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Numerical Analysis of Three-Dimensional MHD Nanofluid Flow over a Stretching Sheet with Convective Boundary Conditions through a Porous Medium

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Additional information is available at the end of the chapter

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Abstract

Numerical analysis has been carried out on the problem of three-dimensional magneto-hydrodynamic boundary layer flow of a nanofluid over a stretching sheet with convective boundary conditions through a porous medium. Suitable similarity transformations were used to transform the governing partial differential equations into a system of ordinary differential equations. We then solved the resultant ordinary differential equation by using the spectral relaxation method. Effects of the dimensionless parameters on velocity, temperature and concentration profiles together with the friction coefficients, Nusselt and Sherwood numbers were discussed with the assistance of graphs and tables. The velocity was found to decrease with increasing values of the magnetic, stretching and permeability parameters. The local temperature was observed to rise as the Brownian motion, thermophoresis and Biot numbers increased. The concentration profiles diminish with increasing values of the Lewis number and chemical reaction parameter.

Keywords: numerical analysis, MHD nanofluid, stretching sheet, convective boundary conditions, porous medium

1. Introduction

Many researchers have over the past few years paid significant attention to the study of boundary layer flow heat and mass transfer over a stretching sheet due to its industrial and engineering applications. These applications include cooling of papers, glass-fibre production, plastic sheets and polymer extrusion, hot rolling wire drawing, metal spinning, stretching of

rubber sheets and crystal growing. The quality and final product formation in these processes are dependent on the rate of stretching and cooling.

Since the pioneering study by Crane [1] who presented an exact analytical solution for the steady two-dimensional flow due to a stretching surface in a quiescent fluid many studies on stretched surfaces have been done [1–5].

Thermal conductivity of nanoparticles has been shown in recent research on nanofluid to change the fluid characteristics. The thermal conductivity of the base liquid with the enhanced conductivity of nanofluid and the turbulence induced by their motion contribute to a remarkable improvement in the convective heat transfer coefficient. This feature of nanofluid makes them attractive to a wide variety of industries, ranging from transportation to energy production and supply to electronics. They can be used in welding equipment, high heat flux and to cool car engines, among other applications. Many researchers [6–10] have studied the boundary layer flow of a nanofluid caused by a stretching surface.

Shateyi and Prakash [11] carried out a numerical analysis on the problem of magneto hydrodynamic boundary layer flow of a nanofluid over a moving surface in the presence of thermal radiation. Kuznetsov and Nield [12] examined the influence of nanoparticles on natural convection boundary layer flow past a vertical plate, using a model in which Brownian motion and thermophoresis are accounted for. Aziz and Khan [13] investigated using a similarity analysis of the transport equations by their numerical computations to the natural convective flow of a nanofluid over a convectively heated vertical plate.

Makinde and Aziz [14] numerically studied the boundary layer flow induced in a nanofluid due to a linearly stretching sheet. Hayat et al. [15] addressed the MHD flow of second grade nanofluid over a nonlinear stretching sheet. Zhao et al. [16] studied the three-dimensional nanofluid bio-convection near a stagnation attachment. Sheikholeslami and Ganji [17] studied two-dimensional laminar-forced convection nanofluids over a stretching surface in a porous medium. The study used different models of nanofluid based on different formulas for thermal conductivity and dynamic viscosity. Nayak et al. [18] did a numerical study on the mixed convection of copper-water nanofluid inside a differentially heated skew enclosure. Recently, Mabood and Das [19] analysed MHD flow and melting heat transfer of a nanofluid over a stretching surface. Naramgari and Sulochana [20] analysed the momentum and heat transfer of MHD nanofluid embedded with conducting dust particles past a stretching surface in the presence of volume fraction of dust particles. Sandeep et al. [21] analysed the unsteady MHD radiative flow and heat transfer characteristics of a dusty nanofluid over an exponentially permeable stretching in the presence of volume fraction of dust and nanoparticles. Sheikholeslami et al. [22] computationally investigated nanofluid flow and heat transfer in a square heated rectangular body. Sheikholeslami and Ganji [23] provided a review of researches on nanofluid flow and heat transfer via semi-analytical and numerical methods. Lastly, Naramgari and Sulochana [20] analysed the three-dimensional MHD Newtonian and non-Newtonian fluid flow over a stretching surface in the presence of thermophoresis and Brownian motion.

The main objective of this chapter is to numerically analyse the influence of convective boundary conditions on the model of three-dimensional magnetohydrodynamic, nanofluid flow over a stretching sheet through a porous medium in the presence of thermophoresis and Brownian

motion as well as thermal radiation. The governing partial differential equations use suitable similarity transformations. The transformed governing equations are solved numerically using the spectral relaxation method (SRM). The effects of dimensionless parameters on velocity components, temperature and concentration profiles together with the skin friction coefficients, local Nusselt and Sherwood numbers are discussed with the aid of tables and graphs.

2. Mathematical formulation

We consider a three-dimensional steady incompressible MHD nanofluid flow, heat and mass transfer over a linearly stretching sheet through a porous medium. The sheet is assumed to be stretched along the xy -plane while the fluid is placed along the z -axis. A uniform magnetic field B_0 is applied normally to the stretched sheet and the induced magnetic field is neglected by assuming very small Reynolds number. We assume that the sheet is stretched with linear velocities $u = ax$ and $v = by$ along the xy -plane, respectively, with constants a and b . Under the above assumptions and the boundary approximation, the governing equations for the current study are given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \nu \frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2}{\rho} u - \frac{v}{k_1} u, \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \nu \frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0^2}{\rho} v - \frac{v}{k_1} v, \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha \frac{\partial^2 T}{\partial z^2} - \frac{\partial q_r}{\partial z} + \tau \left[D_B \frac{\partial C}{\partial z} \frac{\partial T}{\partial z} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial z} \right)^2 \right], \quad (4)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} = D_B \frac{\partial^2 C}{\partial z^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial z^2}, \quad (5)$$

where u, v and w are the velocity components in the x, y and z - directions, respectively, T is the fluid temperature, C is the fluid concentration, k_1 is the permeability, ν is kinematic viscosity, ρ is the fluid density, τ is the ratio of the heat capacitances, D_B and D_T are the Brownian motion and thermophoretic diffusion coefficients and c_p is the specific heat capacity.

The corresponding boundary conditions for the flow model are:

$$u = ax, v = by, w = 0, -k \frac{\partial T}{\partial z} = h_f(T_f - T), -D_B \frac{\partial C}{\partial z} = h_s(C_s - C) \text{ at } z = 0, \quad (6)$$

$$u \rightarrow 0, v \rightarrow 0, w \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty, \text{ as } z \rightarrow \infty. \quad (7)$$

We have h_f as the convective heat transfer coefficient, h_s is the convective mass transfer coefficient and T_f and C_f are the convective fluid temperature and concentration below the moving sheet.

3. Similarity transformation

In order to non-dimensionalise the governing equations, we introduce the following similarity equations [24]. These transformations also transform the partial differential equations into a system of ordinary differential equations which is then solved using the spectral relaxation method:

$$\eta = \sqrt{\frac{a}{v}}z, u = axf'(\eta), v = byg'(\eta), w = -\sqrt{av}[f(\eta) + cg(\eta)], \theta(\eta) = \frac{T-T_\infty}{T_f-T_\infty}, \phi(\eta) = \frac{C-C_\infty}{C_s-C_\infty}. \quad (8)$$

Upon substituting the similarity variables into Eqs. (2)–(5), we obtain the following system of ordinary equations

$$f''' + (f + cg)f'' - f'^2 - (M + K)f' = 0, \quad (9)$$

$$g''' + (f + cg)g'' - g'^2 - (M + K)g' = 0, \quad (10)$$

$$\left(\frac{3 + 4R}{3PrR}\right)\theta'' + (f + cg)\theta' + Nb\theta'\phi' + Nt(\theta')^2 = 0, \quad (11)$$

$$\phi'' + Le(f + cg)\phi' + \frac{Nt}{Nb}\theta'' = 0. \quad (12)$$

The corresponding boundary conditions are

$$f = 0, f' = 1, g = 0, g' = 1, \theta' = -Bi_t(1 - \theta), \phi' = -Bi_c(1 - \phi), \text{ at } \eta = 0, \quad (13)$$

$$f'(\infty) \rightarrow 0, g'(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0. \quad (14)$$

Primes denote differentiation with respect to η and parameters appearing in Eqs (9)–(14) are defined as: $Pr = \nu/\alpha$ is the Prandtl number, $Le = \nu/D_B$ is the Lewis number, $Nb = \tau D_B(C_s - C_\infty)/\nu$ is the Brownian motion parameter, $Nt = \tau D_T(T_f - T_\infty)/\nu T_\infty$ is the thermophoresis parameter, $Bi_t = \frac{h_f}{k} \sqrt{\nu/a}$, $Bi_c = \frac{h_s}{D_B} \sqrt{\nu/a}$ are the Biot numbers and $c = b/a$ is the stretching parameter. The quantities of engineering interest are the skin-friction coefficient C_f along the x - and y -direction (C_{fx} and C_{fy}), the Nusselt number and Sherwood number. These quantities are defined as follows:

$$C_{fx} = \frac{\tau_{wx}}{\rho u_w^2}, C_{fy} = \frac{\tau_{wy}}{\rho u_w^2}, Nu = x \frac{q_w}{k(T_f - T_w)}, Sh = x \frac{q_w}{D_B(C_f - C_w)}, \quad (15)$$

where τ_{wx}, τ_{wy} are the wall shear along x - and y -directions, respectively, and q_w and q_m are the heat flux and mass flux at the surface, respectively. Upon using the similarity variables into the above expressions, we obtain the following:

$$Re^2 C_{fx} = f''(0), Re^{\frac{1}{2}} C_{fy} = g''(0), Re^{\frac{1}{2}} Nu = -\theta'(0), Re^{\frac{1}{2}} Sh = -\phi(0). \quad (16)$$

4. Method

To solve the set of ordinary differential Eqs. (9)–(12) together with the boundary conditions (13) and (14), we employ the Chebyshev pseudo-spectral method known as spectral relaxation method. This is a recently developed method, and the details of the method are found in Motsa et al. [25]. This method transforms sets of non-linear ordinary differential into sets of linear ordinary differential equations. The entire computational procedure is implemented using a program written in MATLAB computer language. The nanofluid velocity, temperature, the local skin-friction coefficient and the local Nusselt and Sherwood numbers are determined from these numerical computations.

