Chapter 12

Theory of Flux Cutting for Type-II Superconducting Plates at Critical State

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Additional information is available at the end of the chapter

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1. Introduction

With the discovery in 1986 of high critical temperature superconductors $T_c \geq 77K$ –which belong to the type-II classification– efforts have been made to recognize which mechanism rules its current carrying capacity in order to expand knowledge of the vortex state and, moreover, devise new and better technological applications. Critical-state phenomenological models for such materials have been a feasible alternative for the theoretical study of the magnetic properties of high- or low-T_c type-II superconductors. Here we present a brief revision of macroscopic critical-state models; following a chronological order, we will begin with the Bean model, moving on with the generalized double-critical state model, the two-velocity hydrodynamic model, and finalizing with the Elliptic Flux-Line Cutting Critical-State Model (ECSM). It will be described further the main features of type-II superconductors, the physical meaning of the critical state and the flux-line cutting phenomenon.

1.1. Type-II superconductor critical state

In 1911 Kammerlingh Onnes discovered the superconductivity of mercury at very low temperature. Nowadays, the characteristics of superconductors are well established: their electric resistance abruptly drops to zero as temperature decreases through a critical temperature value designated as T_c. They show the *Meissner-Ochsenfeld effect*, that is, they completely expel a weak magnetic field as temperature decreases through the transition point. Depending on how this diamagnetic phenomenon is destroyed, superconductors can be classified as type I or II. Type-I superconductors are perfect diamagnets below a critical field H_c. Because their coherence length ξ exceeds the penetration length λ, it is energetically unfavorable for borders to be formed between the normal and superconductive phases. However, when a type-II superconductor is subjected to a magnetic field $\mathbf{H}_a$, free energy can diminish, thus generating normal matter domains that contain trapped flux, with low-energy borders created between the normal core and the superconductive surroundings. When the applied magnetic field exceeds the lower critical field H_{c1}, the magnetic flux penetrates in quantized units Φ_0, forming cylindrical domains called **vortices**. As $\mathbf{H}_a$ increases, vortices will overlap increasing the interior field until the material gently enters the normal state, once $\mathbf{H}_a$ has reached the upper critical field H_{c2}. Between the fields H_{c1} and H_{c2} the superconductor state coexists with the magnetic state in a **mixed state** or **vortex state**.

Another characteristic of superconductors is the presence of a gap, just below the Fermi energy, the energy of conduction electrons. BCS superconductivity theory demonstrated that electrons in the vicinity of the Fermi level are grouped in the so-called Cooper pairs. In addition, the junction of two superconductors –separated by a thin insulating layer– shows the *DC Josephson effect*, in which the superconductor current tunneling is caused by the tunneling of Cooper pairs. This effect demonstrated that the superconductor state is a coherent state, which is associated to a macroscopic uniform-phase wave function; this function corresponds to the order parameter $\kappa = \lambda/\xi$ in the Ginzburg-Landau theory. Finally, they also show the *AC Josephson effect* that describes the relation between the time variation of the macroscopic wave function with the voltage produced across the junction. This voltage arises from the quantized magnetic flux movement, and is identical to the macroscopic voltage observed in type-II superconductors in flux flow state.

Indeed, it is well established that type-II superconductors posses a stationary vortex spatial arrangement only if the total force over each vortex is null. If an electric current is applied with $\mathbf{J}$ density, vortices move at a velocity $\mathbf{v}$ with a direction determined by the Hall angle. If both the Magnus force and the Hall effect on the material are neglected, equilibrium between the Lorentz force $\mathbf{F}_L = \mathbf{J} \times \mathbf{B}$ and the pinning force $\mathbf{F}_p$ will exist:

$$(\mathbf{J} \times \mathbf{B}) - \mathbf{F}_p = 0. \tag{1}$$

In addition to being able to describe the vortex dynamics under the transport current influence, the equation (1) can be used for the time-variable external magnetic case, in absence of transport currents.

Indeed, $\mathbf{F}_p$ opposes the magnetic flux velocity $\mathbf{v}$ due to the local depression of the Gibbs free energy of each vortex. This potential well may be due to inhomogeneities, defects, or material grains. Therefore, magnetic flux movement will occur if the Lorentz force exceed the pinning force. Any electromotive force, even small, causes the vortices to move further into the material, inducing a local current. Initially, this superconductive current flows in regions close to the superconductor surface because pinning centers near the sample surface can catch the vortices in such a way that, in the interior, the Meissner state is preserved, thus the sample is partially penetrated. For higher external magnetic field values, vortices will completely penetrate the material.

