnetic wave propagating in this medium due to absorption of energy.

For the magnetic permeability, as in Ref. [10], a Drude-Lorentz model similar to that given by Eq. (16) can be used:

$$\mu(\omega) = 1 + \frac{\omega_{\rm pm}^2}{\omega_{\rm Te}^2 - \omega^2 - i\Gamma_{\rm dm}} \tag{17}$$

where $\omega_{\rm pm}$ is the magnetic coupling strength, $\omega_{Te}$ is the transverse resonance frequency, and $\Gamma_{\rm dm}$ is the absorption parameter. The real part of $\epsilon(\omega)$ and $\mu(\omega)$ are negative for the following frequency ranges:

$$\omega_{\rm oe}^2 < \omega^2 < \omega_{\rm oe}^2 + \omega_{\rm pe}^2 \text{ for } Re[\epsilon(\omega)] < 0$$

$$\omega_{\rm Tm}^2 < \omega^2 < \omega_{\rm Tm}^2 + \omega_{\rm pm}^2 \text{ for } Re[\mu(\omega)] < 0$$

When both $\epsilon(\omega)$ and $\mu(\omega)$ have negative real parts, the real part of the index of refraction is also negative and the medium has the behaviour of a left-handed material (also known as a double-negative material) with the Poynting vector and the wave vector having opposite directions. For example, see Ref. [11].

### 4.3. Quantum treatment

When an incident pulse enters into a dielectric medium, it undergoes modifications due to dispersion and absorption and in the case of a dielectric slab due to reflections from its surfaces. These modifications give rise to a distortion of the transmitted pulse in comparison with the incident pulse. Furthermore, the transmission of the pulse may be affected by thermal emission from the slab at elevated temperatures.

In classical electrodynamics, the interaction of light with matter is performed in two stages. First, an explicit model of the medium is assumed and its response to an electromagnetic field is calculated. The interaction is represented by the dielectric function, which embodies the optical properties of the material. In the second stage, this dielectric function is used for studying the propagation of the electromagnetic wave through the medium, determining effects as energy absorption and velocity of propagation. Nevertheless, for a finite number of photons, there are effects that cannot be described by a classical approach such as zero average electric field between two conducting plates and electric force between the plates even if the number of photons is zero (the so-called vacuum fluctuation and Casimir forces, respectively). In addition, for a nonclassical pulse propagating in an absorbing and dispersive medium, there are modifications in the correlation properties that can only be described by a quantum theory of the photon.

As in Refs. [12, 13], the formalism for electromagnetic field quantization in a dispersive and absorbing dielectric, in general, includes the following steps:

1. Express Maxwell's equations in terms of transverse electric and magnetic vector operators obtained from a vector potential operator.
2. Express the above vector potential operator as a function of the complex refraction index and of a current operator associated with noise sources coupled with the electromagnetic field in presence of lossy dielectrics.

3. Incorporate boson-type operators and commutation relations between the electromagnetic field operators.

In what follows, we illustrate the application of this procedure for the case of light propagating in the $x$ direction in a dielectric homogeneous medium with index of refraction $n_r = \eta(\omega) + iK(\omega)$, focusing our analysis in the determination of the first-order correlation of the electromagnetic field in two separate points at the same instant.

As shown in Ref. [12] for a state with $N$ photons, the quantum field-field correlations between two points placed over the $x$ axis at positions $x_1$ and $x_2$ are

$$\langle E(x_1,\omega)E(x_2,\omega)\rangle = \frac{N^2\hbar\omega\exp[-K\omega|x_2-x_1|/c]}{\epsilon_0 cS(\eta^2+K^2)}\left\{\eta\cos\left(\frac{\eta\omega|x_2-x_1|}{c}\right)+K\sin\left(\frac{\eta\omega|x_2-x_1|}{c}\right)\right\}. \quad (18)$$

$$\langle E(x_1,\omega)E(x_1,\omega)\rangle = \frac{N^2\hbar\omega}{\epsilon_0 cS(\eta^2+K^2)}\eta \quad (19)$$

We consider the following definition of the spatial first-order coherence function for two points placed over the $x$ axis and separated by a distance $s$ (e.g., see Refs. [14, 15])

$$g^{(1)} = \left|\frac{\langle E(x,\omega)E(x+s,\omega)\rangle}{E(x,\omega)E(x,\omega)}\right| \quad (20)$$

Therefore, from Eqs. (18–20), we obtain

$$g^{(1)} = \frac{\exp[-K\omega|x_2-x_1|/c]}{\eta}\left\{\eta\cos\left(\frac{\eta\omega|x_2-x_1|}{c}\right)+K\sin\left(\frac{\eta\omega|x_2-x_1|}{c}\right)\right\} \quad (21)$$

As an example, we make a comparison of this last result with that obtained calculating the classical coherence function. For that purpose, we consider a beam of light produced by excitation of two linearly polarized waves with frequencies $\omega_1$ and $\omega_2$ propagating in the $x$ direction in a medium with refraction index $n_r(\omega)$ so that the resulting electric field is

$$\vec{E}\,(x,t) = \hat{y}E_1\exp\left[i\frac{\omega_1}{c}\left(n_r(\omega_1)x-ct\right)\right]+\hat{y}E_2\exp\left[i\frac{\omega_2}{c}\left(n_r(\omega_2)x-ct\right)\right] \quad (22)$$

The Fourier transform of this field is

$$\vec{E}\,(x,\omega) = \hat{y}E_1\exp\left[i\frac{n_r(\omega_1)\omega_1}{c}x\right]\delta(\omega-\omega_1)+\hat{y}E_2\exp\left[i\frac{n_r(\omega_2)\omega_2}{c}x\right]\delta(\omega-\omega_2) \quad (23)$$

From the definition of the classical spatial coherence function:

$$\langle E(x,\omega)E(x+s,\omega)\rangle = \frac{1}{L}\int_0^L \mathrm{d}x[E(x,\omega)]^*[E(x+s,\omega)] \quad (24)$$

we obtain that the classical spatial first order coherence function calculated for this case is

$$g_{cl}^{(1)}(\omega) = \exp\left[\frac{-K(\omega)\omega s}{c}\right] \tag{25}$$

Due to the exponential factor of this last equation, the classical coherence function for the considered case has a value lower than 1 meaning that measurements of the electric field at two separated points are partially correlated in a medium with absorption and for $s \to \infty$ the correlation goes to zero.

