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General Hydraulic Geometry

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Abstract

Employing bed load formulae hydraulic geometry relations were derived for stream width, meander wave length, and bed slope. The relations are in terms of friction factor, bed load discharge, bed load diameter, and water discharge. The bed load formulae are those of Engelund and Hansen (1966) [1], Einstein (1950) [2], Shields (1936) [3], and Meyer-Peter and Muller (1948) [4].

Keywords: Hydraulic Geometry, sediment transport, bed load, alluvium

1. Introduction

The water and sediment discharge of a river are primarily determined by the hydrology, geology, and topography of the drainage area. According to the influx water and sediment, the river creates its own geometry, i.e., slope, depth, width, and meandering pattern. Since the slope and meander pattern do not respond quickly enough to follow seasonal variations of discharge, it is natural to invoke some measure of dominant discharge values for these variables (Engelund and Hansen, 1966 [1]; Hansen, 1967 [5]; Kennedy and Alam, 1967 [6]).

In a given river, the water and sediment discharges normally increase in the downstream direction, and so do the depth and the width of the stream. The slope and the grain size usually decrease gradually from the source to estuary. According to Leviavsky (1955) [7], the grain size decreases approximately exponentially in the downstream direction. These observations point towards the existence of relations for the depth, width, and slope as functions of the water and sediment discharge. To that end, a number of “regime” relations have been suggested using it. An account of such relations has been given by Blench (1957[8], 1966[9]). Accordingly,

$$B \sim Q^{1/2} \quad (1)$$

$$D \sim Q^{1/3} \quad (2)$$

$$S \sim Q^{-1/6} \quad (3)$$

where B = the width, D = the depth, S = the slope, and Q = the water discharge. An empirical relation expressing the meander "wave-length" has been suggested by Inglis (1947) [10] as:

$$L \sim Q^{1/2} \quad (4)$$

Equations (1) and (4) describe a direct proportionality between the width of the stream and the meander length. Using the data from a large number of rivers in U.S.A and Indian as well as from several small scale model tests, Leopold and Wolman (1957) [11] derived the following popular relation:

$$L = 10B \quad (5)$$

Engelund and Hansen (1967)[12] derived equation (5) using the similarity principle. Due to the complex mechanics of the bed load and water transport, a number of variations of the above formulae have been proposed in the literature.

The objective of this paper is to derive hydraulic geometry relations using bed load formulae and compare them.

2. Derivation of hydraulic geometry relations

2.1. Engelund and Hansen (1966) [1] Bed Load Formula

The Darcy-Weisbach relation for the energy slope can be expressed as:

$$S = f \frac{V^2}{2gD} \quad (6)$$

where S = energy gradient (slope), f = friction factor, V = mean velocity, g = acceleration due to gravity, D = mean flow depth. Engelund and Hansen (1967) [12] expressed f as

$$f = 0.1\theta^{5/2}/\Phi \quad (7)$$

where θ = the dimensionless form of the bed shear stress τ_0 , and Φ = non-dimensional sediment transport rate, and d = mean fall diameter.

The non-dimensional transport rate is expressed as

$$\Phi = \frac{q_T}{\sqrt{(s-1)gd^3}} \quad (8)$$

where $q_T = Q_T/B$ = sediment discharge per unit width, Q_T = total sediment discharge ($=Q_B + Q_S$), Q_B = bed load discharge, Q_S = suspended load discharge, $s = \gamma_s/\gamma$ = relative density of sediment grains, γ = specific gravity of water, γ_s = specific gravity of sediment grains, B = flow width. Substitution for $q_T = Q_T/B$ in equation (8) yields

$$\Phi = \frac{Q_T}{B\sqrt{(s-1)gd^3}} \quad (9)$$

The bed shear stress τ_0 can be expressed in dimensionless form, θ , as (Shields, 1936)[3]:

$$\theta = \frac{DS}{(s-1)d} \quad (10)$$

Substituting equations (7) – (10) into the Darcy-Weisbach equation (6) one gets

$$S = 0.1 \left[\frac{DS}{(s-1)d} \right]^{5/2} \frac{1}{\Phi} (V)^2 \frac{1}{2g} \frac{1}{D} \quad (11)$$

Substitution of equation (10) for Φ , the non-dimensional transport rate and the continuity equation $V=Q/BD$ in equation (11) yields

$$S = 0.1 \left[\frac{DS}{(s-1)d} \right]^{5/2} \frac{B\sqrt{(s-1)gd^3}}{Q_T} (Q/BD)^2 \frac{1}{2g} \frac{1}{D} \quad (12)$$

