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The Strain Energy Release Rate for Stack of Coated Conductors with Interface Crack in Perpendicular Magnetic Field

Meng Zhao, Huadong Yong and Youhe Zhou

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<http://dx.doi.org/10.5772/64006>

Abstract

Due to the ability of transporting huge current in the stack of high-temperature superconducting conductors, the electromagnetic body force generated by the interaction of magnetic field and current may affect the mechanical stability of structure. In this paper, the fracture behavior of the stack of coated conductors which contains an interface crack is studied for increasing field and decreasing field. The body forces are obtained with variational formulation for the Bean's critical state model. Based on the virtual crack closure technique (VCCT), the strain energy release rate of the stack of coated conductors with an interface crack is determined. The strain energy release rates are compared for different crack positions, crack lengths, magnetic fields and the thicknesses of substrate, respectively. These results may be useful for the practical application.

Keywords: strain energy release rate, stacked conductors, interface crack, magnetic field, superconductor

1. Introduction

Due to the ability of transporting high critical current density and trapping magnetic field, high-temperature superconductors have been expected to be used in the transmission cables, transformers, motors and maglev system [1–5]. A typical application is the stack of high-temperature superconductor coated tapes for power engineering [6]. However, the application of tape is limited for the reason that the superconductor is a brittle material, which cannot withstand large mechanical loadings [7, 8]. Meanwhile, the manufacturing process is complex

for high-temperature superconductor. Thus, the cavity and microcracks are inevitable in superconductor [9, 10]. The electromagnetic body force induced by the interaction of electromagnetic field may result in the extension of crack in superconductor under high magnetic fields, which will finally give rise to the fracture of superconductor [11]. Cracking can reduce the critical current density of superconductor and mechanical stability [12]. Recently, significant efforts have been made to study the mechanical behavior of the superconductors with the development of critical current density.

Since the electromagnetic behavior in superconductor is complex, several different theoretical models were presented to describe the constitutive relation, such as Bean model, Kim model and E-J relation [13–15]. In recent years, the relationships between mechanical response and magnetic field have been concerned [16, 17]. The general procedure is that the current and magnetic field distributions are obtained based on the constitutive model. After deriving the electromagnetic body force, we will find the mechanical characteristics of superconductor. The superconductor will undergo mechanical deformation in magnetic field which is regarded as magnetostriction. The magnetostriction is dependent on the amplitude of magnetic field and critical state model [16, 17]. For the superconductors during the magnetization, the body force induced by the flux pinning evolves from initially compressive to tensile as the external field changes from increasing to decreasing. It was pointed out that the stress and strain induced by flux pinning show obvious irreversible behavior. In other words, the stresses are different for increasing field and decreasing field cases [18–20]. Moreover, two different cooling processes (zero field cooling and field cooling) are discussed. The quantitative analysis on the magneto-elastic problems in bulks and strips was made for different critical state models, such as the Bean model and Kim model [18–31]. For superconducting strip with transport current, the magnetostriction curve is not a closed loop [32]. The stress and magnetostriction in many complex structures also have attracted many attentions [33–37].

The crack problem is another challenge for the superconductors because the electromagnetic body force and thermal stress are important driving forces for crack growth. During the cycles of the applied magnetic field, the tensile and compressive stress will be generated alternately and cracking of superconductor usually occurs as the field is decreased to zero [38]. This is because the crack is prone to advance under the tensile stress. The linear elastic fracture theory was widely used to investigate the fractures behaviors of the thin film and bulk superconductors [39–47]. The collinear cracks, transverse crack and crack-inclusion problems in superconductors were also studied recently [48–50]. Superconductors are usually metal composites which consist of several components. Then, the crack problems in inhomogeneous superconductors were studied extensively [51–57]. Recently, the dynamic fracture behaviors in superconductors were analyzed with the finite element method [58].

Compared to the bulk superconductor, coated conductors have better mechanical strength [59]. As is known to us, the coated conductor structure is made up of several layers deposited onto the substrate. In the coated conductor structure, the interface crack may be generated by lattice mismatch, thermal expansion and chemical reactions during the deposition progress of the coated structure [60]. The delamination of superconducting layer is easy to take place at the interface between dissimilar materials [61]. Unlike the internal or edge crack problems, the

compressive stress can also lead to the propagation of interface crack [62]. Thus, it is necessary to analyze the fracture behavior of interface crack to predict the mechanical instability in the coated conductor structure.

