Chapter 7

A Lie-QED-Algebra and their Fermionic Fock Space in the Superconducting Phenomena

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Additional information is available at the end of the chapter

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1. Introduction

It's created a canonical Lie algebra in electrodynamics with all the "nice" algebraic and geometrical properties of an universal enveloping algebra with the goal of can to obtain generalizations in electrodynamics theory of the TQFT (*Topological Quantum Field Theory*) and the Universe based in *lines and twistor bundles* to the obtaining of *orbital spaces* [1] that will be useful in the study of superconducting phenomena. The obtained object haves the advantages to be an algebraic or geometrical space at the same time. This same space of certain $\pounds$- modules can explain and model different electromagnetic phenomena as superconductor and quantum processes where is necessary an organized transformation of the electromagnetic nature of the space-time and obtain nanotechnology of the space-time and their elements when this is affected by the superconducting fields created by the different *electrodynamic Majorana states* in the matter and space [2]. Then using the second quantizing formalism to the description of the behavior of the particles of the affected space for superconducting fields created by the BSC-theory (*Bardeen-Cooper-Schrieffer-theory*) in condensation of the matter where is conformed a fermionic space called set of *pairs of Cooper*, which are comported as bosons will be the domain of the QED (*Quantum Electro-Dynamics*) transformations of the space to different actions as magnetic levitation, electromagnetic impulse, etc. Then considering the Hilbert space of their multiple fermionic modes we obtain the *fermionic Fock space* which describes the *quantum organized transformation* of the particles to a *Fock space* of an arbitrary number of identical fermions explaining the superconductivity as a Bose-Einstein condensation in this process. The fluidity obtained in the Bose-Einstein condensation obeys to a Bogoliubov transformation which in this case, is our organized transformation required to produce the micro-electromagnetic effects from the actions of the operators of Lie-QED-algebra whose fermionic Fock space is the given for the affecting of the energy space where exist the fermionic modes as a Plasmon resonances.

From this study are mentioned some applications to photonics using the *boson link-wave* to hyper-telecommunications and superconducting materials.

2. A Lie-quantum electrodynamical-algebra and their corollaries

2.1. A Lie Algebra in Electro-dynamics

We consider the electromagnetic field or Maxwell field defined as the differential $2-$ form of the forms space $\Omega^2(\mathbb{R}^4)$;

$$F = F_{ab}dx^a \wedge dx^b, \tag{1}$$

which can be described in the endomorphism space of M, by the matrix (where ^{a}, and ^{b}, are equal to $1,2,3$):

$$F_{ab} = \begin{pmatrix} 0 & B^3 & -B^1 & E^1 \\ -B^3 & 0 & B^1 & E^2 \\ B^2 & -B^1 & 0 & E^3 \\ -E^1 & -E^2 & -E^3 & 0 \end{pmatrix}, \tag{2}$$

where E (respectively B) the corresponding forms of electric field (respectively magnetic field).

We want to obtain a useful form to define the actions of the group $\pounds$, on the space of electromagnetic fields F, which is resulted of generalize to the space $\Omega^2(M)$, as an *anti-symmetric tensor algebra* through from induce to the product in the product , shape that will be useful to the localizing and description of the irreducible unitary representations of the groups $SO(4), O(1,3)$, and *representations of spinor fields* in the space-time furthermore of their characterizing as principal $G-$ bundle of M. In the context of the gauge theories (that is to say, in the context of *bundles with connection* as the principal $G-$bundles) we first observe that F, is an *exact form* and thus there exists a $1-$form A^b (*electromagnetic potential*) that defines a connection in a $U(1)-$ bundle on M, and such that[1]

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1 The anti-symmetric nature of this form results obvius:

$$F_{ab} = \partial_a A_b - \partial_b A_a = -F_{ba} = -(\partial_b A_a - \partial_a A_b).$$

Likewise, the electromagnetic field is the 2-form given by (6) with the property of the transformation

$$F'_{ab} = \partial'_a A'_b - \partial'_b A'_a = a_{ac}a_{bd}(\partial_c A_d - \partial_d A_c)$$
$$= a_{ac}a_{bd}F_{cd}.$$

In $\mathbb{R}^3$, said 2-form match with the $3 \times 3-$ matrix to B_{ab} . Remember that $\boldsymbol{B} = \nabla \times \boldsymbol{A}$.

$$F_{ab} = \partial_a A_b - \partial_b A_a, \tag{3}$$

Consider the $K-$invariant $G-$structure $S_G(M)$, of the differentiable manifold $M \cong \mathbb{R}^4$, with Lorentzian metric (and thus pseudo-Riemannian) g, on $\mathbb{R}^4$, with $Diag(g) = (1,1,1,-1)$, in the system of canonical coordinates

$$\varphi\{U, x, y, z, t\}, \tag{4}$$

and let the spaces $\mathbb{E}, \mathbb{H}$, two free $\mathbb{R}-$ modules (modules belonging to a commutative ring with unit $\mathbb{R}$) such that

$$\mathbb{E} = \{E \in \mathcal{X}(\mathbb{R}^4) \Big| E^b = \iota \frac{\partial}{\partial t} F\}, \tag{5}$$

and

$$\mathbb{H} = \{B \in \mathcal{X}(\mathbb{R}^4) \Big| B^b = -\iota \frac{\partial}{\partial t} F\}, \tag{6}$$

where ^{b}, is Euclidean in $\mathbb{R}^3$, and $*$, is Loretzian in $\mathbb{R}^4$. Such $\mathbb{R}-$ modules are $\pounds-$ modules where $\pounds(M) \cong O(1,3)$, is the orthogonal group of range 4. The two modules in (5) and (6) intrinsically define all electric and magnetic fields E, and B, in terms of F. Thus also their tensor, exterior, and scalar products between elements must be expressed in terms of F. To it we consider the tensor product of (5) and (6) as free $\mathbb{R}-$ modules elements, to know[2],

$$E^b \otimes B^b = \frac{1}{c} F \otimes \overline{F}, \tag{7}$$

where c, is light speed and $\overline{F}$ [3], is the dual electromagnetic tensor of F. Then, what must be $E \otimes B$?

2This is valid since tensor product of free $\mathbb{R}-$ modules is a free $\mathbb{R}-$ module[5]. Here $E = \sum_b E^b dx_b \wedge dt, \quad B = \sum_b B^b dx_b \wedge dt.$

3The Levi-Civita tensor can be used to construct the dual electromagnetic tensor in which the electric and magnetic components exchange roles (conserving the symmetry, characteristic that can be seen in the matrices of the electromagnetic tensor F, and their dual $\overline{F}$):

Proposition 2. 1 (F. Bulnes) [3]. Said $\mathbb{R}-$modules are invariant under Euclidean movements of the group $O(1,3)$, and thus are $\pounds-$modules.

Proof. [3]. □

Now we consider electro-strength field algebra given by the $U(1)-$*gauge field* coupled to a charged spin 0 scalar field that takes the place of the Dirac fermions in "ordinary" QED.

Let $(\otimes\mathbb{E})$, be the tensor algebra generated by the elements $F_1 \otimes F_2 - F_2 \otimes F_1$. Let J, be the two-seated ideal generated by the elements $F_1 \otimes F_2 - F_2 \otimes F_1 - [F_1, F_2]$. Let $\mathfrak{e}$, be the Lie algebra whose composition rule is $[,]$. Its wanted to construct an associative algebra with unity element corresponding to $\mathfrak{e}$, such that

$$[F_1, F_2]_{\otimes} = F_1 \otimes F_2 - F_2 \otimes F_1 \tag{8}$$

We want describe energy flux in liquid and elastic media in a completely generalized diffusion of electromagnetic energy from the source (*particles of the space-time "infected" for this electromagnetic energy*), which must be very seemed as a *multi-radiative tensor insights space* or a *electromagnetic insights tensor space*. This will permits us to express and model the flux of electromagnetic energy through pure tensor product of Maxwell fields F, which will be useful in the symplectic structure subjacent in the quantum version of this algebra and their actions of group. After and inside of the demonstration of one result where is related the structure of this quantum version algebra with the superconducting phenomena , the quantum macroscopic effects obtained result of this inheritance of structure, having that for the energy conservation and the use of Lagrangians [4, 5]:

"The rate of energy transfer (per unit volume) from a region of space equals the rate of work done on a charge distribution plus the energy flux leaved in that region"

Of fact these are elements $E^b \otimes B^b$, that are constructed from a power space given in by $\mathbb{E} \times \mathbb{B}$, and that conforms the *electromagnetic multi-radiative space which will be the region space with the "transmittance" of the fermions effects obtained one time that let be quantized the space* $\mathbb{E} \otimes \mathbb{H}$,

to obtain our QED-Lie algebra necessary whose operators will act in the wrapped space (of the electromagnetic type) to get the superconducting effects accord to the Bogoliubob transformation [6] required to produce the quantum electromagnetic effects (electro-anti-gravitational effects) from the actions of the operators of Lie-QED-algebra whose fermionic

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$$\tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\kappa\lambda}F^{\mu\nu},$$

where $\varepsilon_{\mu\nu\kappa\lambda}$, is the rank-4 Levi-Civita tensor density in Minkowski space.

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Fock space is the given for the affecting of the energy space where exist the fermionic modes as a *Plasmon resonances*.

First we demonstrate the nature of Lie algebra of this space of tensors to electro-physics.

Proposition (F. Bulnes) 3. 1. The electrodynamical space $\mathbb{E}\otimes\mathbb{H}$, is a closed algebra under the composition law $\left[\ ,\ \right]$ of the $U(1)-$ connections.

Proof. Let $F_1=\nabla^1_a A^1_b-\nabla^1_b A^1_a$, and $F_2=\nabla^2_a A^2_b-\nabla^2_b A^2_a$, two elements of $\mathbb{E}\otimes\mathbb{H}$, $\forall A$ an $U(1)-$ connection. Then the composition $[F_1,F_2]$, takes the form in function of the $U(1)-$ connections as

$$
\begin{aligned}
[F_1,F_2]&=(\nabla^1_a A^1_b-\nabla^1_b A^1_a)\otimes(\nabla^2_a A^2_b-\nabla^2_b A^2_a)-\\
&\quad(\nabla^2_a A^2_b-\nabla^2_b A^2_a)\otimes(\nabla^1_a A^1_b-\nabla^1_b A^1_a)\\
&=\nabla^1_a A^1_b\otimes\nabla^2_a A^2_b-\nabla^1_b A^1_a\otimes\nabla^2_a A^2_b-\nabla^1_a A^1_b\otimes\nabla^2_b A^2_a+\\
&\nabla^1_b A^1_a\otimes\nabla^2_b A^2_a-(\nabla^2_a A^2_b\otimes\nabla^1_a A^1_b-\nabla^2_b A^2_a\otimes\nabla^1_a A^1_b-\\
&\nabla^1_b A^1_a\otimes\nabla^2_a A^2_b+\nabla^2_a A^2_b\otimes\nabla^1_b A^1_a)=\\
&=\nabla^1_a A^1_b\otimes(\nabla^2_a A^2_b-\nabla^2_b A^2_a)-\nabla^1_b A^1_a\otimes(\nabla^2_a A^2_b-\nabla^2_b A^2_a)\\
&-\{\nabla^2_a A^2_b\otimes(\nabla^1_a A^1_b-\nabla^1_b A^1_a)-\nabla^2_b A^2_a\otimes(\nabla^1_a A^1_b-\nabla^1_b A^1_a)\}\\
&=\nabla^1_a A^1_b\otimes\nabla^2_a A^2_b-\nabla^1_a A^1_b\otimes\nabla^2_b A^2_a-\nabla^1_b A^1_a\otimes\nabla^2_a A^2_b+\\
&\nabla^1_b A^1_a\otimes\nabla^2_b A^2_a-\nabla^2_a A^2_b\otimes\nabla^1_a A^1_b+\nabla^2_b A^2_a\otimes\nabla^1_a A^1_b+\\
&\nabla^1_b A^1_a\otimes\nabla^2_a A^2_b-\nabla^2_a A^2_b\otimes\nabla^1_b A^1_a\\
&=-\nabla^1_b A^1_a\otimes\nabla^2_a A^2_b+\nabla^2_b A^2_a\otimes\nabla^1_a A^1_b,
\end{aligned}
$$