To apply the SRM to the non-linear ordinary differential equations, we first set $f'(\eta) = p(\eta)$ and $g'(\eta) = q(\eta)$. We then write the equations as follows:

$$f' = p, \quad (17)$$

$$p'' + (f + cg)p' - p^2 - (M + K)p = 0, \quad (18)$$

$$g' = q, \quad (19)$$

$$q'' + (f + cg)q' - q^2 - (M + K)q = 0, \quad (20)$$

$$\left(\frac{3 + 4R}{3PrR}\right)\theta'' + (f + cg)\theta' + Nb\theta'\phi' + Nt\theta'^2 = 0, \quad (21)$$

$$\phi'' + Le(f + cg)\phi' + \frac{Nt}{Nb}\theta'' = 0. \quad (22)$$

The boundary conditions become

$$f(0) = 0, g(0) = 0, p(0) = 1, q(0) = 1, \quad (23)$$

$$\theta'(0) = -Bit(1 - \theta), \phi'(0) = -Bic(1 - \phi), \quad (24)$$

$$p(\infty) = 0, q(\infty) = 0, \phi(\infty) = 0, \theta(\infty) = 0, \quad (25)$$

In view of the SRM, we then obtain the following iterative scheme:

$$f'_{r+1} = p_r, f_{r+1}(0) = 0, \quad (26)$$

$$p''_{r+1} + (f_{r+1} + cg_{r+1})p'_{r+1} - (M + K)p_{r+1} = p_r^2, p_{r+1}(0) = 1, p_{r+1}(\infty) = 0, \quad (27)$$

$$g'_{r+1} = q_r, g_{r+1}(0) = 0, \quad (28)$$

$$q''_{r+1} + (f_{r+1} + cg_{r+1})q'_{r+1} - (M + K)q_{r+1} = q_r^2, q_{r+1}(0) = 1, q_{r+1}(\infty) = 0, \quad (29)$$

$$\left(\frac{3 + 4R}{3PrR}\right)\theta''_{r+1} + (f_{r+1} + cg_{r+1})\theta'_{r+1} = -Nb\theta'_r\phi'_r - Nt\theta_r'^2, \theta'_{r+1}(0) = -Bit(1 - \theta_{r+1}(0)), \phi_{r+1}(\infty) = 0, \quad (30)$$

$$\phi_{r+1}'' + Le(f_{r+1} + cg_{r+1})\phi_{r+1}' = -\frac{Nt}{Nb}\phi_{r+1}'', \phi_{r+1}'(0) = -Bic(1-\phi_{r+1}(0)), \phi_{r+1}(\infty) = 0. \quad (31)$$

The above equations form a system of linear decoupled equations which can be solved iteratively for $r = 1, 2, \dots$. Starting from initial guesses $(p_0(\eta), q_0(\eta), \theta(\eta), \phi(\eta))$. Applying the Chebyshev pseudo-spectral method to the above equations, we obtain

$$A_1 f_{r+1} = B_1, f_{r+1}(\tau N) = 0, \quad (32)$$

$$A_2 p_{r+1} = B_2, p_{r+1}(\tau N) = 1, p_{r+1}(\tau_0) = 0, \quad (33)$$

$$A_3 g_{r+1} = B_3, g_{r+1}(\tau N) = 0, \quad (34)$$

$$A_4 q_{r+1} = B_4, q_{r+1}(\tau N) = 1, q_{r+1}(\tau_0) = 0, \quad (35)$$

$$A_5 \theta_{r+1} = B_4, \theta_{r+1}(\tau N) = \frac{Bit}{1 + Bit}, \theta_{r+1}(\tau_0) = 0, \quad (36)$$

$$A_6 \phi_{r+1} = B_6, \phi_{r+1}(\tau N) = \frac{Bit}{1 + Bit}, \phi_{r+1}(\tau_0) = 0. \quad (37)$$

where, $A_1 = D, B_1 = p_r, A_2 = D^2 + \text{diag}(f_{r+1} + cg_{r+1})D - (M + K)I, B_2 = q_{r+1}^2, A_3 = D, B_3 = q_r, A_4 = D^2 + \text{diag}(f_{r+1} + cg_{r+1})D - (M + K)I, A_5 = (\frac{3+4R}{3PrR})D^2 + \text{diag}(f_{r+1} + cg_{r+1})D, B_5 = -N_6\theta_r'\phi_r' - Nt\theta_r'^2, A_6 = D^2 + \text{diag}[Le f_{r+1} + cLe g_{r+1}]D, B_6 = -\frac{Nt}{Nb}\phi_{r+1}''$,

where I is the identity matrix of size $(N + 1)(N + 1)$. The initial guesses are obtained as:

$$p_0(\eta) = e^{-\eta}, q_0(\eta) = e^{-\eta}, \theta_0(\eta) = \frac{Bite^{-\eta}}{1 + Bit}, \phi_0(\eta) = \frac{Bice^{-\eta}}{1 + Bic}. \quad (38)$$

5. Results and discussion

The system of ordinary differential Eqs. (9)–(12) subject to the boundary conditions (13) and (14) is numerically solved by applying the spectral relaxation method. The SRM results presented in this chapter were obtained using $N = 40$ collocation points, and also the convergence was achieved after as few as six iterations. We also use these default values for the parameters $Pr = 0.71, Nt = Nb = 0.3, Le = 2, R = 1, M = 1, K = 0.5, C = 0.1, Bit = 0.2 = Bic$.

Table 1 displays the validation of the present results with those obtained by the bvp4c results. As can be clearly observed from this table, there is an excellent agreement between the results obtained by bvp4c method giving confidence in the findings of this study. **Table 1** also shows the influence of the magnetic, permeability and stretching parameters on the skin friction coefficients. It is noticed that the skin friction coefficient increase with the increasing values of the parameters.

Table 2 depicts the influence of Brownian motion thermophoresis, parameters and the Biot numbers on the Nusselt and Sherwood numbers. Both the rates of heat transfer and mass

$\langle i \rangle - f''(0) \langle /i \rangle$					$\langle i \rangle - g''(0) \langle /i \rangle$	
M	K	C	Bvp4c	SRM	Bvp4c	SRM
0	0.1	0.1	1.22690	1.22690	0.09495	0.09495
1	0.1	0.1	1.52270	1.52270	0.137834	0.197854
3	0.1	0.1	2.12214	2.12214	0.19750	0.197520
5	0.1	0.1	2.55018	2.55018	0.24290	0.24290
1	0.0	0.1	1.415504	1.415504	0.118270	0.118270
1	0.5	0.1	1.582270	1.582270	0.137834	0.137834
1	1.5	0.1	1.871764	1.871764	0.170320	0.170320
1	3.0	0.1	2.236839	2.236839	0.209800	0.209800
1	0.2	0.2	1.585645	1.585645	0.280922	0.28022
1	0.2	0.4	1.599025	1.599025	0.585064	0.585064
1	0.2	0.6	1.610701	1.610701	0.916198	0.916198
1	0.2	0.8	1.647760	1.647760	0.927968	0.927968

Table 1. Variation of the magnetic, permeability and stretching parameters on the skin friction coefficients.

Nb	Nt	Bit	Bic	$\langle i \rangle - \theta'(0) \langle /i \rangle$	$\langle i \rangle - \phi'(0) \langle /i \rangle$
0.2	0.1	0.2	0.2	0.528756	0.272081
0.4	0.1	0.2	0.2	0.540153	0.283370
0.6	0.1	0.2	0.2	0.627812	0.370495
0.8	0.1	0.2	0.2	0.685700	0.428392
0.1	0.2	0.2	0.2	0.484629	0.277618
0.1	0.4	0.2	0.2	0.418686	0.299623
0.1	0.6	0.2	0.2	0.364816	0.321133
0.1	0.8	0.2	0.2	0.320526	0.342009
0.1	0.1	0.2	0.2	0.523097	0.266480
0.1	0.1	0.4	0.2	0.868307	0.633407
0.1	0.1	0.6	0.2	1.114143	0.926920
0.1	0.1	0.2	0.2	0.523097	0.266480
0.1	0.1	0.2	0.4	0.516229	0.115364
0.1	0.1	0.2	0.6	0.511106	0.002054

Table 2. The influence of the Brownian motion and thermophoresis parameters as well as that of the Biot numbers on the Nusselt and Sherwood numbers.

transfer are increasing functions of the Brownian motion parameter. By definition, thermophoresis is the migration of a colloidal particle in a solution in response to a microscopic temperature gradient. The heat transfer is reduced while the mass transfer increases

with increasing values of the thermophoresis parameter. Lastly, **Table 2** shows the influence of the Biot numbers on the heat transfer and mass transfer rates and they both increase with increasing values of the thermal Biot number. But we noticed opposite effects when the solutal Biot number increases.