The fracture behavior of interface crack between superconducting film or tape and substrate was studied in magnetic field or with transport current [62–64]. However, there are few works on the crack problem in stack of conductors. In this paper, we consider the interface crack in the stack of high-temperature coated tapes. The effect of copper and substrate on the shielding current and magnetic field distributions in the superconducting layer is neglected. Based on the simplified geometrical and material assumptions, linear elastic equation is used. We carry out an in-depth investigation on the strain energy release rate for interface crack in different magnetic fields, crack lengths, substrate thicknesses with virtual crack closure technique (VCCT). The results may provide guidance for the design of stack of coated conductors.

This paper is organized as follows. In Section 2, we introduce VCCT to calculate the strain energy rate at the crack tip. Section 3 computes a bimaterial plate with a central interface crack to verify the accuracy of VCCT. In Section 4, we discuss the body force distribution in superconducting tape. In Section 5, the interface crack in stack of tapes is analyzed under perpendicular magnetic field. In Section 6, we draw a conclusion from the above work.

2. Strain energy release rate of interface crack using VCCT

In this section, we will first verify the accuracy of the numerical computation. Here, we only consider the linear elastic fracture behavior. For the interface crack problem, the stress intensity factor or strain energy release rate can be determined with the virtual crack closure technique (VCCT). VCCT is a well-established method for computing the crack problem [65], which was proposed by Rybicki and Kanninen for two-dimensional problems [66].

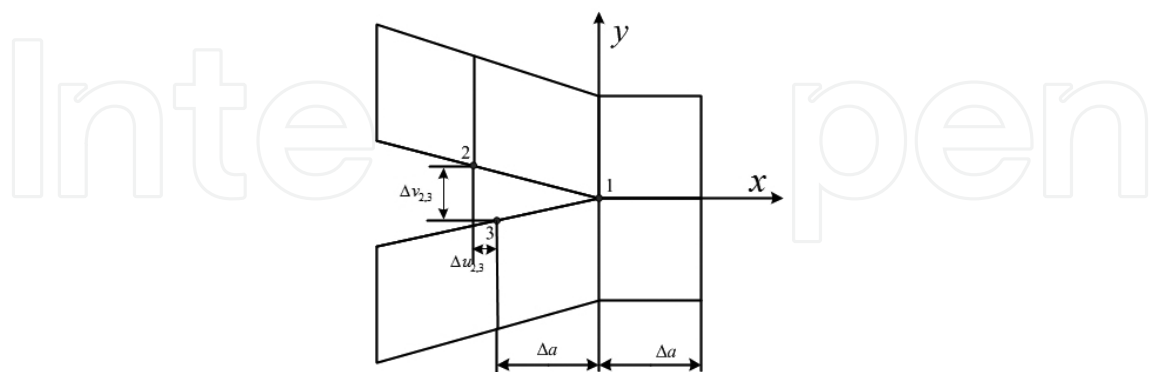


Figure 1. Calculating model of the strain energy release rate using VCCT.

Figure 1 shows the mesh model for computing the strain energy release rate with VCCT, where u and v stand for the displacement along x and y directions, respectively. Each mesh is a square and Δa is the side width. On the basis of assumption given in reference [66], we

define the opening displacement behind the tip of virtual crack ($\Delta v_{2,3}$ and $\Delta u_{2,3}$) as the actual crack opening displacement for determining strain energy release rate with VCCT. The strain energy release rate represents the change of elastic strain energy per unit area of crack extension [67]. Then, the strain energy release rates for different fracture modes can be expressed as follows:

$$G_I = \frac{F_y \Delta v_{2,3}}{2B\Delta a}, \quad G_{II} = \frac{F_x \Delta u_{2,3}}{2B\Delta a} \quad (1)$$

$$G = G_I + G_{II} \quad (2)$$

where F_x and F_y are the nodal forces and B is the thickness of the body with crack. VCCT is an effective method which does not need integration and it is insensitive to mesh.

3. Verification of numerical results

In order to verify the simulation results, we study the fracture behavior for bimaterial plate with an interface crack, as shown in **Figure 2**. It is well known that for the interface crack, there is the coupling between the tensile fracture mode and shear fracture mode even for mode I loadings. In addition, the oscillation of stress singularity will occur at the crack tip [65]. In this model, the interface crack is located in the center of the bimaterial plate and a uniform tension load σ_0 is applied on the top and bottom edges of the plate. The parameters used for numerical analysis are as follows [68]: $2w = 100$ mm, $2l = 200$ mm, $a/w = 0.4$, $E_1 = 2.058 \times 10^5$

MPa, $\frac{E_1}{E_2} = 1 \sim 100$, $v_1 = v_2 = 0.3$, $\sigma_0 = 9.8$ MPa.

