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# Heat Transfer and Entropy

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Additional information is available at the end of the chapter

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## 1. Introduction

When you read a standard textbook on thermodynamics (like [1-3]) as one of the most fundamental formulae you will find

$$\delta Q = T dS \quad (1)$$

indicating that the (process) quantity heat ( $\delta Q$ ) is obviously closely linked to the (state) quantity entropy ( $dS$ ), here both written as infinitesimal quantities.

If, however, you do the same with a standard textbook on heat transfer (like [4] with 1024 pages or [5] with 1107 pages), you will find entropy neither in the index of these books, nor in the text.

There may be two reasons for that: Either entropy has turned out to be irrelevant for a heat transfer analysis or entropy is ignored deliberately in the heat transfer community in spite of its relevance. What is true is a yet open question and can only be answered when thermodynamic considerations are taken into account.

In thermodynamics the relevance of entropy with respect to heat transfer is beyond any controversy, it is the heat transfer community that has to be persuaded of its relevance. This can best be done by showing the advantages of including entropy in a heat transfer analysis as well as showing the disadvantages one has to face when entropy is ignored.

## 2. A thermodynamic view on heat transfer

### 2.1. General considerations

Engineers, using the phrase “heat transfer”, would not be bothered by the view that heat is moved across the boundary of a system and then stored in it, increasing its heat content.

This line of argument, however, violates at least two principles of thermodynamics and misses the crucial point. From a thermodynamics point of view heat is a process quantity that describes a certain way by which energy can be transferred across the boundary of a system. And of course this quantity cannot be stored, only the energy moved by it can be stored.

And the crucial point is: Transferring energy as heat into a system is fundamentally different from doing the same by work. The energy transferred in form of heat and work, though it may be the same amount, has a very different quality once it is part of the energy of the system. To put it in a simple and not yet precise form for the moment: It is not only the amount of energy that counts in energy transfer processes (like heat transfer) but also the quality of the energy and the change in quality during the transfer process. If that is true, there must be a measure for the quality and its potential degradation in energy transfer processes. This is where entropy comes in and plays a crucial role – even and also in heat transfer considerations.

From the very clear principle of energy conservation (thermodynamically formulated as the first law of thermodynamics) we know that energy given as primary energy never gets lost when used in technical devices but finally ends up as part of the internal energy of the ambient. Then, however, it is of no use anymore. Obviously energy has a certain potential that can get lost on the way from being primary energy to being part of the internal energy of the ambient.

In thermodynamics there is a useful definition by which the quality of an energy can be characterized which was first proposed in [6]. This definition primarily refers to an energy which is subject to transfer processes either by work or by heat. According to this definition energy is composed of two parts, *exergy* and *anergy*. Within this concept exergy is the precious part of the energy. It is that part which can be used by work until it is part of the internal energy of the ambient. Sometimes exergy is also called *available work*. The remaining part of the energy is called anergy. According to the second law of thermodynamics exergy can get lost (can be converted to anergy) in irreversible processes but never can be generated. Any transfer of energy by work or by heat thus can either preserve the exergy part of the energy in a reversible process or reduce it in an irreversible one.

As far as heat transfer is concerned there are two aspects that are important: The first is the amount of energy transferred by heat and the second is the amount of exergy lost in this (heat) transfer process. Ignoring entropy means that only the first aspect can be accounted for. For a complete characterization of a heat transfer process both aspects have to be accounted for, i.e. two physical quantities have to be specified. They can be

- the heat flux  $\dot{q} = \delta\dot{Q} / dA$
- a characteristic temperature difference  $\Delta T$

In a heat transfer process both quantities are independent of each other because a certain amount of energy ( $\dot{q}$ ) can be transferred with a different decrease of quality, i.e. with a different degree of irreversibility ( $\Delta T$ ). Here  $\Delta T$  is an indirect measure of the quality decrease of the energy in the transfer process since  $\Delta T = 0$  is the reversible limit of an irreversible process with  $\Delta T > 0$ . When two independent quantities are required then two nondimensional parameters are needed in the context of describing heat transfer processes nondimensionally. In section 3

it will be discussed what is missing when the Nusselt number  $Nu$  alone is used in order to characterize a heat transfer process.

In thermodynamics the two aspects of energy transfer and its devaluation by irreversible processes are quantified by introducing the entropy and its generation in the course of irreversible processes. In this context entropy is a measure of the structure of the system storing the energy under consideration, i.e. energy can be stored in a more or less ordered way. This again can be expressed in terms of exergy versus anergy of the energy transferred and stored.

## 2.2. Change of entropy in energy transfer processes

For most considerations the absolute value of entropy is not of interest but its change during a certain process like a heat transfer process. This change of entropy in a transfer process generally is twofold:

- i. Transfer - change of entropy in a reversible process,
- ii. Generation - change of entropy when the transfer process is not reversible, i.e. irreversible.

In a real (irreversible) process the change of entropy thus always is the sum of both, i.e. (i) + (ii).

