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# **On the New Bivariate Local Linearisation Method for Solving Coupled Partial Differential Equations in Some Applications of Unsteady Fluid Flows with Heat and Mass Transfer**

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Additional information is available at the end of the chapter

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## **Abstract**

This work presents a new numerical approach for solving unsteady two-dimensional boundary layer flow with heat and mass transfer. The flow model is described in terms of a highly coupled and nonlinear system of partial differential equations that models the problem of unsteady mixed convection flow over a vertical cone due to impulsive motion. The proposed method of solution uses a local linearisation approach to decouple the original system of PDEs to form a sequence of equations that can be solved in a computationally efficient manner. Approximate functions defined in terms of bivariate Lagrange interpolation polynomials are used with spectral collocation to approximate the solutions of the decoupled linearised equations. To test the accuracy and to validate the results of the proposed method, numerical error analysis and convergence tests are conducted. The present results are also compared with results from published literature for some special cases. The proposed algorithm is shown to be very accurate, convergent and very effective in generating numerical results in a computationally efficient manner.

On the New Bivariate Local Linearisation Method for Solving Coupled Partial Differential Equations in Some Applications of Unsteady Fluid Flows with Heat and Mass Transfer

**Keywords:** Unsteady boundary layer flow, heat and mass transfer, bivariate spectral collocation, impulsive flow

## 1. Introduction

In this investigation we revisit the problem of unsteady mixed convection flow over a vertical cone due to impulsive motion that was previously discussed in Singh and Roy [7]. In the flow model, the external stream is impulsively set into motion and the surface temperature is suddenly changed from its ambient temperature. The resulting constitutive equations describing the flow properties are a system of highly coupled non-linear partial differential equations (PDEs) which cannot be solved exactly. In [7], the solution of the PDEs system was approximated numerically using implicit finite differences after linearising using the quasi-linearisation (QLM) technique of Bellman and Kalaba [1]. The QLM simplifies the solution process by simply reducing the original PDEs to a linearised form which, nevertheless, is coupled. In view of the coupled nature of the QLM iteration scheme, matrices of very large dimensions are obtained when using any numerical method is used for discretization. For systems of coupled equations, the method proposed by [3] was used in [7] to decouple the governing equations. There is generally a large overhead in computing resources when performing computational tasks on large matrix systems. To avoid dealing with large matrix systems, relaxation methods can be used. However, the disadvantage of relaxation methods is that they converge much slower than quasi-linearisation schemes which are known to converge quadratically.

A linearisation method, termed the local linearisation method (LLM), was recently introduced in [4] as an efficient method for solving coupled systems of non-linear ordinary differential equations that model boundary layer equations. This method was extended to a PDE system in [5] where the Chebyshev spectral collocation method [2, 8] was used for discretization in one independent variable and finite differences was used for discretization in the second independent variable of the PDE. In this study, we present a new approach that seeks to improve on the original LLM approach of [5] by eliminating the need for using finite differences for discretizing in one of the independent variables and, instead, apply spectral collocation independently in all independent variables of the PDE system. Finite differences require very fine grids (with a large number of grid points) to give more accurate solutions. In contrast, spectral methods are computationally less expensive, converge faster and are more accurate than finite difference methods particularly for problems with smooth solutions. The collocation method applied in this work uses bivariate Lagrange interpolation polynomials as basis functions. The proposed method converges fast and gives very accurate results which are obtained in a computationally efficient manner. The accuracy is established through residual and solution error analysis. Further validation of present results is established by comparing with existing results from literature.

## 2. Governing equations

The model under consideration is that of an unsteady mixed convection flow over a vertical cone that is impulsively set into motion to cause unsteadiness in the flow (see [6, 7]). It is

assumed that buoyancy forces are present due to temperature and concentration variation of the fluid flow. The governing boundary layer momentum, energy and concentration equations were reduced to dimensionless form in [7] using transformations that were initially proposed in Williams and Rhyne [9] to give,

$$\begin{aligned} \frac{\partial^3 f}{\partial \eta^3} + \frac{\eta}{2}(1-\xi) \frac{\partial^2 f}{\partial \eta^2} + f \frac{\partial^2 f}{\partial \eta^2} \left[ \xi \left( \frac{m+3}{6} \right) - (1-\xi) \left( \frac{m-3}{6} \right) \log(1-\xi) \right] + \frac{m}{3} \xi \left[ 1 - \left( \frac{\partial f}{\partial \eta} \right)^2 \right] \\ \lambda \xi (\theta + N\phi) = \xi (1-\xi) \frac{\partial^2 f}{\partial \eta \partial \xi} + \frac{\xi}{3} (m-3) (1-\xi) \log(1-\xi) \left[ \frac{\partial^2 f}{\partial \eta^2} \frac{\partial f}{\partial \xi} - \frac{\partial f}{\partial \eta} \frac{\partial^2 f}{\partial \eta \partial \xi} \right], \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{\partial^2 \theta}{\partial \eta^2} + \frac{\eta}{2} Pr (1-\xi) \frac{\partial \theta}{\partial \eta} + Pr f \frac{\partial \theta}{\partial \eta} \left[ \xi \left( \frac{m+3}{6} \right) - (1-\xi) \left( \frac{m-3}{6} \right) \log(1-\xi) \right] - Pr \left( \frac{2m-3}{3} \right) \xi \theta \frac{\partial f}{\partial \eta} \\ = \xi (1-\xi) Pr \frac{\partial \theta}{\partial \xi} + Pr (1-\xi) \log(1-\xi) \xi \left( \frac{m-3}{3} \right) \left[ \frac{\partial \theta}{\partial \eta} \frac{\partial f}{\partial \xi} - \frac{\partial f}{\partial \eta} \frac{\partial \theta}{\partial \xi} \right], \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial^2 \phi}{\partial \eta^2} + \frac{\eta}{2} Sc (1-\xi) \frac{\partial \phi}{\partial \eta} + Sc f \frac{\partial \phi}{\partial \eta} \left[ \xi \left( \frac{m+3}{6} \right) - (1-\xi) \left( \frac{m-3}{6} \right) \log(1-\xi) \right] - Sc \left( \frac{2m-3}{3} \right) \xi \phi \frac{\partial f}{\partial \eta} \\ = \xi (1-\xi) Sc \frac{\partial \phi}{\partial \xi} + Sc (1-\xi) \log(1-\xi) \xi \left( \frac{m-3}{3} \right) \left[ \frac{\partial \phi}{\partial \eta} \frac{\partial f}{\partial \xi} - \frac{\partial f}{\partial \eta} \frac{\partial \phi}{\partial \xi} \right], \end{aligned} \quad (3)$$