Recalling the definition of s and putting the values of $s = 2.65$ and $g = 9.81$ in equation (12), one obtains:

$$S = 0.1 \left[\frac{DS}{(1.65)d} \right]^{5/2} \frac{B\sqrt{(1.65)gd^3}}{Q_T} \left(\frac{Q}{BD} \right)^2 \frac{1}{19.62} \frac{1}{D} \quad (13)$$

A little rearrangement of equation (13) yields

$$\frac{S}{S^{5/2}} = \frac{0.005}{(1.65)^{5/2}} d^{-5/2} D^{5/2} Q^2 / Q_T [(1.65)(9.81)]^{1/2} d^{3/2} BD^{-3} \quad (14)$$

Equation (14) can be simplified as

$$S = \left[0.20 d^{-1} BD^{1/2} (Q^2/Q_T) \right]^{-2/3} \quad (15)$$

The mean depth can be expressed from equation (15) as

$$D = \left[5S^{-3/2} Bd(Q_T/Q^2) \right]^2 \quad (16)$$

The water surface width B can be expressed from equation (16) as

$$B = 0.20 D^{1/2} S^{3/2} d (Q^2/Q_T) \quad (17)$$

or using $L = 10B$, one obtains from equation (17):

$$L = 2.0 D^{1/2} S^{3/2} d (Q^2/Q_T) \quad (18)$$

Shields' (1936) [3] Bed Load Formula:

On the basis of his experimental results, Shields (1936) [3] proposed a dimensionally homogeneous transport function:

$$Q_B \gamma_s / Q \gamma S = 10(\tau_0 - \tau_c) / (\gamma_s - \gamma) d_m \quad (19)$$

where Q_B = bed load transport rate (tons/hour), d_m = the effective diameter of sediment (mm), γ_s = the specific weight of sediment (T/m^3), γ = the specific weight of water (T/m^3), τ_0 = the shear stress (T/m^2); τ_c = critical shear stress (T/m^2), and S = the energy slope.

Shields (1936) [3] hypothesized that the rate of transport was a function of the dimensionless resistance coefficient:

$$\tau_o [(\gamma_s - \gamma)d_m]^{-1} \quad (20)$$

the critical value controlling the incipient motion of the bed load. The computation of the shear stress uses the hydraulic radius R and the bed slope S:

$$\tau_o = \gamma RS \quad (21)$$

whereas the critical shear stress is obtained from Straub's graph (1935) [3] for various sediment sizes. The bed load transport rate obtained from the Shields formula is the mass

of the solid particles. This rate value should be multiplied by a factor 1.60, in accordance with the density of sand, to obtain the volumetric value.

Recalling that

$$\tau_o - \tau_c = \tau = \gamma DS \quad (22)$$

Substituting of equation (22) in equation (19) gives

$$\frac{\gamma_s Q_B}{\gamma QS} = \frac{10\gamma DS}{(\gamma_s - \gamma)d_m} = \frac{10DS}{\left(\frac{\gamma_s}{\gamma} - 1\right)d_m} \quad (23)$$

where $s = \frac{\gamma_s}{\gamma}$. This can be simplified as

$$\frac{\gamma_s Q_B}{\gamma QS^2} = \frac{10D}{\left(\frac{\gamma_s}{\gamma} - 1\right)d_m} \quad (24)$$

Substituting $Q_B = q_B B$ and equation (6) into equation (24), one obtains

$$\frac{q_B B D^4 g^2 B^4 D^4}{Q f^2 Q^4} = \frac{10D}{\frac{\gamma_s}{\gamma} \left(\frac{\gamma_s}{\gamma} - 1\right) d_m} \quad (25)$$

Using the continuity equation $V=Q/A$, one gets from equation (25)

$$Q_B = \frac{10Q^5 f^2}{\frac{\gamma_s}{\gamma} \left(\frac{\gamma_s}{\gamma} - 1 \right) d_m B^4 D^5 4g^2} \quad (26)$$

D is expressed from equation (26) as

$$D = 0.3587 \frac{Q}{Q_B^{0.20}} f^{0.4} d_m^{-0.2} B^{-0.8} \quad (27)$$

Using $L=10B$, one obtains from equation (27)

$$L = 2.776 \frac{Q^{1.25}}{Q_B^{0.25}} f^{0.5} d_m^{-0.25} D^{-1.25} \quad (28)$$

From equation (28) the width B is computed as

$$B = (0.2776) \frac{Q^{1.25}}{Q_B^{0.25}} f^{0.5} d_m^{-0.25} D^{-1.25} \quad (29)$$

