Chapter 5

Robust Adaptive Repetitive and Iterative Learning Control for Rotary Systems Subject to Spatially Periodic Uncertainties

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Additional information is available at the end of the chapter

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Abstract

This book chapter reviews and summarizes the recent progress in the design of spatial-based robust adaptive repetitive and iterative learning control. In particular, the collection of methods aims at rotary systems that are subject to spatially periodic uncertainties and based on nonlinear control paradigm, e.g., adaptive feedback linearization and adaptive backstepping. We will elaborate on the design procedure (applicable to generic nth-order systems) of each method and the corresponding stability and convergence theorems.

Keywords: rotary system, disturbance rejection, robust adaptive control, repetitive control, iterative learning control

1. Introduction

Rotary systems play important roles in various industry applications, e.g., packaging, printing, assembly, fabrication, semiconductor, and robotics. A conspicuous characteristic of such systems is the utilization of actuators, e.g., electric motor, to control the angular position, velocity, or acceleration of the system load. Depending on the occasion of application, simple or complicated motion control algorithm may be used. The increasing complexity in architecture and the high-performance requirement of recent rotary systems have posed a major challenge on conceiving and synthesizing a desirable control algorithm.

Nonlinearities and uncertainties are common issues when designing a control algorithm for a rotary system. Nonlinearities are either intrinsic properties of the system or actuator and sensor dynamics being nonlinear. Uncertainties mainly come from structured/unstructured uncertainties (also known as parametric uncertainty/unmodeled dynamics) and disturbances. For tackling nonlinearities, conventional techniques, e.g., feedback linearization and backstepping, are to employ feedback to cancel all or part of the nonlinear terms. On the contrary, design techniques for conducting disturbance rejection or attenuation in control systems may be roughly categorized with respect to whether or not the techniques generate the disturbance by an exosystem. Representative techniques that resort to the exosystem of the disturbance are internal model design [1,2], which originates from the internal model principle [3], and observer-based design [4,5]. Establishing a suitable mathematical description of the disturbance is an essential step for internal model design techniques. An internal model design for systems in an extended output feedback form and subject to unknown sinusoidal disturbances was addressed in [1]. For observer-based design techniques, an observer is usually employed to estimate the states of the unknown exosystem. Chen [5] showed that the design of the observer can be separated from the controller design. For techniques that do not resort to the exosystem of the disturbance, disturbance observer [6,7] or optimization-based control approaches [8,9] have been shown to work well. In [6], integral phase shift and half-period integration operator were used together to estimate the periodic disturbances. Another type of disturbance observers was introduced in [7]. The proposed disturbance observer may estimate lumped disturbances that comprise unmodeled dynamics and disturbances. However, the performance of the disturbance observer is very sensitive to the adaptation rate of the estimated disturbance components. If the output error of the disturbance observer does not converge sufficiently fast, instability or performance degradation is inevitable.

With measurement of the system states not available, a common technique is to establish a state observer that provides estimates of the states. Unlike state observer for linear systems, no state observer is applicable to general nonlinear systems. Most state observers for nonlinear systems are suited for systems transformable to a specific representation, e.g., normal form [10] or adaptive observer form [11]. One class of observers, known as adaptive state observers, are those having their own update laws adapt the estimated parameters [11,12] or the observer gain [10] to minimize the observer error, i.e., the error between the real states and the estimated states. Marine et al. [11] and Vargas and Hemerly [12] presented a state estimator design for systems subject to bounded disturbances. Bullinger and Allgöwer [10] proposed a high-gain observer design for nonlinear systems, which adapts the observer gain instead of the estimated system parameters. The uncertainties under consideration are nonlinearities of the system. However, the observer error converges to zero only when persistent excitation exists or the disturbance magnitude goes to zero. Moreover, the update law for the observer might have an unexpected interaction with that of the control law. The other type of state observers, e.g., K-filters [13,14] and MT-filters [15,16], does not estimate the system states directly. Specifically, the update law for adapting estimated system parameters (which include both observer and system parameters) is determined from the control law to ensure desired stability and convergence property.

Temporal-based motion control algorithms of various class have been in progress lately. Adaptive control is suited for systems susceptible to uncertain but constant parameters.

Moreover, repetitive and iterative learning control [17–21] is capable of dealing with systems affected by periodic disturbances or in need of tracking periodic commands. Lately, adaptive control has been adopted to adapt the period of the repetitive controller [22,23]. Adaptive and iterative learning control has consolidated and been studied by researchers (see [17] and references therein). The integration immediately gains benefits, such as perfect tracking over finite time, dealing with time-varying parameters, and nonresetting of initial condition. As indicated by Chen and Chiu [19] and Chen and Yang [24], most temporal-based control algorithms for rotary systems of variable speed do not explore the characteristics of most uncertainties being spatially periodic. Analyzing and synthesizing such control system in time domain will mistakenly admit those spatially periodic disturbances/parameters as nonperiodic/time-varying ones. This often results in a design either with complicated time-varying feature or with degraded performance.

Spatial-based control algorithms have been studied by researchers recently. The initial step is to reformulate the given system model into the one in spatial domain. Because the reformulation renders those spatial uncertainties stationary in spatial domain, position-invariant control design can be performed to achieve the desired performance regardless of the operating speed. A spatial-based repetitive controller synthesizes its kernel (i.e., e^{-Ls} with positive feedback) and operates in accordance with spatial coordinate, e.g., angular displacement. Therefore, its ability for spatially periodic disturbances or references rejection/tracking will not deteriorate as the system operates at variable speed. A regular repetitive controller is composed of repetitive (i.e., a kernel) and nonrepetitive (e.g., a stabilizing controller) parts. With the kernel synthesized with respect to spatial coordinate and given a time-domain system, designing the nonrepetitive portion that interfaces with the repetitive kernel properly poses a challenge. For spatially periodic disturbance rejection, Nakano et al. [18] reformulated a given linear time-invariant (LTI) system in an angular position domain. The resulting nonlinear system was linearized around an operating speed. Coprime factorization is then used to synthesize a stabilizing controller with repetitive kernel for the acquired linear model. A more sophisticated design based on linearization and robust control was proposed by Chen et al. [25]. Design approaches for linearized systems are straightforward. However, the overall system might lack the stability of operating at a variable speed or coping with large velocity fluctuation. For tracking of spatially periodic references, Mahawan and Luo [26] have validated the idea of operating the repetitive kernel in angular domain and the stabilizing controller in time domain. Doing so does not require reformulation of the open-loop system. For experimental verification, however, the approach involves solving an optimization problem to synchronize the hardware (time) and software (angular position) interruptions. To further limit the applicability, the mapping between time and angular position has to be known *a priori*. The problem formulation made by Nakano and Mahawan assumed the simplest scenario, i.e., the open-loop system is LTI without nonlinearity and modeling uncertainty. Chen and Chiu [19] reported that a class of nonlinear models can be reformulated into a quasi-linear parameter varying (quasi-LPV) system. An LPV gain-scheduling controller was synthesized subsequently to address unmodeled dynamics, actuator saturation, and spatially periodic disturbances. The approach could lead to conservative design if the number of varying parameters rises, the parametric space is nonconvex, or the modeling uncertainties are significant. The restraint and conservatism of modeling uncertainties was relieved by Chen and Yang [24] by formulating a spatial-based repetitive control system with the adoption of adaptive feedback linearization. However, this method is only applicable to systems with measurement of all states available in real time.

The design of spatial-based repetitive control has been sophisticated enough to cope with a class of uncertain nonlinear systems. On the contrary, existing spatial-based iterative learning controls [27,28] are still primitive and aim at only linear systems. It is not apparent whether those methods can be generalized to be applicable for nonlinear and high-order systems. Knowing that spatial uncertainties in rotary systems may be tackled as periodic disturbances or periodic parameters [29–31], treating the uncertainties as disturbances seem to be more prevalent in literatures.

This book chapter reviews and summarizes the recent progress in the design of spatial-based robust adaptive repetitive and iterative learning control. In particular, the collection of methods aims at rotary systems that are subject to spatially periodic uncertainties and based on nonlinear control paradigm, e.g., adaptive feedback linearization and adaptive backstepping. We will elaborate on the design procedure (applicable to generic nth-order systems) of each method and the corresponding stability and convergence theorems. The outline of the chapter is as follows.

Section 2 presents a spatial-based robust repetitive control design that builds on the design paradigm of feedback linearization. This design basically evolves from the work of Chen and Yang [24]. The proposed design resolves the major shortcoming in their design, i.e., which requires full-state feedback, by the incorporation of a K-filter-type state observer. The system is allowed to operate at varying speed, and the open-loop nonlinear time-invariant (NTI) plant model identified for controller design is assumed to have both unknown parameters and unmodeled dynamics. To attain robust stabilization and high-performance tracking, we propose a two-degrees-of-freedom control configuration. The controller consists of two modules, one aiming at robust stabilization and the other tracking performance. One control module applies adaptive feedback linearization with projected parametric adaptation to stabilize the system and account for parametric uncertainty. Adaptive control plays the role of tuning the estimated parameters, which differs from those methods (e.g., [22,23]), where it was for tuning the period of the repetitive kernel. The other control module comprises a spatial low-order and attenuated repetitive controller combined with a loop-shaping filter and is integrated with the adaptively controlled system. The overall system may operate in variable speed and is robust to model uncertainties and capable of rejecting spatially periodic and nonperiodic disturbances. The stability of the design can be proven under bounded disturbance and uncertainties.

Section 3 presents another spatial-based robust repetitive control design that resorts to the design paradigm of backstepping. This design basically builds on the work of Yang and Chen [32]. The method has been extended to a category of nonlinear systems (instead of just LTI systems). Furthermore, the main deficiency of requiring full-state feedback in Yang and Chen's design is resolved by incorporating a K-filter-type state observer. To achieve robust stabilization and high-performance tracking, a two-module control configuration is constructed. One of the module using adaptive backstepping with projected parametric adaptation to robustly stabilize the system. The other module incorporates a spatial-based low-order and attenuated repetitive controller cascaded with a loop-shaping filter to improve the tracking performance. The overall system incorporating the state observer can be proven to be stable under bounded disturbance and system uncertainties.

Section 4 introduces a spatial-based iterative learning control design that is suited for a generic class of nonlinear rotary systems with parameters being unknown and spatially periodic. Fundamentally, this design borrows the feature of parametric adaptation in adaptive control and integrates it with iterative learning. Note that the theoretical success of the integration is not immediate because the stability and tracking performance of the overall system is in need of further justification. Control input and periodic parametric tuning law are specified by establishing a sensible Lyapunov-Krasovskii functional (LKF) and rendering its derivative negative semidefinite. The synthesis of the control input and parametric tuning law and stability/convergence analysis established for this design is distinct from that in [17]. Moreover, unlike a typical adaptive control, the proposed periodic parametric tuning law can cope with unknown parameters of stationary or arbitrarily fast variation.