For a heat transfer process between two temperature levels  $T_a$  and  $T_b$  the two parts (i) and (ii) are

$$d_t \dot{S} = \frac{\delta \dot{Q}}{(T_a + T_b)/2} \quad (2)$$

$$d_g \dot{S} = \delta \dot{Q} \left( \frac{1}{T_a} - \frac{1}{T_b} \right) = \delta \dot{Q} \frac{T_a - T_b}{T_a T_b} = \delta \dot{Q} \frac{\Delta T}{T_a T_b} \quad (3)$$

Equation (2) corresponds to eq. (1) in the introduction, now in terms of rates for a continuous process. Equation (3) states that entropy generation leads to an increase of entropy when the energy is transferred from one system (a) with high temperature (i.e. low entropy) to another system (b) with low temperature (i.e. high entropy). Thus the overall change of entropy in such a process is

$$d\dot{S} = d_t \dot{S} + d_g \dot{S} \quad (4)$$

In figure 1 such a process is illustrated for the convective heat transfer from a flow in system (a) with  $\dot{m}_a$  to a flow in system (b) with  $\dot{m}_b$ . The wall between both flows is diabatic, walls to the ambient are adiabatic.

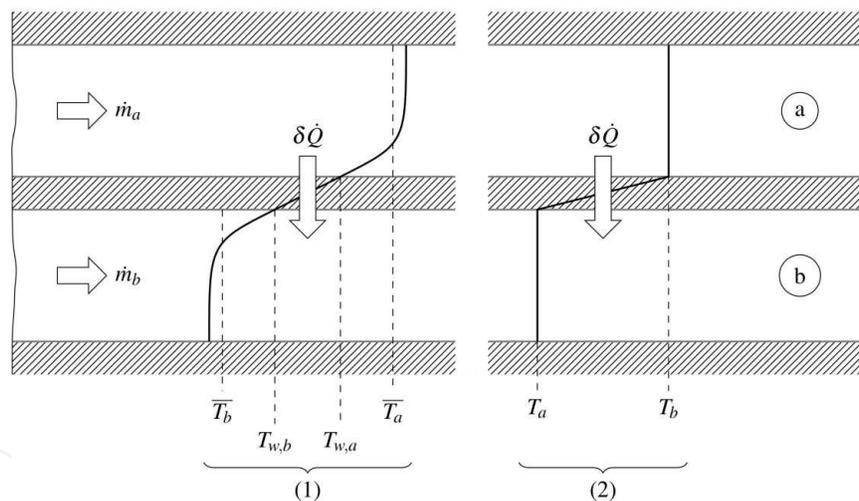
The generation-change of entropy in eq. (3) strictly speaking is an approximation only. It is based on the assumption that in (a) and in (b) the real temperature distributions can be approximated by their (constant) mean values and that the temperature drop from (a) to (b) completely happens in the wall between both systems, see figure 1 for an illustration of this approximation. In section 4 the real temperature distribution is accounted for in order to determine the generation-change of entropy without approximation.

Though it is not the topic of this chapter it should be mentioned what (i) and (ii) are for an energy transfer by work:

$$d_t \dot{S} = 0 \quad (5)$$

$$d_g \dot{S} = \delta \dot{\Phi} / T \quad (6)$$

with  $\delta \dot{\Phi}$  as dissipation rate of mechanical energy in the flow field involved in the transfer process. That always  $d_t \dot{S} = 0$  holds for a work transfer of energy shows the fundamental difference of the two ways to transfer energy, i.e. by heat or by work, c.f. eq. (2) for the energy transfer by heat.



**Figure 1.** Convective heat transfer from a flow in (a) to a flow in (b) over a surface element  $dA$  (1) Real temperature distribution (2) Mean temperature model

### 2.3. Energy devaluation in a heat transfer process and the entropic potential concept

When in an energy transfer process exergy gets lost the “value” of the energy is reduced, since exergy as the precious part of the energy is reduced. This is called *energy devaluation* during a transfer process and is immediately linked to the generation-change of entropy, c.f. eq. (3).

Exergy lost and entropy generated are interrelated by the so-called Gouy-Stodola theorem, see for example [7]. It reads

$$d\dot{E}_l^e = T_\infty d\dot{S}_g \quad (7)$$

Here  $T_\infty$  is the ambient temperature and  $\dot{E}_l^e$  is the loss of the exergy rate  $\dot{E}^e$  of the energy rate  $\dot{E}$ , after subdividing  $\dot{E}$  into an exergy and an anergy part,  $\dot{E}^e$  and  $\dot{E}^a$ , respectively.

For a single transfer operation indicated by  $i$  then there is the finite exergy loss

$$\dot{E}_{l,i}^e = T_\infty \dot{S}_{g,i} \quad (8)$$

with  $\dot{S}_{g,i}$  as entropy generation in the transfer operation  $i$ . This entropy generation can and should be seen in the context of those devaluations of the energy transfer rate  $\dot{E}$  that happened prior to the transfer operation  $i$  and that will happen afterwards. This idea takes into account that a certain energy (rate) always starts as primary energy being exergy as a whole and finally ends up as part of the internal energy of the ambient, then being anergy as a whole. In [8] this has been described as the “devaluation chain” with respect to the energy transfer rate  $\dot{E}$  with the process  $i$  being one link of this chain.

For the sum of all single transfer operations that completely devaluates the energy from 100% exergy to 100% anergy then

$$\dot{E}_l^e = \dot{E} = T_\infty \dot{S}_g \quad (9)$$

holds. Here  $\dot{S}_g$  is the overall entropy generation (rate), i.e. the entropy increase of the ambient, when  $\dot{E}$  becomes part of its internal energy.