Comparing Eqs. (21) and (25), it can be seen that if absorption is neglected the classical model predicts total coherence, while the quantum treatment in this case gives an oscillatory behaviour of the spatial coherence function with respect to the distance between the points considered, with null partial coherence for some values of this distance. It is worth to note that this result is also valid for a left-handed medium if absorption may be neglected.

## 5. Propagation of photons through a waveguide

When the propagation of an electromagnetic wave of a given frequency $\omega$ is restricted to a region bounded by conducting walls, as is the case of a waveguide, the photon appears to acquire an effective mass. For example, see Ref. [16]. For flow along the waveguide axis, the action of confinement may be viewed as yielding longitudinal photons propagating with a mass proportional to the cut-off frequency of the corresponding electromagnetic mode.

This is illustrated considering the propagation of a transverse electric (TE) mode in a rectangular waveguide with transversal section having dimensions $a$ and $b$ in the plane $xy$. We consider propagation along the $z$ direction taken parallel to the axis of the waveguide. As known, the magnetic field in the TE mode of order $nl$ in a rectangular waveguide has a component parallel to the direction of propagation of the guided light (axial component), while the electric field has only transverse components. As shown in Ref. [17], each component of the electromagnetic field satisfies a Klein-Gordon-like equation:

$$\left[\frac{1}{c^2}\frac{\partial}{\partial t^2} - \frac{\partial^2}{\partial z^2} + \frac{m_\gamma^2 c^2}{\hbar^2}\right]\psi_j = 0 \tag{26}$$

where $m_\gamma = \hbar k_c/c = \hbar\pi\sqrt{(n/a)^2 + (l/a)^2}/c$ is the effective mass acquired by the photon.

For a relativistic fermion of mass $m$, Klein-Gordon's equation can be obtained from Dirac's equation. For example, see Ref. [18]. This makes natural to wonder what is the form that the Dirac-like equation takes for a photon moving along a waveguide. This can be determined writing the Dirac-like equation in the following form:

$$i\hbar\alpha^{\mu}\partial_{\mu}\psi = 0 \tag{27}$$

with $\alpha^{\mu} = (\alpha^0, -\vec{\alpha})$, $\partial_{\mu} = \partial/\partial x^{\mu}$, $x^{\mu} = (t, -\vec{r})$, $\partial_0 = \partial/\partial(ct)$ and

$$\alpha^0 = \begin{pmatrix} I_{3\times3} & 0 \\ 0 & -I_{3\times3} \end{pmatrix} \tag{28}$$

For a photon moving along the waveguide, let us write the wave functions as

$$\psi(t, \vec{r}) = \varphi(t, z)\exp[-i(k_x x + k_y y)] \tag{29}$$

From Eqs. (27) and (29), we get

$$\frac{i\hbar}{c}\alpha^o\frac{\partial\varphi}{\partial t} + i\hbar\alpha_z\frac{\partial\varphi}{\partial z} + (\hbar k_x\alpha_x + \hbar k_y\alpha_y)\varphi = 0 \tag{30}$$

This last equation may be recast as

$$i\hbar\alpha^{\mu}\left(\partial_{L\mu} - i\frac{p_{T\mu}}{\hbar}\right)\varphi = 0 \tag{31}$$

For propagation along the $z$ direction, as it is considered in this case:

$$\partial_{L\mu} = \left(\frac{\partial}{c\partial t}, \frac{\partial}{\partial z}, 0, 0\right); p_{T\mu} = (0, p_x, p_y, 0)$$

Equation (31) is the Dirac-like equation for photons moving along the waveguide and it can be shown that leads to the Klein-Gordon equation applying the operator $\partial_L^{\mu}$ by the left and considering that $p_{T\mu}p^{T\mu} = -(m_{\gamma}c)^2$.

## 6. Some technological applications

On the last years, several articles about the properties of a special kind of materials, known as metamaterials, have been published. These materials can exhibit negative values on their permittivity or permeability. They are also named "left handed materials" and can have negative refraction index, which leads to interesting phenomena for the wave propagation. The effect of negative refraction was predicted in 1968 by Veselago in Ref. [19], principle that has led to many technological applications. For example, see Ref. [20]. Nowadays is possible to build artificial metamaterials with different geometries. These arrays can achieve negative values of permittivity or permeability, achieving either single negative material (SNG), where $\epsilon$ or $\mu$ are negative, or double negative material (DNG), where both $\epsilon$ and $\mu$ are negative. On the other hand, natural medias, such as plasmas, can behave as an SNG media, depending on its physical characteristics.

http://dx.doi.org/10.5772/66673

Metamaterials constructed by circuit arrays are based on a group of elements organized periodically, and designed in order to respond to an impinging electromagnetic field. The size and spacing of each element of the array must be much lower than the wavelength of the wave interacting with the array. This will allows that the impinging wave interacts with the artificial material as a homogeneous material with certain $\epsilon$ and $\mu$ characteristics.

There are different type of structures to obtain negative permittivity and permeability. To obtain a negative permittivity, periodic structures based on wire arrays that are based on the Drude-Lorentz model for dielectric constant are used. For example, see Refs. [21–23]. On the other hand, to obtain negative permeability values, split ring resonators (SRR) and the induced current on wire structures are used, as can be seen in Refs. [21, 24, 25]. Currently, we can find two kind of artificial metamaterials that can exhibit negative refraction: photonic crystals, as shown in Refs. [26, 27], and composite materials as shown in Ref. [28]. Composite materials exhibit simultaneously negative permittivity and permeability within a certain frequency range. This immediately leads to a negative index of refraction. Dielectric photonic crystals are composed of materials with positive $\epsilon'$ and $\mu'$ but exhibit negative refraction because of peculiarities of dispersion characteristics at some frequencies, as shown in Ref. [29].