1.2. Bean model for a type-II superconductor in critical state

Half a century ago, Charles Bean approached — with great physics intuition and from a macroscopic point of view — the study of the magnetic properties of superconductors made with impure metals or alloys. Bean modeled the spatial distribution of the magnetic flux, for partial and totally penetrated states, a couple of years before having experimental evidence of the mixed or vortex state predicted by A. A. Abrikosov.

He relied on *Mendelssohn sponge model* to describe the magnetic behaviour of such superconductors, supposing they possessed a filamental structure capable of maintaining a maximum macroscopic current J_c, without energy dissipation in form of Joule heating, that he called it *critical current density*. Due to the J_c dependence on the magnetic field, he considered that such currents were extended into the material, preserving the magnitude J_c.

Bean argued that the macroscopic current is a consequence of the magnetic induction gradient penetrating the material, governed by Ampère's law $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$. He also argued that the current originates as the Lorentz force drives the magnetic flux into the interior of the material. Therefore, he considered that any local region where an electric field (related to heat dissipation) is perceived during the process of magnetization, it would originated a critical current density J_c which flows in the electric field direction and it keep flowing even if it the electric field was null [6–9]. He synthesized these ideas in the material law:

$$J = J_c(B)\,\mathrm{sign}(E), \tag{2}$$

which is valid for slabs or infinite cylinders subjected to a magnetic field parallel to the superconductor's surface (this is the so-called *parallel geometry*).

To exemplify the Bean model, Figure (1) shows the profile evolution of magnetic induction for a PbBi plate at mixed-state, as it increased (blue curves) or decreased (red curves) the external magnetic field magnitude $\mathbf{H}_a$ parallel to its surface. The hysteresis cycle of the average static magnetization is shown when the $\mathbf{H}_a$ varies a full cycle, from -0.3T to 0.3T. The superconducting plate has a 8mm thickness and a penetration field $\mu_0 H_p = 0.1015$T. We have considered the reliance on B of the critical current density $J_c(B)$.

Subsequently, Bean studied energy dissipation in materials subjected to a magnetic field that rotates in the specimen's plane. He extended his arguments assuming the current density $\mathbf{J}$ and the electric field $\mathbf{E}$ vectors would be parallel to each other [9]. For this case, the material equation can be written as follows:

$$\mathbf{J} = J_c(B)\frac{\mathbf{E}}{E(J)}, \tag{3}$$

where it is necessary to model the $E(J)$ function form. The Bean model, corresponding to the material equation (2) or (3), together with the Ampère's law, have been used to calculate magnetic induction profiles, hysteresis of magnetization cycles, and the energy dissipation of several type-II superconductor materials. In the search for new superconducting alloys that would produce more intense magnetic fields, or a greater current conduction capacity, these materials were simultaneously subjected to a magnetic field and a transport current parallel to each other. It was desirable that the electric current density $\mathbf{J}$ and the magnetic induction $\mathbf{B}$ established a force-free configuration, that is, a zero Lorentz force $\mathbf{F}_L = \mathbf{J} \times \mathbf{B} = 0$. However, experimental evidence showed that even if $\mathbf{F}_L$ could be considered null, a significant voltage