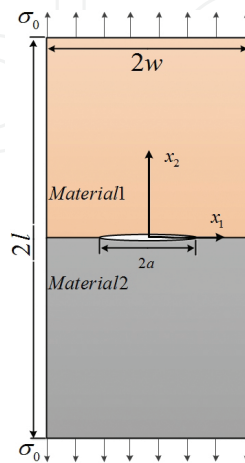


Figure 2. A finite bimaterial plate with interface crack under a uniform tension.

We use finite element software Abaqus to solve the interface crack problem. A 4-node bilinear plane stress quad-dominated structured element is used and mesh width is given as $\Delta a = 1\text{mm}$. It is convenient for us to use VCCT to compute the strain energy release rate. For the sake of comparing our results with those of Yu et al. [69], Nagashima et al. [70], Miyazaki et al. [71], we use the formulation proposed by Suo and Hutchinson [72]. For bimaterial interface crack problem, the relation between stress intensity factor and strain energy release rate is as follows [72]

$$G = \frac{c_1 + c_2}{16 \cosh^2 \pi \varepsilon} |K|^2 \quad (3)$$

where subscripts 1 and 2 represent the material 1 and material 2, respectively, and

$$K^2 = K_1^2 + K_2^2 \quad (4)$$

$$c_1 = \frac{\kappa_1 + 1}{\mu_1}, \quad c_2 = \frac{\kappa_2 + 1}{\mu_2} \quad (5)$$

$$\varepsilon = \frac{1}{2\pi} \ln \frac{1 - \beta}{1 + \beta}, \quad \kappa = (3 - \nu)/(1 + \nu) \quad (6)$$

$$\beta = \frac{\mu_1(\kappa_2 - 1) - \mu_2(\kappa_1 - 1)}{\mu_1(\kappa_2 + 1) + \mu_2(\kappa_1 + 1)} \quad (7)$$

in which β is the parameter proposed by Dundurs to describe the mismatch of material 1 and material 2 [73], and μ is the shear modulus.

Table 1 shows the comparison of strain energy release rates for present work and the results given by others when $a/w = 0.4$. These results show that there is little difference between our work using VCCT and other methods. The difference is smaller than 1% and this verifies the accuracy of the numerical method.

E1/E2	Yu	Nagashima	Miyazaki	This work
1	0.0361	0.0361	0.0362	0.0358
2	0.0531	0.0529	0.0231	0.0526
3	0.0688	0.0685	0.0690	0.0683
4	0.0841	0.0835	0.0843	0.0834
10	0.1725	0.1699	0.1727	0.1711
100	1.4652	1.4264	1.4642	1.4529

Table 1. Comparison of the strain energy release rate G (N/mm).

4. Body force distribution in tape under magnetic field

Next, we turn our attention to the body force distributions in superconducting tape. **Figure 3** shows the configuration of the stack of superconducting conductors. The conductors consist of three tapes and these tapes are separated by the insulation layers (green). Generally, the superconducting layer (black) is attached to the substrate in each tape. In addition, copper layer is deposited on the superconducting layer which can also transport current. The thicknesses of the insulation layer, copper layer, superconducting layer and substrate are $h_{Ins} = 25 \mu\text{m}$, $h_{Cop} = 40 \mu\text{m}$, $h_{SC} = 1 \mu\text{m}$, $h_{Sub} = 50 \mu\text{m}$, respectively [74]. Then, the total thickness of stacked conductors is $h = 373 \mu\text{m}$ and the width is $200 \mu\text{m}$. The mechanical behavior is based on the electromagnetic body force and we will first calculate the field and current distributions. For simplicity, we use the Bean's critical state model, i.e., the critical current density is a constant which is independent of the magnetic field.

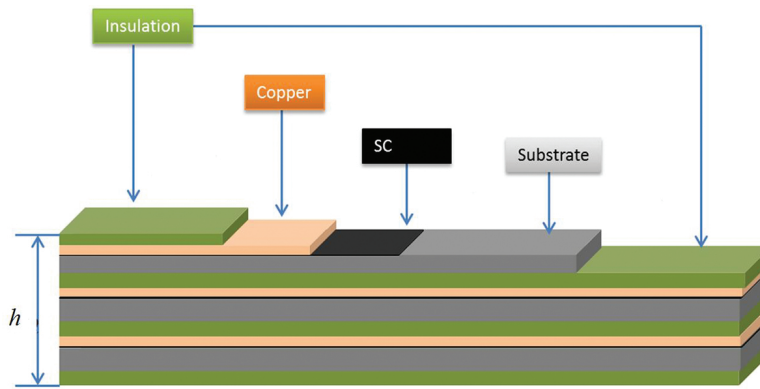


Figure 3. The configuration of the stack of superconducting conductors.