where  $Pr$  is the Prandtl number,  $Sc$  is the Schmidt number,  $f$  is the dimensionless stream function,  $\theta$  and  $\phi$  are the dimensionless temperature and concentration,  $N = \lambda / \lambda^*$  is the ratio of the thermal ( $\lambda$ ) and concentration  $\lambda^*$  buoyancy parameter.

The boundary conditions are

$$\begin{aligned} f(0, \xi) = 0, \quad \frac{\partial f(0, \xi)}{\partial \eta} = 0, \quad \theta(0, \xi) = 1, \quad \phi(0, \xi) = 1, \\ \frac{\partial f(\infty, \xi)}{\partial \eta} = 1, \quad \theta(\infty, \xi) = 0, \quad \phi(\infty, \xi) = 0. \end{aligned} \quad (4)$$

Other quantities of physical interest include the local skin friction coefficient, Nusselt number and Sherwood numbers which are given, in dimensionless forms as,

$$Re_L^{1/2} C_f = 2 \xi^{-1/2} f''(0, \xi), \quad (5)$$

$$Re_L^{-1/2} Nu = -\xi^{-1/2} \theta'(0, \xi), \quad (6)$$

$$Re_L^{-1/2} Sh = -\xi^{-1/2} \phi'(0, \xi). \quad (7)$$

### 3. Derivation of solution method

The derivation of the solution method is described for a general system of non-linear PDEs in this section. Consider a system of 3 coupled PDEs in  $f(\eta, \xi)$ ,  $\theta(\eta, \xi)$  and  $\phi(\eta, \xi)$

$$\Omega_k[F, T, H] = 0, \quad \text{for } k = 1, 2, 3, \quad (8)$$

where  $\Omega_1$ ,  $\Omega_2$  and  $\Omega_3$  are non-linear operators that represent the non-linear PDEs (1), (2) and (3), respectively and  $F, T, H$  are defined as

$$\begin{aligned} F &= \left\{ f, \frac{\partial f}{\partial \eta}, \frac{\partial^2 f}{\partial \eta^2}, \frac{\partial^3 f}{\partial \eta^3}, \frac{\partial^2 f}{\partial \xi \partial \eta} \right\}, \\ T &= \left\{ \theta, \frac{\partial \theta}{\partial \eta}, \frac{\partial^2 \theta}{\partial \eta^2}, \frac{\partial \theta}{\partial \xi} \right\}, \\ H &= \left\{ \phi, \frac{\partial \phi}{\partial \eta}, \frac{\partial^2 \phi}{\partial \eta^2}, \frac{\partial \phi}{\partial \xi} \right\}. \end{aligned}$$

Equation (8) can be simplified and decoupled by using the following algorithm called local linearisation method (LLM):

#### LLM Algorithm

1. Solve for  $F$  in 1st equation assuming that  $T$  and  $H$  are known from previous iteration.  $\rightarrow F_{r+1}$ .
2. With  $F_{r+1}$  now known, solve for  $T$  in 2nd equation, assuming that  $H$  is known from previous iteration.  $\rightarrow T_{r+1}$ .
3. Solve for  $H$  in 3rd equation  $\rightarrow H_{r+1}$ .

The LLM algorithm was recently reported in [4] as an efficient method for solving coupled system of non-linear ordinary differential equations that model boundary layer equations. The method was later extended to partial differential equations in [5]. Applying the algorithm in equations (8) gives

$$F \cdot \nabla_f \Omega_1[F_r, T_r, H_r] = F_r \cdot \nabla_f \Omega_1[F_r, T_r, H_r] - \Omega_1[F_r, T_r, H_r], \quad (9)$$

$$T \cdot \nabla_{\theta} \Omega_2 [F_{r+1}, T_r, H_r] = T_r \cdot \nabla_{\theta} \Omega_2 [F_{r+1}, T_r, H_r] - \Omega_2 [F_{r+1}, T_r, H_r], \quad (10)$$

$$H \cdot \nabla_{\phi} \Omega_3 [F_{r+1}, T_{r+1}, H_r] = H_r \cdot \nabla_{\phi} \Omega_3 [F_{r+1}, T_{r+1}, H_r] - \Omega_3 [F_{r+1}, T_{r+1}, H_r]. \quad (11)$$

