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Implications of Quantum Informational Entropy in Some Fundamental Physical and Biophysical Models

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Additional information is available at the end of the chapter

<http://dx.doi.org/10.5772/59203>

1. Introduction

Complex systems are a large multidisciplinary research theme that has been studied using a combination of fundamental theory, derived especially from physics and computational modeling. This kind of systems is composed of a large number of elemental units that interact with each other, being called “agents” [1, 2, 62]. Examples of complex systems can be found in human societies, the brain, internet, ecosystems, biological evolution, stock markets, economies and many others.

The manner in which such a system manifests can't be predicted only by the behavior of individual elements or by adding their behavior, but is determined by the way the elements interact in order to influence global behavior. Very important properties of complex systems are those of emergence, self-organization, adaptability etc. [3, 4, 62].

An example of a complex system is represented by polymers. [Their structures present a multitude of organized networks starting from simple, linear chains of identical structural units to very complex sequences of amino acids that are chained together, thus forming the fundamental units of living fields. Probably one of the most interesting biological complex system is DNA that generates cells by employing a simple but very efficient code. It is the striking way in which individual cells organize into complex systems, such as organs and, subsequently, organisms. Research in the field of complex systems could provide new information on the realistic dynamics of polymers, solving troublesome problems such as protein folding. We note that the dynamics of such complex systems implies the quantum formalism] [1-4, 62].

Correspondingly, the theoretical models that describe the complex systems dynamics become more and more advanced [1-4]. For all that, this problem can be solved by taking into account that the complexity of the interaction process implies various temporal resolution scales, and the pattern evolution implies different degrees of freedom [5].

[In order to develop new theoretical models we must state the fact that the complex systems displaying chaotic behavior are recognized to acquire self-similarity (space-time structures can appear) in association with strong fluctuations at all possible space-time scales [1-4]. Afterwards, for temporal scales that are large with respect to the inverse of the highest Lyapunov exponent, the deterministic trajectories are replaced by a set of potential trajectories and the concept of definite positions by that of probability density] [62]. An interesting example is the collisions processes in complex systems, where the dynamics of the particles can be described by non-differentiable curves.

Since non-differentiability can be considered a universal property of complex systems, it is mandatory to develop a non-differentiable physics. In this way, by considering that the complexity of the interaction processes is replaced by non-differentiability, using the entire range of quantities from the standard physics (differentiable physics) is no longer required [19].

This topic was developed in the Scale Relativity Theory (SRT) [6, 7] and in the non-standard Scale Relativity Theory (NSSRT) [8-22]. [In the framework of SRT or NSSRT we assume that the movements of complex system entities take place on continuous but non-differentiable curves (fractal curves) so that all physical phenomena involved in the dynamics depend not only on the space-time coordinates but also on the space-time scales resolution. In this conjecture, the physical quantities that describe the dynamics of complex systems can be considered as fractal functions. In addition, the entities of the complex system may be reduced to and identified with their own trajectories. In this way, the complex system's behavior will be identical to the one of a special interaction-less "fluid" by means of its geodesics in a non-differentiable (fractal) space] [6, 7, 62].

In such context notions as informational entropy, Onicescu informational energy etc become important in the Nature description. These notions will be correlated with the fractal part of the physical quantities that describe the dynamics of complex systems.

2. Informational entropy and energy

Independently of scale resolution, the motion, either on infragalactic scale (for instance, the planetary motion), or on atomic scale (for instance, the motion of the electron around its nucleus) takes place on conics (ellipses). Such motion is invariant with respect to the $SL(2R)$ group. In what follows, we shall consider this invariance only with respect to the motion on atomic scale.

2.1. $SL(2R)$ invariance and canonic formalism

$SL(2R)$ group is the group of transformations [23-26]

$$x' = \alpha x + \beta y, \quad y' = \gamma x + \delta y, \quad \alpha\delta - \beta\gamma = 1 \quad (1)$$

which makes invariant the areas in the phase space (x, y) .

Choosing

$$\alpha = 1 + \frac{1}{2}a_2, \quad \beta = a_1, \quad \gamma = -a_3, \quad \delta = 1 - \frac{1}{2}a_2, \quad (2)$$

the infinitesimal transformations of the group have the expressions

$$x' = x + ya_1 + \frac{p}{2}a_2, \quad y' = y - \frac{1}{2}ya_2 - xa_3 \quad (3)$$

Then the Lie algebra associated to the group becomes

$$[\hat{L}_1, \hat{L}_2] = \hat{L}_1; \quad [\hat{L}_2, \hat{L}_3] = \hat{L}_3; \quad [\hat{L}_3, \hat{L}_1] = -2\hat{L}_2 \quad (4)$$

where

$$\hat{L}_1 = y \frac{\partial}{\partial x}, \quad \hat{L}_2 = \left(x \frac{\partial}{\partial x} - y \frac{\partial}{\partial y} \right), \quad \hat{L}_3 = -x \frac{\partial}{\partial y} \quad (5)$$

are the vectors of the Lie base.

The general vector of the algebra (4) is given by the linear combination

$$\hat{L} = c\hat{L}_1 + 2b\hat{L}_2 + a\hat{L}_3, \quad a, b, c = \text{const.} \quad (6)$$

The hamiltonian H results as an invariant function along the tangent trajectories to the vector (6). Precisely, it is a solution of the equation

$$\hat{L}H = 0 \quad (7)$$

According to (5), relation (7) becomes

$$(bx + cy) \frac{\partial H}{\partial x} - (ax + by) \frac{\partial H}{\partial y} = 0 \quad (8)$$

whence the characteristic differential system

$$\frac{dx}{bx + cy} = -\frac{dy}{(ax + by)} = dt \quad (9)$$

admits the integral

$$H(x, y) = \frac{1}{2}(ax^2 + 2bxy + cy^2) \quad (10)$$

We notice that the differential system (9) is Hamilton's system of equations [25]

$$\dot{x} = \frac{\partial H}{\partial y}, \quad \dot{y} = -\frac{\partial H}{\partial x} \quad (11)$$

associated to the hamiltonian (1), where the symbol “ \cdot ” refers to the derivative with respect to the time.

Among the solutions of the equation (1), we have also the Gaussian

$$\rho(x, y) = A \exp[-H(x, y)], \quad A = \text{const.} \quad (12)$$

In consequence, all invariant functions on the group (7) will be functions of the hamiltonian (10) and particularly, of the Gaussian (12).

If the quadratic form (10) is positive definite, that is, the condition

$$a > 0, \quad \Omega = ac - b^2 > 0 \quad (13)$$

is fulfilled, then, by deriving the relations

$$\begin{aligned} \dot{x} &= bx + cy \\ \dot{y} &= -(ax + by) \end{aligned} \quad (14)$$

with respect to the time and eliminating \dot{p} and \dot{q} , based on the relations (14), we obtain the symmetric equations

$$\begin{aligned} \ddot{x} + \Omega x &= 0 \\ \ddot{y} + \Omega y &= 0 \end{aligned} \quad (15)$$

These equations are formally equivalent to the equations of two linear oscillators of coordinates x, y .

Then the 2-form

$$\omega = dx \wedge dy \quad (16)$$

has the meaning of the elementary surface in phase space (x, y) and the transformations (1) are canonic because they maintain the 2-form (16) (Liouville's theorem [25]). Simultaneously, the Gaussian (12) can be considered as a probabilistic density in phase space (x, y) . In this situation, the parameters (a, b, c) can get statistical significance (see also [71]).

