Chapter 4

Computer-Aided Techniques for Geometry Assurance

Andrea Corrado and Wilma Polini

Additional information is available at the end of the chapter

http://dx.doi.org/10.5772/65474

Abstract

Geometry assurance can be described as a number of activities, all contributing to minimize the effect of geometrical variation in a final product. This work aims to introduce a new type of comparison between a computer-aided technique for geometry assurance and some models proposed by the literature. In particular, the aim of this work is to solve, through a computer-aided technique, some case studies that were already solved by different methods of the literature. The different case studies that have been introduced and solved in this paper aim to highlight the potentialities and the limits in using a computer-aided technique for geometry assurance. Because this type of comparison is not present in the literature yet, this work wants to place the emphasis on the fact that till now there is not a unique approach to solve problems of geometry assurance and no approach can be defined as better than another, in terms of results.

Keywords: geometry assurance, tolerance analysis, computer-aided tolerancing, assembly, Monte Carlo simulation

1. Introduction

Manufacturing often involves geometrical variation that propagates and accumulates in assembly processes, thus giving products that do not satisfy functional, esthetical or assembly conditions. Quality problems due to the geometrical variation are often discovered late, thus involving huge cost for changes and delays in delivery time. To simulate the geometry problems allows to discover them early and, therefore, to optimize the applied tolerances and the assembly sequence.

Geometry assurance consists of all the activities that minimize the effect of geometrical variation in the final product [1]. Activities are in all the different steps of the product development.

In the concept phase, the effect of manufacturing variation is tested virtually by means of the available production data in order to analyze different alternative concepts. The robustness of the product concept and the appearance of the product are optimized, and then, they are verified by means of statistical tolerance analysis assuming a given production system. The product tolerances are allocated down to the components of the product. In the verification phase, the product and the production system are physically tested and verified. Adjustments are made to both product and production system to correct errors and prepare full production. In this phase, computer-aided inspection planning of coordinate measurement machines and scanning equipment takes place to decide inspection strategies and sensor path. In the production step, all needed adjustments are carried out on production processes. Inspection data are collected to control the production. Ebro et al. [2] states that unclear concepts or tolerances typically cause 60% of all late changes in the development of a new product. Late changes involve high costs for a company; therefore, it has a great potential to anticipate the changes in order to prevent failure, by using more robust concepts.

In this context, tolerance analysis is a key tool for product and process developers to predict the effects of inevitable part deviations on functional key characteristics (KCs) of mechanical assemblies and to assess the consequences of variation on product quality [3, 4]. Tolerance analysis is widely used in engineering design to verify the functionality, the manufacturability, and the perceived quality of a product. Assembly requirements depend on individual geometric tolerances, and it means that assembly requirements and performance vary with the constituent tolerances. The individual tolerances have their own distributions for mass production; their random assembly gives the distribution of the assembly requirement. Tolerance analysis aims to estimate the resultant variation of the assembly requirement, once defined the tolerances of the individual parts and the functional relationship between the individual tolerances and the assembly requirement [5]. If the range of the assembly requirement is set, the fraction of random assemblies that falls inside the limits refer to the total number of sampled assemblies may be estimated as a necessary step in any design evaluation. When working with tolerance analysis, the main aim is to find out the functional relationship between independent and dependent variables:

$$y = f\left(x_1, x_2, \ldots, x_n\right) \tag{1}$$

where y is the dependent variable aimed to evaluate, while x_i with $i = 1,\ldots, n$ is the list of assigned deviations (independent variables). Function f takes into account all phenomena, which may occur during the assembly process, such as assembly constraints, assembly sequence, flexibility of parts being assembled, variability of fixturing and tooling systems, and nonlinear stack-up conditions.

In the literature, various approaches for computer-aided tolerance analysis were proposed with specific advantages and disadvantages. Most of these approaches are not completely conformed to international standards for the geometric product specification (GPS) and verification, as deeply discussed in the following.

This contribution presents the quantitative assessment of a computer-aided tolerance analysis tool in comparison with established tolerance analysis methods, where the focus is laid upon rigid mechanical assemblies. This comparison is performed employing some tolerance analysis case studies of the literature.

2. State of the art

Tolerancing limits geometric deviations of parts, which inevitably are due to the manufacturing [6], to ensure the product to assemble and to function rightly [3, 7]. Tolerance analysis predicts the effects of geometric part deviations on assembly requirements, that is, "the objective of tolerance analysis is to check the extent and the nature of the variation of an analyzed dimension or geometric feature of interest for a given Geometric Dimensioning and Tolerancing (GD&T) scheme" [8] (see **Figure 1**).

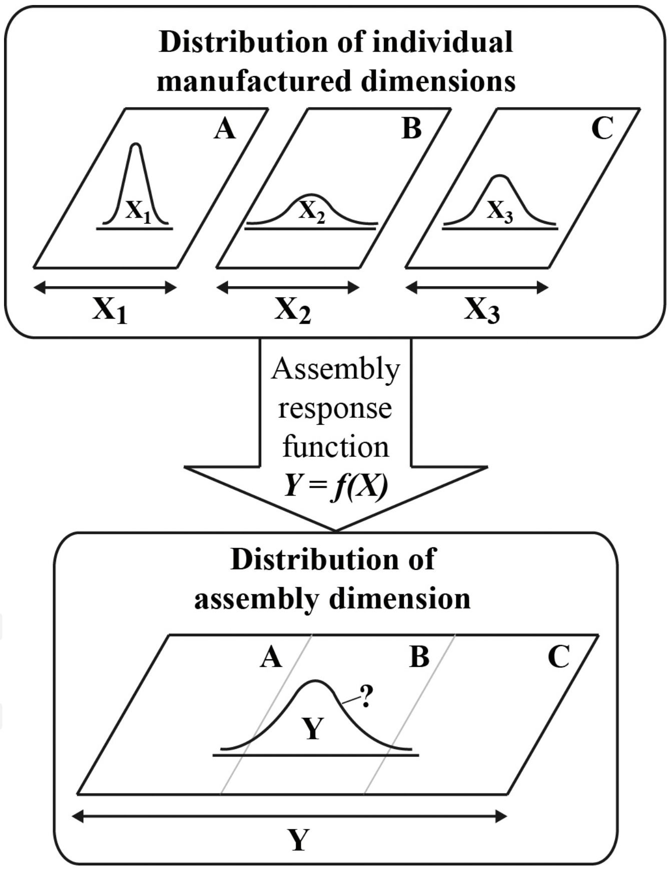

Figure 1. Scheme of tolerance analysis for mechanical assemblies.