Figures 1 and **2** display the effect of permeability parameter on the velocity profiles. We observe that the tangential velocity profiles decrease as the values of the permeability parameter. Also, the transverse velocity ($f'(\eta)$) is reduced by the increasing values of the permeability parameter as more nanofluid is taken away from the boundary layer. This explains the thinning of the velocity boundary layers as the values of K increases (**Figure 1**). **Figures 3** and **4** depict the effect of the magnetic field parameter on the velocity profiles. As expected, we observe that both velocity components are greatly reduced as the values of the magnetic parameter increase. This is because physically increasing the values of magnetic field strength produces a drag-like force known as the Lorentz force. This force acts against the flow when the magnetic field is applied in the normal direction, as in this chapter. **Figures 5** and **6** display the influence of the stretching parameter on the velocity fields. It is seen from **Figure 5** that the tangential velocity profiles $f'(\eta)$ are reduced by increasing values of the stretching parameter c . The transverse velocity is enhanced with the increasing values of the stretching parameter.

Figure 7 displays the influence of the Biot number Bi , on the temperature profiles. It is clearly observed on this figure that the nanofluid temperature field rapidly increases near the boundary with increasing values of the Biot number, Bi , . It is also observed that as the Biot number increases the convective heating of the sheet also increases.

Figures 8 and **9** reveal the effect of the stretching ratio parameter c on the temperature and concentration profile. It is observed that the temperature and concentration profiles are

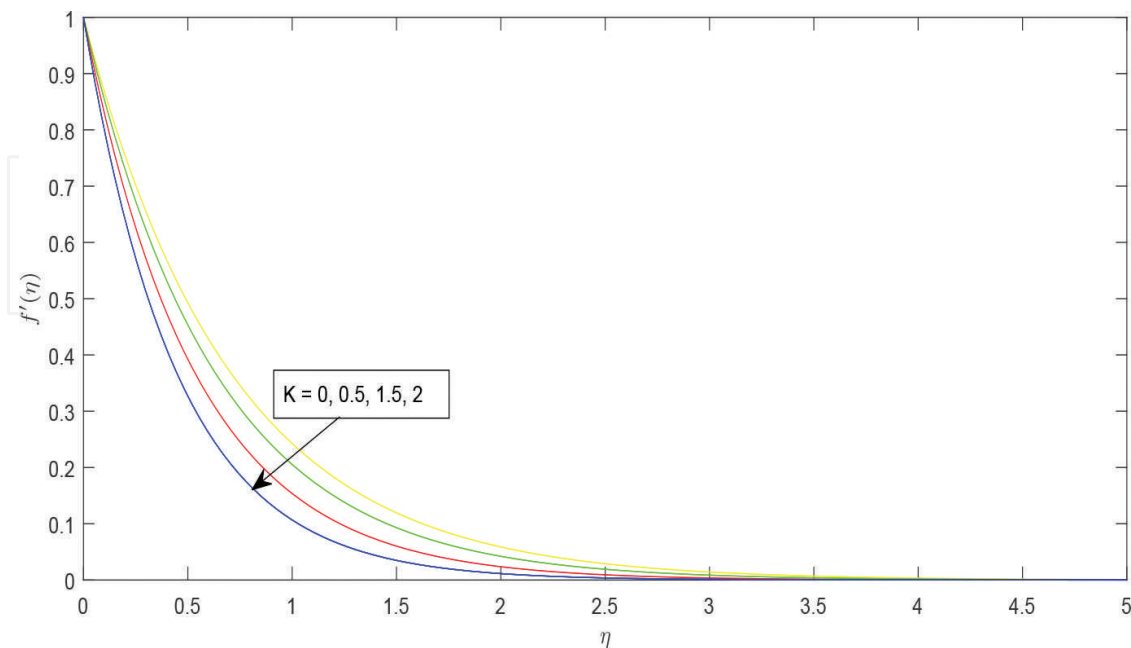


Figure 1. Effect of permeability parameter on the tangential velocity profiles.

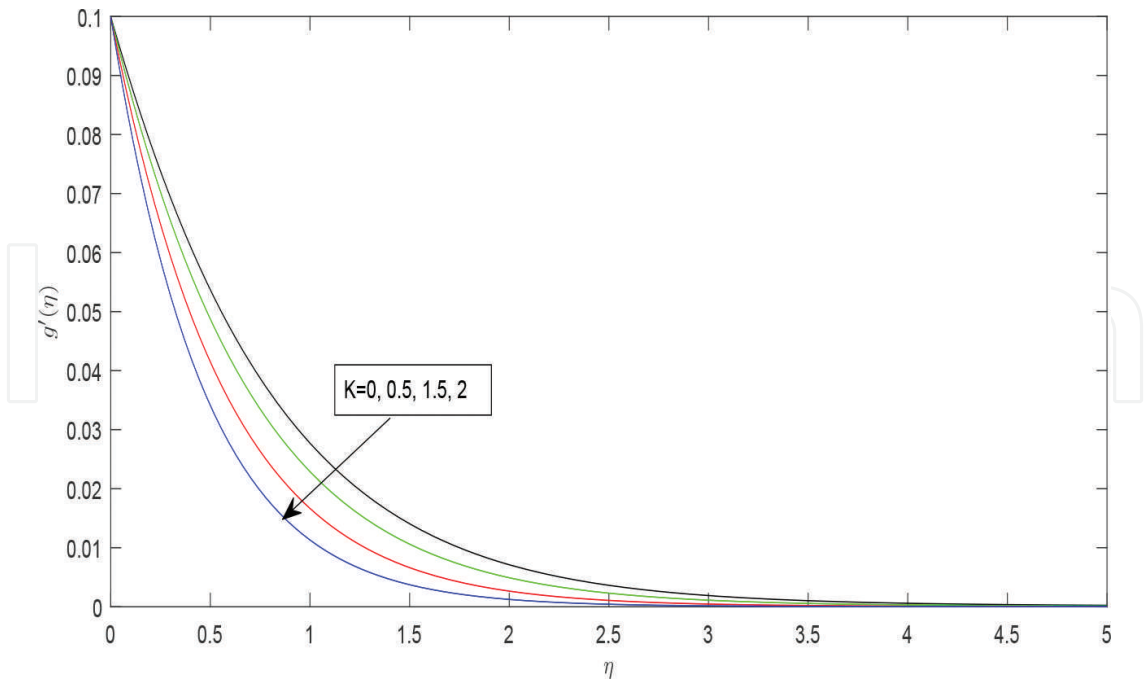


Figure 2. Effect of permeability parameter on the transverse velocity profiles.

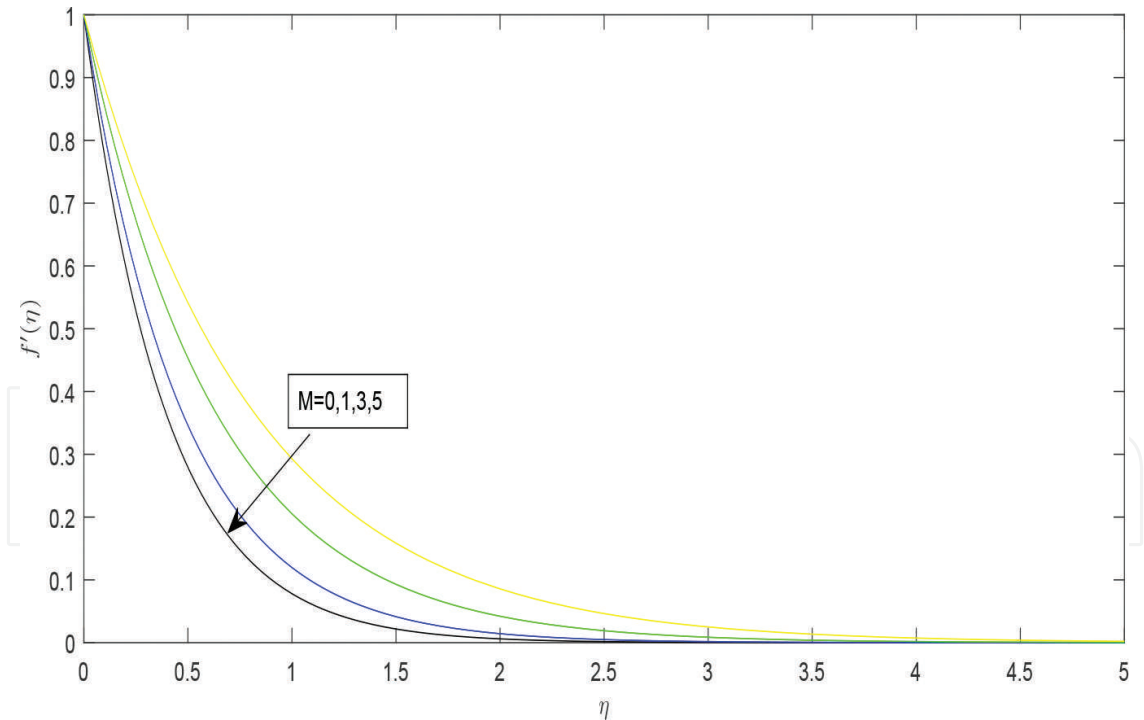


Figure 3. Influence of the magnetic parameter on the tangential velocity.