2.2. Shannon's informational entropy and transitivity manifolds

[In standard quantum mechanics, the impossibility of determining the variances of the position coordinate Δy_i and of the conjugate momentum component Δx_i ($x_i = -i\hbar \nabla_i$) with arbitrary accuracy is widely accepted as being caused by the unavailable perturbation exerted on the particle by the measuring process. Because the measuring apparatus is most often not defined quantitatively and its perturbation can be very large, the uncertainty relation is formulated as a larger-than-or equal to equation

$$\Delta x_i \Delta y_i \geq \frac{1}{2} \hbar$$

Relating to this, it is unusual that the definitive nonzero variances Δx_i and Δp_i can be obtained for quantum system which are not exposed to a measuring device. This has been shown using the so called negative-result experiments. Furthermore, it can be noticed that we could theoretically obtain the nonzero variance Δx_i and Δy_i of quantum systems without including in the analysis perturbations from or in presence of a measuring device at all] [65].

In such a conjecture the uncertainty relations result in a quite natural way from the momentum perturbations associated with the fractal potential, i.e. with the Shannon's information.

Indeed, let be the probability density in the phase space, $\rho(x, y)$ with the constraints [27-32, 62]

$$\begin{aligned} \iint y \rho(x, y) dx dy &= \langle y \rangle \\ \iint x \rho(x, y) dx dy &= \langle x \rangle \\ \iint (y - \langle y \rangle) \rho(x, y) dx dy &= (\delta y)^2 \\ \iint (x - \langle x \rangle) \rho(x, y) dx dy &= (\delta x)^2 \\ \iint (y - \langle y \rangle)(x - \langle x \rangle) \rho(x, y) dx dy &= cov(x, y) \end{aligned} \quad (17)$$

where $\langle y \rangle$ is the mean value of the position, $\langle x \rangle$ is the mean value of the momentum, δy is the position standard deviation, δx is the momentum standard deviation and $cov(x, y)$ is the covariance of the random variables (x, y) [62].

Now, we introduce Shannon's informational entropy [27]:

$$\bar{H} = \iint \rho(x, y) \ln[\rho(x, y)] dx dy. \quad (18)$$

Through Shannon's maximum informational entropy principle

$$\delta \bar{H} = 0 \quad (19)$$

with constraints (17), we get the normalized Gaussian distribution:

$$\rho(x - \langle x \rangle, y - \langle y \rangle) = \frac{\sqrt{ac - b^2}}{2\pi} \exp[-H(x - \langle x \rangle, y - \langle y \rangle)] \quad (20)$$

with

$$\begin{aligned} H(x - \langle x \rangle, y - \langle y \rangle) &= \frac{1}{2} [\bar{a}(x - \langle x \rangle)^2 + \\ &\quad + 2\bar{b}(x - \langle x \rangle)(y - \langle y \rangle) + \bar{c}(y - \langle y \rangle)^2] \\ \bar{a} &= \frac{(\delta y)^2}{D}, \quad \bar{b} = -\frac{cov(x, y)}{D}, \quad \bar{c} = \frac{(\delta x)^2}{D} \\ D &= (\delta x)^2(\delta y)^2 - cov^2(x, y) \end{aligned} \quad (21)$$

[We must note that the set of parameters $(\bar{a}, \bar{b}, \bar{c})$ has statistical significance given by relations (21).

In such context, the statistical hypothesis are specified through a particular choice of the set of parameters $(\bar{a}, \bar{b}, \bar{c})$ of the quadratic form the first Eq (21). Their class is given by the restriction [70]:

$$H(x', y') = H(x' - \langle x \rangle, y' - \langle y \rangle) = H(x - \langle x \rangle, y - \langle y \rangle) \quad (22)$$

where

$$\begin{aligned} H(x' - \langle x \rangle, y' - \langle y \rangle) &= \\ &= \frac{1}{2} [\bar{a}'(x' - \langle x \rangle)^2 + 2\bar{b}'(x' - \langle x \rangle)(y' - \langle y \rangle) + \bar{c}'(y' - \langle y \rangle)^2] \end{aligned} \quad (23)$$

If $(x' - \langle x \rangle, y' - \langle y \rangle)$ and $(x - \langle x \rangle, y - \langle y \rangle)$ are dependent through the unimodular transformations (1), we get that (22) imposes through $(\bar{a}, \bar{b}, \bar{c})$, the group of three parameters] (see [71] for details)

$$\begin{aligned}\bar{a}' &= \delta^2 \bar{a} - 2\gamma \delta \bar{b} + \gamma^2 \bar{c} \\ \bar{b}' &= -\beta \delta \bar{a} - (\beta \gamma + \alpha \delta) \bar{b} - \alpha \gamma \bar{c} \\ \bar{c}' &= \beta^2 \bar{a} - 2\alpha \beta \bar{b} + \alpha^2 \bar{c}\end{aligned}\quad (24)$$

If for the group (24) we choose the same parameterization as the one given by relations (2), then the corresponding infinitesimal transformations

$$\begin{aligned}\bar{a}' &= \bar{a} - \bar{a} a_2 + 2\bar{b} a_3 \\ \bar{b}' &= \bar{b} - \bar{a} a_1 + \bar{c} a_3 \\ \bar{c}' &= \bar{c} - 2\bar{b} a_1 + \bar{c} a_2\end{aligned}\quad (25)$$

can be considered as an incompatible algebraic system in the unknowns a_1, a_2, a_3 . In consequence, there cannot exist a transformation able to ensure the correspondence

$$(\bar{a}', \bar{b}', \bar{c}') \rightarrow (\bar{a}, \bar{b}, \bar{c}) \quad (26)$$

Thus, the action of the group (24) in the space of variables $(\bar{a}, \bar{b}, \bar{c})$ is intransitive and, therefore, there exists a relation among the parameters $(\bar{a}, \bar{b}, \bar{c})$, which remains invariant to the action of the group (24). This relation is called transitivity manifold [26] (see also [71]).

The Lie algebra associated to the group (24) is

$$[\hat{A}_1, \hat{A}_2] = \hat{A}_1; \quad [\hat{A}_2, \hat{A}_3] = \hat{A}_3; \quad [\hat{A}_3, \hat{A}_1] = -2\hat{A}_1 \quad (27)$$

where

$$\begin{aligned}\hat{A}_1 &= -\bar{a} \frac{\partial}{\partial \bar{b}} - 2\bar{b} \frac{\partial}{\partial \bar{c}} \\ \hat{A}_2 &= -\bar{a} \frac{\partial}{\partial \bar{a}} + \bar{c} \frac{\partial}{\partial \bar{c}} \\ \hat{A}_3 &= 2\bar{b} \frac{\partial}{\partial \bar{a}} + \bar{c} \frac{\partial}{\partial \bar{b}}\end{aligned}\quad (28)$$

are the vectors of the base Lie. By the conditions

$$\hat{A}_1 F = 0, \quad \hat{A}_2 F = 0, \quad \hat{A}_3 F = 0 \quad (29)$$

[where F is an arbitrary function, we can obtain the transitivity manifolds of the group in the form

$$\bar{a} \bar{c} - \bar{b}^2 = \text{const.} \quad (30)$$

If H has energy significance, then condition (30) shows that a representative point from space (x, y) (which is in motion on a surface of constant energy (22)), can be also found on a surface of constant probabilistic density (ergodic condition) in Stoler's sense [33]:

$$\frac{\sqrt{\bar{a}' \bar{c}' - \bar{b}'^2}}{2\pi} e^{-H(x', y')} = \frac{\sqrt{\bar{a} \bar{c} - \bar{b}^2}}{2\pi} e^{-H(x, y)} \quad (31)$$

Therefore, the “class” of statistical hypothesis associated to the Gaussians having the same mean, is given by the ergodic condition. This highlights the strong relationship existing among the energetic issues and the probabilistic ones] (see [71]).