In the literature, the three main issues on tolerance analysis are to represent geometric deviations, geometric specifications, and geometric requirements by means of mathematical relationships, to model the effects of these geometric deviations on the assembly and the product working, and to solve these models by means of worst-case or statistical techniques

[9]. The worst-case model was first introduced by Fortini [10]. It considers all tolerances of the chain to assume simultaneously the extreme values, and therefore, it describes the worst assembly. The worst-case model has a mathematical nature, so it may be applied for low production volume and for short tolerance chain. Fortini [10] also introduced statistical models to perform tolerance stack-up functions. The most simple of the statistical models is the root sum square (RSS): it is a linear variation approach and it assumes Gaussian probability density function for each tolerance of the assembly. The RSS model is very optimistic: it is applied to high volume production and to long tolerance chains. Another statistical model is the Monte Carlo simulation: it is a nonlinear variation approach that was applied to tolerance analysis for the first time by Knappe [11]. Shan et al. [12] use a Monte Carlo approach to perform 3D statistical tolerance analysis. In the last decades, various models have been proposed for tolerance analysis, which can be divided into deviation accumulation methods that express the functional requirements in terms of geometric part deviations, and tolerance accumulation approaches that express the tolerance zones to analyze as subsets of multidimensional spaces [3]. For these two categories, several models exist in the literature: parametric tolerance analysis [13], simple tolerance stacks [8], solid offsets [14], vector loops [15], direct linearization method [16], small displacement torsor [17, 18], tolerance-maps® [19], deviation domains [20], and polytopes [21].

Furthermore, a considerable number of review papers highlighting the similarities and differences of the aforementioned approaches exist, such as [8, 13, 19, 22–25]. Most of the proposed tolerance analysis methods of the literature are not completely conformed to geometric product specification (GPS) standards and are not able to take into account the combination of 3D tolerance zones, the envelope and the independence principles, the material condition modifiers, and the datum precedence. Moreover, these methods are not able to handle point cloud representation of the variant parts. Computer-aided technologies (CAT) for tolerance analysis based on point cloud representation is highly desirable, since assembly and measurement tasks give point clouds as output. Therefore, to deal with point clouds allows to connect design, manufacturing, and inspection in a unique way. Recently, a new model for tolerance analysis, based on the representation of nonideal workpieces employing point clouds, has been presented, which is known as skin model [26, 27].

3. Computer-aided techniques for geometry assurance

Many are the computer-aided techniques (CATs) for tolerance analysis that allows to assure the geometry of the assembly product by analyzing the geometrical interferences. These tolerance analysis software packages are based on the approaches cited in the previous paragraph [8, 28], such as MECAMaster®, which is based on the SDT, 3DCS®, VSA®, and CeTol®, which use parametric approaches (CeTol® used vector loops in former versions), and PolitoCAT®, which employs polytopes. They efficiently deal with simple geometrical feature of mechanical assemblies, such as plane and hole-pin, while they hardly treat of sculptured surfaces to connect. Moreover, they are not completely true to the GPS standards. To use these

http://dx.doi.org/10.5772/65474

tools, the user should understand the packages' theoretical base of modeling to build a valid model and to have accurate results.

Requirements	Computer-aided techniques for tolerance analysis		
	CeTol®	VSA®	3DCS®
Tolerancing scheme			
Dimensional tolerances	Yes	Yes	Yes
Geometric product specification (GPS)	Yes	Yes	Yes
Automatic utilization of CAD model, once defined GPS data	No	No	No
Tolerance analysis			
Worst-case approach	Yes	Yes	Yes
Statistical approach	Yes	Yes	Yes
Sensitivity analysis	Yes	Yes	Yes
Uncertainty qualification methods			
Monte Carlo	Yes	Yes	Yes
Simplifying assumptions			
Rigid body	Yes	Yes	Partial
Limit on variation size	No	No	No
Further considerations			
Compatible CAD tools	SolidWorks, Pro/E	CATIA, NX I-deas, Pro/E, NX	CATIA, Unigraphics, STEP, IGES
Distributed/parallel computing	No	No	No
Integration with external CAE modeling tools	No	No	No
Accommodation of assembly loads	No	No	Limited

Table 1. Comparison among computer-aided techniques for tolerance analysis.

A number of comparative review and survey works of computer-aided techniques for tolerance analysis were presented in the literature. Initial works focused on the limitations of the two-dimensional geometry capabilities of the contemporary systems [29]. Other review works focused on CATs for tolerance analysis developed from a research perspective [23]. A number of recent reviews offer a more detailed overview of the nature of contemporary commercial CATs for tolerance analysis [13, 28]. A particularly comprehensive review of some of the most popular CATs for tolerance analysis has noted a number of common capabilities as well as shortcomings [28]. Additional investigation carried out in association with this dissertation has identified changes in the CAT capabilities, since the previously published reviews. Furthermore, current limits associated with uncertainty quantification method capabilities and accommodations of assemblies under loading have been identified. **Table 1** summarizes the results of three CATs for tolerance analysis. **Table 1** shows how the CATs for tolerance analysis may adopt a "tolerance scheme" that models only dimensional tolerances, GPS, but they do not automatically extract tolerance information by model, even if it has been specified in Computer Aided Design (CAD) environment. Moreover, CATs for tolerance analysis may use different approaches for "tolerance analysis": worst-case and/or statistical one. They may carry out a sensitivity analysis to identify the tolerances mainly affecting assembly requirements. They may use Monte Carlo simulation, they are based on rigid body assumption, and they do not have any limits about size variation.

Despite the extensive capabilities of commercial CATs for tolerance analysis, some notable limits remain (see the last three lines of **Table 1**):

1. GPS data defined in the CAD model may not be automatically imported into the CAT due to the limits of the CAD geometry translator standards, such as Standard for The Exchange of Product model data (STEP) or Initial Graphics Exchange Specification (IGES);

2. none of the currently available tools offer distributed/parallel computing capabilities, which can offer reduced analysis times by distributing simulations over multiple computers;

3. lack of ability to accommodate general tolerance analysis and synthesis problems involving assemblies under loading. CAT tools have been identified that accommodate a limited subset of physical phenomena, such as deformation of sheet assemblies. However, in general, the abstracted geometric model employed in current CAT systems becomes incompatible when dealing with tolerance analysis and synthesis involving a general class of problem requiring the numeric simulation of assemblies under loading conducted on Computer Aided Engineering (CAE) models (such as FEA, CFD, or multi-body dynamics simulations).

This paper considers a typical CAT for tolerance analysis, called VSA® of Siemens; it involves a set of nine steps (see **Figure 2**):

1. Define CAD models for each part of the product assembly.

2. Import CAD model into CAT environment and interactively create tolerance modeling geometry superimposed on the original CAD data.

3. Specify tolerance types for features of interest on each part in the assembly.

4. Define part relationships that constitute the assembly, such as assembly sequence and mating conditions.

5. Specify Key Product Characteristics (KPC), e.g., assembly clearances, which must be satisfied in order to fulfill design requirements.

6. Simulate the effect of part tolerances on Key Product Characteristics using a stochastic or worst-case tolerance analysis approach.

