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Measurements of Stationary Josephson Current between High-T_c Oxides as a Tool to Detect Charge Density Waves

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1. Introduction

Since an unexpected and brilliant discovery of high-T_c superconductivity in cuprates in 1986 [8], experts have been trying to find the origin of superconductivity in them, but in vain. There are several problems that are interconnected and probably cannot be solved independently. But they are so complex that researchers are forced to consider them separately in order to find the key concepts and express key ideas explaining the huge totality of experimental data. General discussion and the analysis of high-T_c-oxide superconductivity can be found in comprehensive reviews [21, 25, 42, 50, 53, 65, 73, 75, 78, 80, 86, 89, 104]. In particular, the main questions to be solved are as follows: (i) Is superconductivity in cuprates a conventional one based on the Cooper pairing concept? (ii) If the answer to the first question is positive, what is the mechanism of superconductivity, i.e., what are the virtual bosons that glue electrons in pairs? (iii) Which is the symmetry of the superconducting order parameter? This question remains unanswered, although the majority of the researchers in the field think believe that the problem is already resolved (namely, $d_{x^2-y^2}$-one, see, e.g., Refs. [54, 90])? (iv) What is the role of the intrinsic disorder and non-stoichiometry in the superconducting properties [2, 28, 43, 69, 70, 75, 103, 105]? (v) What is the origin of the symmetry loss and, specifically, the emerging nematicity [28, 58, 70, 75, 88, 103]? (vi) What is the origin of the so-called pseudogap [42, 68, 75, 94, 98, 104]? (vii) What is the role of spin- and charge- density waves (SDWs and CDWs) both in the normal and superconducting states of cuprates? The role of various electron spectrum instabilities competing with the Cooper pairing below the critical temperature T_c is a part of the more general problem: How can certain anomalous high-T_c oxide properties above T_c be explained, e.g., the linear behavior of the resistivity [66, 91]? In this connection, a quite reasonable viewpoint was expressed that if one understands the normal state of cuprates, the superconducting state properties will be perceived [42, 46]. Here, it is also worth to mention a possible failure [91, 92] of the

Fermi liquid concept belonging to Landau [1] and the role of strong electron correlations [18, 48, 61, 62].

During last decades we have been developing a phenomenological theory to elucidate the influence of CDWs on superconductivity of high-T_c oxides, since the CDWs were observed in a number of those materials [23, 33, 35, 37–39]. We identified the CDW energy gap with the pseudogap mentioned above. Such an identification is based, in particular, on the appearance of CDWs only below the approximate border of the pseudogapped region in $La_{2−x}Sr_xCuO_4$ [14] and $YBa_2Cu_3O_{7−\delta}$ [5, 45]. Moreover, the symmetry of the pseudogap order parameter (isotropic) differs from that for the superconducting one ($d_{x^2−y^2}$) in $Bi_2Sr_2CaCuO_{8+\delta}$ [79], superconductivity in $Bi_2Sr_{2−x}La_xCuO_{6+\delta}$ emerges with doping when the (nodal) pseudogap disappears [72], the pseudogap competes with the superconducting gap at antinodes in $(Bi,Pb)_2(Sr,La)_2CuO_{6+\delta}$ [43], and the interplay of pseudogapping and superconductivity among different members of the oxide family $(Ca_xLa_{1−x})(Ba_{1.75−x}La_{0.25+x})Cu_3O_y$ is not the same for varying dopings x [15]. It is worthy of note that both angle-resolved photoemission spectroscopy (ARPES) and scanning tunnel microscopy (STM) experiments allow one to measure only overall energy gaps whatever their microscopic origin. That is why it is usually difficult to distinguish for sure between superconducting, SDW, and CDW gaps even in the case when they manifest themselves separately in certain momentum ranges each [11, 42].

As for direct experiments confirming the existence of CDWs competing with superconductivity in cuprates, CDWs have been shown to be a more important factor in this sense than SDWs, the remnants of which survive far from the antiferromagnetic state appropriate to zero-doped samples of superconducting families [20]. It is useful to shortly summarize the main new findings in this area.

X-ray scattering experiments in $YBa_2Cu_3O_{6+x}$ revealed the CDW ordering at temperatures lower than those of the pseudogap formation, giant phonon anomalies, and elastic central peak induced by nanodomain CDWs [9, 10, 45, 59]. The CDW correlation length increases with the temperature, T, lowering. However, the competing superconducting order parameter, which emerges below T_c, so depresses CDWs that the true CDW long-range order does not develop, as was shown by Raman scattering [5]. Suppression of CDWs by Cooper pairing was also found in x-ray measurements of $La_{2−x}Sr_xCuO_4$ [14].

The well-known CDW manifestations in $Bi_2Sr_{2−x}La_xCuO_{6+\delta}$ were recently confirmed by complex X-ray, ARPES, and STM studies [13]. Those authors associate CDWs with pseudogapping, but argue that the CDW wave vector connects the Fermi arc tips rather than the antinodal Fermi surface (FS) sections, as stems from the Peierls-insulator scenario [27, 41]. This conclusion, if being true, makes the whole picture even more enigmatic than in the conventional density-wave approach to pseudogaps either in the mean-field approximation or taking into account fluctuations.