The distributions of shielding current are plotted in **Figure 4**. As the conductor is under magnetic field, the current will be induced to shield the external field. From **Figure 4 (a)**, we can find that the tape is divided into two parts and the current direction is opposite in two parts for increasing field case. As the external field is decreased, the shielding current will redistribute. The conductor can be divided into four parts for decreasing field case. However, it is interesting to see that the direction of shielding current is antisymmetric.

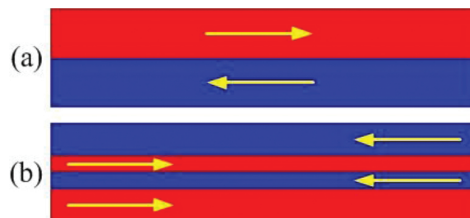


Figure 4. The current distribution in superconducting layer under magnetic field. (a) Increasing field case and (b) decreasing field case.

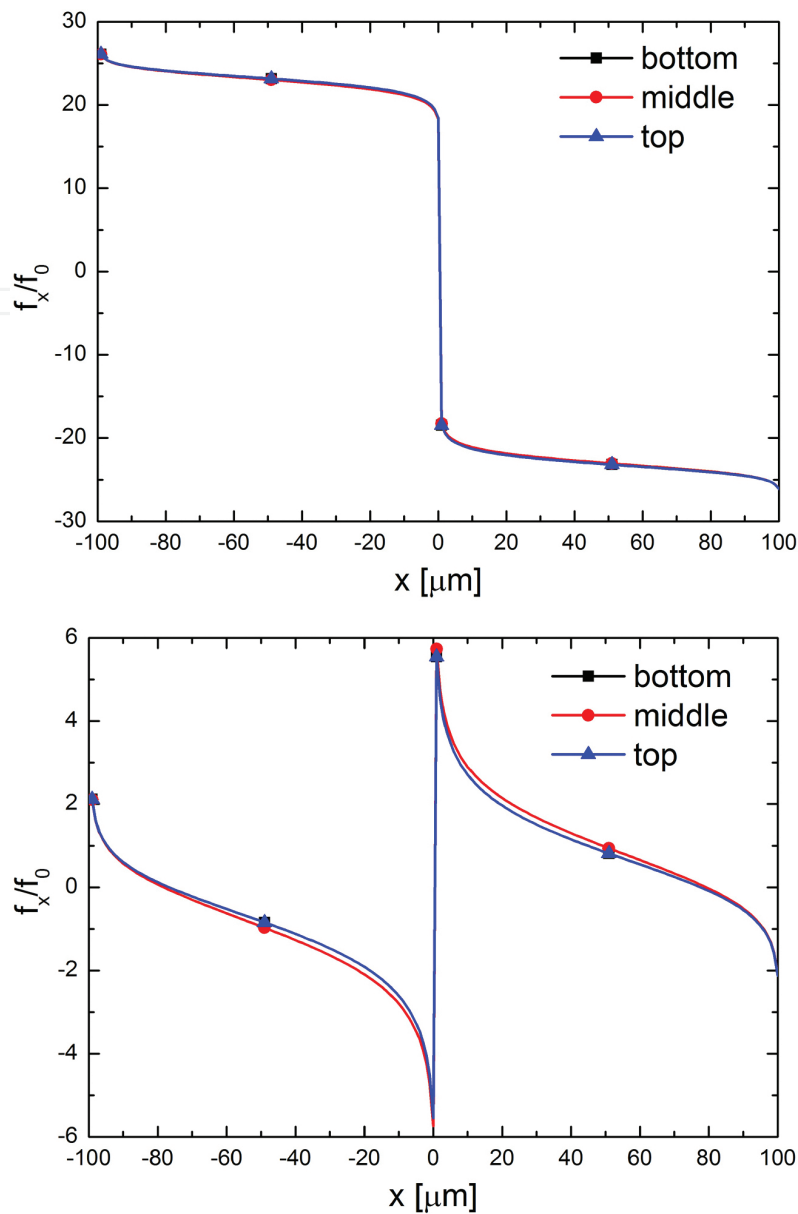


Figure 5. The distributions of body force along x direction. The magnetic field is increased from zero to B_e (upper). The field is decreased from B_e to zero (lower).

Using the variational formulation derived by Prigozhin [75], the electromagnetic responses of superconducting layer are obtained in the external field. The body force $\mathbf{f} = \mathbf{j} \times \mathbf{B}$ can be determined with the flux density and current. **Figures 5 and 6** present the body forces f_x and f_y per unit volume for various stages of magnetization process. It is obvious that the body forces along x direction are very close for the bottom, middle and top layers. Moreover, the value of f_x for the increasing field is larger than that for decreasing field. The body force f_y is much smaller than f_x . This is due to the reason that the magnetic field is mainly along y direction. Thus, it can be expected that shear loading is important than the tensile loading. As external field B_e is larger than the full penetration field, the value of f_y for increasing and decreasing field is also very close.

