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# Light Propagation in Optically Dressed Media

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Additional information is available at the end of the chapter

<http://dx.doi.org/10.5772/55830>

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## 1. Introduction

Light propagation in material media has been the subject of interest for centuries. In particular it has been known that the refraction index, defined as the ratio of the phase velocities in vacuum and in a medium, depends on the light colour, which is known as dispersion. The theory of electromagnetic waves based on Maxwell electrodynamics provided a coherent description of light propagation, which in a material medium is influenced by the medium's polarization. To describe the latter a theory of the atomic medium is required. A simple model of the medium, consisting of classical damped oscillators allowed one to describe the medium's linear response to the propagating field in terms of an electric susceptibility, being in the crudest approximation a Lorentzian function of the light frequency. A causal character of the propagation process implied the analytical properties of the susceptibility. This in turn allowed for drawing important conclusions about the dynamics of propagation and the evolution of the pulse shape, including the presence of the pulse precursors [1].

The birth of quantum mechanics made it possible to describe the atomic structure of the medium in a more sophisticated way. A description of an atom in terms of a wave function, being a superposition of eigenfunctions of a free atom (restricted to a subspace of the states accessible really of virtually due to interaction with light), and of atomic eigenenergies provided a more modern approach to resonant transitions. To account for relaxation, first of all for spontaneous emission, it appeared useful to generalize the quantum formalism by admitting density matrices which after introducing relaxation terms fulfill the optical Bloch equations. The latter equations, completed with the Maxwell equations for the propagating field, treated either classically or quantum-mechanically, constitute a full description of the propagation [2].

A new epoch in the history of studies on light propagation has begun when one realized that by irradiating the medium by an additional (control) field or fields one can completely alter the conditions for the propagation of the probe beam. A striking and important example is the electromagnetically induced transparency (EIT) [3, 4] which consists in making the

medium transparent for a pulse resonant with some atomic transition by switching on a strong control laser field, coupling two unpopulated levels. During the last twenty or so years hundreds of papers have been published, studying, both theoretically and experimentally, more and more sophisticated variations of EIT. They include atoms with more active states, coupled by more control fields in various configurations. By admitting the control fields to adiabatically change in time it has become possible to dynamically change the optical properties of the medium while the probe pulse travels inside it [5]. In particular one can reduce the pulse group velocity and finally stop the pulse by switching the control field off; by switching it on again one can release the stored pulse, preserving the phase relations. One can also increase the group velocity or even make it negative. If the control field is in the form of a standing wave, the optical properties of the medium become periodic so the medium resembles a solid state structure [5], which can thus be created on demand with optical means. All those fascinating ways of a precise dynamical control of the optical properties of the medium by optical means reveal new aspects of quantum optics and are supposed to lead to constructing efficient tools for photonics, e.g., quantum memories, quantum switches, multiplexers or, as optimists believe, to designing optically based quantum computers.

Those developments justify the present work the aim of which is to give an introductory review of some of optically dressed atomic systems and to present a method of a theoretical description of their optical properties and of a pulse propagation in such media. In the following chapters we first give a short general theory of wave propagation in atomic media with a few active states. We consider the particular cases of the two-level system, the so-called  $\Lambda$  system, the tripod system and the double  $\Lambda$  system. We show in particular how light propagation and storage can be described in terms of so-called dark state polaritons, being a joint atom+field excitations. We also discuss the situation in which the probe field is described quantum-mechanically which is necessary in the case of a few-photon quantum pulse. In a separate chapter we present atomic models allowing for superluminality, i.e. the pulse's group velocity being negative or larger than the light velocity in vacuum. The final part is devoted to periodic media, a kind of metamaterials, created by optical means. [6].

In our work we remain within the paradigm of the probe field being weak enough to be treated in the linear approximation while the control fields, treated nonperturbatively, are strong enough or couple unpopulated levels so that propagation effects for them can be neglected. We thus leave aside a great part of nonlinear quantum optics dealing with nonlinear effects for the propagating fields.

The bibliography of the field includes hundreds of papers and quickly grows; reviews of various aspects of light propagation in coherently driven atomic media are also available [4, 5, 7, 8]. The list of papers cited here, though obviously not complete, should provide the reader with good tracks for further studies.

## 2. General theory of light propagation

Consider a quasi-one-dimensional propagation of a probe light pulse in an atomic medium. It is governed by the wave equation stemming from the Maxwell equations

$$\frac{\partial^2 \mathbf{E}_1(z, t)}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}_1(z, t)}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}_1(z, t)}{\partial t^2}, \quad (1)$$

where  $\mathbf{E}_1(z, t) = \mathbf{e}_1 E_1(z, t)$  is the electric field of the probe pulse propagating along the  $z$  axis and having polarization  $\mathbf{e}_1$ ,  $\mathbf{P}_1(z, t) = \mathbf{e}_1 P_1(z, t)$  is the medium polarization, i.e. the induced dipole moment per unit volume and  $\mu_0$  is the vacuum magnetic permeability. Both the electric field and the polarization can be expressed in terms of slowly varying complex amplitudes  $\epsilon_1(z, t)$  and  $p_1(z, t)$  and a term rapidly oscillating in time and space

$$\begin{aligned} E_1(z, t) &= \epsilon_1(z, t) \exp[i(k_1 z - \omega_1 t)] + c.c., \\ P_1(z, t) &= p_1(z, t) \exp[i(k_1 z - \omega_1 t)] + c.c., \end{aligned} \quad (2)$$

where  $\omega_1$  is the pulse's central frequency and  $k_1 = \omega_1/c$ ; the latter relation, exact for the propagation in vacuum, means that only dilute media are considered, in which the refraction index does not differ much from unity. For not too short pulses one can make the slowly varying envelope approximation (SVEA) [9] which consists in discarding second-order derivatives of  $\epsilon_1$  and first- and second-order derivatives of  $p_1$ , which leads to the propagation equation of the form

$$\frac{\partial \epsilon_1(z, t)}{\partial t} + c \frac{\partial \epsilon_1(z, t)}{\partial z} = i \frac{\omega_1}{2\epsilon_0} p_1(z, t), \quad (3)$$

where  $\epsilon_0$  is the vacuum electric permittivity. If the medium response to the pulse is linear and spatially local, its polarization  $p_1(z, t)$  is expressed by a memory time integral

$$p_1(z, t) = \epsilon_0 \int_{-\infty}^t \chi(t - t') \epsilon_1(z, t') dt', \quad (4)$$

or, after the Fourier transformation with respect to time,

$$p_1(z, \omega) = \epsilon_0 \chi(\omega) \epsilon_1(z, \omega), \quad (5)$$

where  $\omega$  is the Fourier variable and  $\chi$  is the electric susceptibility. The refraction index is  $n(\omega) = \sqrt{1 + \chi(\omega)} \approx 1 + \frac{1}{2}\chi(\omega)$ . Note that the functions in the time or frequency domains are here distinguished only by their arguments. The propagation equation in the Fourier picture takes the form

$$(-i\omega + c \frac{\partial}{\partial z}) \epsilon_1(z, \omega) = i \frac{\omega_1}{2} \chi(\omega) \epsilon_1(z, \omega). \quad (6)$$

The last equation can be easily solved and, after returning to the time domain, the solution reads

$$\epsilon_1(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \epsilon_1(0, \omega) \exp(-i\omega t) \exp\left[i\frac{\omega z}{c} + i\frac{\omega_1 z}{2c} \chi(\omega)\right] d\omega. \quad (7)$$

In the case of spectrally not too wide pulses one can approximate the susceptibility by the lowest terms of its Taylor expansion at the line centre

$$\chi(\omega) \equiv \chi'(\omega) + i\chi''(\omega) \approx \chi'(0) + i\chi''(0) + \frac{d\chi'(0)}{d\omega} \omega. \quad (8)$$

In such a case one can write the pulse as

$$\epsilon_1(z, t) = \exp\left(i\frac{\omega_1\chi'(0)z}{2c} - \frac{\omega_1\chi''(0)z}{2c}\right)\epsilon_1\left(0, t - \frac{z}{v_g}\right), \quad (9)$$

where the group velocity of the pulse is

$$v_g = c\left(1 + \frac{\omega_1}{2} \frac{d\chi'(0)}{d\omega}\right)^{-1}. \quad (10)$$

This means that the pulse moves with the velocity  $v_g$ , with its shape essentially unchanged apart from an exponential modification of its height (Lambert-Beer law) and an overall phase shift. The group velocity is approximately the velocity of the pulse maximum (exactly if there is no damping). A positive value of  $\chi''(0)$  corresponds to an exponential damping (absorption) while its negative value - to a negative absorption (gain). Note that derivative of the real part of the susceptibility may be positive (normal dispersion) or negative (anomalous dispersion); in the latter case the group velocity may exceed the light velocity in vacuum or even become negative.

The medium total polarization, due to the total laser field applied to the system, can be expressed in terms of the quantum-mechanical mean value of the dipole moment  $\mathbf{d}$

$$\mathbf{P}_1(z, t) = N\text{Tr}\rho(z, t)\mathbf{d}, \quad (11)$$

where  $N$  is the number of atoms per unit volume while  $\rho(z, t)$  is the atomic density matrix. It is assumed that atoms, which do not interact with each other, are distributed in a continuous way, with their position along the sample denoted by  $z$ .