The electron-hole asymmetric CDW ordering was demonstrated by STM and resonant elastic x-ray scattering measurements [17] for $Bi_2Sr_2CaCuO_{8+\delta}$ samples, with the pseudogapping in the antinodal momentum region. As was shown in those experiments, CDWs and concomitant periodic crystal lattice distortions, PLDs can be observed directly, whereas their interplay with superconductivity manifestations can be seen only indirectly, e.g., as anticorrelations between T_c and the structural, T_s, or CDW, T_{CDW}, transition temperature.(There is a viewpoint [19] that the strong interrelation between electronic CDW modulations and PLDs [27], inherent, e.g., to the Peierls model of the structural phase transition [41], does not exist, and PLDs can emerge without electronic contributions,

which seems strange in the context of indispensable Coulomb forces.). This fact is well known, say, for superconducting transition metal dichalcogenides [49] or pseudoternary systems $(Lu_{1-x}Sc_x)_5Ir_4Si_{10}$ [102]. Therefore, it seems interesting to propose such studies of superconducting properties, which would demonstrate manifestations of CDW existence, although the CDW gapping is an insulating rather than a superconducting one. In a number of publications, we suggested that certain measurements of the stationary Josephson critical current, I_c, between quasi-two-dimensional CDW superconductors with the $d_{x^2-y^2}$ order parameter symmetry (inherent to cuprates) can conspicuously reveal such dependences that would reflect CDW gapping as well or at least demonstrate that the actual gapping symmetry differs from the pure $d_{x^2-y^2}$ one [29–31, 34, 35]. Below, we present further theoretical studies in this direction, which put forward even more effective experiments.

2. Formulation

Following the dominating idea (see our previous publications [29–31, 34, 35, 95, 96] and references therein) concerning the electron spectrum of high-T_c oxides identified as partially gapped CDW superconductors, CDWSs, we restrict our consideration to the two-dimensional case with the corresponding FS shown in Fig. 1a. The superconducting d-wave order parameter Δ is assumed to span the whole FS, whereas the s-wave mean-field dielectric (CDW) order parameter Σ develops only on the nested (dielectrized, d) FS sections. There are $N = 4$ or 2 of the latter (the checkerboard and unidirectional configurations, respectively), and they are connected in pairs by the CDW-vectors $\mathbf{Q}$'s in the momentum space. The non-nested sections remain non-dielectrized (nd). The orientations of $\mathbf{Q}$'s are assumed to be fixed with respect to the crystal lattice. In particular, they are considered to be directed along the $\mathbf{k}_x$- and $\mathbf{k}_y$-axes in the momentum space (anti-nodal nesting) [39, 67, 74]. The same orientation along $\mathbf{k}_x$- and $\mathbf{k}_y$-axes is also appropriate to Δ-lobes, so that we confine ourselves to the $d_{x^2-y^2}$-wave symmetry of the superconducting order parameter as the only one found in the experiments for cuprates. Hence, the profile of the d-wave superconducting order parameter over the FS is written down in the form

$$\bar{\Delta}(T,\theta) = \Delta(T)f_\Delta(\theta). \tag{1}$$

The function $\Delta(T)$ is the T-dependent magnitude of the superconducting gap, and the angular factor $f_\Delta(\theta)$ looks like

$$f_\Delta(\theta) = \cos 2\theta. \tag{2}$$

In the case $N = 4$, the experimentally measured magnitudes of the CDW order parameter Σ in high-T_c oxides are identical in all four CDW sectors, and the corresponding sector-connecting $\mathbf{Q}$ vectors are oriented normally to each other. Therefore, we assume the CDWs to possess the four- (the checkerboard configuration) or the two-fold (the unidirectional configuration) symmetry [3, 23, 24, 30, 39, 44, 93]. The latter is frequently associated with the electronic nematic, smectic or more complex ordering [16, 22, 26, 28, 52, 70, 83, 84, 88, 97, 99, 100]). The opening angle of each CDW sector, where $\Sigma \neq 0$, equals 2α. Such a profile of Σ over the FS can also be described in the factorized form
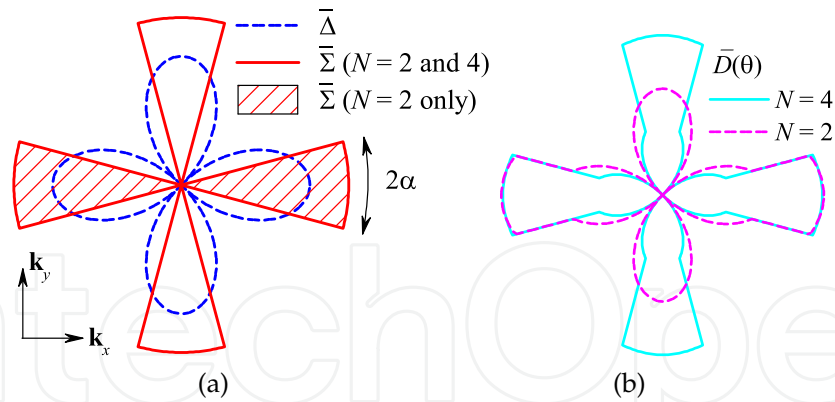
Figure 1. (a) Superconducting, $\bar{\Delta}(\theta)$, and dielectric, $\bar{\Sigma}(\theta)$, order parameter profiles of the partially gapped d-wave charge-density-wave (CDW) superconductor. N is the number of CDW sectors with the width 2α each. (b) The corresponding energy-gap contours (gap roses).

as

$$\bar{\Sigma}(T,\theta) = \Sigma(T)f_\Sigma(\theta), \tag{3}$$

where $\Sigma(T)$ is the T-dependent CDW order parameter, and the angular factor

$$f_\Sigma(\theta) = \begin{cases} 1 \text{ for } |\theta - k\Omega| < \alpha \text{ (d section)}, \\ 0 \text{ otherwise} \qquad \text{(nd section).} \end{cases} \tag{4}$$

Here, k is an integer number, and the parameter $\Omega = \pi/2$ for $N = 4$ and π for $N = 2$.