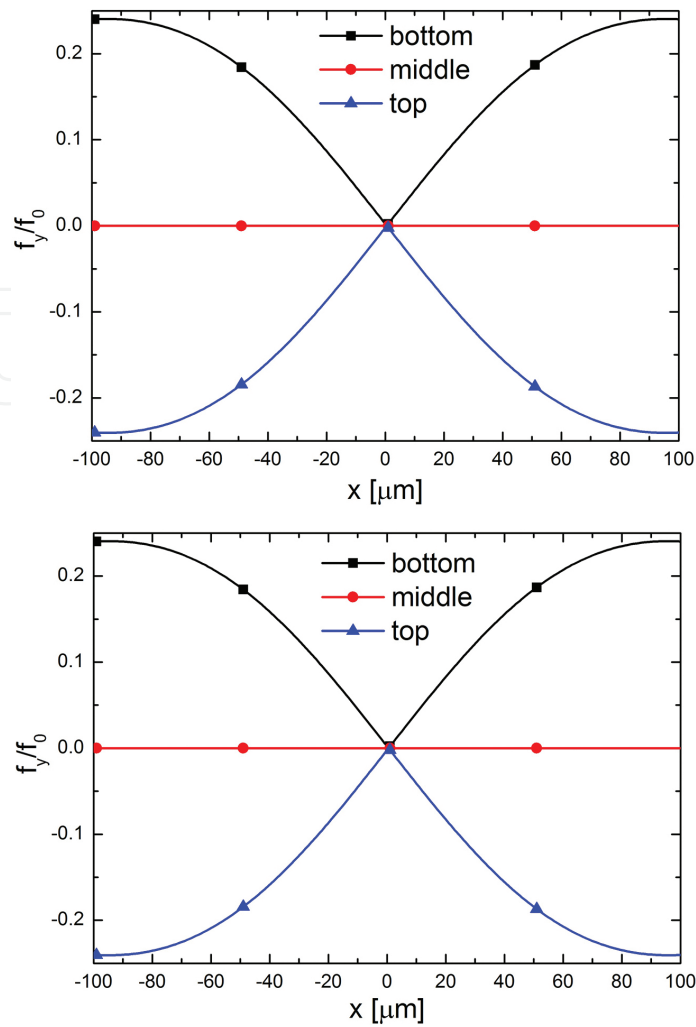


Figure 6. The distributions of body force along y direction. The magnetic field is increased from zero to B_e (upper). The field is decreased from B_e to zero (lower).

5. Interface crack problem for the stacked conductors structure

The structure of the stack of high-temperature superconducting conductors in perpendicular magnetic field is shown in **Figure 7**. The lower insulation layer which is located at the bottom is fixed. That is to say that the displacements of x and y directions at the bottom of conductors are zero. The stacked conductors consist of three superconducting tapes. The central interface crack is located between the superconducting layer and substrate, and is assumed to be through the length of conductor.

Although copper is an elasto-plastic material, we neglect the plastic deformation. Thus, the focus will be on the linear fracture mechanics problem. All constituent materials are isotropic, linearly elastic and homogeneous. The effects of copper and substrate on the distributions of magnetic field and current density in the superconductor layer can be ignored and the stack

of superconducting conductors is placed in a magnetic field B parallel to the y direction. Since the interface crack is parallel to the direction of current flowing, the crack will not disturb the current density and field distributions. Moreover, the width of the stacked conductors is much smaller than the length of the tape along z direction. For the 2D dimensional analysis, a plane strain condition can be assumed.