The atom-field interaction for the atomic system irradiated by the probe field and possibly other control fields is described in the electric dipole approximation, so the hamiltonian for the atom in the position  $z$  reads

$$H = H_{at} - \mathbf{E}_1(z, t)\mathbf{d} \quad (12)$$

and the time evolution of  $\rho$  is given by the von Neumann equation with some additional phenomenological relaxation terms, known in this context as optical Bloch equation

$$i\hbar\dot{\rho} = [H, \rho] - i\Gamma\rho. \quad (13)$$

The set of Maxwell-Bloch equations provide a complete description of a weak pulse propagation in a dispersive medium. It is assumed that an atom can be represented by a model including a few states  $a, b, c, \dots$  and each laser field couples a pair of them. Let the probe field 1 be resonant with the transition  $a - b$ , i.e. it couples the states  $a$  (upper) and  $b$  (lower) of energies  $E_a$  and  $E_b$  such that  $E_a - E_b \approx \hbar\omega_1$ . Due to a lack of resonance or selection rules this field does not couple any other pair of states. Then the part of medium polarization responsible for propagation effects for the field 1 is  $P_1 = N(\rho_{ab}d_{ba} + \rho_{ba}d_{ab})$ . In the matrix

elements of the density matrix one can separate the factor quickly oscillating in time and space, i.e.  $\rho_{ab}(z, t) = \sigma_{ab}(z, t) \exp[i(k_1 z - \omega_1 t)]$ . A comparison of the terms including the same quickly oscillating factors (the rotating wave approximation - RWA) allows one to write the propagation equation as

$$\left(\frac{\partial}{\partial t} + c \frac{\partial}{\partial z}\right) \epsilon_1(z, t) = i \frac{N \omega_1 d_{ba}}{2 \epsilon_0} \sigma_{ab}(z, t), \quad (14)$$

or in the Fourier picture

$$\left(-i\omega + c \frac{\partial}{\partial z}\right) \epsilon_1(z, \omega) = i \frac{N \omega_1 d_{ba}}{2 \epsilon_0} \sigma_{ab}(z, \omega). \quad (15)$$

Instead of using the field amplitude  $\epsilon_1$  one often introduces the so-called Rabi frequency  $\Omega_1 \equiv \epsilon_1 d_{ab} / \hbar$ , which accounts for the strength of the atom-field coupling for a given transition. The propagation equation Eq. (15) reads then

$$\left(-i\omega + c \frac{\partial}{\partial z}\right) \Omega_1(z, \omega) = i \frac{N \omega_1 |d_{ba}|^2}{2 \epsilon_0 \hbar} \sigma_{ab}(z, \omega) \equiv i \kappa_1^2 \sigma_{ab}(z, \omega). \quad (16)$$

The density matrix element  $\rho_{ab}$  or equivalently  $\sigma_{ab}$  (so-called atomic coherence) is obtained from the Bloch equations for a particular atom and coupling model. One need not write down the complete set of those equations due to the assumption of a perturbational treatment of the coupling field  $E_1$ . If there were no other sources of this coherence this means that the populations remain unchanged in this approximation and the coherences involving two initially unpopulated states are equal to zero.

### 3. Light propagation in a few-level atomic media

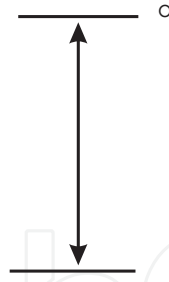
In this section we review the most important atom-field configurations in the case of which coherent interactions modify the optical properties of an atomic medium. Each of them reveals new physical phenomena and provides one with new means to control the propagation of the probe pulse.

#### 3.1. Two-level atom

In the simplest case of a two-level atom irradiated by the probe field alone [9] (see Figure 1), the only essential equation (i.e. such that it contributes in the first-order perturbation theory with respect to the probe field) is

$$i\hbar \dot{\rho}_{ab}(z, t) = (E_a - E_b - i\hbar \gamma_{ab}) \rho_{ab}(z, t) - E_1(z, t) d_{ab} (\rho_{bb} - \rho_{aa}), \quad (17)$$

where  $\rho_{aa}$  and  $\rho_{bb}$  are initial (and unchanging) populations of the excited state ( $a$ ) and ground state ( $b$ ), respectively, and  $\gamma_{ab}$  is the relaxation rate for the coherence  $\rho_{ab}$ . In the simplest case,



**Figure 1.** The level and coupling scheme for a two-level system.

in which the spontaneous emission is the only mechanism of relaxation,  $\gamma_{ab}$  is one half of the relaxation rate for the population of the excited state [2].

After separating the rapidly oscillating terms and introducing the Rabi frequency one obtains

$$i\dot{\sigma}_{ab}(z, t) = (-\delta_{ab} - i\gamma_{ab})\sigma_{ab}(z, t) - \Omega_1(\sigma_{bb} - \sigma_{aa}), \quad (18)$$

where  $\delta_{ab} \equiv (E_b + \hbar\omega_1 - E_a)/\hbar$  is the laser field detuning,  $\sigma_{aa} \equiv \rho_{aa}$  and  $\sigma_{bb} \equiv \rho_{bb}$ . After the Fourier transformation one obtains the coherence in the frequency domain (we assume that there is no contribution to  $\sigma_{ab}$  other than that due to the probe field)

$$\sigma_{ab}(z, \omega) = -\frac{\Omega_1(z, \omega)}{\omega + \delta_{ab} + i\gamma_{ab}}(\sigma_{bb} - \sigma_{aa}). \quad (19)$$

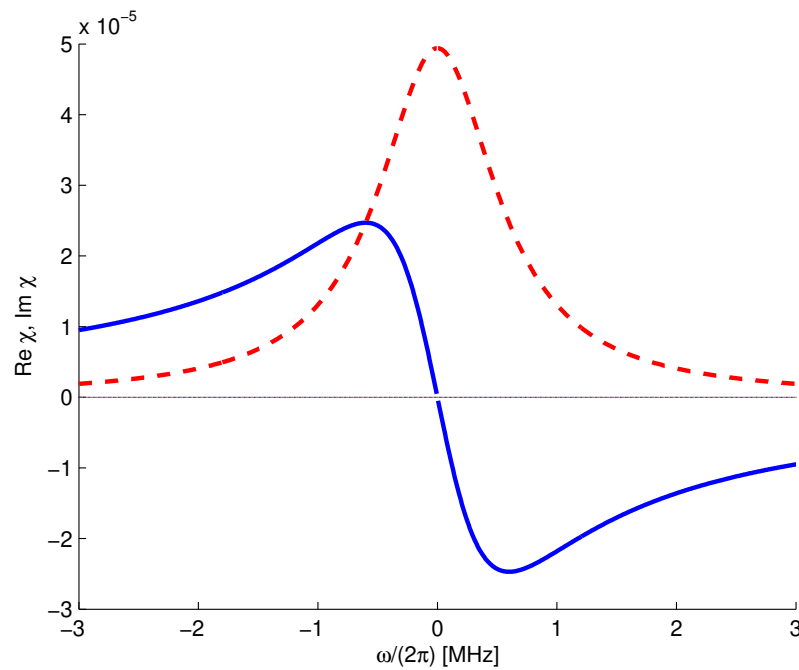
Thus the propagation equation reads

$$(-i\omega + c\frac{\partial}{\partial z})\Omega_1(z, \omega) = \frac{i\omega_1}{2}\chi(\omega)\Omega_1(z, \omega), \quad (20)$$

with the electric susceptibility given by

$$\chi(\omega) = -\frac{N|d_{ab}|^2}{\epsilon_0\hbar} \frac{1}{\omega + \delta_{ab} + i\gamma_{ab}}(\sigma_{bb} - \sigma_{aa}). \quad (21)$$

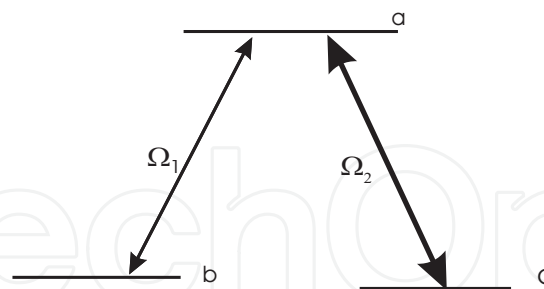
Note that in the typical case of an unprepared medium, i.e. one being initially in the ground state,  $\sigma_{bb} = 1$ ,  $\sigma_{aa} = 0$ . Then one has to do with absorption and anomalous dispersion. A typical susceptibility is shown in Figure 2; this plot as well as the following plots of the electric susceptibility are shown for typical values of atomic parameters. The group velocity calculated according to Eq. (10) is larger than  $c$  or even negative but absorption at the line center is so strong that the pulse is absorbed just after it has entered the medium and its peak does not even travel inside it. If one has the population inversion, i.e.  $\sigma_{aa} > \sigma_{bb}$  the pulse becomes amplified during its propagation. However, one must remember that after it has become strong enough the populations are changed until saturation occurs; such nonperturbative effects are not taken into account in this work.



**Figure 2.** The real (solid, blue line) and imaginary (dashed, red line) parts of the electric susceptibility for a two-level system.