The possibility of having SNG or DNG metamaterials opens a huge number of new applications that these physical characteristics can offer. Some examples of these applications are invisible materials or cloaking, as shown in Refs. [30, 31], phase control of propagating modes on waveguides, as shown in Refs. [32, 33], antenna miniaturization as shown in Refs. [34, 35], and superlens, as shown in Refs. [36, 37].

For each application, the design parameters of the metamaterials are an important issue. If the parameters that determine the permittivity and permeability of the material are known beforehand, it is possible to predict the behaviour of the electromagnetic wave that propagates on the media. The following study is intended to analyse the possible variations of the wavenumber of metamaterials, depending on their design parameters.

### 6.1. Wavenumber on SNG media depending on metamaterial parameters

The first analysis to be developed is the variation of the wavenumber in terms of the material properties. The complex relative permittivity and permeability of the metamaterials (either real or artificial) can be modelled by adopting a simplified Drude's model, which uses the following expressions based in Ref. [38]:

$$\epsilon_r(\omega) = 1 - \frac{{\omega_{\mathrm{pe}}}^2}{\omega(\omega - i\Gamma_{\mathrm{e}})} \tag{32}$$

$$\mu_{\mathrm{r}}(\omega) = 1 - \frac{{\omega_{\mathrm{pm}}}^2}{\omega(\omega - i\Gamma_{\mathrm{m}})} \tag{33}$$

where $\omega_{\mathrm{pe}}$ and $\omega_{\mathrm{pm}}$ correspond to the plasma frequency, $\Gamma_{\mathrm{e}}$ and $\Gamma_{\mathrm{m}}$ correspond to the damping frequencies, and $\omega$ is the angular frequency of the impinging wave on the metamaterial. On the design of metamaterials, it is important to know the behaviour of these parameters in order to characterize the impinging wave on the media. In the analysis of artificial metamaterials, the damping frequencies are not considered or neglected, which implies that are considered as lossless metamaterials. In other materials, such as cold plasmas, we cannot ignore this parameter because implies that we are ignoring the plasma collision frequency, which is fundamental for the plasma generation, and depends on the gas parameters of the plasma. Nevertheless, it is important to consider the losses in the design of any metamaterial in order to have a better estimation of the behaviour of the impinging wave on the metamaterial, and knowing beforehand for which frequencies the metamaterial will behave as a SNG or DNG material.

One way to describe the behaviour of the wave is to describe the permittivity o the permeability as function of a parameter ratio, and use this relation on the wavenumber k. Let us take for example the permittivity described in Eq. (48) and express it on terms of $\omega_{\mathrm{pe}}/\omega$ and $\Gamma_{\mathrm{e}}/\omega$ in the form:

$$\epsilon_{\mathrm{r}} = 1 - \frac{\left(\frac{\omega_{\mathrm{pe}}}{\omega}\right)^2}{1+\left(\frac{\Gamma_{\mathrm{e}}}{\omega}\right)^2} - i\frac{\left(\frac{\omega_{\mathrm{pe}}}{\omega}\right)^2\left(\frac{\Gamma_{\mathrm{e}}}{\omega}\right)}{1+\left(\frac{\Gamma_{\mathrm{e}}}{\omega}\right)^2} \tag{34}$$

**Figure 1** shows the real and imaginary part of the relative permittivity as function of $\omega_{\mathrm{pe}}/\omega$ and $\Gamma_{\mathrm{e}}/\omega$.

From **Figure 1(a)** and **(b)**, we can notice the necessary ratios of $\omega_{\mathrm{pe}}/\omega$ and $\Gamma/\omega$ for achieving a negative permittivity on a lossy media. Considering those ratios also, we can estimate the losses due to the increment of the imaginary part of the relative permittivity of the SNG media. For example, if we want a negative permittivity of $\varepsilon_{\mathrm{r}} = -10$, is possible to obtain it only within the ratios $\omega_{\mathrm{pe}}/\omega > 3$ and $\Gamma/\omega < 3$. By knowing these ratios, it is possible to see where the imaginary part of the permittivity will be lower, and so, the losses. Therefore, it is possible to extend this analysis on a media with negative permeability instead of negative permittivity.

After the analysis of the permittivity or permeability on a media than can achieve SNG characteristics, we can express the complex wavenumber $k$ of a wave propagating on an infinite media where its permittivity can have negative values, normalized by the wavenumber on free space $k_0$ (**Figure 2**). By knowing the complex wavenumber of the wave

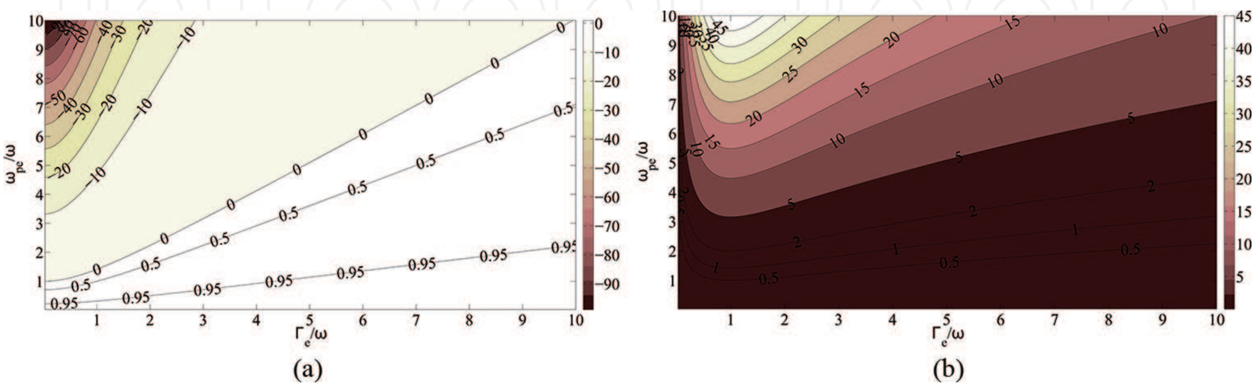


**Figure 1.** Complex relative permittivity of a media depending on the $\omega_{\mathrm{pe}}/\omega$ and $\Gamma/\omega$ ratios. (a) Real part of the relative permittivity. (b) Imaginary part of the relative permittivity.