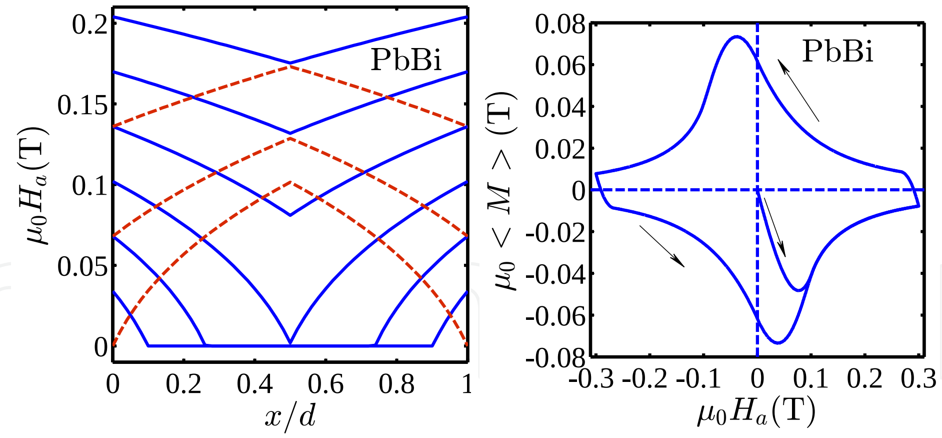

Figure 1. Theoretical curves of a superconductor plate, with thickness $d = 8$mm and penetration field $\mu_0 H_p = B_p = 0.1015$T, obtained with the Bean critical-state model and considering $J_c = J_c(B)$. (Left) Evolution of magnetic induction B as it increases (continuous blue lines) and decreases (red discontinuous lines) the magnitude of the external magnetic field $\mathbf{H}_a$. When $\mu_0 H_a = \mu_0 H_p$, B at the center of the plate is null. (Right) Average static magnetization cycle $\mu_0\langle M\rangle$ against the applied magnetic field $\mu_0 H_a$. Since this material is an irreversible type-II superconductor, $\mu_0\langle M\rangle$ describes a hysteresis.

would arise from the material. Therefore, if the vortex velocity is equal to zero in a force-free configuration, what is going on in this type of configuration? This question could not be answered using the Bean model, so it was considered that another phenomenon might be occurring. The answer to this query is the so-called *flux-line cutting* or *flux crossing*, which will be discussed in the next section.

1.3. Flux-Line cutting

D.G. Walmsley [10] measured the magnetization and the axial resistance of a type-II superconductor –in mixed state and with cylindrical geometry– by subjecting it simultaneously to a magnetic axial field and a transport current, parallel to each other. The objective was to prove under what the Lorentz force density could be null. He found that, at low currents, the potential difference between the extremes of the sample was negligible ($\sim 10\mu$V). However, when the superconducting sample conduced a sufficiently high current, it measured a voltage (or a longitudinal electric field), as well as a paramagnetic moment, that is, a positive average magnetization. He then suggested that the force-free structure could not be established on the material's surface, which originated the flux-flow and, consequently, a voltage in the rest of the material.

He intuited that the measurement of a paramagnetic moment suggested a helicoidal vortex distribution. Nonetheless, the voltage produced by the flux flow would imply a permanent increase in the longitudinal magnetic field. For this reason, he supposed the existence of a non-stationary process in which the magnetic flux lines would continually divide each other, only to reconnect afterward.

As a solution for the flux-flow movement contradiction in a force-free configuration, Clem was the first to suggest that the helicoidal instabilities were precisely the precursors of vortex cutting or crossing; that is, considering the elastic properties of flux lines, he proposed that they could stay fixed but they would be able to bend, calling this phenomenon *flux-line cutting*.

Even though Josephson had already established that cutting or crossing of vortices could not occur because they were energetically too expensive, theoretical calculations done by Brandt, Clem, and Walmsley — using the London and Ginzburg-Landau theories — proved that the threshold of cutting of a pair of rigid flux lines was possible since the characteristics energies of a type-II superconductor.

Subsequently, Brandt and Sudbo extended these results for the case of a pair of twisted flux lines, they considered the tension and interaction between each flux-line or vortex. Given that cutting is energetically plausible, they found that flux-line cutting is an effective disentanglement mechanism of flux lines; and that the cutting energy barrier, in the case of twisted flux lines, is lower for the rigid two-flux-lines case [11, 12]. Several groups, for example, M.A.R. LeBlanc *et. al* [13–15], have done experiments in the last decades that have shown flux-cutting presence in materials with low or high κ, and low or high T_c. In the Clem *et. al* [16–18] and Brandt [19] papers, we can see flux-line cutting diagrams for the case of rigid-vortices arrangement. Furthermore, it includes diagrams for the first theoretical formulations for this often-studied and not completely understood phenomenon.

More recent theoretical studies have studied the scattering dynamics of vortices, and the resulting topology after a collision between two flux lines generated by an applied current. Using the time-dependent Ginzburg-Landau equations, numerical results yielded two generic collision types dependent on the initial angle: one local collision that induces changes in topology through recombination, and a double collision that can occur due to geometrical restrictions. The second case leads to a vortex-crossing type configuration, that is, it seems as if two vortices, while interacting, would cut themselves and join again. This can be seen in the simulations shown in paper [20]. Experiments have been proposed using a magnetic force microscope to monitor vortex-line dynamics and prove if these cut through each other when they are in a liquid-vortex phase [21].

In 2008, A. Palau *et al.* reported results with superconductive heterostructures subjected to an external magnetic field at a θ angle respect to the sample's normal. For this, they designed a device made out of a thin film of low-pinning amorphous material ($Mo_{82}Si_{18}$), sandwiched between two Nb films– a material characterized for strongly pinning the vortices.