The both gapping mechanisms (superconducting and CDW-driven) suppress each other, because they compete for the same quasiparticle states near the FS. As a result, a combined gap (the gap rose in the momentum space, see Fig. 1b)

$$\bar{D}(T,\theta) = \sqrt{\bar{\Sigma}^2(T,\theta) + \bar{\Delta}^2(T,\theta)}, \tag{5}$$

arises on the FS. The actual $\Delta(T)$- and $\Sigma(T)$-values are determined from a system of self-consistent equations. The relevant initial parameters, besides N and α, include the constants of superconducting and electron-hole couplings recalculated into the pure BCS (no CDWs) and CDW (no superconductivity) limiting cases as the corresponding Δ_0 and Σ_0 order parameters at $T = 0$. It should be emphasized that our model is a simplified, generic one, because real CDWs are complex objects, which behave differently on the crystal surfaces and in the bulk [77]. Thus, it is quite natural that they are not identical for various high-T_c oxides [15]. Nevertheless, the presented model allows the main features of the materials concerned to be taken into account. For brevity, we mark the CDW d-wave superconductor with N CDW sectors as S_{CDWN}^d.

The *s*-wave BCS superconductor is described in the framework of the standard BCS theory. Its characteristic parameter is the value of the corresponding superconducting order parameter Δ_{BCS} at $T = 0$. Also for the sake of brevity, it will be marked below as S^s_{BCS}.

In the tunnel Hamiltonian approximation, the stationary Josephson critical current is given by the formula [7, 57, 85]

$$I_c(T) = 4eT \sum_{\mathbf{pq}} \left| \widetilde{T}_{\mathbf{pq}} \right|^2 \sum_{\omega_n} \mathsf{F}^+(\mathbf{p};\omega_n) \mathsf{F}'(\mathbf{q};-\omega_n). \tag{6}$$

Here, $\widetilde{T}_{\mathbf{pq}}$ are the tunnel Hamiltonian matrix elements, $\mathbf{p}$ and $\mathbf{q}$ are the transferred momenta; $e > 0$ is the elementary electrical charge, and $\mathsf{F}(\mathbf{p};\omega_n)$ and $\mathsf{F}'(\mathbf{q};-\omega_n)$ are Gor'kov Green's functions for superconductors to the left and to the right, respectively, from the tunnel barrier (hereafter, all primed quantities are associated with the right hand side electrode). The internal summation is carried out over the discrete fermionic "frequencies" $\omega_n = (2n + 1)\pi T$, $n = 0, \pm 1, \pm 2, \ldots$. Below, we consider tunnel junctions of two types: symmetric $S^d_{\text{CDW}N} - I - S^d_{\text{CDW}N}$ between two identical CDWSs, and nonsymmetric $S^d_{\text{CDW}N} - I - S^s_{\text{BCS}}$ between a CDWS as the left electrode and an *s*-wave BCS superconductor as the right one (here, I stands for the insulator). Expressions for the corresponding Green's functions can be found elsewhere [30, 35]. Since CDWS electrodes are anisotropic, their orientations with respect to the junction plane will be characterized by the angles γ and γ' (the latter appears only in the symmetric case), i.e. the deflections of the "positive" Δ- and Δ'-lobes from the normal $\mathbf{n}$ to the junction (Fig. 2). Accordingly, the angular dependences $f_\Delta(\theta)$ and $f_\Sigma(\theta)$ of the corresponding order parameters (see formulas (2) and (4, respectively) should be modified by changing θ to $\theta - \gamma$ or $\theta - \gamma'$.
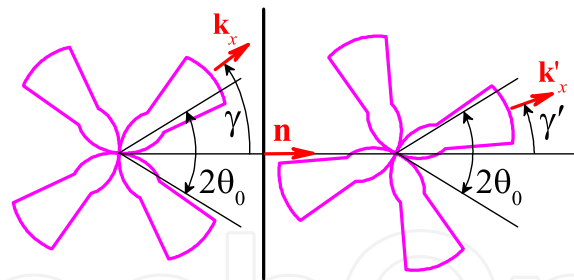

Figure 2. Configuration of symmetric Josephson junction between identical S^d_{CDW4}'s. See further explanations in the text.

An important issue while calculating the Josephson current is tunnel directionality [101], which should be taken into consideration in the tunnel Hamiltonian $\widetilde{T}_{\mathbf{pq}}$. Indeed, if we calculate I_c between, e.g., pure BCS *d*-wave superconductors, S^d_{BCS}, making no allowance for this factor, formula (6) would produce an exact zero. It is so because, owing to the alternating signs of superconducting lobes, the current contributions from the FS points described by the angles θ and $\theta + \dfrac{\pi}{2}$ would exactly compensate each other in this case. The same situation also takes place in the case of a junction with S^d_{CDW4}. For a junction with S^d_{CDW2}, it is not so, but, in the framework of the general approach, we have to introduce tunnel directionality in this case as well.