reduced with increasing values of the stretching ration parameter. **Figures 10 and 11** display the effects of thermophoresis parameter on the dimensionless temperature and concentration profiles. It is observed that the temperature and concentration profiles increase as the values of

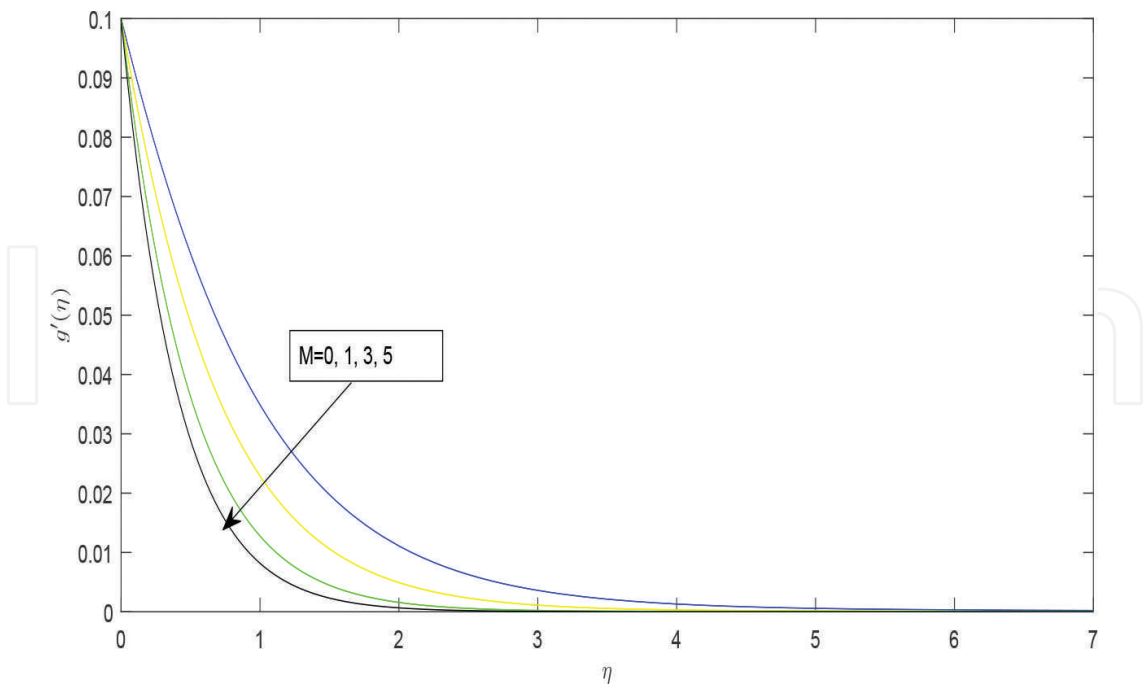


Figure 4. Influence of the magnetic parameter on the transverse velocity.

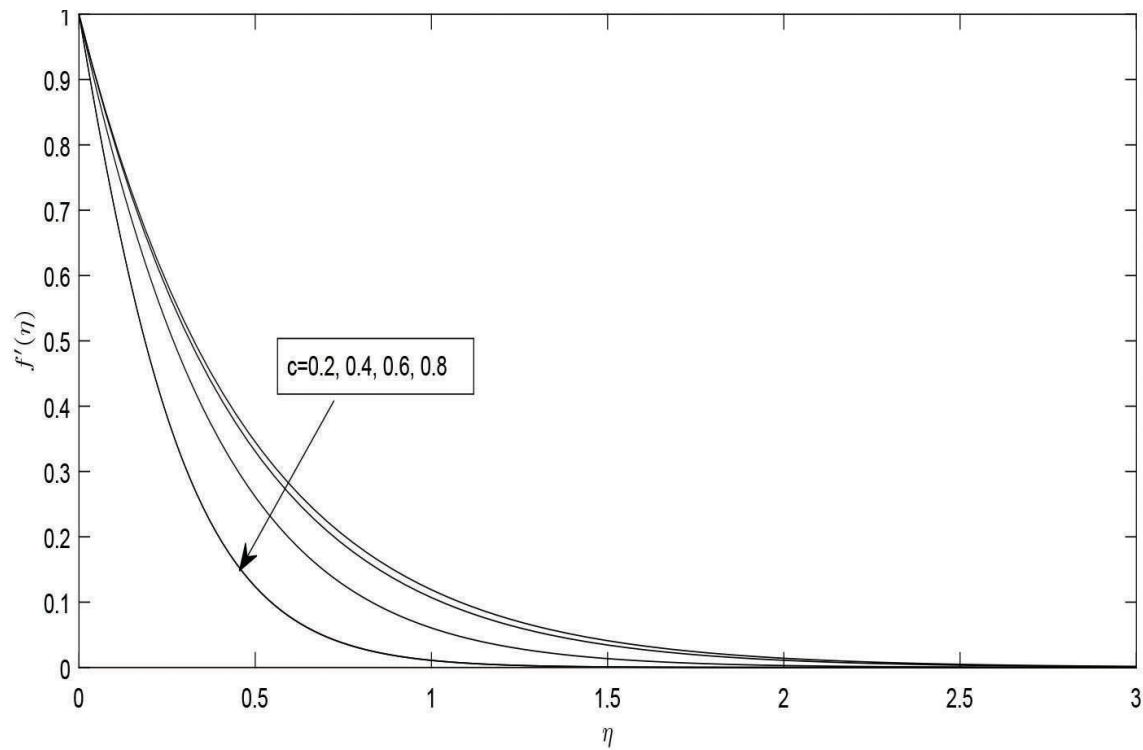


Figure 5. Influence of the stretching parameter on the tangential velocity.

the thermophoresis Nt increase. **Figure 12** depicts the influence of the Brownian motion parameter Nb on the temperature profiles. Increasing the values of the Brownian motion parameter Nb results in thickening of the thermal boundary layer, thus enhancing the

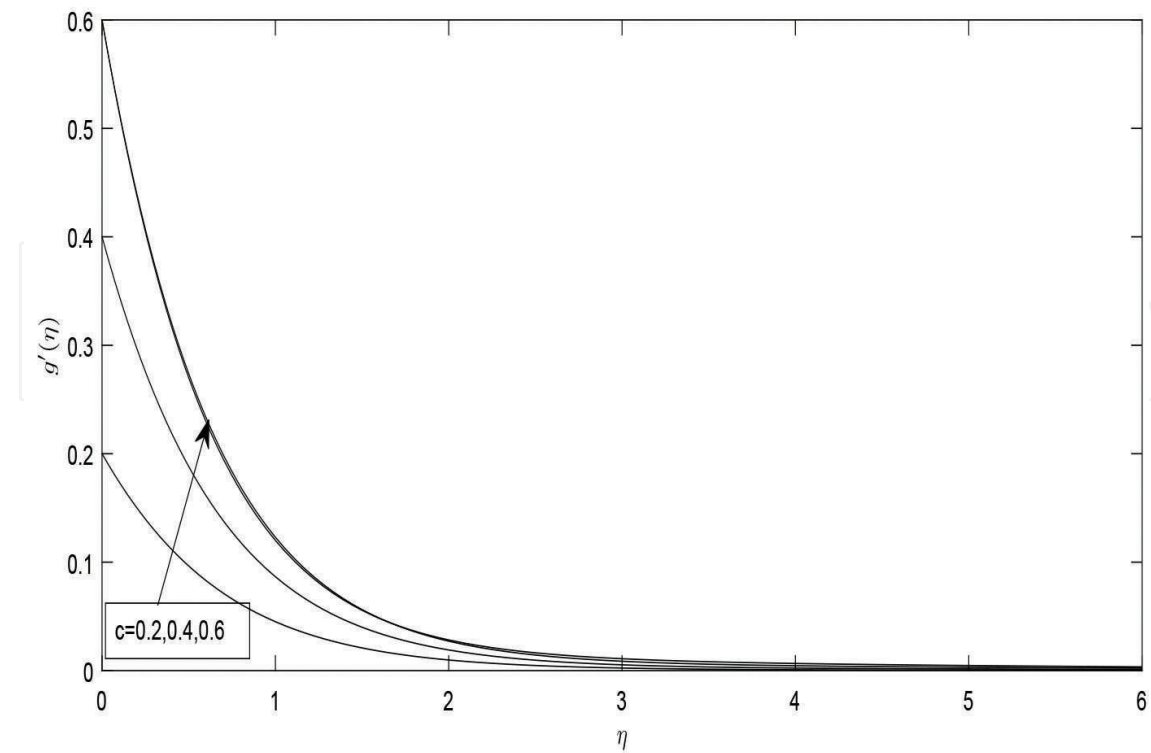


Figure 6. Variation of the stretching parameter on the velocity.