Since $F_{ab}=-F_{ba}$, in $\mathbb{R}^4$, then

$$
\begin{aligned}
&-\nabla^1_b A^1_a\otimes\nabla^2_a A^2_b+\nabla^2_b A^2_a\otimes\nabla^1_a A^1_b=\nabla^1_a A^1_b\otimes\\
&\nabla^2_a A^2_b-\nabla^2_b A^2_a\otimes\nabla^1_b A^1_a\in\mathbb{E}\otimes\mathbb{H},
\end{aligned}
$$

Thus $[F_1,F_2]\in\mathbb{E}\otimes\mathbb{H},\forall F_1,F_2\in\Omega^2(M)$. $\square$

Due to that we are using a *torsion-free connection* (e.g. the *Levi Civita connection*), then the partial derivative ∂_a, used to define F, can be replaced with the covariant derivative ∇_a. The Lie derivative of a tensor is another tensor of the same type, i.e. even though the individual terms in the expression depend on the choice of coordinate system, the expression as a whole result in a tensor in $\mathbb{R}^4$.

Proposition (F. Bulnes) 3. 2. The closed algebra $(\mathbb{E}\otimes\mathbb{H},[,])$, is a Lie algebra.

Proof. Consider

$$[F,F]=\nabla_a A_b \otimes \nabla_b A_a - \nabla_b A_a \otimes \nabla_a A_b - [\nabla_a A_b \otimes \nabla_b A_a - \nabla_b A_a \otimes \nabla_a A_b] = 0,$$

Then the other properties of Lie algebra are trivially satisfied. Thus $\mathrm{E}\otimes\mathrm{H}$, haves structure of Lie algebra under the operation $[,]$. □

2.2. Lie-QED-algebra to superconducting phenomena

If first, we consider the Maxwell tensors given by $F=\nabla_a A_b - \nabla_b A_a$, and thus of the Lie-EM-algebra given in the before section, these comply the following variation principle given by the Maxwell Lagrangian.

$$\mathcal{L}_{\mathrm{MAXWELL}} = -\frac{1}{4} F_{ab} F^{ab}, \tag{9}$$

Then the due action to this Maxwell Lagrangian is

$$\begin{aligned}\Im &= -\frac{1}{4}\int dx^4 F_{ab}F^{ab} = -\frac{1}{4}\int dx^4 (\nabla_a A_b - \nabla_b A_a)(\nabla^a A^b - \nabla^b A^a) \\ &= \frac{1}{2}\int d^4k A_a(k)[-k^2 g^{ab} + k^a k^b] A_b(-k),\end{aligned} \tag{10}$$

where is expected that the inner product $\langle A_a(k), A_a(k')\rangle$, must be equivalent to an expression where the inverse of the differential operator defined by $[-k^2 g^{ab} + k^a k^b]$,[4] appears

[4]The general formula for the Gaussian integral of the last integral of (10) takes the form:

$$\int [d\varphi]\exp\left\{-\frac{1}{2}(\varphi k_{ab}\varphi)+(J\varphi)\right\} \approx \frac{1}{\sqrt{\det K}}\exp(JK^{-1}K),$$

However in our case the operator $K=k_{ab}$, comes given by

$$K=k_{ab}(x-y)=[-k^2 g^{ab} - k^a k^b]\delta^4(x-y),$$

Has the property of the projection operator, that is to say,

$$\int d^4y k_{ab}(x-y)k_\lambda^\nu(y-z) \propto k_{\nu\lambda}(x-y),$$

and has not inverse. This means that the Gaussian integral diverges. The reason that the free-field part of the action integral given in (10) is singular is due to the gauge invariance which projects out the transverse gauge fields. In the path integral for the free-field part given by

$$\int (d^4 A_a)\exp\left\{i\int d^4x[\mathcal{L}_0 + J_a A^a]\right\},$$

in the product given in the covariant rules of Feynman diagrams. The field equation that must be to solve is

$$[-k^2 g^{ab} + k^a k^b] k_b \alpha(k) = [-k^2 k^a + k^2 k^a]\alpha = 0,$$

which is particular case of the Bulnes´s equation in the *curvature context* [4] to $G = U(1)$. Here $k^a = \nabla^a$, $k_a = \nabla_a$, and $-k = \Box$.

We consider the model that consists of a complex scalar field $\varphi(x)$, [5]minimally coupled to a gauge field given by $1-$forms ($U(1)-$ gauge field) "coupled to a charged spin 0 scalar field" and that satisfy:

$$\mathcal{L} = \frac{1}{2}(D_a\varphi)^* D^a\varphi - U(\varphi^*\varphi) - \frac{1}{4}F_{ab}F^{ab}, \tag{11}$$

where F_{ab}, has been defined in the section 2. 1. We define to $D_a\varphi = (\partial_a\varphi - ieA_a\varphi)$, as the covariant derivative of the field φ, also e, is the electric charge and $U(\varphi,\varphi^*)$, is the potential for the complex scalar field. This model is invariant under gauge transformations parametrized by $\lambda(x)$, that is to say, are had the following transformations to the fields:

$$\varphi'(x) = e^{ie\lambda(x)}\varphi(x), \qquad A_a^{'}(x) = A_a(x) + \partial_a\lambda(x), \tag{12}$$

If the potential is such that their minimum occurs at non-zero value of $|\varphi|$, this model exhibits the Higgs mechanism. This can be seen studying the fluctuations about the lowest energy configuration, one sees that gauge field behaves as a massive field with their mass proportional to the e, times the minimum value of $|\varphi|$. As shown by Nielsen and Olesen [7], this model, in $2+1$, dimensions, admits time-independent finite energy configurations corresponding to vortices carrying magnetic flux. The magnetic flux carried by these

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we have summed over all field configurations including "orbits" that are related by gauge transformations. This over counting is the root of the divergent integral. Thus we have to remove this "volume" of the orbit in this quantization. In the case where the quantizing is realized by the scalar field theory to our superconducting phenomena, the "orbits" are considered as part of the interactions spin-orbit, and the Lie-QED structure to the orbits will be conserved.

[5]In a complex scalar field theory, the scalar field takes values in the complex numbers, rather than the real numbers. The action considered normally takes the form

$$\Im = -\frac{1}{4}\int dx^{d-1}xdt\mathcal{L} = \int d^{d-1}xdt[\eta^{ab}\nabla_a\varphi^*\nabla_b\varphi - V(|\varphi|^2)],$$

This has a $U(1)$, or, equivalently $SU(2)$, symmetry, whose action on the space of fields rotates $\varphi \mapsto e^{i\alpha}\varphi$, for some real phase angle α.

vortices is quantized (in units of $\frac{2\pi}{e}$ [6]) and appears as a topological charge associated with the topological current [8]:

$$J^a_{top} = \in^{abc} F_{bc}, \tag{13}$$

These vortices are similar to the vortices appearing in type-II superconductors. These in the superconducting theory are acquaintance as *fluxoids*. There exist some thermo-dynamical conditions established to the existence the superconductor of type II[7].

Theorem. (F. Bulnes) [1, 9, 10] 2. 2. 1. We consider $F = (H_i H_k - \frac{1}{2}H^2\delta_{ik}) / 4\pi\sigma\mu,$ [8]and $F_S = -H^2_{ext}\vec{n} / 8\pi\sigma\mu,$ with the Hamiltonian foreseen in the *Appendix A* given by (A. 12), of the

[6]If $\Gamma \subset \mathfrak{S}$, is away from of the borders and rounds to the hollow (see the Figure 1) and suppose that is have applied a magnetic field to this superconductor $\mathfrak{S}$, then

$$\oint_\Gamma \left(\frac{m^*}{\hbar n_s e^*} J + \frac{e^*}{\hbar c} A \right) dl = n2\pi,$$

But $J = 0,$ (inside the superconductor (or ring in the experimental Figure 1) there not are currents) then

$$\oint_\Gamma A dl = n\frac{\hbar c}{e^*},$$

For the Stokes theorem is had that

$$\oint_\Gamma A dl = \iint_{\partial\mathfrak{S}} B dS = \Phi,$$

To it is necessary remember that the superconducting current J_s , haves an unique value in each point, which is equivalent to that the density of superconductor electrons is injective in each point. This bring as consequence that in a close circuit Γ , of length 2π, we have

$$\phi(2\pi) - \phi(0) = n2\pi,$$

For the circulation around a close circuit Γ , and considering that

$$A = -\nabla^a \phi,$$

we have that on the close circuit Γ ,

$$\oint_\Gamma \nabla^a \varphi dl = n2\pi,$$

that in our case is

$$\Phi = n\Phi_0.$$

[7]Its considered the superconductor of the type I, in the intermediate state $\boldsymbol{H}_c(1-n) < \boldsymbol{H} < \boldsymbol{H}_c,$ (ellipsoidal superconductor) and we calculate the transition to type II, with $\lambda(T) > \xi(T)$[9,10].

[8]Where the term $\frac{1}{2}H^2\delta_{ik},$ is the term of the tensor *F*, corresponding to the free or total energy of the magnetic field of the superconductor, which involves the thermodynamic effects foreseen in (9), for "compression", to which is subject the surface of the object *O* (**see theAppendix A**).

Lemma. A. 1., and their proof, where is satisfied the inequality on magnetic energy necessary to all magnetic process of superconducting

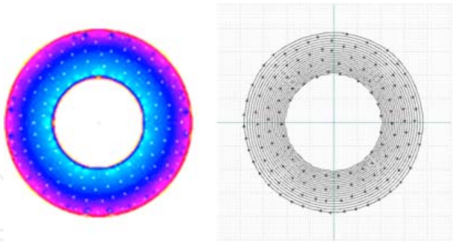

Figure 1. Ring region where is applied a magnetic field in a superconductor $\mathfrak{S}$. This magnetic field could realize iso-rotations and levitation force and impulse. The magnetic flow is intense in this region and the lines of intensity of magnetic field behave like it is pointed out. Inside of hollow the current is zero. The region of *fluxoids* will be generated from the inner of the ring where not exists current but yes magnetic flows doing it in discrete form.

$$8\pi\Phi_0 n H_v \geq \int_O (H(1-h\nabla^2\vec{H}))^2 dV \geq 8\pi\int_O H^2 dV, \tag{14}$$

Then a sensorchip to magnetic flux (pressure on the surface of O) to super-currents is defined by the inequality

$$\int_V j_s(x,y)\delta(z)dV \leq \int_S BdS \leq n\Phi_0, \tag{15}$$

where Φ_0, is a fluxoid ($=\hbar c / e^* = 2.07\times10^{-7} gauss \bullet cm^2$), being $e^* = 2e$, where e, is the charge of electron.