The above equation forms a system of three decoupled linear PDEs that are to be solved iteratively for  $f(\eta, \xi)$ ,  $\theta(\eta, \xi)$  and  $\phi(\eta, \xi)$ , where  $\nabla$  is a vector of partial derivatives defined as

$$\begin{aligned} \nabla &= \{\nabla_f, \nabla_{\theta}, \nabla_{\phi}\}, \\ \nabla_f &= \left\{ \frac{\partial}{\partial f}, \frac{\partial}{\partial f'}, \frac{\partial}{\partial f''}, \frac{\partial}{\partial f'''}, \frac{\partial}{\partial f'_{\xi}}, \frac{\partial}{\partial f'_{\xi}} \right\}, \\ \nabla_{\theta} &= \left\{ \frac{\partial}{\partial \theta}, \frac{\partial}{\partial \theta'}, \frac{\partial}{\partial \theta''}, \frac{\partial}{\partial \theta'''} \right\}, \\ \nabla_{\phi} &= \left\{ \frac{\partial}{\partial \phi}, \frac{\partial}{\partial \phi'}, \frac{\partial}{\partial \phi''}, \frac{\partial}{\partial \phi'''} \right\}. \end{aligned}$$

Applying the LLM iteration scheme (9) - (11) on the governing non-linear PDE system (1) - (3) gives

$$f_{r+1}''' + a_{1,r}(\eta, \xi) f_{r+1}'' + a_{2,r}(\eta, \xi) f_{r+1}' + a_{3,r}(\eta, \xi) f_{r+1} + a_{4,r}(\eta, \xi) \frac{\partial f_{r+1}}{\partial \xi} = R_{1,r}(\eta, \xi), \quad (12)$$

$$\theta_{r+1}'' + b_{1,r}(\eta, \xi) \theta_{r+1}' + b_{2,r}(\eta, \xi) \theta_{r+1} + b_{3,r}(\eta, \xi) \frac{\partial \theta_{r+1}}{\partial \xi} = R_{2,r}(\eta, \xi), \quad (13)$$

$$\phi_{r+1}'' + c_{1,r}(\eta, \xi) \phi_{r+1}' + c_{2,r}(\eta, \xi) \phi_{r+1} + c_{3,r}(\eta, \xi) \frac{\partial \phi_{r+1}}{\partial \xi} = R_{3,r}(\eta, \xi), \quad (14)$$

where the primes denote partial differentiation with respect to  $\eta$  and the coefficients  $a_{i,r}, b_{i,r}$  and  $c_{i,r}$  ( $i=1,2,3,4$ ) are known coefficients whose quantities are known from the previous iteration level  $r$ . The definition of the coefficients is given in the Appendix.

The boundary conditions are given by

$$f_{r+1}(0, \xi) = 0, \quad f'_{r+1}(0, \xi) = 0, \quad \theta_{r+1}(0, \xi) = 1, \quad \phi_{r+1}(0, \xi) = 1, \quad (15)$$

$$f'_{r+1}(\infty, \xi) = 1, \quad \theta_{r+1}(\infty, \xi) = 0, \quad \phi_{r+1}(\infty, \xi) = 0. \quad (16)$$

Starting from given initial approximations, denoted by  $f_0(\eta, \xi)$ ,  $\theta_0(\eta, \xi)$  and  $\phi_0(\eta, \xi)$ , equations (12) – (14) are solved iteratively for  $r=1, 2, \dots$ , until approximate solutions that are consistent to within a certain tolerance level are obtained.

To solve the linearised equations (12) – (16), an approximate solution defined in terms of bivariate Lagrange interpolation polynomials is sought. For example, the approximate solution for  $f(\eta, \xi)$  takes the form

$$f(\eta, \xi) \approx \sum_{m=0}^{N_x} \sum_{j=0}^{N_t} f(\tau_m, \zeta_j) L_m(\tau) L_j(\zeta), \quad (17)$$

where the functions  $L_m(\tau)$  and  $L_j(\zeta)$  are the well-known characteristic Lagrange cardinal polynomials. The function (17) interpolates  $f(\eta, \xi)$  at the collocation points (known as Chebyshev-Gauss-Lobatto) defined by

$$\tau_i = \cos\left(\frac{\pi i}{N_x}\right), \quad \zeta_j = \cos\left(\frac{\pi j}{N_t}\right), \quad i = 0, 1, \dots, N_x; \quad j = 0, 1, \dots, N_t. \quad (18)$$

The choice of collocation points (18) makes it possible for the Chebyshev spectral collocation method to be used as a solution procedure. The Chebyshev collocation method requires that the domain of the problem be transformed to  $[-1, 1] \times [-1, 1]$ . Accordingly, linear transformations have been used to transform  $\eta \in [0, \eta_\infty]$  and  $\xi \in [0, L_t]$  to  $\tau \in [-1, 1]$  and  $\zeta \in [-1, 1]$ , respectively. Here  $\eta_\infty$  is a finite value that is introduced to facilitate the application of the numerical method at infinity and  $L_t$  is the largest value of  $\xi$ .

Substituting equation (17) in equation (12)-(14) and making use of the derivatives formulas for Lagrange functions at Gauss-Lobatto points given in [2, 8], results in

$$A_i F_{r+1,i} + a_{4,i} \sum_{j=0}^{N_t} d_{i,j} D F_{r+1,j} + a_{5,i} \sum_{j=0}^{N_t} d_{i,j} F_{r+1,j} = R_{1,i}, \quad (19)$$

$$B_i Q_{r+1,i} + b_{3,i} \sum_{j=0}^{N_t} d_{i,j} Q_{r+1,j} = R_{2,i}, \quad (20)$$

$$C_i F_{r+1,i} + c_{3,i} \sum_{j=0}^{N_t} d_{i,j} F_{r+1,j} = R_{3,i}, \quad (21)$$