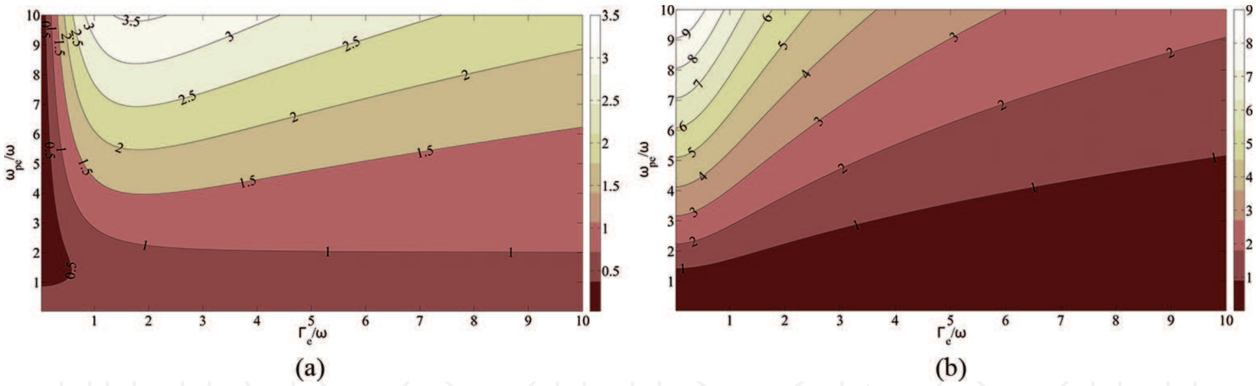


**Figure 2.** Variation of the wavenumber $k$ in a lossy media with complex permittivity depending on the $\omega_{pe}/\omega$ and $\Gamma/\omega$ ratios. (a) Real part of k normalized by $k_0$ (rad/m). (b) Imaginary part k normalized by $k_0$ (Np/m).

propagating on a media, we can determine for example the cut-off frequency of a waveguide filled with the material.

If we consider the wavenumber $k = \omega\sqrt{\epsilon\mu}$, where only $\epsilon$ or $\mu$ can be negative and the other parameter remains positive (SNG media), there are two interesting cases. If we consider a lossless media ($\Gamma_e = 0$), the wavenumber expression becomes:

$$k = k_0\sqrt{1 - \frac{\omega_{pe}^2}{\omega^2}} \tag{35}$$

When $\omega \ll \omega_{pe}$, and considering a lossless media, the wavenumber becomes purely imaginary and there is no propagation on the media ($k \approx ik_0\, \omega_{pe}/\omega$). On the other hand, when $\omega \gg \omega_{pe}$, the waves propagates on the media with a wavenumber with a value near $k_0$. When the media presents losses, there is always propagation on the media, even if $\omega \ll \omega_{pe}$. The propagation in this case will occur with an attenuation, that depending of the values of $\Gamma_e$, can be important.

Equation (35) may be related with the description in terms of massive photons propagating in a plasma: As shown in Ref. [16], the presence of the plasma decreases the rate of electromagnetic energy flow, reaching a zero speed when $\hbar\omega = \hbar\omega_p = m_\gamma c^2$, a photon energy below which the propagation is not possible.

### 6.2. Wavenumber on DNG media depending on metamaterial parameters

It is possible to do a further analysis considering now a variation of the permittivity and permeability where both takes negatives values (DNG media). In this case, we will consider a material with its permittivity an permeability following the Drude's model expressed in Eqs. (20) and (21), and its parameters changes equally in terms of the ratios $\omega_p/\omega$ and $\Gamma/\omega$ ratios ($\omega_{pe}/\omega = \omega_{pm}/\omega$ and $\Gamma_e/\omega = \Gamma_m/\omega$). **Figure 3** shows the real and complex values of the wavenumber $k$ of the described material. Using this last figure we can relate the values of the wavenumber and the ratios $\omega_p/\omega$, $\Gamma/\omega$ when both permittivity an permeability are negative. For these calculations, some values of $k''$ can result on negative values (remember that here

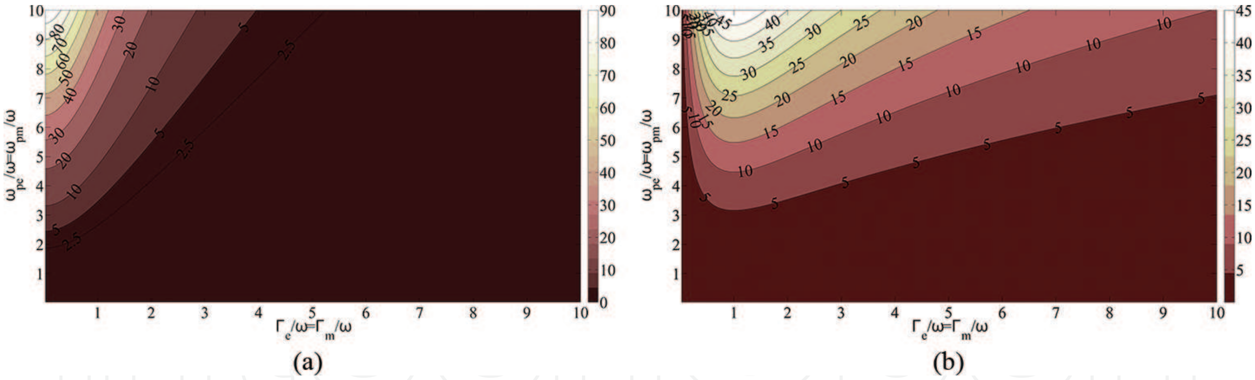


**Figure 3.** Variation of the wavenumber k in a lossy media where $\epsilon$ and $\mu$ can be negative (DNG) depending on the $\omega_{\mathrm{pe}}/\omega = \omega_{\mathrm{pm}}/\omega$ and $\Gamma_{\mathrm{e}}/\omega = \Gamma_{\mathrm{m}}/\omega$ ratios. (a) Real part of k normalized by $k_0$ (rad/m). (b) Imaginary part k normalized by $k_0$ (Np/m).

both permittivity and permeability can be negative). As this is not possible, because the imaginary part must be positive due to the conservation of energy, as shown in Ref. [39], there is a change of sign for those values. For these calculations, some values of $k''$ can result on negative values (remember that here both permittivity and permeability can be negative).