7. Record outcomes such as yield and associated tolerance cost.

8. Possible sensitivity analysis to determine the most influential part tolerances contributing to variation in assembly Key Product Characteristic.

9. Subsequently, based on analysis outcomes and CAT tool capabilities, reallocate part feature tolerances to target the total allowable variation in Key Product Characteristics. The allocation may be achieved manually, or automated through tolerance synthesis aimed at maximizing yield and/or minimizing tolerance cost.

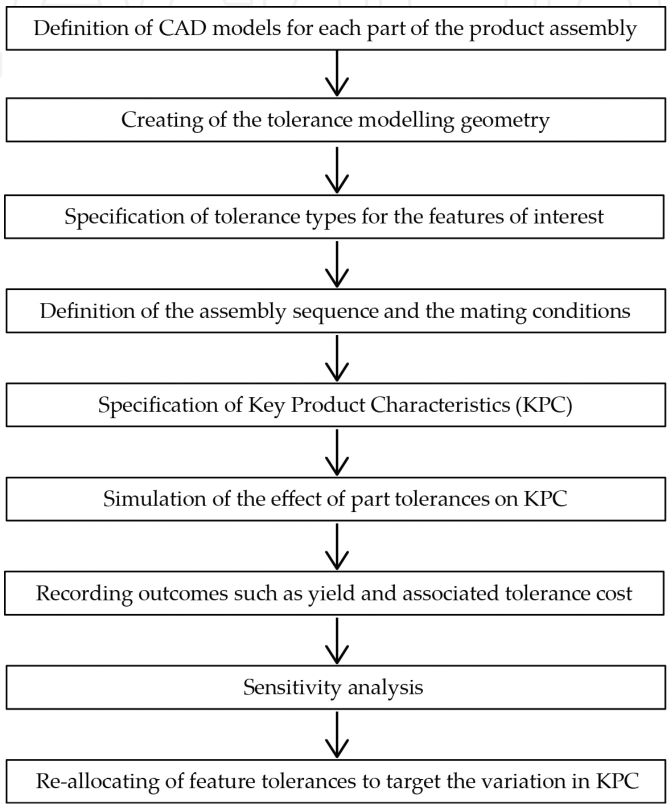

Figure 2. Flow chart of a computer-aided technique for tolerance analysis.

The present work gives the application of the VSA® software to four case studies from the literature. The four case studies have been chosen since they were used in the literature to apply five important approaches for tolerance analysis: the tolerance charting, the Jacobian model, the vector loop model, the proprietary software, and the skin model. The results due to the application of the VSA® software have been compared with those of the literature for each case study, as deeply described in the following.

4. Case study 1: R-A assembly

The first case study is the R-A assembly (see **Figure 3**); it consists of two nominally parallel shafts (Item 3) mounted into a housing (Item 1). During assembly, the bushings (Item 2) have a slight interference fit with the holes of the housing and a small amount of clearance with the shafts in order to allow the shafts to rotate. Retaining rings (Item 4) do not slide the shaft out of the housing along the axial direction. This assembly is simple, but it represents many common products in industry, such as blowers, gear boxes, and pumps.

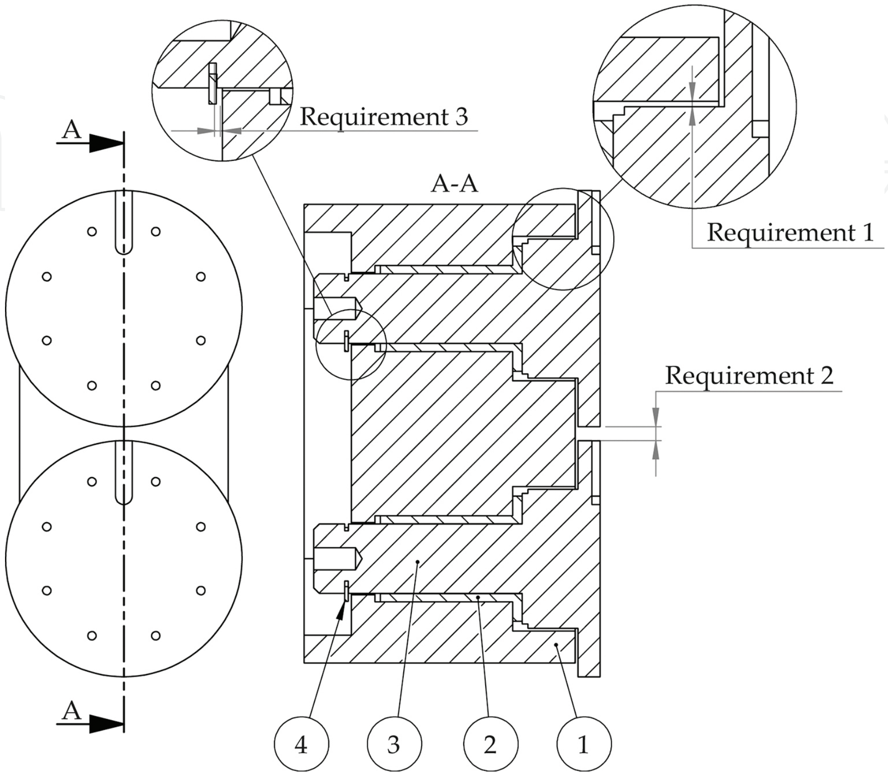

Figure 3. First case study: R-A assembly.

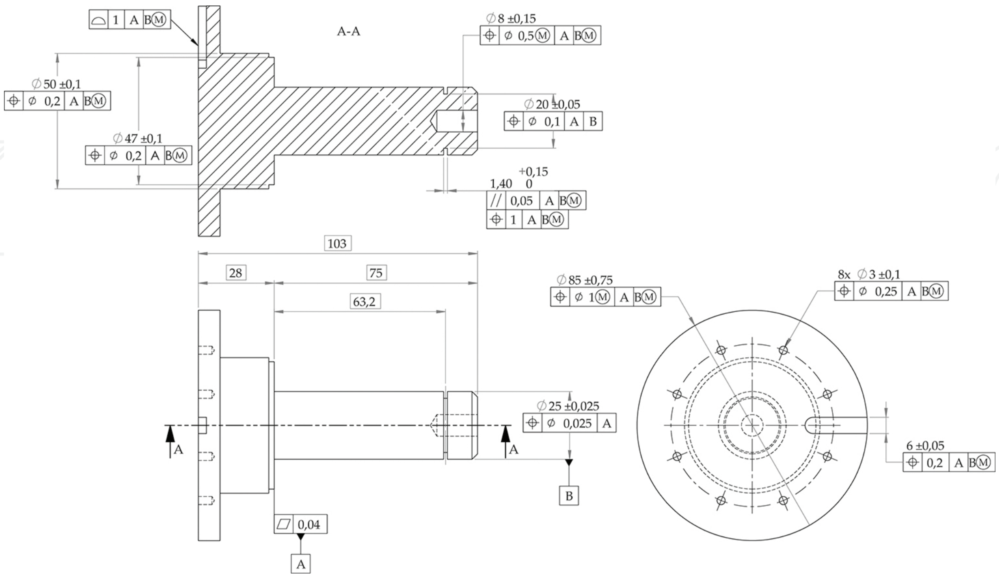

Figure 4. R-A assembly: shaft drawing.