Here, we briefly consider three factors responsible for tunnel directionality (see a more thorough discussion in Ref. [31]). First, the velocity component normal to the junction should be taken into account. This circumstance is reflected by the $\cos\theta$-factor in the integrand and an angle-independent factor that can be incorporated into the junction normal-state resistance R_N [51, 64]. Second, superconducting pairs that cross the barrier at different angles penetrate through barriers with different effective widths [12] (the height of the junction barrier is assumed to be much larger than the relevant quasiparticle energies, so that this height may be considered constant). Since the actual θ-dependences of $\widetilde{T}_{\mathbf{pq}}$ for realistic junctions are not known, we simulate the barrier-associated directionality by the phenomenological function

$$w(\theta) = \exp\left[-\left(\frac{\tan\theta}{\tan\theta_0}\right)^2 \ln 2\right], \tag{7}$$

This means that the effective opening of relevant tunnel angles equals $2\theta_0$. The barrier transparency is normalized by the maximum value obtained for the normal tunneling with respect to the junction plane and included into the junction resistance R_N. Hence, $w(\theta = 0) = 1$. The multiplier $\ln 2$ in (7) was selected to provide $w(\theta = \theta_0) = \frac{1}{2}$. Third, we use the model of coherent tunneling [12, 56, 60], when the superconducting pairs are allowed to tunnel between the points on the FSs of different electrodes characterized by the same angle θ.

As a result of the standard calculation procedure [7, 57] applied to formula (6) and in the framework of the approximations made above, we obtain the following formula for the stationary Josephson critical current across the tunnel junction:

$$\begin{aligned}
I_c(T, \gamma, \gamma') &= \frac{1}{2eR_N} \\
&\times \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \cos\theta\, w(\theta)\, P(T, \theta, \gamma, \gamma') \mathrm{d}\theta,
\end{aligned} \tag{8}$$

where [32, 40]

$$P(T, \theta, \gamma, \gamma') = \bar{\Delta}\bar{\Delta}' \int\limits_{\min\{\bar{D},\bar{D}'\}}^{\max\{\bar{D},\bar{D}'\}} \frac{\tanh\frac{x}{2T}\mathrm{d}x}{\sqrt{(x^2 - \bar{D}^2)(\bar{D}'^2 - x^2)}}. \tag{9}$$

Here, for brevity, we omitted the arguments in the dependences $\bar{\Delta}(T, \theta - \gamma)$, $\bar{\Delta}'(T, \theta - \gamma')$, $\bar{D}(T, \theta - \gamma)$, and $\bar{D}'(T, \theta - \gamma')$. Integration over θ in Eq. (8) is carried out within the interval $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, i.e. over the "FS semicircle" turned towards the junction plane. If any directionality and CDW gapping are excluded (so that the integration over θ is reduced to a factor of π) and the angular factors f_Δ and f'_Δ remain preserved, we arrive at the Sigrist–Rice model [81].

3. Results and their discussion

The influence of various problem parameters on the critical stationary Josephson curent in the symmetric, $S_{\mathrm{CDWN}}^d - I - S_{\mathrm{CDWN}}^d$, and nonsymmetric, $S_{\mathrm{CDWN}}^d - I - S_{\mathrm{BCS}}^s$, junctions was analyzed in detail in works [30, 31]. Here, we attract attention to the problem of CDW detection in high-T_c oxides.

The number of problem parameters can be diminished by normalizing the "order parameter" quantities by one of them. For such a normalization, we selected the parameter Δ_0 and introduced the dimensionless order parameters $\sigma_0 = \Sigma_0/\Delta_0$ and $\delta_{\mathrm{BCS}} = \Delta_{\mathrm{BCS}}(T \to 0)/\Delta_0$ (for the superconducting order parameter of CDWS, $\delta_0 = \Delta_0/\Delta_0 = 1$). With regard to experimental needs, we also introduced the reduced temperature $\tau = T/T_c$. Here T_c is the actual critical temperature of the CDWS. In the framework of our theory, it has to be found from the system of equations for the CDWS indicated above. For the Josephson current amplitude I_c, we introduced the dimensionless combination $i_c = I_c e R_N / \Delta_0$.

One more preliminary remark concerns the parameter of effective tunnel directionality θ_0 (see formula (7)). Our calculations [30, 31] showed that its choice is very important. On the one hand, large values of this parameter correspond to thin junctions and large values of the tunnel current, which is beneficial for the experiment. However, in this case, the predicted phenomena become effectively smoothed out up to their disappearance. On the other hand, narrow tunnel cones (small θ_0-values) provide well pronounced effects, but correspond to thick interelectrode layers and, as a result, small tunnel currents. Hence, in the real experiment, a reasonable compromise should be found between those two extremes.