Section 5 concludes the chapter and points out issues and future research directions relevant to spatial-based robust adaptive repetitive and iterative learning control.

2. Spatial-based output feedback linearization robust adaptive repetitive control (OFLRARC)

Consider the state-variable model of an nth-order single-input single-output NTI system with model uncertainties and output disturbance, i.e.,

$$\begin{aligned}\dot{x}(t) &= \left[f_t\left(x(t),\phi_f\right)+\Delta f_t\left(x(t),\phi_f\right)\right]+\left[g_t\left(x(t),\phi_g\right)+\Delta g_t\left(x(t),\phi_g\right)\right]u(t)\\ y &= \Psi x(t)+d_y(t)=x_1(t)+d_y(t)\end{aligned} \tag{1}$$

where $x(t)=[x_1(t) \ \cdots \ x_n(t)]^T$, $\Psi=[1 \ 0 \ \cdots \ 0]$, $u(t)$, and $y(t)$ correspond to the control input and measured output angular velocity of the system, respectively.

Assumption 2.1

(1) $d_y(t)$ is a class of bounded signals with (dominant) spatially periodic and band-limited (or nonperiodic) components.

Here, band-limited disturbances are signals with Fourier transform or power spectral density being zero above a certain finite frequency. The number of distinctive spatial frequencies and the spectrum distribution are the only available information of the disturbances.

(2) $f_t(x(t), \ \phi_f)$ and $g_t(x(t), \ \phi_g)$ are known vector-valued functions with unknown but bounded system parameters, i.e., $\phi_f=[\phi_{f1} \ \cdots \ \phi_{fk}]$ and $\phi_g=[\phi_{g1} \ \cdots \ \phi_{gl}]$.

(3) $\Delta f_t(x(t),\ \phi_f)$ and $\Delta g_t(x(t),\ \phi_g)$ represent unmodeled dynamics, which are also assumed to be bounded.

Consider an alternate variable $\theta=\lambda(t)$, i.e., the angular displacement, instead of time t as the independent variable. Because $\lambda(t)=\int_0^t \omega(\tau)d\tau+\lambda(0)$ where $\omega(t)$ is the angular velocity, the following condition

$$\omega(t)=\frac{d\theta}{dt}>0,\ \forall\ \mathrm{t}>0 \tag{2}$$

will ensure that $\lambda(t)$ is strictly monotonic, so that $t=\lambda^{-1}(\theta)$ exists. Hence, all the time-domain variables can be transformed into their counterparts in the θ-domain, i.e.,

$$\hat{x}(\theta)=x(\lambda^{-1}(\theta)),\ \hat{y}(\theta)=y(\lambda^{-1}(\theta)),\ \hat{u}(\theta)=u(\lambda^{-1}(\theta)),\ \hat{d}(\theta)=d(\lambda^{-1}(\theta)),\ \hat{\omega}(\theta)=\omega(\lambda^{-1}(\theta))$$

where we denote $\hat{\bullet}$ as the θ-domain representation of $\bullet$. Note that, in practice, (2) can usually be satisfied for most rotational motion system where the rotary component rotates only in one direction. Because

$$dx(t)/dt=d\theta/dt\cdot d\hat{x}(\theta)/d\theta=\hat{\omega}(\theta)\cdot d\hat{x}(\theta)/d\theta$$

(1) can be rewritten as

$$\begin{aligned}&\hat{\omega}(\theta)\frac{d\hat{x}(\theta)}{d\theta}=\left[f_t\left(\hat{x}(\theta),\phi_f\right)+\Delta f_t\left(\hat{x}(\theta),\phi_f\right)\right]+\left[g_t\left(\hat{x}(\theta),\phi_g\right)+\Delta g_t\left(\hat{x}(\theta),\phi_g\right)\right]\hat{u}(\theta)\\&\hat{y}(\theta)=\Psi\hat{x}(\theta)+\hat{d}_y(\theta)=\hat{x}_1(\theta)+\hat{d}_y(\theta).\end{aligned} \tag{3}$$

Equation (3) is an nonlinear position-invariant (NPI; as opposed to the definition of time-invariant) system with the θ as the independent variable. Note that we define the Laplace transform of a signal $\hat{g}(\theta)$ in the angular displacement domain as $\hat{G}(\tilde{s})=\int_0^\infty \hat{g}(\theta)e^{-\tilde{s}\theta}d\theta$.

This definition will be useful for describing the linear portion of the overall control system.

Drop the θ notation and rewrite (3) in the form

$$\dot{\hat{x}}=f(\hat{x},\phi_f)+g(\hat{x},\phi_g)\hat{u}+\hat{d}_s,\ \hat{y}=h(\hat{x})+\hat{d}_y=\hat{\omega}+\hat{d}_y \tag{4}$$

where terms involving unstructured uncertainty are merged into $\hat{d}_s = \Delta f(\hat{x}, \phi_f) + \Delta g(\hat{x}, \phi_g)\hat{u}$ with $\Delta f(\hat{x}, \phi_f) = \Delta f_t(\hat{x}, \phi_f)/\hat{x}_1$, $\Delta g(\hat{x}, \phi_g) = \Delta g_t(\hat{x}, \phi_f)/\hat{x}_1$. In addition, we have

$$f(\hat{x}, \phi_f) = f_t(\hat{x}, \phi_f)/\hat{x}_1\,,\ g(\hat{x}, \phi_g) = g_t\left(\hat{x}, \phi_g\right)/\hat{x}_1\,,\ h(\hat{x}) = \hat{\omega} = \hat{x}_1$$

The state variables have been specified such that the angular velocity $\hat{\omega}$ is equal to $\hat{x}_1$, i.e., the undisturbed output $h(\hat{x})$. To proceed, we will adopt the definitions and notations given in [24] for Lie derivative, relative degree, diffeomorphism.

It can be verified that (4) has the same relative degree in $D_0 = \{\hat{x} \in \mathbb{R}^n \mid \hat{x}_1 \neq 0\}$ as the NTI model in (1). If (4) has relative degree r, the following nonlinear coordinate transformation can be defined as

$$\hat{z} = T(\hat{x}) = \left[\psi_1(\hat{x}) \quad \cdots \quad \psi_{n-r}(\hat{x}) \mid h(\hat{x}) \quad \cdots \quad L_f^{r-1}h(\hat{x})\right]^T \triangleq \left[\begin{array}{c}\hat{z}_2 \\ \hline \hat{z}_1\end{array}\right]$$

where ψ_1 to ψ_{n-r} are chosen such that $T(\hat{x})$ is a diffeomorphism on $D_0 \subset D$ and

$$L_g\psi_i(\hat{x}) = 0,\ 1 \le i \le n-r$$

$\forall \hat{x} \in D_0$. With respect to the new coordinates, i.e., $\hat{z}_1$ and $\hat{z}_2$, (4) can be transformed into the so-called normal form, i.e.,

$$\begin{aligned}
\dot{\hat{z}}_2 &= L_f\psi(\hat{x})\Big|_{\hat{x}=T^{-1}(\hat{z})} + \hat{d}_{so} \triangleq \Psi(\hat{z}_1, \hat{z}_2) \\
\dot{\hat{z}}_1 &= A_c\hat{z}_1 + B_c\left[L_gL_f^{r-1}h(\hat{x})\Big|_{\hat{x}=T^{-1}(\hat{z})}\right]\left[\hat{u} + \frac{L_f^r h(\hat{x})}{L_gL_f^{r-1}h(\hat{x})}\Bigg|_{\hat{x}=T^{-1}(\hat{z})}\right] + \hat{d}_{si},\ \hat{y} = C_c\hat{z}_1 + \hat{d}_y
\end{aligned} \tag{5}$$

where $\hat{d}_{so}$ and $\hat{d}_{si} = [\hat{d}_{si_1} \ \cdots \ \hat{d}_{si_r}]^T$ come from $\hat{d}_s$ going through the indicated coordinate transformation. $\hat{z}_1 = [\hat{z}_{11} \ \cdots \ \hat{z}_{1r}]^T \in \mathbb{R}^r$, $\hat{z}_2 \in \mathbb{R}^{n-r}$, and (A_c, B_c, C_c) is a canonical form representation of a chain of r integrators. The first equation in (5) is the internal dynamics and not affected by the control $\hat{u}$. By setting $\hat{z}_1 = 0$, we obtain $\dot{\hat{z}}_2 = \Psi(0, \hat{z}_2)$, which is the zero dynamics of (4) or (5). The system is called minimum phase if the zero dynamics has an asymptotically stable equilibrium point in the domain of interest. To allow us to present the proposed algorithm and stability analysis in a simpler context, we will make the following assumptions for the subsequent derivation.

Assumption 2.2

(1) $f(\hat{x}(\theta),\ \phi_f)$ and $g(\hat{x}(\theta),\ \phi_g)$ are linearly related to those unknown system parameters, i.e.,

$$f\left(\hat{x}(\theta),\phi_f\right)=\phi_{f1}f_1\left(\hat{x}(\theta)\right)+\ldots+\phi_{fk}f_k\left(\hat{x}(\theta)\right),\ g\left(\hat{x}(\theta),\phi_g\right)=\phi_{g1}g_1\left(\hat{x}(\theta)\right)+\ldots+\phi_{gl}g_l\left(\hat{x}(\theta)\right) \tag{6}$$

(2) (4) is exponentially minimum phase, i.e., the zero dynamics is exponentially stable;

(3) The output disturbance is sufficiently smooth [i.e., $\hat{d}_y,\ \cdots,\ \hat{d}_y^{(r)}$ exists];

(4) $\hat{d}_{si_1}^{(r-1)},\ \hat{d}_{si_2}^{(r-2)},\ \cdots,\ \hat{d}_{si_{r-1}}$ exist, i.e., the transformed unstructured uncertainty is sufficiently smooth; and

(5) The reference command $\hat{y}_m$ and its first r derivates are known and bounded. Moreover, $\hat{y}_m^{(r)}$ is piecewise continuous.