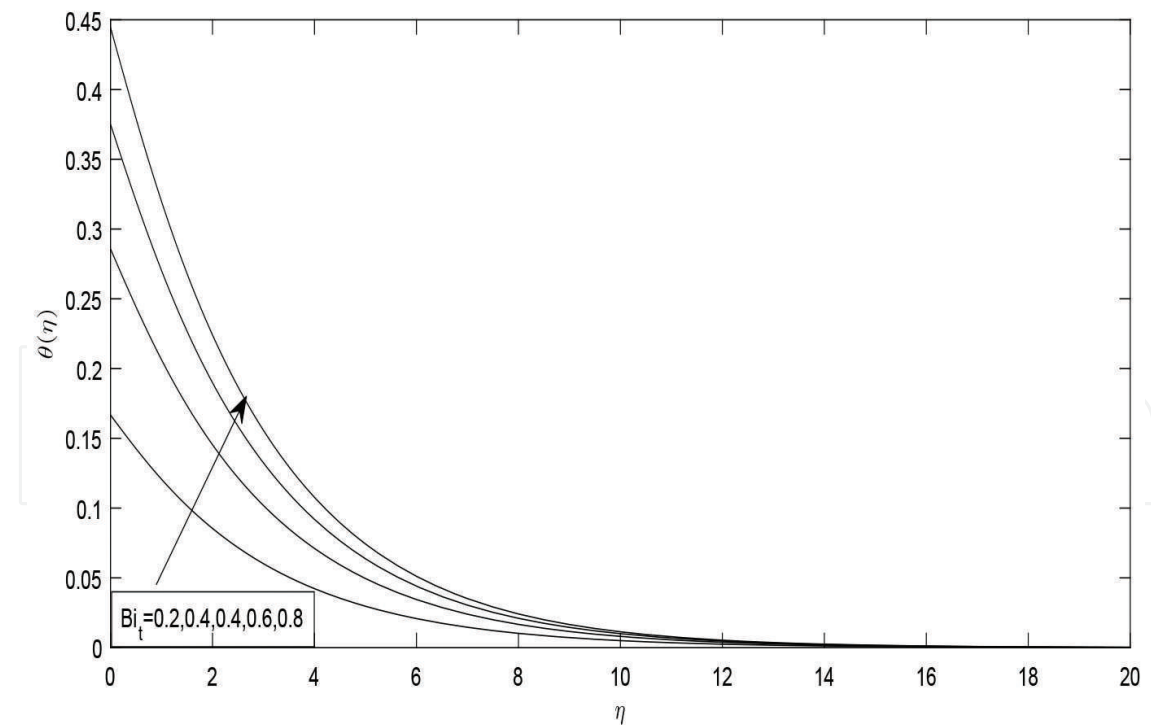


Figure 7. Effect of the Biot number on the temperature.

temperature of the nanofluid. **Figures 13 and 14** are plotted to depict the influence of the permeability K and magnetic M , parameters on the temperature profiles. The temperature of

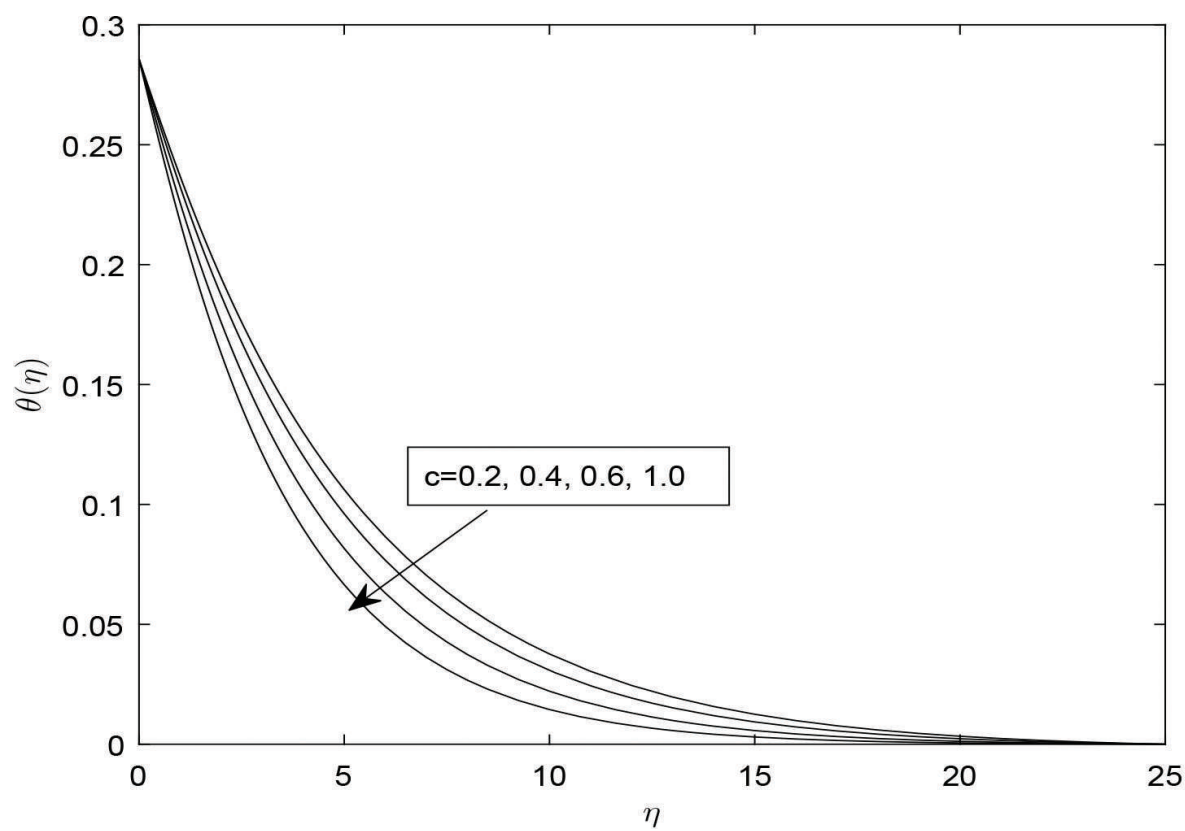


Figure 8. Effect of varying the stretching parameter on the temperature.

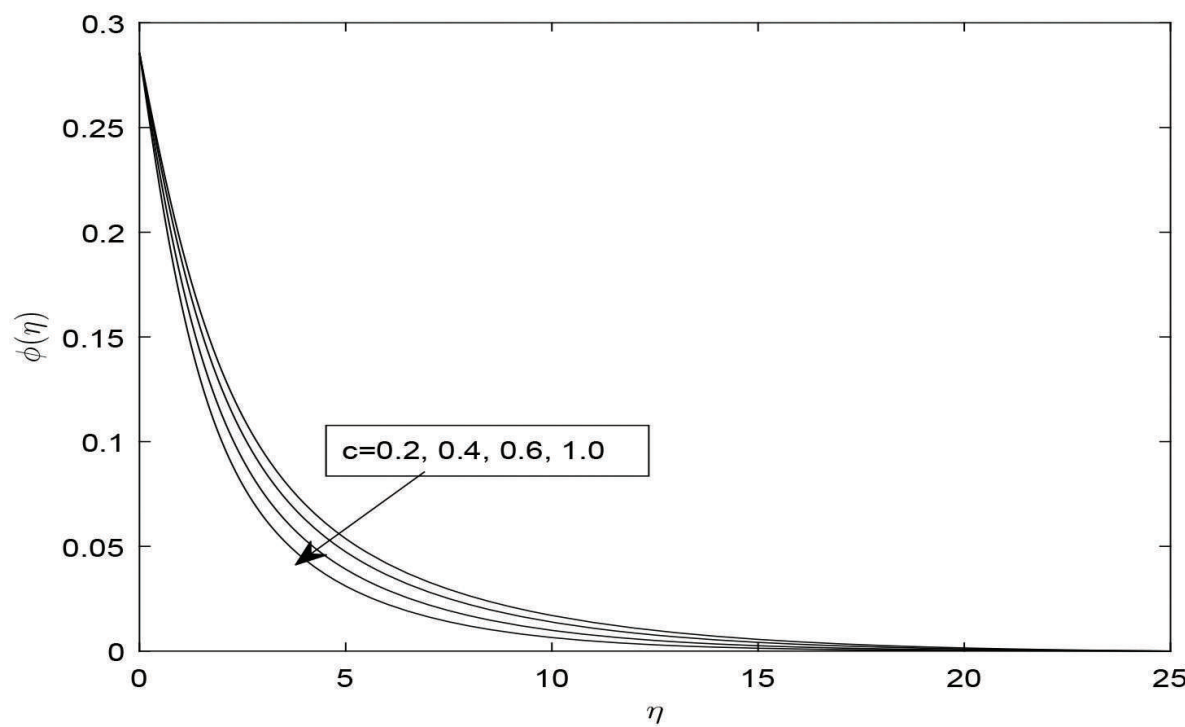


Figure 9. Effect of varying the stretching parameter on the temperature.