In a more practical way, both ratios are not necessarily modified in the same way. Normally, the periodical structures can present with different parameters of design its negative permittivity or permeability. This means that $\omega_{\mathrm{pe}}/\omega$ ratio does not change in the same way that $\omega_{\mathrm{pm}}/\omega$ and $\Gamma_{\mathrm{e}}/\omega$ do not change in the same way that $\Gamma_{\mathrm{m}}/\omega$. For solving this problem, we can rewrite Eqs. (32) and (33) in terms of other parameters as follows, so we can relate the different changes of permittivity and permeability.

$$\epsilon_{\mathrm{r}} = 1 - \frac{\left(\frac{\omega_{\mathrm{pe}}}{\Gamma_{\mathrm{e}}}\right)^2}{1+\left(\frac{\omega}{\Gamma_{\mathrm{e}}}\right)^2} - i\frac{\left(\frac{\omega_{\mathrm{pe}}}{\Gamma_{\mathrm{e}}}\right)^2}{\left(1+\left(\frac{\omega}{\Gamma_{\mathrm{e}}}\right)^2\right)\left(\frac{\omega}{\Gamma_{\mathrm{e}}}\right)} \tag{36}$$

$$\mu_{\mathrm{r}} = 1 - \frac{\left(\frac{\omega_{\mathrm{pm}}}{\Gamma_{\mathrm{m}}}\right)^2}{1+\left(\frac{\omega}{\Gamma_{\mathrm{m}}}\right)^2} - i\frac{\left(\frac{\omega_{\mathrm{pm}}}{\Gamma_{\mathrm{m}}}\right)^2}{\left(1+\left(\frac{\omega}{\Gamma_{\mathrm{m}}}\right)^2\right)\left(\frac{\omega}{\Gamma_{\mathrm{m}}}\right)} \tag{37}$$

Expressing the complex permittivity and complex permeability in terms of $\omega_{\mathrm{pe}}/\Gamma_{\mathrm{e}}$, $\omega_{\mathrm{pm}}/\Gamma_{\mathrm{m}}$, $\omega/\Gamma_{\mathrm{e}}$ and $\omega/\Gamma_{\mathrm{e}}$ allows to describe in a more independent way the effects of the material parameters on the design of the metamaterial and the resultant permittivity or permeability. Other important thing to consider is that in practice, the angular frequency of the impinging wave on the media is higher that the damping frequencies ($\omega \gg \Gamma$) due to the applications involved on the use of metamaterials. If we fix a ratio of $\omega/\Gamma_{\mathrm{e}}$ and $\omega/\Gamma_{\mathrm{m}}$, we can analyse the variation of the permittivity and permeability, and so the wavenumber, depending on the $\omega_{\mathrm{pe}}/\Gamma_{\mathrm{e}}$ and $\omega_{\mathrm{pm}}/\Gamma_{\mathrm{m}}$ ratios. **Figure 4** shows the real and imaginary part of the relative permittivity as function of $\omega_{\mathrm{pe}}/\Gamma_{\mathrm{e}}$ with a $\omega/\Gamma_{\mathrm{e}}$ ratio equals to 5 and $\omega/\Gamma_{\mathrm{e}}$ ratio equals to 10.

http://dx.doi.org/10.5772/66673

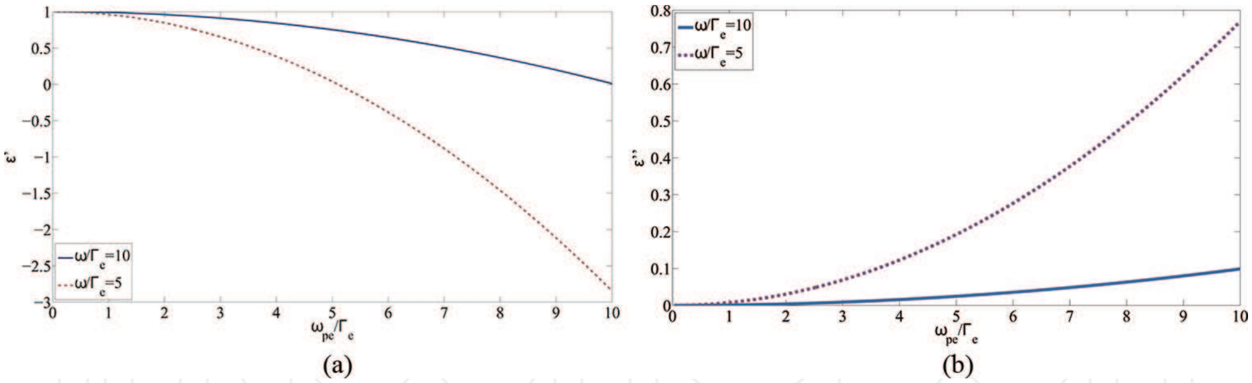


**Figure 4.** Complex relative permittivity of a media depending on the $\omega_{pe}/\Gamma_e$. (a) Real part of the relative permittivity. (b) Imaginary part of the relative permittivity.