In [8] this quantity is called the *entropic potential*:

$$\dot{S}_g = \frac{\dot{E}}{T_\infty} \quad (10)$$

of the energy  $\dot{E}$  involved in an energy (here: heat) transfer process. Taking this as a reference quantity the so-called energy devaluation number

$$N_i \equiv \frac{\dot{S}_{g,i}}{\dot{S}_g} = \frac{T_\infty \dot{S}_{g,i}}{\dot{E}} \quad (11)$$

indicates how much of the entropic potential of the energy is used in a certain transfer process  $i$  with  $N_i=0$  for a reversible process. Examples will be given afterwards.

### 3. An engineering view on heat transfer

As mentioned before, engineers trained to solve heat transfer problems with books like [4] care little or not at all about entropy. They characterize heat transfer situations by the heat transfer coefficient

$$h = \frac{\dot{q}_w}{\Delta T} \quad (12)$$

or in a more systematic way by the Nusselt number

$$\text{Nu} = \frac{\dot{q}_w L}{k \Delta T} = h \frac{L}{k} \quad (13)$$

In both cases  $\dot{q}_w$  and  $\Delta T$  are combined within one assessment quantity so that the two independent aspects of heat transfer

- the amount, associated with  $\dot{q}_w$  and
- the change of quality, associated with  $\Delta T$

are not captured separately. A second assessment quantity is required for a comprehensive characterization of a heat transfer situation. This can be the energy devaluation number  $N_i$  according to eq. (11).

When  $N_i$  accounts for the quality of heat transfer the Nusselt number  $\text{Nu}$  covers the quantitative aspect in the following sense. Often either  $\Delta T$  or  $\dot{q}_w$  are prescribed as a thermal boundary condition. Then the Nusselt number quantifies the heat transfer by providing the heat flux that occurs or the temperature difference that is required, respectively. Both are quantitative aspects leaving the question about the quality still open. This then is addressed by the energy devaluation number  $N_i$ .

Since the Nusselt number  $\text{Nu}$  is well established in the heat transfer community, but the energy devaluation number  $N_i$  is not,  $N_i$  will be further explained with respect to its physical background in the following section.

### 4. The physics behind the energy devaluation number

According to Fourier's law of heat conduction, see for example [4] or [9],

$$\delta \vec{Q} = -k(\text{grad } T) dA \quad (14)$$

i.e. a heat flux occurs along the (negative) gradient of temperature. The energy transferred in this way reduces its exergy part because this exergy part is

$$\dot{Q}^e = \eta_c \dot{Q} \quad (15)$$

with the Carnot factor

$$\eta_c = 1 - \frac{T_\infty}{T} \quad (16)$$

Here again  $T_\infty$  is the ambient temperature, so that the exergy part of  $\dot{Q}$  once its temperature level  $T$  has reached the ambient temperature, is zero.

This permanent exergy loss when heat transfer occurs with  $\text{grad}T > 0$  (irreversible heat transfer) according to the Gouy-Stodola theorem (7) is accompanied by entropy generation which here can be written as

$$\dot{S}_g''' = \frac{k}{T^2} (\text{grad } T)^2 \quad (17)$$

or after integrating the local entropy generation rate  $\dot{S}_g'''$  as

$$d_g \dot{S} = \frac{k}{T^2} (\text{grad } T)^2 dV \quad (18)$$

which in Cartesian coordinates reads

$$d_g \dot{S} = \frac{k}{T^2} \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 + \left( \frac{\partial T}{\partial z} \right)^2 \right] dV \quad (19)$$

Note that this eq. (19) reduces to eq. (3) when there is a linear temperature distribution in  $x$  - direction only so that  $\partial T / \partial x = \Delta T / \Delta x$ ,  $dV = dA \Delta x$  and  $\partial \dot{Q} = -k(\Delta T / \Delta x) dA$ .

Comparing eqs. (3) and (19) shows that

$$\dot{S}_g = \int d_g \dot{S} \quad (20)$$

in the mean temperature model according to eq. (3) and figure 1(2) is an integration with respect to  $\delta \dot{Q}$  while with the real temperature distribution according to eq. (19) and figure 1 (1) it is an integration with respect to the volume accounting for the local entropy generation rate.

In both cases  $\dot{S}_{g,i}$  is determined which is the overall entropy generation due to heat conduction in a transfer process  $i$ . The energy devaluation number refers this to the entropic potential of  $\dot{Q}$ , i.e. to  $\dot{Q}/T_\infty$ , so that

$$N_i = \frac{k T_\infty}{\dot{Q}} \int_V \frac{1}{T^2} \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 + \left( \frac{\partial T}{\partial z} \right)^2 \right] dV \quad (21)$$

is that percentage used of the entropic potential of the energy  $\dot{E}$  which in a process  $i$  is transferred as heat  $\dot{Q}$ . Note that part of the entropic potential has been used already on the way of  $\dot{E}$  starting as primary energy to the situation in which it is transferred as heat and that the remaining part of the entropic potential after the heat transfer process  $i$  can be used in subsequent energy transfer processes. This may illustrate why it is important to see a certain transfer process  $i$  in the context of the overall devaluation chain of an energy starting as primary energy and ending as part of the internal energy of the ambient, for more details of this concept see [8].