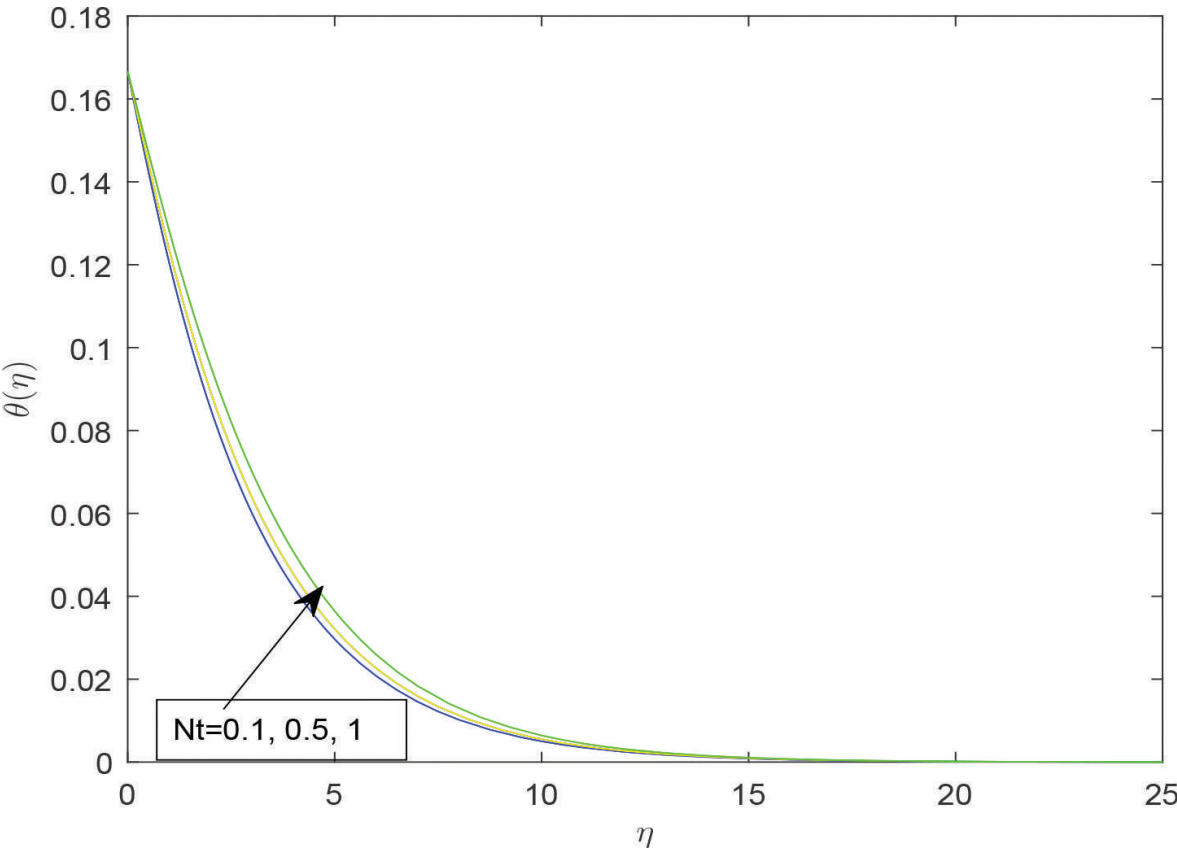


Figure 10. Effect of thermophoresis parameter on the temperature profiles.

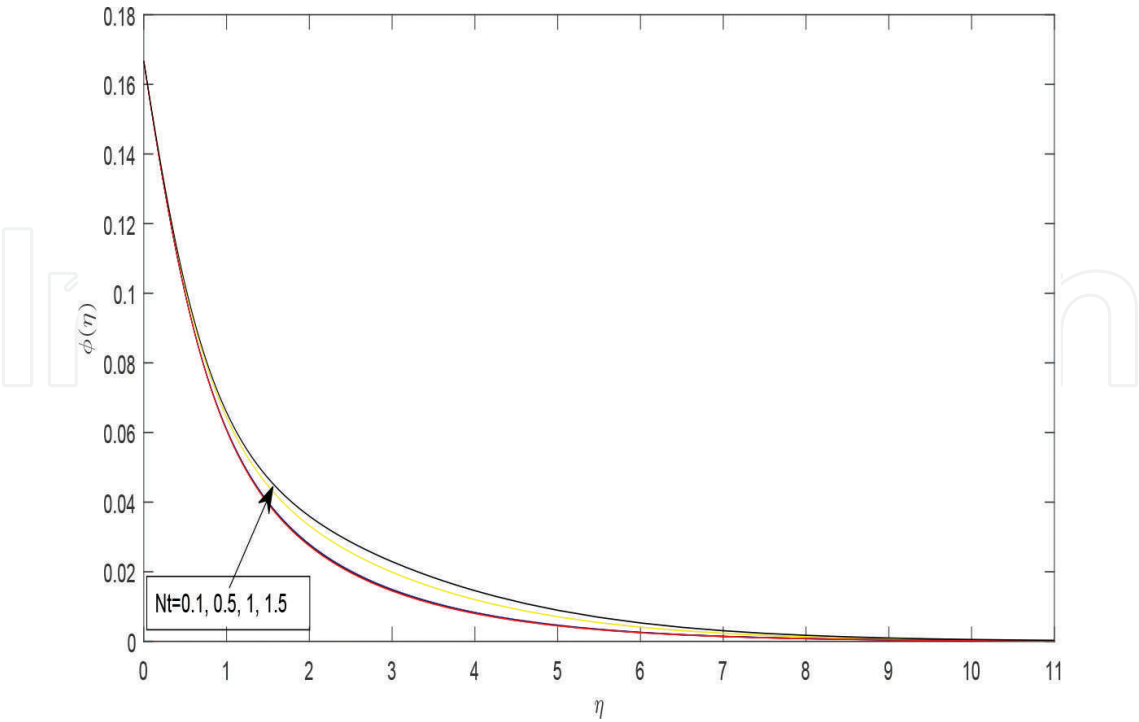


Figure 11. Effect of thermophoresis parameter on the concentration profiles.

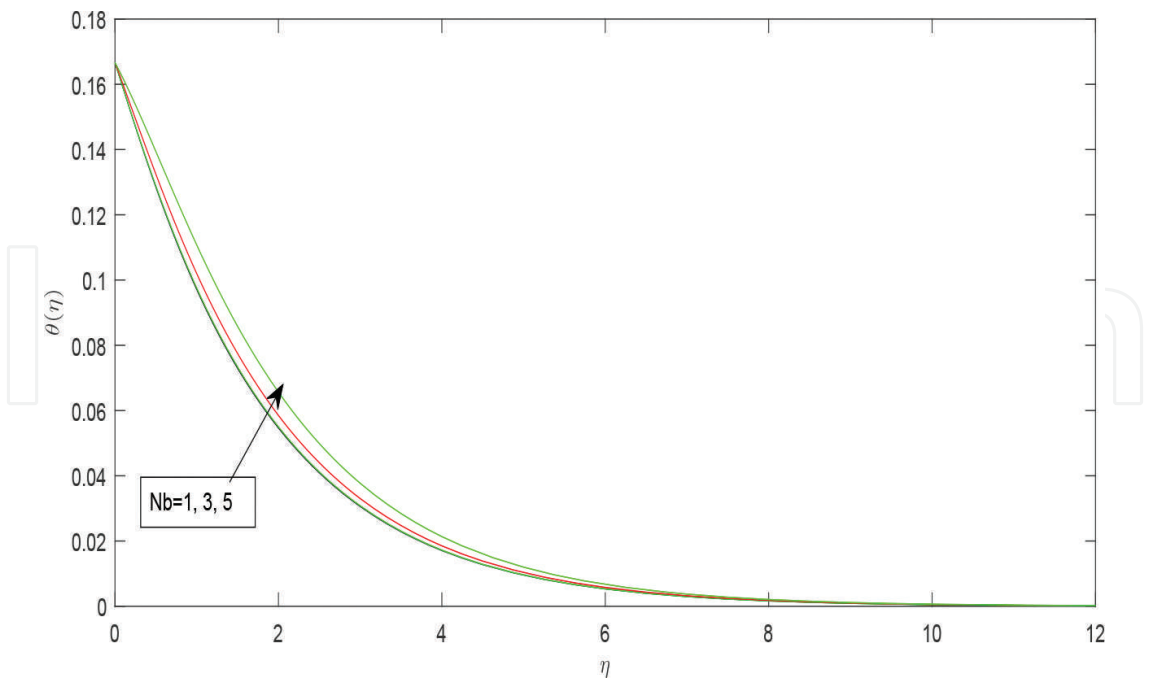


Figure 12. Effect of Brownian motion parameter on the temperature profiles.