Proof. [9]. □

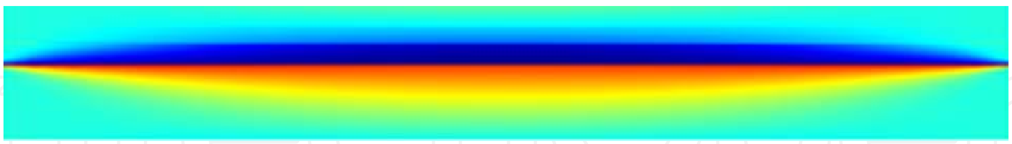

Figure 2. One has a view of profile of the magnetic flow of a plate under magnetic field (this simulation was published in the Proceedings of Fluid Flow, Heat Transfer and Thermal Systems of ASME in the paper IMECE2010-37107, British Columbia, Canada with all rights reserved ® [9]).

Developing these topological electromagnetic elements using the tensor $\in^{abc}$, we have to two Maxwell tensors:

$$J_{\alpha\beta} = \nabla_b^1 A_c^1 \otimes \nabla_b^2 A_c^2 - \nabla_c^2 A_b^2 \otimes \nabla_c^1 A_b^1 = (F_1 \otimes F_2 - F_2 \otimes F_1), \tag{16}$$

precisely is our tensor algebra given in the *proposition 3. 1.*, with their conserved Lie structure.

The essential difference between both versions consists in the coupling to a charged $spin0$, scalar field, that in this case is a scalar magnetic field corresponding to a magnetic flow associated to the supercurrent J_s.

Considering the supercurrent J_s in presence of magnetic field of vector potential, this takes the form

$$J_s = \frac{e^*\hbar}{2m^*i}(\psi^*\nabla\psi - \psi\nabla\psi^*) - \frac{(e^*)^2}{m^*c}|\psi|^2 A, \tag{17}$$

where ψ, is a function very general of complex type that are changing spatially and that in an any point this function depends of the order parameter (as coherent length, penetration length, etc parameters that are useful to characterize a superconductor [11]) and $|\psi|^2 = n_s$, is the density of the superconducting electrons.

Considering the action (10) and the proper to the Lagrangian (11), the electromagnetic total action to all electromagnetic phenomena (included the given in ordinary electromagnetism) using the scalar field theory takes the form:

$$\begin{aligned} S_\xi = \int \varphi^*[-\nabla_a\nabla^a - m^2]\varphi + \frac{1}{2}A_a[g^{ab} - (1-\frac{1}{\xi})\nabla^a\nabla^b]A_b \\ + ieA_a((\nabla^a\varphi^*)\varphi - \varphi^*\nabla^a\varphi) + e^2 A_a A^a \varphi^*\varphi, \end{aligned} \tag{18}$$

in where and under certain physical conditions of symmetries we can establish the following relations between the complex functions ψ, and ψ^*, and their covariant derivatives $\nabla\psi$, $\nabla\psi^*$, and the proper to the scalar fields given by φ, $(\nabla^a\varphi^*)$, φ^*, and $\nabla^a\varphi$ obtaining the pure action to superconductors given by the integral:

$$S_s = \int ieA_a(\psi^*\nabla\psi - \psi\nabla\psi^*) + e^2 A_a A^a \nabla\psi^*\nabla\psi, \tag{19}$$

which appears in the superconductor under energy regime given by their corresponding Hamiltonian defined in (A. 12) of the *appendix A*. The representation spaces that appear are the Fock states spaces and are corresponded in the superconductivity with the photon states spaces where there exist the interaction *photon-phonon-photon* [12] under frame study of the microscopic theory of the superconductivity. In a second affirmation, is necessary consider that electrons in superconductivity are moving in a very special enthrone, that is the crystalline net formed by ions that constitute the solid which we want that be superconductor under the application of field actions as given in (19) on the solid of object O.

We consider a particles system, all identical, that is to say, undistinguishable, that is to say, the interchange of two of them not change the measurable properties of the system. Let e_1, and e_2, two electrons and we suppose that the electron e_1, is in a state that comes represented by the function ψ_n, (*wave function*), for other side, the electron e_2, comes represented by the wave function ψ_m, and we suppose for last that the direct interaction between these two electrons is quasi-vanishing. Then we can describe the system of these two electrons by the wave function $\psi_n(e_1)\psi_m(e_2)$. Remember that the two particles are identical, thus the interchange of the two let us equal the system. Then also is valid as before that $\psi_n(e_2)\psi_m(e_1)$. Then the total wave function with the interchanges realized by the two electrons takes the form:

$$\Psi = \frac{1}{\sqrt{2}}[\psi_n(e_1)\psi_m(e_2) + \psi_n(e_2)\psi_m(e_1)], \tag{20}$$

and their conjugated

$$\Psi^* = \frac{1}{\sqrt{2}}[\psi_n(e_1)\psi_m(e_2) - \psi_n(e_2)\psi_m(e_1)], \tag{21}$$

But if the two states are the same state, then only $\Psi \neq 0$, since $\Psi^* = 0$. Then the system is anti-symmetric in the interchange of two particles, that is to say, the wave function is anti-symmetric under the interchange of electron coordinates. But by the *Pauli Exclusion Principle* the situation described in the total wave function Ψ, is incorrect, being the correct by Ψ^*.

But the before help us to establish the anti-symmetric structure of the interaction between pair of particles in the microscopy superconductivity theory and reflected this anti-symmetry property also in every *spin-orbit interaction* of every wave function to the two pair electrons that satisfy the total wave function Ψ^*.

Considering to an electron field, a representation $\xi : \mathfrak{E} \rightarrow V$, where V, , is a Hilbert space and whose correspondence rule is

$$e \mapsto \xi(e), \tag{22}$$

and let J, [9]the two-sided ideal in the tensor algebra defined in the section 2. 1, $(\mathfrak{E}, \otimes)$, generated by the elements of the form $e_1 \otimes e_2 - e_2 \otimes e_1$, where $e_1, e_2 \in \mathfrak{E}$.

Proposition 2. 2. 1. There is a natural one-to-one correspondence between the set of all representations of $\mathfrak{E}$, on V, and the set of all representations of $\mathfrak{E} \otimes \mathfrak{E} / J$, on V.. If ξ, is a representation of $\mathfrak{E}$, on V, and , ξ^*, is a representation of $\mathfrak{E} \otimes \mathfrak{E} / J$, on V, then

$$\xi(e) = \xi^*(e^*), \qquad \forall\, e \in \mathfrak{E}, \tag{23}$$

[9]Remember that J, from a point of view of the superconductors is a topological current associated with the topological charge defined related with the magnetic flux carried by the fluxoinds.

Proof. Let ξ, be a representation of $\mathfrak{E}$, on V. Then there exists a unique representation $\bar{\xi}$, of $(\mathfrak{E}, \otimes)$, on V , satisfying that $\bar{\xi}(e) = \xi(e), \quad \forall\, e \in \mathfrak{E}$. Then mapping $\bar{\xi}$,, vanishes on the ideal J,, became

$$\bar{\xi}(e_1 \otimes e_2 - e_2 \otimes e_1 - [e_1, e_2]) = \xi(e_1)\xi(e_2) - \xi(e_2)\xi(e_1) - \xi([e_1, e_2]) = 0, \tag{24}$$

Thus we can define a representation ξ^*, of factor algebra $\mathfrak{E} \otimes \mathfrak{E} / J$, on V , by the condition $\xi^* \circ \pi = \bar{\xi}$. [10]Then (23) is satisfied and determines ξ^*, uniquely. Also is unique, indeed suppose other representation, for example σ , $\mathfrak{E} \otimes \mathfrak{E} / J$ of on V . If $e \in \mathfrak{E}$, we put $\xi(e) = \sigma(e^*)$. Then the mapping $e \mapsto \xi(e)$, is linear and is a representation of $\mathfrak{E}$ on V , since

$$\begin{aligned} \xi([e_1, e_2]) &= \sigma([e_1, e_2]^*) = \sigma(\pi(e_1 \otimes e_2 - e_2 \otimes e_1)) \\ &= \sigma(e_1^* e_2^* - e_2^* e_1^*) = \xi(e_1)\xi(e_2) - \xi(e_2)\xi(e_1). \square \end{aligned}$$

Considering in particular the representation given for Ψ^*, [11]we have that Lie-QED-algebra structure to the spin and orbits is conserved. Indeed the spin and orbit parts can be separated in each wave function as for example, from (24) we have that if $\psi(e_1) = \phi_n(r_1)\alpha(s_1)$, and $\psi(e_2) = \phi_m(r_2)\alpha(s_2)$, then (24) defines Ψ^* . Remember that the electron is a *fermion* and the electrons are described for the anti-symmetric wave functions. This property will be fundamental in the section relative to the construction of the fermionic Foch space corresponding to the Lie-QED-algebra.

Def. 2. 2. 1. [10**]. A** $\mathfrak{E} \otimes \mathfrak{H}$ - field is an element of a bi-sided ideal of the Maxwell fields [1, 6]. Explicitly is the formal space

$$\mathfrak{E} \otimes \mathfrak{H} = \{(F_1, F_2) \in \Omega^2(O) \times \Omega^2(O) \big| F_1 \otimes F_2 - F_2 \otimes F_1 - [F_1, F_2], \text{with } \otimes = \otimes_{\mathbb{R}} \}, \tag{25}$$

Before of this, we pass to the fundamental lemma to characterize the algebra $\mathfrak{E} \otimes \mathfrak{H}$, as the fundamental algebra of all movements and electromagnetic phenomena (*for example, magnetic levitation, electromagnetic matter condensation, Eddy currents, etc*) produced to

---

[10] Its realized the following descend map π, from $\bar{\xi} : \mathfrak{E} \to \mathfrak{E} \otimes \mathfrak{E}$, to $\xi^* : \mathfrak{E} \to \mathfrak{E} \otimes \mathfrak{E} / J$.

[11]It is the Slater determinant (that helps to construct wave functions to start of the expressions $\phi_n(r_1)\alpha(s_1)\phi_m(r_2)\beta(s_2)$ where $r_i, s_i (i = 1, 2)$ are the radius of the orbit and spin respectively) , for example:

$$\Psi^* = \frac{1}{\sqrt{2}} \begin{vmatrix} \phi_n(r_1)\alpha(s_1) & \phi_n(r_2)\alpha(s_2) \\ \phi_m(r_1)\alpha(s_1) & \phi_m(r_2)\alpha(s_2) \end{vmatrix} = \frac{1}{\sqrt{2}} [\phi_n(r_1)\alpha(s_1)\phi_m(r_2)\alpha(s_2) -$$
$$\phi_m(r_1)\alpha(s_1)\phi_n(r_2)\alpha(s_2)]$$

quantum level by their electromagnetic fields satisfying the variation principle in their field actions.

Lemma (F. Bulnes) [10] 2. 2. 1. All electromagnetic actions and their effects (microscopic and macroscopic) on the superconductor object O, comes from the $\mathfrak{E}\otimes\mathfrak{H}$- fields.