where  $d_{i,j}$  ( $i, j=0, 1, \dots, N_t$ ) are entries of the standard Chebyshev differentiation matrix  $d = [d_{i,j}]$  of size  $(N_t + 1) \times (N_t + 1)$  (see, for example [2, 8]),  $D2\eta_e[D_{r,s}]$  ( $r, s=0, 1, 2, \dots, N_x$ ) with  $[D_{r,s}]$  being an  $(N_x + 1) \times (N_x + 1)$  Chebyshev derivative matrix. The vectors and matrices are defined as

$$\begin{aligned} F_{r+1,i} &= [f_{r+1,i}(\tau_0), f_{r+1,i}(\tau_1), \dots, f_{r+1,i}(\tau_{N_x})]^T, \\ Q_{r+1,i} &= [\theta_{r+1,i}(\tau_0), \theta_{r+1,i}(\tau_1), \dots, \theta_{r+1,i}(\tau_{N_x})]^T, \\ F_{r+1,i} &= [\phi_{r+1,i}(\tau_0), \phi_{r+1,i}(\tau_1), \dots, \phi_{r+1,i}(\tau_{N_x})]^T, \\ A_i &= D^3 + a_{1,i}D^2 + a_{2,i}D + a_{3,i}, \\ B_i &= D^2 + b_{1,i}D + b_{2,i}, \\ C_i &= D^2 + c_{1,i}D + c_{2,i}, \end{aligned}$$

$a_{m,r}(\xi_i)$  ( $m=1,2,3,4,5$ ) is the diagonal matrix of the vector  $[a_{m,r}(\tau_0), a_{m,r}(\tau_1), \dots, a_{m,r}(\tau_{N_x})]^T$ . The matrices  $b_{1,i}, b_{2,i}, c_{1,i}, c_{2,i}$  are also diagonal matrices corresponding to  $b_{1,r}, b_{2,r}, c_{1,r}, c_{2,r}$  evaluated at collocation points. Additionally,  $R_{1,i}, R_{2,i}$  and  $R_{3,i}$  are  $(N_x + 1) \times 1$  vectors corresponding to  $R_{1,r}, R_{2,r}$  and  $R_{3,r}$ , respectively, evaluated at the collocation points.

After imposing the boundary conditions for  $i=0,1, \dots, N_t$ , equation (20) can be written as the following matrix equation:

$$\begin{bmatrix} A_{0,0} & A_{0,1} & \cdots & A_{0,N_t} \\ A_{1,0} & A_{1,1} & \cdots & A_{1,N_t} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N_t,0} & A_{N_t,1} & \cdots & A_{N_t,N_t} \end{bmatrix} \begin{bmatrix} F_{r+1,0} \\ F_{r+1,1} \\ \vdots \\ F_{r+1,N_t} \end{bmatrix} = \begin{bmatrix} R_{1,0} \\ R_{1,1} \\ \vdots \\ R_{1,N_t} \end{bmatrix}, \quad (22)$$

where

$$\begin{aligned} A_{i,j} &= A_i + a_{4,i}d_{i,i}D + a_{5,i}d_{i,i}I, \\ A_{i,j} &= a_{4,i}d_{i,i}D + a_{5,i}d_{i,i}I, \quad \text{when } i \neq j, \end{aligned}$$

where  $I$  is an  $(N_x + 1) \times (N_x + 1)$  identity matrix.

Applying the same approach to (20) and (21) gives

$$\begin{bmatrix} B_{0,0} & B_{0,1} & \cdots & B_{0,N_t} \\ B_{1,0} & B_{1,1} & \cdots & B_{1,N_t} \\ \vdots & \vdots & \ddots & \vdots \\ B_{N_t,0} & B_{N_t,1} & \cdots & B_{N_t,N_t} \end{bmatrix} \begin{bmatrix} Q_{r+1,0} \\ Q_{r+1,1} \\ \vdots \\ Q_{r+1,N_t} \end{bmatrix} = \begin{bmatrix} R_{2,0} \\ R_{2,1} \\ \vdots \\ R_{2,N_t} \end{bmatrix}, \quad (23)$$

$$\begin{bmatrix} C_{0,0} & C_{0,1} & \cdots & C_{0,N_t} \\ C_{1,0} & C_{1,1} & \cdots & C_{1,N_t} \\ \vdots & \vdots & \ddots & \vdots \\ C_{N_t,0} & C_{N_t,1} & \cdots & C_{N_t,N_t} \end{bmatrix} \begin{bmatrix} F_{r+1,0} \\ F_{r+1,1} \\ \vdots \\ F_{r+1,N_t-1} \end{bmatrix} = \begin{bmatrix} R_{3,0} \\ R_{3,1} \\ \vdots \\ R_{3,N_t} \end{bmatrix}, \quad (24)$$



where

$$\begin{aligned} B_{i,j} &= B_i + b_{3,i} d_{i,i} I, \\ B_{i,j} &= b_{3,i} d_{i,j} I, \quad \text{when } i \neq j, \\ C_{i,j} &= C_i + c_{3,i} d_{i,i} I, \\ C_{i,j} &= c_{3,i} d_{i,j} I, \quad \text{when } i \neq j, \end{aligned}$$

Starting from a given initial guess, the approximate solutions for  $f(\eta, \xi)$ ,  $\theta(\eta, \xi)$  and  $\phi(\eta, \xi)$  are obtained by iteratively solving the matrix equations (22), (23) and (24), in turn, for  $r=0, 1, 2, \dots$ . Convergence to the expected solution can be affirmed by considering the norm of the difference between successive iterations. If this quantity is less than a given tolerance level, the algorithm is assumed to have converged. The following error norms are defined for the difference between values of successive iterations,

$$E_f = \max_{0 \leq i \leq N_t} \|F_{r+1,i} - F_{r,i}\|_{\infty}, \quad E_{\theta} = \max_{0 \leq i \leq N_t} \|Q_{r+1,i} - Q_{r,i}\|_{\infty}, \quad E_{\phi} = \max_{0 \leq i \leq N_t} \|F_{r+1,i} - F_{r,i}\|_{\infty}. \quad (25)$$