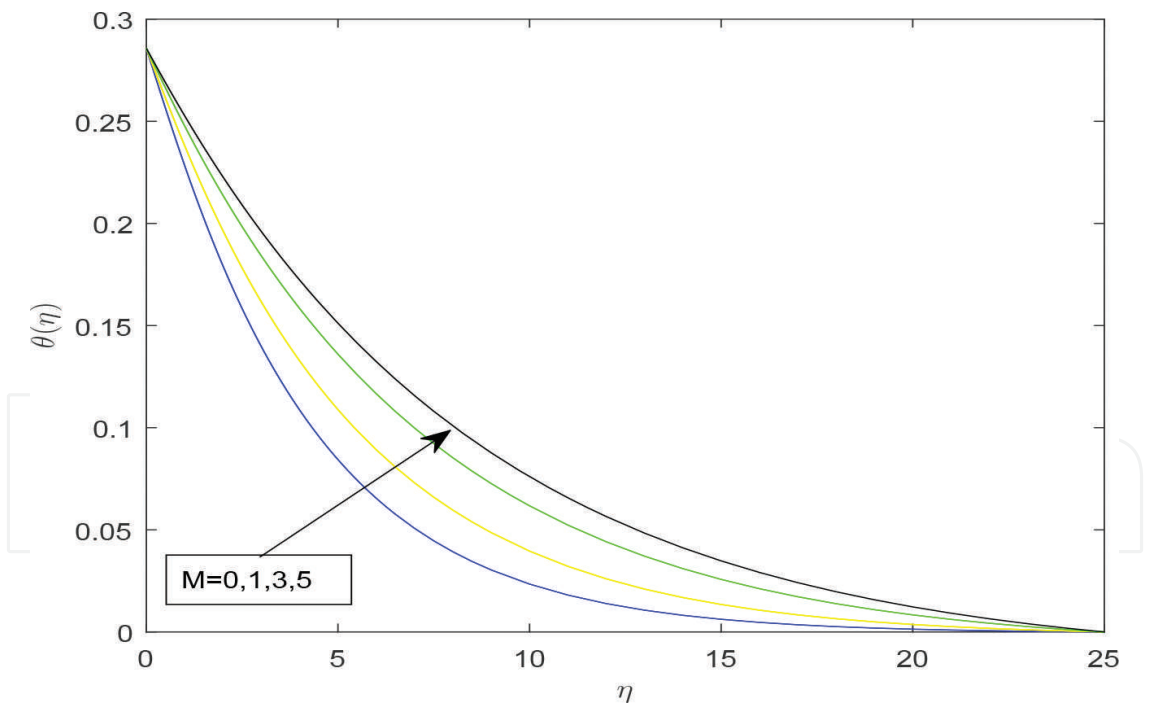


Figure 13. Effect of magnetic parameter on the temperature profiles.

the nanofluid increases with increases values of the permeability parameter. From **Figure 14**, we observe that the temperature profiles increase with the increasing values of the magnetic field parameter. **Figure 15** displays the effect of thermal radiation parameter R on the

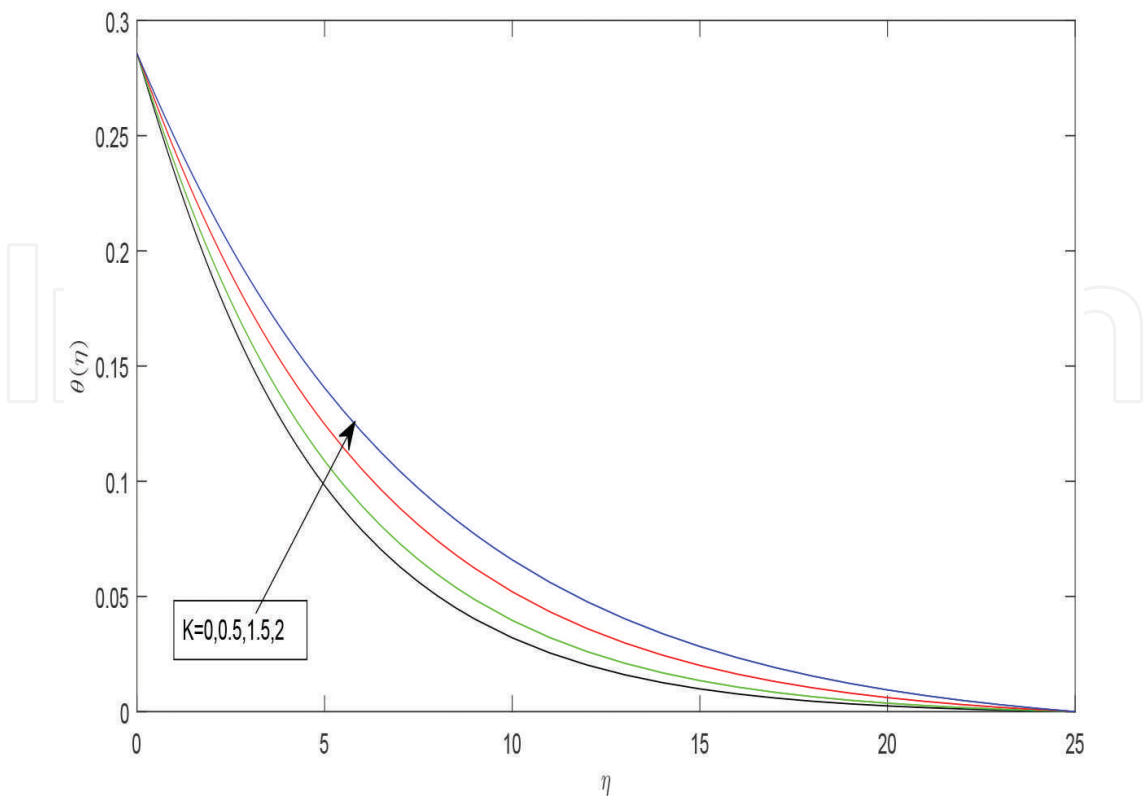


Figure 14. Influence of the permeability parameter on the nanofluid temperature.

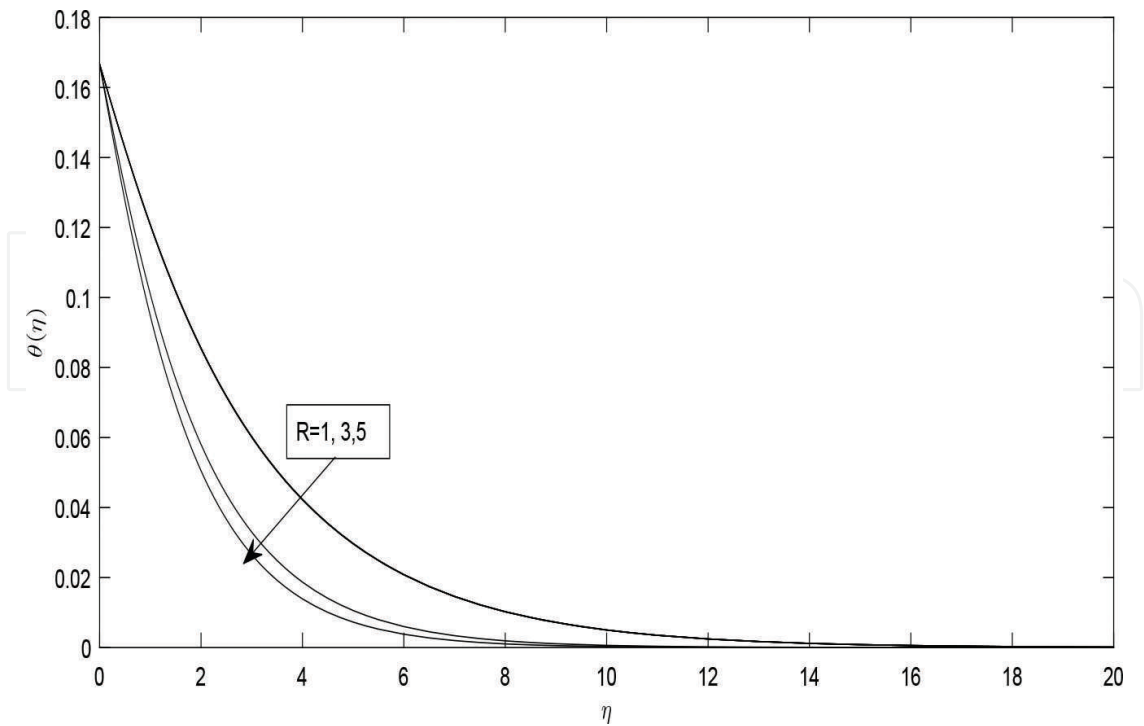


Figure 15. Influence of thermal radiation on the temperature.

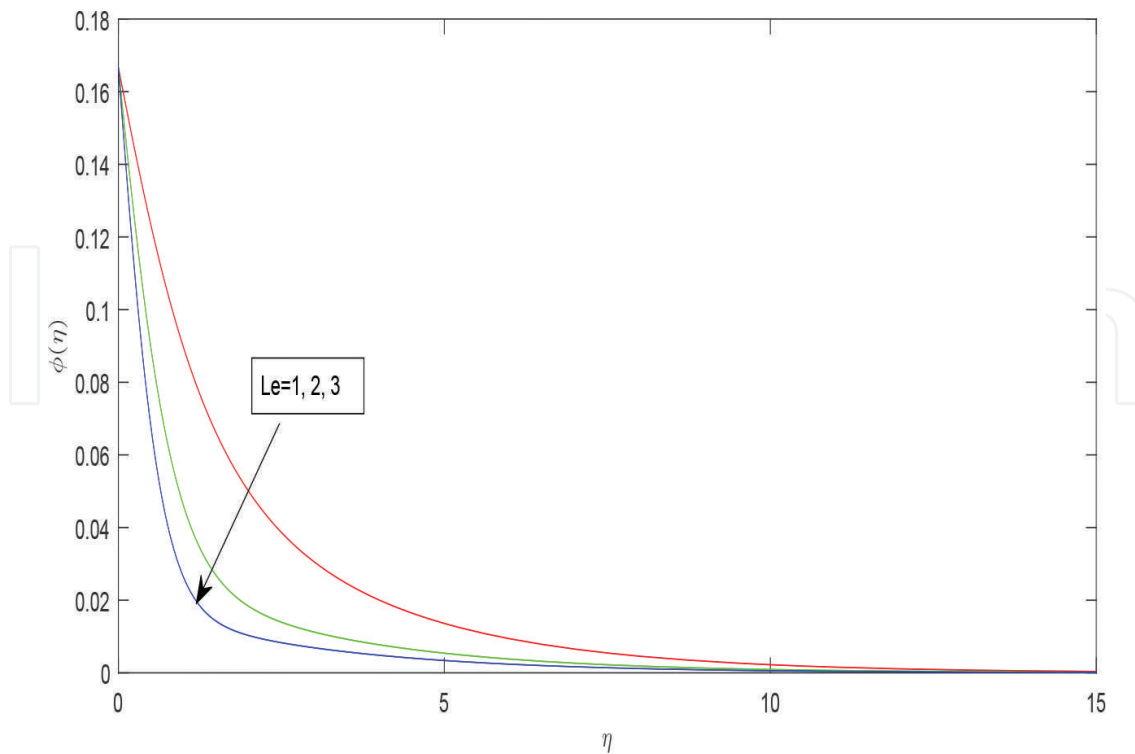


Figure 16. Influence of the Lewis number on the concentration.

temperature profiles. We observe in this figure that increasing the values of the thermal radiation produces a significant reduction in the thermal condition of the fluid flow.