They measured the critical current density J_c obtained as a function of θ, the applied field $\mu_0 H_a$, and temperature T. Once obtained, they calculated the balance force between the Lorentz force, the pinning force, and a so-called breaking force. They found that the latter was necessary in order to consider vortex deformation and destabilization. Results showed that the breaking force is independent of B, and that the following cross joining neither limit vortex movement nor increase J_c. Even if a flux-line segment is strongly pinned to the area where Nb material is found, the cut induces other vortex segments to be liberated, thus reducing J_c [22].

Furthermore, Campbell's revision paper can be consulted to know the state of the art about experiments and critical state theories for flux-cutting in superconductors [23]. He included the last proposal of Clem to determine the electric field direction, for the flux transport regime in a type-II superconductor.

Here it is presented three critical state models created for the phenomenological study of type-II superconductors subject to magnetic fields that vary not only in magnitude, but also in direction. All models consider that flux pinning and flux-line cutting govern their answer. It is undeniable that both phenomena can occur in cases when a sample oscillates in presence of a static magnetic field, or when it is subjected to a DC magnetic field and a transversal sweeping magnetic field is superimposed.

2. Other critical state models

2.1. Generalized Double Critical-State Model

LeBlanc *et. al* proposed a model containing two critical-state equations based on their experimental observations on the magnetic response of a disc oscillating at low frequency, in presence of a magnetic field [24]:

$$\frac{dB}{dx} = \pm\frac{F_p(B)}{B}, \qquad \frac{d\alpha}{dx} = \pm k(B)\frac{dB}{dx}. \tag{4}$$

This pair of equations is known as the Double Critical-State Model (DCSM). In their construction, the fact that the magnetic induction B and the orientation α of flux-lines varied spatially was considered. They assumed as well that gradients existed in critical states. In his model, $F_p(B)$ is a parameter that characterizes pinning intensity; k is associated to the shearing coefficient of the flux lattice in the superconductive sample.

Clem and Perez-Gonzalez extended the DCSM based on the assumption that intersection and cross-joining of adjacent non-parallel vortices generate a electric field different to the electric field $\mathbf{E} = \mathbf{B} \times \mathbf{v}$. The latter field is associated to the flux flow with velocity $\mathbf{v}$, for the case in which $\mathbf{J}$ is perpendicular to $\mathbf{B}$. For this, they proposed a pair of constitutive laws of the form:

$$\begin{aligned} J_\perp &= J_{c,\perp}\,\mathrm{sign}\,E_\perp \\ J_\parallel &= J_{c,\parallel}\,\mathrm{sign}\,E_\parallel, \end{aligned} \tag{5}$$

where parameters $J_{c,\perp}$ and $J_{c,\parallel}$ correspond to the depinning and the flux-line cutting thresholds, respectively. They considered that the electric field $\mathbf{E}$ components obey independently the vertical laws:

$$E_\perp = \begin{cases} \rho_\perp[|J_\perp| - J_{c,\perp}]\,\mathrm{sign}\,(J_\perp), & |J_\perp| > J_{c,\perp} \\ 0, & 0 \leqslant |J_\perp| \leqslant J_{c,\perp} \end{cases} \tag{6}$$

$$E_\parallel = \begin{cases} \rho_\parallel[|J_\parallel| - J_{c,\parallel}]\,\mathrm{sign}\,(J_\parallel), & |J_\parallel| > J_{c,\parallel} \\ 0, & 0 \leqslant |J_\parallel| \leqslant J_{c,\parallel}; \end{cases} \tag{7}$$

here, $\rho_\perp$ and $\rho_\parallel$ are the resistivities caused by flux transport and superconductor flux-line cutting, respectively. The group of equations (5)-(7) constitutes the Generalized Double Critical-State Model (GDCSM.) Clem and Perez-Gonzalez did numerical calculations considering as possible values for magnitude J_c all those defined within a rectangular region of $J_{c,\parallel}$ and $J_{c,\perp}$ sides. The model reproduced successfully the experimental distributions of magnetic induction and magnetization, when the magnetic field oscillates at great amplitudes [25].

The GDCSM was also used to try to reproduce *Magnetization Collapse* and *Paramagnetism*, phenomena encountered when a type-II superconductor is subjected first to a DC magnetic field on which an oscillating low-frequency magnetic field is superposed, perpendicular to the former.