S is derived from equation (23)

$$S = 0.6612 \left(\frac{Q_B}{Q} \right)^{0.5} d^{0.5} D^{-0.5} \quad (30)$$

2.2. Einstein's (1950) [2] bed load formula

The bedload formula due to Einstein (1950) [2] can be expressed as

$$q_{sbi} = P_i \Phi_* \gamma_s / \left\{ \left[\frac{\gamma}{\gamma_s} - \gamma \right] / g d_{si}^3 \right\}^{1/2} \quad (31)$$

where d_{si} = the grain diameter for which si per cent is finer, q_{sbi} = the intensity of bed load movement of size class i, Φ_* = the bed load intensity, γ = the unit weight of water; γ_s = the unit weight of sediment particles, g = the gravitational acceleration, P_i = the particle availability parameter $\sim G_{di}/G_d$ (bed surface gradation), G_d = the total weight of sediment in the bed surface layer, and G_{di} = the weight of the i th size class in the bed surface layer.

The bed surface layer in this equation is a zone near the bed surface called the “active layer”. The surface elevation is changed as sediment is deposited into or scoured out of this layer.

Setting the particle availability parameter, P_1 , equal to 1. One obtains from equation (31)

$$\frac{Q_B}{B} = \frac{\Phi_* \gamma_s}{\left[(\gamma / \gamma_s - \gamma) / g d_{si}^3 \right]^{1/2}} \quad q_s = \frac{Q_B}{B} \quad B = \text{water surface width} \quad (32)$$

where $q_s = P_1 \Sigma q_{sbi}$ and q_{sbi} = the intensity of bed load movement of size class i.

The water surface width can finally be expressed from the equation (32) as

$$B = Q_B \Phi_*^{-1} \gamma_s^{-1} \left[(\gamma^2 / (s-1) \gamma_s) / g d_{si}^3 \right]^{1/2} \quad (33)$$

Using $L=10B$, one obtains from the equation (36) as

$$L = 10 Q_B \Phi_*^{-1} \gamma_s^{-1} \left[(\gamma^2 / (s-1) \gamma_s) / g d_{si}^3 \right]^{1/2} \quad (34)$$

The Darcy-Weisbach relation, equation (6) can be written as

$$S = f \frac{Q^2}{B^2 D^3} \frac{1}{2g} \quad (35)$$

Substituting the water surface width B from the equation (32) into the equation (35) and using the continuity equation $Q=V B D$,

D is expressed from Equation (35) as

$$D = 0.2529 f^{0.571} Q^{0.571} B^{-0.571} V^{0.28571} \quad (36)$$

The mean velocity is derived from equation (36) as

$$V = 122.93 f^{-1.998} Q^{-1.998} B^{1.998} D^{3.5} \quad (37)$$

and the slope S is derived from the equation (38) as

(38)

$$S = fQ^2B^20.0509D^{-3}$$

2.3. Bed load formula of Meyer-Peter and Muller (MPM) model (1948) [4]

This formula was derived from experiments using a laboratory flume with a maximum width of 2 m, very different from the conditions encountered in large channels. The formula depends primarily on the grain diameter and water discharge. They derived a formula for bed load discharge with the aim to develop a more practical formula. Bogardi (1978) indicated several difficulties that are encountered in application of this formula.

According to the Meyer-Peter and Muller(1948) [4] (MPM) Model:

$$\gamma DS = 0.047(\gamma_s - \gamma)d_s + 0.25\rho^{1/3}q_b^{1/3} \quad (39)$$

where γ = specific gravity of water, γ_s = specific gravity of sediment grains,

D = mean depth, S = energy gradient (slope), d_s = mean sediment diameter,

ρ = density of water, q_b = sediment discharge per unit width.

From equation (39) the slope S is computed as

$$S = (\gamma D)^{-1} \left[0.047(\gamma_s - \gamma)d_s + 0.25\rho^{1/3}q_b^{1/3} \right] \quad (40)$$

Putting equation (6) into the equation (40)

$$\gamma f \frac{Q^2}{B^2 D^2} \frac{1}{2g} = 0.047(\gamma_s - \gamma)d_s + 0.25\rho^{1/3}q_b^{1/3} \quad (41)$$

Equation (41) is rearranged for computing the width B as

$$B^{-2} = 19.62D^2 \left[0.047(\gamma_s - \gamma)d_s + 0.25\rho^{1/3}q_b^{1/3} \right] f^{-1}\gamma^{-1}Q^{-2} \quad (42)$$