## 5. Convective heat transfer

Often convective heat transfer occurs in technical applications like power plants and heating or cooling systems. Then a second energy flux is involved which is the flow work rate that is needed to maintain the flow into which or out of which the heat transfer occurs. This energy flux applied as work is pure exergy which gets lost in the dissipation process during the convective heat transfer.

### 5.1. Losses due to dissipation of mechanical energy

In fluid mechanics losses in a flow field usually are characterized by a drag coefficient  $c_D$  for external flows and a head loss coefficient  $K$  for internal flows, which are a nondimensional drag force  $F_D$  and a nondimensional pressure loss  $\Delta p$ , respectively. In table 1 both definitions are shown together with an alternative approach based on the entropy generation rate  $\dot{S}_{g,D}$  due to the dissipation of mechanical energy (index:  $D$ ). For details of this alternative approach see [10]. Since both coefficients,  $c_D$  and  $K$ , account for the dissipation rate in the flow field and according to eq. (6)  $\delta\dot{\Phi} = T d_g \dot{S}$  the dissipation of mechanical energy corresponds to the loss of exergy only when  $T = T_\infty$ , c.f. eq. (7). Whenever the flow occurs on a temperature level which is not that of the ambient temperature  $T_\infty$ ,  $c_D$  and  $K$  account for the dissipation but not for the lost exergy in the flow.

Then a second coefficient is needed which best is defined as an *exergy destruction number*  $N^E$  analogous to the energy devaluation number, eq. (11), i.e.

$$N^E = \frac{T_\infty \dot{S}_{g,D}}{\dot{E}} \quad (22)$$

	conventional approach	alternative approach
external flow	$c_D = \frac{F_D}{\frac{\rho}{2} u_\infty^2 A}$	$c_D = \frac{T}{\frac{\rho}{2} u_\infty^3 A} \dot{S}_{g,D}$
internal flow	$K = \frac{\Delta p}{\frac{\rho}{2} u_m^2}$	$K = \frac{T}{\frac{\rho}{2} u_m^3 A} \dot{S}_{g,D}$

**Table 1.** Drag and head loss coefficients; conventional and alternative definitions, from [10].  $u_\infty$ : free-stream velocity,  $u_m$ : cross-section averaged velocity

which for an external flow with  $\dot{E} = \frac{u_\infty^2}{2} \dot{m} = \frac{\rho}{2} u_\infty^3 A$  is (c.f. table 1):

$$N^E = \frac{T_\infty}{T} c_D \text{ (exergy destruction number)} \quad (23)$$

and for an internal flow with  $\dot{E} = \frac{u_m^2}{2} \dot{m} = \frac{\rho}{2} u_m^3 A$  is (c.f. table 1):

$$N^E = \frac{T_\infty}{T} K \text{ (exergy destruction number)} \quad (24)$$

Note that  $N^E$  is not an energy devaluation number in the sense of its definition in eq. (11) because the reference quantity  $\dot{E}$  in eq. (22) is not an energy transfer rate (that might be devaluated during the transfer process). Instead it is the kinetic energy involved in the convective process. It serves as a reference quantity for the flow work required to maintain the flow.

Different from  $N_i$  according to eq. (11), for which by definition always  $0 \leq N_i \leq 1$  holds,  $N^E$  is not restricted to this range. For example  $N^E = 3$  for an internal flow means that the exergy loss (exergy destructed) during this process is three times higher than the kinetic energy involved in the convective process. Note that it is not the kinetic energy that is devaluated but the energy that enters the system as flow work, being pure exergy at the beginning and partly or totally converted to energy by the dissipation process.

## 5.2. Assessing convective heat transfer

Since both energies in a convective heat transfer process (flow work needed and thermal energy transferred) are subjected to devaluation they should both be accounted for when a convective heat transfer process is assessed, for example for the purpose of its optimization.

In terms of losses what counts is the lost exergy in both energies that are involved in the convective heat transfer process. These exergy losses are characterized by the corresponding entropy generation rates  $\dot{S}_{g,i}$  in eq. (11) and  $\dot{S}_{g,D}$  in eq. (22). They can be added to provide the overall entropy generation rate in a convective heat transfer process and serve as a target quantity in an optimization procedure. This is a reasonable criterion for all those cases in which the exergy part of energy transfer process counts like for a power cycle. In such a process exergy lost ahead of the turbine cannot be converted to mechanical energy in the turbine and thus reduces the efficiency of the power cycle.

When the entropy generation rates should be determined from detailed numerical solutions of a convective heat transfer process,  $\dot{S}_{g,i}$  follows from eqs. (19), (20) while  $\dot{S}_g$  due to dissipation is determined by

$$\dot{S}_g = \int d_g \dot{S} \quad (\text{exergy destruction number}) \quad (25)$$

with

$$d_g \dot{S} = \frac{\mu}{T} \left( 2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial z} \right)^2 \right] + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 \right) dV \quad (26)$$

When the flow is turbulent,  $d_g \dot{S}$  according to eqs. (19) and (26) are adequate only for a direct numerical simulation (DNS) approach with respect to the turbulence, as for the example shown in [11]. Since DNS solutions with their extraordinary computational demand cannot be used for solving technical problems, the time-averaged equations (Reynolds-averaged Navier-Stokes: RANS) are solved instead. Then, also  $d_g \dot{S}$  has to be time averaged, leading to:

$$d_g \dot{S}_C = d_g \dot{S}_{\bar{C}} + d_g \dot{S}'_C \quad (27)$$

and

$$d_g \dot{S}_D = d_g \dot{S}_{\bar{D}} + d_g \dot{S}'_D \quad (28)$$

with  $d_g \dot{S}_{\bar{C}}$  and  $d_g \dot{S}_{\bar{D}}$  for the entropy generation in the time-averaged temperature and velocity field as well as  $d_g \dot{S}'_C$  and  $d_g \dot{S}'_D$  for the time averaged contributions of the corresponding fluctuating parts.