### 3.2. $\Lambda$ configuration

Compared with the two-level system, the  $\Lambda$  system includes one additional long-living lower state  $c$ . The states  $b$  and  $c$  may for example be hyperfine or Zeeman states in the lowest atomic electron state. An additional strong laser field  $E_2$  (control field) couples resonantly or almost resonantly the unpopulated states  $a$  and  $c$  [2]. The level and coupling scheme is shown in Figure 3.



**Figure 3.** The coupling and level scheme for a  $\Lambda$  system.

The amplitude of the control field is assumed constant both in space and in time. This means that the propagation effects for the control field are neglected; this is justified for a strong control field coupling unpopulated states. The control field is written as

$$E_2(z, t) = E_{20} \exp[i(k_2 z - \omega_2 t)] + c.c. \quad (22)$$



In the case of the only populated state  $b$ ,  $\sigma_{bb}(z, t) = 1$  and the set of essential Bloch equations includes two of them

$$\begin{aligned} i\hbar\dot{\rho}_{ab} &= (E_a - E_b - i\hbar\gamma_{ab})\rho_{ab} - E_1d_{ab} - E_2d_{ac}\rho_{cb}, \\ i\hbar\dot{\rho}_{cb} &= (E_c - E_b - i\hbar\gamma_{cb})\rho_{cb} - E_2d_{ca}\rho_{ab}. \end{aligned} \quad (23)$$

After separating the rapidly oscillating factor, i.e setting  $\rho_{cb} = \sigma_{cb} \exp[i((k_1 - k_2)z - (\omega_1 - \omega_2)t)]$  the equations take the form

$$\begin{aligned} i\dot{\sigma}_{ab} &= (-\delta_{ab} - i\gamma_{ab})\sigma_{ab} - \Omega_1 - \Omega_2\sigma_{cb}, \\ i\dot{\sigma}_{cb} &= (-\delta_{ab} + \delta_{ac} - i\gamma_{cb})\sigma_{cb} - \Omega_2^*\sigma_{ab}, \end{aligned} \quad (24)$$

where the Rabi frequency connected with the control field is given by  $\Omega_2 = E_2d_{ac}/\hbar$ ,  $\delta_{ac} = (E_c + \hbar\omega_2 - E_a)/\hbar$  is the detuning of the control field and  $\gamma_{cb}$  is the relaxation rate for the coherence between the lower states; it is due to collisions between the atoms and collisions of atoms with the walls of the cell and is usually smaller by a few orders of magnitude than  $\gamma_{ab}$ .

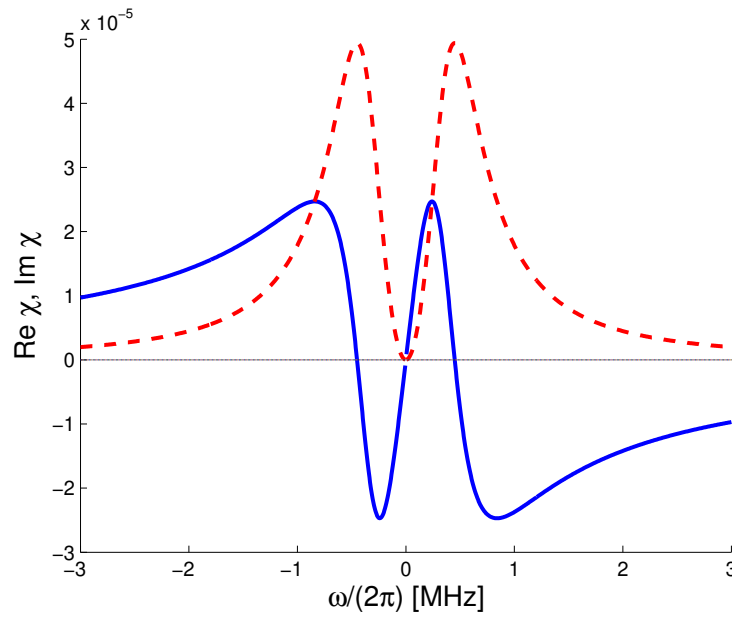
In the Fourier picture the equations read

$$\begin{aligned} (\omega + \delta_{ab} + i\gamma_{ab})\sigma_{ab}(z, \omega) &= -\Omega_1(z, \omega) - \Omega_2\sigma_{cb}(z, \omega), \\ (\omega + \delta_{ab} - \delta_{ac} + i\gamma_{cb})\sigma_{cb}(z, \omega) &= -\Omega_2^*\sigma_{ab}(z, \omega), \end{aligned} \quad (25)$$

From the above equations one can calculate the coherence  $\sigma_{ab}(z, \omega)$  and then the susceptibility which takes the form

$$\chi(\omega) = -\frac{N|d_{ab}|^2}{\epsilon_0\hbar} \frac{1}{\omega + \delta_{ab} + i\gamma_{ab} - \frac{|\Omega_2|^2}{\omega + \delta_{ab} - \delta_{ac} + i\gamma_{cb}}}. \quad (26)$$

A comparison of the susceptibilities for two-level and  $\Lambda$  systems (see Figures 2 and 4) reveals that switching the control field on leads to producing a dip in the Lorentzian absorption profile, called a transparency window. This means that a resonant probe beam which otherwise would be strongly absorbed, is now transmitted almost without losses. Such a process is known as electromagnetically induced transparency (EIT) [3]. The dispersion inside the transparency window becomes normal, with the slope which increases for a decreasing control field. For a negligible relaxation rate  $\gamma_{cb}$  the absorption dip reaches zero. This means that the medium has become transparent for the probe pulse which travels with a reduced group velocity. The width of the transparency window is proportional to the square of the control field amplitude.



**Figure 4.** The real (solid, blue line) and imaginary (dashed, red line) parts of the electric susceptibility for a  $\Lambda$  system.

The Maxwell-Bloch equations in the simplest case of resonant ( $\delta_{ab} = \delta_{ac} = 0$ ) and relaxationless conditions ( $\gamma_{ab} = \gamma_{cb} = 0$ ) read

$$\begin{aligned} \left(\frac{\partial}{\partial t} + c \frac{\partial}{\partial z}\right) \Omega_1(z, t) &= i \kappa_1^2 \sigma_{ab}(z, t), \\ i \dot{\sigma}_{ab}(z, t) &= -\Omega_1(z, t) - \Omega_2(t) \sigma_{cb}(z, t), \\ i \dot{\sigma}_{cb}(z, t) &= -\Omega_2(t) \sigma_{ab}(z, t), \end{aligned} \quad (27)$$

where for simplicity  $\Omega_2$  is assumed real.

In the adiabatic approximation ( $\dot{\sigma}_{ab} = 0$ ) their solution can be written in terms of the so-called dark state polariton  $\Psi(z, t)$  [10] which is a joint atom-field excitation and provides an illustrative insight into the mechanism of propagation

$$\Psi(z, t) = \Omega_1(z, t) \cos \theta(t) - \kappa_1 \sigma_{cb}(z, t) \sin \theta(t), \quad (28)$$

where the mixing angle  $\theta(t)$  is defined by the relation  $\tan \theta(t) = \kappa_1 / \Omega_2(t)$ . The polariton satisfies the equation

$$\left(\frac{\partial}{\partial t} + c \cos^2 \theta(t) \frac{\partial}{\partial z}\right) \Psi(z, t) = 0, \quad (29)$$

the solution of which is

$$\Psi(z, t) = \Psi\left(z - c \int_0^t \cos^2 \theta(\tau) d\tau, t = 0\right). \quad (30)$$

The probe field Rabi frequency and the atomic coherence are expressed by the polariton  $\Psi$

$$\begin{aligned}\Omega_1(z, t) &= \Psi(z, t) \cos \theta(t), \\ \sigma_{cb}(z, t) &= -\frac{1}{\kappa_1} \Psi(z, t) \sin \theta(t).\end{aligned}\quad (31)$$

This means that the polariton travels inside the medium without changing its shape, with the velocity dependent on the current value of the control field amplitude. For a decreasing control field the share of the probe field in the polariton decreases and that of the atomic coherence - grows. A gradual switch-off of the control field implies a slowdown of the pulse which can be "stopped" in the limit of  $\Omega_2 \rightarrow 0$ . The width of the transparency window gradually decreases, but the pulse's spectral width is compressed as well [10] so it remains inside the transparency window all the time. Switching the control field on again maps the coherence back into the electromagnetic field, with the phase relations being preserved. This is true provided the relaxation at the storage stage, represented by the relaxation rate  $\gamma_{cb}$ , has not destroyed the phase relations. In practice the times for which a pulse can be stored range from milliseconds in hot gases to seconds in solids. In the language of polaritons light slowdown, stopping and release mean that one changes the ratio of the field and atomic components of the polariton: for a strong control field  $\cos \theta$  is almost unity so the polariton is built mainly of the field component; on the opposite, for the control field being switched off  $\sin \theta = 1$ , which means that the polariton has become purely atomic. One should stress that the expression "light stopping", though illustrative, is a semantic misuse: when the pulse is "stopped", photons constituting the pulse do not exist any more. They have been mapped into an atomic excitation represented by the coherence  $\sigma_{cb}$ ; their energy has not been absorbed by atoms but has rather been pumped to the control field. The pulse can be restored by switching the control field on, which provides an inverse mapping of the atomic coherence back into photons. Therefore "light storage" is a more proper term which is commonly used in this context. The successful experiments with stopped light were performed by Liu *et al.* in cold gases [11], Phillips *et al.* in atomic vapours [12] and Turukhin *et al.* in a solid state [13].