From the equation (42), is computed the width B is computed

$$B = 0.226D^{-1} \left[0.047(\gamma_s - \gamma)d_s + 0.25\rho^{1/3}q_b^{1/3} \right]^{1/2} f^{1/2}\gamma^{1/2}Q^{-1} \quad (43)$$

Using $L=10B$, one obtains the wave length L as

$$L = 2.26D^{-1} \left[0.047(\gamma_s - \gamma)d_s + 0.25\rho^{1/3}q_b^{1/3} \right]^{1/2} f^{1/2}\gamma^{1/2}Q^{-1} \quad (44)$$

The water depth D is derived from equation (42) as

$$D = 0.226B^{-1} \left[0.047(\gamma_s - \gamma)d_s + 0.25\rho^{1/3}q_b^{1/3} \right]^{1/2} f^{1/2}\gamma^{1/2}Q^{-1} \quad (45)$$

3. Discussion

The hydraulic geometry relationships, B , D , S , L computed with DuBoys (1879) [13] sediment transport formula (Huang and Nanson, 2000) [14] were similar, those computed with Einstein's (1950) [2] model and Shields' (1936) [3] model, but MPM (1948) [4] model have differences by using the specific gravity of water and sediment grains, and density of water. Although there are four dependent variables (width, depth, velocity and slope) with only three basic flow relations of continuity, resistance and sediment transport. This study finds that the long-recognized problem of no closure can be solved directly in terms of the analytical approach advocated here for understanding the self-adjusting mechanism of alluvial channels.

With stable canal flow relations (Lacey's flow resistance relation and DuBoys' (1879) [13] sediment transport formula) and rectangular sections, introducing a channel form factor (width/depth ratio) as a dependent variable identifies an optimum condition for sediment transport by adjusting width/depth ratio for a given flow discharge, channel slope and sediment size (Huang and Nanson, 2000) [14].

Theoretically derived channel geometry relations are highly consistent with their counterparts obtained from "Darcy-Weisbach relation", except that 'threshold theory' provides a larger value of friction factor coefficient. This may be due to the use of rectangular cross-sections in our analytical study. Furthermore, the maximum friction value is greater in natural rivers using relationships not so dependent on canal data.

A comparison of the averaged channel geometry relations with downstream hydraulic geometry relations developed by Huang and co-workers (Huang and Warner, 1995 [15]; Huang and Nanson, 1995 [16], 1998 [17]) and by Julien and Wargadalam (1995) [18] based on numerous sets of field observations, reveals high level of consistency. When sediment concentration varies in a limited range, the averaged relationships are very similar to empirical regime formulations ('regime theory') (Huang and Nanson, 2000) [4].

Table I presents a comparison of equations with downstream hydraulic geometry relations obtained by Huang and co-workers (2000) and by Julien and Wargadalam (1995) [18], based on numerous sets of field observations.

Huang and co-workers' model (1995)[16]	Julien and Wargadalam's model(1995)[18]
$B \propto Q^{0.501} S^{-0.156}$	$B \propto Q^{0.4-0.5} S^{-(0.2-0.25)}$
$D \propto Q^{0.299} S^{-0.206}$	$D \propto Q^{0.4-0.25} S^{-(0.2-0.125)}$
$V \propto Q^{0.200} S^{0.362}$	$V \propto Q^{0.2-0.25} S^{0.4-0.375}$
Huang and Nanson (2000) [14]	Engelund and Hansen (1967) model [12]
$B \propto Q^{0.478} S^{0.076}$	$B \propto Q^2 (Q^{-4} S^{-3})^{0.5} S^{1.5}$
$D \propto Q^{0.289} S^{-0.350}$	$D \propto Q^{-4} S^{-3}$
$V \propto Q^{0.233} S^{0.274}$	
Shields (1936) model [3]	
$B \propto Q^{1.25} \Gamma^{0.5} D^{-1.25} d_m^{-0.25}$	
$D \propto (Q/Q_B)^{0.20} \Gamma^{0.4} d_m^{-0.2} B^{-0.8}$	
Einstein's (1950) model [2]	
$B \propto \Phi_*^{-1} d_{si}^{1.5}$	
$D \propto S^{-0.33} \Gamma^{0.33} Q^{0.66} (\Phi_*^{-1} d_{si}^{1.5})^{0.66}$	
MPM (1948) model [4]	
$B \propto Q^{-1} \Gamma^{0.5} D^{-1}$	
$D \propto Q^{-1} \Gamma^{0.5} B^{-1}$	

Table 1. Downstream hydraulic geometry relations defined as the functions of flow discharge and channel slope

To reflect how channel geometry adjusts within the range, this study is only able to present acceptably averaged relationships for channel geometry by assigning the four variables depth, width, slope and wave length.