Drawings for the shaft, bushing, and housing are shown in **Figures 4–6**. These parts are dimensioned and toleranced using GPS, and the GPS scheme reflects the functional requirements of the parts and the assembly. Three tolerance stack-ups are included in the following material: two radial tolerance stack-ups and one axial tolerance stack-up.

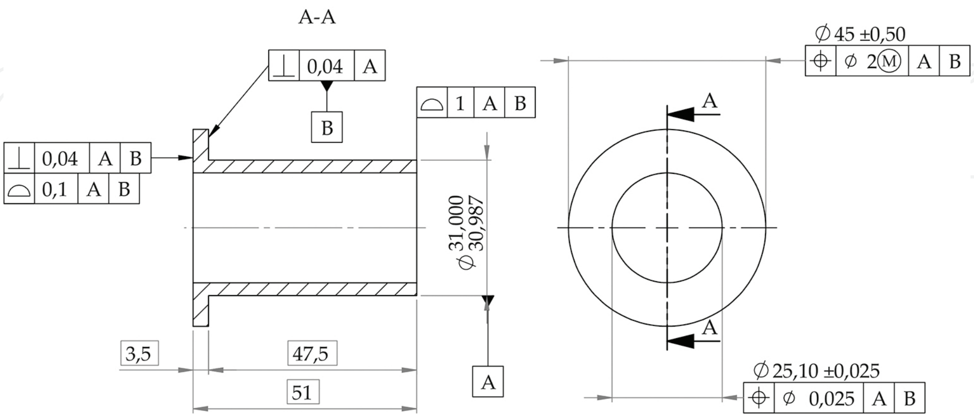

Figure 5. R-A assembly: bushing drawing.

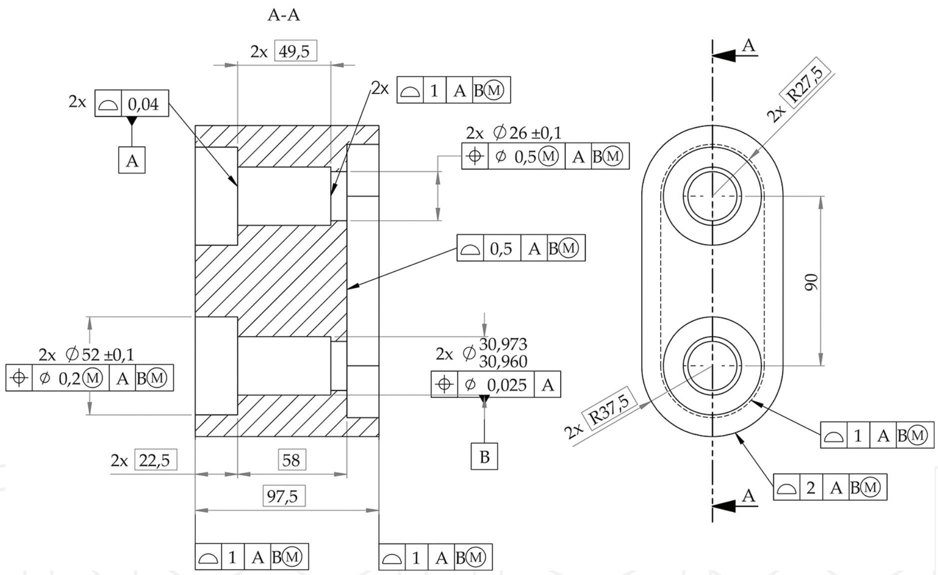

Figure 6. R-A assembly: housing drawing.

It is assumed that the inside diameter (ID) of the bushing does not deform when its outside diameter (OD) is pressed into the housing. The ID of the bushings continues to satisfy the dimension and the tolerance specifications shown in **Figure 5**. This is possible by machining the ID of the bushings to their stated diameter and tolerance after installation. The functional requirements to check are shown in **Figure 3**.

The tolerance charting approach has been applied to this case study in Ref. [30] by using a root sum squared method without or with a multiplier factor. The VSA® software of Siemens has been used in this work. The results are shown in **Table 2**. It can be observed that the results due to VSA software stay always between those due to the two tolerance charting methods.

Approach		**[mm]**	**Requirement 1**	**Requirement 2**	**Requirement 3**
Tolerance charting [30]	RSS	μ	1.00	5.00	0.96
		σ	0.07	0.30	0.19
	RSSx multiplier	μ	1.00	5.00	0.96
		σ	0.10	0.45	0.28
VSA® software		μ	1.01	5.02	1.09
		σ	0.07	0.41	0.23
Differences	(VSA®-tolerance charting)/tolerance charting	$\Delta\mu$	+1.14%	+0.41%	+13.72%
		$\Delta\sigma$	+7.64 ÷ −28.25%	+37.90 ÷ −8.07%	+18.82 ÷ −20.78%

Table 2. Results of the first case study.

5. Case study 2: an assembly of three blocks

The second case study is an assembly that contains three parts. The functional requirement is the gap shown on the leftmost side of the assembly (see **Figure 7**). Two analytical approaches that are the Jacobian and the vector loop have been applied to this case study in Ref. [31], and the obtained results are given in the first two lines of **Table 3**.

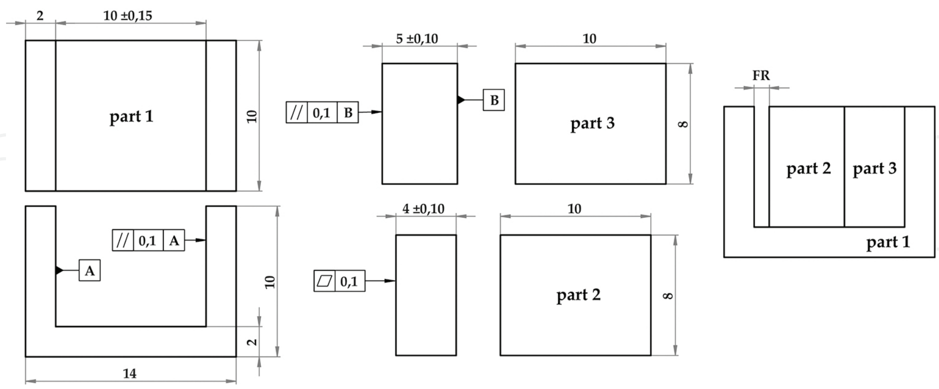

Figure 7. Second case study.