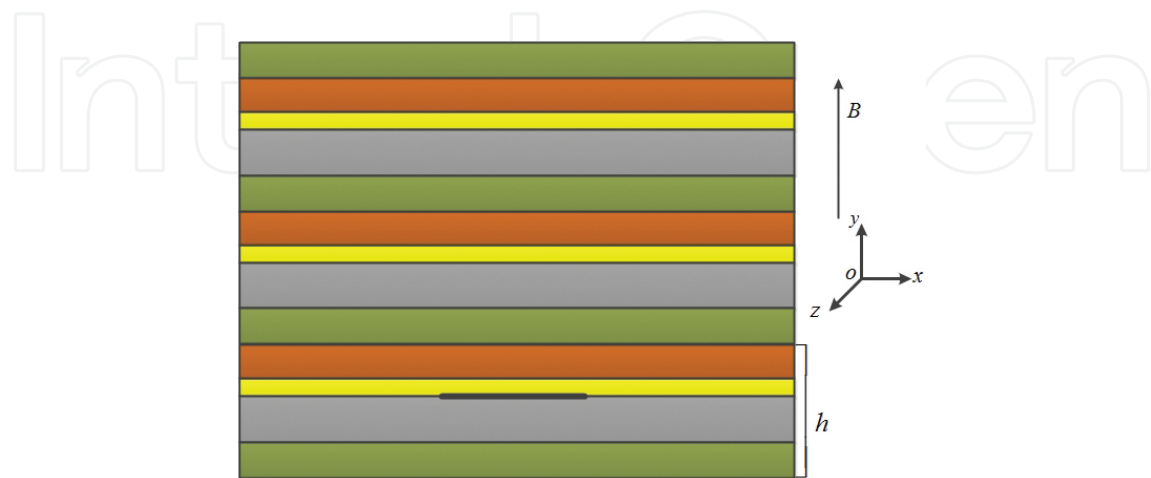


Figure 7. Schematic drawing of the stacked conductors with an interface crack in the perpendicular magnetic field.

During the following simulations, the used parameters are given as follows. We assume that the current density j is no larger than the critical current value $j_c = 3 \times 10^4$ A/mm². Besides, the applied maximum field is equal to $B_e = 8\mu_0 j_c t$, t is the thickness of the total superconducting layers and μ_0 is permeability in vacuum. The strain energy release is normalized by

$$G_0 = \frac{(1 - \nu^2)\pi a}{E_{sc}} \left(\frac{\mu_0 j_c^2 t w}{\pi} \right)^2.$$

	$E(\text{MPa})$	Poisson's ratio
Copper	117×10^3	0.35
YBCO	178×10^3	0.3
Hastelloy	200×10^3	0.3
Insulation	3.5×10^3	0.3

Table 2. Parameters of the materials of the coated tapes [74].

We consider the magnetization process in which the field is increased from zero to maximum and subsequently decreased to zero. As discussed earlier, the body force direction will change for increasing field and decreasing field. Since the total strain energy release rate for increasing field is larger, we mainly discuss the increasing field case. We can assume $G_I = 0$ when the interface crack is closed [62]. For the sake of simplicity, the contact and friction effects between crack surfaces are not considered. The parameters used are given in **Table 2**.

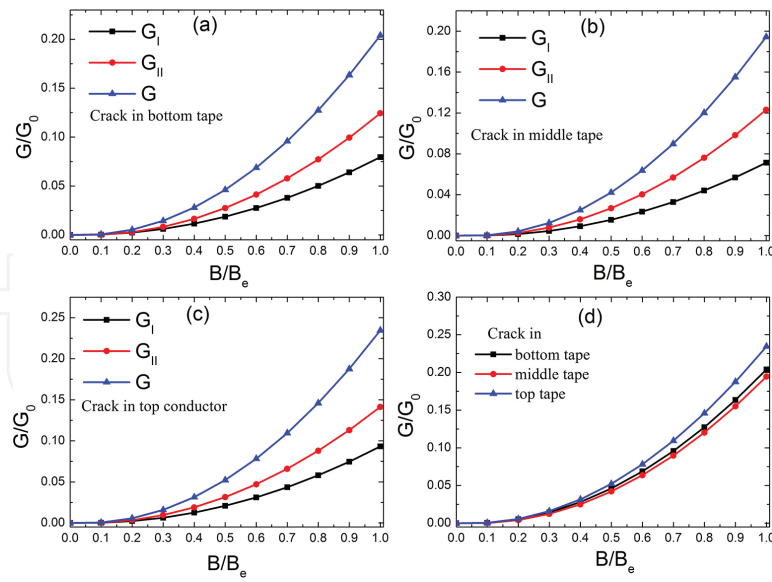


Figure 8. The variation of strain energy release rates with magnetic field as the interface crack is in (a) top tape, (b) middle tape and (c) top tape. The strain energy release rates are compared in (d).