Although as stated earlier τ_c and the other coefficients are determined only by sediment size d , their combined effects on natural channel geometry are much more complicated with both bank strength and channel roughness (or sediment size) being often particularly examined (Millar and Quick, 1993 [19]; Huang and Nanson, 1998 [17]). When the constant terms and the terms related to sediment size d are ignored, the equations of Einstein's (1950) [2] and Shields' (1936) [3] agree very closely with the widely observed empirical regime channel relationships, when sediment concentration (Q_s/Q) remains unchanged or varies within a limited range. This is consistent with the study by Simons and Albertson (1960) [20]. In an analysis of numerous observations that were collected from stable canals in different parts of the world, Simons and Albertson (1960) [20] identified that the relations closest to this equations occur only when sediment concentration varies in a limited range (less than 500 ppm).

In many circumstances, sediment discharge Q_s is unknown and consequently channel slope S is used as an alternative. The consistency of the theoretical results of equations (Engelund and Hansen, 1967 [12]; Shields [3], 1936; Einstein, 1950 [2]; MPM model, 1948 [4]) with the studies

based on direct observations suggests strongly that most natural alluvial channels are able to adjust their channel form, so as to reach an optimum state. This must be a general principle for flow in rivers and canals that causes channels in very different environments to exhibit remarkably similar hydraulic geometry relations.

The physical relationships of flow continuity, flow resistance and sediment transport determine the degree of channel adjustment and consequently illustrate a condition of maximum sediment transporting capacity, subject to the conditions of flow discharge, channel slope and sediment size.

Mathematically, it can be defined a minimum slope subject to the conditions of flow discharge, sediment discharge and sediment size; or a minimum flow discharge subject to the conditions of sediment discharge, channel slope and sediment size; or an optimum sediment size subject to the conditions of flow and sediment discharges, and channel slope. This formulation provides the maximum sediment transporting capacity per unit of approximate total stream power (actual total stream power is γQS where γ is the specific weight of water). This concept includes the following specific optimum conditions that have long been hypothesized and applied for practical problem solving:

1. for fixed S and Q , $Q_s =$ a maximum as proposed by Pickup (1976) [21], Kirkby (1977) [22] and White et al. (1982) [23];
2. for fixed Q_s and Q , $S =$ a minimum as proposed by Chang (1979 a [24], b [25], 1988 [26]);
3. for fixed Q_s , $S^{7/11}Q^{8/11} =$ a minimum or $SQ^{1.142} =$ a minimum, close to $\gamma QS =$ a minimum as proposed by Chang (1980 a [27], b [28], 1986 [29], 1988 [26]).

Hence, the use of the analytical approaches proposed by Pickup (1976) [21], Kirkby (1977) [22] and White et al. (1982) [23] with the maximum sediment transport capacity, and by Chang (1979 a [24], b [25], 1980 a [27], b [28], 1988 [26]) based on the minimum stream power hypothesis should produce consistent stable channel geometry relations with the theoretical approach advocated in this study.

This study shows that the optimum condition for sediment transport with regard to downstream hydraulic geometry relations for a given flow discharge, channel slope and sediment size results from the general condition of maximum flow efficiency according to Huang and Nanson, 2000 [14], defined as the maximum sediment transporting capacity per unit available stream power. Maximum flow efficiency is an internal optimum condition and includes the conditions of maximum sediment transporting capacity and minimum stream power as proposed by Pickup (1976) [21], Kirkby (1977) [22] and White et al. (1982) [23], and by Chang (1979 a [24], b [25], 1980 a [27], b [28], 1986 [29], 1988 [26]) respectively.

Despite criticism of the use of extremal hypotheses (Griffiths, 1984)[30], this study offers strong support for the use of bed-load concepts of different authors for hydraulic-geometry relation derivations and for understanding natural channel-form adjustment.

Finally, this study indicates that the general principle of sediment transport approaches in the variational theory of mechanics is able to provide a physical explanation for the existence of

the optimum conditions of natural channel-form adjustment. However, these findings are of a preliminary nature and further detailed research is ongoing.

4. Conclusion

Strictly speaking, Engelund and Hansen's equation should be applied to flows with dune beds in accordance with the similarity principle. According to Yang(1987) [31], a river can adjust its roughness, geometry, profile and pattern through the processes of sediment transport. Qualitative descriptions of these dynamic adjustments of natural streams have been made mainly by geologists and geomorphologists. Empirical regime types of equation have been developed by engineers to solve design problems. Attempts have been made in recent years to explain these adjustments based on different extremal theories and hypotheses.

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