## 4. Results and discussion

The local linearisation method (LLM) described by equations (12) - (14) was used to generate approximate numerical solutions for the governing systems of equations (1) - (3). The linearised equations were subsequently solved using the bivariate spectral collocation method as described in the previous section. The whole solution process is termed bivariate spectral local linearisation method (BSLLM). In this section, we present the results of the numerical computations for the various flow profiles. The BSLLM results are compared with published results from literature. In computing the numerical results presented in this paper,  $N_x=60$  and  $N_t=15$  collocation points in the  $\eta$  and  $\xi$  domain, respectively, were found to be sufficient to give accurate and consistent results. The minimum number of iterations required to give consistent results to within a tolerance level of  $10^{-7}$  were determined using equation (25).

The effects of buoyancy parameter ( $\lambda$ ) and Prandtl number ( $Pr$ ) on velocity and temperature profiles are shown in Figs. 1 and 2, respectively for when  $N=0.5$ ,  $Sc=0.94$  at  $\xi=0.5$ . It can be seen from the figures that the results from the present work match those reported in [7] exactly. We remark that in [7], the quasi-linearisation approach was used with implicit finite differences as a solution method. Details of the analysis of the effects of various governing parameters on the governing equations (1) - (3) can be found in [7] and have not been repeated here.

Figs. 3, 4 and 5 show the effect of buoyancy ratio  $N$  on the velocity, temperature and concentration profiles, respectively, when  $Pr=0.7$ ,  $Sc=0.94$  and  $\lambda=5$  at  $\xi=0.5$ . The graphs are qualitatively similar to those reported in [7]. These figures are a qualitative validation of the numerical results generated using the proposed BSLLM.

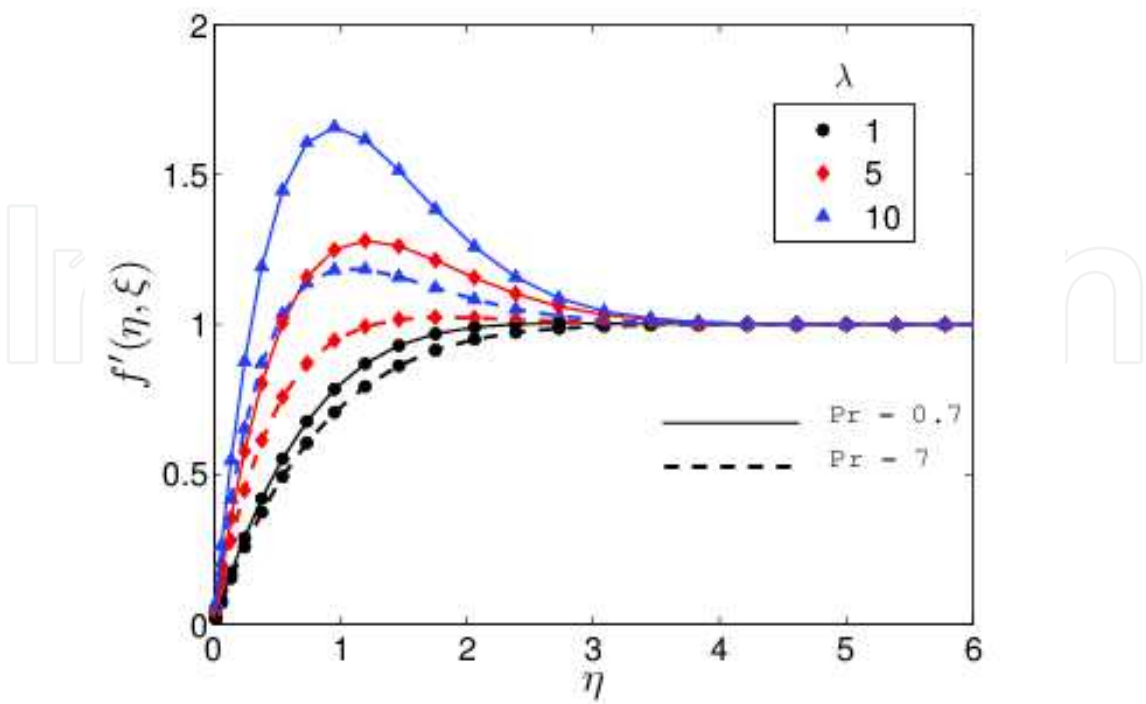


Figure 1. Effect of  $\lambda$  and  $Pr$  on the velocity profile  $f'(\eta, \xi)$