With Assumption 2, the design of a nonlinear state observer may focus on the external dynamics of (5), i.e.,

$$\dot{\hat{z}}_1=A_c\hat{z}_1+B_c\left[L_gL_f^{r-1}h(\hat{x})\Big|_{\hat{x}=T^{-1}(\hat{z})}\right]\left[\hat{u}+\frac{L_f^r h(\hat{x})}{L_gL_f^{r-1}h(\hat{x})}\Bigg|_{\hat{x}=T^{-1}(\hat{z})}\right]+\hat{d}_{si} \tag{7}$$

2.1 State observer design

In this section, we show how to establish a state observer for the transformed NPI system (5). Because $f(\hat{x})$ and $g(\hat{x})$ are assumed to be linearly related to system parameters, $L_gL_f^{r-1}h(\hat{x})$ and $L_gL_f^{r-1}h(\hat{x})$ can be expressed as

$$L_f^r h(\hat{x})=\Theta^T W_f\left(\hat{x}\right),\ L_gL_f^{r-1}h(\hat{x})=\Theta^T W_g\left(\hat{x}\right)$$

where $W_f(\hat{x})$ and $W_g(\hat{x})$ are two nonlinear functions, and

$$\Theta=\begin{bmatrix}\phi_{f1} & \cdots & \phi_{fk} & \phi_{g1} & \cdots & \phi_{gl} & \cdots\end{bmatrix}^T=\begin{bmatrix}\phi_1 & \cdots & \phi_\ell\end{bmatrix}^T\in\mathbb{R}^\ell.$$

where ℓ denotes the number of unknown parameters. Hence, (7) can be rewritten as

$$\dot{\hat{z}}_1=A_c\hat{z}_1+B_c\left[\Theta^T W_g\left(\hat{x}\right)\hat{u}+\Theta^T W_f\left(\hat{x}\right)\right]+\hat{d}_{si} \tag{8}$$

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Equation (8) can be further written in the form

$$\dot{\hat{z}}_1 = A_0\hat{z}_1 + \bar{k}\,\hat{z}_{11} + B_c\left[\Theta^T W_g(\hat{x})\hat{u} + \Theta^T W_f(\hat{x})\right] + \hat{d}_{si}, \tag{9}$$

where $A_0 = \begin{bmatrix} -k_1 & I_{(r-1)\times(r-1)} \\ \vdots & \\ -k_r & 0_{1\times(r-1)} \end{bmatrix}$ and $\bar{k} = [k_1 \quad \cdots \quad k_r]^T$.

By properly choosing $\bar{k}$, the matrix A_0 can be made Hurwitz. Next, we adopt the following observer structure:

$$\dot{\bar{z}}_1 = A_0\bar{z}_1 + \bar{k}\hat{y} + B_c\left[\Theta^T \bar{W}_g(\hat{y})\hat{u} + \Theta^T \bar{W}_f(\hat{y})\right] \tag{10}$$

where $\bar{z}_1 = [\bar{z}_{11} \quad \cdots \quad \bar{z}_{1r}]^T$ is the estimate of $\hat{z}_1$ and $\bar{W}_f(\hat{y})$ and $\bar{W}_g(\hat{y})$ are nonlinear functions with the same structure as $W_f(\hat{x})$ and $W_g(\hat{x})$, except that each entry of $\hat{x}$ is replaced by $\hat{y}$. Equation (10) can be further expressed as

$$\dot{\bar{z}}_1 = A_0\bar{z}_1 + \bar{k}\hat{y} + F(\hat{y},\hat{u})^T\Theta \text{ with } F(\hat{y},\hat{u})^T = \begin{bmatrix} 0_{(r-1)\times\ell} \\ \bar{W}_f^T(\hat{y}) + \bar{W}_g^T(\hat{y})\hat{u} \end{bmatrix} \in \mathbb{R}^{r\times\ell} \tag{11}$$

Define the state estimated error as $\varepsilon \triangleq [\varepsilon_{\hat{z}_{11}} \quad \cdots \quad \varepsilon_{\hat{z}_{1r}}]^T \triangleq \hat{z}_1 - \bar{z}_1$. The dynamics of the estimated error can be obtained by subtracting (10) from (9), i.e.,

$$\dot{\varepsilon} = A_0\varepsilon + \Delta \quad \Delta = -\bar{k}\hat{d}_y + B_c\Theta^T\left[W_g(\hat{x}) - \bar{W}_g(\hat{y})\right]\hat{u} + B_c\Theta^T\left[W_f(\hat{x}) - \bar{W}_f(\hat{y})\right] + \hat{d}_{si}. \tag{12}$$

Here, we further assume that

Assumption 2.3

(9) $W_g(\hat{x}) - \bar{W}_g(\hat{y})$ and $W_f(\hat{x}) - \bar{W}_f(\hat{y})$ are bounded to ensure the boundness of the estimated error. To see this, note that the solution of (12) may be viewed as sum of zero input response ε_u and zero state response ε_s, i.e., $\varepsilon = \varepsilon_u + \varepsilon_s$. The zero input response $\dot{\varepsilon}_u = A_0\varepsilon_u$ will decay to zero exponentially, as A_0 is Hurwitz, and the zero state response ε_s will be bounded due to the bounded disturbance $\hat{d}_y$, $W_g(\hat{x}) - \bar{W}_g(\hat{y})$, and $W_f(\hat{x}) - \bar{W}_f(\hat{y})$.

Equation (10) or (11) cannot be readily implemented due to the unknown parametric vector Θ, but it motivates the subsequent mathematical manipulation. Define the state estimate as $\bar{z}_1 \triangleq \xi + \Omega^T\Theta$ such that $\xi = [\xi_{11} \quad \cdots \quad \xi_{1r}]^T \in \mathbb{R}^r$ and $\Omega^T \in \mathbb{R}^{r\times\ell}$ and employ the following two K-filters:

$$\dot{\xi} = A_0\xi + \overline{k}\hat{y},\ \dot{\Omega}^T = A_0\Omega^T + F(\hat{y},\hat{u})^T. \tag{13}$$

It can be easily verified that (13) is equivalent to (11). Hence, (13) may replace the role of (11) for providing the state estimate. With $\Omega^T \triangleq [v_1 \ \cdots \ v_\ell]$, the second equation of (13) may be further decomposed into

$$\dot{v}_j = A_0 v_j + e_r\sigma_j,\ j = 1,2,\cdots,\ell \tag{14}$$

where $e_r = [0 \ \cdots \ 0 \ 1] \in \mathbb{R}^r$ and $\sigma_j = w_{1j} + w_{2j}\hat{u}$ with w_{1j} and w_{2j} are the j^{th} columns of $\overline{W}_f^T(\hat{y})$ and $\overline{W}_g^T(\hat{y})$, respectively. Equation (13) is still not applicable due to Θ. However, with the definition of the state estimated error ε, the state estimate, the first equation of (13), and (14), we acquire the following relationship that is not available from (11):

$$\begin{aligned}\hat{z}_{11} &= \overline{z}_{11} + \varepsilon_{\hat{z}_{11}} = \xi_{11} + \sum_{j=1}^{\ell} v_{j,1}\phi_j + \varepsilon_{\hat{z}_{11}},\ \ldots,\ \hat{z}_{1r} \\ &= \overline{z}_{1r} + \varepsilon_{\hat{z}_{1r}} = \xi_{1r} + \sum_{j=1}^{\ell} v_{j,r}\phi_j + \varepsilon_{\hat{z}_{1r}}\end{aligned} \tag{15}$$

where $\bullet_{j,i}$ denotes the i^{th} row of $\bullet_j$. Equation (15) will be used in the subsequent design.

2.2 Output feedback robust adaptive repetitive control system

In this section, we show how to incorporate the state observer established in the previous section into an output feedback adaptive repetitive control system. The control configuration consists of two layers. The first layer is the adaptive feedback linearization, which tackles system nonlinearity and parametric uncertainty. The second layer is a repetitive control module of a repetitive controller and a loop-shaping filter. This layer not only enhances the ability of the overall system for rejection of disturbance, sensitivity reduction to model uncertainty, and state estimated error but also improves the robustness of the parametric adaptation. Although inclusion of the state observer relieves the design of the need of full-state feedback, it actually introduces extra dynamics into the system. Hence, the stability of the resulting system needs to be further justified.

Suppose that (4) has relative degree r. To perform input/output feedback linearization, differentiate the output $\hat{y}$ until the control input $\hat{u}$ appears to obtain

$$\hat{y}^{(r)} = \hat{z}_{11}^{(r)} + \hat{d}_y^{(r)} = \dot{\hat{z}}_{1r} + \hat{d}_y^{(r)} = \dot{\overline{z}}_{1r} + \dot{\varepsilon}_{\hat{z}_{1r}} + \hat{d}_y^{(r)} \tag{16}$$

Substituting the r^{th} state equation of (10) into (16), we have

$$\hat{y}^{(r)} = \dot{\bar{z}}_{1r} + \dot{\varepsilon}_{\hat{z}_{1r}} + \hat{d}_y^{(r)} = -k_r \bar{z}_{11} + k_r \hat{y} + \Theta^T \bar{W}_f\left(\hat{y}\right) + \Theta^T \bar{W}_g\left(\hat{y}\right)\hat{u} + \dot{\varepsilon}_{\hat{z}_{1r}} + \hat{d}_y^{(r)} z \tag{17}$$

To put the previously developed state observer into use, we substitute the first equation of (15) into (17) and arrive at

$$\hat{y}^{(r)} = -k_r\left(\xi_{11} + \sum_{j=1}^{\ell} v_{j,1}\phi_j\right) + k_r \hat{y} + \Theta^T \bar{W}_f\left(\hat{y}\right) + \Theta^T \bar{W}_g\left(\hat{y}\right)\hat{u} + \dot{\varepsilon}_{\hat{z}_{1r}} + \hat{d}_y^{(r)} \tag{18}$$

Define the estimated parametric vector of Θ as

$$\tilde{\Theta} = \begin{bmatrix} \tilde{\phi}_{f1} & \cdots & \tilde{\phi}_{fk} & \tilde{\phi}_{g1} & \cdots & \tilde{\phi}_{gl} & \cdots \end{bmatrix}^T = \begin{bmatrix} \tilde{\phi}_1 & \cdots & \tilde{\phi}_\ell \end{bmatrix}^T \in \mathbb{R}^{\ell}.$$