3.1. Electrode rotation

While examining Fig. 2, it becomes clear that the clearest way to prove that electrons in high-T_c oxides undergo an additional pairing of some origin besides the d-wave BCS one is to demonstrate that the gap rose differs from that in the S_{BCS}^d superconductor. The case in question concerns pairing symmetries, which may be different from the d-wave one or/and extend over only certain FS regions. In the framework of the tunnel technique, the most direct way to perform the search is to fix one electrode and rotate the other one (e.g., $\gamma' = \mathrm{const}$ and $\gamma = \mathrm{var}$). In the case of $S_{\mathrm{BCS}}^d - I - S_{\mathrm{BCS}}^d$ junction, the corresponding $i_c(\gamma)$ dependences are known to have a cosine profile stemming from dependence (2) for the superconducting order parameter Δ and, since any other gapping is absent, for the corresponding gap rose ($\bar{D}(T, \theta) = |\Delta(T, \theta)|$). Any deviations of the gap rose from this behavior will testify in favor of the existence of additional order parameter(s). Certainly, averaging the current over the FS will smooth the relevant peculiarities and making allowance for tunnel directionality will distort them. Nevertheless, the proposed method will be sufficient to detect the competing pairing without its ultimate identification.

In Fig. 3, the corresponding normalized $i_c(\gamma)$ dependences calculated for the symmetric $S_{\mathrm{CDWN}}^d - I - S_{\mathrm{CDWN}}^d$ junction and the CDW geometries $N = 2$ and 4, as well as the reference d-wave BCS curve, are shown. The tunnel directionality parameter $\theta_0 = 10°$ was assumed. A more detailed analysis of $i_c(\gamma)$ dependences and their relations with other problem parameters can be found in work [30]. The results obtained testify that the formulated task is feasible. An attractive feature of this technique is that, instead of the fixed S_{CDWN}^d electrode, we may use the S_{BCS}^s one as well, which might be more convenient from the experimental point of view.
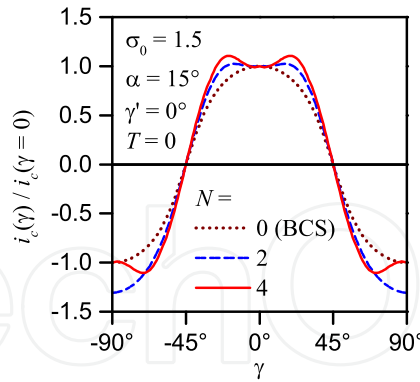
Figure 3. Orientation dependences of the reduced critical Josephson current for the symmetric junction.

3.2. Anomalous temperature dependence of I_c

The measurement of the temperature dependences of the critical Josephson tunnel current $I_c(T)$ seems to be the most easily realizable method of those proposed in this work. The dependence $I_c(T)$ in the symmetric $S^s_{BCS} - I - S^s_{BCS}$ junctions has a monotonic convex shape. Among other things, this fact is associated with the constant sign of order parameter over the whole FS. However, in the case of symmetric $S^d_{BCS} - I - S^d_{BCS}$ junctions, the situation may change. Indeed, for junctions involving $YBa_2Cu_3O_{7-\delta}$, nonmonotonic $I_c(T)$-dependences and even the change of I_c sign, i.e. the transformation of the 0-junction into the π-one or vice versa were observed [47, 87]. Such a phenomenon was not found for other cuprates. However, it is extremely difficult to produce Josephson junctions made of other materials than $YBa_2Cu_3O_{7-\delta}$. Therefore, further technological breakthrough is needed to make sure that the non-monotonic behavior is a general phenomenon inherent to all high-T_c oxides with d-wave superconducting order parameter.

It should be noted that, in the measurements concerned, the electrodes remained fixed, so that the peculiar behavior of $I_c(T)$ could not result from the change of overlapping between the superconducting lobes with different signs. There is an explanation based on the existence of the bound states in the junction due to the Andreev–Saint-James effect [51, 64]. This theory predicts that the current $I_c(T)$ between d-wave superconductors must exhibit a singularity at $T \to 0$. Nevertheless, the latter has not been observed experimentally until now. Probably, this effect is wiped out by the roughness of the interfaces in the oxide junctions [6, 76] and therefore may be of academic interest.

Earlier we suggested a different scenario [36]. Namely, we showed that, at some relative orientations of $S^d_{BCS} - I - S^d_{BCS}$ junction electrodes, one of them can play a role of differential detector, which enables tiny effects connected with the thermally induced repopulation of quasiparticle levels near the FS to be observed. In our approach, no zero-T singularity of the current could arise.

A similar situation takes place for CDWSs. Although we cannot assign a definite sign to the combined gap $\bar{D}$ (see Eq. (5)), the corresponding unambiguously signed Δ enters the expression for the calculation of I_c (formulas (8) and (9)). In this sense, the FS of the CDWS "remembers" the specific Δ-sign at every of its points and, thus, can also serve as a differential detector of the current at definite electrode orientations. As a result, the dependences $I_c(T)$

both for symmetric $S^d_{\text{CDW}N} - I - S^d_{\text{CDW}N}$ and nonsymmetric $S^d_{\text{CDW}N} - I - S^d_{\text{BCS}}$ junctions can also by nonmonotonic and even sign-changing functions. Unlike the $S^d_{\text{BCS}} - I - S^d_{\text{BCS}}$ junctions, for which the $I_c(T)$-behavior could depend only on the orientation angles of both electrodes (γ and γ'), now the other parameters responsible for the superconducting and combined gaps—these are σ_0 and α—become relevant. In Figs. 4 and 5, the $i_c(\tau)$ dependences are shown for various fixed α and σ_0, respectively, both for the "checkerboard" and "unidirectional" CDW geometry. We would like to attract attention to the fact that those dependences are rather sensitive to the electrode orientations (see the relevant illustration in Fig. 6), so that it might be laborious to find a suitable experimental configuration.