The application of the VSA® software to the case study is required to calculate five measurement operations, as shown in **Figure 8**, to obtain the functional requirement that is the distance between Blocks 1 and 2, once assembled. The results about the mean and the standard deviation of the five measurements are shown in **Table 4**; then, the mean values of the means and the standard deviations have been calculated and reported in the last line of **Table 4** and in the third line of **Table 3**. It can be observed that the values due to VSA® software are lower than those due to the two analytical models. However, some interesting considerations may be made by comparing the values due to the dimensional tolerances only with those due to the dimensional and the geometrical tolerances together. The two analytical approaches give a value of the standard deviation that increases when the geometrical tolerances are applied together with the dimensional ones, whereas it should decrease, since some further constraints are added to the part. This is probably due to the fact that the two analytical approaches add the contribution of geometrical tolerances to that of dimensional tolerances. VSA® software instead gives the same result with and without the geometrical tolerance, and this is probably due to the fact that it is not able to deal with the form tolerances applied to the planes of the components.

Approaches	[mm]	Dimensional tolerances	Dimensional and geometric tolerances
Jacobian [31]	μ	1	1
	σ	0.069	0.073
Vector loop [31]	μ	1	1
	σ	0.069	0.075
VSA® software	μ	0.891	0.956
	σ	0.066	0.060
(VSA®-Jacobian)/Jacobian	$\Delta\mu$	−10.85%	−4.45%
	$\Delta\sigma$	−4.13%	−16.83%
(VSA®-vector loop)/vector loop	$\Delta\mu$	−10.85%	−4.45%
	$\Delta\sigma$	−4.13%	−18.94%

Table 3. Results of the second case study.

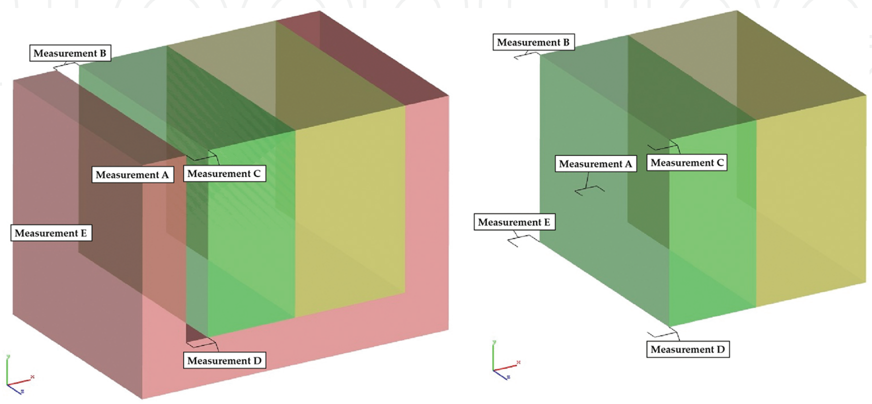

Figure 8. Measurements required by VSA® software.

Measurements	Dimensional tolerances		Dimensional and geometric tolerances	
	μ [mm]	σ [mm]	μ [mm]	σ [mm]
A	0.891	0.050	0.956	0.053
B	0.886	0.066	0.953	0.061
C	0.898	0.073	0.958	0.064
D	0.898	0.074	0.958	0.064
E	0.885	0.067	0.953	0.062
Mean value	0.891	0.066	0.956	0.060

Table 4. Results of measurements of VSA software.

6. Case study 3: three-part assembly

The third case study is a three-part assembly, as shown in **Figures 9** and **10**. The aim is to analyze the minimum distance "D_f" and the angle "A_f", between the pin of the part C and the hole of the part A. Mate conditions have been established between part A and part B ("mate1," "mate2," and "mate3"). The pin/hole joint has been modeled through a contact constraint ("contact1")—no penetration allowed. Moreover, a "minimum distance" contact constraint ("contact2") has also been defined to assure part C and part B was close as much as possible to each other. Tolerances were defined for each part according to **Figure 9**. Each tolerance has been modeled with a statistical normal distribution (natural tolerance range = 6). Monte Carlo method has been used to generate random variational features (number of simulation = 1000). The following assembly sequence has been assigned: mate1 + mate2 + mate3 + mate4 + contact1 + contact2. The Monte Carlo simulation has been carried out by means of a proprietary

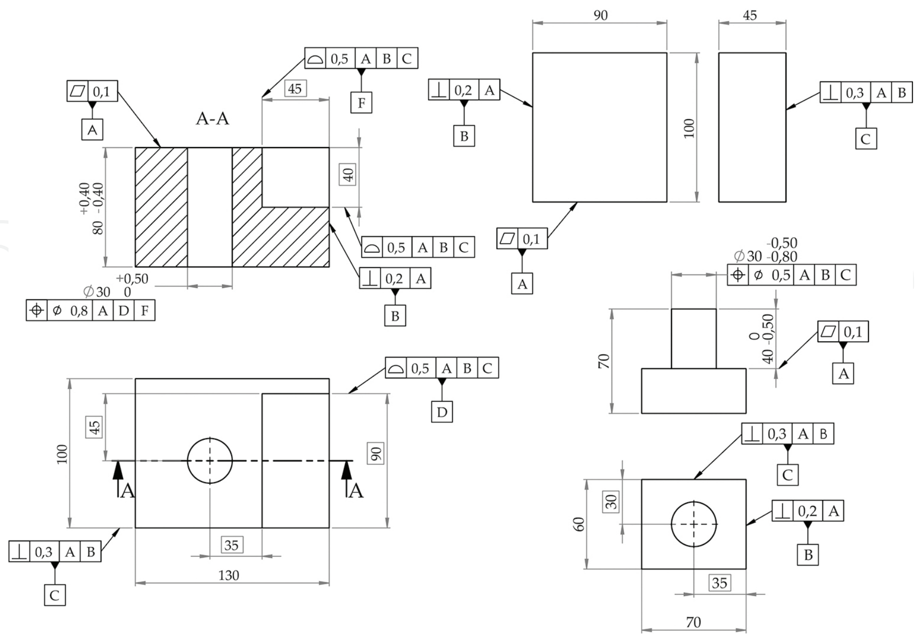

Figure 9. Tolerance specification of the third case study.

software presented in Ref. [32] in the literature, and the obtained results are reported in the first two lines of **Table 5**, respectively, for the two functional requirements. As expected, the minimum value of the functional requirement is right zero (see **Table 5**), since assembly features cannot penetrate each other. Moreover, the maximum value is about 0.7 mm, which is lower than the maximum radial gap (1.3 mm) between pin and hole features.