Proof. [10]. □

One important fact inside the demonstration of the lemma 2. 2. 1, was consider the bi-sided ideal given by the space (25) whose actions are extended to all space from the superconductor O, until the infinite (ambient of O) through the gauge transformations used by the Lie-QED-algebra . Then by the lemma A. 1 (F. Bulnes), given in the *appendix A*, of this work [9], the quantum effects underlying in superconducting phenomena satisfies that

$$H(A,\Im_M) = \Im_M - H(A,B) = \int_O \left[F_i F_k - \frac{1}{2} H^2 \delta_{ik} \right] dV / 4\pi$$
$$= \int_O L_M(x(s)) d(x(s)) - \int_O H^2 / 8\pi dV,$$

where $H^2/8\pi$, is the free magnetic energy and the integral of the Lagrangian, of the expulsion by the action , and that is useful to establish the macroscopic wave functions that give place to a microscopic quantum current Js. By the mathematical electrodynamics [3] we can define using the structure of $\mathfrak{E}\otimes\mathfrak{H}$, (that is to say, a module of the exterior algebra which is deduced by the universal map applied to each term of the element $F_1\otimes F_2 - F_2\otimes F_1 - [F_1, F_2]$) that:

$$(\psi, J) = \int_{\partial O} (\psi^* \wedge J \text{ - } J \wedge \psi^*) dS - \int_{\partial O} \langle \psi, \psi^* \rangle dS, \tag{26}$$

where $J = \nabla\psi$, , and $\psi^* = conj(\psi)$, , such that $|\psi|^2 = \langle \psi, \psi^* \rangle$, which is (19) in the supercurrents modality. As the last integral (18) measures effects due to the macroscopic actions to level quantum, this proved the affirmation of the lemma 2. 2. 1, in microscopic theory of superconductivity and also the macroscopic effects due to the Eddy currents must satisfy the magnetic force equation [9] to magnetic levitation.

2.3. Photon spin algebra from $\mathfrak{E}\otimes\mathfrak{H}$

The same Lie structure is conserved to the electromagnetic spin algebra. The Lie structure of the macroscopic level given and demonstrated to the space $\mathfrak{E}\otimes\mathfrak{H}$, in the before subsection

2.1, and using after through the path integral quantization on Lie bracket $[e_1, e_2]$, given in the *subsection 2. 2* can establish a version of this quantized Lie algebra to quantum spin number (parts $\alpha(s_i)$) associated to the photons that interact in the superconducting phenomena on the quantum macroscopic effects generated by the superconducting currents (for quantized electromagnetic fields). Then the QED- algebra obtained conserves the Lie structure to spin electromagnetic operators.

The photon can be assigned a triplet spin with spin quantum number $s = 1$, accord to the classification of particles and their spin. This is similar to, say, the nuclear spin of the N isotope, but with the important difference that the state with $M_s = 0$, is zero, only the states with $M_s = \pm 1$, are non-zero. We consider the electro-spin operator as the vector with their Pauli matrices associated[12]:

$$S_k = -i\hbar\varepsilon_{ijk} = \frac{\hbar}{2}\sigma_k, (k = 1,2,3), \tag{27}$$

Then we can define the analogous of $\mathbb{E} \otimes \mathbb{H}$ to the quantum spin context as the algebra:

$$\mathfrak{e} \otimes \mathfrak{h} = \left\{ S_k \in \middle| [S_i, S_j] = -i\hbar\varepsilon_{ijk} \right\}, \tag{28}$$

which is closed under the bracket $[,]_\otimes$, operation . Indeed, we consider two elements $S_i, S_j \in \mathfrak{e} \otimes \mathfrak{h}$, given by the relations

$$S_i = -i\hbar(\varepsilon_j \otimes \varepsilon_k - \varepsilon_k \otimes \varepsilon_j), \qquad S_j = -i\hbar(\varepsilon_k \otimes \varepsilon_i - \varepsilon_i \otimes \varepsilon_k), \tag{29}$$

satisfying the cyclically rule $i \to j \to k \to i$. Then their operation under $[,]$, is

$$\begin{aligned}
&[S_i, S_j] = -\hbar^2(\varepsilon_j \otimes \varepsilon_k - \varepsilon_k \otimes \varepsilon_j)(\varepsilon_k \otimes \varepsilon_i - \varepsilon_i \otimes \varepsilon_k) + \\
&\hbar^2(\varepsilon_k \otimes \varepsilon_i - \varepsilon_i \otimes \varepsilon_k)(\varepsilon_j \otimes \varepsilon_k - \varepsilon_k \otimes \varepsilon_j) = i\hbar\left[-i\hbar(\varepsilon_i \otimes \varepsilon_j - \varepsilon_j \otimes \varepsilon_i)\right] = i\hbar S_k \in \mathfrak{e} \otimes \mathfrak{h}.
\end{aligned}$$

For simple inspection it follows that

$$-i\hbar(\varepsilon_i \otimes \varepsilon_j - \varepsilon_j \otimes \varepsilon_i) \bullet \varepsilon^{(\mu)} = \mu\varepsilon^{(\mu)}, \tag{30}$$

---

[12]In the special case of spin $-1/2$ particles, σ_x, σ_y, and σ_z, are the three Pauli matrices given by:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

with $\mu = +1$ or -1, and therefore labels the photon spin, $S_z|\mathbf{k},\mu\rangle = \mu|\mathbf{k},\mu\rangle$, with $\mu = +1$ or -1. Then due the vector potential is a transverse field the photon has no forward ($\mu = 0$) spin component.

3. Fermionic Fock spaces of the superconductivity

By the BSC-theory we have a Cooper pair is a magnitude whose spin is zero. But spin zero are bosons, then is easy fall in the temptation of treat a Cooper pairs as bosons. Furthermore, we have indicated that few many more Cooper pairs, better energy will be the process of superconductivity. However, the Pauli principle remains in force and the state formed, for example by $(k\uparrow, -k\downarrow)$, cannot be occupied for more than a pair of electrons at the same time (see figure 3).

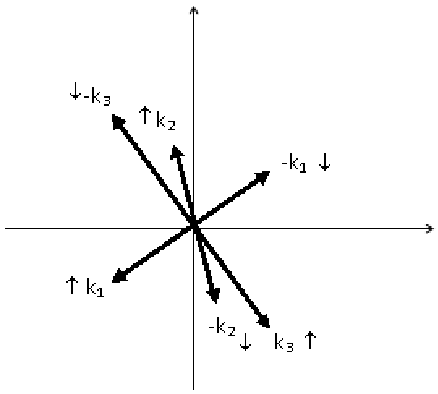

Figure 3. BSC fundamental state with three Cooper pairs.

Also the Hamiltonian in the BSC-theory is constructed by operators (that is to say, their formalism with that is calculated the energy of the *fundamental superconductor state are anti-commutative*) follows anti-commutative rules as was discussed in the introduction of this paper. But involve a boson that is created an interaction between fermions[13]. The wave function such and as is proposed by the BSC-theory to Ξ, electrons (*foreseen in* (21) *to the case of only pairs of electrons*) is the product of wave functions of pair conveniently anti-symmetrized, that is to say:

$$\psi(1,2,\ldots,\Xi) \approx \psi(1,2)\psi(2,3)\cdots\psi(1-\Xi,\Xi), \tag{31}$$

If we not write explicitly the part of spin and only we do with the orbital part we have:

$$\psi(1,2,\ldots,\Xi) \approx \sum_{k_1}\sum_{k_2}\cdots\sum_{k_{\Xi/2}}\zeta_{k_1}\cdots\zeta_{k_{\Xi/2}}e^{i(k_1r_1-k_2r_2+\ldots+k_{\Xi/2}r_{\Xi-1}-k_{\Xi/2}r_{\Xi})}, \tag{32}$$

[13]To difference of the London superconductivity where a charged gas of bosons produces naturally a Meissner effect.

where each term of this wave function describes a configuration where the $\Xi,$ electrons is grouped in $\Xi/2,$ pairs that are:

$$(k_1,-k_1)\cdots(k_{\Xi/2},-k_{\Xi/2}), \tag{33}$$

The spin part is immediate, each electron of each pair haves opposite spines. The wave function is a complicated function that covers all the related pairs between them. This takes the form from excited states are obtained as linear combinations of the ground state excited by some creation operators $\prod_{k=1}^{n} a_{i_k}^{\dagger} \left|0\right\rangle = 0$,to the wave functions as:

$$\psi = \prod_k \psi_k, \tag{34}$$

Using the second quantizing formalism to the Fock space in *appendix B,,* we have that the potential energy to said pairs is:

$$V = \sum_{k,k'} V_{k,k'} b_k^{\dagger} b_k, \tag{35}$$

The term of kinetic energy of the corresponding Hamiltonian considering the energy in the Fermi level is:

$$E = \sum_{k,k'} 2\varepsilon_k b_k^{\dagger} b_k + \Delta b_k^{\dagger} b, \tag{36}$$

where to the states of exited electrons (super-electrons) appear the trenches (to break the pairs and get to superconductivity peak state (see the figure 4A) and 4B)).

The fermionic Fock space is (B. 1) where for second approximation we have

$$\mathcal{F} = \mathcal{H}_0 \oplus \mathcal{H}_1 \oplus \mathcal{H}_2, \tag{37}$$

whose energies of the electron are $\varepsilon_k,$ and $\varepsilon_{k-q},$ with momentums to two electrons in superconducting states $hk,$ and $h(k-q)$ respectively. This interaction is negative since is attractive, Then the potential is:

$$V(k,k',q) =, \frac{A^2 \hbar\omega_q}{(\varepsilon_k - \varepsilon_{k-q})^2 - (\hbar\omega_q)^2}, \tag{38}$$

where A, is a coupling electron-fonon. Then

$$\left|\varepsilon_{k\pm q} - \varepsilon_k\right| < \hbar\omega_q, \tag{39}$$

Then after of realize some calculations in the fermionic Fock space is had that

$$\varepsilon_{\mathcal{F}} = (\Delta^2 + \varepsilon_k{}^2)^{1/2}, \tag{40}$$

where Δ, is the minimum energy of excitation, that is to say, the value of energy trenches that appears in the superconductor state (*see the* Figure 11A, *in the last section*). *These trenches* have a big variation with respect to the absolute temperature of the material. The energy ε_k, is the quasi-particle energy, that is to say, the energy of the holes of the fermions distribution when happen the excited states. The trenches energy as the excited states shape the orthogonal space of infinite dimension separated by the Fermi momentum.

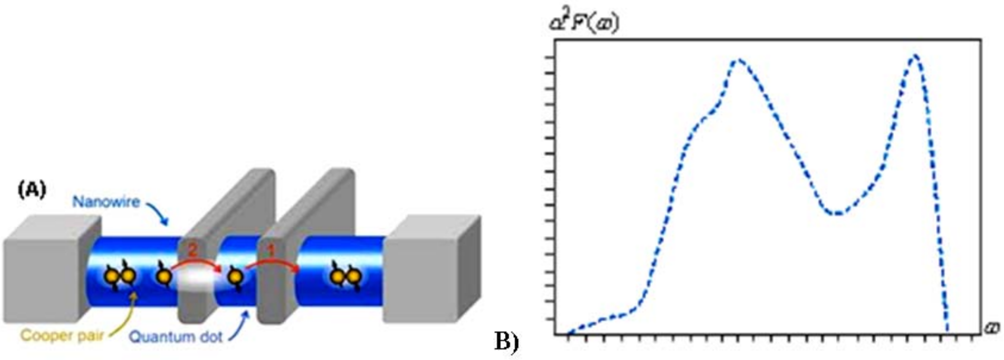

Figure 4. A) Nano-wire device to break Cooper pairs. The Cooper pairs must be break to obtain the maximum superconductor state. The super-electrons are transformed in Fermi liquid which established the required transformation of the immediate region of the space-time where must be executive to transformation due superconductivity [2, 13, 14, 15]. B). Spectral density of electron-phonon.