The key issue is that the parameters σ_0 and/or α can be (simultaneously) varied by doping. Hence, doping CDWS electrodes and keeping their orientations fixed, we could change even the character of the $I_c(T)$ dependence: monotonic, nonmonotonic, and sign-changing. Provided the corresponding set of parameters, we could transform the same junction, say, from the 0-state into the π-one by varying the temperature only.

3.3. Anomalous doping dependence of I_c

Now, let the electrode orientations be fixed by the experimentalist [55, 82] and the temperature be zero (for simplicity), but the both parameters α and σ_0 can be varied (by doping). In Figs. 7 and 8, the dependences of the dimensionless order parameters $\delta(0) = \Delta(T = 0)/\Delta_0$ and $\sigma(0) = \Sigma(T = 0)/\Delta_0$ on α and σ_0 are exhibited for both analyzed CDW structures. One can see that, in every cross-section $\alpha = \text{const}$ or $\sigma_0 = \text{const}$, both $\delta(0)$ and $\sigma(0)$ profiles are monotonic. At first glance, the Josephson tunnel current should also demonstrate such a behavior. However, our previous calculations [30, 31, 35] showed that it is so when the orientations of $S^d_{\text{CDW}N}$ electrodes in the $S^d_{\text{CDW}N} - I - S^d_{\text{CDW}N}$ junction are close or rotated by about $90°$ with respect to each other, i.e. when the superconducting lobes strongly overlap in the momentum space and make contributions of the same sign to the current. But if they are oriented in such a way that mutually form a kind of differential detector for monitoring the states at the gapped and non-gapped FS sections, contributions with different signs cancel each other and more tiny effects become observable. Such a conclusion can already be made from Figs. 4 and 5.

Really, as is illustrated by Figs. 7 and 8, in the limiting cases—$\sigma_0 \to \infty$ for both kinds of CDWs, and, if $\sigma_0 \geq \sqrt{e}/2 \approx 0.824$ (here, e is the Euler constant), $\alpha \to \pi/4$ at $N = 4$ or $\pi/2$ at $N = 2$ [24]—we have $\delta(0) \to 0$. Then, according to formulas (8) and 9), I_c also vanishes. Therefore, if the current crosses the point $i_c = 0$ at some values of parameters α or σ_0 different from their limiting ones, (i) the current behavior becomes nontrivial, because larger values of α and σ_0, which are accompanied by smaller values of the superconducting order parameter δ, lead to the current enhancement. Nevertheless, as α or σ_0 grows further towards its corresponding limiting values, the current must sooner or later begin to decrease by the absolute value.

This conclusion is confirmed by Figs. 9 and 10, where the dependences $i_c(\sigma_0, \alpha = \text{const})$ and $i_c(\alpha, \sigma_0 = \text{const})$ at $T = 0$ are shown. While analyzing those figures, the following consideration should be taken into account. Namely, we suppose that gradual doping monotonically affects the parameters α and σ_0 of $S^d_{\text{CDW}N}$ superconductors. Specific calculations (Figs. 9 and 10) were made assuming that only one of the control parameters, α
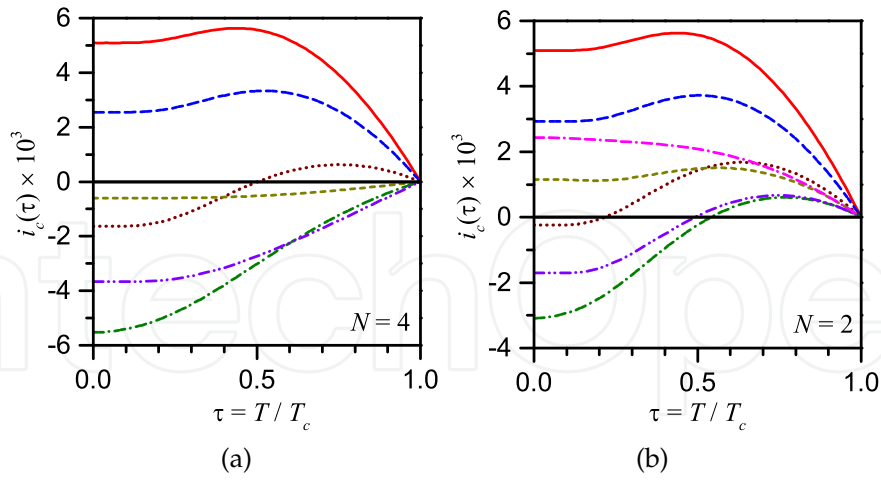
Figure 4. Temperature dependences of the Josephson current for various numbers of CDW sectors $N = 4$ (a) and 2 (b), and their widths $\alpha = 0$ (solid), 5 (dashed), 10 (dotted), 15 (dash-dotted), 20 (dash-dot-dotted), 25 (short-dashed), and 30° (dash-dash-dotted). $\sigma_0 = 1.3$, $\gamma = 15°$, $\gamma' = 45°$, $\theta_0 = 10°$. See further explanations in the text.
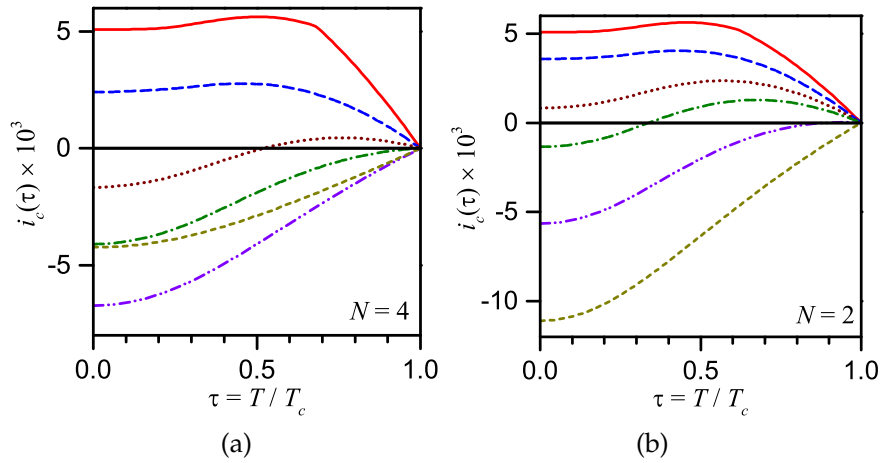