All four parts are

$$d_g \dot{S}_{\bar{c}} = \frac{k}{T^2} \left[ \left( \frac{\partial \bar{T}}{\partial x} \right)^2 + \left( \frac{\partial \bar{T}}{\partial y} \right)^2 + \left( \frac{\partial \bar{T}}{\partial z} \right)^2 \right] dV \quad (29)$$

$$d_g \dot{S}_{\bar{c}'} = \frac{k}{T^2} \left[ \overline{\left( \frac{\partial T'}{\partial x} \right)^2} + \overline{\left( \frac{\partial T'}{\partial y} \right)^2} + \overline{\left( \frac{\partial T'}{\partial z} \right)^2} \right] dV \quad (30)$$

$$d_g \dot{S}_{\bar{D}} = \frac{\mu}{T} \left[ 2 \left[ \left( \frac{\partial \bar{u}}{\partial x} \right)^2 + \left( \frac{\partial \bar{u}}{\partial y} \right)^2 + \left( \frac{\partial \bar{u}}{\partial z} \right)^2 \right] + \left( \frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x} \right)^2 + \left( \frac{\partial \bar{u}}{\partial z} + \frac{\partial \bar{w}}{\partial x} \right)^2 + \left( \frac{\partial \bar{v}}{\partial z} + \frac{\partial \bar{w}}{\partial y} \right)^2 \right] dV \quad (31)$$

$$d_g \dot{S}_{\bar{D}'} = \frac{\mu}{T} \left[ 2 \left[ \overline{\left( \frac{\partial u'}{\partial x} \right)^2} + \overline{\left( \frac{\partial u'}{\partial y} \right)^2} + \overline{\left( \frac{\partial u'}{\partial z} \right)^2} \right] + \overline{\left( \frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x} \right)^2} + \overline{\left( \frac{\partial u'}{\partial z} + \frac{\partial w'}{\partial x} \right)^2} + \overline{\left( \frac{\partial v'}{\partial z} + \frac{\partial w'}{\partial y} \right)^2} \right] dV \quad (32)$$

with the results for a turbulent flow field from RANS equations,  $d_g \dot{S}_{\bar{c}}$  and  $d_g \dot{S}_{\bar{D}}$  can be determined, but not  $d_g \dot{S}_{\bar{c}'}$  and  $d_g \dot{S}_{\bar{D}'}$ . For these terms turbulence models are needed, as for examples discussed in [12].

### 5.3. Nondimensional parameters

When the whole process of a convective heat transfer should be assessed (comprising the exergy loss in the temperature and in the flow field) that again should be done by means of nondimensional parameters. The nondimensional parameters introduced so far are:

- *Nusselt number*  $Nu$  / eq. (13), indicating the strength of heat transfer versus its irreversibility;
- *Energy devaluation number*  $N_i$  / eq. (11), indicating the loss of entropic potential of the transferred energy;
- *Head loss coefficient*  $K$  / table 1, indicating the dissipation rate in the flow field;
- *Exergy destruction number*  $N^E$  / eq. (24), indicating the loss of exergy in the flow field.

If now the overall exergy loss for a convective heat transfer process is of interest this basically is the sum of the effects covered by  $N_i$  and  $N^E$ . Since both parameters are not nondimensionalized in the same way, however, they cannot simply be added. Note that  $N_i$  refers to the transferred energy which for the convective heat transfer is  $\dot{Q}$ , whereas in  $N^E$  the kinetic energy of the fluid flow is used as a reference quantity.

For an overall assessment of a convective heat transfer process we now refer the sum of the exergy losses (in the temperature and in the flow field) to the exergy transferred in the process, which is  $\eta_c \dot{Q}$ , c.f. eq. (15), and thus introduce the

$$\text{overall exergy loss number : } \hat{N}^E = \frac{T_\infty (\dot{S}_{g,C} + \dot{S}_{g,D})}{\eta_c \dot{Q}} \quad (33)$$

With the help of eq. (33) it can be decided whether the increase of the Nusselt number by a certain technique to improve the heat transfer, like adding turbulence promoters, roughening of the wall or simply increasing the flow rate, is beneficial from the perspective of exergy conservation. When  $\hat{N}^E$  is decreased, more available work is left and the increase of Nu is beneficial.

Since a device with a small  $\hat{N}^E$  obviously is more efficient than one with a larger  $\hat{N}^E$ , an

$$\text{overall efficiency factor : } \hat{N}_E = 1 - \hat{N}^E \quad (34)$$

was introduced in [13] which is  $\hat{\eta}_E = 1$  for a perfect (thermodynamically reversible) process without any exergy loss and  $\hat{\eta}_E = 0$  for a process in which all exergy gets lost because it is converted to energy.

## 6. Examples

Two examples will be given in which the parameters that were introduced above will be used in order to characterize the heat transfer situation. With these examples it should become obvious that entropy and/or its generation should not be ignored when heat transfer processes are considered in practical industrial applications.