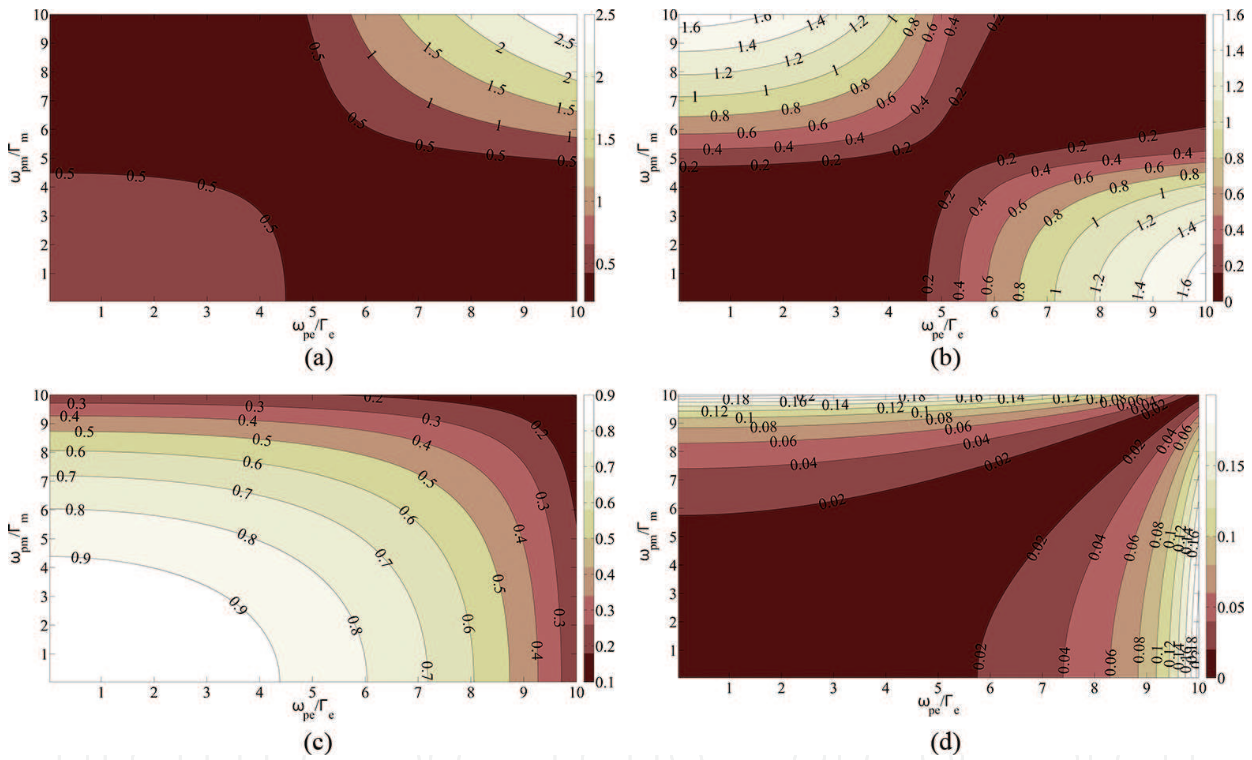


**Figure 5.** Variation of the wavenumber $k$ in a lossy media where $\epsilon$ and $\mu$ can be negative (DNG) depending on the $\omega_{pe}/\Gamma_e$ and $\omega_{pm}/\Gamma_m$ ratios, having a fixed $\omega/\Gamma$ ratio ($\omega/\Gamma_e = \omega/\Gamma_m$= 5 and $\omega/\Gamma_e = \omega/\Gamma_m$= 10). (a) Real part of $k$ normalized by $k_0$ (rad/m) when $\omega/\Gamma_e = \omega/\Gamma_m = 5$. (b) Imaginary part $k$ normalized by $k_0$ (Np/m) when $\omega/\Gamma_e = \omega/\Gamma_m = 5$. (c) Real part of $k$ normalized by $k_0$ (rad/m) when $\omega/\Gamma_e = \omega/\Gamma_m = 10$. (d) Imaginary part $k$ normalized by $k_0$ (Np/m), when $\omega/\Gamma_e = \omega/\Gamma_m = 10$.

From **Figure 4**, we can observe when the material have negative or positive values of $\epsilon$. Extending this analysis to $\mu_r$, we can determine the ratios of $\omega_p/\Gamma$, where the material will exhibit a DNG behaviour. Having this information, as shown in **Figure 5**, we can trace the wavenumber in function of the different variations of $\omega_{pe}/\Gamma_e$ and $\omega_{pm}/\Gamma_m$ having a fixed $\omega/\Gamma$ ratio ($\omega/\Gamma_e = \omega/\Gamma_m$= 5 and $\omega/\Gamma_e = \omega/\Gamma_m$= 10).

From this analysis, we can notice that the values of the attenuation and of the phase constant will vary depending of the design parameters of $\omega_{pe}/\Gamma_e$ and $\omega_{pm}/\Gamma_m$. However, it is important to consider the ratio between $\omega/\Gamma_m$, in terms that depending of this value, we can achieve or not the DNG behaviour of the material.

In conclusion, for the design of metamaterials, it is important to know how the parameters that are described on the Drude's model vary, in order to predict the behaviour of the wave that propagates on the media.

## 7. Discussion

From the point of view of unification of electromagnetic fields and relativistic quantum theory, it is useful to study the dynamics of a photon in a form comparable with the case of a particle like an electron. As a first stage towards this unification, an important result is that the Dirac-like equation allows to write Maxwell's equations in a compact form and that for light propagating in an homogeneous medium this equation has energy solutions similar to those obtained by Dirac's equation for fermions in the limit of zero mass, except that in this case there is no a solution with zero energy. Nevertheless, this must be considered only as a formal analogy since photons have spin 1 and Dirac's equation is applicable for particles of spin ½.

Among other issues related with the behaviour of photons that have been a matter of discussion in several publications, it is worth to mention those concerned with localizability, Zitterbewegung and its relation with spin.

The localizability of massless photons was first examined in Ref. [40] by Newton and Wigner and later by Wightman in Ref. [41], showing that there is no position operator for a massless particle with spin higher than ½ leading many authors to conclude that it is not possible to define a wave function for a photon, which has zero mass and spin (or helicity) 1. Wightman has proved that the only localizable massless elementary system has spin zero and that a free photon is not localizable.

However, Bialynicki-Birula in Refs. [42, 43] and, independently, Sipe in Ref. [44] introduced a function of the position and time coordinates that completely describes the quantum state of a photon. Such function may be referred to as the photon wave function. The wave equation for this function can be derived from the Einstein kinematics for a particle with spin 1 and zero mass in the same way that the Dirac equation is obtained for a massive particle with spin ½. For example, see Refs. [45, 46]. A strong argument in favour of this photon wave function formulation is that the corresponding wave equation is completely equivalent to the Maxwell equations in vacuum. In addition in Ref. [1], a position operator has been postulated whose eigenvectors form bases of localized states.