The control law using the estimated system parameters and states is

$$\hat{u} = \frac{1}{\tilde{\Theta}^T \bar{W}_g\left(\hat{y}\right)}\left(-\tilde{\Theta}^T \bar{W}_f\left(\hat{y}\right) + k_r\left(\xi_{11} + \sum_{j=1}^{\ell} v_{j,1}\tilde{\phi}_j\right) - k_r \hat{y} + \tilde{\hat{v}}_d + \hat{u}_{\hat{R}}\right), \tag{19}$$

where we introduce two designable inputs, $\tilde{\hat{v}}_d$ and $\hat{u}_{\hat{R}}$. Specify $\tilde{\hat{v}}_d$, the estimate of $\hat{v}_d$, as

$$\tilde{\hat{v}}_d = \hat{y}_m^{(r)} + \alpha_1(\hat{y}_m^{(r-1)} - \tilde{\hat{y}}^{(r-1)}) + \cdots + \alpha_{r-1}(\dot{\hat{y}}_m - \dot{\tilde{\hat{y}}}) + \alpha_r(\hat{y}_m - \hat{y}), \tag{20}$$

where $\hat{y}_m$ is a prespecified reference trajectory, $\tilde{\hat{y}}^{(k)}$ denotes the estimate of $\hat{y}^{(k)}$, and α_i's are adjustable parameters. Substituting (19) back into (18) and defining the tracking error $\hat{e} \triangleq \hat{y} - \hat{y}_m$, we arrive at the following error equation:

$$\begin{aligned} &\hat{e}^{(r)} + \alpha_1 \hat{e}^{(r-1)} + \cdots + \alpha_{r-1}\dot{\hat{e}} + \alpha_r \hat{e} = \Phi^T W + \hat{u}_{\hat{R}} + \hat{d}_y^{(r)} + \dot{\varepsilon}_{\hat{z}_{1r}} \\ &+\alpha_1\left(\hat{d}_y + \varepsilon_{\hat{z}_{11}}\right)^{(r-1)} + \cdots + \alpha_{r-1}\left(\dot{\hat{d}}_y + \dot{\varepsilon}_{\hat{z}_{11}}\right), \end{aligned} \tag{21}$$

where $\Phi = \Theta - \tilde{\Theta}$ and W is a function of ξ, v, and $\tilde{\Theta}$. If we denote $M(\tilde{s}) = 1/(\tilde{s}^r + \alpha_1 \tilde{s}^{r-1} + \cdots + \alpha_r)$, (21) implies that

$$\frac{1}{M(\tilde{s})}\hat{E}(\tilde{s}) = \Phi^T W + \hat{U}_{\hat{R}}(\tilde{s}) + \left(\tilde{s}^r + \alpha_1 \tilde{s}^{r-1} + \cdots + \alpha_{r-1}\tilde{s}\right)\hat{d}_y \\ + \left(\alpha_1 \tilde{s}^{r-1} + \cdots + \alpha_{r-1}\tilde{s}\right)\varepsilon_{\hat{z}_{11}} + \tilde{s}\varepsilon_{\hat{z}_{1r}}. \tag{22}$$

Neglecting the details of $\Phi^T W$, we can view (21) or (22) as a linear system (with the output $\hat{e}$) subject to five inputs. We propose adding another control loop between $\hat{E}(\tilde{s})$ and $\hat{U}_{\hat{R}}(\tilde{s})$. This control loop provides an additional degree-of-freedom for reducing the effect of the unstructured uncertainty, the state estimated error, and the output disturbance. The tracking error $\hat{E}(\tilde{s})$ and the control input $\hat{U}_{\hat{R}}(\tilde{s})$ is related by

$$\hat{U}_{\hat{R}}(\tilde{s}) = -\hat{R}(\tilde{s})\hat{C}(\tilde{s})\hat{E}(\tilde{s}),\ \hat{R}(\tilde{s}) = \prod_{i=1}^{k} \frac{\tilde{s}^2 + 2\zeta_i \omega_{ni}\tilde{s} + \omega_{ni}^2}{\tilde{s}^2 + 2\xi_i \omega_{ni}\tilde{s} + \omega_{ni}^2} \text{ (low-order repetitive controller)} \tag{23}$$

where k is the number of periodic frequencies, ω_{ni} is the ith disturbance frequency in rad/rev, and ξ_i and ζ_i are damping ratios with $0<\xi_i<\zeta_i<1$. The gain of $\hat{R}(\tilde{s})$ at those periodic frequencies may be varied by adjusting the values of ξ_i and ζ_i. Furthermore, $\hat{C}(\tilde{s})$ is a controller that should ensure the stability of the overall system. Substitute (23) back into (22), we obtain

$$\left[1/M(\tilde{s}) + \hat{R}(\tilde{s})\hat{C}(\tilde{s})\right]\hat{E}(\tilde{s}) = \Phi^T W + \left(\tilde{s}^r + \alpha_1 \tilde{s}^{r-1} + \cdots + \alpha_{r-1}\tilde{s}\right)\hat{d}_y \\ + \left(\alpha_1 \tilde{s}^{r-1} + \cdots + \alpha_{r-1}\tilde{s}\right)\varepsilon_{\hat{z}_{11}} + \tilde{s}\varepsilon_{\hat{z}_{1r}} \tag{24}$$

Define

$$\bar{M}(\tilde{s}) \triangleq \left[1/M(\tilde{s}) + \hat{R}(\tilde{s})\hat{C}(\tilde{s})\right]^{-1}, \tag{25}$$

Equation (24) becomes

$$\hat{e} = \bar{M}(\tilde{s})\Phi^T W + \hat{d}_{\bar{M}},\ \hat{d}_{\bar{M}} = \bar{M}(\tilde{s})\Big[\left(\tilde{s}^r + \alpha_1 \tilde{s}^{r-1} + \cdots + \alpha_{r-1}\tilde{s}\right)\hat{d}_y \\ + \left(\alpha_1 \tilde{s}^{r-1} + \cdots + \alpha_{r-1}\tilde{s}\right)\varepsilon_{\hat{z}_{11}} + \tilde{s}\varepsilon_{\hat{z}_{1r}}\Big] \tag{26}$$

where

$$\hat{d}_{\bar{M}} = \bar{M}(\tilde{s})\Big[\left(\tilde{s}^r + \alpha_1 \tilde{s}^{r-1} + \cdots + \alpha_{r-1}\tilde{s}\right)\hat{d}_y + \left(\alpha_1 \tilde{s}^{r-1} + \cdots + \alpha_{r-1}\tilde{s}\right)\varepsilon_{\hat{z}_{11}} + \tilde{s}\varepsilon_{\hat{z}_{1r}}\Big]$$

Because $\hat{e}, \hat{\dot{e}}, \cdots, \hat{e}^{(r-1)}$ cannot be measured directly, the so-called augmented error scheme will be used. The augmented error is defined as

$$\hat{e}_1 = \hat{e} + \left(\Phi^T \bar{M}(\tilde{s})W - \bar{M}(\tilde{s})\Phi^T W\right). \tag{27}$$

Substituting (26) into (27), we obtain

$$\hat{e}_1 = \Phi^T \bar{\varsigma} + \hat{d}_{\bar{M}}, \tag{28}$$

where $\bar{\varsigma} = \bar{M}(\tilde{s})W$. The parametric adaptation law to be used is modified from the normalized gradient method proposed in [33], i.e.,

$$\dot{\tilde{\Theta}} = -\dot{\Phi} = \begin{cases} \dfrac{\rho \hat{e}_1 \bar{\varsigma}}{1+\bar{\varsigma}^T\bar{\varsigma}} & \text{if } |\hat{e}_1| > \hat{d}_{\bar{M}_0} \text{ and } \tilde{\Theta} \in w^0, \\ P_R\left(\dfrac{\rho \hat{e}_1 \bar{\varsigma}}{1+\bar{\varsigma}^T\bar{\varsigma}}\right) & \text{if } |\hat{e}_1| > \hat{d}_{\bar{M}_0}, \tilde{\Theta} \in \partial w, \text{ and } \hat{e}_1 \bar{\varsigma}^T \tilde{\Theta}_{perp} > 0, \\ 0 & \text{if } |\hat{e}_1| \le \hat{d}_{\bar{M}_0}, \end{cases} \tag{29}$$

where w is the allowable parametric variation set (compact and convex) with its interior and boundary denoted by w^0 and ∂w, respectively, $\hat{d}_{\bar{M}_0}$ is an upper bound for the magnitude of $\hat{d}_{\bar{M}}$, and ρ is an adjustable adaptation rate that affects the convergence property. If the magnitude of $\hat{e}_1$ is small and dominated by the magnitude of $\hat{d}_{\bar{M}}$, the adaptation law is disabled to prevent the parameters from being adjusted based on the disturbance. If $\hat{e}_1$ is greater than $\hat{d}_{\bar{M}}$ magnitude-wise, two scenarios need to be considered. If the current estimated parametric vector locates within the allowable parametric set, regular adaptation law is applied. If the current estimated parametric vector is on the boundary of the allowable parametric set, the projected adaptation law is employed to stop the parametric vector from leaving the variation set.

In the following, we present stability theorem for the proposed spatial-based OFLRARC system. The theorem extends the results in the literature [33,34] to take into account the addition of the repetitive control module. It will be seen that the overall OFLRARC system will stay stable and the tracking error will be bounded as long as a stable and proper loop-shaping filter stabilizes a certain feedback system.

Theorem 2.1 The error equation (28) with the parametric update law (29) leads to $\Phi \in L_\infty$, $\dot{\Phi} \in L_2 \cap L_\infty$, and $\| \Phi^T \bar{\varsigma}(\theta) \|_2 \le \gamma(1 + \| \bar{\varsigma} \|_{TL_\infty})$ for all θ.

Proof: Follow the same steps for proof of Theorem 3.1 in [24].

Theorem 2.2 Consider an exponentially minimum-phase nonlinear system with parameter uncertainty and subject to output disturbance as given by (4), which is augmented with a state observer (or K-filters) described by (13) [35]. Specify the control laws as (19), (20), and (23). Let Assumptions (1) to (9) be satisfied. Assume that $\hat{y}_m, \hat{y}_m, \cdots, \hat{y}_m^{(r-1)}$ (where r is the relative degree) and $\hat{d}_{\bar{M}}$ are bounded with an upper bound $\hat{d}_{\bar{M}_0}$, f, g, h, $L_f^k h$, $L_g L_f h$ are Lipschitz continuous functions, and W has bounded derivative with respect to ξ, v, and $\widetilde{\Theta}$. In addition, assume that a stable and proper controller $\hat{C}(\tilde{s})$ is specified such that the feedback system shown in **Figure.1** is stable. Then, the parametric adaptation law given by (29) yields the bounded tracking error, i.e., $|\hat{y}(\theta)-\hat{y}_m(\theta)| < \hat{d}_{\bar{M}_0}$ as $\theta \to \infty$.

Proof: Follow the same steps for proof of Theorem 3.2 in [24] with some differences.