Figure 5. The same as in Fig. 4 but for $\alpha = 15°$ and various $\sigma_0 = 0.9$ (solid), 1 (dashed), 1.1 (dotted), 1.3 (dash-dotted), 1.5 (dash-dot-dotted), and 3 (short-dashed).

or σ_0, changes, which is most likely not true in the real experiment. However, the presented results testify that each of those parameters differently affects the current. Moreover, underdoping is usually accompanied by the increase of both α and Σ (proportional to the structural phase transition temperature, i.e. the pseudogap appearance temperature, T^*) [39, 42, 63, 94]. Therefore, the situation when the doping-induced simultaneous changes in the values of α and Σ_0 would lead to their mutual compensation seems improbable. Accordingly, we believe that the proposed experiments may be useful in one more, this time indirect, technique to probe CDWs in high-T_c oxides. In particular, the oscillating dependences $i_c(\alpha)$ depicted in Fig. 10b, if reproduced in the experiment, will be certain to prove the interplay between the superconducting order parameter and another, competing, one; here, the latter is considered theoretically to be associated with CDWs.
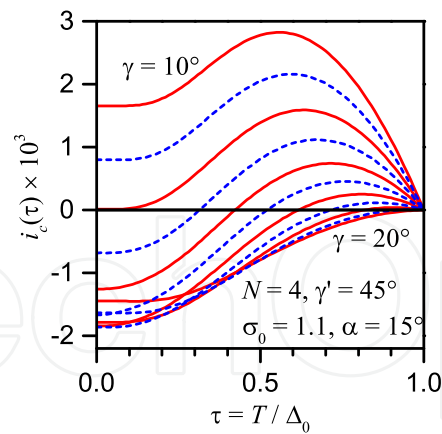
Figure 6. The same as in Fig. 4a but for $\alpha = 15°$ and $\sigma_0 = 1.1$ and various $10° \leq \gamma \leq 20°$.

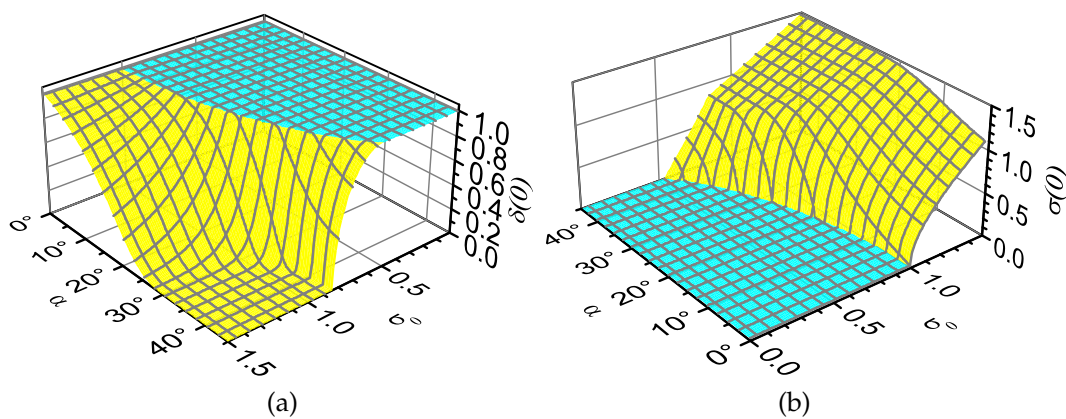(a)   (b)

Figure 7. Dependences of the normalized zero-temperature order parameters $\delta(0)$ (a) and $\sigma(0)$ (b) for the S_{CDW4}^d superconductor on α and σ_0.
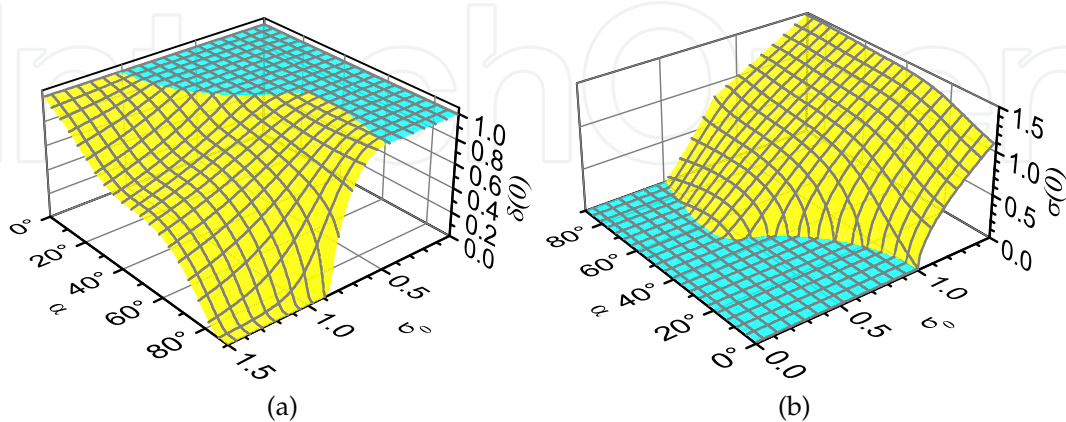