### 6.1. Fully developed pipe flow with heat transfer

This simple example may illustrate how important it is to account for entropy generation which is the crucial aspect in the *energy devaluation number*  $N_i$  according to its definition (11).

What usually can be found as the characterization of the heat transfer performance of a fully developed pipe flow is the Nusselt number Nu. Let's assume it is  $Nu = 100$  and it occurs on the

upper temperature level of a power cycle, i.e. ahead of the turbine of this energy conversion device. Let's also assume that this heat transfer situation with  $Nu = 100$  and a heat flux  $\dot{q}_w = 10^3 \text{ W/m}^2$  on a length  $L = 0,1 \text{ m}$  occurs in two different power cycles:

- A steam power cycle (SPC) with water as the working fluid and an upper temperature level  $T_{m,u} = 900 \text{ K}$ .
- An organic Rankine cycle (ORC) with ammonia  $\text{NH}_3$  as working fluid and an upper temperature level  $T_{m,u} = 400 \text{ K}$ .

When in both cycles  $Nu$ ,  $\dot{q}_w$  and  $L$  are the same, the temperature difference  $\Delta T$  in  $Nu$  according to eq. (13) is larger by a factor 2.6 for ammonia compared to water. This is due to the different values of the thermal conductivity  $k$  of water (at  $T_{m,u} = 900 \text{ K}$  and  $p = 250 \text{ bar}$ ) and ammonia (at  $T_{m,u} = 400 \text{ K}$  and  $p = 25 \text{ bar}$ ), assuming typical values for the temperature and pressure levels in both cycles.

For a further comparison note that the energy devaluation number according to eq. (11) in this case with  $d_g \dot{S}_i$  according to eq. (3) and integrated to obtain

$$\dot{S}_{g,i} = \dot{Q}_{w,i} \left( \frac{1}{T_w} - \frac{1}{T_{m,u}} \right) \approx \dot{Q}_{w,i} \frac{\Delta T}{T_{m,u}^2} \quad (35)$$

with  $\dot{E} = \dot{Q}_w$  is

$$N_i = \frac{T_\infty \Delta T}{T_{m,u}^2} \quad (36)$$

Table 2 shows the *energy devaluation number*  $N_i$  for both cases according to this approximation. It shows that only 0.37 % of the entropic potential is used for the heat transfer in the SPC-case, but almost 5% in the ORC-case “though” both heat transfer situations have the same Nusselt number  $Nu = 100$  and the same amount of energy is transferred. Note that only that part of the entropic potential that is not yet used is available for further use after the process under consideration.

Cycle/fluid	$\frac{k}{\text{W/mK}}$	$\frac{T_\infty}{\text{K}}$	$\frac{T_{m,u}}{\text{K}}$	$\frac{\Delta T}{\text{K}}$	$N_i$
SPC/water	0.1	300	900	10	0.0037
ORC/ammonia	0.038	300	400	26	0.049

**Table 2.** Heat transfer with  $Nu = 100$ ,  $\dot{q}_w = 10^3 \frac{\text{W}}{\text{m}^2}$ ,  $L = 0.1 \text{ m}$  in two different power cycles

## 6.2. Using CFD to assess a heat exchanger

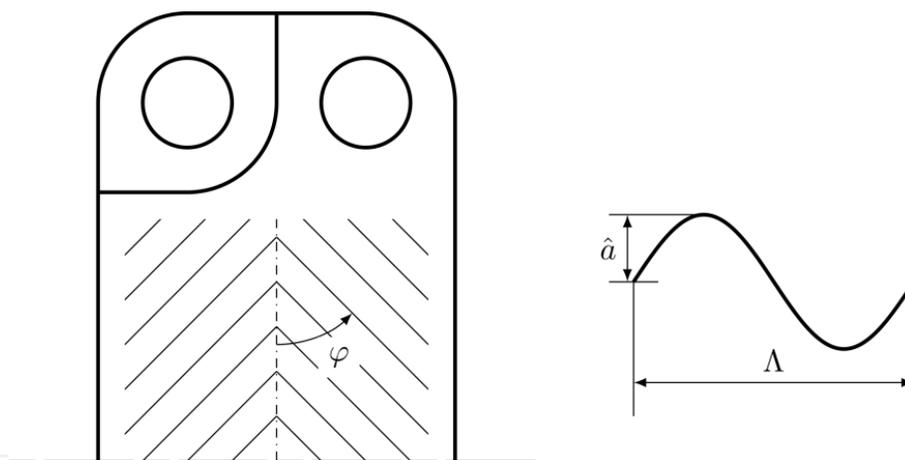
In the previous example, two similar processes at two different temperature levels were considered. Such a pipe flow with heat transfer is part of the heat transfer situation illustrated in figure 1: the cold side (b) is heated.

In the second example computational fluid dynamics (CFD) is used to assess the heating of a fluid in a passage within a plate heat exchanger, trying to find the best point of operation for the device. We will first describe the device and how it is modeled, and then discuss the results and how to use them. Further details can be found in [14].

### 6.2.1. Geometry of the device

Plate heat exchangers are made of corrugated plates which are arranged in a plate stack, forming channels between the plates. The plates are designed in such a way that two fluids are separated from each other on their way through adjacent channels.