Indeed, consider a system of fermions with an one-body Hamiltonian of the form (accord to (32) and (34)):

$$\hat{H} = \sum_k \varepsilon_k \hat{a}_k^\dagger \hat{a}_k + E_0, \tag{41}$$

When all particle energies ε_k, are positive, the ground state of the system is the vacuum state $\left|\text{vac}\right\rangle$, with all $n_k = 0$. In terms of the creation and annihilation operators said state $\left|\text{vac}\right\rangle$, can be identified as the unique state killed by the the annihilation operators, that is to say, $\hat{a}_k\left|\text{vac}\right\rangle = 0, \forall k$. The excited states of the Hamiltonian (50) are $\Xi-$ particle states which obtain by applying creation operators to the vacuum, $\left|k_1,\ldots,k_\Xi\right\rangle = \hat{a}_{k_\Xi}^\dagger \cdots \hat{a}_{k_1}^\dagger \left|\text{vac}\right\rangle$, the energy of such a state is $E = E_0 + \varepsilon_{k_1} + \ldots + \varepsilon_{k_\Xi} > E_0$.

Now suppose for a moment that all the particle energies ε_k, are negative instead of positive. In this case, adding particles decreases the energy, so the ground state of the system is not the vacuum but rather the full-to-capacity state

$$\left|\text{full}\right\rangle = \left|\text{all } n_k = 1\right\rangle \prod_{\text{all } k} \hat{a}_k^\dagger \left|\text{vac}\right\rangle, \tag{42}$$

with energy

$$E_{\text{full}} = E_0 + \sum_{\text{full } k} \varepsilon_k, \tag{43}$$

Never mind whether the sum here is convergent; if it is not, we may add an infinite constant to the E_0, cancel the divergence. What's important to us here are the energy difference between this ground and the excited states.

The excited states of the system are not completely full but have a few *holes*. If we consider $n_{k_1} = \ldots = n_{k_\Xi} = 0$, for some $\Xi -$ modes $(k_1, \ldots, k_\Xi)$, while all the other $n_l = 1$. The energy of such a state is

$$E = E_0 + \sum_{l \neq k_1, \ldots, k_\Xi} \varepsilon_l = E_{\text{full}} - \sum_{i=1}^{\Xi} \varepsilon_{k_i} > E_{\text{full}}, \tag{44}$$

In other words, an un-filled hole in mode k, carries a positive energy $-\varepsilon_k$.

In terms of the operator algebra, the $\left|\text{full}\right\rangle$, state is the unique killed by all the creation operators, $\hat{a}_k^\dagger \left|\text{full}\right\rangle, \forall k$. The holes can be obtained by acting on the $\left|\text{full}\right\rangle$, state with the annihilation operators that remove one particle at a time. Thus,

$$\left|\text{1hole at } k\right\rangle = \left|\hat{n}_k \text{ other } n = 1\right\rangle = \hat{a}_k \left|\text{full}\right\rangle, \tag{45}$$

and likewise

$$\left|N \text{ holes at } k_1, k_2 \ldots, k_\Xi\right\rangle = \hat{a}_{k_\Xi} \cdots \hat{a}_{k_1} \left|\text{full}\right\rangle, \tag{46}$$

Altogether, when the ground state is $\left|\text{full}\right\rangle$, the creation and annihilation operators Exchange their roles. Indeed, the $\hat{a}_k$, make extra holes in the full or almost-full states, while the $\hat{a}_k^\dagger$, operators annihilate those holes (by filling them up). Also the algebraic definition of the $\left|\text{full}\right\rangle$, and $\left|\text{vac}\right\rangle$, states are related by the exchange: $\hat{a}_k \left|\text{vac}\right\rangle = 0, \forall k, \text{vs } \hat{a}_k^\dagger \left|\text{full}\right\rangle, \forall k.$

To make this exchange manifest, let us define a new family of fermionic creation and annihilation operators, to know,

$$\hat{b}_k = \hat{a}_k^\dagger, \qquad \hat{b}_k^\dagger = \hat{a}_k, \tag{47}$$

Unlike the bosonic commutation relations, the fermionic anti-commutation are symmetric between $\hat{a}, \hat{a}^\dagger$, so the $\hat{b}, \hat{b}^\dagger$, satisfy exactly the same anti-commutation relations as the $\hat{a}_k$, and $\hat{a}_k^\dagger$,

$$\{\hat{b}_k, \hat{b}_l\} = 0, \{\hat{b}_k^\dagger, \hat{b}_l^\dagger\} = 0, \{\hat{b}_k, \hat{b}_l^\dagger\} = \delta_{k,l}, \tag{48}$$

Physically, the $\hat{b}_k^\dagger$, operators *create holes* while the $\hat{b}_k$, operators *annihilate holes* and the holes obey exactly the same Fermi statistics (as given in the Figure 4B) as the original particles. In condensed-matter terminology, the holes are *quasi-particles*, but the only distinction between the quasi-particles and true particles is that the later may exist outside the condensed matter. When viewed from the inside of condensed matter, this distinction becomes irrelevant.

Anyhow, from the hole point of view, the $\left|\text{full}\right\rangle$, state is the *hole vacuum* which is the unique state with no holes at all, algebraically defined by $\hat{b}_k\left|\text{full}\right\rangle = 0, \forall k$. The excitations are $\Xi-$ hole states obtained by acting with *hole-creation operators* $\hat{b}_k^\dagger$, on the hole-vacuum, $\left|\text{ holes at } k_1, k_2 \ldots, k_\Xi\right\rangle = b_{k_\Xi}^\dagger \cdots b_{k_1}^\dagger \left|\text{full}\right\rangle$. Then the Hamiltonian operator (32) of the system becomes[14]

$$\hat{H} = E_0 + \sum_k \varepsilon_k (1 - \hat{b}_k^\dagger \hat{b}_k) = E_{\text{full}} + \sum_k (-\varepsilon_k) \hat{b}_k^\dagger \hat{b}_k, \tag{49}$$

in accordance with individual holes having positive energies $-\varepsilon_k > 0$.

The k, modes are eigenspaces of some conserved quantum numbers such as momentum or spin (or rather $\hat{S}_z$). When one makes a hole by removing a particle from mode $(\mathbf{p}, s)$, the net momentum of the system changes by $-\mathbf{p}$, while the net S_z, changes by $-s$, so one can say that the hole in that mode has momentum $-\mathbf{p}$, and $S_z = -s$. Consequently the hole operators are usually defined as

$$\hat{b}_{\mathbf{p},s} = \hat{a}^\dagger_{-\mathbf{p},-s}, \qquad \hat{b}^\dagger_{\mathbf{p},s} = \hat{a}_{-\mathbf{p},-s}, \tag{50}$$

[14] $\hat{a}_k^\dagger \hat{a}_k = \hat{b}_k \hat{b}_k^\dagger = (1 - \hat{b}_k^\dagger \hat{b}_k)$.

which leads to

$$\hat{\mathbf{P}}_{\text{Tot}} = \mathbf{P}_{\text{full}} + \sum_{\mathbf{p},s} \mathbf{p} \times \hat{b}^{\dagger}_{\mathbf{p},s} \hat{b}_{\mathbf{p},s}, \tag{51}$$

and likewise

$$\hat{S}^{z}_{\text{Tot}} = S^{z}_{\text{full}} + \sum_{\mathbf{p},s} s \times \hat{b}^{\dagger}_{\mathbf{p},s} \hat{b}_{\mathbf{p},s}, \tag{52}$$

Finally, consider a system where the energies ε_k, take both signs: positive for some modes k, but negative for other modes. For example, a free fermion gas with a positive chemical potential μ, and free-energy operator

$$\hat{H} = \sum_{\mathbf{p},s} (\frac{\mathbf{p}^2}{2m} - \mu) \hat{a}^{\dagger}_{\mathbf{p},s} \hat{a}_{\mathbf{p},s}, \tag{53}$$

has positive ε, for $|\mathbf{p}| > p_f$, where p_f, is the Fermi momentum defined by the threshold $\frac{p_f^2}{2m} - \mu = 0$ (see figure 4). For this system the ground state is the *Fermi sea* where

$$n_{\mathbf{p},s} = \Theta(|\mathbf{p}| < p_f) = \begin{cases} 1 & \text{for } |\mathbf{p}| < p_f \\ 0 & \text{for } |\mathbf{p}| > p_f \end{cases}, \tag{54}$$

In terms of the creation and annihilation operators, the Fermi sea is the state (directly from $\prod_{k=1}^{n} a^{\dagger}_{i_k} |0\rangle = 0$):

$$|\text{Fermi sea}\rangle = \prod_{\mathbf{p},s}^{|\mathbf{p}|<p_f \text{ only}} \hat{a}^{\dagger}_{\mathbf{p},s} |\text{vac}\rangle, \tag{55}$$

which satisfies

$$\hat{a}_{\mathbf{p},s} |\text{Fermi sea}\rangle = 0, \tag{56}$$

for $|\mathbf{p}| > p_f$, and

$$\hat{a}^{\dagger}_{\mathbf{p},s} |\text{Fermi sea}\rangle = 0, \tag{57}$$

for $|\mathbf{p}| < p_f$.

We may treat this state as a quasi-particle vacuum if we re-define all the operators killing the $|\text{Fermi sea}\rangle$, as annihilation operators. Thus we define new (59) for $|\mathbf{p}| < p_f$. only. But keep the original operators $\hat{a}_{\mathbf{p},s}$, and $\hat{a}^{\dagger}_{\mathbf{p},s}$ acting with momentum outside the Fermi surface. Despite the partial exchange the complete set or creation and annihilation operators satisfies the fermionic anticommutation relations having:

$$\begin{aligned} &\text{all}\{\hat{a},\hat{a}\} = \{\hat{b},\hat{b}\} = \{\hat{a},\hat{b}\} = 0, \\ &\text{all}\{\hat{a}^{\dagger},\hat{a}^{\dagger}\} = \{\hat{b}^{\dagger},\hat{b}^{\dagger}\} = \{\hat{a}^{\dagger},\hat{b}^{\dagger}\} = 0, \\ &\text{all}\{\hat{a},\hat{b}^{\dagger}\} = \{\hat{b}^{\dagger},\hat{a}^{\dagger}\} = 0, \end{aligned} \tag{58}$$

and also

$$\{\hat{a}_{\mathbf{p},s},\hat{a}^{\dagger}_{\mathbf{p},s}\} = \delta_{\mathbf{p},\mathbf{p}'}\delta_{s,s'}, \qquad \{\hat{b}_{\mathbf{p},s},\hat{b}^{\dagger}_{\mathbf{p},s}\} = \delta_{\mathbf{p},\mathbf{p}'}\delta_{s,s'}, \tag{59}$$

if we restrict the $\hat{b}_{\mathbf{p},s}$, and $\hat{b}^{\dagger}_{\mathbf{p},s}$, to $|\mathbf{p}| < p_f$, only and the $\hat{a}_{\mathbf{p},s}$, and $\hat{a}^{\dagger}_{\mathbf{p},s}$, to $|\mathbf{p}| > p_f$.

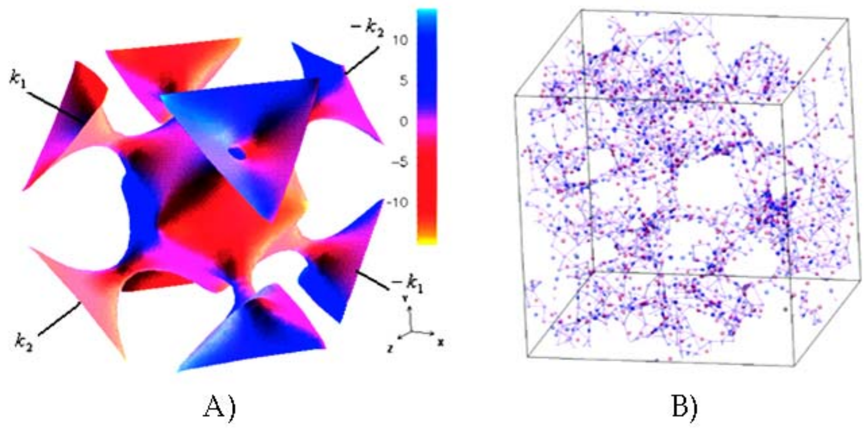

Figure 5. A). Fermi surface to the gold (Au). All Au-quasi-particles must shape this surface with the number of pairs corresponding to the metal to the superconducting state. This surface in the BCS-theory shapes the quantum nucleus of the interaction electron-fonon-electron corresponding to the fermionic Fock superconducting space [16, 17].. B). Fermionic Fock superconducting space conformed with for Cooper pairs: (red particle $k\uparrow$, blue particle $-k\downarrow$). The net is obtained by the adding of quantum Hilbert spaces respectively.

The Fermi sea $|\text{Fermi sea}\rangle$, is the quasi-particle vacuum state of these fermionic operators. The two types of creation operators $\hat{a}^{\dagger}_{\mathbf{p},s}$, and $\hat{b}^{\dagger}_{\mathbf{p},s}$, create two distinct types of quasi-particles (*respectively the extra fermions above the Fermi surface and the holes below the surface*).

Both types of quasi-*particles* have positive energies. Then in terms of our new fermionic operators, the Hamiltonian takes the form:

$$\hat{H} = E_{\text{Fermi Surface}} + \sum_{\mathbf{p},s} (\frac{\mathbf{p}^2}{2m} - \mu > 0) \times \hat{a}^{\dagger}_{\mathbf{p},s} \hat{a}_{\mathbf{p},s} + \sum_{\mathbf{p},s} (\mu - \frac{\mathbf{p}^2}{2m} > 0) \times \hat{b}^{\dagger}_{\mathbf{p},s} \hat{b}_{\mathbf{p},s}, \tag{60}$$

with the domains to every sum $|\mathbf{p}| > p_f$, only (in the first sum) and $|\mathbf{p}| > p_f$, only (in the second sum).

Clearly (60) describes elements of the fermionic Fock space given in (37).

4. Fermionic C*-Lie-QED-algebra

Theorem (F. Bulnes) 5. 1. The *electro-anti-gravitational effect*s produced from superconductivity have that to be governed by the actions of the *superconducting Lie*-QED-*algebra* $\mathfrak{E} \otimes \mathfrak{H}$.

To demonstrate the before result is necessary to define the electro-anti-gravity in the formalism of the Lie-QED-algebra and their C*-algebras associated to her. The electro-anti-gravity is obtained through of experiments where a fast rotating superconductor reduces the gravitational effect. Of fact the rotation is fundamental and necessary to the complementing of the anti-gravity effects searched through the magnetic levitation (see the Figure 6. where were realized many experiments with rotating geometrical pin, using the high intense magnetic field).

Figure 6. Magnetic levitation by fast rotating magnet.

The demostration of this theorem has been realized in part, by the lemma 2. 2. 1. However we require other additional lemmas that have that to see with other aspects, as the iso-rotations (condition established and illustrated in the experiments realized in the Figure 6) and the condensation effects of the matter required in the transmission process through a "bosonic cloud" of the plasmons in the quantum transmission of the electro-anti-gravity effect. The concluyent aspect in this last digression is that the rotation movement to very fast velocity and the superconducting phenomena must be togheter and due to the Lie structure of our QED-algebra, this rotations will be inside the $\mathfrak{E} \otimes \mathfrak{H}$ - fields as images of orthogonal transformations of the special orthogonal group $SO(2)$, [15]and their topological essency let to see the inherent geometrical properties to an application of our superconductor electro-fields as was demostrated in the following lemma [9] and mentioned in [18]:

Lemma (F. Bulnes) 5. 1. Let $\mathfrak{S} = G^{\mathbb{C}} / C(T)$, be with $C(T)$, a space of orbits (*hypersurfaces*), generated by the electro-fields on O, by the realization of movements given for $SU(2)$, through of the action of their Maxwell fields F, given by $F = (H_i H_k - \frac{1}{2} H^2 \delta_{ik}) / 4\pi\sigma\mu$, in the superconductor. Then the orbits engendered by the actions $\Im_M$, on M, are magnetic torus engendered by rotations $SO(2)x(s), \forall x \in M$, generates by fluxoids Φ_0, in the vortex zone [8].

Proof. [9]. □

Then an analogous to QED of the fields F, will have that consider in the states generated in a Fock space $\mathcal{F}$, the corresponding transformation of a subgroup of $O(n)$, that is to say, the automorphism of the group must act on fermionic states of the space, where the electro-anti-gravity comes established to change of spin-orbit from $M_s = +1$, to $M_s = -1$, (or viceversa), in a bose-Einstein distribution in the matter condensation phenomena to produce an electro-anti-garavity wrapping of the object O .

One important fact is that there exist orthogonal invariance of the CAR-algebra on a Fock space, that is to say, the Fermionic Fock space is invariant under rotations, that is tosay, $O(n)\psi = \psi, \forall \psi \in \mathcal{F}$, where explicitely the orthogonal group is:

$$O(n,K) = \{Q \in GL(n,K) \big| Q^T Q = QQ^T = I\}, \tag{61}$$

If we consider the subgroup $SO(2)$, of $O(2,K)$, we have that the group $U(1)$, of the 2-forms F^{ab}, satisfy:

$$U(1) = Spin(2) = SO(2), \tag{62}$$

[15] $SO(2) = \left\{ A \in GL_2 \middle| A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \forall \theta \in [0, 2\pi] \right\}.$

having the considered in the section 2. 3. Then all particle represented for their energy (by their wave function ψ) can change their behavior using a gauge group as $U(1)$, , or $SU(2)$. This last enclose the all electromagnetic phenomena around of the superconductivity that we want cover. Remember that we required to obtain anti-gravity from the $\mathfrak{E} \otimes \mathfrak{H}$ - fields of our superconducting Lie-QED-algebra.

Then considering two elements of the group $SO(2)$, , for example $e_1, e_2 \in \mathfrak{E} \otimes \mathfrak{H}$, the representation fulfils (by proposition 2. 2. 1) is

$$\zeta(e_1)\zeta(e_2) - \zeta(e_2)\zeta(e_1) = \zeta(e_1 \otimes e_2 - e_2 \otimes e_1), \tag{63}$$

and the field is transformed as

$$\Psi \mapsto \Psi', \tag{64}$$

where explicitly the image $\Psi' = \zeta(J_{\alpha\beta})\Psi$. From this always is possible construct a second representation defined by:

$$\zeta^*(J_{\alpha\beta}) = \zeta((J_{\alpha\beta}{}^T)^{-1}), \tag{65}$$

which belongs to the charge-conjugated particle. The anti-particle is obtained of accord to the contragradient $\bar{\zeta}$, representation, which is:

$$\bar{\zeta}(J_{\alpha\beta}) = \zeta(J_{\alpha\beta}^{-1}), \tag{66}$$

There are not charge-conjugated in gravity, since if the gauge group is Lorentz group $SO(3,1)$, then elements $J_{\alpha\beta}^{-1} = J_{\alpha\beta}^{T}$, , which means that the second representation ζ^*, is equivalent to ζ .

But we need affect the immediate space-time at least locally through of these $\mathfrak{E} \otimes \mathfrak{H}$ - fields, such that we will have the anti-particles given in (75). Also we need a mapping that involves and include in their image the spin connection that is involved in this anti-gravity process from superconductivity.

We define the field Ψ , as a vector field whose application is as given in (64)

$$\Psi' = \Gamma\Psi, \qquad \bar{\Psi}' = \bar{\Psi}\Gamma^{-1}, \tag{67}$$

under a general diffeomorphism Γ, that is to say, the mapping belonging to the space $Diff(TM, TM^*)$, where TM, is the dual to T^*M. But we required local transformations at least in the immediate enthrone of object O, such that be anti-gravitational and this local enthrone acts with the space-time to create levitation in O.

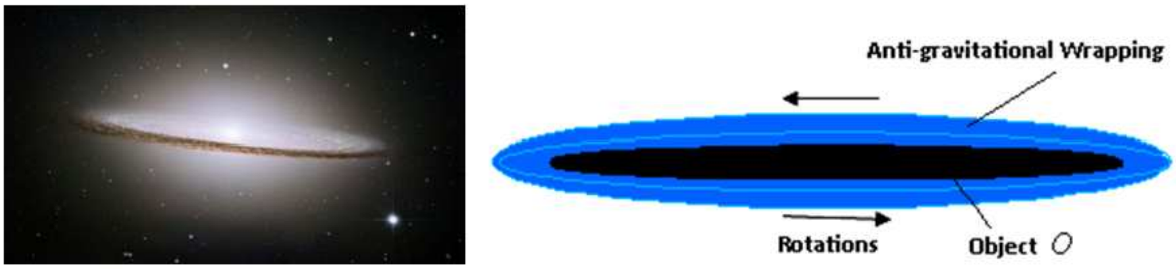

Figure 7. Rotations and anti-gravitational wrapping energy. An example of this idea is the cloud energy in the formatting iso-rotations in a galaxy, in condensed matter to sidereal objects with autonomous energy.

Then the principal equivalence requires that the fields on our manifold locally transform be as in special relativity, that is to say, if , is an element of the Lorentz group ,[16] the fields are transformed like Lorentz-vectors. Of fact this property is extended to all electro-physical modules $\mathfrak{E}$, and $\mathfrak{H}$, like $\mathcal{L}-$modules[17].

However, the generalization to a general diffeomorphism is not unique. We could have chosen the field Ψ, as a vector field whose applications $\Gamma \in Diff(\underline{TM}, \underline{TM}^*)$ are

$$\underline{\Psi}' = (\Gamma^T)^{-1}\underline{\Psi}, \qquad \underline{\bar{\Psi}}' = \underline{\bar{\Psi}}\Gamma^T, \tag{68}$$

But as Γ, is an element of $\mathcal{L}$, that is to say $\Gamma^{-1} = \Gamma^T$, both representations (67) and (68) agree. For general diffeomorphism that will not be the case, although introducing a new field that have a modified scaling behavior, this can be possible to affected to the space-time by $\mathfrak{E} \otimes \mathfrak{H}$- fields. Then is considered $\tau \in \mathrm{Isom}(TM, \underline{TM})$, such that to fields $\underline{\Psi} = \tau\Psi$, $\Psi' = \tau'\overline{\Psi}'$, one finds the behavior

$$\tau' = (\Gamma^T)^{-1}\tau\Gamma^{-1}, \tag{69}$$

It will be useful to clarify the emerging picture of space-time properties by having a close look at a contravariant vector field Ψ^{κ}, as depicted in the wrapping energy around O, (see

[16] $\mathcal{L} = \{\xi \in GL(\mathbb{R}^4) \,|\, g(\xi p, \xi q) = g(p, q), \forall p, q \in \mathbb{R}^4\}$,

[17] **Proposition 2.1 (F. Bulnes) [3].** $\mathfrak{E}$, and $\mathfrak{H}$, like $\mathbb{R}-$modules are invariant under Euclidean movements of the group $O(1,3)$, and thus are $\mathcal{L}-$modules.

the figure 7). This field in blue is a cut in the tangent bundle, that is the set of tangent spaces $T_pM, \forall p \in M$, which describes our space-time. The field is mapped to their covariant field Ψ_ν, which is a cut in the cotangent bundle T_qM^*, , by the metric tensor [19]

$$\Psi_\nu = g_{\kappa\nu}\Psi^\nu, \tag{70}$$

Newly introducing the fields $\underline{\Psi}^\kappa$ (from here anti-graviting) this is transformed under the local Lorentz transformations like a Lorentz-vector in special relativity[18]. Then we can have (after of involve the relations of $\mathrm{Isom}(TM^*,\underline{TM}^*)$):

$$\tau = (\Lambda^T)^{-1}\hat{\tau}\Lambda^{-1} = (\Lambda\Lambda^T)^{-1}, \tag{71}$$

where $|\tau_{\kappa\underline{\kappa}}| = 1$, in the space $\mathrm{Isom}(TM,\underline{TM})$. Then $1_{SO(1,3)} = g\underline{g}$, $|\tau^{\underline{\kappa}}_{\kappa}| = |g|$, and $|\tau^{\kappa}_{\underline{\kappa}}| = |\underline{g}|$, thus the properties of the vector fields are transformed directly to those of fermionic fields by using the fermionic representatives and the transformation of, in this case is the mapping $(\ )^\dagger\gamma^0$, [19] instead of the metric, is used to relate a particle to the particle transforming under the contravariant or contragradient representation.

Then using the notation ∇, to covariant derivative we have:

$$\nabla_{\underline{\kappa}} = \tau^{\kappa}_{\underline{\kappa}}\nabla_\kappa, \tag{72}$$

which is a new connection. Then the *Maxwell-anti-gravity Lagrangian* (that is to say, for anti-gravitational pendants A^a, of gauge fields) is introduced via the field tensors:

$$\underline{F}^a_{\underline{\kappa\nu}} = \nabla_{\underline{\kappa}}A^\kappa_{\underline{\nu}} - \nabla_{\underline{\nu}}A^a_{\underline{\kappa}} + ef^{abc}A^b_{\underline{\kappa}}A^c_{\underline{\nu}}, \tag{73}$$

[18] The underlined indices on these quantities do not refer to the coordinates of the manifols, but to the local basis in the tangential. All of these fields still are functions of the space-time coordinates x_ν. As diffeomorphism τ, maps the basis of one space into the other. We can expand it as $\tau = \tau^{\underline{\kappa}}_{\nu}dx^\nu\partial_{\underline{\kappa}}$, or $\tau = \tau^{\nu}_{\underline{\kappa}}dx^{\underline{\kappa}}\partial_{\underline{\nu}}$, respectively, such that (have inverses):

$\tau^{\underline{\kappa}}_{\nu}\tau^{\nu}_{\underline{\varepsilon}} = \delta^{\underline{\kappa}}_{\underline{\varepsilon}}, \qquad \tau^{\varepsilon}_{\underline{\kappa}}\tau^{\underline{\kappa}}_{\nu} = \delta^{\varepsilon}_{\nu},$

Then for completeness, let us also define the combined mappings through the relations:

$\tau_{\nu\underline{\nu}} = \tau^{\underline{\kappa}}_{\nu}g_{\underline{\kappa\nu}}, \qquad \tau^{\nu\underline{\nu}} = g^{\nu\kappa}\tau^{\underline{\nu}}_{\kappa}.$

[19] γ^0, is the canonical Dirac matrix $\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$.

Staying an Lagrangian of the type $[\eta^{ab}\nabla_a\varphi^*\nabla_b\varphi - V(|\varphi|^2)]$ (see the section 2. 2. 1). Here f^{abc}, are the structure constants of the group and e, is the charge electron coupling with the Planck scale. Then the corresponding electro-anti-gravitational-Lie-QED-algebra is that with supercurrents

$$J^{ab}_{\underline{\kappa\nu}} = \underline{F}^a_{\underline{\kappa\nu}} \otimes \underline{F}^b_{\underline{\kappa\nu}} - \underline{F}^b_{\underline{\kappa\nu}} \otimes \underline{F}^a_{\underline{\kappa\nu}} - \left\{\underline{F}^a_{\underline{\kappa\nu}}, \underline{F}^b_{\underline{\kappa\nu}}\right\}, \tag{74}$$

Then the Lagrangian of fermionic fields can now be composed from the new ingredients as [19]:

$$\mathcal{L}_{\mathcal{ELECTRO\text{-}ANTI\text{-}GRAVITATIONAL}} = \mathcal{L}_F + \underline{\mathcal{L}}_{\underline{F}}, \tag{75}$$

where using the fields Ψ , $\overline{\Psi}$, $\underline{\Psi}$, and $\overline{\underline{\Psi}}$, the Lagrangian $\mathcal{L}_F, \mathcal{L}_{\underline{F}}$, take the form (using of Feynman symbols):

$$\mathcal{L}_F = (\not{D}\overline{\Psi})\Psi + \overline{\Psi}(\not{D}\Psi), \qquad \underline{\mathcal{L}}_{\underline{F}} = (\not{D}\overline{\underline{\Psi}})\underline{\Psi} + \overline{\underline{\Psi}}(\not{D}\underline{\Psi}), \tag{76}$$

where $\underline{\mathcal{L}} = \mathcal{L}(g_{\kappa\nu} \mapsto \underline{g_{\kappa\nu}}, \Psi \mapsto \underline{\Psi})$, and $\mathcal{L} = \underline{\mathcal{L}}(\underline{g}_{\underline{\kappa\nu}} \mapsto g_{\kappa\nu}, \underline{\Psi} \mapsto \Psi)$. This prove the *theorem 5. 1.*

Testing the Lagrangian we can see that there not is direct interaction between gravitational and anti-gravitational particles. However, both of the particles-species will interact with the gravitational field, which mediates an interaction between them. But this coupling is suppressed with the Planck scale. Thus the production of anti-gravitational matter (which is not observable today) can be is explained as ones condensation matter obtained in the scattering process when the anti-gravitational wrapping is created. This usually could see as a cloud or other haze type.

What happen with the energy states Fock space?

States of the Fermionic particles entering go interact through of the corresponding C*-CAR-algebra [20, 21]. Likewise, for example if we consider the anti-symmetric Fock space $\mathcal{F}_{\mathrm{a}}(\mathcal{H})$, and let p_a, the othogonal projection on to anti-symmetric vectors then C*-CAR-algebra is represented on $\mathcal{F}_{\mathrm{a}}(\mathcal{H})$, by settings

$$b^*(\varphi)p_a(\psi_1 \otimes \psi_2 \otimes \ldots \otimes \psi_n) = p_a(\varphi \otimes \psi_1 \otimes \psi_2 \otimes \ldots \otimes \psi_n), \tag{77}$$

This means that the action of orthogonal group $O(2)$, stay restricted to the Hilbert space corresponding to the C*-CAR-algebra becoming the immediated finite region of the space-time in a fermionic Fock space that is mixture of particles and anti-particles (*at least until that* *is converted all space*). We could call to this restrinction of orthogonal group as $O(2,\mathcal{H})$, where a new operator is obtained by the composition $T^{\wedge} = b^*(\varphi) \circ p_a$, acting on a module Fock space that we can write as $\mathcal{A}^{\mathfrak{a}^{\otimes 2}}(\mathcal{H})$, [22] [20] which represent the new energy space whose elements are the second side of (77). Using the CAR-algebra of creation and annhilitation operators and $D_\varphi = b^\dagger(\varphi) - b(\varphi), \forall \varphi \in \mathcal{H}_{\mathfrak{a}\otimes^2}$. The canonical anti-commutation relations are equivalent to the commutator relation:

$$[D_\varphi, D_\psi] = D_\varphi D_\psi - D_\psi D_\varphi = -2i\omega(\varphi,\psi), \tag{78}$$

with the anti-symmetrical form of the Weyl relations given by $\omega(\varphi,\psi)$. If we extend the operators before to linear R-operators on Hilbert space $\mathcal{H}_{\mathfrak{a}\otimes^2}$, we obtain the relation $\omega(A\varphi, A\psi) = \omega(\varphi,\psi)$, which defines a fermionic orthogonal group

$$\mathcal{L}^{\otimes_2} = \{A \in O(2,\mathcal{H}_{\mathbb{R}}) \big| \omega(A\varphi, A\psi) = \omega(\varphi,\psi), \forall \varphi,\psi \in \mathcal{H}_{\mathfrak{a}\otimes^2}\}, \tag{79}$$

where appear the Bogoliubov transformation[21].

Finally, the orbital spaces created by the superconductivity in the quantum regime satisfy the corresponding orbital integrals due F. Bulnes [17] to cuspidal surfaces in the generating chirality inversion through a Dirac node(with Hamiltonian $H = \psi^\dagger(i\upsilon\nabla - \mu)\psi + \Delta\psi^\dagger{}_\uparrow\psi^\dagger{}_\downarrow + \Delta^*\psi_\uparrow\psi_\downarrow$ [23]):

$$J_t(E) = \int\limits_{\underline{N}_F} a_F(a_t \underline{n} a_{-t})^{\rho+\mu} < \sigma(m_F(a_t \underline{n} a_{-t}))^{-1} m_F(\underline{n}) g(k(a_t \underline{n} a_{-t})) > d\underline{n}, \tag{80}$$

where E, is the total Fermi energy in all Fermi surface including the proper kinetic energies, the term $k(a_t \underline{n} a_{-t})$,is the momentum created in the chirality inversion through the node of automorphism $\underline{n} \in \underline{N}_F$, where the space $\underline{N}_F$ is the normal subgroup defined to the action created by fermions in the transit electron-phonon-electron, which is normed by the product of logarithms of the actions of their automorphisms [24].

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[20] $\mathcal{A}^{\mathfrak{a}^{\otimes 2}}(\mathcal{H})$, is a algebra of operators from $\mathcal{H}$, in the super-algebra $\mathfrak{a}^{\otimes_2}$.

[21] The Bogoliubov transformation is a canonical transformation of these operators. To find the conditions on the constants s, and t, such that the transformation remains canonical, the commutator is expanded:

$[\hat{b},\hat{b}^\dagger] = [s\hat{a} + t\hat{a}^\dagger, s^*\hat{a}^\dagger + t^*\hat{a}] = \ldots\ldots = (|s|^2 + |t|^2)[\hat{a},\hat{a}^\dagger]$.