Another way of explaining electromagnetically induced transparency applies the notion of dressed states. Consider the subspace spanned by the states  $a$  and  $c$  (the energy  $E_c$  of the latter being moved by the photon energy  $\hbar\omega_2$ ), coupled by the interaction  $\Omega_2$ . The dressed states are eigenvectors of the hamiltonian restricted to this subspace. The eigenenergies are shifted from their bare values; if the control field is at resonance the shift is equal to  $\pm\Omega_2$ . If the probe photon's frequency  $\omega_1$  is tuned right in the middle between the dressed eigenenergies, the transition amplitudes from the state  $b$  interfere destructively. This can be clearly seen if one writes the electric susceptibility for the  $\Lambda$  system in the resonant and relaxationless conditions as

$$\chi(\omega) = -\frac{N|d_{ab}|^2}{2\epsilon_0\hbar} \left( \frac{1}{\omega + \Omega_2} + \frac{1}{\omega - \Omega_2} \right), \quad (32)$$

where the dipole matrix elements between the state  $b$  and the dressed states have been expressed by those between  $b$  and the bare states (this is where the factor  $\frac{1}{2}$  has come from). Indeed, at the centre of the lineshape ( $\omega = 0$ ) the susceptibility takes zero value.

This theory can also be formulated in the fully quantum version [10]. The probe electromagnetic field is quantized

$$\hat{\epsilon}_1(z, t) = \hat{\epsilon}^+(z, t) + \hat{\epsilon}^-(z, t) = \sum_k g_k \hat{a}_k \exp(i[(k - k_1)z - (\omega_k - \omega_1)t]) + h.c., \quad (33)$$

where  $g_k = \sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}}$ ,  $V$  being the quantization volume,  $\hat{a}_k$  and  $\hat{a}_k^\dagger$  are photon annihilation and creation operators in the mode  $k$  and resonance has been assumed:  $E_a - E_b = \hbar\omega_1$ . The atomic excitations are also quantized: for an atom in the position  $z$  define the flip operator

$$\hat{\sigma}_{bc}(z, t) = |b\rangle\langle c| \exp(-i[(k_1 - k_2)z - (\omega_1 - \omega_2)t]), \quad (34)$$

and similarly for other pairs of the indices  $a, b, c$ . The time evolution is now governed by the Heisenberg equations of motion. They can be completed by relaxation terms and the corresponding Langevin forces; the latter effect can however be neglected in the timescale of the process [10]; also the relaxation terms will be skipped in the ideal picture presented below. The equations have the same form as Eqs.(24) except that the matrices  $\sigma$  should be transposed. In the adiabatic approximation the solutions can again be written in terms of the polariton field operator

$$\hat{\Psi}(z, t) = \frac{1}{g\sqrt{L}} (\hat{\epsilon}_1^+ \cos \theta - \frac{\hbar\kappa_1}{d_{ab}} \hat{\sigma}_{bc} \sin \theta), \quad (35)$$

where  $\theta$  is the same mixing angle as before,  $g$  is the value of  $g_k$  for the central frequency,  $L$  is the length of the sample and  $d_{ab}$  and  $\Omega_2$  are supposed to be real. Note that it was necessary to adopt a different normalization than previously to assure fulfilling the commutation relations typical of creation and annihilation operators

$$[\hat{\Psi}(z, t), \hat{\Psi}^\dagger(z', t)] = \delta(z - z'), \quad (36)$$

valid provided the relation  $\hat{\sigma}_{bb} \approx 1$  holds, which is true in the first-order of perturbation. The quantum polariton can thus be interpreted as a quasiparticle, being a mixture of the electromagnetic and atomic excitations, the shares of which depend on the current value of the control field. The quantum polariton field operator satisfies the same equation as the classical one (cf. Eq. (29)) and the solution in the adiabatic approximation is

$$\hat{\Psi}(z, t) = \hat{\Psi}(z - c \int_0^t \cos^2 \theta(\tau) d\tau, t = 0). \quad (37)$$

This means that the evolution of the polariton field operator consists just in changing its position. In particular, switching the control field first off and then on effects in turning the photon into the atomic excitation and back into the photon with certainty and without changing the photon state. Instead of describing creation and annihilation of a quasiparticle

at point  $z$  one can introduce a family of orthonormal wavepackets  $f_j(z)$  and introduce the corresponding operators

$$\hat{\Psi}(j, t) = \int dz f_j^*(z) \hat{\Psi}(z, t). \quad (38)$$

The propagation, storage and release of a single photon accompanied by the atomic excitation characterized by the wave packet  $f_j$  are described by the state

$$|1j(t)\rangle = \hat{\Psi}^\dagger(j, t)|0\rangle, \quad (39)$$

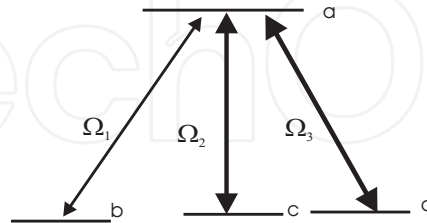
where  $|0\rangle$  is the vacuum state and  $|1j(t)\rangle$  is a one-polariton state corresponding to the wave packet  $f_j$ . This picture constitutes a basis for possible applications to quantum information processing. An information coded in a single photon state can be written down (stored) as an atomic excitation by switching the control field off and later read out through releasing the photon due to switching the control field on again. In this way one obtains a kind of a quantum memory. The information may be processed inside the memory if the atomic excitation is subject to some controlled transformation during the storage stage. In this particular case of a three-level system it is only the photon phase which can be changed. However, admitting additional active atomic states and additional control fields enables one to perform more sophisticated information processing (see below).

### 3.3. Tripod configuration

Compared with the  $\Lambda$  configuration, the tripod configuration includes another additional long-living lower state  $d$  and a second control field  $E_3$  coupling that state with  $a$  (see Figure 5), that is

$$E_3 = E_{30} \exp[i(k_3 z - \omega_3 t)] + c.c., \quad (40)$$

and the corresponding Rabi frequency is  $\Omega_3 = E_{30}d_{ad}/\hbar$ . Note that it is also possible to consider such a system with two probe fields and one control field or even three fields of comparable intensities but those problems will not be discussed here.



**Figure 5.** The level and coupling scheme for a tripod system; the field 1 is the probe field, the fields 2 and 3 are control fields.

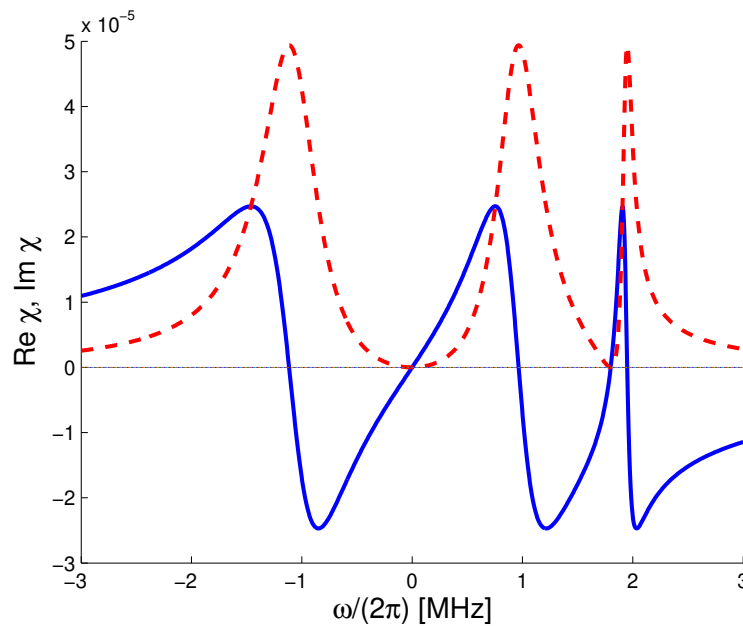
The essential Bloch equations in the Fourier domain have the form

$$\begin{aligned} (\omega + \delta_{ab} + i\gamma_{ab})\sigma_{ab}(z, \omega) &= -\Omega_1(z, \omega) - \Omega_2\sigma_{cb}(z, \omega) - \Omega_3\sigma_{db}(z, \omega), \\ (\omega + \delta_{ab} - \delta_{ac} + i\gamma_{cb})\sigma_{cb}(z, \omega) &= -\Omega_2^*\sigma_{cb}(z, \omega), \\ (\omega + \delta_{ab} - \delta_{ad} + i\gamma_{db})\sigma_{db}(z, \omega) &= -\Omega_3^*\sigma_{db}(z, \omega), \end{aligned} \quad (41)$$

where  $\sigma_{db}$  is the density matrix element after separating the rapidly oscillating factor, i.e.  $\rho_{db} = \sigma_{db} \exp[i((k_1 - k_3)z - (\omega_1 - \omega_3)t)]$ ,  $\delta_{ad} = (E_d + \hbar\omega_3 - E_a)/\hbar$  is the detuning of the second control field and  $\gamma_{db}$  is the relaxation rate for the coherence  $\sigma_{db}$ . The coherence  $\sigma_{ab}(z, \omega)$  can be obtained to yield the susceptibility

$$\chi(\omega) = -\frac{N|d_{ab}|^2}{\epsilon_0 \hbar} \frac{1}{\omega + \delta_{ab} + i\gamma_{ab} - \frac{|\Omega_2|^2}{\omega + \delta_{ab} - \delta_{cb} + i\gamma_{cb}} - \frac{|\Omega_3|^2}{\omega + \delta_{ab} - \delta_{db} + i\gamma_{db}}}. \quad (42)$$

The above susceptibility for the tripod system exhibits in general two transparency windows of different widths and different slopes of the normal dispersion curve (see Figure 6, cf. also Ref. [14]); the latter means that the group velocity in the two windows is different. This asymmetry depends on the model parameters. Speaking in terms of the dressed states one can say that now the subspace in which prediagonalization performed is three dimensional (it is spanned by the states  $a, c, d$ ), so there are three dressed levels and two frequency regions for which the destructive interference is observed. In general one can analyze configurations including a populated ground state and  $n$  unpopulated lower levels, coupled by  $n$  control fields with the upper short-living level. The probe field will then experience  $n$  transparency windows. If the initial state of the medium is coherently prepared one can observe new effects: two resonant pulses may parametrically generate a third one [15].