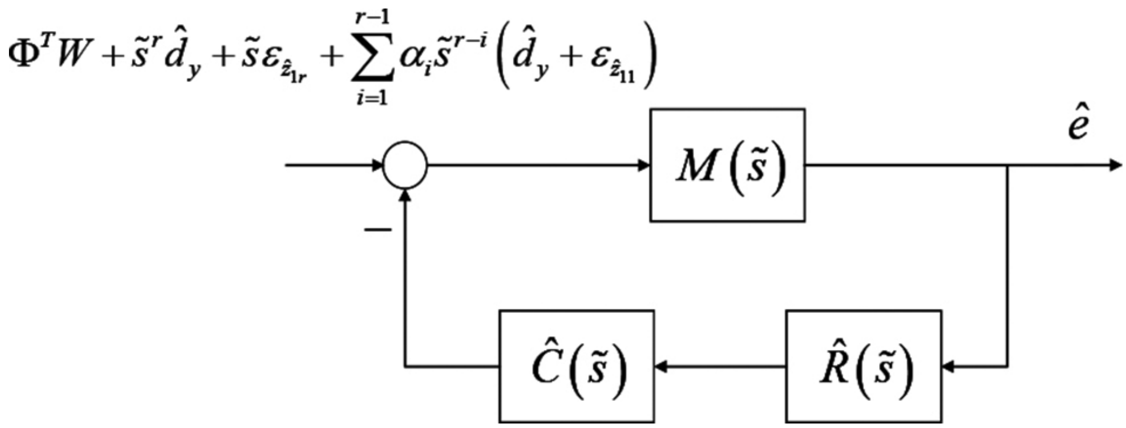

Figure 1. Repetitive controller and stabilizing compensator.

3.Spatial-based output feedback backstepping robust adaptive repetitive control (OFBRARC)

Consider the same NPI model (3), which is transformed from the NTI model (1), under the same set of assumptions (Assumptions 2.1 and 2.2). The NPI model will be used for the subsequent design and discussion.

3.1 Nonlinear state observer

Drop the θ notation and note that (3) can be expressed as a standard nonlinear system:

$$\dot{\hat{y}} = \dot{\xi}_{11} + \left(-k_1 v_{01} + v_{02}\right) b_0 + \dot{\Xi}_1^T a + \dot{\varepsilon}_{\hat{x}_1} + \dot{\hat{d}}_y, \; \dot{v}_{02} = -k_2 v_{01} + \sigma \hat{u}. \tag{30}$$

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where terms involving unstructured uncertainty are merged into $\hat{d}_s = \Delta f(\hat{x}, \phi_f) + \Delta g(\hat{x}, \phi_g)\hat{u}$ with

$$\Delta f\left(\hat{x}, \phi_f\right) = \frac{\Delta f_t(\hat{x}, \phi_f)}{\hat{x}_1}, \Delta g\left(\hat{x}, \phi_g\right) = \frac{\Delta g_t(\hat{x}, \phi_g)}{\hat{x}_1}$$

In addition, we have

$$f(\hat{x}, \phi_f) = \frac{f_t(\hat{x}, \phi_f)}{\hat{x}_1}, g(\hat{x}, \phi_g) = \frac{g_t\left(\hat{x}, \phi_g\right)}{\hat{x}_1}, h(\hat{x}) = \hat{\omega} = \hat{x}_1$$

The state variables have been specified such that the angular velocity $\hat{\omega}$ is equal to $\hat{x}_1$, i.e., the undisturbed output $h(\hat{x})$. It is not difficult to verify that (30) has the same relative degree in $D_0 = \{\hat{x} \in \mathbb{R}^n \mid \hat{x}_1 \neq 0\}$ as the NTI model in (1). If (30) has relative degree r, we can use the same nonlinear coordinate transformation defined previously. With respect to the new coordinates, i.e., $\hat{z}_1$ and $\hat{z}_2$, (30) can be transformed into the so-called normal form, i.e., (5). With zero dynamics being assumed to be asymptotically stable, we may focus on designing a nonlinear state observer for external dynamics of (5), i.e., (7).

Because $f(\hat{x})$ and $g(\hat{x})$ are linearly related to system parameters, $L_g L_f^{r-1} h(\hat{x})$ and $L_g L_f^{r-1} h(\hat{x})$ can be written as $L_f^r h(\hat{x}) = \Theta^T W_f(\hat{x})$ and $L_g L_f^{r-1} h(\hat{x}) = \Theta^T W_g(\hat{x})$, where $W_f(\hat{x})$ and $W_g(\hat{x})$ are two nonlinear functions, and $\Theta = [\phi_{f1} \ \cdots \ \phi_{fk} \ \phi_{g1} \ \cdots \ \phi_{gl} \ \cdots]^T = [\phi_1 \ \cdots \ \phi_\ell]^T \in \mathbb{R}^\ell$, where ℓ is the number of unknown parameters. Next, we adopt the following observer structure: $\dot{\bar{z}}_1 = A_0 \bar{z}_1 + \bar{k} y + F(y, u)^T \Theta$, where $\bar{z}_1 = [\bar{z}_{11} \ \cdots \ \bar{z}_{1r}]^T$ is the estimate of z_1 and $\overline{W}_f(y)$ and $\overline{W}_g(y)$ are nonlinear functions with the same structure as $W_f(x)$ and $W_g(x)$, except that each entry of x is replaced by y. Furthermore, $A_0 = \begin{bmatrix} -k_1 & I_{(r-1)\times(r-1)} \\ \vdots & \\ -k_r & 0_{1\times(r-1)} \end{bmatrix}$, $\bar{k} = [k_1 \ \cdots \ k_r]^T$, and $F(y, u)^T = \begin{bmatrix} 0_{(r-1)\times\ell} \\ \overline{W}_f^T(y) + \overline{W}_g^T(y) u \end{bmatrix} \in \mathbb{R}^{r\times\ell}$.

By properly choosing $\bar{k}$, the matrix A_0 can be made Hurwitz. Define the state estimated error as $\varepsilon \triangleq [\varepsilon_{z_{11}} \ \cdots \ \varepsilon_{z_{1r}}]^T \triangleq z_1 - \bar{z}_1$. The dynamics of the estimated error can be obtained as $\dot{\varepsilon} = A_0 \varepsilon + \Delta$, where $\Delta = -\bar{k} d_y + B_c \Theta^T [W_g(x) - \overline{W}_g(y)] u + B_c \Theta^T [W_f(x) - \overline{W}_f(y)] + d_{si}$. To proceed, the role of the state observer is replaced by $\bar{z}_1 \triangleq \xi + \Omega^T \Theta$ and the following two K-filters:

$$\dot{\xi} = A_0 \xi + \bar{k} y, \ \dot{\Omega}^T = A_0 \Omega^T + F(y, u)^T \tag{31}$$

such that $\xi = [\xi_{11} \quad \cdots \quad \xi_{1r}]^T \in \mathbb{R}^r$ and $\Omega^T \triangleq [v_1 \quad \cdots \quad v_\ell] \in \mathbb{R}^{r\times\ell}$. Decompose the second equation of (31) into $\dot{v}_j = A_0 v_j + e_r \sigma_j,\ \ j=1,2,\ \cdots,\ \ell$, where $e_r = [0 \quad \cdots \quad 0 \quad 1] \in \mathbb{R}^r$ and $\sigma_j = w_{1j} + w_{2j} u$ with w_{1j} and w_{2j} are the j^{th} columns of $\overline{W}_f^T(y)$ and $\overline{W}_g^T(y)$, respectively. With the definition of the state estimated error ε, the state estimate $\bar{z}_1$, and (31), we acquire the following set of equations that will be used in the subsequent design:

$$z_{1k} = \overline{z}_{1k} + \varepsilon_{z_{1k}} = \xi_{1k} + \sum_{j=1}^{\ell} v_{j,k}\phi_j + \varepsilon_{z_{1k}},\ k = 1,\dots,r \tag{32}$$

where $\bullet_{j,i}$ denotes the i th row of $\bullet_j$.

3.2 Spatial domain output feedback adaptive control system

To apply adaptive backstepping method, we first rewrite the derivative of output $\hat{y}$ as

$$\dot{\hat{y}} = \dot{\hat{z}}_{11} + \dot{\hat{d}}_y = \hat{z}_{12} + \hat{d}_{si_1} + \dot{\hat{d}}_y = \overline{z}_{12} + \varepsilon_{\hat{z}_{12}} + \hat{d}_{si_1} + \dot{\hat{d}}_y \tag{33}$$

With the second equation in (32), (33) can be written as

$$\dot{\hat{y}} = \overline{z}_{12} + \varepsilon_{\hat{z}_{12}} + \hat{d}_{si_1} + \dot{\hat{d}}_y = \xi_{12} + v_{\ell,2}\phi_\ell + \overline{\omega}^T\Theta + \varepsilon_{\hat{z}_{12}} + \hat{d}_{si_1} + \dot{\hat{d}}_y$$

where $\overline{\omega}^T = [v_{1,2} \quad \cdots \quad v_{\ell-1,2} \quad 0]$.

In view of designing output feedback backstepping with K-filters, we need to find a set of K-filter parameters, i.e., $v_{\ell,2},\ \cdots,\ v_{1,2}$, separated from $\hat{u}$ by the same number of integrators between $\hat{z}_{12}$ and $\hat{u}$. From (31), we see that $v_{\ell,2},\ \cdots,\ v_{1,2}$ are all candidates if w_{2j} are not zero. In the subsequent derivation, we assume that $v_{\ell,2}$ is selected. Therefore, the system incorporated the K-filters can be represented by

$$\begin{aligned}&\dot{\hat{y}} = \xi_{12} + v_{\ell,2}\phi_\ell + \overline{\omega}^T\Theta + \varepsilon_{\hat{z}_{12}} + \hat{d}_{si_1} + \dot{\hat{d}}_y,\ \dot{v}_{\ell,i} = v_{\ell,i+1} - \\ &k_i v_{\ell,1},\ i = 2,\cdots,r-1,\ \dot{v}_{\ell,r} = -k_r v_{\ell,1} + w_{1\ell} + w_{2\ell}\hat{u}\end{aligned} \tag{34}$$

To apply adaptive backstepping to (34), a new set of coordinates will be introduced

$$z_1 = \hat{y} - \hat{y}_m,\ z_i = v_{\ell,i} - \alpha_{i-1},\ i = 2,\cdots,r \tag{35}$$

where $\hat{y}_m$ is the prespecified reference output, and α_{i-1} is the virtual input to be used for stabilizing each state equation. For simplicity, we define $\partial\alpha_0/\partial\hat{y} \triangleq -1$ for subsequent derivations.