Depending on the plate corrugation the channels have constantly changing cross sections, but there is a repeating geometric pattern. Figure 2 (left) shows part of a plate with such a pattern; in this case it is a symmetric fish-bone pattern with a sinusoidal corrugation (see figure 2, right).



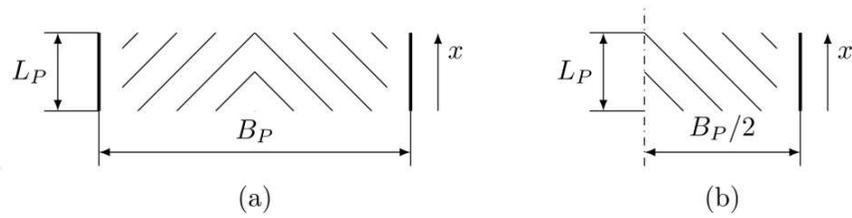
**Figure 2.** Heat exchanger plate geometry: the plate has a symmetric fishbone pattern with the corrugation angle  $\varphi$ , amplitude  $\hat{a}$  and period  $\Lambda$ ; c.f. [15]

### 6.2.2. Modeling of the device

The first simplification made in order to facilitate the simulations is that the plate (and therefore the heat exchanger) is assumed to have an infinite length. Thus effects on the flow caused by the inlet or outlet areas can be neglected: the flow is hydraulically developed. This has two consequences:

- the channel can be modeled as an endlessly repeating stripe of finite length, see figure 3 (a),
- only half of the channel must be simulated, see figure 3 (b).

The resulting domain geometry is shown in figure 4.



**Figure 3.** Simplified geometry of the heat exchanger: (a) symmetric stripe; (b) solution domain due to the symmetry assumption.



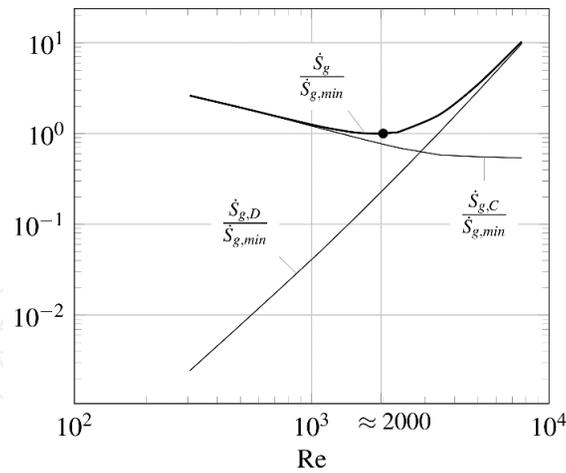
**Figure 4.** view of the simulated plate heat exchanger stripe.

The second simplification made here is that the heat exchanger is operated with a balanced counter-flow: The capacity flow rate  $\dot{m}c_p$  is the same on the hot and the cold side, so that the temperature difference between them as well as the heat flux  $\dot{q}_w$  are the same at every point between the inlet and the outlet.

### 6.2.3. Boundary conditions

Based on the assumptions made above, periodic boundary conditions can be applied to the flow field in main flow direction  $x$  (see figure 3). The boundary condition applied with respect to the pressure field is a so-called “fan” boundary condition that sets a constant pressure drop between the inlet and outlet patch. In the symmetry plane a symmetry boundary condition is imposed, and no-slip boundary conditions hold at all walls.

The temperature field has a fan boundary condition with a positive temperature difference  $\Delta T_{io}$  between the inlet and the outlet patch. This results in a heating of the fluid as it passes through the simulated passage. The boundary condition used for the top and bottom walls is a linearly increasing temperature profile in mean flow direction. The increase in temperature  $\Delta T_{\omega,io}$  is the same as  $\Delta T_{io}$ . Together, these two boundary conditions model the balanced counter-flow configuration of the heat exchanger. A zero-gradient boundary condition is used for the gasket, which is modeled as an adiabatic wall.



**Figure 5.** Overall entropy generation rate  $\dot{S}_g$ , entropy generation rate due to dissipation  $\dot{S}_{g,D}$  and entropy generation rate due to conduction  $\dot{S}_{g,C}$  (normalized with the minimum entropy generation rate at  $Re \approx 2000$ ) at varying Reynolds numbers, for the simulated heat exchanger passage.

Changing the pressure drop leads to different mean flow velocities. In order to keep the heat flux  $\dot{q}_w$  fixed, it was necessary to adjust the temperature difference between inlet and outlet ( $\Delta T_{w,io} = \Delta T_{io} = \dot{q}_w A / \dot{m} c_p$ ) accordingly.

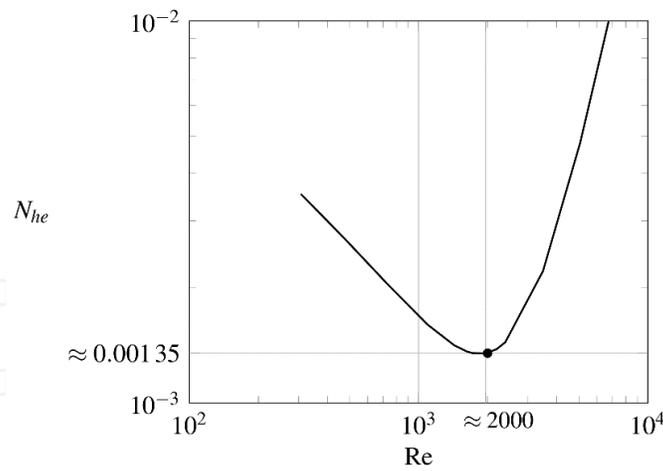
#### 6.2.4. Simulation results

The results obtained from CFD simulations give access to the velocity, pressure and temperature fields  $u$ ,  $p$  and  $T$ . They can be used to calculate the heat transfer coefficient and the head loss coefficient for the convective heat transfer under consideration.