**Figure 6.** The real (solid, blue line) and imaginary (dashed, red line) parts of the electric susceptibility for a tripod system.

The polariton description in the case of the tripod configuration, both in the classical and quantum versions, is similar to that for the  $\Lambda$  system but requires some modifications [16, 17]. Two polaritons are necessary to describe the adiabatic evolution: they are now built of the electromagnetic component and two atomic excitations. In the quantum version they are

represented by the operators

$$\begin{aligned}\hat{\Psi}_1(z, t) &= \frac{1}{g\sqrt{L}}[\hat{\epsilon}_1^+ \cos \theta - \frac{\hbar\kappa_1}{d_{ab}}(\hat{\sigma}_{bc} \cos \phi + \hat{\sigma}_{bd} \sin \phi) \sin \theta], \\ \hat{\Psi}_2(z, t) &= \frac{1}{g\sqrt{L}}\frac{\hbar\kappa_1}{d_{ab}}(\hat{\sigma}_{bc} \sin \phi - \hat{\sigma}_{bd} \cos \phi) \sin \theta,\end{aligned}\quad (43)$$

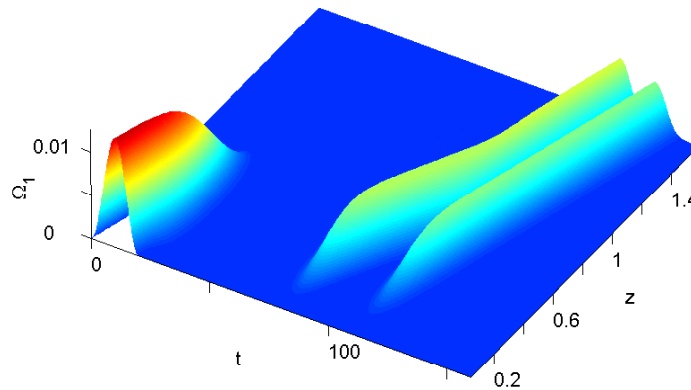
where  $\tan \phi = \frac{\Omega_3}{\Omega_2}$  and for simplicity it has been assumed again that the Rabi frequencies of the control fields are real. The equations of motion for the two polaritons read

$$\begin{aligned}\left(\frac{\partial}{\partial t} + c \cos^2 \theta(t) \frac{\partial}{\partial z}\right) \hat{\Psi}_1(z, t) &= \dot{\phi} \hat{\Psi}_2(z, t) \sin \theta, \\ \frac{\partial}{\partial t} \hat{\Psi}_2(z, t) &= \dot{\phi} \hat{\Psi}_1(z, t) \sin \theta.\end{aligned}\quad (44)$$

Thus if the control fields change at the same rate, i.e.  $\phi = \text{const}$ , the polaritons evolve uncoupled; the first one travels with the velocity  $c \cos^2 \theta$  otherwise unchanged, while the second one is constant in time. Changing the control fields so that  $\phi \rightarrow \frac{\pi}{2} - \phi$  while  $\theta = \frac{\pi}{2}$  means an exchange of the polaritons. In particular the sample may serve as a beam splitter in the time domain [17]: one has to store a single photon in a combination of the atomic excitations using a configuration of the control fields corresponding to some angle  $\phi = \phi_0 = \text{const}$ . If a combination of the control field corresponding to a different angle  $\phi = \phi_1 = \text{const}$  is applied the photon will be released with the probability  $\cos^2(\phi_1 - \phi_0)$ . The second part of the release operation corresponding to the angle  $\frac{\pi}{2} - \phi_1$  will liberate the photon with probability  $\sin^2(\phi_1 - \phi_0)$ . The whole operation becomes even more flexible if one admits changing the phases of the control fields. An interesting extrapolation of this idea is a suggestion of a two-photon interference experiment of the Hong-Ou-Mandel [19] type in the time domain. It consists in an independent storing of two photons in two successive steps and in releasing them, also in two steps, using different combinations of the control fields than at the storage stage. For a special combination of the control fields the result is the photon coalescence in one of the two release channels [17], which can be considered an analogue of the Mandel dip in the standard realization. Note that quantum statistical properties, usually concerning photons, can be investigated and modified for the quasiparticles represented by polaritons. In particular, one can store light, perform an operation on the atomic excitations, which changes the statistical properties of the polariton (being a purely atomic excitation at the storage stage), and release light of modified statistical properties.

The picture becomes even more complicated if one applies nonproportional control fields, such that  $\dot{\phi} \neq 0$ . Figure 7 shows an example the pulse's space-time dependence in the case in which the pulse has been stopped by proportional fields but the releasing field  $\Omega_3$  precedes  $\Omega_2$ . It can be seen that the pulse is released in two stages: in the first one the field  $\Omega_3$  liberates the part of the pulse trapped in  $\sigma_{db}$  while in the second one both control fields liberate the pulse from both atomic excitations. Note that the velocities of the two pulse components become equal only after the amplitude of the second control field has reached its final value. A part of the excitation remains in general trapped inside the medium.



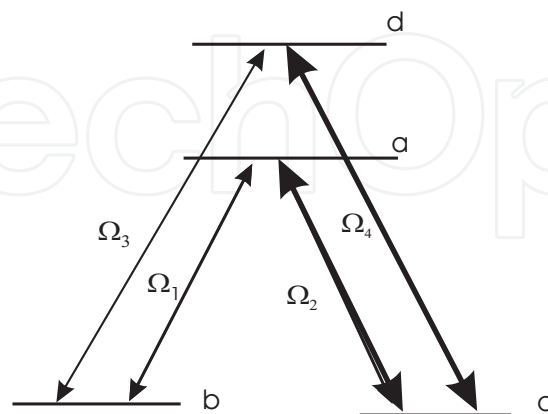


**Figure 7.** The space-time evolution of the probe field in the case of storing a pulse by proportional control fields and its release by the control fields of the same shape but shifted in time.

Another interesting application was proposed by Wang *et al.* [18], who demonstrated how to obtain a one-photon time-entangled state by storing a single photon and later releasing it in successive steps using different combinations of the control fields. Light storage in a medium of rubidium atoms in the tripod configuration has recently been realized experimentally [20].

### 3.4. Double $\Lambda$ configuration

Compared with the single  $\Lambda$  system, the double  $\Lambda$  system considered here includes an additional upper state  $d$  coupled with the ground, populated, state  $b$  with a second weak probe field  $E_3 = \epsilon_3(z, t) \exp[i(k_3z - \omega_3t)] + c.c.$ , and with the unpopulated state  $c$  by a second control field of a constant amplitude  $E_4 = E_{40} \exp[i(k_4z - \omega_4t)] + c.c.$  (see Figure 8).



**Figure 8.** The level and coupling scheme for the double  $\Lambda$  system.

One thus has to do with two  $\Lambda$ 's:  $b - a - c$  and  $b - d - c$ . Light propagation in a medium of such a configuration has been investigated in a number of papers [21–25]. It was



shown in particular that an adiabatic propagation was possible only for such pulses that  $\Omega_1/\Omega_2 = \Omega_3/\Omega_4$ . In the general case pulse matching occurs during the first stage of propagation. The essential Bloch equations read

$$\begin{aligned} i\hbar\dot{\rho}_{ab} &= (E_a - E_b - i\hbar\gamma_{ab})\rho_{ab} - E_1d_{ab} - E_2d_{ac}\rho_{cb}, \\ i\hbar\dot{\rho}_{db} &= (E_d - E_b - i\hbar\gamma_{db})\rho_{db} - E_3d_{db} - E_4d_{dc}\rho_{cb}, \\ i\hbar\dot{\rho}_{cb} &= (E_c - E_b - i\hbar\gamma_{cb})\rho_{cb} - E_2d_{ca}\rho_{ab} - E_4d_{cd}\rho_{db}, \end{aligned} \quad (45)$$

where use has been made of the fact that  $\rho_{bb} = 1$ . Again one can separate the rapidly oscillating factors by substituting additionally  $\rho_{db} = \sigma_{db} \exp[i(k_3z - \omega_3t)]$ , introduce the detuning  $(E_b + \hbar\omega_3 - E_d)/\hbar \equiv \delta_{db}$  and the Rabi frequencies  $\Omega_3(z, t) \equiv \epsilon_3(z, t)d_{db}/\hbar$  and  $\Omega_4 \equiv E_4d_{dc}/\hbar$ . The above equations take the form

$$\begin{aligned} i\dot{\sigma}_{ab} &= (-\delta_{ab} - i\gamma_{ab})\sigma_{ab} - \Omega_1 - \Omega_2\sigma_{cb}, \\ i\dot{\sigma}_{db} &= (-\delta_{db} - i\gamma_{db})\sigma_{db} - \Omega_3 - \Omega_4\sigma_{cb} \exp(i\phi), \\ i\dot{\sigma}_{cb} &= (-\delta_{ab} + \delta_{ac} - i\gamma_{cb})\sigma_{cb} - \Omega_2^*\sigma_{ab} - \Omega_4^*\sigma_{db} \exp(-i\phi), \end{aligned} \quad (46)$$

where  $\phi \equiv (k_1 - k_2 - k_3 + k_4)z - (\omega_1 - \omega_2 - \omega_3 + \omega_4)t$  is a time- and space-dependent phase factor. The analysis of the propagation in the general case of an arbitrary  $\phi$  would require Floquet expansions with respect to the four-wave detuning; in what follows it will be assumed that  $\phi = 0$ . In this case the equations in the frequency domain read

$$\begin{aligned} (\omega + \delta_{ab} + i\gamma_{ab})\sigma_{ab}(z, \omega) &= -\Omega_1(z, \omega) - \Omega_2\sigma_{cb}(z, \omega), \\ (\omega + \delta_{db} + i\gamma_{db})\sigma_{db}(z, \omega) &= -\Omega_3(z, \omega) - \Omega_4\sigma_{cb}(z, \omega), \\ (\omega + \delta_{ab} - \delta_{ac} + i\gamma_{cb})\sigma_{cb}(z, \omega) &= -\Omega_2^*\sigma_{ab}(z, \omega) - \Omega_4^*\sigma_{db}(z, \omega). \end{aligned} \quad (47)$$

The above equations can be solved with respect to the matrix elements of  $\sigma$ . The solutions can be used in the propagation equations for the two probe fields  $\Omega_1$  and  $\Omega_3$

$$\begin{aligned} (-i\omega + c\frac{\partial}{\partial z})\Omega_1(z, \omega) &= i\kappa_1^2\sigma_{ab}(z, \omega), \\ (-i\omega + c\frac{\partial}{\partial z})\Omega_3(z, \omega) &= i\kappa_3^2\sigma_{db}(z, \omega), \end{aligned} \quad (48)$$

where  $\kappa_1^2 \equiv \frac{N\omega_1|d_{ab}|^2}{2\epsilon_0\hbar}$  and  $\kappa_3^2 \equiv \frac{N\omega_3|d_{db}|^2}{2\epsilon_0\hbar}$ . One finally obtains coupled propagation equations for the two fields

$$(-i\omega + c\frac{\partial}{\partial z})\Omega_j(z, \omega) = i \sum_{k=1,3} M_{jk}(\omega)\Omega_k(z, \omega), \quad j = 1, 3, \quad (49)$$

where

$$\begin{aligned} M_{11}(\omega) &= -\frac{\kappa_1^2}{W(\omega)} [(\omega + \delta_{db} + i\gamma_{db})(\omega + \delta_{ab} - \delta_{ac} + i\gamma_{cb}) - |\Omega_4|^2], \\ M_{33}(\omega) &= -\frac{\kappa_3^2}{W(\omega)} [(\omega + \delta_{ab} + i\gamma_{ab})(\omega + \delta_{ab} - \delta_{ac} + i\gamma_{cb}) - |\Omega_2|^2], \end{aligned} \quad (50)$$

$$\begin{aligned} M_{13}(\omega) &= -\frac{\kappa_1^2}{W(\omega)} \Omega_2 \Omega_4^*, \\ M_{31}(\omega) &= -\frac{\kappa_3^2}{W(\omega)} \Omega_2^* \Omega_4, \end{aligned} \quad (51)$$

and

$$\begin{aligned} W(\omega) &= (\omega + \delta_{ab} + i\gamma_{ab})(\omega + \delta_{db} + i\gamma_{db})(\omega + \delta_{ab} - \delta_{ac} + i\gamma_{cb}) \\ &\quad - (\omega + \delta_{ab} + i\gamma_{ab})|\Omega_4|^2 - (\omega + \delta_{db} + i\gamma_{db})|\Omega_2|^2. \end{aligned} \quad (52)$$

The propagation equations may be decoupled by a linear  $\omega$ -dependent transformation diagonalizing the matrix  $M$

$$U(\omega)^{-1} M(\omega) U(\omega) = M^d(\omega), \quad (53)$$

where  $M^d$  is the diagonal matrix with

$$M_{11}^d = \frac{1}{2}(M_{11} + M_{33}) + \sqrt{\frac{1}{4}(M_{11} - M_{33})^2 + M_{13}M_{31}}, \quad (54)$$

$$M_{33}^d = \frac{1}{2}(M_{11} + M_{33}) - \sqrt{\frac{1}{4}(M_{11} - M_{33})^2 + M_{13}M_{31}}. \quad (55)$$

$$(56)$$

As a consequence, after taking the inverse Fourier transform the solutions  $\Omega_j(z, t)$  in the time domain can be written as

$$\begin{aligned} \Omega_j(z, t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \exp\left(-i\omega\left(t - \frac{z}{c}\right)\right) \sum_{k,m=1,3} U_{jk}(\omega) \exp\left(\frac{iz}{c}(M_{kk}^d(\omega))\right) U_{km}^{-1}(\omega) \Omega_m(z=0, \omega), \\ &\quad j = 1, 3. \end{aligned} \quad (57)$$

Thus each of the probe pulses' amplitudes can be considered a superposition of two components, each of which propagates as an analogue of a single pulse with  $M_{kk}^d$  playing the role of  $\frac{\omega_1}{2}\chi(\omega)$ . It appears that one of those "susceptibilities" resembles that for a two-level system while the other one - that for a  $\Lambda$  system. This can be seen from their analytical

form in some special cases. For example when  $\kappa_1 = \kappa_3$ ,  $\delta_{ab} = \delta_{db}$  and  $\gamma_{ab} = \gamma_{db}$  the expressions for  $M_{kk}^d$  are identical (apart from the factor  $\omega_1$ ) with the expressions for the susceptibilities given by Eqs. (26) and (21). One can also see that for  $\gamma_{cb} = 0$  at the line centre  $\omega = 0$  the "susceptibility"  $M_{33}^d(\omega = 0) = 0$  so the absorption is zero. This means that one of the superpositions of the pulse amplitudes quickly disappears while the other can propagate unchanged in the conditions of the electromagnetically induced transparency. Disappearance of the former means a transformation of a part of each of the pulses into the other one rather than absorbing the electromagnetic field by the medium. One can also say that pulse matching has occurred which means that the shapes of the two pulses have become adjusted. A description of pulse propagation for such a system can also be formulated in terms of classical [23, 24] or quantum polaritons [25].

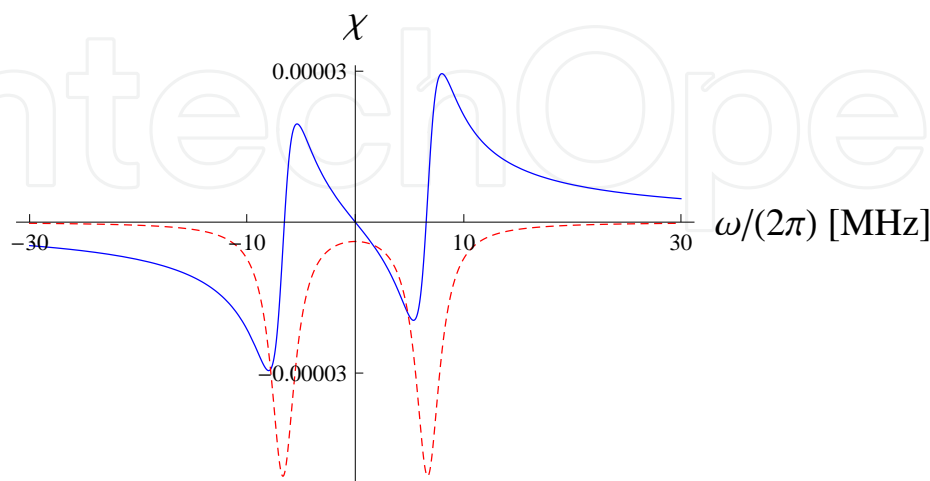
#### 4. Superluminal pulses

The notion of the group velocity may still make sense if it happens that its value exceeds that of the light velocity in vacuum or is negative; this depends on the sign and value of the derivative  $d\chi'(\omega)/d\omega$  [1, 26] (see Eq. (10)). Remember that in the case of a two-level atom the absorption is so strong that the group velocity does not correspond to the velocity of the pulse maximum. The situation may be different in the case of the so-called gain doublet [27, 28]. This is a configuration in which there are two closely spaced upper states coupled with the ground state with the probe pulse and the system is prepared so that population inversion occurs.

The electric susceptibility (see Figure 9) resembles then that for the  $\Lambda$  system in the dressed states version

$$\chi(\omega) = \frac{C}{\omega + \delta + i\gamma} + \frac{C}{\omega - \delta + i\gamma}, \quad (58)$$

where, in contradistinction to the case discussed previously,  $C$  is now a positive constant and  $\omega = 0$  means laser tuning in the middle of the doublet.



**Figure 9.** The real (solid, blue line) and imaginary (dashed, red line) parts of the electric susceptibility for a gain doublet.

In this case the dispersion is anomalous but the gain (negative absorption) is not too large. For positive  $v_g$  it is almost the velocity of the pulse maximum which moves more quickly than light.

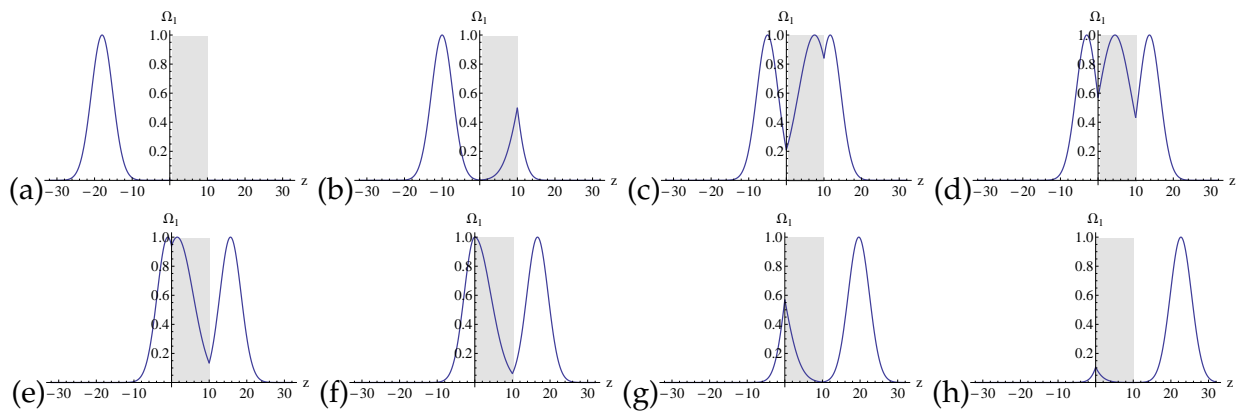
The case of a negative pulse velocity requires more care. It is nonintuitive because the pulse does not any more remain a compact structure. Consider a sample ranging from  $z = 0$  to  $z = L$ . The pulse amplitude should be written as

$$\Omega_1(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \Omega_1(z=0, \omega) \exp \left[ -i\omega \left( t - \frac{z}{c} \right) + i \frac{\omega_1}{2c} \int_0^z \chi(z', \omega) dz' \right] \quad (59)$$

where the susceptibility is constant inside the sample but is zero outside it. Performing the integration yields

$$\Omega_1(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \Omega_1(z=0, \omega) \exp \left[ -i\omega \left( t - \frac{z}{c} \right) + i \frac{\omega_1}{2c} \chi(\omega) \Theta(z) (z\Theta(L-z) + L\Theta(z-L)) \right], \quad (60)$$

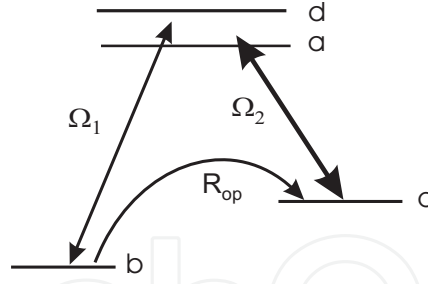
where  $\Theta$  is the step function. The pulse history is presented in the series of plots in Figure 10 (see also Ref. [29]). When an incoming pulse approaches the entrance of the sample a



**Figure 10.** Snapshots of the pulse in arbitrary units, at time instants -18 (a), -10 (b), -5 (c), -3 (d), -1 (e), 0 (f), 3 (g), 6 (h); the sample ranges from  $z = 0$  to  $z = 10$ , the pulse velocity in vacuum  $v_g = 1$  while in the medium  $v_g = -1.5$ . The area of the sample has been shaded.

pair of pulses is created at its exit: one inside the sample and the other outside it. The latter moves away from the sample while the former moves backwards with the velocity  $v_g < 0$ . At the entrance the incoming pulse vanishes and so does that moving backwards inside. The final result is that the pulse has left the medium earlier than would a pulse traveling in vacuum.

The simplest case in which such effects can in principle be observed is again the  $\Lambda$  system in which however it is the state  $c$  which is occupied. Other possible realizations are, e.g., the  $\Lambda$  system with two fields of slightly different frequencies, coupling the the populated state  $c$  with  $a$  (see the experimental work of Dogariu *et al.* [27]), or the double  $\Lambda$  system with two closely spaced upper levels  $a$  and  $d$  coupled with the level  $c$  by a single control field, with an additional incoherent pump transferring the population from  $b$  to  $c$  (see Figure 11), or the  $N$ -system [30] .



**Figure 11.** A possible level and coupling scheme for a system with gain doublet.  $R_{op}$  is the strength of an incoherent pump.

It is very important to stress that superluminality does not violate causality. It has been shown that neither energy [31] nor information [32] are transferred more quickly than light in vacuum. The proof is based on noticing that the energy of the pulses created at the end of the sample is in some sense "borrowed" from the energy stored inside the medium and not from the energy of the incoming pulse. Another point is that a transfer of information requires a nonanalytical pulse, the spectrum of which is very wide, so it contains a part, built of components of frequencies far from resonance, which propagates unaffected by the medium.

## 5. Standing-wave control field

New effects occur in a  $\Lambda$  system if the control field is taken in the form of a standing or quasi-standing wave, i.e. one has

$$E_2(z, t) = [E_{20+} \exp(ik_2 z) + E_{20-} \exp(-ik_2 z)] \exp(-i\omega_2 t) + c.c., \quad (61)$$

where the subscripts  $\pm$  correspond to the direction propagation parallel or antiparallel to the  $z$  axis. Such a field makes the optical properties of the medium periodic in space [6, 33, 34]. If the induced lattice fits the incident wave, i.e.  $k_1 = k_2$ , which also means  $\omega_1 = \omega_2$ , then in addition to transmission and absorption the incident probe field can be reflected and the methods of describing the propagation are adopted from the solid state physics (Bragg scattering). The probe field including now both the forward and backward propagating components can be written in the two-mode approximation [34]

$$E_1(z, t) = [\epsilon_{1+}(z, t) \exp(ik_2 z) + \epsilon_{1-}(z, t) \exp(-ik_2 z)] \exp(-i\omega_1 t) + c.c., \quad (62)$$

where  $\epsilon_{1\pm}$  are slowly varying. After considerations similar to those presented in the case of the typical  $\Lambda$  system (the only difference is that now one has to transform off rapid oscillations in time but not in space) one obtains the electric susceptibility rapidly varying in space in the form (cf. eq. (26))

$$\chi(z, \omega) = -\frac{N|d_{ab}|^2}{\epsilon_0 \hbar} \frac{1}{\omega + \delta_{ab} + i\gamma_{ab} - \frac{\Omega_{2+}^2 + \Omega_{2-}^2 + 2\Omega_{2+}\Omega_{2-} \cos 2k_2 z}{\omega + \delta_{ab} - \delta_{ac} + i\gamma_{cb}}}, \quad (63)$$

where  $\Omega_{2\pm} = d_{ac}E_{20\pm}/\hbar$ .

The above function can be expanded into the Fourier series

$$\chi(z, \omega) = \chi_0(\omega) + \sum_{j=1}^{\infty} \chi_{2j}(\omega) (\exp(2ijk_2 z) + \exp(-2ijk_2 z)). \quad (64)$$

The coupled propagation equations for the two slowly varying components of the probe field read in the frequency domain

$$\begin{aligned} \left( i \frac{\partial}{\partial z} + \frac{\omega}{c} + \frac{\omega_1}{2c} \chi_0(\omega) \right) \Omega_{1+}(z, \omega) + \frac{\omega_1}{2c} \chi_2(\omega) \Omega_{1-}(z, \omega) &= 0, \\ \left( -i \frac{\partial}{\partial z} + \frac{\omega}{c} + \frac{\omega_1}{2c} \chi_0(\omega) \right) \Omega_{1-}(z, \omega) + \frac{\omega_1}{2c} \chi_2(\omega) \Omega_{1+}(z, \omega) &= 0, \end{aligned} \quad (65)$$

where use has been made of the fact that  $\omega_1 \approx \omega_2$  and the Rabi frequencies for both components of the probe field have been introduced  $\Omega_{1\pm} = d_{ab}\epsilon_{1\pm}/\hbar$ . The solutions of the above equations read

$$\begin{aligned} \Omega_{1+}(z, \omega) &= \Omega_{1+}^+(\omega) \exp(iQz) + \Omega_{1+}^-(\omega) \exp(-iQz), \\ \Omega_{1-}(z, \omega) &= \Omega_{1-}^+(\omega) \exp(iQz) + \Omega_{1-}^-(\omega) \exp(-iQz), \end{aligned} \quad (66)$$

where the wavevector  $Q(\omega)$ , the so-called Bloch vector, is given by

$$Q(\omega) = \frac{1}{c} \sqrt{\left( \omega + \frac{\omega_1}{2} \chi_0(\omega) \right)^2 - \left( \frac{\omega_1}{2} \chi_2(\omega) \right)^2}, \quad (67)$$

and the superscripts  $\pm$  distinguish between the two solutions of the differential equations.