2.2. Two-velocities Hydrodynamic Model (TVHM)

This macroscopic model considers that electrodynamics of a type-II superconductor depends on the translation of vortex planes and the interaction between them. It establishes two vortex subsystems, assuming they posses no elastic properties and that the flux cutting consists of the disappearance of interacting vortices, creating new vortices on a plane with an orientation different to the previous one. Gibbs energy varies through small disturbances on the vortices' coordinates considering the following: 1) magnetic energy; 2) work done by pinning forces given the translation of the vortex network; and 3) the work done by the pinning forces to straighten a vortex after its crossing [26–28]. Thus, the model is conformed by a continuity equation for total vortex density $n(x,t)$, and the average angular distribution $\alpha(x,t)$ of the vortex planes:

$$\frac{\partial n}{\partial t} = -\frac{\partial}{\partial x}\left[n\frac{V_A+V_B}{2}\right], \tag{8}$$

$$\frac{\partial (n\alpha)}{\partial t} = -\frac{1}{2}\frac{\partial [n\alpha(V_A+V_B)]}{\partial x} - \frac{1}{4}\frac{\partial [n\alpha(V_A-V_B)]}{\partial x}, \tag{9}$$

where

$$V_A = V + \frac{U}{2}, \qquad V_B = V - \frac{U}{2}, \tag{10}$$

correspond to the velocities of subsystems A and B, $V(x,t)$ is the mean hydrodynamic velocity, and $U(x,t)$ is the relative velocity. The TVHM requires additionally two equations obtained from force balance conditions in a superconductor, defined for the magnetic induction gradient and for angular distribution:

$$\frac{\partial B}{\partial x} = -\frac{\mu_0 J_{c,\perp}}{2}[F(V_A)+F(V_B)], \tag{11}$$

$$\begin{aligned} \Delta\alpha B\frac{\partial \alpha}{\partial x} &+ p\sqrt{\frac{n}{8}}\,[B-\mu_0 H_a\cos(\alpha-\alpha_0)]\Delta\alpha^2\,\mathrm{sign}(V_A-V_B), \\ &= -\mu_0 J_{c\perp}[F(V_A)-F(V_B)+p\,\mathrm{sign}(V_A-V_B)], \end{aligned} \tag{12}$$

here p corresponds to the probability that flux-line cutting occurs. Finally, to resolve the equation system (8)-(12) for variables V_A, V_B, $B = n\Phi_0, \alpha$ and $\Delta\alpha$, it is required to introduce a phenomenological equation that relates $\Delta\alpha$ to the mean orientation's spatial derivative of the form:

$$\Delta\alpha = -l\,\mathrm{sign}(V_A - V_B)\,\frac{\partial\alpha}{\partial x}, \tag{13}$$

where l is the vortex mean free path between two successive cuttings or crossings.

3. Theoretical Description of the ECSM

Now we introduce the characteristics of the Elliptic Critical-State Model (ECSM) used in this chapter.

3.1. Geometrical aspects of a superconductor plate

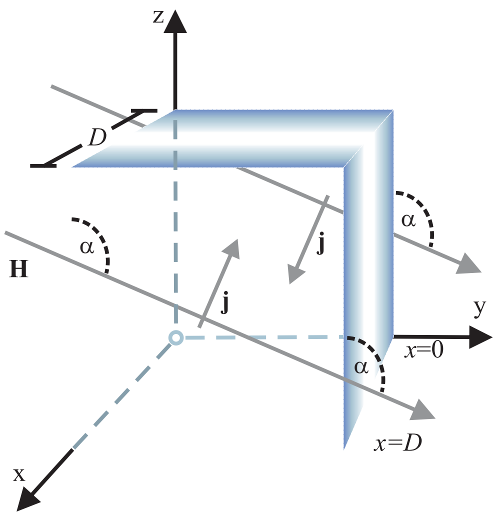

Figure 2. Diagram of a portion of infinite superconducting plate. It is shown, in an instant of time t, the orientation of the external magnetic field $\mathbf{H}_a$, which is always parallel to plane yz.

The study system is a superconducting plate possessing an infinite surface parallel to a plane yz and a finite thickness $0 \le x \le D$, as it is shown in Figure (2). The plate is subjected to a magnetic field $\mathbf{H}_a$ parallel to plane yz given by the expression:

$$\mathbf{H}_a = H_{ay}\hat{\mathbf{y}} + H_{az}\hat{\mathbf{z}} = H_a(\sin\alpha_a\hat{\mathbf{y}} + \cos\alpha_a\hat{\mathbf{z}}), \tag{14}$$

where α_a is an angle measured relative to z axis . This problem pertains to the *parallel geometry*. Demagnetization effects are not present, the current density $\mathbf{J}$, electric field $\mathbf{E}$, and magnetic induction $\mathbf{B}$ vectors are all coplanar with their components y and z depending only on variable x and time t. Given the applied magnetic field $\mathbf{H}_a$, local magnetic induction in a superconducting sample is: