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Chapter 3

Modeling the Production and Replenishment Decisions in a Supply Chain when the Vendor Has Limited Space

Sheng-Pen Wang

Additional information is available at the end of the chapter

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1. Introduction

This research was motivated by the problems encountered by Chang Gung Memorial Hospital (CGMH), a medical center in Taiwan affiliated with the author's university. Started since 1976, there are now up to 8 CGMHs in Taiwan, including three medical centers and one children's hospital. CGMH is the biggest hospital chain in Asia, with around 10,576 beds and more than 26,000 outpatient visits every day in 2011. Currently, the main supplier of beds and stretchers to all the group hospitals is Chang Gung Medical Technology Co. Ltd. This subsidiary company of CGMH, established in 2009, is a small specialist manufacturer with little storage space available. Managers in the hospital's supply chain department are asked to investigate a better cooperation and coordination mechanism with suppliers. In particular, as both the hospital (buyer) and the manufacturer (vendor, or supplier) are in the same organization, therefore they can be seen as two parties in a vertically integrated supply chain or members working towards the common goal. The department general manager may act as a central decision maker and, in a hope, to find a win-win paradigm for both parties and for the associated medical group.

Maximizing customer value and profit for each supply chain member requires effective and efficient management of product and service flows through information sharing and coordinated decision making. In most of the studies in production and distribution coordination, researchers consider no limit for the capacity of a warehouse. This however, can be one of the most important issues in a real-world problem. A new business on high street where space is very expensive can be one example. Though the main goal of supply chain and inventory management in a hospital is to reduce the cost of healthcare without sacrificing customer service, a limited warehouse capacity at the side of any party in a supply chain can differentiate the production and inventory decisions. Most recently, Priyan and Uthayakumar (2014)

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proposed an integrated inventory model for pharmaceutical products in a two-layer supply chain consisting of a pharmaceutical company and a hospital; Gebicki et al. (2014) investigated different management approaches for a medication inventory system and found that policies that incorporate drug characteristics in ordering decisions can address the tradeoff between patient safety and cost. However, they did not address the important issue of space constraints.

In any joint economic lot sizing (JELS) model, there is a central decision maker and those involved parties follow the centralized decisions in implementing inventory management. Literature has shown that if both the manufacturer and the buyer are willing to cooperate and jointly plan their inventory control decisions, the resulted system cost can be less than that of planning separately. An extensive review on JELS model and its variants up to 1989 can be found in Goyal and Gupta (1989). Ben-Daya et al. (2008) presented another comprehensive review on JELS and provided some extensions of this important problem. Glock (2012) included the most updated review of existing works on JELS models while focusing on coordinated inventory replenishment decisions between buyer and vendor and their impact on the performance of the supply chain. Goyal (1977) started analyzing the single buyer single vendor integrated production-inventory problem under the assumption of having infinite production rate for the vendor and lot-for-lot policy for the shipments. If n stands for the number of transportation batches in a production cycle, the lot-for-lot policy means n=1. In this policy, each production lot is sent to the buyer as a single shipment; this implies that the entire production lot should be ready before shipment. Banerjee (1986) generalized Goyal's model by incorporating a finite production rate for the vendor while retaining the lot-for-lot policy. This study is actually the one that coined the term JELS. Goyal (1988) further generalized Banerjee's model by relaxing the lot-for-lot policy. He assumed the vendor delivers an integral number ($n > 1$) of equal-sized shipments to the buyer, but only after the entire lot has been produced.

In the models discussed so far, it was assumed that shipments are delayed until the whole production cycle is completed. In contrast, the non-delayed shipments policy means the production processes exist where partial shipments to the buyer can be made. Lu (1995) first admitted the non-delayed equal-sized shipments in the JELS problems; it allows the vendor to deliver shipments during production, which is the relaxation of the assumption made by Goyal (1988) about completing a lot before starting shipments. A similar treatment of partial shipments to integrated inventory-production problems can be found in Agrawal and Raju (1996), Kim and Ha (2003), Wee and Chung (2006), and Teng (2009). Partial shipment obviously helps the inventory holding costs decrease in n (n >1) since shipping batches to the buyer leads to an earlier depletion of inventory at the vendor.

On the other hand, it is clear that space constraints could have some major effects on inventory operations and decisions in a supply chain, restrictions have to be considered when determining order and production quantity. However, because the addition of capacity constraints dramatically increases the difficulty in solving integrated production distribution models, the advancement of capacity-limited inventory research has been surprisingly slow. Hoque and Goyal (2000) developed an optimal solution procedure for an integrated production inventory system with unequal shipments from the vendor to the buyer, and under the capacity

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constraint of transport equipment. Lee and Wang (2008) studied the impact of buyer's available warehouse capacity on the inventory decisions under a consignment stock agreement. In such a scenario, the buyer pays for items only when they are used and further has the guarantee that the inventory never drops below a predetermined level. Wang and Lee (2013) proposed a general JELS model in determining the production and shipment policy for a two-layer supply chain, and attained a specific threshold value which defines those existing JELS problems as a special case of the proposed model, while the supplier's warehouse capacity exceeds it. Most recently, Hariga et al. (2014) considered a supply chain where the single vendor manages its multiple retailers' inventory under a contract that specifies maximum stock levels allowed by the retailers. They proposed a heuristic and found a near optimal delivery schedule with unequal shipments in an iterative approach.

Since an equal-shipment policy is attractive and easy to implement, we assume a non-delayed equal-sized shipment policy in the studied vertically integrated supply chain where the hospital (buyer) observes a deterministic demand of a certain type of medical items and order lots from the manufacturer (vendor also the supplier). The vendor manufacturers the requested products in lots and delivers shipments during production to the buyer in batches with equal-size and partial shipment policy. While taking the vendor's limited warehouse capacity into account, this research models the case as a generic JELS problem and finds the optimal number of shipments and the production lot and delivery batch size such that the joint vendor-buyer cost is minimized. The present study then extends Wang and Lee's (2013) and specifies the closed-form expressions of capacity thresholds, which define those existing JELS problems are just special cases of the proposed model once the manufacturer's warehouse capacity exceeds either the lower bound or the upper limit.

Our main contribution lies in the development of inventory replenishment decisions for the members of a hospital supply chain working together towards a reduction of total system costs; the proposed general JELS model can be applied to accommodate the warehouse capacity restriction of any supplier in manufacturing or service industry. The remainder of this chapter is organized as follows. In Section 2 we present the assumptions and notations, and develop the general JELS model. In the third section we characterize the optimal lot sizing decisions subject to the manufacturer's warehouse capacity limit. In the fourth section a numerical example is used to observe how the vendor's warehouse capacity influences replenishment policies such as the manufacturer's production lot sizes. Finally, conclusions are given in Section 5.

2. The capacitated JELS model

In this chapter, as the single vendor and the single buyer are in the same organization, a JELS model with non-delayed equal-shipments is formulated to study the inventory decisions under vendor's warehouse capacity constraints. For easy tractability with Wang and Lee (2013), we use the same assumptions and notations defined as follows.

D demand rate seen by the buyer, units/year

P supplier (the manufacturer) production rate, units/year

S_b buyer order processing cost per replenishment, $/order

S_s supplier production setup cost, $/setup

h_b buyer's annual holding cost per item, $/unit-year

h_s supplier's annual holding cost per unit of finished goods, $/ unit-year

W the manufacturer's warehouse capacity, units

$I_{max,m}$ maximum manufacturer inventory level, units

$I_{max,b}$ maximum buyer inventory level, units

TC total relevant costs of the system per time unit, $/year

Q quantity transported per delivery to the buyer, units/order

n number of transport operations per production batch, a positive integer

u number of shipments delivered in each cycle before manufacturer's inventory level reaches $I_{max,m}$, an non-negative integer

v number of shipments delivered in each cycle after manufacturer's inventory level hits W till production run stops, an non-negative integer

The following assumptions are made in deriving the proposed JELS models.

a. The demand rate is constant, as the case hospital is a medical center which observes large and stable demand for a particular medical supply.

b. $P>D$ or the utilization rate $\rho=D/P<1$; which ensures the problem is not trivial.

c. The just-in-time delivery in the same organization justifies the assumption of zero lead time for replenishments delivered from the supplier to the buyer.

d. No shortages and backorders are allowed to sustain high healthcare quality.

e. As in Hill (1999), this study assmes $h_b > h_s$, which is reasonable, as stock value usually increases as a product moves down the supply chain and the associated holding costs increase accordingly. This implies that, before completing production and turning to utilize buyer's space, the manufacturer has the incentive to store products in his own warehouse as much as possible.

The decision variables are $Q>0$, and non-negative integers n, u, and v. Figure 1 illustrates the trend of both parties' inventory levels where the manufacturer's warehouse capacity is given at W. Specifically, in each production cycle a vendor incurs a setup cost S_s and manufacturers a product in lots at a rate P; each production lot (Q_s, units/batch) is delivered to the designated buyer in n shipments ($Q_s=nQ$). The buyer's order cost is S_b per shipment. The vendor and the buyer incur inventory holding costs h_b and h_s, respectively. Note that once the manufacturer's maximal inventory level ($I_{\max,m}$) reaches the ceiling (W) after u replenishments in a cycle, the

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replenished interval might be no longer confined to Q/D due to the manufacturer's limited space.

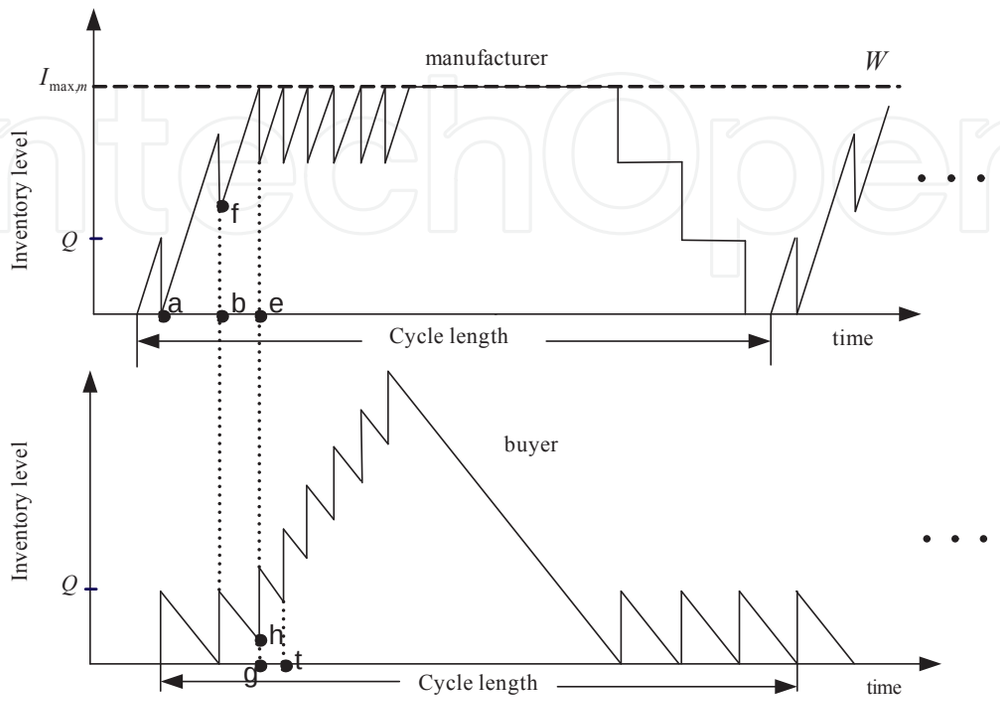


Figure 1. Trend of inventory levels of two parties when the manufacturer warehouse capacity is not "big enough," where $u=2$, $v=6$, and $n=11$.

The objective in the proposed generic JELS model is to view the system as an integrated whole and determine the production lot size and shipments schedule that minimizes TC, the annual total relevant cost in the supply chain. The annual total relevant cost consists of four components, and three among them are on the supplier's side. First, as there is D/Q_s production batch in a year, and it costs S_s per setup, the manufacturer production setup cost is $\frac{D}{nQ}S_s$. Secondly, since the quantity transported per delivery is Q, the manufacturer can fulfill the buyer's annual demand by D/Q replenishments; given that each order processing costs S_b, the replenishment cost thus is $\frac{D}{Q}S_b$. The third component on the supplier's side is the manufacturer's finished goods inventory holding cost, which depends on both the number of shipments before reaching the maximum inventory level (u) and the number of delivery since the manufacturer's warehouse is full (v) and can be expressed as

$$\{\frac{Q}{2}[(1-\rho)(n-\frac{v^2}{n}-\frac{2uv}{n}+\frac{v}{n}-1)-(\frac{2v}{n}-1)\rho]+\frac{I_{\max,m}}{n}v\rho\}h_s, \quad (1)$$

where the term inside the bracket and before h_s is the manufacturer's time-weighted inventory over a cycle. Finally, the only one component accounted for the buyer is the inventory holding cost incurred on the buyer's side, which is

$$\{\frac{Q}{2}[(1-\rho)(\frac{v^2}{n}+\frac{2uv}{n}-\frac{v}{n}+1)+(\frac{2v}{n}+1)\rho]-\frac{I_{\max,m}}{n}v\rho\}h_b. \tag{2}$$

Readers are referred to Wang and Lee (2013) for the detailed proofs of (1) and (2), where mathematical derivations are analogous to Joglekar's (1988, p. 1395). By adding up three cost components in the supplier's side and the only buyer's cost (2), the total annual relevant cost, *TC*, can be expressed as

$$TC\left(n,u,v,Q\right) = \frac{D}{Q}(\frac{S_s}{n}+S_b)+\frac{Q}{2}\Psi(n,u,v)-\frac{I_{\max,m}}{n}v\rho(h_b-h_s), \tag{3}$$

where

$$\Psi(n,u,v)=[(1-\rho)(\frac{v(v+2u-1)}{n}+1)+\frac{2v\rho}{n}](h_b-h_s)+(1-\rho)nh_s+\rho(h_b+h_s). \tag{4}$$

The proposed JELS model with manufacturer's warehouse capacity limits thus is a constrained optimization problem as to minimize (3) subject to the following three constraints:

$$I_{\max,m} \le W, \tag{5}$$

$Q \le I_{\max,m}$, and

$$u+v<n. \tag{6}$$

The manufacturer's maximal inventory level, $I_{\max,m}$, is naturally at least as much as the quantity transported per delivery to the buyer (Q), and not greater than the given limited space in (5). Instead, constraint (6) specifies that, once the manufacturer's inventory level reaches $I_{\max,m}$ after u replenishments, there will be at most $(n-1)$ shipments delivered until the production run stops in each cycle. *Lemma 1* spells out u in terms of $I_{\max,m}$ and Q as follows.

Lemma 1: The nonnegative integer u equals $\left\lfloor \frac{\frac{I_{\max,m}}{Q}-1}{\frac{1}{\rho}-1} \right\rfloor$, where $\lfloor x \rfloor$ denotes the largest integer not greater than x.

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Proof of *Lemma 1*: Since u is the number of shipments delivered in each cycle before manufacturer's inventory level reaches $I_{\max,m}$, and during this period the replenishment interval is confined to Q/D, the manufacturer's maximal inventory level thus satisfies both inequalities (i) $(u-1)\frac{Q}{D}P-(u-2)Q<I_{\max,m}$, and (ii) $(u)\frac{Q}{D}P-(u-1)Q\geq I_{\max,m}$. These hold both $(u-1)(\frac{1}{\rho}-1)<\frac{I_{\max,m}}{Q}-1$, and $u(\frac{1}{\rho}-1)\geq\frac{I_{\max,m}}{Q}-1$. It concludes the proof that the nonnegative integer u satisfies $\frac{\frac{I_{\max,m}}{Q}-1}{\frac{1}{\rho}-1}\leq u<\frac{\frac{I_{\max,m}}{Q}-1}{\frac{1}{\rho}-1}+1$. □

Intuitively, the total relevant cost decreases as the manufacturer's warehouse capacity increases. This observation is justified as the following *Lemma 2*, and whose proof can be found in Wang and Lee (2013).

Lemma 2: TC in (3) is non-increasing with respect to W.

Lemma2 states the annual total relevant cost decreases as the manufacturer's warehouse space becomes bigger until it is "big enough." If the manufacturer's warehouse capacity is not "big enough," the resulting JELS policy will raise the system cost to keep up the inflexibilities on the buyer's always constant replenishment interval and maximal inventory level. From the definition of v, the fact that $v\neq 0$ happens only in the case the manufacturer's limited warehouse is not "big enough." By complementarities in constrained optimization, the constraint (5) will be binding, or equivalently, $I_{\max,m}=W$. This is also justified by the assumption that $h_b>h_s$ because lower unit holding cost on the supplier side will drive more utilization in manufacturer's warehouse. The total annual cost of the proposed model in (3) will be modified as

$$TC(n,\ u,\ v,\ Q)=\frac{D}{Q}(\frac{S_s}{n}+S_b)+\frac{Q}{2}\Psi(n,\ u,\ v\neq 0)-\frac{W}{n}v\rho(h_b-h_s)$$

And the resulting $\frac{\partial TC}{\partial W}=-\frac{1}{n}v\rho(h_b-h_s)<0$, which justifies *Lemma 2* that the total cost is strictly decreasing when the vendor's warehouse capacity is not "big enough."

3. Characteristics of the proposed model

The vendor's warehouse capacity is definitely a key factor to the objective of the capacitated JELS model. Figure 2 depicts the trends of the vendor's and buyer's inventory levels where the manufacturer's warehouse space imposes no constraints on the JELS model. The is the basic counterpart presented in Goyal (1988), Lu (1995) and Pibernik et al. (2011), to mention just a few. Note that in this JELS model without warehouse capacity constraints, the maximum buyer's inventory level is always constant at Q, and the replenishment intervals are always constant with length Q/D, and total cost function for the uncapacitated JELS model is known as

$$TC_U(n,\ Q) = \frac{D}{Q}\left(\frac{S_s}{n} + S_b\right) + \frac{Q}{2}\left\{[(n-1)-(n-2)\rho]\cdot h_s + h_b\right\}. \tag{7}$$

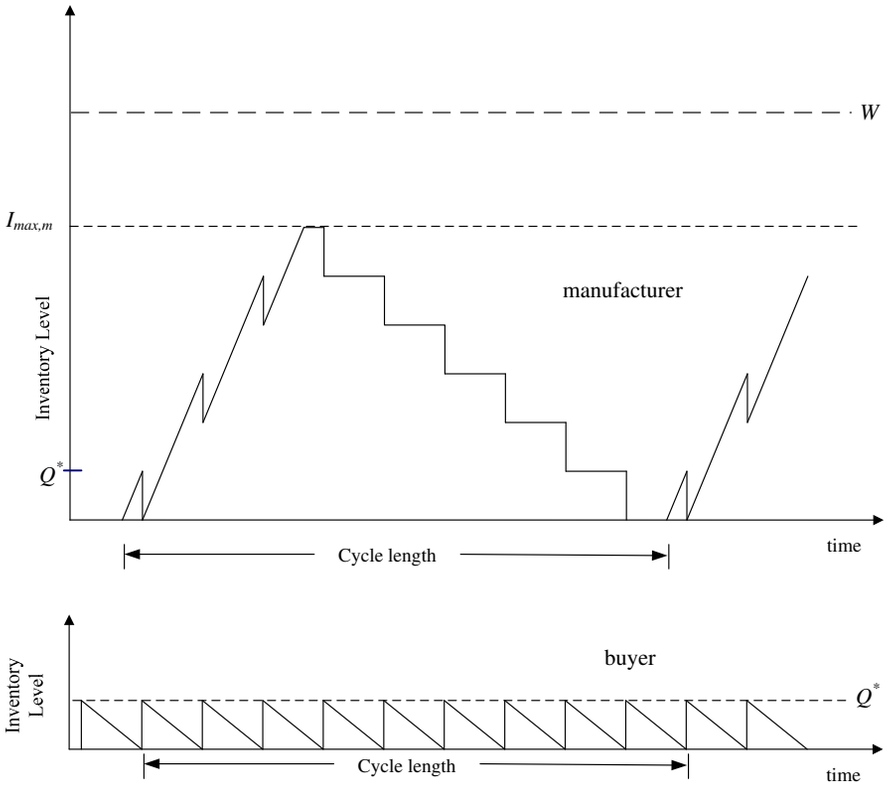


Figure 2. Trend of inventory levels of two parties when the manufacturer warehouse capacity is "big enough" or unlimited, where $u=3,\ v=0,$ and $n=9$.

Readers can refer to Kim and Ha (2003) and Wang and Lee (2013) about the proofs for the following facts: total cost (7) in the uncapacitated JELS model is a convex function of $(n,\ Q)$, and its minimum is uniquely attained at

$$n_U^* = \left\lfloor \frac{1}{2}\left(1 + \sqrt{1 + 4\frac{S_s\left[\left(2\rho-1\right)h_s + h_b\right]}{\left(1-\rho\right)S_b h_s}}\right)\right\rfloor, \tag{8}$$

and

$$Q_U^* = \sqrt{\frac{2D\cdot\left(\frac{S_s}{n_U^*} + S_b\right)}{[(n_U^*-1)-(n_U^*-2)\rho]\cdot h_s + h_b}}. \tag{9}$$

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We then state in *Lemma 3* the vendor's maximum inventory levels while the manufacturer's warehouse capacity is "big enough" as shown in Figures 2. Proof of *Lemma 3* can be, again, referred to Wang and Lee (2013).

Lemma 3: In the JELS model without warehouse capacity constraints, the manufacturer's maximum inventory levels, $I_{\max,m}$, is

$$Max\{[n_U^* - 1 - \lfloor (n_U^* - 1)\rho \rfloor]Q_U^*,\ [\lfloor (n_U^* - 1)\rho \rfloor(\frac{1}{\rho} - 1) + 1]Q_U^*\}, \tag{10}$$

where $(n_U^*,\ Q_U^*)$ are explicitly expressed in (8) and (9), respectively.

As in *Lemma 3*, the vendor's maximum inventory level helps to identify whether or not the given supplier's warehouse capacity is "big enough," or if the constraint $I_{\max,m} \le W$ is active. For simplicity, we denote (10) as the upper bound of capacity threshold, *UB*, that is,

$$UB = Max\left\{[n_U^* - 1 - \lfloor (n_U^* - 1)\rho \rfloor]Q_U^*,\ \ [\lfloor (n_U^* - 1)\rho \rfloor(\frac{1}{\rho} - 1) + 1]Q_U^*\right\}.$$

Case 1: $W \ge UB$

In this case the given manufacturer's warehouse capacity is "big enough," constraint (5) is not binding and the corresponding Lagrange multiplier will be zero, or $v = 0$. Furthermore, from (4), $\Psi(n,\ u,\ v)$can be simplified as

$$\Psi(n,\ u,\ v = 0) = [n - 1 - (n - 2)\rho] \cdot h_s + h_b,$$

which is independent of u and v. Accordingly, the cost function (3) can be rewritten as

$$TC(n, u, v = 0, Q) = \frac{D}{Q}(\frac{S_s}{n} + S_b) + \frac{Q}{2}\{[n - 1 - (n - 2)\rho]h_s + h_b\}. \tag{11}$$

Note that (11) is exactly the counterpart presented in (7), which depends only on n and Q. Once $W > UB$, any more space of the manufacturer's warehouse does not reduce the total cost in (11); similarly, the maximal inventory levels of both parties are kept constant. We summarize the results in *Theorem 1* and, again, the proof can be referred to Wang and Lee (2013).

Theorem 1: If the manufacturer's warehouse capacity is "big enough," as shown in Figure 2, or if $W > UB$, then the vendor's and buyer's maximal inventory levels are $I_{\max,m} = Max\left\{[n_U^* - 1 - \lfloor (n_U^* - 1)\rho \rfloor]Q_U^*,\ \ [\lfloor (n_U^* - 1)\rho \rfloor(\frac{1}{\rho} - 1) + 1]Q_U^*\right\}$, and $I_{\max,b} = Q_U^*$, respectively, and $(n_U^*,\ Q_U^*)$ are in (8) and (9).

However, if the manufacturer's warehouse capacity is not "big enough," in this case, constraint (5) is active and the decision variables (n_U^*, Q_U^*) as (8) and (9) will no longer be necessarily optimal to the capacitated JELS problems. In fact, lower unit holding cost in the supplier side drives full utilization in manufacturer's warehouse under the assumption of $h_b > h_s$. In the following, we turn to the situation that the vendor's warehouse capacity is "too small."

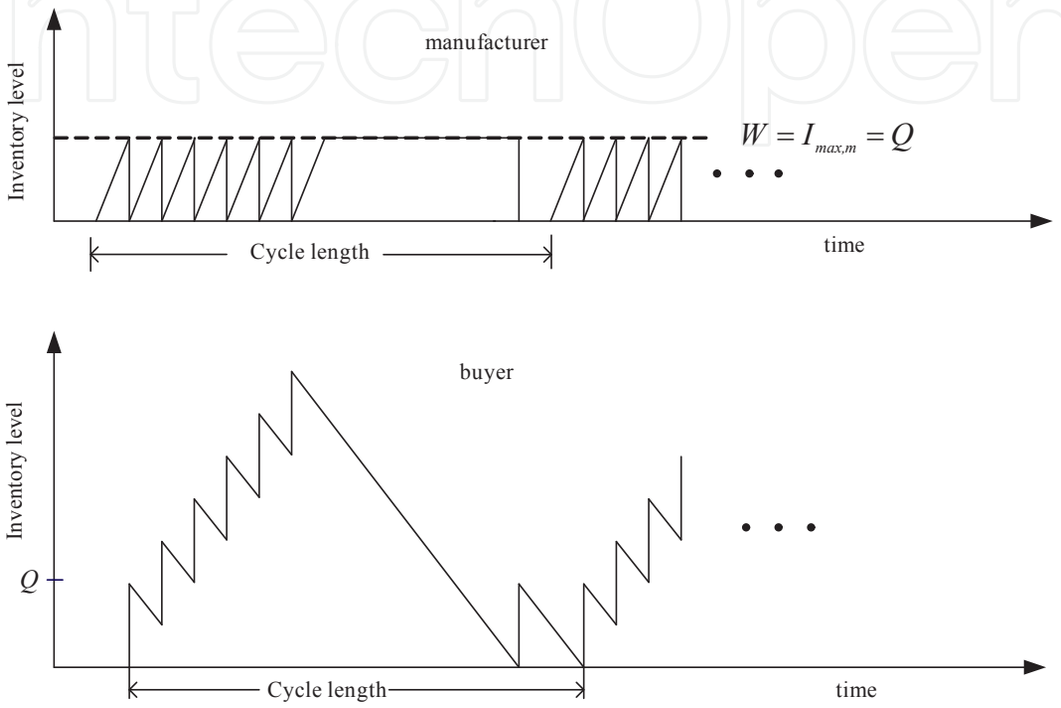


Figure 3. Trend of inventory levels of two parties when the manufacturer warehouse capacity is "too small" or $Q = I_{\max,m} = W$, where $u = 0$, $v = 6$, and $n = 7$.

Case 2: $W \le LB$

In case the supplier's warehouse capacity is "too small" as illustrated in Figure 3, where, except for the last replenishment, the manufacturer delivers a lot size consecutively to the buyer as soon as the manufacturer's warehouse is full of inventory. Under this circumstance, either quantity transported per delivery or the vendor's maximum inventory level is as small as the warehouse capacity, or $Q = W$ and $I_{\max,m} = W$. This implies that $u = 0$ and $v = n - 1$, the largest integer satisfying (6) on the number of replenishments in each cycle. Expression (4) can be represented in terms of the only variable n as the follows.

$$\Psi(n,\ u=0,\ v=n-1)=\left[\left(2-\frac{2}{n}\right)-\left(3-\frac{4}{n}\right)\rho\right]\cdot h_s+\left[\left(n-2+\frac{2}{n}\right)-\left(n-5+\frac{4}{n}\right)\rho\right]\cdot h_b.$$

In this case that vendor's warehouse capacity is "too small," the annual cost function (3) for the proposed capacitated JELS model can be accordingly rewritten as

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$$TC_L(n,Q)=\frac{D}{Q}\left(\frac{S_s}{n}+S_b\right)+\frac{Q}{2}\left\{\left[2-\frac{2}{n}-\left(1-\frac{2}{n}\right)\rho\right]h_s+\left[n-2+\frac{2}{n}-\left(n-3+\frac{2}{n}\right)\rho\right]h_b\right\} \tag{12}$$

Note that (12) is independent of u and v, and is close to the cost function in Braglia and Zavanella's (2003, p. 3798) consignment contract model. Using the same argument as showing the convexity of $TC_U(n,\ Q)$ in (7), it is straightforward to prove that $TC_L\ (n,\ Q)$ in (12) is also a convex function of (n, Q), and there exists an unique optimal solution, denoted as $(n_L^*\ ,\ Q_L^*)$. Let

$$\Omega(n)=\frac{S_s}{n}+S_b,$$

and

$$H(n)=[2-\frac{2}{n}-(1-\frac{2}{n})\rho]h_s+[n-2+\frac{2}{n}-(n-3+\frac{2}{n})\rho]h_b,$$

then (12) can be rewritten as

$$TC_L(n,Q)=\frac{D}{Q}\Omega(n)+\frac{Q}{2}H(n). \tag{13}$$

Taking the first derivative of (13) with Q and setting it equal zero, as that in Grubbstrom and Erdem (1999), then

$$Q_L^*=\sqrt{\frac{2D\Omega(n)}{H(n)}}. \tag{14}$$

Inputting (14) to (13), we obtain $TC_L\ (n,\ Q)=\sqrt{2D\Omega(n)H(n)}$, which is without variable Q. Thus, the optimal solution n_L^* is an integer that minimizes $\Omega(n)H(n)$. As detailed in *Lemma 4*, solving for n_L^* involves with solving a cubic polynomial function; additional assumptions have to make to guarantee the existence of exactly one positive root.

Lemma 4: Total cost (12) is a convex function of $(n,\ Q)$, and its minimum is uniquely attained at $(n_L^*\ ,\ Q_L^*)$ under some conditions.

Proof of *Lemma 4*: We first treat n as a continuous variable and take the partial derivative of $Z(n)=\Omega(n)H(n)\ :\ Z'(n)=\frac{1}{n^3}(An^3+Bn^2+Cn+D)$. Let $f(n)=An^3+Bn^2+Cn+D$, and the coefficients are A= $(1-\rho)S_bh_b$, B=0, C= $[(2-3\rho)S_s-2(1-\rho)S_b]h_b-[(2-\rho)S_s-2(1-\rho)S_b]h_s$, and $D=-4S_s(1-\rho)(h_b-h_s)$. If the following three assumptions are made:

i. $0 < \rho < \frac{2}{3}, \Rightarrow (2-3\rho) > 0;$

ii. $S_s > \frac{2(1-\rho)}{2-3\rho} S_b \Rightarrow (2-3\rho)S_s - 2(1-\rho)S_b > 0$; and

iii. $h_b > \left[1 + \frac{2\rho S_s}{(2-3\rho)S_s - 2(1-\rho)S_b}\right] h_s,$

then the signs for these coefficients are: $A>0$, $B=0$, $C>0$, and $D<0$.

Since $Z'(n \to 0^+) = \lim_{n \to 0^+} \frac{1}{n^3} \cdot D < 0$, and $Z'(n \to +\infty) = A + \lim_{n \to +\infty} \left(\frac{C}{n^2} + \frac{D}{n^3} \right) > 0$, the intermediate value theorem implies that there exists at least a positive n^* such that $Z'(n^*) = 0$. But the determinant for $f(n)$ is $\Delta = -4B^3D + B^2C^2 - 4AC^3 + 18ABCD - 27A^2D^2 = -4AC^3 - 27A^2D^2$, which is negative and it implies that cubic equation $f(n) = 0$ has exactly one real root n^* and a pair of complex conjugate roots.

Let $Q = \frac{C}{3A} > 0$, and $R = \frac{-D}{2A} > 0$, then the unique real root of $f(n) = 0$ can be expressed as $n^* = \sqrt[3]{R + \sqrt{Q^3 + R^2}} + \sqrt[3]{R - \sqrt{Q^3 + R^2}}$. The optimal solution n_L^* minimizing $Z(n) = \Omega(n)H(n)$ is either $\lfloor n^* \rfloor$ or $\lfloor n^* \rfloor + 1$. □

Note that the integral solution n_L^* cannot be formulated in a closed form as n_U^* shown in (8). Therefore, the complexity makes it impossible to express Q_L^* in terms of the given parameters. For the two-layer supply chain where the vendor's warehouse capacity is "too small," we denote Q_L^* as the lower bound of capacity threshold, *LB*, that is, $LB = Q_L^*$. Note that, unlike the definition of *UB*, here *LB* is independent of n_L^*.

Theorem 2: If $W \le LB$, or the manufacturer's warehouse capacity is "too small," then the maximal inventory levels of the two parties are $I_{\max,m} = W = Q^*$, and $I_{\max,b} = \begin{cases} W, & \text{if } n^* = 1 \\ [(n^* - 1) - (n^* - 2)\rho] \cdot W, & \text{if } n^* \ge 2 \end{cases}$, respectively, where n^* is optimal to minimize $TC_L(n, Q=W)$ in (12).

Proof of *Theorem 2*: Referring to Figure 3, the manufacturer will ship a lot size $Q = W$ to the buyer once the accumulative product reaches full warehouse, that is, $I_{\max,m} = W$. When $n = 1$, the manufacturer adopts the lot-for-lot policy and the buyer's maximal inventory level is the same as that of the manufacturer, which is *W*. If $n \ge 2$, it takes the manufacturer *Q*/*P* to make one batch of *Q*, while during the same time the buyer's inventory level reduces by (*Q*/*P*) *D*. Consequently, the buyer's inventory level increases (*Q*-*DQ*/*P*) each time a new batch *Q* arrives. Since there are (*n*-2) shipments before the last delivery, the buyer's maximum inventory level is $(n-2)(Q - DQ/P) + Q$, which equals

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$$[(n-1)-(n-2)\rho]Q. \tag{15}$$

We complete the proof by replacing in (15) the optimal Q with the vendor's warehouse capacity W, while the optimal n^* is attained from minimizing $TC_L(n, Q=W)$ by *Lemma 4*. □

Case 3: $LB < W < UB$

If the vendor's warehouse capacity is smaller than the ceiling threshold (UB), then constraint (5) will be binding by complementarities in constrained optimization, or equivalently, $I_{\max,m}=W$, and thus the auxiliary integral variable $v \geq 1$. In addition, if the vendor's warehouse capacity is greater than the floor threshold (LB), then $u \geq 1$ as well. The total annual cost of the proposed model in (3) will be modified as

$$TC(n,u,v,Q)=\frac{D}{Q}\left(\frac{S_s}{n}+S_b\right)+\frac{Q}{2}\Psi(n,u,v\neq 0)-\frac{W}{n}v\rho(h_b-h_s), \tag{16}$$

where it is obvious to see both the variables u and v influence the replenishment decisions in the capacitated JELS model. Readers are referred to Wang and Lee (2013) for the note that $TC(n, u, v, Q)$ in (16) is not a convex function of W since the marginal costs are not monotonic, and for the detailed proof of the following *Theorem 3* on how the supplier's limited warehouse capacity affects the maximum inventory levels of two parties while achieving the optimal replenishment policy.

Theorem 3: If $LB < W < UB$, or the manufacturer's warehouse capacity is neither "big enough" nor "too small," then the maximal inventory levels of two parties are $I_{\max,m}=W$, and $I_{\max,b}=Q^*[(u^*+v^*)(1-\rho)+2\rho]-\rho W$, respectively, where (Q^*, u^*, v^*) are optimal to minimize cost function (16).

4. Numerical illustration

For illustration, this chapter considers a similar example used by Banerjee (1986), Goyal (1988), Lee (2005), and Wang and Lee (2013), in which the parameters are S_s=3000, S_b=25, h_s=4, h_b=5, P=3200, and D=1000. When there is no manufacturer's warehouse capacity constraint imposed in the traditional model, then $n_U^*=12$ from (8). Input n_U^* to (9) to obtain the optimal replenishment lot size $Q_U^*=122.75$ units/order. Consequently, the manufacturer's production batch size, $Q_s=nQ$, is 1,473.05 units/batch, and the annual system cost for the uncapacitated JELS model, $TC_U(n_U^*, Q_U^*)$ in (7), is \$4,480.51 per year. From (10) in *Lemma 3*, the manufacturer's maximal inventory level, $I_{\max,m}$, will be 982.03 units, which also defines the upper bound of capacity threshold UB=982.03. It means that in the proposed general JELS model, a manufacturer's warehouse space is called "big enough" only if it exceeds 982.03 units, and in this case, $v=0$. The total annual cost for this two-party supply chain cannot be reduced further even when

the manufacturer possesses a larger warehouse capacity than the threshold *UB*. This is similar to the influence of warehouse capacity to the profitability of the steel firm in Italy (Zanoni and Zavanella, 2005).

On the other hand, if the vendor's warehouse capacity fails to be "big enough," the capacity constraint $I_{\max,m} \le W$ is binding, and the annual total cost strictly increases as the manufacturer's warehouse capacity decreases. The facts that $u \ge 1$, $v \ge 1$, and hence $Q^* < W$ remain true until the vendor's warehouse capacity is "too small." As depicted in Figure 3, the lot-for-lot delivery policy is adopted while the vendor's warehouse capacity is "too small," we proceed to define the lower bound of capacity threshold $LB = 122.75$ units. It means that in the proposed general JELS model, a manufacturer's warehouse space is called "too small" only if $W < LB$, and in this case, $u = 0$and $I_{\max,m} = Q^* = W$; that is, the vendor's maximal inventory level is bounded by the warehouse capacity, which also defines the optimal quantity transported per delivery to the buyer.

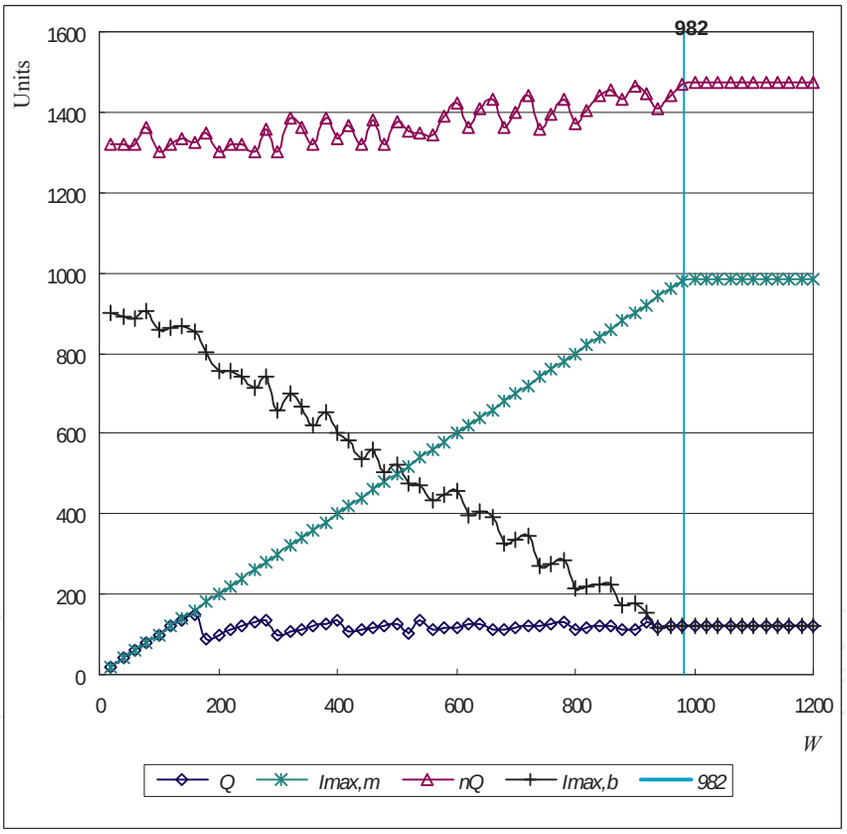


Figure 4. Impact of *W* on the optimal *Q*, $I_{\max,m}$, $nQ = Q_s$, and $I_{\max,b}$, respectively.

While achieving the optimal production and distribution decisions for the proposed general JELS model, Figure 4 shows the impact of *W* on the manufacturer's product batch size and each replenishment lot size, as well as on the buyer's maximum inventory levels of two parties, respectively. As the manufacturer's warehouse capacity increases to be "big enough," some

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interesting findings against our intuition are: (i) neither the manufacturer's production batch size (Q_s) nor the buyer's replenishment lot size (Q) is increasing monotonically; (ii) while the buyer's maximum inventory level ($I_{\max,b}$) is not decreasing monotonically, it comes down with the same size of one replenishment (Q) when the manufacturer's warehouse capacity is greater than 982.03 units; and (iii) instead the vendor's maximal inventory level is increasing monotonically and with the same size of warehouse capacity ($I_{\max,m}=W$), while it equals the replenishment lot size ($I_{\max,m}=Q$) when the vendor's warehouse capacity is less than 122.75 units.

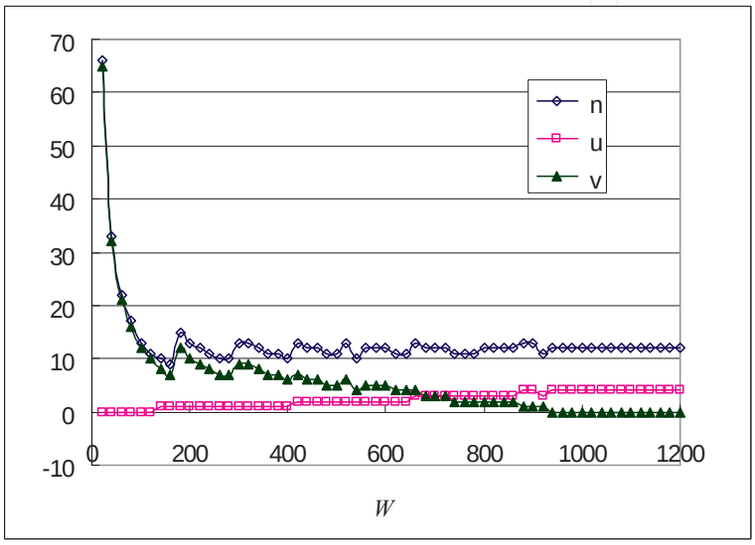


Figure 5. Impact of W on the integral decision variables n, u, and v, respectively.

In addition, Figure 5 depicts the impact of W on the integral decision variables, n, u, and v, and justifies the facts that $v=0$ when $W \geq UB=982.03$, and $u=0$ when $W \leq LB=122.75$ units, respectively. Note that the number of transport operations per production batch, n, does not show any monotonic property with respect to the vendor's warehouse capacity. As shown in the figure, u, the number of shipments delivered in each cycle before manufacturer's inventory level reaches $I_{\max,m}$, has one tilt down from $u=4$ to $u=3$ somewhere as takes the test value at 920 units. This phenomenon might be due to multiple solutions for u and v, since the integral decision variables are discrete and not continuous. Though we conjecture that the auxiliary variable u might increase monotonically as W increases from LB to UB, investigation on the mathematical properties is too complex to be completed in the present study and we leave it as future research.