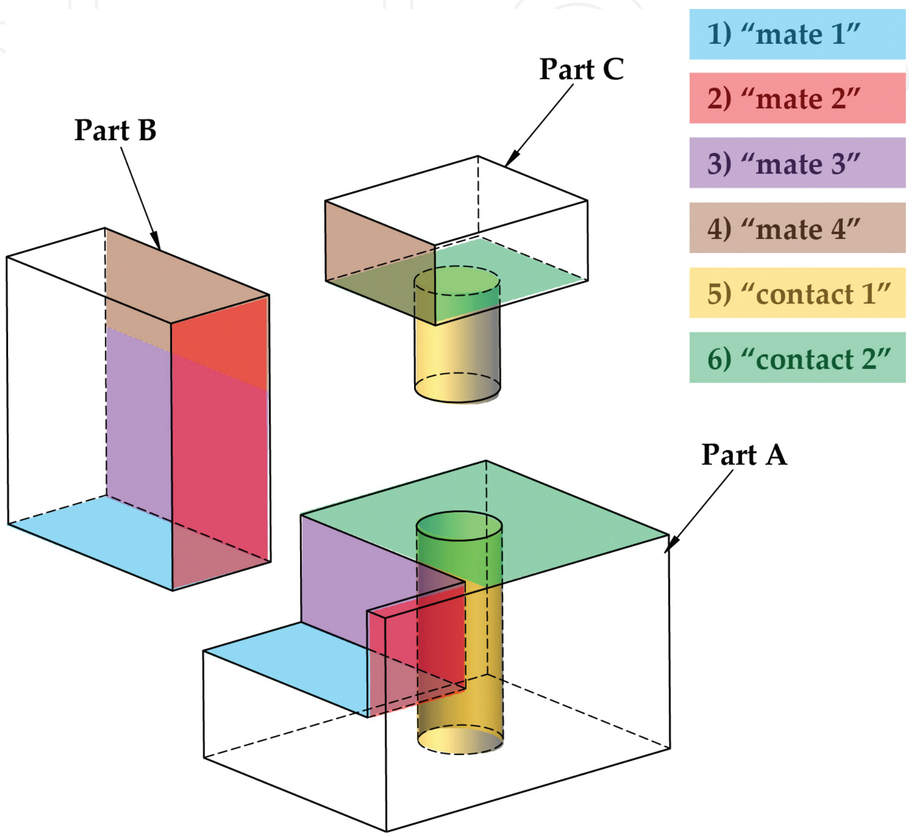

Figure 10. Assembly features of the components belonging to the third case study.

Approaches	Values	A_f [°]	D_f [mm]
Proprietary software [32]	μ	179.36	0.35
	σ	0.41	0.16
	Max	179.99	0.77
	Min	177.91	0.00
VSA® software	μ	179.99	0.41
	σ	0.32	0.13
	Max	180.94	1.06
	Min	179.04	0.06
(VSA®-proprietary software)/proprietary software	$\Delta\mu$	+0.35%	+17.14%
	$\Delta\sigma$	−21.95%	−18.75%

Table 5. Results of the third case study.

The VSA® software has been applied to the case study by using the same assembly sequence and the same normal distribution of the applied tolerances. The obtained results are shown in the last two lines of **Table 5**. The proprietary and the commercial software packages give different results for the two functional requirements. This is probably due to the different ways the two software packages simulate the assembly constraints.

7. Case study 4: two flat plates and a cube

The fourth case study is an assembly comprising two flat plates and a cube, as shown in **Figure 11**. It has been used to illustrate the skin model shapes approach in Ref. [27]. It is needed to evaluate the seven distances between the top and the bottom plates and the tilt angles between the point pairs AC-A′C′ (α) and AE-A′E′ (β). These are the key characteristics (KCs). The skin model shape of the cube is generated by a Gaussian probability density function with $\mu = 0.00$ mm, $\sigma = 0.01$ mm, correlation length: $\rho = 5.00$ mm.

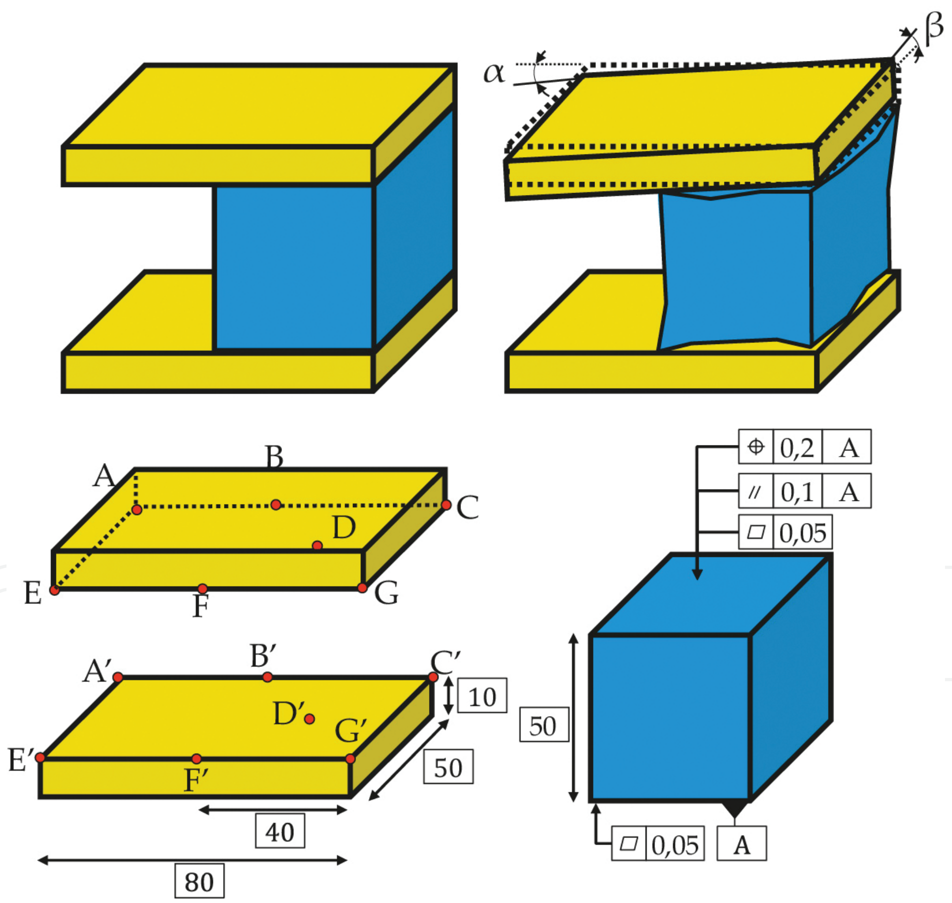

Figure 11. Fourth case study.

The KCs results of the example for 1000 skin model shapes are given in the first two lines of **Table 6**.

http://dx.doi.org/10.5772/65474

Approach	[mm]	AA′	BB′	CC′	GG′	FF′	EE′	DD′	α	β
Skin model [27]	μ	0.21	0.13	0.15	0.10	0.21	0.14	0.15	2.4	1.7
	σ	0.046	0.032	0.035	0.030	0.048	0.033	0.035	0.60	0.40
VSA® software	μ	0.25	0.19	0.18	0.19	0.18	0.23	0.16	3.5	2.9
	σ	0.041	0.031	0.030	0.032	0.030	0.039	0.027	0.50	0.40
(VSA®-skin model)/skin model	$\Delta\mu$	+17%	+42%	+19%	+90%	−13%	+67%	+9%	+45%	+68%
	$\Delta\sigma$	−11%	−4%	−15%	+5%	−37%	+18%	−22%	−23%	+9%

Table 6. Results of the fourth case study.