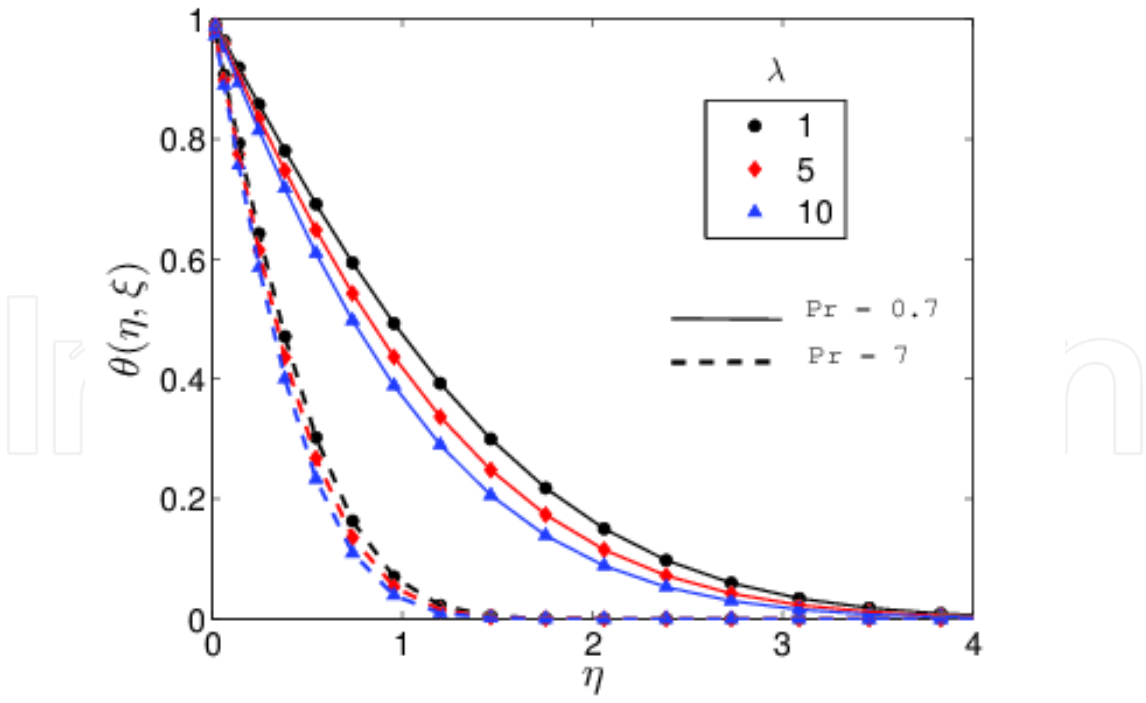


Figure 2. Effect of  $\lambda$  and  $Pr$  on the temperature profile  $\theta(\eta, \xi)$