Lastly, the effect of the Lewis number on the concentration profiles is depicted on **Figure 16**. Large values of the Lewis number implies increased values of the Schmidt number which results in the thinning of the solutal boundary layer.

6. Conclusion

A three-dimensional magnetohydrodynamic nanofluid, heat and mass transfer over a stretching surface with convective boundary conditions through a porous medium. The transformed governing equations are solved numerically using the spectral relaxation method. The accuracy of the SRM was validated against the MATLAB in-built bvp4c routine for solving boundary value problems. The following conclusions are driven from this study:

- The effect of increasing the magnetic field parameter is to reduce the momentum boundary layer there and to increase the thermal and solutal boundary layer thickness. The same effect on the flow characteristics is also experienced by increasing values of the stretching parameter (c).
- We observed that the local temperature rises as the Brownian motion, thermophoresis, permeability parameter and Biot numbers intensify. But opposite influences are observed when the values of the thermal radiation and stretching parameters increase.

- Increasing values of the Lewis number (Le) diminishes the concentration of the nanoparticles.
- The rise in the stretching ratio parameter increases the Nusselt and Sherwood number.
- Lastly, the Nusselt number decreases, while the Sherwood number increases as the Brownian motion and thermophoresis effects increase.

Nomenclature

a, b	positive constants
B_0	uniform magnetic field strength
B_{it}, B_{ic}	Biot numbers
c	stretching parameter
C	fluid concentration
C_f	skin friction coefficient
C_s	convective fluid concentration below the moving sheet
C_w	concentration on the wall
C_∞	free stream concentration
D_B	Brownian motion coefficient
D_T	thermopheric diffusion coefficient
f	dimensionless stream function
g	acceleration due to gravity
h_f	convective heat transfer coefficient
h_s	convective mass transfer
k_1	permeability of the porous medium
k	thermal conductivity
K	permeability parameter
Le	Lewis number
M	magnetic parameter
N_b	Brownian motion parameter
N_t	thermophoresis parameter
Nu	Nusselt number

Pr	Prandtl number
q_w, q_m	heat and mass fluxes at the surface
q_r	radiative heat flux
R	thermal radiation
Re	Reynolds number
Sh	Sherwood number
T	fluid temperature
T_f	convective fluid temperature
T_w	temperature
u, v, w	velocity components
x, y, z	Cartesian coordinates

Greek symbols

α	thermal expansion coefficient
ρ	fluid density
ν	kinematic viscosity
σ	electrical conductivity
θ	dimensionless temperature
ϕ	dimensionless concentration
τ	ratio of heat capacities
τ_{wx}, τ_{wy}	wall shears

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References

- [1] Crane, L. J. Flow past a stretching plate. *Zeitschrift für Angewandte Mathematik und Physik*. 1970; 21(4), 645–647.
- [2] Gupta, P. S., Gupta, A. S. Heat and mass transfer on a stretching sheet with suction or blowing. *The Canadian Journal of Chemical Engineering*. 1977; 55, 744–746.
- [3] Mohan Krishna, P., Sandeep, N., Sugunammai, V. Effects of radiation and chemical reaction of MHD convective flow over a permeable stretching surface suction and heat generation. *Walaliak Journal of Science and Technology*. 2015; 12, 831–847.
- [4] Sandeep, N., Sulochana, C. Dual solution for unsteady mixed convection flow of MHD micropolar fluid over a stretching/shrinking sheet with non-uniform heat source. *International Journal of Engineering Science and Technology*. 2015; 18, 1–8.
- [5] Bhattacharya, K. Boundary layer stagnation point flow of Casson fluid and heat transfer towards a shrinking/stretching sheet. *Front Heat Mass Transfer*. 2013; 4, 1–9. <http://dx.doi.org/10.5098/hmt.v4.2.3003>.
- [6] Sheikholeslami, M., Ellahi, R., Ashorynejad, H., Domairry, G., Hayat, T. Effects of heat transfer in flow of nanofluids over a permeable stretching wall in a porous medium. *Journal of Computational and Theoretical Nanoscience*. 2014; 11, 486–496.
- [7] Hayat, T., Asad, S., Alsaed, A. Flow of Casson fluid with nanoparticles. *Applied Mathematics and Mechanics*. 2016; 37(4), 459–470.
- [8] Ellahi, R., Aziz, S., Zeeshan, A. Non-Newtonian nanofluid flow through a porous medium between two coaxial cylinders with heat transfer and variable viscosity. *Journal of Porous Media*. 2013; 16, 205–216.
- [9] Buongiorno, J. Convective transport in nanofluids. *Journal of Heat Transfer*. 2006; 128, 240–250.
- [10] Zeeshan, A., Baig, M., Ellahi, R., Hayat, T. Flow of viscous nanofluid between the concentric cylinders. *Journal of Computational and Theoretical Nanoscience*. 2014; 11, 646–654.
- [11] Shateyi, S., Prakash, J., A new numerical approach for MHD laminar boundary layer flow and heat transfer of nanofluids over a moving surface in the presence of thermal radiation. *Boundary Value Problems*. 2014; 2014, 2. <http://www.boundaryvalueproblems.com/content/2014/1>.
- [12] Kuznetsov, A. V., Nield, D. A. Natural convective boundary-layer flow of a nanofluid past a vertical plate. *International Journal of Thermal Sciences*. 2010; 49, 243–247.
- [13] Aziz, A., Khan, W. A., Natural convective boundary layer flow of a nanofluid past a convectively heated vertical plate. *International Journal of Thermal Sciences*. 2012; 53, 83–90.

- [14] Makinde, O. D., Aziz, A., Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. *International Journal of Thermal Sciences*. 2011; 50, 1326–1332.
- [15] Hayat, T., Aziz, A., Muhammad, T., Ahmad, B. On magnetohydrodynamic flow of second grade nanofluid over a nonlinear stretching sheet. *Journal of Magnetism and Magnetic Materials*. 2016; 408, 99–106.
- [16] Zhao, Q., Xu, H., Tau, L., Raees, A., Sun, Q. Three-dimensional free bio-convection of nanofluid near stagnation point on general curved isothermal surface. *Applied Mathematics and Mechanics*. 2016; 37(4), 417–432.
- [17] Sheikholeslami, M., Ganji, D. D. Heated permeable stretching surface in a porous medium using nanofluids. *Journal of Applied Fluid Mechanics*. 2014; 7(3), 535–542.
- [18] Nayak, R. K., Bhattacharyya, S., Pop, I. Numerical study on mixed convection and entropy generation of Cu–water nanofluid in a differentially heated skewed enclosure. *International Journal of Heat and Mass Transfer*. 2015; 85, 620–634.
- [19] Mabood, F., Das, K. Melting heat transfer on hydromagnetic flow of a nanofluid over a stretching sheet with radiation and second-order slip. *European Physical Journal Plus*. 2016; 131, 3.
- [20] Naramgari, S., Sulochana, C. MHD flow of dusty nanofluid over a stretching surface with volume fraction of dust particles. *Ain Shams Engineering Journal*. 2016; 7, 709–716.
- [21] Sandeep, N., Sulochana, C., Kumar, B. R. Unsteady MHD radiative flow and heat transfer of a dusty nanofluid over an exponentially stretching surface. *Engineering Science and Technology, an International Journal*. 2016; 19(1), 227–240.
- [22] Sheikholeslami, M., Ashorynejad, H. R., Rana, P. Lattice Boltzmann simulation of nanofluid heat transfer enhancement and entropy generation. *Journal of Molecular Liquids*. 2016; 214, 86–95.
- [23] Sheikholeslami, M., Ganji, D. D. Nanofluid convective heat transfer using semi analytical and numerical approaches: a review. *Journal of the Taiwan Institute of Chemical Engineers*. 2016; 65, 42–77.
- [24] Sulochana, C., Ashwinkumar, G. P., Sandeep, N. Similarity solution of 3D Casson nanofluid flow over a stretching sheet with convective boundary conditions. *Journal of the Nigerian Mathematical Society*. 2016; 35, 128–141.
- [25] Motsa, S. S., Dlamini, P. G., Khumalo, M. Solving hyperchaotic systems using the spectral relaxation method. *Abstract and Applied Analysis*. 2012; doi:10.1155/2012/203461.