The concept of spin of a photon and its relation with Zitterbewegung is still a matter of discussion and deserves further research. In Ref. [47], it has been postulated that the spin of the photon can be considered as a consequence of the orbital angular momentum due to the photon's Zitterbewegung. This postulate is based on a Schrödinger-like equation, having a velocity operator that undergoes oscillations in a direction orthogonal to its momentum, effect known as Zitterbewegung, with a spatial amplitude equal to the classical wavelength. The spin of the photon would be the orbital angular momentum due to the Zitterbewegung. Nevertheless, up to now, this is a theoretical postulate which results from an equation of the form of Schrödinger's equation which is known, was formulated for a nonrelativistic particle. In this context it seems formally more suitable to use a Dirac-like equation.

## 8. Conclusion

We have presented a conceptual frame for understanding the propagation of light through a dispersive and absorptive medium, considering both the classical description based on electromagnetic waves and a quantum description considering photons as elementary excitations of the quantized electromagnetic field.

A semiclassical description of the dynamics of a photon propagating freely in an unbounded medium has been presented using a Dirac-like equation, discussing the solutions for the energy and comparing with those corresponding to a free electron as given by the Dirac's equation.

The interaction of light with the medium of propagation has been analysed using both a classical and a quantum treatment. In particular and as a specific example, the first-order field-field spatial correlation for a beam of light produced by the excitation of two linearly polarized waves has been calculated, comparing results between the classical and quantum model. For this specific case, it is concluded that in absence of absorption, the classical model predicts total coherence at all points, while the quantum treatment predicts that for some distances between the considered points there is null coherence.

The propagation of light along a rectangular waveguide has been studied showing how the Dirac-like equation previously studied is modified due to the bounding conditions in the propagation imposed by the conducting walls verifying the result obtained in several publications that in this condition photons appear to acquire an effective mass.

As an application to communication engineering, we have analysed conditions for which the dielectric permittivity and magnetic permeability of a medium filled with plasma behaves as a metamaterial.

## Author details

Patricio Robles* and Francisco Pizarro

*Address all correspondence to: probles@ucv.cl

Escuela de Ingeniería Eléctrica, Pontificia Universidad Católica de Valparaíso, Chile

## References


[1] Hawton M. Photon wave mechanics and position eigenvectors. Phys. Rev. A. 2007; **75**: 062107–062120. DOI: 10.1103/PhysRevA.75.062107

[2] Kobe D.H. A relativistic Schrödinger-like equation for a photon and its second quantization. Found. Phys. 1999; **29**: 1203–1231. DOI: 10.1023/A:1018855630724

[3] Barnett S.M. Optical Dirac equation. New J. Phys. 2014; **16**: 093008-1–093008-25. DOI: 10.1088/1367-2630/16/9/093008

[4] Mohr P. Solutions of the Maxwell equations and photon wave functions. Ann. Phys. 2010; **325:** 607–663. DOI: 10.1016/j.aop.2009.11.007

[5] Robles P. An introduction to relativistic quantum mechanics: a round trip between Schrödinger and Dirac equations. Int. J. Eng. Tech. Res. (IJETR). 2016; **4**: 125–132. ISSN: 2321-0869 (P)

[6] Thaller B. The Dirac Equation. 1st ed. Berlin, Heidelberg, New York: Springer-Verlag, 2000

[7] Zhi-Yong W., Cai-Dong X., Bing H.E. Quantum mechanical description of waveguides. Chin. Phys. B. 2008; **17**: 3985–3997. DOI: 10.1088/1674-1056/17/11/008

[8] Feynman R.P. Quantum Electrodynamics. New York: Benjamin, 1962, pp. 66–70.

[9] Feynman R.P. The theory of positrons. Phys. Rev. 1949; **76**: 749–759. DOI: 10.1103/PhysRev.76.749

[10] Ruppin R. Electromagnetic energy density in a dispersive and absorptive material. Phys. Lett. A. 2002; **299**: 309–312. DOI: 10.1016/S0375-9601(01)00838-6

[11] Dung Ho T., Buhmann S.Y., Knöll L., Welsh D.G., Scheel S. Electromagnetic-field quantization and spontaneous decay in left-handed material. Phys. Rev. A. 2003; **68**: 043816–043832. DOI: 10.1103/PhysRevA.68.043816

[12] Matloob R., Loudon R., Barnett S.M., Jeffers J. Electromagnetic field quantization in absorbing dielectrics. Phys. Rev. A 1995; **52**:4823–4838. DOI: 10.1103/PhysRevA.52.4823

[13] Huttner B., Barnett S.M. Quantization of the electromagnetic field in dielectrics.Phys. Rev. A 1992; **46**: 4306–4322. DOI: 10.1103/PhysRevA.46.4306

[14] Loudon R. The Quantum Theory of Light. 3rd. Ed. Oxford, UK: Oxford University Press, 2000.

[15] Gerry C.G., Knight P.L. Introductory Quantum Optics. Cambridge UK: Cambridge University Press, 2005.

[16] Robles P., Claro F. Can there be massive photons? A pedagogical glance at the origin of mass. Eur. J. Physics. 2012; **33**: 1217–1226. DOI: 10.1088/0143-0807/33/5/1217

[17] Rojas R., Robles P. Teaching on the common aspects in mechanical, electromagnetic and quantum waves at interfaces and waveguides. Eur. J. Phys. 2011; **32**: 1–11. DOI: 10.1088/0143-0807/32/6/019

[18] Greiner W. Relativistic Quantum Mechanics Wave Equations. 3rd. Ed. Berlin, Heidelberg, New York: Springer-Verlag, 2000.

[19] Veselago V. The electrodynamics of substances with simultaneously negative values of ε and μ. Soviet Physics Uspekhi. 1968; **10**: 509–514. DOI: 10.1070/PU1968v010n04ABEH003699

[20] Hippler R., Kersten H., Schmidt M., Schoenbach K.H. Low Temperature Plasmas: Fundamentals, Technologies and Techniques, Wiley, 2nd Ed., 2010, Germany, ISBN: 978-3-527-40673-9.