5. Conclusions

This chapter intends to extend the study in Wang and Lee (2013). Specifically, this chapter proposes a general JELS model for a supply chain with one buyer and one vendor, both

belonging to the same organization. The vendor is also the manufacturer and has limited warehouse capacity, which greatly influences the production and distribution decisions. By taking into account the vendor's warehouse capacity constraints, this research aims to minimize the annual total costs in the two-layer supply chain, which consist of the production setup, replenishment cost, and the inventory holding cost of finished goods on both the manufacturer's and buyer's sides. This study specifies the upper and lower capacity threshold values, *UB* and *LB*, for classifying the manufacturer's warehouse space as "big enough" or "too small," respectively. In particular, the traditional basic JELS problems, where no capacity constraints are imposed, are just special cases of the proposed model with inactive capacity constraint.

The proposed model characterizes the maximal inventory levels of both parties in three distinct situations whether the supplier's given warehouse space is either "big enough," or "too small," or "medium in between." Impact of the vendor's warehouse capacity constraint on the inventory control policy is also investigated. Smaller space available on the supplier side drives higher total cost in the supply chain; however, neither the manufacturer's production lot nor the replenishment lot size is monotonically increasing as the manufacturer's available space increases. If the vendor's warehouse capacity is "too small," the supplier's optimal replenishment lot size equals her warehouse capacity. Once the supplier has "big enough" warehouse space, the total cost is kept flat and both the manufacturer's optimal production and replenishment lot sizes are kept constant as well. This chapter can offer an insight for the supply-chain managers seeking for better coordination and cooperation, in particular, when the supplier's warehouse capacity is scarce. This chapter can also provide a guideline for planners regarding whether their companies should expand or reduce their current warehouse capacity to maintain the supply chain system economically.

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Author details

Sheng-Pen Wang*

Address all correspondence to: wangsp@mail.cgu.edu.tw

Department of Industrial and Business Management, Chang Gung University, Kwei-Shan, Taoyuan 333, Taiwan

References

[1] Agrawal K, Raju A. Improved Joint Economic Lot Size Model for a Purchaser and A Vendor. In: Khan MK, Wright CS, Whalley R. (eds.) Advanced Manufacturing Processes, Systems, and Technologies. London: ASME Books; 1996. p579-587.

[2] Banerjee A. A Joint Economic-Lot Size Model for Purchaser and Vendor. Decision Science 1986;17(3) 292-311.

[3] Ben-Daya M, Darwish M, Ertogral K. The Joint Economic Lot Sizing Problem: Review and Extensions. European Journal of Operational Research 2008;185(2) 726-742.

[4] Braglia M, Zavanella L. Modelling an Industrial Strategy for Inventory Management in Supply Chains: The 'Consignment Stock' Case. International Journal of Production Research 2003;41(16) 3793-3808.

[5] Gebicki M, Mooney E, Chen S-J, Mazur LM. Evaluation of Hospital Medication Inventory Policies. Health Care Management Science 2014;17(3) 215-229.

[6] Glock CH. The Joint Economic Lot Size Problem: A Review. International Journal of Production Research 2012;135(2) 671-686.

[7] Goyal SK. An Integrated Inventory Model for a Single Supplier-Single Customer Problem. International Journal of Production Research 1977;15(1) 107-111.

[8] Goyal SK. A Joint Economic-Lot-Size Model for Purchaser and Vendor: A Comment. Decision Science 1988;19(1) 236-241.

[9] Goyal SK, Gupta Y. Integrated Inventory Models: The Buyer-Vendor Coordination. European Journal of Operational Research 1989;41(3) 261-269.

[10] Grubbstrom RW, Erdem A. The EOQ with Backlogging Derived without Derivatives. International Journal of Production Economics 1999;59(1) 529-530.

[11] Hariga M, Gumus M, Daghfous A. Storage Constrained Vendor Managed Inventory Models with Unequal Shipment Frequencies. Omega 2014;48 94-106.

[12] Hill RM. The Optimal Production and Shipment Policy for the Single-Vendor Single-Buyer Integrated Production-Inventory Problem. International Journal of Production Research 1999;37(11) 2463-2475.

[13] Hoque MA, Goyal SK. An Optimal Policy for a Single-Vendor Single-Buyer Integrated Production-Inventory System with Capacity Constraint of the Transport Equipment. International Journal of Production Economics 2000;65(3) 305-315.

[14] Joglekar PN. Comments on a Quantity Discount Pricing Model to Increase Vendor Profits. Management Science 1988;34(11) 1391-1398.

[15] Kim SL, Ha D. A JIT-Splitting Model for Supply Chain Management: Enhancing Buyer-Supplier Linkage. International Journal of Production Economics 2003;86(1) 1-10.

[16] Lee W. A Joint Economic Lot Size Model for Raw Material Ordering, Manufacturing Setup, and Finished Goods Delivering. Omega 2005;33(2) 163-174.

[17] Lee W, Wang S-P. Managing Level of Consigned Inventory with Buyer's Warehouse Capacity Constraint. Production Planning and Control 2008;19(7) 677-685.

[18] Lu L. A One-Vendor Multi-Buyer Integrated Inventory Model. European Journal of Operational Research 1995;81(2) 312-323.

[19] Pibernik R, Zhang Y, Kerschbaum F, Schropfer A. Secure Collaborative Supply Chain Planning and Inverse Optimization-The JELS Model. European Journal of Operational Research 2011;208(1) 75-85.

[20] Priyan S, Uthayakumar R. Optimal Inventory Management for Pharmaceutical Company and Hospital Supply Chain in a Fuzzy-Stochastic Environment. Operations Research for Health Care 2014; http://dx.doi.org/10.1016/j.orhc2014.08.001

[21] Teng J-T. A Simple Method to Compute Economic Order Quantities. European Journal of Operational Research 2009;198(1) 1289-1293.

[22] Wang S-P, Lee W. 2013. A Joint Economic Lot-Sizing Model for the Hospital's Supplier with Capacitated Warehouse Constraint. Journal of Industrial and Production Engineering 2013;30(3) 202-210.

[23] Wee H-M, Chung C-J. 2006. A Note on the Economic Lot Size of the Integrated Vendor-Buyer Inventory System Derived without Derivatives. European Journal of Operational Research 2006;177(2) 1289-1293.

[24] Zanoni S, Zavanella L. Model and Analysis of Integrated Production-Inventory System: The Case of Steel Production. International Journal of Production Economics 2005;93-94(Spec.Iss.) 197-205.

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