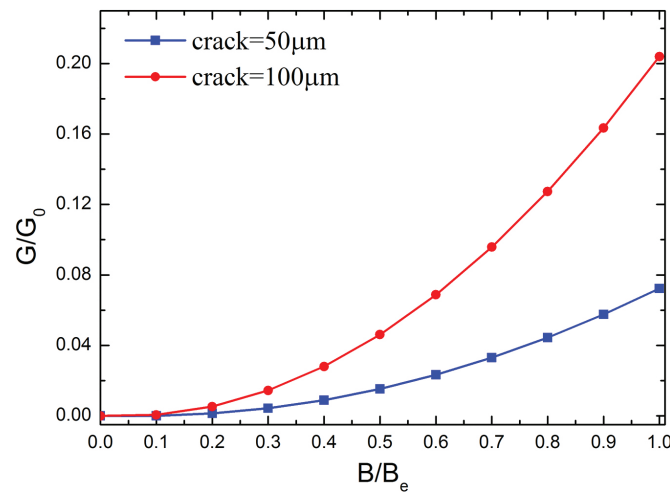


Figure 9. The strain energy release rates for the crack lengths of 50 μm and 100 μm during increasing magnetic field.

Next, we will calculate the strain energy release rates for the stacked conductors with the interface crack. **Figure 8(a)–(c)** shows the variation of strain energy release rate with the magnetic field, i.e., the field is increased from zero to maximum. The central interface crack is located at the bottom, middle or top tape. It can be found that all the strain energy release rates increase with the increasing field. Although the body force is compressive for increasing field case, G_I is smaller than G_{II} and this trend is similar to the results given in reference [62]. The strain energy release rates with interface crack in different tapes are compared in **Figure 8(d)**. It can be found that the strain energy release rate reaches the largest value as the interface crack is in the top tape during the increasing of field, while the value is the lowest as the crack is in the middle tape. We can draw a conclusion that the central interface crack in the top tape is

most dangerous during an increasing magnetic field. This may be due to the reason that the penetration field is largest in the top tape. In **Figure 9**, it is to be noted that the strain energy release rate for the crack length of 100 μm is always larger than that of 50 μm when the magnetic field rises. However, since the shielding current reverses, the trend of strain energy release rate becomes complicated for decreasing field case, as shown in **Figure 10**. At the beginning of the decreasing field, both the strain energy release rates for the crack lengths of 100 μm and 50 μm reduce. It is obvious that the reduction for longer crack length is faster. With the decreasing external field, the strain energy release rate reaches the local minimum and begins to increase. The longer crack firstly reaches the local minimum at $B/B_e = 0.88$. Finally, the strain energy release rate will decrease again, and the value is very close to zero as external field vanishes. Comparing the values for increasing field and decreasing field (see **Figure 11**), we can find that the strain energy release rate is larger for increasing field case. Thus, the highest possibility of cracking may happen during the increasing field, which is opposite to the bulk superconductor.

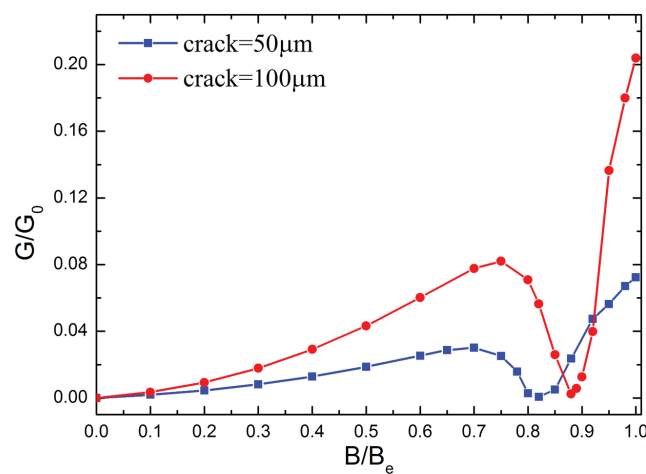


Figure 10. The strain energy release rates for the crack lengths of 50 μm and 100 μm during decreasing magnetic field.

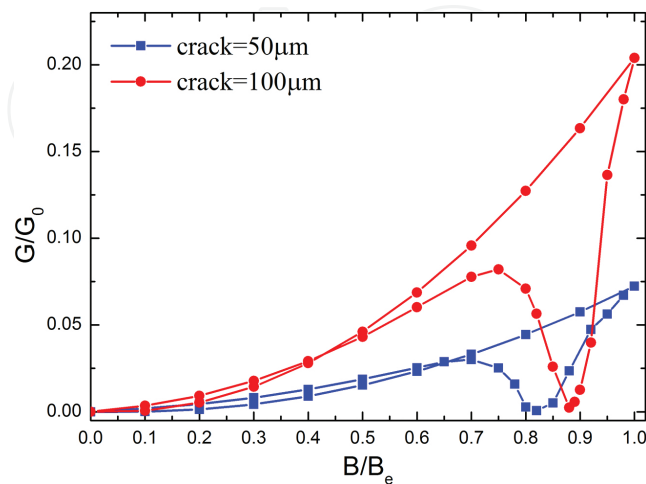


Figure 11. The comparison of strain energy release rates for increasing field and decreasing field.