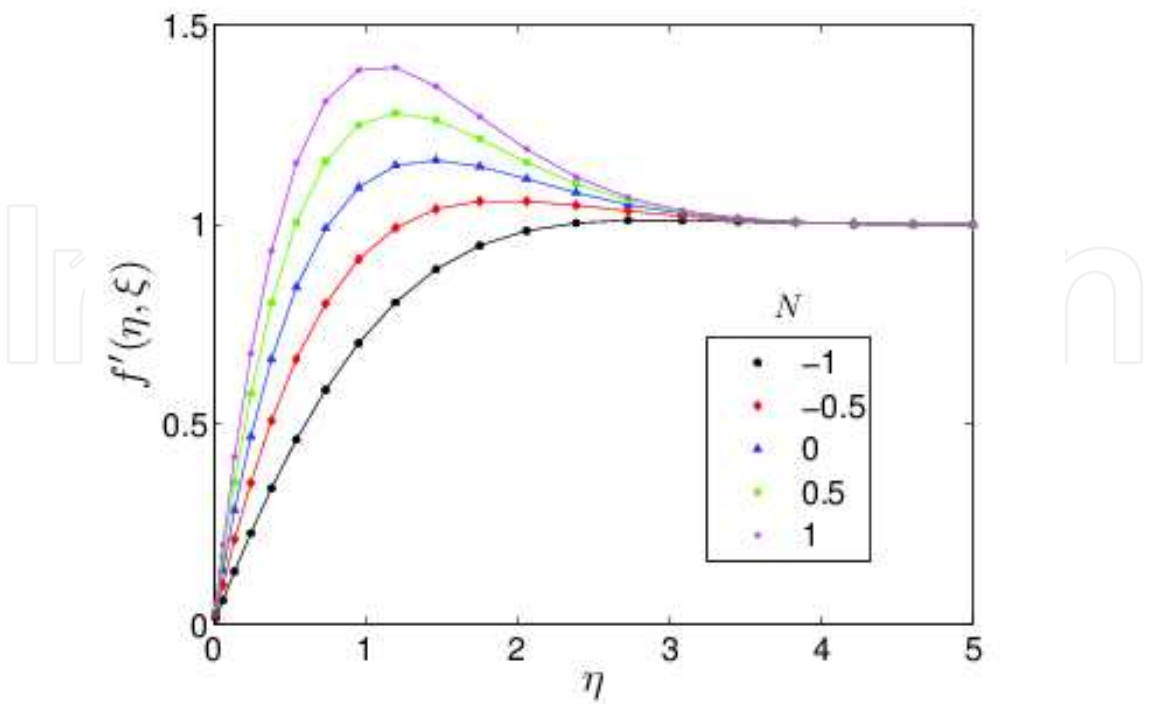


Figure 3. Effect of  $N$  velocity profile  $f'(\eta, \xi)$

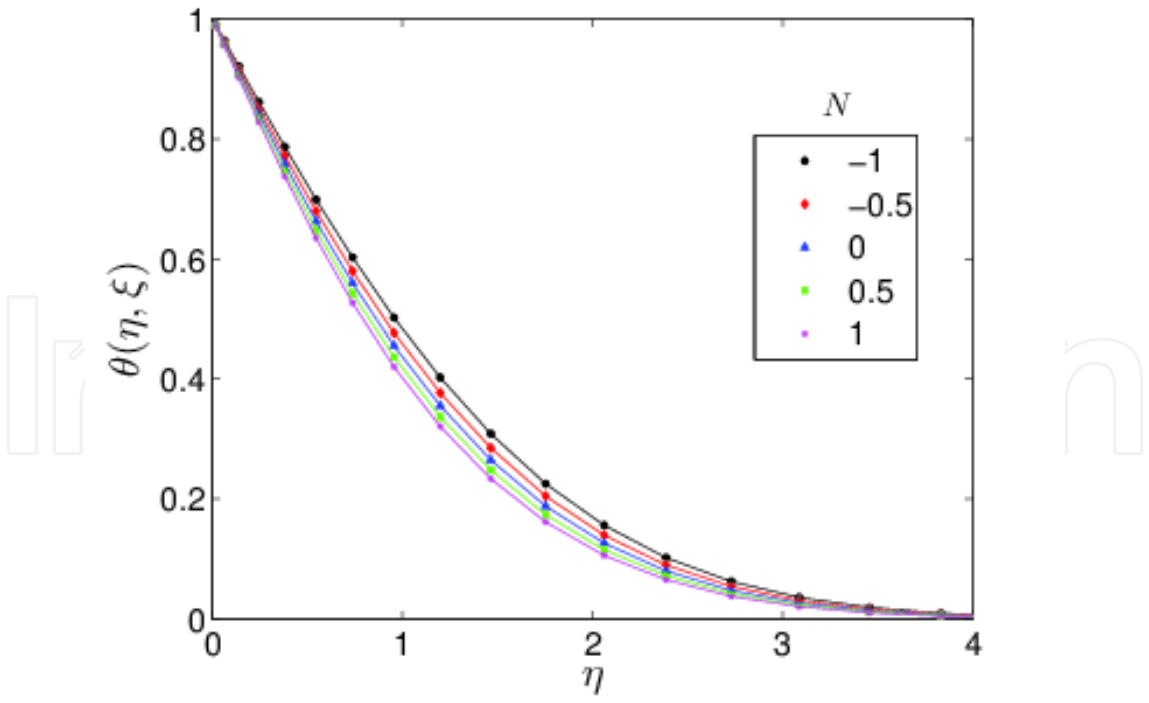


Figure 4. Effect of  $N$  on the temperature profile  $\theta(\eta, \xi)$

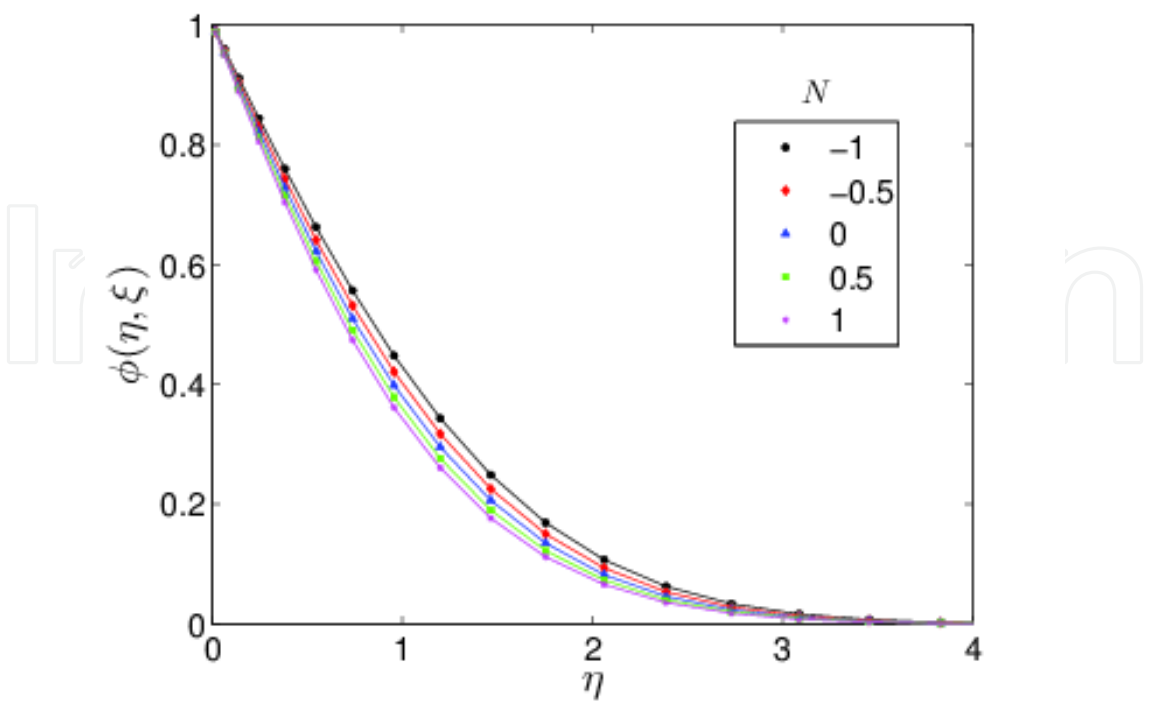


Figure 5. Effect of  $N$  concentration profile  $\phi(\eta, \xi)$

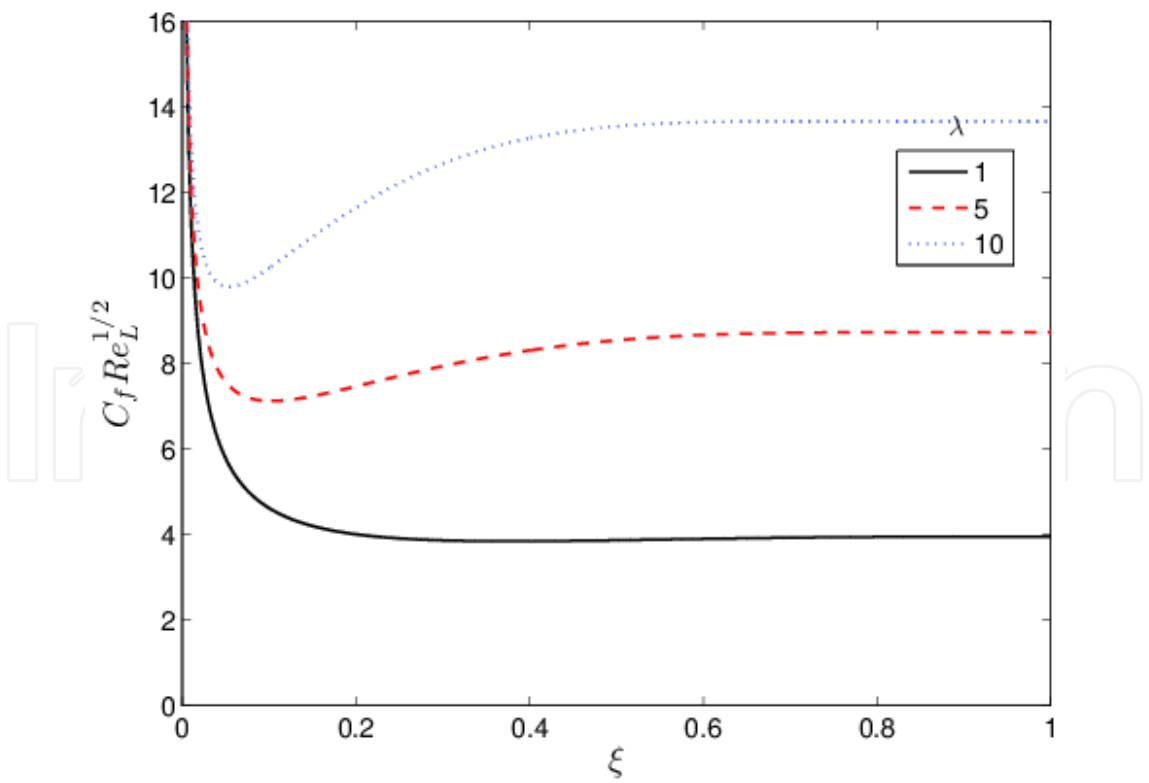


Figure 6. Effect of  $\lambda$  on the skin friction coefficient  $C_f Re_L^{1/2}$

The effect of  $\lambda$  on the skin friction coefficient is shown in Fig. 6 for  $Pr=0.7$ ,  $Sc=0.22$  and  $N=0.5$ . Again, it is observed that the results match those of [7]. In particular, the oscillating trend in the skin friction coefficient for near the stagnation region can be observed.

## 5. Conclusion

The purpose of the current study was to develop a bivariate Lagrange polynomial based spectral collocation method for solving system of coupled non-linear partial differential equations. The applicability of the proposed method, termed bivariate spectral local linearisation method (BSLLM) was tested on the well-known problem of unsteady mixed convection flow over a vertical cone due to impulsive motion. The validity of the approximate numerical results was confirmed against known results from literature. The results of this study were found to be qualitatively congruent with those from published literature. The study revealed that the proposed method can be used as a viable approach for solving coupled partial differential equations with fluid mechanics applications.