Calculating the pressure and velocity fields is the computationally expensive part of the simulation. When all fluid properties are assumed to be constant, i.e. pressure and temperature independent, the temperature field can even be modeled as a passive scalar, which comes at very little computational cost. The four parts of the entropy generation ( $\dot{S}_{g,\bar{c}}$ ,  $\dot{S}_{g,C}$ ,  $\dot{S}_{g,\bar{D}}$ ,  $\dot{S}_{g,D}$ , see eqs. (29) to (32) in section 5.2.) are post-processing quantities: they can be obtained from the  $u$ -,  $p$ - and  $T$ -fields without solving further differential equations. This is beneficial for the assessment of a certain process operating on different temperature levels.

The entropy generation rates due to dissipation, conduction and the sum of both are shown in figure 5 for different Reynolds numbers. For increasing Reynolds numbers,  $\dot{S}_{g,D}$  increases, while  $\dot{S}_{g,C}$  decreases. An optimal point of operation can be identified at about  $Re = 2000$ . The same optimum can be identified in figure 6 for the energy devaluation number of the heat exchanger,  $N_{he}$ , since in eq. (11) the heat flux, wall area and ambient temperature are the same for all calculations.

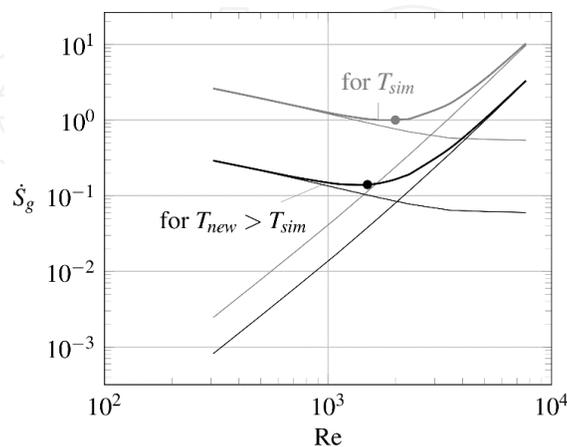
Note that the curves for  $\dot{S}_{g,C}$  and  $\dot{S}_{g,D}$  in figure 5 are almost straight lines, especially for higher Reynolds numbers. Therefore only two simulations are necessary in order to roughly estimate



**Figure 6.** Energy devaluation number  $N_{he}$  for the simulated plate heat exchanger passage.

an optimum point of operation. From the two straight lines for  $\dot{S}_{g,C}$  and  $\dot{S}_{g,D}$  the sum of both results as a curve with the minimum at the optimal Reynolds number.

As mentioned before the entropy generation is a post-processing quantity. This can be leveraged to assess the simulated heat transfer situation at different temperature levels. If the overall change in temperature between inlet and outlet is not too large, an approximation can be done by simply scaling the results accordingly. The entropy generation due to dissipation  $\dot{S}_{g,D,new}$  at the temperature level  $T_{new}$  is (compared to the entropy generation in an existing simulation result)  $\dot{S}_{g,D,new} / \dot{S}_{g,D,sim} = T_{sim} / T_{new}$ . If the new temperature level is higher,  $\dot{S}_{g,D,new}$  will be smaller than  $\dot{S}_{g,D,sim}$ . Similarly, for entropy generation due to conduction, the relationship is  $\dot{S}_{g,C,new} / \dot{S}_{g,C,sim} = (T_{sim} / T_{new})^2$ . Again, if the new temperature level is higher,  $\dot{S}_{g,C,new}$  will be smaller than  $\dot{S}_{g,C,sim}$ . The optimum point of operation shifts to a lower Reynolds number (see figure 7), because the effect of a temperature level change on  $\dot{S}_{g,C}$  is larger than the effect on  $\dot{S}_{g,D}$ .



**Figure 7.** Entropy generation rates for a heat transfer at different temperature levels. For higher temperatures, the optimum point of operation shifts to lower Reynolds numbers.

## 7. Conclusions

Despite its apparently low popularity, entropy generation is a crucial aspect of every heat transfer process. Every real technical process includes the generation of entropy, which at some point has to be discharged to the ambient. It has been shown that every energy flow has an entropic potential, which is the amount of entropy that can be discharged to the ambient along with the energy flow. It therefore sets the limit for all wanted processes associated with this energy flow. Based on this, the *energy devaluation number* has been introduced, which quantifies the part of the entropic potential which is lost in a transfer process. The *energy devaluation number* is applicable to all processes in which energy is transferred and is recommended for their assessment especially with regard to sustainability.

In the examples it has also been shown how different heat transfer situations can be compared with each other. Such comparisons can be made on very different levels, reaching from system assessment (i.e. to compare different systems) to more detailed studies regarding the optimization of subsystems which are part of an overall heat transfer system. It has also been shown how existing simulation results can be reused at different temperature levels, effectively lowering the cost of CFD simulations.

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