Step 1:$i=1$With (35), the first state equation in (34) can be expressed as

$$\dot{z}_1 = \xi_{12} + z_2\phi_\ell + \alpha_1\phi_\ell + \overline{\omega}^T\Theta + \varepsilon_{\hat{z}_{12}} + \hat{d}_{si_1} + \dot{\hat{d}}_y - \dot{\hat{y}}_m \tag{36}$$

Consider a Lyapunov function $V_1=(1/2)z_1^2$ and calculate its derivative

$$\dot{V}_1 = z_1\dot{z}_1 = z_1\left(\xi_{12} + z_2\phi_\ell + \alpha_1\phi_\ell + \overline{\omega}^T\Theta + \varepsilon_{\hat{z}_{12}} + \hat{d}_{si_1} + \dot{\hat{d}}_y - \dot{\hat{y}}_m\right) \tag{37}$$

Define the estimates of ϕ_i as $\tilde{\phi}_i$ and $\Phi=[\Phi_1 \ \cdots \ \Phi_\ell]=\Theta-\tilde{\Theta}$, where $\tilde{\Theta}=[\tilde{\phi}_{f1} \ \cdots \ \tilde{\phi}_{fk} \ \tilde{\phi}_{g1} \ \cdots \ \tilde{\phi}_{gl} \ \cdots]^T=[\tilde{\phi}_1 \ \cdots \ \tilde{\phi}_\ell]^T \in \mathbb{R}^\ell$. Note that Θ is the "true" parameter vector, whereas $\tilde{\Theta}$ is the estimated parameter vector. Design the virtual input α_1 as $\alpha_1=\bar{\alpha}_1/\tilde{\phi}_\ell$ and specify

$$\bar{\alpha}_1 = \frac{1}{z_1}\left(-z_1\xi_{12} - z_1z_2\tilde{\phi}_\ell - z_1\overline{\omega}\tilde{\Theta} + z_1\dot{\hat{y}}_m - c_1z_1^2 - d_1z_1^2 - g_1z_1^2\right) \tag{38}$$

where c_i, d_i, g_i are variables. Therefore, (37) becomes

$$\dot{V}_1 = -c_1z_1^2 - d_1z_1^2 - g_1z_1^2 + \tau_1\Phi + z_1\left(\varepsilon_{\hat{z}_{12}} + \hat{d}_{si_1} + \dot{\hat{d}}_y\right) \tag{39}$$

where $\tau_1\Phi=z_1z_2\Phi_\ell + \alpha_1\Phi_\ell + z_1\overline{\omega}^T\Phi$.

Step 2:$i=2, \ \cdots, \ r-1$With respect to the new set of coordinates (35), the second equation of (34) can be rewritten as

$$\begin{aligned}\dot{z}_i = z_{i+1} + \alpha_i - k_i v_{\ell,1} - \Bigg[&\frac{\partial\alpha_{i-1}}{\partial\hat{y}}\left(\xi_{12} + v_{\ell,2}\phi_\ell + \overline{\omega}^T\Theta + \varepsilon_{\hat{z}_{12}} + \hat{d}_{si_1} + \dot{\hat{d}}_y\right) + \\ &\frac{\partial\alpha_{i-1}}{\partial\xi}\left(A_0\xi + \bar{k}\hat{y}\right) + \frac{\partial\alpha_{i-1}}{\partial\tilde{\Theta}}\dot{\tilde{\Theta}} \\ &\sum_{j=1}^{\ell}\frac{\partial\alpha_{i-1}}{\partial v_j}\left(A_0v_j + e_r\sigma_j\right) + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{y}_m^{(j-1)}}\hat{y}_m^{(j)}\Bigg]\end{aligned}$$

Consider a Lyapunov function $V_i = \sum_{j=1}^{i-1} V_j + \frac{1}{2} z_i^2$. Specify

$$\alpha_i = \frac{1}{z_i}\Bigg\{ -z_i z_{i+1} + z_i k_i v_{\ell,1} + z_i \Bigg[\frac{\partial \alpha_{i-1}}{\partial \hat{y}} \left(\xi_{12} + v_{\ell,2}\tilde{\phi}_\ell + \bar{\omega}^T \tilde{\Theta} \right) + \frac{\partial \alpha_{i-1}}{\partial \xi} \left(A_0 \xi + \bar{k}\hat{y} \right) + \frac{\partial \alpha_{i-1}}{\partial \tilde{\Theta}} \dot{\tilde{\Theta}}$$
$$\sum_{j=1}^{\ell} \frac{\partial \alpha_{i-1}}{\partial v_j} \left(A_0 v_j + e_r \sigma_j \right) + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{y}_m^{(j-1)}} \hat{y}_m^{(j)} \Bigg] - c_i z_i^2 - d_i \left(\frac{\partial \alpha_{i-1}}{\partial \hat{y}} \right)^2 z_i^2 - g_i \left(\frac{\partial \alpha_{i-1}}{\partial \hat{y}} \right)^2 z_i^2 \Bigg\}$$

The derivative of V_i becomes

$$\dot{V}_i = -\sum_{j=1}^{i-1} \left(c_j z_j^2 + d_j \left(\frac{\partial \alpha_{j-1}}{\partial \hat{y}} \right)^2 z_j^2 + g_j \left(\frac{\partial \alpha_{j-1}}{\partial \hat{y}} \right)^2 z_j^2 \right) + \tau_i \Phi - \sum_{j=1}^{i-1} z_j \frac{\partial \alpha_{j-1}}{\partial \hat{y}} \left(\varepsilon_{\hat{z}12} + \hat{d}_{si_1} + \dot{\hat{d}}_y \right)$$

where $\tau_i \Phi = \tau_1 \Phi - \sum_{j=2}^{i-1} \frac{\partial \alpha_{j-1}}{\partial \hat{y}} \left(z_j v_{\ell,1} \Phi_\ell + z_j \bar{\omega}^T \Phi \right)$.

Step 3:

With respect to the new set of coordinates (35), the third equation of (34) can be written as

$$\dot{z}_r = -k_r v_{\ell,1} + w_{1\ell} + w_{2\ell}\hat{u} - \Bigg[\frac{\partial \alpha_{r-1}}{\partial \hat{y}} \left(\xi_{12} + v_{\ell,2}\phi_\ell + \bar{\omega}^T \Theta + \varepsilon_{\hat{z}12} + \hat{d}_{si_1} + \dot{\hat{d}}_y \right) + \frac{\partial \alpha_{r-1}}{\partial \xi} \left(A_0 \xi + \bar{k}\hat{y} \right) + \frac{\partial \alpha_{r-1}}{\partial \tilde{\Theta}} \dot{\tilde{\Theta}}$$
$$\sum_{j=1}^{\ell} \frac{\partial \alpha_{r-1}}{\partial v_j} \left(A_0 v_j + e_r \sigma_j \right) + \sum_{j=1}^{r-1} \frac{\partial \alpha_{r-1}}{\partial \hat{y}_m^{(j-1)}} \hat{y}_m^{(j)} \Bigg]$$

The overall Lyapunov function may now be chosen as

$$V_r = \sum_{j=1}^{r-1} V_j + \frac{1}{2} z_r^2 + \frac{1}{2} \Phi^T \Gamma^{-1} \Phi + \sum_{j=1}^{r} \frac{1}{4d_j} \varepsilon^T P \varepsilon \tag{40}$$

where Γ is a symmetric positive definite matrix, i.e., $\Gamma = \Gamma^T > 0$. With the definition of state estimated error ε, we can obtain that

$$\dot{V}_r = \sum_{j=1}^{r-1}\dot{V}_j + z_r\begin{cases}-k_r v_{\ell,1} + w_{1\ell} + w_{2\ell}\hat{u} \\ -\left[\dfrac{\partial\alpha_{r-1}}{\partial\hat{y}}\left(\xi_{12} + v_{\ell,2}\phi_\ell + \bar{\varpi}^T\Theta + \varepsilon_{\hat{z}_{12}} + \hat{d}_{si_1} + \dot{\hat{d}}_y\right) + \dfrac{\partial\alpha_{r-1}}{\partial\xi}\left(A_0\xi + \bar{k}\hat{y}\right)\right.\end{cases}$$
$$\left.\left. + \frac{\partial\alpha_{r-1}}{\partial\tilde{\Theta}}\dot{\tilde{\Theta}}\sum_{j=1}^{\ell}\frac{\partial\alpha_{r-1}}{\partial v_j}\left(A_0 v_j + e_r\sigma_j\right) + \sum_{j=1}^{r-1}\frac{\partial\alpha_{r-1}}{\partial\hat{y}_m^{(j-1)}}\hat{y}_m^{(j)}\right]\right\}$$
$$+\dot{\Phi}^T\Gamma^{-1}\Phi - \sum_{j=1}^{r}\frac{1}{4d_j}\varepsilon^T\varepsilon + \sum_{j=1}^{r}\frac{1}{4d_j}\left(\varepsilon^T P\Delta + \Delta^T P\varepsilon\right)$$

Specify the control input as

$$\hat{u} = \frac{1}{z_r w_{2\ell}}\left\{ z_r k_r v_{\ell,1} - z_r w_{1\ell} + z_r\left[\frac{\partial\alpha_{r-1}}{\partial\hat{y}}\left(\xi_{12} + v_{\ell,2}\tilde{\phi}_\ell + \bar{\varpi}^T\tilde{\Theta}\right) + \frac{\partial\alpha_{r-1}}{\partial\xi}\left(A_0\xi + \bar{k}\hat{y}\right) + \frac{\partial\alpha_{r-1}}{\partial\tilde{\Theta}}\dot{\tilde{\Theta}}\right.\right.$$
$$\left.\left.\sum_{j=1}^{\ell}\frac{\partial\alpha_{r-1}}{\partial v_j}\left(A_0 v_j + e_r\sigma_j\right) + \sum_{j=1}^{r-1}\frac{\partial\alpha_{r-1}}{\partial\hat{y}_m^{(j-1)}}\hat{y}_m^{(j)}\right] - c_r z_r^2 - d_r\left(\frac{\partial\alpha_{r-1}}{\partial\hat{y}}\right)^2 z_r^2 - g_r\left(\frac{\partial\alpha_{r-1}}{\partial\hat{y}}\right)^2 z_r^2 + z_r\hat{u}_{\hat{R}}\right\} \tag{41}$$

where $\hat{u}_{\hat{R}}$ is an addition input that will be used to target on rejection of uncertainties.