## Appendix

### Definition of coefficients

$$a_{1,r} = \frac{\eta}{2}(1-\xi) + \beta_1(\xi)f_r - \beta_2(\xi)\frac{\partial f_r}{\partial \xi}, \quad (26)$$

$$a_{2,r} = -\frac{2m}{3}\xi f_r' + \beta_2(\xi)\frac{\partial f_r'}{\partial \xi}, \quad (27)$$

$$a_{3,r} = \beta_1(\xi)f_r'', \quad (28)$$

$$a_{4,r} = -\xi(1-\xi) + \beta_2(\xi)f_r', \quad (29)$$

$$a_{5,r} = -\beta_2(\xi)f_r'', \quad (30)$$

$$R_{1,r} = \beta_1(\xi)f_rf_r'' - \frac{m}{3}\xi[1+(f_r')^2] - \lambda\xi(\theta_r + N\phi_r) - \beta_2(\xi)\left[f_r''\frac{\partial f_r}{\partial \xi} - f_r'\frac{\partial f_r'}{\partial \xi}\right], \quad (31)$$

$$\beta_1(\xi) = \xi\left(\frac{m+3}{6}\right) - (1-\xi)\left(\frac{m-3}{6}\right)\log(1-\xi), \quad (32)$$

$$\beta_2(\xi) = \xi(1 - \xi) \log(1 - \xi) \frac{(m-3)}{3}, \quad (33)$$

$$b_{1,r} = \frac{\eta}{2} Pr(1 - \xi) + Pr\beta_1(\xi)f_r - Pr\beta_2(\xi)\frac{\partial f_r}{\partial \xi}, \quad (34)$$

$$b_{2,r} = -Pr \frac{(2m-3)}{3} \xi f_r', \quad (35)$$

$$b_{3,r} = -\xi(1 - \xi)Pr + Pr\beta_2(\xi)f_r', \quad (36)$$

$$c_{1,r} = \frac{\eta}{2} Sc(1 - \xi) + Sc\beta_1(\xi)f_r - Sc\beta_2(\xi)\frac{\partial f_r}{\partial \xi}, \quad (37)$$

$$c_{2,r} = -Sc \frac{(2m-3)}{3} \xi f_r', \quad (38)$$

$$c_{3,r} = -\xi(1 - \xi)Sc + Sc\beta_2(\xi)f_r', \quad (39)$$

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