[21] Pendry J.B., Smith D.R. Reversing light with negative refraction. Phys. Today. 2004; **57**(6): 37–43. DOI: 10.1063/1.1784272

[22] Hrabar S., Bonefacic D., Muha D. Application of wire-based metamaterials for antenna miniaturization. 2009 3rd European Conference on Antennas and Propagation, Berlin, 2009, pp. 620–623. DOI: 10.1109/LMWC.2003.822563

[23] Dutta Gupta S. Subliminal to superluminal propagation in a left-handed medium. Phys. Rev. B. 2004; **69**(11): 113104. DOI: 10.1103/PhysRevE.91.033206

[24] Smith D.R., Padilla W.J., Vier D.C., Nemat-Nasser S.C., Schultz S. Composite medium with simultaneously negative permeability and permittivity. Phys. Rev. Lett. 2000; **84**(18): 4184–4187. DOI: 10.1103/PhysRevLett.84.4184

[25] Shelby R.A., Smith D.R., Schultz S. Experimental verification of a negative index of refraction. Science. 2001; **292**(5514): 77–79. DOI: 10.1103/PhysRevLett.90.107401

[26] Grigorenko A.N., Geim A.K., Gleeson H.F., Zhang Y., Khrushchev I.Y., Petrovic J. Nanofabricated media with negative permeability at visible frequencies. Nature. 2005; **438**: 335–338. DOI: 10.1038/nature04242

[27] Di Gennaro E., Parimi P.V., Lu W.T., Sridhar S., Derov J.S., Turchinetz B. Slow microwaves in left-handed materials. Phys. Rev. B. 2005; **72**(3): 033110. DOI: 10.1103/PhysRevB.72.033110

[28] Podolskiy V.A., Narimanov E.E. Strongly anisotropic waveguide as a nonmagnetic lefthanded system. Phys. Rev. B. 2005; **71**(20): 201101. DOI: 10.1103/PhysRevB.71.201101

[29] Veselago V., Braginsky L., Shklover V., Hafner C. Negative refractive index materials. J. Comput. Theor. Nanosci. 2006; **3**: 1–30.

[30] Zhang X., Yao J., Liu Z., Liu Y., Wang Y., Sun C., Bartal G., Stacy A.M. Optical negative refraction in bulk metamaterials of nanowires. Science. 2008; **321**(5891): 930. DOI: 10.1126/science.1157566

[31] Zhang X., Valentine J., Zentgraf T., Ulin-Avila E., Genov D.A., Bartal G. Three dimensional optical metamaterial with a negative refractive index. Nature. 2008; **455**(7211): 376. DOI: 10.1063/1.3656286

[32] Alù A., Engheta N. Guided modes in a waveguide filled with a pair of single-negative (SNG), double-negative (DNG), and/or double-positive (DPS) layers. IEEE Trans. Microwave Theory Tech. 2004; **MTT-52**: 199–210. DOI: 10.1109/TMTT.2003.821274

[33] Alù A., Engheta N. Pairing an epsilon-negative slab with a Mu-negative slab: anomalous tunneling and transparency. IEEE Trans. Antennas Propag. 2003; **AP-51**: 2558–2570. DOI: 10.1109/TAP.2003.817553

[34] Ziolkowski R.W., Kipple A. Application of double negative metamaterials to increase the power radiated by electrically small antennas. IEEE Trans. Antennas Propag. 2003; **AP-51**: 2626–2640. DOI: 10.1109/TAP.2003.817561

[35] Ziolkowski R.W., Kipple A.D. Reciprocity between the effects of resonant scattering and enhanced radiated power by electrically small antennas in the presence of nested metamaterial shells. Phys. Rev. E. 2005; **72**: 036602. DOI: 10.1103/PhysRevE.72.036602

[36] Iyer A.K., Eleftheriades G.V. Free-space imaging beyond the diffraction limit using a Veselago-Pendry transmission-line metamaterial superlens. IEEE Trans. Antennas Propag. 2009; **57**(6): 1720–1727. DOI: 10.1109/TAP.2009.2019890

[37] Fang N., Zhang X. Imaging properties of a metamaterial superlens, Nanotechnology, 2002. IEEE-NANO 2002. In: Proc. 2002 2nd IEEE Conference on, 2002, pp. 225–228. DOI: 10.1063/1.1536712

[38] Engheta N., Ziolkowsky R. (Ed.). Metamaterials: Physics and Engineering Explorations. Wiley, IEEE Press, 2006, United States

[39] Lee D.H., Park W.S. A new material classification of lossy metamaterials. Microw. Opt. Technol. Lett. 2011; pp. 445–447, **53**(2).

[40] Newton T.D., Wigner E.P. Localized states for elementary systems. Rev. Mod. Phys. 1949; **21**: 400–406. DOI: 10.1103/RevModPhys.21.400

[41] Wightman A.S. On the localizability of quantum mechanical systems. Rev. Mod. Phys. 1962; **34**: 845–872. DOI: 10.1103/RevModPhys.34.845

[42] Bialynicki-Birula I. On the wave function of the photon. Acta Phys. Pol. A. 1994; **86**: 97–116. DOI: 10.12693/APhysPolA.86.97

[43] Bialynicki-Birula I. Photon wave function. Progr. Optics. 1996; **XXXVI**: 245–294. arXiv: quant-ph/0508202

[44] Sipe J.E. Photon wave functions. Phys. Rev. A. 1995; **52**: 1875–1883. DOI: 10.1103/PhysRevA.52.1875

[45] Raymer M.G., Smith B.J. The Maxwell wave function of the photon. In: Proc. SPIE 5866, The Nature of Light: What Is a Photon? August 04, 2005, pp. 293–297. DOI: 10.1117/12.619359

[46] Smith B.J., Raymer M.G. Photon wave functions, wave-packet quantization of light, and coherence theory. New J. Phys. 2007; **9**: 1–37. DOI: 10.1088/1367-2630/9/11/414

[47] Kobe D.H. Zitterbewegung of a photon. Phys. Lett. A. 1999; **253**: 7–11. DOI: 10.1016/S0375-9601(99)00011-0

IntechOpen