2.3. Informational energy in the sense of Onicescu and uncertainty relations

For the informational energy we shall use Onicescu's relation [34, 62, 71]:

$$E = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho^2(x, y) dx dy \quad (32)$$

Thus, the informational energy corresponding to the normed Gaussians (20), which is subject to conditions (22), becomes

$$E(\bar{a}, \bar{b}, \bar{c}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho^2(x, y) dx dy \quad (33)$$

where $H(x, y) > 0$ is a condition imposed by the existence of the integral (33).

Thus we get

$$E(\bar{a}, \bar{b}, \bar{c}) = \frac{\sqrt{\bar{a} \bar{c} - \bar{b}^2}}{2\pi} \quad (34)$$

and therefore, if H has energetic significance, it results (see [62] and [71] for details):

- i. The informational energy indicates the dispersion distribution (20) because the quantity

$$A = \frac{2\pi}{\sqrt{\bar{a} \bar{c} - \bar{b}^2}} \quad (35)$$

is a measure of the ellipses' areas of equal probability $H(p, q) = \text{const.}$, in the manner that the normed Gaussians are even more clustered the more their informational energy is higher;

- ii. The class of statistical hypothesis which are specific to the Gaussians having the same mean is given by the constant value of the informational energy;
- iii. The constant informational energy is equivalent to the ergodic condition;
- iv. If the informational energy is constant, then the relations (21) and (34) give the egalitarian uncertainty relation

$$(\delta x)^2 (\delta y)^2 = \frac{1}{4\pi^2 E^2 (\bar{a}, \bar{b}, \bar{c})} + \text{cov}^2(x, y) \quad (36)$$

or the non-egalitarian one

$$\delta x \delta y \geq \frac{1}{2\pi E (\bar{a}, \bar{b}, \bar{c})}. \quad (37)$$

In such context we can show that the constant value of the Onicescu informational energy implies, in the case of a linear oscillator, the Planck's quantification condition.

2.4. Quantum mechanics and informational energy – Generalized uncertainty relations

[The original theory of de Broglie on the wave-corpusele duality was developed using a theorem found in Lorentz's transformation [35]. This theorem interlinks the local horologes cyclic frequency (in each point of a spatial domain) with a progressive wave frequency in phase with the horologes. This wave gives determines the distribution of the oscillators' phases on the respective spatial domain. We desire to show that a distribution of this kind, in a true sense, can be determined without resorting to Lorentz's transformation [67].

The concept imagined by de Broglie, of equal pulsation horologes, can be evidenced by a periodic field, which is described by the local oscillators of equation

$$\ddot{Q} + \Omega^2 Q = 0 \quad (38)$$

where $Q = y + ix / m\Omega$ is the relevant complex coordinate of the field and Ω is its pulsation. The general solution of (38) can be written as [23]:

$$Q(t) = ze^{i(\Omega t + \varphi)} + \bar{z}e^{-i(\Omega t + \varphi)} \quad (39)$$

where z is a complex amplitude, \bar{z} its complex conjugate and φ is a specific phase. The quantities z and \bar{z} give the initial conditions, which are not the same for any point from the space. Precisely, at a time, the various oscillators corresponding to the points of the space are in different states and have different phases. A problem arises: can we apriori indicate a relationship among the parameters z , \bar{z} and $e^{i(\Omega t + \varphi)}$ of the various oscillators at a given momentum? Because (39) is a solution of the equation (38) gives us an affirmative answer to this problem because (38) possesses a “hidden” symmetry that can be expressed by the homographic group: the ratio $\tau(t)$ of two solutions of the equation (38) is a solution of Schwartz's equation] [71] (see also [36, 67]).

$$\left(\frac{\tau''}{\tau'}\right)' - \frac{1}{2}\left(\frac{\tau''}{\tau'}\right)^2 = 2\Omega^2 \quad (40)$$

[This equation is invariant to the homographic transformation of $\tau(t)$: any homographic function of τ is itself a solution of (40). Since projections on the line can be characterized by the homography, we can assert that the ratio of two solutions of the equation (38) is a projective parameter for the class of the oscillators of the same pulsation from a given spatial region. Thus, one can define with ease a convenient, suitable projective parameter that should be in bi-univocal correspondence with the oscillator] [69, 71]. First, we observe a “universal” projective parameter: the ratio of the fundamental solutions of (38):

$$k = e^{2i(\Omega t + \varphi)} \quad (41)$$

Any homographic function of this ratio will be again a projective parameter [67]. Among all other, the function

$$\tau(t) = \frac{z + \bar{z}k}{1 + k} \quad (42)$$

has primarily the advantage of being specific to each oscillator. But not only that: let be another function

$$\tau'(t) = \frac{z' + \bar{z}'k'}{1 + k'} \quad (43)$$

which is specific to another oscillator. Since (42) and (43) are solutions of the equation (40), there exists a homographic relation between them:

$$\tau' = \frac{a'\tau + b'}{c'\tau + d'} \quad (44)$$

which, made explicit, leads to the Barbilian group equations [37]:

$$\begin{aligned} z' &= \frac{a^1 z + b^1}{c^1 z + d^1} \\ k' &= \frac{c^1 \bar{z} + d^1}{c^1 z + d^1} k \end{aligned} \quad (45)$$

[The group may be considered as a 'synchronization' group among various oscillators, a process in which the values of each take part, meaning that not only their phases, but also their amplitudes are correlated. The usual synchronization, manifested through the difference among the oscillators' phases as a whole, represents here just a particular case. Indeed, the group is involved for z , \bar{z} and k , and also for (45), which indicates the fact that, indeed, the phase of k is only shifted with a value depending on the oscillator's amplitude, during passage between various members of the assembly and, moreover, the oscillator's amplitude is homographically affected.

When taking into consideration, for the group (45), the parameterization from [23], the following infinitesimal generators of the above-mentioned group will be obtained] [63]:

$$\begin{aligned} \hat{B}_1 &= \frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}} \\ \hat{B}_2 &= z \frac{\partial}{\partial z} + \bar{z} \frac{\partial}{\partial \bar{z}} \\ \hat{B}_3 &= z^2 \frac{\partial}{\partial z} + \bar{z}^2 \frac{\partial}{\partial \bar{z}} + (z - \bar{z})k \frac{\partial}{\partial k} \end{aligned} \quad (46)$$

the following commutation relations

$$[\hat{B}_1, \hat{B}_2] = \hat{B}_1; \quad [\hat{B}_2, \hat{B}_3] = \hat{B}_3; \quad [\hat{B}_3, \hat{B}_1] = -2\hat{B}_2 \quad (47)$$

being involved. [Thus, a structure near-identical to group $SL(2R)$'s Lie algebra is shown. In consequence, the Lie algebra of the group (45) is, again, a result of group $SL(2R)$'s Lie algebra. Actually, as can be easily observed, the group (45) represents only another action of the group $SL(2R)$, performed in variables z , \bar{z} , k] [63].

Once we fulfill the conditions of the theorem [38], the invariant functions can be found, simultaneously to the actions of the groups (5) and (46) as solutions of the equation

$$\hat{L}_i F(x, y, z, \bar{z}, k) + \hat{B}_i F(x, y, z, \bar{z}, k) = 0, \quad i = 1, 2, 3 \quad (48)$$

Explaining this equation by means of Equations (5) and (46) leads to their simple solution, by successive reduction, while simultaneously obtaining the invariant functions in the form

$$f(\mu, \nu) = \text{const.} \quad (49)$$

where μ and ν are expressed as (see [63]):

$$\begin{aligned} \mu &= \frac{-i(z - \bar{z})}{(x - zy)(x - \bar{z}y)} \\ \nu &= k \frac{x - \bar{z}y}{x - zy} \end{aligned} \quad (50)$$

ν being a unimodular complex and μ a real one. A particular class of such invariant functions is represented by linear combinations of the type

$$\bar{p}\mu = m\left(\nu + \frac{1}{\nu}\right) + 2n \quad (51)$$

where m, n and \bar{p} represent three arbitrary real constants.

If considering Eq (50), then Eq (51) takes the form

$$mk^{-1}z^2 + 2nz\bar{z} + mk\bar{z}^2 \equiv \bar{p} \quad (52)$$

where the following notation has been used:

$$z = \frac{x - \bar{z}y}{\sqrt{-i(z - \bar{z})}} \quad (53)$$

We also noticed that

$$-i(z - \bar{z}) > 0 \quad (54)$$

Eq (52) represents a family of conical shapes from the phase space (x, y) . They represent ellipses if

$$m^2 - n^2 > 0 \quad (55)$$

a condition also fulfilled if

$$\begin{aligned} m &= \bar{Q} \sinh(2r) \\ n &= \bar{Q} \cosh(2r) \end{aligned} \quad (56)$$

where \bar{Q} is a real constant and r is a real variable.

Quite an interesting case appears when z is completely imaginary, with no restriction concerning the generality value $z=i$. Thus, the quadratic form (52) may be identified with $H(p, q)$ from (10), resulting

$$\begin{aligned} \bar{a} &= \bar{Q}[\cosh(2r) + \sinh(2r) \cos \varphi] \\ \bar{b} &= -\bar{Q} \sinh(2r) \sin \varphi \\ \bar{c} &= \bar{Q}[\cosh(2r) - \sinh(2r) \cos \varphi] \end{aligned} \quad (57)$$

where φ is the value of k , assumed fixed. The square value of \bar{Q} represents the value of the constant from (30), which determines the transitivity manifolds of the group (1) (see [63]).

The Gaussian distribution value obtained in such a manner represents only a particular case of the distribution that may occur, assuming in addition the obligation of satisfying the maximum principle of informational entropy under quadratic restrictions. The solutions of Eq (48) could be, however, much more general, being possibly selected from criteria involving group theory. In this context, the informational energy becomes

$$E(\bar{a}, \bar{b}, \bar{c}) = \frac{\bar{Q}}{4\pi} = \text{const.} \quad (58)$$

while the uncertainty relation (36) is

$$(\delta x)^2 (\delta y)^2 = \frac{1}{\bar{Q}^2} (1 + \sinh^2(2r) \sin^2 \varphi) \quad (59)$$

resulting that the concept of uncertainty is minimum only for $\varphi=0$, i.e., all the oscillators of the assembly possess the same initial phase of zero. Based on this simplified hypothesis, at any moment of time subsequent to the initial one, the uncertainty relation (59) gives up its condition of minimum, along with the assembly's covariance which differs from zero [63].

When the creation and annihilation operators refer to a harmonic oscillator, the uncertainty relations have the form from [33, 63] with $\bar{Q}=2/\hbar$ and \hbar the reduced Planck constant. In this situation, the “synchronization” is achieved through Stoler’s group [33] (the parameter r is exactly equal to the frequency ratio).

Onicescu informational energy can be correlated with the standard quantum mechanics and the second quantification (which indicates its utility, for instance in NDA-NRA dynamics) [39].

2.5. Gravity and information

The structure of the group (45) is given by the equations (46) in the manner that the only non-zero structure constants are [26]:

$$C_{12}^1 = C_{23}^3 = -1, C_{31}^2 = -2 \quad (60)$$

Therefore, the invariant quadratic form is given by the “quadratic” tensor of the group,

$$C_{\alpha\beta} = C_{\alpha\nu}^{\mu} C_{\beta\mu}^{\nu} \quad (61)$$

or, more explicit, by (60),

$$C_{\alpha\beta} = \begin{pmatrix} 0 & 0 & -4 \\ 0 & 2 & 0 \\ -4 & 0 & 0 \end{pmatrix} \quad (62)$$

This yields that the invariant metric of the group is given by the relation [50]

$$\frac{ds^2}{k_0^2} = \omega_0^2 - 4\omega_1\omega_2 \quad (63)$$

where k_0 is an arbitrary factor and ω_α , three differential 1-forms, which are absolutely invariant through the group.

These 1-forms have the expressions:

$$\begin{aligned} \omega_0 &= i \left(\frac{dk}{k} - \frac{dz + d\bar{z}}{z - \bar{z}} \right) \\ \omega_1 &= \bar{\omega}_2 = \frac{dz}{k(z - \bar{z})} \end{aligned} \quad (64)$$

in which case the metric (63) becomes

$$\frac{ds^2}{k_0^2} = - \left(\frac{dk}{k} - \frac{dz + d\bar{z}}{z - \bar{z}} \right)^2 + 4 \frac{dz d\bar{z}}{(z - \bar{z})^2} \quad (65)$$

It should be mentioned here a property related to integral geometry: the group (45) is measurable. Indeed, it is simply transitive and, since his structure vector $C_\alpha = C_{\nu\alpha}^\nu$ is identically zero, which can be seen from (60), it means that he possesses the invariant function

$$F(z, \bar{z}, k) = - \frac{1}{k(z - \bar{z})^2} \quad (66)$$

that is, the inverse of the module of the linear system's determinant obtained through the infinitesimal transformations of the group (45). Therefore, in the field variables space (z, \bar{z}, k) , one can build an a priori probabilities theory [40], based on the elementary probability

$$dP(z, \bar{z}, k) = - \frac{dz \wedge d\bar{z} \wedge dk}{(z - \bar{z})^2 k} \quad (67)$$

where \wedge defines the external product of the 1-forms (64).

We now analyze the metric (65): it reduces to the metric of the Lobacevski's plan in Poincaré representation [26]:

$$\frac{ds^2}{k_0^2} = 4 \frac{dz d\bar{z}}{(z - \bar{z})^2} \quad (68)$$

for $\omega_0=0$. Specifying ω_0 from (64) by the aid of the usual relations

$$\bar{z} = u + iv, \quad k = e^{i\varphi} \quad (69)$$

it results

$$\omega_0 = - \left(d\varphi + \frac{du}{v} \right) \quad (70)$$

and so, the condition $\omega_0=0$ becomes

$$d\varphi = -\frac{du}{v} \quad (71)$$

Since by this restriction, the metric (68) in the variables (69) reduces to the Lobachevski's one in Beltrami's representation:

$$\frac{ds^2}{k_0^2} = -\frac{du^2 + dv^2}{v^2} \quad (72)$$

the condition (71) defines a parallel transport of vectors in a Levi-Civita meaning: the application point of the vector moves on the geodesic, the vector always making a constant angle with the tangent to the geodesic in the current point. Truly, using the fact that the plan's metric is conformal Euclidean, the angle between the initial vector and the vector transported through parallelism can be calculated as the integral of the equation (see [36, 68, 71] for details):

$$d\varphi = \frac{1}{2} \left(\frac{\partial \ln E}{\partial v} du - \frac{\partial \ln E}{\partial u} dv \right) \quad (73)$$

along the transport curve. $E(u, v)$ denotes here the conformity factor of the respective metric, in our case $E(u, v) = 1/v^2$. Substituting it in (73), we get (71).

Now the variables (z, \bar{z}, k) can be considered as amplitudes of a gravitational field. Thus, let us admit that we describe the gravitational field through the variables y_i for which we "discovered" the metric

$$h_{ij} dy^i dy^j \quad (74)$$

in an ambient space of the metric

$$\gamma_{\alpha\beta} dx^\alpha dx^\beta \quad (75)$$

Then the field equations derive from the variational principle [41]

$$\delta \int L \gamma^{\frac{1}{2}} d^3x = 0 \quad (76)$$

relative to the Lagrange function

$$L = \gamma^{\alpha\beta} h_{ij} \frac{\partial y^i}{\partial x^\alpha} \frac{\partial y^j}{\partial x^\beta} = \gamma^{\alpha\beta} \frac{\frac{\partial z}{\partial x^\alpha} \cdot \frac{\partial \bar{z}}{\partial x^\beta}}{(z - \bar{z})^2} = \frac{z'' \bar{z}}{(z - \bar{z})^2} \quad (77)$$

In such a context, Einstein's equations with $z = i\varepsilon$ become Ernst's ones for the gravitational field of vacuum [42, 43]

$$\begin{aligned} -\frac{1}{2}(\varepsilon + \bar{\varepsilon})''^2 \varepsilon &= \varepsilon'' \bar{\varepsilon} \\ -(\varepsilon + \bar{\varepsilon})^2 R_{\alpha\beta}(\gamma) &= \partial_\alpha \varepsilon \partial_\beta \bar{\varepsilon} + \partial_\beta \varepsilon \partial_\alpha \bar{\varepsilon} \end{aligned} \quad (78)$$

($R_{\alpha\beta}$ is here the Ricci tensor of the three-dimensional metric $\gamma_{\alpha\beta}$).

Thus, adopting as a starting point the variational principle (76), the essential goal of the analysis in the gravitational field domain is to produce metrics of Lobachevski's plan or metrics related to them. All these can be directly related to Einstein's equations (78). Moreover, by substituting the principle of independence of the simultaneous actions, in the form of linear composition in a point of various fields intensities (through the apriori invariance of fields' action with respect to a certain group), we may conceive a gravity theory that has none of the contradictions inherently and commonly present in the current theory [44]. We observe that the $SL(2R)$ group parameters can be interpreted as field amplitudes in a supergravitation model [23] (see also [71] for details).

2.6. Extracellular vesicles convection in haptotaxis with hydrodynamical dissipation, a novel mechanism for vesicle migration

2.6.1. On the vesicle role

In the field of cell's biology, we call vesicles those small bags wrapped in a membrane forming part of eukaryotic cell organelles. They are involved in proteins or enzymes transport and absorption, or meet other needs of the cell. Inside the membrane bag of a vesicle, there are macromolecules which require the ability to move outside the cell walls. The membrane encircling the bag merges with the outer wall of the cell to allow such macromolecules to penetrate the wall. The vesicles are important parts of the human cells, although they are also found in other multicellular organisms [66].

Cells found in humans, plants and animals use a variety of types of vesicles, depending on the type of cell and its specific intended function. For example, one type of vesicles, lysosomes, are necessary for the process of digestion. Lysosomes contain enzymes that breakdown food cells. With food absorption, a lysosome vesicle bonds to the food holding cell and releases

enzymes by a process called phagocytosis. These enzymes break down food cells into smaller parts that can be better absorbed by other cells.

Secretory vesicles are frequently associated with nerve cells in humans or animals. Their membranes sacs contain neurotransmitters. Nervous system through hormonal signals activates these components. Through the process of exocytosis, the secretory vesicle's outer membrane adheres to the nerve terminal and releases neurotransmitters in the area of the nerve endings, named the synaptic cleft. Neurotransmitters transport information from one nerve terminal to the next, across the entire central nervous system, way up to the brain [66].

Vesicles, in their role as cellular mechanism are internally appointed for transport, uptake and storage of numerous imperative bodily functions. Without these tiny bags wrapped in membranes, cells could not make the exchange of materials necessary to maintain their healthy development and other crucial processes. As a conclusion, with no vesicles, humans and other pluricellular organisms could not have existed, because the essential cellular chemical processes would have no other method to pass onto another key materials [66].

Since there is increasing support that vesicle trafficking, including the release of EVs, is a highly important process in tumorigenesis, embryogenesis and tissue remodeling, in this paragraph we present an extensive discussion on the EVs convection in haptotaxis with hydrodynamical dissipation (i.e., a novel mechanism for vesicle migration).

2.6.2. *Mathematical model*

Vesicles are closed membranes floating in an aqueous solution (see Fig. 1). These membranes act as a barrier that efficiently controls permeability. The vesicles mimic maybe the most primitive and mechanically flexible dividing interfaces between the inside and the outside of a cell. Generally, the fluid enclosed by the membrane is incompressible in order that the vesicle evolves at a constant volume. Moreover, the membrane exchanges no phospholipid molecules with the solution, its area remaining constant as time passes [64]. Helfrich [45] described very well the vesicle's bending energy in its equilibrium state, which is compatible with the constraints above, i.e. constant volume and area. Even if the model is relatively simple it generates various equilibrium profiles, such as, discocytes (resembling red blood cells), stomatocytes, as well as forms presenting higher topologies (such as n -genus torus) that have been also observed experimentally [46]. We identify works studying alignments of vesicle in shear flows [47], fluctuations out of equilibrium [48], lift forces [49, 50], migration of vesicle in the proximity of a substrate [51, 52] or in gravity fields [53] and also vesicle tumbling [54]. One may note several recent experiments dealing with vesicle migration [55-58].

Considering the vesicle migration, we acknowledge that hydrodynamical dissipation in the neighboring fluid as well as inside the vesicle is present, and, in principle, between the two mono-layers which may glide with respect to each other. Furthermore, during motion on the substrate the dynamics of a vesicle may be restricted not only by the hydrodynamical flow but also by bonds breaking and restoring mechanisms that occur on the substrate (see [64]). It is obvious that the slowest mechanism limits the motion. Here we focus on a situation where

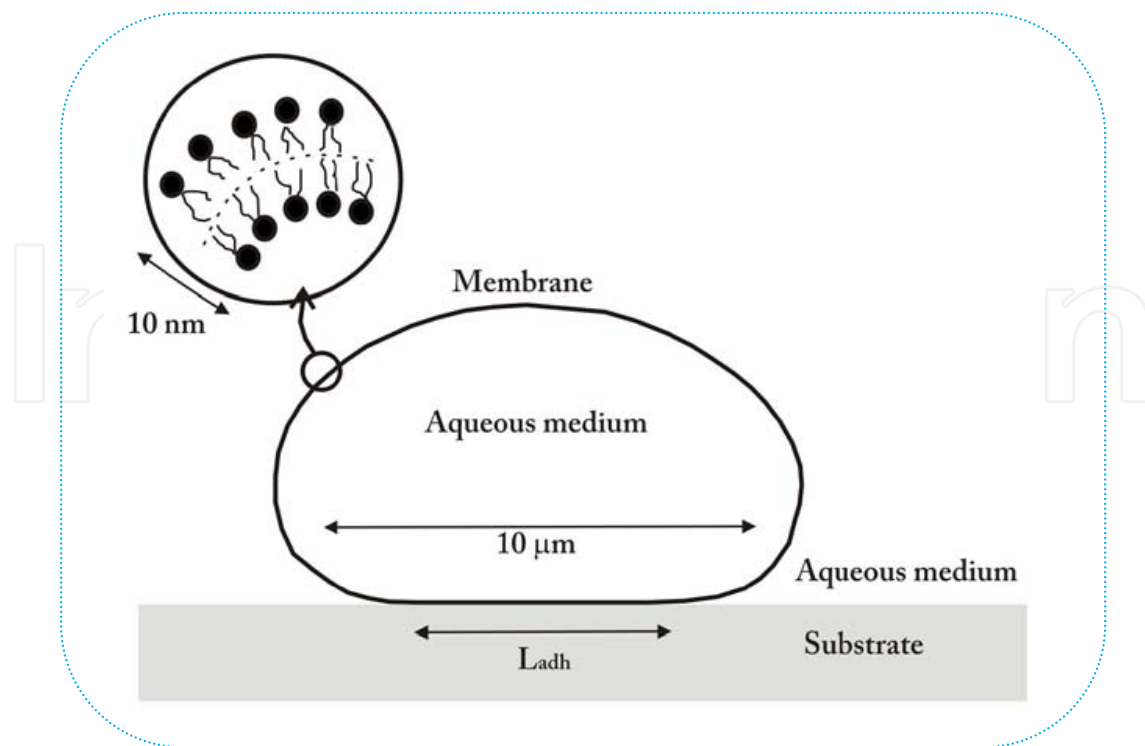


Figure 1. Schematic view of a vesicle emphasizing its microscopic structure: a bilayer of phospholipidic molecules. L_{adh} is the adherence length of the membrane

hydrodynamics are the limiting factors and we give out dissipation associated with bonds on the substrate.

Let us imagine a vesicle that initially adheres on a flat surface. We then consider an adhesion gradient along the substrate. The vesicle then moves in the direction of increasing adhesion energy (see Fig. 2) - it is named haptotaxis (a motion induced by an adhesion gradient).

A highly permeable vesicle can be pulled into a fluid without opposing any resistance (and without modifying the inner area), whereas an impermeable one would be subjected to a drag force [64]. The assumption of local impermeability is legitimate. This entails that the fluid velocity at the membrane is equal that of the membrane itself [59].

On a vesicle's scale ($R \sim 10 \mu\text{m}$) and for the expected velocities ($V \sim 1 \mu\text{m/s}$), the dissipative processes fully dominate the dynamics. The energy added instantly dissipates in various degrees of freedom. Local dissipation caused by molecular reorganization, characterized by Leslie's coefficient, is negligible with respect to the hydrodynamics modes [60].

If dissipation is dominated by bulk effects, as shown in [59], we are in the position to write down the basic governing equations for convective vesicles in a geometry depicted in Fig. 3, since we also know and it was proved the velocity field obeys Stokes equations [59].

In an original atmospheric system, the non-even distribution of ascending water droplets is determined by the interplay between solar energy-induced thermal gradients, thermal diffusivity, friction, and gravity. Ultimately, the mathematics of this model shapes the

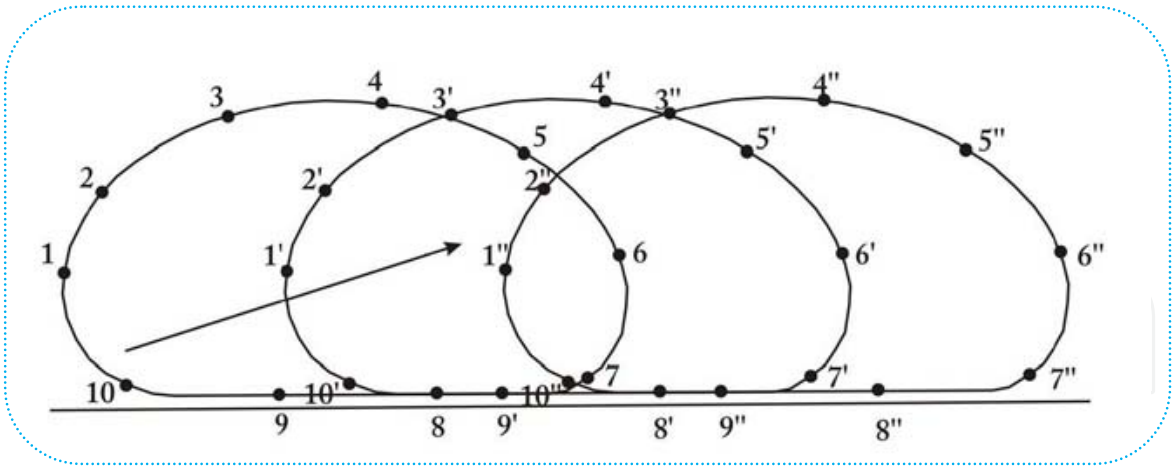


Figure 2. Stationary vesicle profiles are depicted. The vesicle is moving from the left (smaller adhesion) to the right (stronger adhesion); a few discretization points are represented and the arrow allows following one of these at three successive times. One can observe here the rolling and sliding components of the vesicle’s motion.

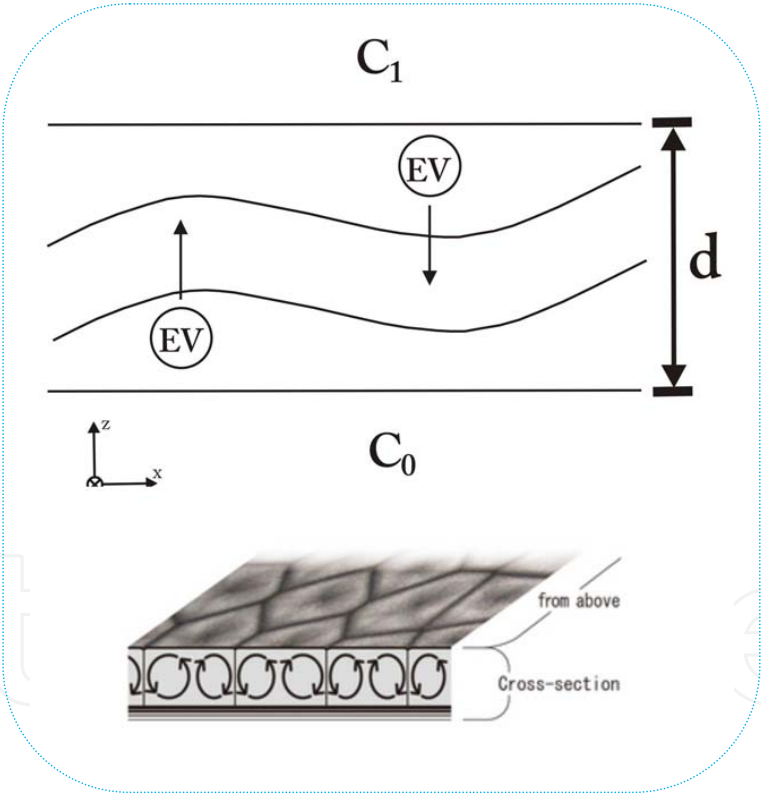


Figure 3. Convective extracellular vesicles (EVs) geometry. A fluid layer of thickness d of EVs, adherent on an extracellular matrix (ECM), is subjected to a gradient of concentration, where $\Delta C = C_1 - C_0 > 0$ is the difference of concentration between the front and back boundaries of the fluid layer.

umbrella-like or budding appearance of structures like cumulonimbus clouds. This model can better or uniquely describe those types of structural dynamics not explained under fractal, simple/linear and several other types of models.

Acknowledging that similar patterns occur in various biological spaces, we think that the same mathematical determinism can be ascribed. Thus, some histoarchitectural prototypic structures, like the capillary sprouting, embryologic organ, or even tumor buds of some types of cancer lesions might be in fact sculpted in that shape because gradients of molecular cues called morphogens can be deployed within the same manner water droplets can organize within nascent clouds.

Assuming that this organization also applies in biological systems, and that the EVs release can be considered among various processes organizing the budding tissue pattern, we think that the Lorenz model can govern their dynamics too. EVs would be particularly interesting as controllers of the tissue shape specification because they can include enzymatically active components (not found in conventional molecular morphogens), and thus might actively interact with the ECM fibers within their migration. Deployment of certain matrix degrading enzymes (MDEs) by EVs can modify this space while diffusing (event not produced by simple morphogens, attractive chemokines or repulsive semaphorins). This activity changes the topography of the ECM and creates spatial gradients directing the migration of subsequent EVs by haptotaxis - a mechanism better described for cell migration.

Let us consider the following thought biological experiment, equivalent to the Bénard experiment: a fluid layer of extracellular vesicles adherent on an ECM, in a haptotactic gradient. The fluid layer presents an unstable stratification of the potential density in a field of forces: the dense fluid is placed in front of the less dense one. We assume that in the basic state the layer of fluid of thickness d is subjected to a gradient of concentration

$$\beta = \frac{C_1 - C_0}{d} > 0 \quad (79)$$

$\Delta C = C_1 - C_0 > 0$ is the difference of concentration between the front and back boundaries of the fluid layer. The regime with the fluid at rest and a non-perturbed distribution of concentration, belongs to the thermodynamic branch, which is continuously linking the non-equilibrium stationary state ($\Delta C \neq 0$) with the equilibrium state ($\Delta C = 0$) (see Fig. 3).

We examine the evolution of a concentration fluctuation θ around the non-perturbed concentration profile $C_0(z)$.

Two dissipative processes tend to maintain the fluid at rest:

- friction (motion amortization through viscosity);
- ECM degradation subsequent to MDE's activity allowing vesicle trespassing - which lowers the concentration of the ECM, thus diminishing the forward, or advancing force.

The instability cannot be developed unless the EV is accelerated enough to overcome the effect of these dissipative processes. The gradient of concentration β which is the control parameter of this instability has to surpass a critical value β_c . Over this critical value, an organized structure of convection cells may appear.

For a one component fluid, the mass, momentum and internal energy equations are the expressions (see the fractal - nonfractal transition method [8-22]):

$$\begin{aligned}\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0 \\ \frac{\partial(\rho \mathbf{v})}{\partial t} + \nabla \cdot (\vec{\Pi} + \rho \mathbf{v} \mathbf{v}) &= \rho \mathbf{g} \\ \frac{\partial(\rho \varepsilon)}{\partial t} + \nabla \cdot (\rho \varepsilon \mathbf{v} + \mathbf{j}_d) &= -\vec{\Pi} \otimes (\nabla \mathbf{v})\end{aligned}\quad (80)$$

where ρ represents the mass density of the fluid, v its speed, \mathbf{g} acceleration of a field of forces, ε the internal energy of the unit volume, and \mathbf{j}_d the flux of ECM degraded by signals received from EVs. Here $\vec{\Pi}$ is the stress tensor and \otimes denotes the product of two tensors,

$$\vec{A} \otimes \vec{B} = A_{ij} B_{ji}$$

and we use Einstein's summation convention (implicit sum over repeating indices). The stress tensor can be written

$$\vec{\Pi} = \vec{\Pi}^e + \vec{\Pi}^v \quad (81)$$

$\vec{\Pi}^e$ is the equilibrium part and depends on the state of the system. $\vec{\Pi}^v$ represents the non-equilibrium part and is named, viscous stress tensor. At equilibrium, this part vanishes. For an isotropic medium at rest,

$$\vec{\Pi}^e = \begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix} = p \vec{I} \quad (82)$$

where p is the hydrostatic pressure. For viscous systems at non-equilibrium, the viscous stress tensor is not null. According to Eq. (81) and Eq. (82), the stress tensor will be, for homogeneous and isotropic viscous systems, at non-equilibrium

$$\vec{\Pi} = p \vec{I} + \vec{\Pi}^v \quad (83)$$

We start with the following assumptions:

- a. the fluid is Newtonian; as a result the stress tensor is given by Eq. (83), where the viscous stress tensor is [8-22]

$$\Pi_{\alpha\beta}^v = -\eta \left(\frac{\partial v_\alpha}{\partial x_\beta} + \frac{\partial v_\beta}{\partial x_\alpha} - \frac{2}{3} \delta_{\alpha\beta} (\nabla \cdot \mathbf{v}) \right) - \zeta \delta_{\alpha\beta} (\nabla \cdot \mathbf{v})$$

with coefficients η and ζ independent of velocity, the tangential (shear) and bulk viscosity, respectively;

- b.** ECM degrading by MDEs from EVs is described by the Fourier equation

$$\mathbf{j}_d = -\lambda \nabla C \quad (84)$$

where λ is the haptotactic coefficient;

- c.** haptotactic energy expansion is linear

$$\delta\rho = \rho - \rho_0 = -\rho_0 \alpha \varepsilon = -\rho_0 \alpha k_h (C - C_0) = -\rho_0 \chi (C - C_0) \quad (85)$$

where we used the expression of the haptotactic energy

$$\varepsilon = k_h (C - C_0)$$

k_h being the haptotactic energy constant. In Eq. (85) α is the haptotactic energy expansion constant and $\chi = \alpha k_h$ is the haptotactic expansion constant;

- d.** the fluid satisfies a state equation: consequently, its internal energy is (up to a constant factor)

$$\varepsilon = k_b C \quad (86)$$

where k_b is the state constant;

- e.** in most liquids, thermal expansion is small. We choose everywhere a constant density, denoted by ρ_0 , except the momentum equation.

With these approximations, the system of Eqs. (80) leads to the Boussinesq type system of equations

$$\begin{aligned} \nabla \cdot \mathbf{v} &= 0 \\ \rho_0 \left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] + \nabla p &= (\rho_0 + \delta\rho) \mathbf{g} + \eta \nabla^2 \mathbf{v} \\ \frac{\partial C}{\partial t} + (\mathbf{v} \cdot \nabla) C &= \frac{\lambda}{\rho_0 k_b} \nabla^2 C \end{aligned} \quad (87)$$

where ρ is the perturbed density

$$\rho = \rho_0 + \delta\rho \quad (88)$$

The first equation (87) represents the incompressibility condition for the fluid.

Convection occurs in the fluid layer when the forward, or advancing force, resulted from energy expansion, surpasses the viscous forces. We may define now a Rayleigh type number identical with the Eqs associated to fractal - nonfractal transition [61]

$$R = \frac{|\mathbf{F}_{asc}|}{|\mathbf{F}_{visc}|} \approx \frac{\left| \frac{\delta\rho \mathbf{g}}{\rho_0} \right|}{\left| \frac{\eta \nabla^2 \mathbf{v}}{\rho_0} \right|} \quad (89)$$

The density perturbation satisfies, according to Eq. (85)

$$\frac{\delta\rho}{\rho_0} \approx \chi \Delta C \quad (90)$$

On the other side, from the internal energy equation Eq. (79), it results

$$v \approx \frac{\lambda}{\rho_0 k_b} \frac{1}{d} \quad (91)$$

Replacing Eqs. (90) and (91) in Eq. (89), and taking into account Eq. (79), we get a biological Rayleigh number

$$R = \frac{\chi \beta \rho_0 k_b g}{\nu \lambda} d^4 \quad (92)$$

where $\nu = \eta / \rho_0$ is the cinematic viscosity. For the Bénard convection, the biological Rayleigh number plays the part of a control parameter. The convection occurs for

$$R > R_{critical}$$

Most of the time, R is controlled by β , the gradient of concentration.

Within a biological context, \mathbf{g} can be specified by polar/linear topography of semaphorins or/and chemokines, signals typically creating stable gradients to which EVs can respond.

We choose as reference state the rest stationary state ($v_s=0$), for which the last two equations in the system of Eqs. (87) reduce to

$$\begin{aligned}\nabla p_s &= -\rho_s g \hat{z} = -\rho_0 [1 - \chi(C_s - C_0)] g \hat{z} \\ \nabla^2 C_s &= 0\end{aligned}\quad (93)$$

where \hat{z} is the versor of the vertical direction. We assume pressure and concentration varies only along the vertical direction, due to the geometry of the experiment. For concentration, the boundary conditions read

$$C(x, y, 0) = C_0; \quad C(x, y, d) = C_1$$

Integrating the second Eq. (93) with these boundary conditions, it results that, in the stationary reference state, the profile of the concentration in the vertical direction is linear

$$C_s = C_0 - \beta z \quad (94)$$

with β , the gradient of concentration. Replacing Eq. (94) in first Eq. (93) and integrating, we get

$$p_s(z) = p_0 - \rho_0 g \left(1 + \frac{\chi \beta z}{2} \right) z \quad (95)$$

The characteristics of the system in this state are independent of the kinetic coefficients η and λ which appear in Eqs. (87). We study the stability of the reference state using the small perturbations method. The perturbed state is characterized by

$$\begin{aligned}C &= C_s(z) + \theta(\mathbf{r}, t) \\ \rho &= \rho_s(z) + \delta\rho(\mathbf{r}, t) \\ p &= p_s(z) + \delta p(\mathbf{r}, t) \\ \mathbf{v} &= \delta\mathbf{v}(\mathbf{r}, t) = (u, v, w)\end{aligned}\quad (96)$$

As can be seen from Eqs. (96), the perturbations are functions of coordinate and time. Replacing Eqs. (96) in the evolution equations of the Boussinesq approximation Eqs. (87) and taking into account Eq. (94) and Eq. (95), we get, in the linear approximation, the following equations for the perturbations

$$\begin{aligned}
\nabla \cdot \delta \mathbf{v} &= 0 & \text{a} \\
\frac{\partial \delta \mathbf{v}}{\partial t} &= -\frac{1}{\rho_0} \nabla \delta p + \nu \nabla^2 \delta \mathbf{v} + g \chi \theta \hat{z} & \text{b} \\
\frac{\partial \theta}{\partial t} &= \beta w + K \nabla^2 \theta & \text{c}
\end{aligned} \tag{97}$$

where $K = \frac{\lambda}{\rho_0 k_b}$ is a coefficient. We pass to non-dimensional variables in Eqs. (97), using the transformations: $\mathbf{r}' = \frac{\mathbf{r}}{d}$; $t' = \frac{t}{d^2/K}$; $\theta' = \frac{\theta}{\left(\frac{\nu K}{g \chi d^3}\right)}$; $\delta \mathbf{v}' = \frac{\delta \mathbf{v}}{K/d}$ and $\delta p' = \frac{\delta p}{\left(\frac{\rho_0 \nu K}{d^2}\right)}$.

Using the standard method from [8-22], it results the biological Lorentz system.

2.6.3. Validity of theoretical model

Some results are evident:

- we build the first Lorenz model for extracellular vesicles migration;
- in [50], and similar other references ([64] etc.), the EVs behavior, under shear flow close to a substrate, was proved to be quite similar to the one encountered in two dimensional simulations, so we are confident that the 2D assumptions captures the essential features of the 3D EVs;
- different control parameter values for the Lorenz system can create shape distributions similar to the cordonal appearance of fingerprints (see Fig. 4, A), or complex skin tissue tiles like scale appendages in the amphibian covering (see Fig. 4, B);
- the biological thought experiment equivalent to Bénard's experiment, involving a fluid layer of extracellular vesicles adherent to an extracellular matrix, in a haptotactic gradient can be *checked experimentally* today to a high degree of accuracy. We think that suitable test systems would be the *embryological ones* (i.e., the development of branched vessels in membranes - avian eggshell membranes, serous membranes of the peritoneal cavity; or the budding development of lung alveoli, or of fingerprints), and, similar, *inflammatory ones* (i.e., the emergence of neoangionetic vessels driven by inflammatory proximities) - all of which apparently start as point like spots displayed in a comb-like appearance along a rectilinear or arched origin;
- we analyze the problem of EVs migration in haptotaxis, though most of the reasoning applies to chemotaxis (migration of cells biased towards a gradient of diffusible MDEs) as well as to a variety of driving forces - all of which include the possibility to specify an active parameter value within the model;
- the resulted system of equations exhibits complex behavior, hard to control, the two occurring convective rolls: either going in one direction, or in the opposite one - means patterning the EVs spreading.

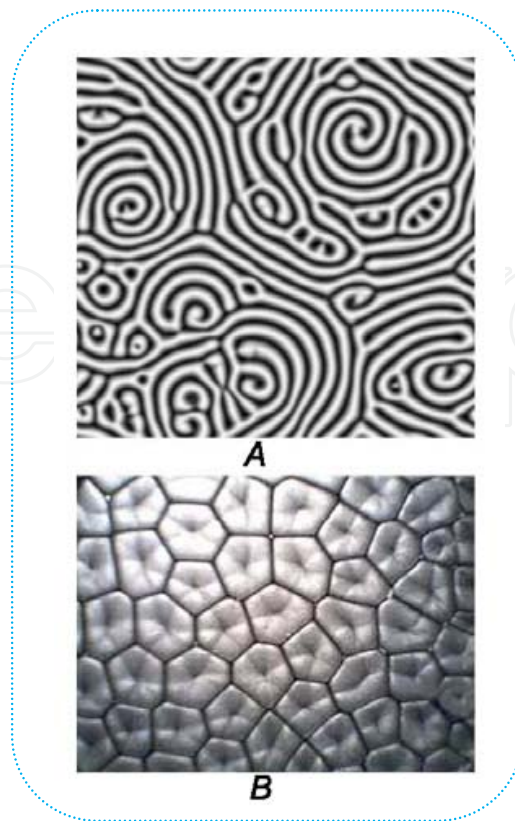


Figure 4. Bénard-Rayleigh model patterns representative for biological instances: A) for fingerprint like distribution of skin cells, B) for fish or amphibian scales.

3. Conclusions

Considering the above, we can write the following conclusions :

- i. We establish a relationship between the $SL(2R)$ group and the canonic formalism. It particularly results that all invariant functions on the $SL(2R)$ group will be functions on the hamiltonian and on the Gausssian;
- ii. We establish the statistical significations for coefficients of the hamiltonian by using Shannon's maximum informational entropy principle. Any statistical assumption is specified by particularly selecting the hamiltonian coefficients and the class of all these hypothesis by the transitivity manifolds of the group. In this manner, if the hamiltonian has energy significance, then through the transitivity manifolds, the motions of a representative point from the phase space on a surface of constant energy are in the same time on a surface of constant probability density (the ergodic hypothesis). Therefore, the class of the statistic hypothesis (which are characteristic to the Gaussians of the same average) is given by the ergodic hypothesis. In this way, we establish a fundamental relationship between energy and probability;

- iii. We prove that informational energy (in the sense of Onicescu) is a measure of the dispersion of a distribution. The class of the statistic hypothesis that are characteristic to the Gaussians of the same average is characterized by the constant value of the informational energy. In addition, it is equivalent to the ergodic hypothesis. For a constant value of the informational energy, we obtain uncertainty egalitarian relationships and, particularly, for the linear harmonic oscillator, we show that the informational energy is quantified;
- iv. Assuming that de Broglie's theory is materialized through a periodic field in a complex coordinate, we prove that it has a "hidden symmetry", which is expressed by the homographical transformations group in three parameters. This group (also an achievement of $SL(2R)$) functions as a synchronization group both in phase and in amplitude, among the oscillators of the same ensemble. The simultaneous invariance related to two different achievements of $SL(2R)$ implies (integrally through the invariant functions on the groups) an uncertainty relation in egalitarian form and the Stoler group of synchronization among oscillators from different ensembles (i.e., the second quantification when the creation and annihilation operators refer to a harmonic oscillator);
- v. The synchronization group among the oscillators of the same ensemble admits three differentiable 1-forms and one differentiable 2-form which is absolutely invariant on the group. The existence of a parallel transport in Levi-Civita's sense, in which case the 2-form in Lobachevski's metric form in Poincare representation, implies through a variation principle, equations of Ernst type for the gravitational field of vacuum;
- vi. Complex measures in the study of certain physical systems dynamics need the use of a space-time endowed with a special topology, namely, the fractal space-time [6, 7] (see also [71] for details).

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