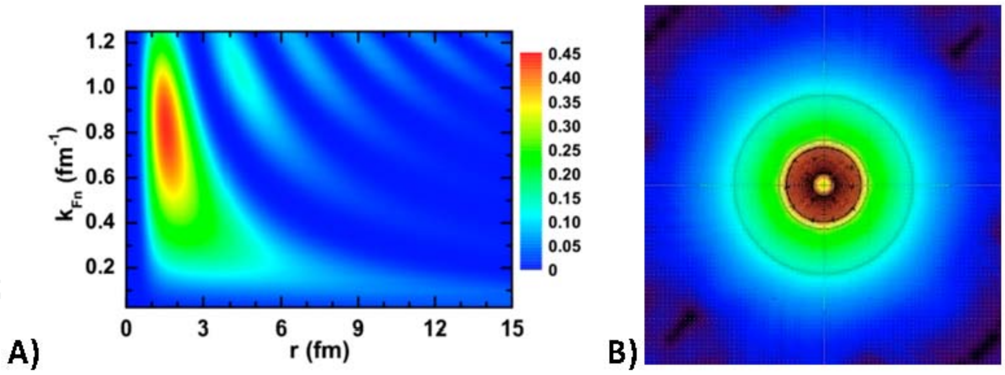

Figure 8. A). Fermionic distribution of probability density $r2|\Psi\text{pair}(r)|2$ of the neutron Cooper pairs as a function of the neutron Fermi momentum *kFn* and the relative distance *r* between the pair partners in symmetric nuclear matter [22]. This can produces Eddy currents with the property anti-gravitational current given in (83). B). The part colored in blue determines the absence of magnetic flow, such like is wanted that it happens for the super-currents existence on the surface of a levitating vehicle, making that this everything behaves as a diamagnetic, except in the central ring colored in red and yellow where exists an intense magnetic field (*this simulation was published in the Proceedings of Fluid Flow, Heat Transfer and Thermal Systems of* ASME *in the paper* IMECE2010-37107, *British Columbia, Canada with all rights reserved* ® [9]).

One example of this automorphisms $a_t \underline{n} a_{-t}$, in action are the quantum operators given by the product $a^{\dagger}_{k+q\uparrow} a^{\dagger}_{-k-q\downarrow} a_{-k\downarrow} a_{k\uparrow}$, , which acts on pairs and not change the electrons in k, and $-k$, and transits to $k+q$, and $-k-q$, letting equals spins. The energy E, is given by $E=\sum_{k,k'} 2\varepsilon_k e^{\dagger}_{k} e_k$, where $e^{\dagger}_k(f) = a^{\dagger}_{k\uparrow} a^{\dagger}_{-k\downarrow}$, $\quad e_k(g) = a_{-k\downarrow} a_{k\uparrow}, \forall f, g \in \mathcal{H}$.

Relating the meaning of these operators with the Debye energy to photons given by $\hbar\omega_D$, , we can to obtain a complete criteria to the energies given by (40) considering the Coulumbian repulsion, obtaining a precise wide measure of trenches where to some real superconductor we consider the term $N(0)V \approx 0.3$, is the magnetic momentum developed by the free electrons in the formatting of the Fermi liquid [25, 26, 27]. The integral $J_t(E)$, is bounded [1, 25].

5. Applications

Proposition (F. Bulnes) 6. 1. Using organized transformations as given in $\sigma_1 \mathcal{T}(\mathbb{M}) \otimes \ldots \otimes \sigma_n \mathcal{T}(\mathbb{M}) \otimes \ldots$, we can to establish that the state of all particles in set, is their corresponding *Fock image* [15, 28].

Inside of the Fock space begins a realization of the potential of the superconductivity, since the Fock pure state involves all the states of particles of the space, object of the transformation [15], that in this case is *superconducting state*. We want organize the particles in two the phases that define our Fock space then the proposition is the shape to do it!

Theorem V. 1 (F. Bulnes, R. Goborov). The organized transformation given by [15]

$$\sigma_1 \mathcal{T}(\mathbb{M}) \otimes \ldots \otimes \sigma_n \mathcal{T}(\mathbb{M}) \otimes \ldots, \tag{81}$$

to *electro-anti-gravitational effect* produced from superconductivity must have a fermionic Fock space [10][22]

$$\mathcal{H}_1 \otimes \mathcal{H}_2 = \{\left| \hat{e}_k^{\dagger} \hat{e}_k \right\rangle \left| \{\hat{e}_k, \hat{e}_l\} = 0, \{\hat{e}_k^{\dagger}, \hat{e}_l^{\dagger}\} = 0, \{\hat{e}_k^{\dagger}, \hat{e}_l\} = \delta_{k,l} \right\}, \tag{82}$$

with rule of transformation in an inherent context of the space-time with Hamiltonian (*transforming each particle around of the source that produces this transformation*):

$$\hat{H} = E_{\text{Fermi Surface}} + \sum_{\mathbf{p},s} \left(\frac{\hbar^2 k_F}{m^*} - \mu > 0\right) \times \hat{e}_{\mathbf{p},s}^{\dagger} \hat{e}_{\mathbf{p},s} + \sum_{\mathbf{p},s} \left(\mu - \frac{\hbar^2 k_F}{m^*} > 0\right) \times \hat{e}_{\mathbf{p},s}^{\dagger} \hat{e}_{\mathbf{p},s}, \tag{83}$$

where k_F, is the Fermi sphere radios (their super-electron momentum) given by

$$k_F = \left(\frac{2mE_{\text{Fermi Surface}}}{\hbar^2} \right)^{1/2}, \tag{84}$$

Their demonstration of the theorem needs more studies and experimental results. This theorem is by way of *conjecture*. But we think that fermionic Fock space of electromagnetic nature can be who can express the phase change in all particles beginning from the structure of metal and transmitting to the immediate ambient space of the metal object (see figure 9B)).

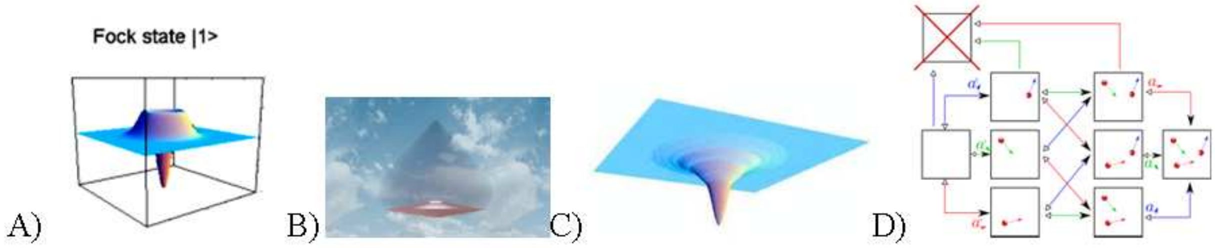

Figure 9. A) The quasi-particle region: holes. The fermionic Fock superconducting space for one particle: observe the two phases of fermion spaces, upper surface corresponds to the holes zone. Of fact this zone is like volcano, since in their interior are holes. The below surface is the free fermions whose behavior is seemed to the Bose-Einstein distribution. B). Structure of the ship transmitting the change phase of the particles that come from of the ship reactor [29, 30]. C) Electro-twistor generated by the magnetic field-superconducting interaction [9]. D) Appearing of the creation operators that shape the wrapping space over structure of the ship. This is defined by a fermionic Fock space, for example under the ship as the impeller twistor [23].

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[22]A electromagnetic case is given bythe algebra: $(\mathfrak{E} \otimes \mathfrak{H})^* = \left\{ (\Psi, \Psi^*) \middle| \Psi \otimes \Psi^* - \Psi^* \otimes \Psi - [\Psi, \Psi] \right\}$.

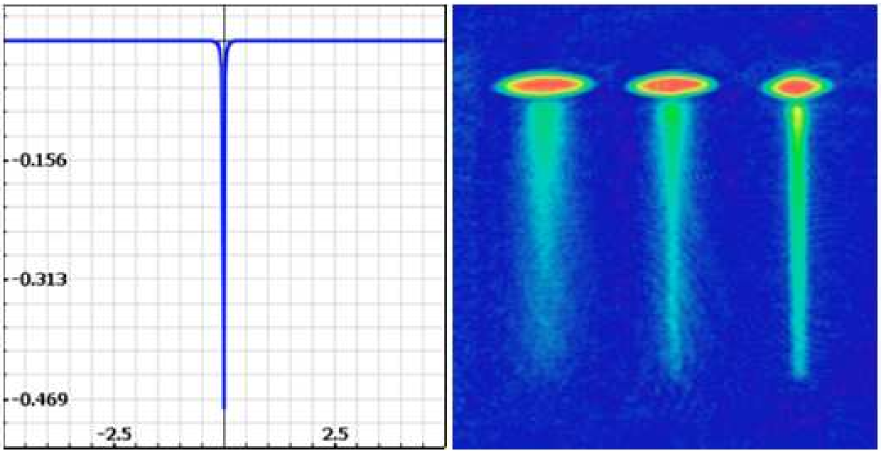

Figure 10. Electro-anti-gravitational Fields $\mathfrak{E}_t$, to levitation (see also figure 11 B)) [18]. Disk (thermal cloud forming the saucer) experiments to magnetic levitation showing the fermionic Fock space in nano-seconds [31] from the orbit-spin interaction.

6. Conclusions

The *sections 4 and 5*, establishes the general conditions to construct the fermionic Fock superconducting space which born from organized transformation of fermions and bosons from the actions of the Lie-QED-algebra $\mathfrak{E} \otimes \mathfrak{H}$, [3, 18, 32] of this way in the next section we establish this and also their transformation to obtain the two phases signed in the Hamiltonian (83) (see figure 9 A)).

Important is do note, that the energy of a quasi-particle depends of the distribution of all the other quasi-particles that haves in the system. Simplified, is can to say, that a free electron, or "naked electron", that is to say, (*outside of interactions*); have as difference with a quasi-particle, or electron with interactions the different masses. The principal effect of the interaction between electrons in normal state consists in change the effective mass of the electron; for example, the specific heat of a Fermi liquid have formally the same expression that the of a ideal Fermi gas changing so only the effective mass, m^*, for the mass of the free electron m.

The fermionic Fock space is a useful topological space picture to describe the interaction obtained for electrons and their link-wave as fonon (boson to describe the quasi-particles) inside the Fermi fluid. In the next work we need to demonstrate this interaction and related with the proposed Hamiltonian in (83). The following region (figure 9 A)), must be the free fermions that realize the transformation in the particles of immediate space moving their spins. Of fact, this change of phase happens inside the superconductor material where the superconductor phenomena happen.

One careful analysis establish certain relations between orbit-spin, saying orbit to the two surfaces that begin in certain step of superconducting process as Rasbha Effect (see figure 4 a)), from the Majorana field produced for this coupling in intermediate state of semi-conductor-superconductor. This can help to design an inter-phase with the reactor of a vehicle of magnetic levitation [23].

Appendices

A. Variation principles to EM in superconductors

Lemma. A. 1. (Bulnes, F) [24, 33]. The energy of action given by E_M, of the O, all like a diamagnetic given by $\Im_M$, satisfies to all vector magnetic potential A, ($1-$form of the corresponding Maxwell equations to the levitation: $rot\boldsymbol{B} = 4\pi\boldsymbol{j}(1/c)$, and $\boldsymbol{B}=\boldsymbol{0}$, [33]), the following Hamiltonian

$$H(A,\Im_M) = \int_O \left\{L_M - H^2/8\pi\right\} dV, \tag{85}$$

Proof. [33, 34].

B. Fermionic Fock space

Now suppose there is an infinite but discrete set of fermionic modes α, corresponding to some $1-$particle quantum states $|\alpha\rangle$, with wave functions $\psi_\alpha(\vartheta)$. In the vector ϑ, we include the spin and other non-spatial quantum numbers into $\vartheta=(x,y,z,spin,etc\ldots)$. In this case, the fermionic Hilbert space is

$$\mathcal{F} = \bigotimes_a \mathcal{H}_{\text{mode}\,\alpha}(\text{spanning}\,|n_\alpha = 0\rangle\,\text{and}\,|n_\alpha = 1\rangle), \tag{86}$$

which has infinite dimension and we may interpret this as a Fock space or arbitrary number of identical fermions. This is our space of study to organized transformations that we require [15, 35, 36]].

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