Substituting (41) into $\dot{V}_r$ and writing $\tau_r\Phi = \tau_{r-1}\Phi - \frac{\partial\alpha_{r-1}}{\partial\hat{y}}\left(z_r v_{\ell,1}\Phi_\ell + z_r\bar{\omega}^T\Phi\right)$, we arrive at

$$\dot{V}_r = -\sum_{j=1}^{r}\left(c_j z_j^2 + d_j\left(\frac{\partial\alpha_{j-1}}{\partial\hat{y}}\right)^2 z_j^2 + g_j\left(\frac{\partial\alpha_{j-1}}{\partial\hat{y}}\right)^2 z_j^2\right) + \left(\tau_r + \dot{\Phi}^T\Gamma^{-1}\right)\Phi + z_r\hat{u}_{\hat{R}}$$
$$-\sum_{j=1}^{r} z_j\frac{\partial\alpha_{j-1}}{\partial\hat{y}}\left(\varepsilon_{\hat{z}_{12}} + \hat{d}_{si_1} + \dot{\hat{d}}_y\right) - \sum_{j=1}^{r}\frac{1}{4d_j}\varepsilon^T\varepsilon + \sum_{j=1}^{r}\frac{1}{4d_j}\left(\varepsilon^T P\Delta + \Delta^T P\varepsilon\right) \tag{42}$$

From (42), we may specify the parameter update law to cancel the term $\left(\tau_r + \dot{\Phi}^T\,\Gamma^{-1}\right)\Phi$. To guarantee that the estimated parameters will always lie within allowable region w, a projected parametric update law will be specified as

$$\dot{\tilde{\Theta}} = \begin{cases}\Gamma\tau_r^T & \text{if } \tilde{\Theta}\in w^0, \\ P_R(\Gamma\tau_r^T) & \text{if } \hat{\Theta}\in\partial w \text{ and } \tau_r\Gamma\tilde{\Theta}_{perp} > 0,\end{cases} \tag{43}$$

where w is the allowable parametric set. It is compact and convex with its interior and boundary denoted by w^0 and ∂w, respectively. If the current estimated parametric vector locates within the allowable parametric set, the regular update law is used. If the current estimated parametric vector is on the boundary of the allowable parametric set, the projected update law denoted by $P_R(.)$ is employed to stop the parametric vector from leaving the set. With (43), add and subtract terms $\sum_{j=1}^{r}\frac{1}{4g_j}|\hat{d}_{si_1}+\hat{d}_y|^2$ to (42), we have

$$
\begin{aligned}
\dot{V}_i \le & -\sum_{j=1}^{r} c_j z_j^2 - \sum_{j=1}^{r} d_j\left(\frac{\partial\alpha_{j-1}}{\partial\hat{y}} z_j + \frac{1}{2d_j}\varepsilon_{\hat{z}_{12}}\right)^2 \\
& -\sum_{j=1}^{r} g_j\left(\frac{\partial\alpha_{j-1}}{\partial\hat{y}} z_j + \frac{1}{2g_j}\left|\hat{d}_{si_1}+\dot{\hat{d}}_y\right|\right)^2 \\
& +\sum_{j=1}^{r}\frac{1}{4g_j}\left|\hat{d}_{si_1}+\dot{\hat{d}}_y\right|^2 + \sum_{j=1}^{r}\frac{1}{4d_i}\left(\varepsilon^T P\Delta + \Delta^T P\varepsilon\right) \\
& -\sum_{j=1}^{r}\frac{1}{4d_j}\left(\varepsilon_{\hat{z}_{11}}^2 + \varepsilon_{\hat{z}_{13}}^2 + \cdots + \varepsilon_{\hat{z}_{1r}}^2\right) + z_r\hat{u}_{\hat{R}}
\end{aligned}
\tag{44}
$$

The tracking error $Z_1(\tilde{s})$ and the control input $\hat{U}_{\hat{R}}(\tilde{s})$ are related by

$$\hat{U}_{\hat{R}}(\tilde{s}) = -\hat{R}(\tilde{s})\hat{C}(\tilde{s})Z_1(\tilde{s}) \tag{45}$$

where we have chosen $\hat{R}(\tilde{s})$ as a low-order and attenuated-type internal model filter, i.e.,

$$\hat{R}(\tilde{s}) = \prod_{i=1}^{k}\frac{\tilde{s}^2 + 2\zeta_i\omega_{ni}\tilde{s} + \omega_{ni}^2}{\tilde{s}^2 + 2\xi_i\omega_{ni}\tilde{s} + \omega_{ni}^2} \tag{46}$$

where k is the number of periodic frequencies, ω_{ni} is the ith disturbance frequency in rad/rev, and ξ_i and ζ_i are damping ratios satisfying $0<\xi_i<\zeta_i<1$. The gain of $\hat{R}(\tilde{s})$ at those periodic frequencies can be varied by adjusting the values of ξ_i and ζ_i.

Theorem 3.1

Consider the control law of (41) and (45) employed to a nonlinear system with unmodeled dynamics, parametric uncertainty, and output disturbance given by (30). Suppose that $\hat{y}_m, \hat{y}_m, \cdots, \hat{y}_m^{(r)}$ (where r is the relative degree) and $\hat{d}_y, \hat{d}_y, \cdots, \hat{d}_y^{(r)}$ are known and bounded, $\hat{d}_{si_1}^{(r-1)}, \hat{d}_{si_2}^{(r-2)}, \cdots, \hat{d}_{si_{r-1}}$ are sufficiently smooth, f, g, h, $L_f^r h$, $L_g L_f^{r-1} h$ are Lipschitz continuous functions, and at least one column of $\overline{W}(\hat{y})$ is bounded away from zero. Moreover, suppose that a loop-shaping filter $\hat{C}(\tilde{s})$ is specified to stabilized the feedback system. Then, the parametric update law given by (43) yields the bounded tracking error.

Proof: Refer to [36].

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4. Spatial-based adaptive iterative learning control of nonlinear rotary systems with spatially periodic parametric variation

Consider an NTI system described by

$$\dot{x}(t)=f_t\left(x,\varphi_f(\theta)\right)+B_c g_t\left(x,\varphi_g(\theta)\right)u(t),\; y(t)=\Psi x(t) \tag{47}$$

where

$$x(t)=\begin{bmatrix}x_1(t) & \cdots & x_n(t)\end{bmatrix}^T,\; \Psi=\begin{bmatrix}0 & \cdots & 0 & 1\end{bmatrix},\; B_c=\begin{bmatrix}0 & \cdots & 0 & 1\end{bmatrix}^T$$

$y(t)$ is the system output, $u(t)$ is the control input, and $\varphi_f(\theta)=[\varphi_1(\theta) \;\; \cdots \;\; \varphi_p(\theta)]$ and $\varphi_g(\theta)$ are system parameters that are periodic with respect to angular position θ (i.e., spatially periodic). Using the aforementioned change of coordinate, we may transform (47) in the time domain into

$$\hat{\omega}(\theta)\dot{\hat{x}}(\theta)=f_t\left(\hat{x},\varphi_f(\theta)\right)+B_c g_t\left(\hat{x},\varphi_g(\theta)\right)\hat{u}(\theta),\; \hat{y}(\theta)=\Psi\hat{x}(\theta) \tag{48}$$

in the θ-domain. If $\hat{\omega}(\theta)$ equals one of the state variables, (48) is an NPI system in the θ-domain.

Remark 4.1. As mentioned previously, uncertainties for rotary systems may be treated as periodic disturbances or periodic parameters. Periodic parametric variation is, in fact, a sensible and practical assumption.

4.1 Definitions and assumptions

In this section, we list and present the definitions and assumptions to be used in the subsequent sections.

Definition 4.1. (Lie derivative) The Lie derivative is defined as

$$L_f^0 h(x)=h(x),\, L_f h(x)=\frac{\partial h}{\partial x}f(x),L_f^2 h(x)=L_f L_f h(x)=\frac{\partial\left(L_f h\right)}{\partial x}f(x),L_g L_f h(x)=\frac{\partial\left(L_f h\right)}{\partial x}g(x),\ldots$$

Definition 4.2. (Diffeomorphism) A diffeomorphism is considered as a mapping $T(.):D\subset R^n\rightarrow R^n$ being continuously differentiable on D and has a continuously differentiable inverse $T^{-1}(.)$.

Definition 4.3. (Adaptation rate) Instead of constant adaptation rate in regular adaptive control, a varying adaptation rate will be used. Consider a matrix $\Gamma(\theta,\ \varphi_c)$ defined by

$$\Gamma(\theta,\varphi_c)=\begin{cases} 0, & \theta=0 \\ \alpha(\theta), & 0<\theta<\varphi_c \\ \beta, & \varphi_c\le\theta \end{cases} \tag{49}$$

where φ_c is the lowest common multiple of the parametric periods, $\beta = diag\{\beta_1 \ \cdots \ \beta_\ell\}$ with nonzero positive constant β_i, and $\alpha(\theta) = diag\{\alpha_1(\theta) \ \cdots \ \alpha_\ell(\theta)\}$ with $\alpha_i(\theta)$ a strictly increasing function, $\alpha_i(0)=0$, and $\alpha_i(\varphi_c)=\beta_i$.

Assumption 4.1. The desired trajectory (or reference command signal) y_m is sufficiently smooth or $y_m^{(n)}, \ y_m^{(n-1)}, \ \cdots, \ \dot{y}_m$ exists.

Assumption 4.2. For a θ-domain NPI system described by

$$\dot{\hat{x}}(\theta)=f\left(\hat{x}(\theta),\varphi_f(\theta)\right)+B_c g\left(\hat{x}(\theta),\varphi_g(\theta)\right)\hat{u}(\theta),\ \hat{y}(\theta)=\Psi\hat{x}(\theta)$$

the nonlinear functions $f(\hat{x}(\theta))$ and $g(\hat{x}(\theta))$ are assumed to linearly relate to the system parameters φ_f and φ_g, i.e.,

$$f\left(\hat{x}(\theta),\varphi_f(\theta)\right)=\sum_{i=1}^{p}\varphi_i(\theta)f_i\left(\hat{x}(\theta)\right),\ g\left(\hat{x}(\theta),\varphi_g(\theta)\right)=\varphi_g(\theta)g\left(\hat{x}(\theta)\right)$$

Remark 4.2. Assumption 1 may be satisfied by considering a reference trajectory without sudden change of slope. Assumption 2 may be satisfied by many systems, e.g., LTI and NTI systems.