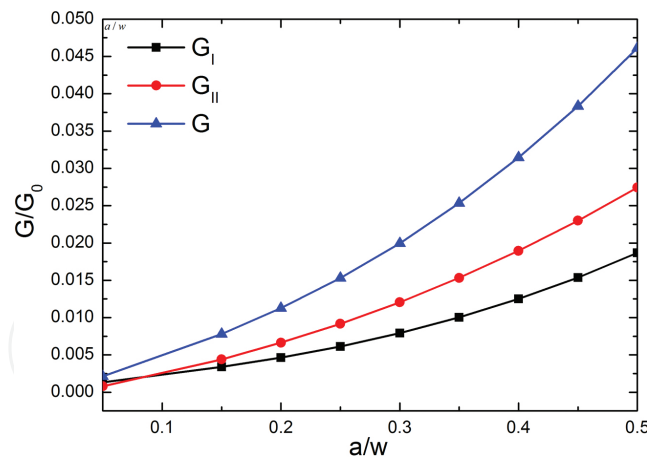


Figure 12. The change of strain energy release rate with the crack length and as the field is increased from zero to $0.5B_e$.

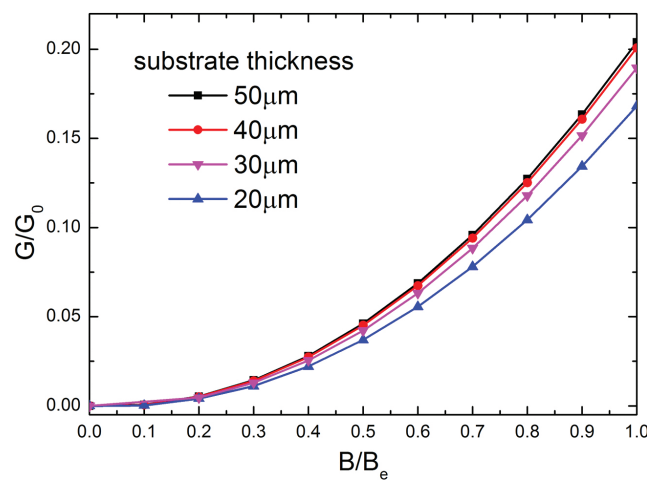


Figure 13. The strain energy release rates with different thicknesses of substrate.

For the central interface crack problem, it can be expected that the strain energy release rate increases with the length of the crack for a fixed magnetic field, as shown in **Figure 12**. The effect of substrate thickness on the strain energy release rate during increasing field is presented in **Figure 13**. One can find that with the increase of substrate thickness from 20 μm to 50 μm , the strain energy release rate also rises. In addition, the increasing strain energy release rate is more obvious in high field. However, the difference is very small as the thickness of substrate is larger than 40 μm . Thus, it is reasonable to reduce the thickness of substrate to improve the mechanical stability of stacked conductors structure.

6. Conclusions

In this paper, we considered the interface crack in the stack of high-temperature superconductor coated conductors with VCCT. A simple interfacial crack problem in bimaterial was

calculated to verify the accuracy of numerical method. On the basis of the linear elastic fracture theory, the strain energy release rates for the stack of coated conductors with interface crack were presented for different magnetic fields, crack lengths, substrate thicknesses. The strain energy release rate increases monotonically with the increasing external field. However, the trend for decreasing field case is not monotonic. When the interface crack is located in the top tape, the strain energy release rate is the largest. In addition, the thickness of hastelloy substrate also has an effect on the strain energy release rate, and larger thickness leads to the higher energy release rate. When the thickness of the substrate is larger than 40 μm , the effect of thickness becomes negligible. This study may provide a better understanding on the mechanical instability of the stack of coated conductor structure during the magnetization process.

Acknowledgements

We acknowledge the support from the Innovative Research Group of the National Natural Science Foundation of China (11421062), National Natural Science Foundation of China (Nos. 11202087 and 11472120), the National Key Project of Magneto-Constrained Fusion Energy Development Program (No. 2013GB110002), the National Key Project of Scientific Instrument and Equipment Development (No. 11327802) and New Century Excellent Talents in University of Ministry of Education of China (NCET-13-0266).

Author details

Meng Zhao, Huadong Yong* and Youhe Zhou

*Address all correspondence to: yonghd@lzu.edu.cn

Key Laboratory of Mechanics on Environment and Disaster in Western China, The Ministry of Education of China, Department of Mechanics and Engineering Sciences, College of Civil Engineering and Mechanics, Lanzhou University, Lanzhou, Gansu, PR China

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