(a)   (b)

Figure 8. The same as in Fig. 7 but for the S_{CDW2}^d superconductor.
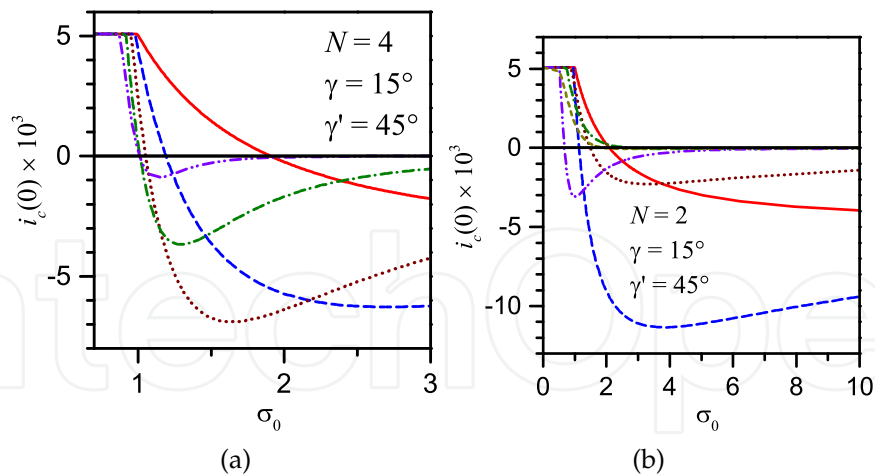
Figure 9. Dependences of the normalized zero-temperature Josephson current on σ_0 for $N = 4$ (a) and $N = 2$ (b) CDW configurations and various α's: (a) $\alpha = 5$ (solid), 10 (dashed), 15 (dotted), 20 (dash-dotted), and 25° (dash-dot-dotted); (b) $\alpha = 5$ (solid), 15 (dashed), 25 (dotted), 35 (dash-dotted), 45 (dash-dot-dotted), and 55° (short-dashed).
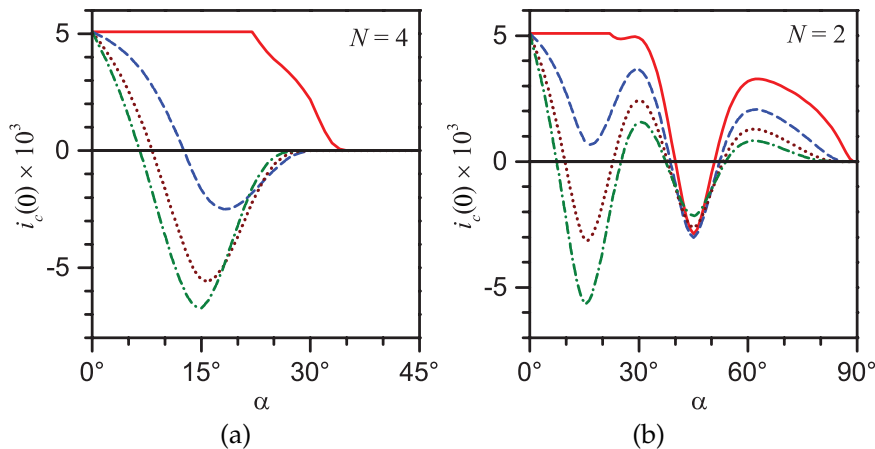

Figure 10. Dependences of the normalized zero-temperature Josephson current on α for $N = 4$ (a) and $N = 2$ (b) CDW configurations and various $\sigma_0 = 0.9$ (solid), 1,1 (dashed), 1.3 (dotted), 1.5 (dash-dotted). $\gamma = 15°$ and $\gamma' = 45°$.

4. Conclusions

In the two-dimensional model appropriate for cuprates, we calculated the dependences of the stationary critical Josephson tunnel current I_c in junctions involving d-wave superconductors with CDWs on the temperature, the CDW parameters, and the electrode orientation angles with respect to the junction plane. It was shown that the intertwining of the CDW and superconducting order parameters leads to peculiar dependences of I_c, which reflect the existence of CDW gapping. The peculiarities become especially salient when the crystal configurations on the both sides of the sandwich make the overall current extremely sensitive to the overlap between the superconducting lobes and the CDW sectors. In this case, the

whole structure can be considered as a differential tool suitable to detect CDWs. Doping serves here as a control process to reveal the CDW manifestations. Such configurations have already been created for $YBa_2Cu_3O_{7-\delta}$ [55, 82] and may be used to check the predictions of our theory.

Acknowledgements

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