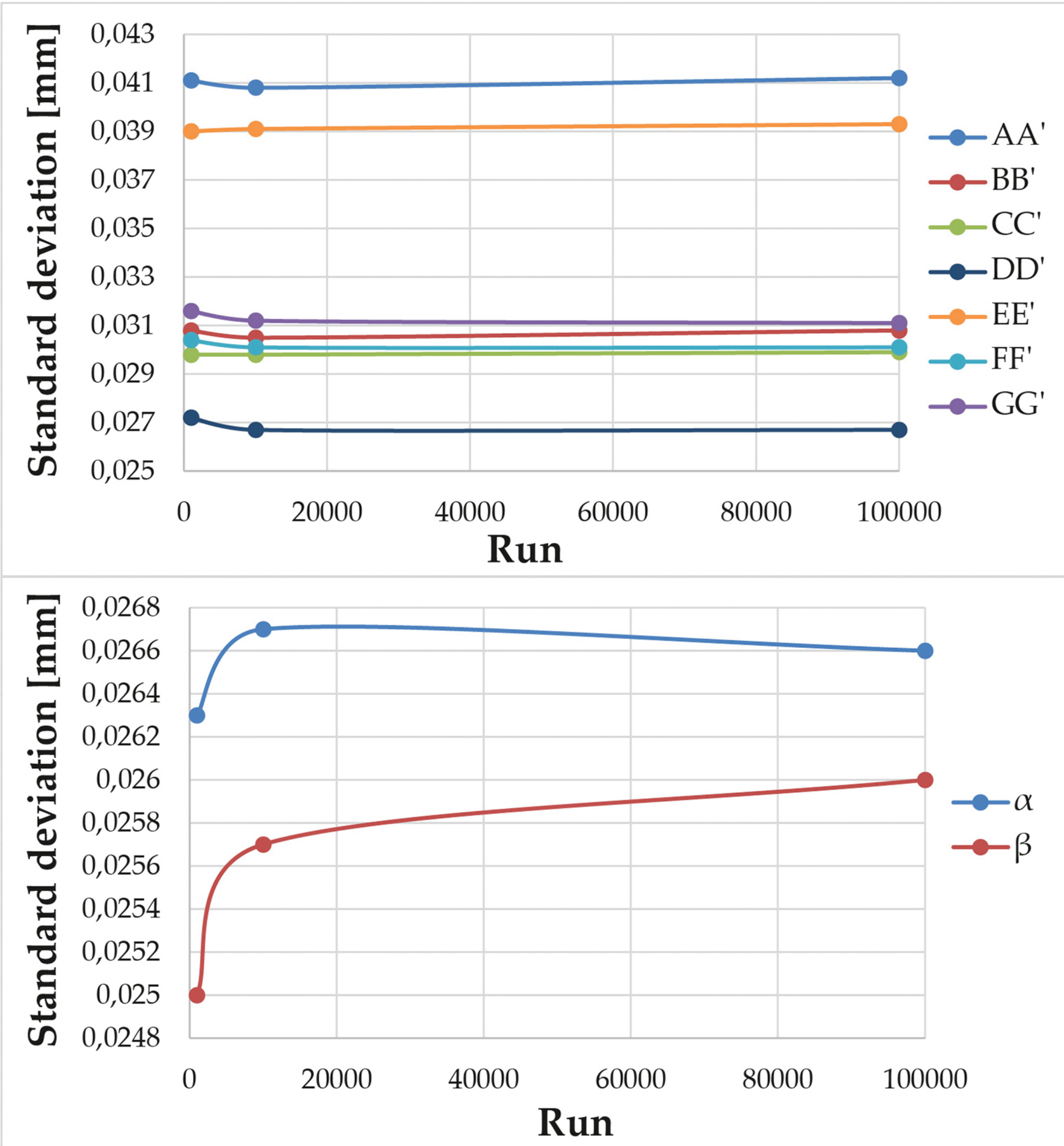

Figure 12. Deviation standard of the KPC refers to the number of simulation runs.

The case study has been solved by the VSA® software of Siemens, and the results obtained by a Monte Carlo simulation with 1000 runs are reported in the second two lines of **Table 6**. To evaluate the stability of the results further Monte Carlo simulations have been performed by varying the number of runs up to 100,000 runs, and the results are shown in **Figure 12**. It is evident that the standard deviation remains the same with the increase in the number of the simulation runs.

The results due to VSA® software are very different from those due to the skin model. The difference between the two approaches is the largest among all the four case studies. This difference is due to the simulation of the form tolerances. VSA® considers only the roto-translation of the nominal feature to which a form tolerance is applied, while the skin model considers each feature as a set of control points whose movement depends on the applied form tolerance. Moreover, the point of a plane is considered geometrically correlated. It is evident that the skin model represents the form deviations in agreement with reality.

8. Results discussion and conclusions

Many are the computer-aided techniques for tolerance analysis. These software packages allow to assure the geometry of the assembly product by analyzing the geometrical interferences. They efficiently deal with simple geometrical feature of mechanical assemblies, such as plane and hole-pin, while they hardly treat of sculptured surfaces to connect. Moreover, they are not completely true to the GPS standards. To use these tools, the user should understand the packages' theoretical base of modeling to build a valid model and to have accurate results.

This work underlines that GPS data, defined in the CAD model, may not be automatically imported into the CATs due to the limits of the CAD geometry translator standards, such as STEP or IGES. None of the currently available CATs offer distributed/parallel computing capabilities, and they lack of ability to accommodate general tolerance analysis and synthesis problems involving assemblies under loading.

The present work shows the application of one of these computer-aided technique for tolerance analysis, knows as VSA® by Siemens, to four case studies from the literature. The four case studies have been chosen since they were used in the literature to apply five important approaches for tolerance analysis: the tolerance charting, the Jacobian model, the vector loop model, the proprietary software, and the skin model. The results due to the application of the VSA® software are different from those due to all of the five methods of the literature for many reasons. The CATs for tolerance analysis have a 3D nature, and they are not able to consider 1D or 2D case studies. They are not able to simulate a form tolerance according to international standard for GPS, or more tolerances applied to the same surface, or assembly constraints refer to actual assembly operations. This paper demonstrates how computer-aided technologies for tolerance analysis need further knowledge of the assembly problems and further development. More efforts are needed to clarify the limits and the potentialities of the computer-aided technologies for tolerance analysis in order to better define their domain of applicability.

Acknowledgements

This research received no specific grant from any funding agency.