4.2 Spatial-based adaptive iterative learning control

For tidy presentation, the θ notation will be dropped from most of the equations in the sequel. Rewrite (48) as

$$\dot{\hat{x}}=f\left(\hat{x},\varphi_f\right)+B_c g\left(\hat{x},\varphi_g\right)\hat{u},\ \hat{y}=\hat{\omega}=\Psi\hat{x} \tag{50}$$

where the output $\hat{y}$ is equal to the angular velocity $\hat{\omega}$, which is set to be the first state of the system. Also note that

$$f\left(\hat{x},\varphi_f\right)=f_t\left(\hat{x},\varphi_f\right)/\hat{x}_1 \text{ and } g\left(\hat{x},\varphi_g\right)=g_t\left(\hat{x},\varphi_g\right)/\hat{x}_1$$

The system (50) is valid within the set $D_0=\{\hat{x}\in R \mid \hat{x}_1\neq 0\}$. Within this set, a diffeomorphism $T(\hat{x}): D_0\subset D$ (as defined previously) exists and may be described by

$$\hat{z}=T(\hat{x})=\left[L_f^0 h(\hat{x}) \quad L_f h(\hat{x}) \quad \cdots \quad L_f^{n-1}h(\hat{x})\right]^T \tag{51}$$

where $\hat{z}=[\hat{z}_1 \quad \cdots \quad \hat{z}_n]^T$. Using (51), we may transform (50) into

$$\dot{\hat{z}}=A_c\hat{z}+B_c\left[L_f^n h(\hat{x})+L_g L_f^{n-1}h(\hat{x})\hat{u}\right]_{\hat{x}=T^{-1}(\hat{z})},\ \hat{y}=\hat{z}_1 \tag{52}$$

where

$$A_c=\begin{bmatrix}0 & I_{(n-1)\times(n-1)}\\ 0 & 0_{1\times(n-1)}\end{bmatrix}$$

According to Assumption 3.2, we may rewrite (52) as

$$\dot{\hat{z}}=A_c\hat{z}+B_c\left[\rho(\hat{z})+\Theta^T W_f(\hat{z})+\varphi_g W_g(\hat{z})\hat{u}\right],\ \hat{y}=\hat{z}_1 \tag{53}$$

where $\Theta=[\varphi_1 \quad \cdots \quad \varphi_p \quad \varphi_g \quad \cdots]^T$ is the actual parametric vector, φ_g is a parameter mapped via the diffeomorphism, $W_f(\hat{z})$ is a vector of nonlinear terms, and $\rho(\hat{z})$ and $W_g(\hat{z})$ are two nonlinear functions.

Consider a reference trajectory $y_m(t)$ satisfying Assumption 3.1, which may be transformed into its counterpart in the θ-domain, i.e., $\hat{y}_m(\theta)=y_m(\lambda^{-1}(\theta))=y_m(t)$. Define another state or coordinate transformation:

$$\hat{z}_{1r}=\hat{y}_m(\theta),\ \hat{z}_{2r}=\dot{\hat{y}}_m(\theta),\cdots,\ \hat{z}_{nr}=\hat{y}_m^{(n-1)}(\theta).$$

We may form a state space model, which produces the reference trajectory, as

$$\dot{\hat{z}}_r=A_c\hat{z}_r+B_c\hat{y}_m^{(n)} \tag{54}$$

where $\hat{z}_r=[\hat{z}_{1r} \quad \cdots \quad \hat{z}_{nr}]^T$. Define the tracking error as $\hat{e}=\hat{z}-\hat{z}_r$. Then, the error dynamics can be obtained using the first equation of (53) and (54), i.e.,

$$\dot{\hat{e}} = A\hat{e} + B_c\left[\Theta^T W_f + \rho + \sigma - \hat{y}_m^{(n)} + \varphi_g W_g \hat{u}\right] \tag{55}$$

where $\sigma = c\hat{e}$ with $c = [c_1 \ \cdots \ c_{n-1} \ 1]$, and

$$A = \begin{bmatrix} 0_{(n-1)\times 1} & I_{(n-1)\times(n-1)} \\ & c \end{bmatrix}$$

Next, specify an LKF as

$$V = \sigma^2 / 2\varphi_g + 1/2\int_{\theta-\varphi_c}^{\theta} \Phi^T(\tau)\beta^{-1}\Phi(\tau)d\tau \tag{56}$$

where $\Phi = \bar{\Theta} - \tilde{\bar{\Theta}}$ and $\bar{\Theta}$ is a vector of parameters (to be defined later). $\tilde{\bar{\Theta}}$ is the estimate of $\bar{\Theta}$. The objective for the following steps is to establish a suitable control input and parametric update law rendering the derivative of the LKF negative semidefinite. Calculating the derivative of V, we obtain

$$\dot{V} = V_1 + V_2\ ,\ V_1 = \frac{\sigma}{\varphi_g}c\dot{\hat{e}} - \frac{\sigma^2}{2\varphi_g^2}\dot{\varphi}_g,\ V_2 = 1/2\left[\Phi^T(\theta)\beta^{-1}\Phi(\theta) - \Phi^T(\theta-\varphi_c)\beta^{-1}\Phi(\theta-\varphi_c)\right]. \tag{57}$$

Substituting the error dynamics (55) into V_1 and recalling that $\sigma = c\hat{e}$, we have

$$V_1 = \sigma\left[\frac{1}{\varphi_g}\left(\bar{c}\hat{e} + \rho - \hat{y}_m^{(n)}\right) + \frac{\Theta^T}{\varphi_g}W_f - \frac{\dot{\varphi}_g}{\varphi_g}\frac{\sigma^2}{2} + W_g\hat{u}\right]. \tag{58}$$

where $\bar{c} = [0 \ \ c_1 \ \cdots \ c_{n-1}]$. Hence, we may specify $\hat{u}$ as

$$\hat{u} = -1/W_g\left(k\sigma + \tilde{\bar{\Theta}}^T W\right), \tag{59}$$

where k is a positive variable, $\tilde{\bar{\Theta}}$ is the corresponding estimate of

$$\bar{\Theta} = \begin{bmatrix} \frac{1}{\varphi_g} & \frac{\Theta^T}{\varphi_g} & \frac{\dot{\varphi}_g}{\varphi_g} \end{bmatrix}^T, \text{ and } W = \begin{bmatrix} \left(\bar{c}\hat{e} + \rho - \hat{y}_m^{(n)}\right) & W_f & -\frac{\sigma^2}{2} \end{bmatrix}^T.$$

This will simplify V_1, i.e.,

$$V_1 = -k\sigma^2 + \sigma\Phi^T W \tag{60}$$

Using the periodicity of $\bar{\Theta}(\theta)=\bar{\Theta}(\theta-\varphi_c)$, we may rewrite V_2 as

$$V_2 = 1/2\left[\left(\bar{\Theta}-\tilde{\bar{\Theta}}\right)^T \beta^{-1}\left(\bar{\Theta}-\tilde{\bar{\Theta}}\right)-\left(\bar{\Theta}-\tilde{\bar{\Theta}}(\theta-\varphi_c)\right)^T \beta^{-1}\left(\bar{\Theta}-\tilde{\bar{\Theta}}(\theta-\varphi_c)\right)\right]. \tag{61}$$

According to the following algebraic relationship,

$$(a-b)^T \beta(a-b)-(a-c)^T \beta(a-c)=1/2(c-b)^T \beta\left[2(a-b)+(b-c)\right]$$

where a, b, and c are vectors, (61) implies that

$$V_2 = 1/2\left(\tilde{\bar{\Theta}}(\theta-\varphi_c)-\tilde{\bar{\Theta}}\right)^T \beta^{-1}\left[2\left(\bar{\Theta}-\tilde{\bar{\Theta}}\right)+\left(\tilde{\bar{\Theta}}-\tilde{\bar{\Theta}}(\theta-\varphi_c)\right)\right] \tag{62}$$

Therefore, we may specify a periodic parametric update law as

$$\tilde{\bar{\Theta}}(\theta)=\tilde{\bar{\Theta}}(\theta-\varphi_c)+\Gamma(\theta,\varphi_c)W\sigma;\ \tilde{\bar{\Theta}}(\theta)=0 \text{ if } -\varphi_c \le \theta \le 0 \tag{63}$$

Recall that $\Gamma(\theta,\ \varphi_c)$ is the adaptation rate as defined in (49). For $\varphi_c \le \theta$, V_2 becomes

$$V_2 = -\sigma\Phi^T W - 1/2\,\sigma^T W^T \beta W\sigma \tag{64}$$

With (60) and (64), we conclude that

$$\dot{V} = -k\sigma^2 - 1/2\,\sigma^T W^T \beta W\sigma \le -k\sigma^2 \tag{65}$$

The objective is achieved. The main results are summarized in the following theorem.

Theorem 4.1 Consider a spatial-based nonlinear system (50) with spatially periodic parameters satisfying Assumption 3.2. The error dynamics described by (55) exists under Assumption 3.1. Assume that the control input is determined by (59) along with the periodic parametric adaptation law (63). Then, the tracking error $\hat{e}$ will converge to 0 with the performance characteristics described by

$$\int_{\theta-\varphi_C}^{\theta} \|\hat{e}\|^2 \, d\tau \to 0, \; as \; \theta \to \infty$$

Proof: Refer to [37].

5. Conclusion

Adaptive fuzzy control (AFC) has been investigated for coping with nonlinearities and uncertainties of unknown structures [38–40]. The major distinctions between AFC techniques and the ones described in Sections 2 and 3 are (a) time-based (AFC) versus spatial-based design (OFLRARC/OFBRARC) and (b) less information assumed on the nonlinearities/uncertainties (AFC) versus more information on the nonlinearities/uncertainties (OFLRARC/OFBRARC). Because, in spatial-based design, a nonlinear coordinate transformation is conducted to change the independent variable from time to angular displacement, the systems under consideration in AFC and OFLRARC/OFBRARC are distinct. Next, AFC design techniques claim being able to tackle systems with a more generic class of nonlinearities/uncertainties, which relies on incorporating a fuzzy system to approximate those nonlinearities/uncertainties. It is not clear how to determine the required structure complexity of the fuzzy system (e.g., number of membership functions) to achieve desired control performance with reasonable control effort. Generally speaking, known characteristics of the uncertainties or disturbances should be incorporated as much as possible into the control design to improve performance, avoid conservativeness, and produce sensible control input. Therefore, instead of assuming the disturbances to be of generic type (as done by AFC), the methods presented in this chapter aim at a category of disturbances prevalent in rotary systems and explore the spatially periodic nature of the disturbances to design a specific control module and integrate into the overall control system.

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