Author details

Andrea Corrado* and Wilma Polini

*Address all correspondence to: a.corrado@unicas.it

Department of Civil and Mechanical Engineering, University of Cassino and Southern Lazio, Cassino, Italy

References

[1] Söderberg R, Lindkvist L, Wärmefjord K, Carlson JS. Virtual geometry assurance process and toolbox. Procedia CIRP. 2016;*43*:3–12. doi:10.1016/j.procir.2016.02.043

[2] Ebro M, Olesen J, Howard TJ. Robust Design Impact Metrics: measuring the effect of implementing and using Robust Design. In: Proceedings of the 1st International Symposium on Robust Design (ISoRD14); 14–15 August 2014; Copenhagen. Denmark. 2014. p. 1–9.

[3] Dantan JY, Qureshi AJ. Worst-case and statistical tolerance analysis based on quantified constraint satisfaction problems and Monte Carlo simulatio. Comput-Aided Des. 2009;*41*(1):1–12. doi:10.1016/j.cad.2008.11.003

[4] Söderberg R, Lindkvist L, Dahlstrŏm S. Computer-aided robustness analysis for compliant assemblies. J Eng Des. 2006;*17*(5):411–428. doi:10.1080/09544820500275800

[5] Bjorke O, editor. Computer aided tolerancing. 2nd ed. New York, NY: ASME Press; 1989. p. 216

[6] Srinivasan V. Computational metrology for the design and manufacture of product geometry: a classification and synthesis. J Comput Inf Sci Eng. 2006;*7*(1):3–9. doi: 10.1115/1.2424246

[7] Weill R, Clément A, Hocken R, Farmer L, Gladman C, Wirtz A, et al. Tolerancing for function. CIRP Ann. 1988;*37*(2):603–610. doi:10.1016/S0007-8506(07)60757-4

[8] Shen Z, Ameta G, Shah JJ, Davidson JK. A comparative study of tolerance analysis methods. J Comput Inf Sci Eng. 2005;*5*(3):247–256. doi:10.1115/1.1979509

[9] Dantan JY, Gayton N, Dumas A, Etienne A, Qureshi AJ. Mathematical issues in mechanical tolerance analysis. In: Proceedings of the 13th Colloque National AIP PRIMECA; 2012.

[10] Fortini E.. Dimensioning for Interchangeable Manufacture. New York: Industrial Press; 1967. 276 p

[11] Knappe L.F.. A technique for analyzing mechanism tolerances. Mach Des. 1963;25:155–157.

[12] Shan A, Roth NR, Wilson JR. A new approach to statistical geometrical tolerance analysis. Int J Adv Manuf Tech. 1999;*15*:222–230. doi:10.1007/s001700050060

[13] Shah JJ, Ameta G, Shen Z, Davidson J. Navigating the tolerance analysis maze. Comput-Aided Des Appl. 2007;*4*(5):705–718. doi:10.1080/16864360.2007.10738504

[14] Requicha AAG. Toward a theory of geometric tolerancing. Int J Robot Res. 1983;*2*(4): 45–60. doi:10.1177/027836498300200403

[15] Gao J, Chase KW, Magleby SP. Generaized 3-D tolerance analysis of mechanical assemblies with small kinematic adjustments. IIE Trans. 1998;*30*(4):367–377. doi: 10.1023/A:1007451225222

[16] Wittwer JW, Chase KW, Howell LL. The direct linearization method applied to position error in kinematic linkage. Mech Mach Theory. 2004;*39*(7):681–693. doi:10.1016/j.mechmachtheory.2004.01.001

[17] Bourdet P, Mathieu L, Lartigue C, Ballu A. The concept of the small displacement torsor in metrology. Ser Adv Math Appl Sci. 1996;40:110–122.

[18] Li H, Zhu H, Li P, He F. Tolerance analysis of mechanical assemblies based on small displacement torsor and deviation propagation theories. Int J Adv Manuf Technol. 2014;*72*(1–4):89–99. doi:10.1007/s00170-014-5630-0

[19] Ameta G, Serge S, Giordano M. Comparison of spatial math models for tolerance analysis: tolerance maps, deviation domain and TTRS. J Comput Inf Sci Eng. 2011;*11*(2): 021004. doi:10.1115/1.3593413

[20] Giordano M, Samper S, Petit J. Tolerance analysis and synthesis by means of deviation domains, axi-symmetric cases. In: Davidson J, editor. Models for Computer Aided Tolerancing in Design and Manufacturing. Netherlands: Springer; 2007. p. 85–94

[21] Homri L, Teissandier D, Ballu A. Tolerance analysis by polytopes: taking into account degrees of freedom with cap half-spaces. Comput-Aided Des. 2015;*62*:112–130. doi: 10.1016/j.cad.2014.11.005

[22] Farmer L, Gladman C. Tolerance technology—computer-based analysis. CIRP Ann. 1986;*35*(1):7–10. doi:10.1016/S0007-8506(07)61827-7

[23] Chase K, Parkison A. A survey of research in the application of tolerance analysis to the design of mechanical assemblies. Res Eng Des. 1991;*3*(1):23–37. doi:10.1007/BF01580066

[24] Nigam SD, Turner JU. Review of statistical approaches to tolerance analysis. Comput-Aided Des. 1995;*27*(1):6–15. doi: 10.1016/0010-4485(95)90748-5

[25] Polini W. Taxonomy of models for tolerance analysis in assembling. Int J Prod Res. 2012;*50*(7):2014–2029. doi:10.1080/00207543.2011.576275

[26] Schleich B, Anwer N, Mathieu L, Wartzack S. Skin model shapes: a new paradigm shift for geometric variations modelling in mechanical engineering. Comput-Aided Des. 2014;*50*:1–15. doi:10.1016/j.cad.2014.01.001

[27] Anwer N, Schleich B, Mathieu L, Wartzack S. From solid modelling to skin model shapes: shifting paradigms in computer-aided tolerancing. CIRP Ann. 2014;*63*(1):137–140. doi:10.1016/j.cirp.2014.03.103

[28] Prisco U, Giorleo G. Overview of current cat systems. Integr Comput-Aid Eng. 2002;*9*:373–387.

[29] Turner JU, Gangoiti A. Tolerance analysis approaches in commercial software. Concurr Eng. 1991;*1*:11–23.

[30] Fischer B.R.. Mechanical tolerance stackup and analysis. 2nd ed. Boca Raton: CRC Press; 2011. 508 p.

[31] Bo C, Yang Z, Wang L, Chen H. A comparison of tolerance analysis models for assembly. Int J Adv Manuf Technol. 2013;*68*:739–754. doi:10.1007/s00170-013-4795-2

[32] Franciosa P, Gerbino S, Patalano S. A sequential constraint solver to simulate assembling operations for tolerance analysis. Procedia CIRP. 2013;*10*:169–177. doi:10.1016/j.procir.2013.08.028