The  $z$ -independent functions  $\Omega_{1\pm}^{\pm}$  in the above equations should be chosen to guarantee fulfilling the boundary conditions, usually  $\Omega_{1+}(0, \omega) = \Omega_{10}(\omega)$ ,  $\Omega_{1-}(L, \omega) = 0$ , which corresponds to an incoming wave of the amplitude  $\Omega_{10}(\omega)$  entering the sample at  $z = 0$  and to the reflected wave equal to zero at its end  $L$ . The solutions of Eqs. (65) then read

$$\begin{aligned} \Omega_{1+}(z, \omega) &= \Omega_{10}(\omega) \frac{N_2 \exp[iQ(z-L)] - N_1 \exp[-iQ(z-L)]}{N_2 \exp[-iQL] - N_1 \exp[iQL]}, \\ \Omega_{1-}(z, \omega) &= \Omega_{10}(\omega) \frac{\omega_2}{2c} \chi_2(\omega) \frac{-\exp[iQ(z-L)] + \exp[-iQ(z-L)]}{N_2 \exp[-iQL] - N_1 \exp[iQL]}, \end{aligned} \quad (68)$$

where  $N_1 = -Q + \frac{\omega}{c} + \frac{\omega_2}{2c} \chi_0$  and  $N_2 = Q + \frac{\omega}{c} + \frac{\omega_2}{2c} \chi_0$ . The above expressions can be used to obtain the transmission and reflection spectra for the probe beam

$$T(\omega) = \left| \frac{\Omega_{1+}(L, \omega)}{\Omega_{10}(\omega)} \right|^2, \quad R(\omega) = \left| \frac{\Omega_{1-}(0, \omega)}{\Omega_{10}(\omega)} \right|^2, \quad (69)$$

which, together with the dispersion relation  $Q = Q(\omega)$ , are subject of an experimental verification. A band structure has been created in the medium. In the frequency ranges where  $\Re(Q)$  is small and  $\Im(Q)$  is considerable, one has to do with band gaps. For incident pulses of such frequencies transmission is forbidden while reflection is strong.

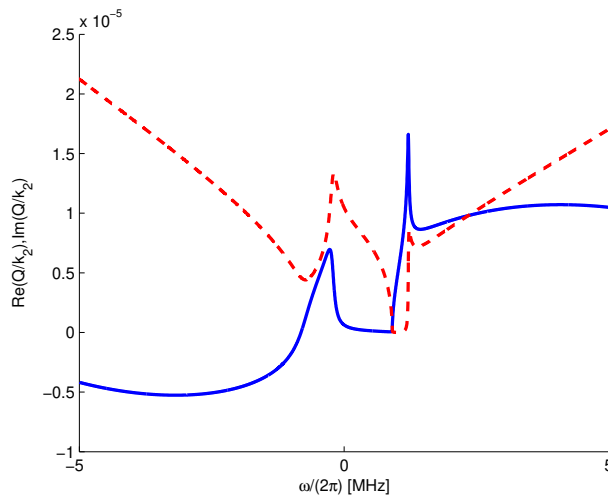
In a more general case, in which the wavenumbers  $k_1$  and  $k_2$  are different, one can make the lattice fit the incident wave by inclining the control beams by an angle  $\beta$  with respect to the  $z$  axis so that  $k'_2 \equiv k_2 \cos \frac{\beta}{2} \approx k_1$  [6]. This means that one should use  $k'_2$  instead  $k_2$  in the Fourier expansion of the electric susceptibility and in the two-mode expansion of the probe field. The above formulae take then the form

$$Q(\omega) = \frac{1}{c \cos \frac{\beta}{2}} \sqrt{\left(\omega \frac{\omega_1}{\omega_2} + \frac{\omega_1^2 - \omega_2^2 \cos^2 \frac{\beta}{2}}{2\omega_2} + \frac{\omega_1^2}{2\omega_2} \chi_0(\omega)\right)^2 - \left(\frac{\omega_1^2}{2\omega_2} \chi_2(\omega)\right)^2}, \quad (70)$$

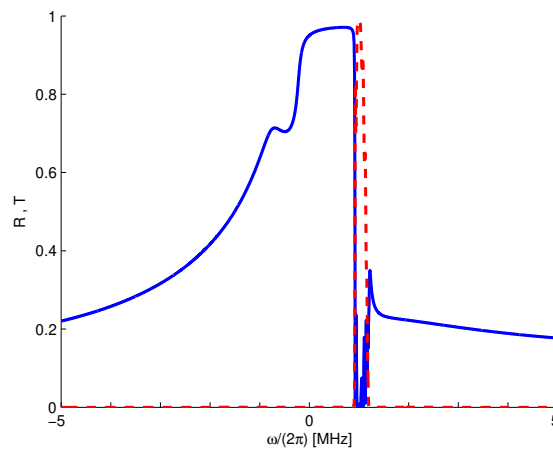
and

$$\begin{aligned} N_1 &= -Q + \frac{1}{c} \left( \omega \frac{\omega_1}{\omega_2} + \frac{\omega_1^2 - \omega_2^2 \cos^2 \frac{\beta}{2}}{2\omega_2} + \frac{\omega_1^2}{2\omega_2} \chi_0(\omega) \right), \\ N_2 &= Q + \frac{1}{c} \left( \omega \frac{\omega_1}{\omega_2} + \frac{\omega_1^2 - \omega_2^2 \cos^2 \frac{\beta}{2}}{2\omega_2} + \frac{\omega_1^2}{2\omega_2} \chi_0(\omega) \right). \end{aligned} \quad (71)$$

A typical dispersion relation  $Q(\omega)$  and the corresponding transmission and reflection spectra are shown in Figures 12 and 13. In the interval (stop band) in which the dispersion is almost zero with nonzero absorption the reflection is almost perfect while a narrow transition peak reaches unity where  $\Im(Q) \approx 0$ . In analogy to the solid state we thus have to do with metamaterials, the optical properties of which can be created on demand with optical methods.



**Figure 12.** The real (solid, blue line) and imaginary (dashed, red line) parts of the electric susceptibility for a lambda system with control field in the form of quasi-standing wave and a small beam deflection angle  $\beta$ .



**Figure 13.** The transmission (dashed, red line) and reflection (solid, blue line) spectra.

## 6. Conclusions

Various situations - generalizations of electromagnetically induced transparency - have been reviewed in which a weak probe beam of light propagates in an atomic medium which coherently interacts with an additional control field or fields. A unified theoretical description of the particular cases, based on Maxwell-Bloch equations, has been given both for classical and quantum probe fields. The considered cases include in particular EIT, light storage, light processing at the storage stage, pulse matching, superluminality and Bragg scattering on an optically created structure.

## Author details

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## References

- [1] Jackson J D. Classical Electrodynamics. New York: Wiley; 1975.
- [2] Scully M O, Zubairy M S. Quantum Optics. Cambridge: University Press; 1997.
- [3] Harris S. Electromagnetically Induced Transparency. *Physics Today* 1997; 50(7): 36-42.
- [4] Fleischhauer M, Imamoğlu A, Marangos J P. Electromagnetically induced transparency: Optics in coherent media. *Reviews of Modern Physics* 2005; 77: 633-673.
- [5] Andre A, Eisaman M D, Walsworth R L, Zibrov A S, Lukin M D. Quantum control of light using electromagnetically induced transparency. *Journal of Physics B: Atomic, Molecular and Optical Physics* 2005; 38: S589-S604.



- [6] Artoni M, la Rocca G C. Optically Tunable Photonic Stop Bands in Homogeneous Absorbing Media. *Physical Review Letters* 2006; 96(7): 073905 1-4.
- [7] Matsko A B, Kocharovskaya O, Rostovtsev Y, Welch G R, Zibrov A S, Scully M O. Slow, Ultraslow, Stored and Frozen Light. *Advances in Atomic, Molecular and Optical Physics* 2001; 46: 191-242
- [8] Milonni P W. *Fast Light, Slow Light and Left-Handed Light*. Bristol, Philadelphia: Institute of Physics; 2005.
- [9] Allen L, Eberly J H. *Optical Resonance and Two-Level Atoms*. Dover Publications; 1987.
- [10] Fleischhauer M, Lukin M D. Dark-State Polaritons in Electromagnetically Induced Transparency. *Physical Review Letters* 2000; 84(22): 5094-7.
- [11] Liu C, Dutton Z, Behroozi C H, Hau L V. Observation of coherent optical information storage in an atomic medium using halted light pulses. *Nature* 2001; 409: 490-3.
- [12] Phillips D F, Fleischhauer A, Mair A, Walsworth R L, Lukin M D. Storage of Light in Atomic Vapor. *Physical Review Letters* 2001; 86: 783-6.
- [13] Turukhin A V, Sudarshanam V S, Shahriar M S, Musser J A, Ham B S, Hemmer P R. Observation of Ultraslow and Stored Light Pulses in a Solid. *Physical Review Letters* 2002; 88(2): 023602 1-4.
- [14] Paspalakis E, Knight P L. Transparency, slow light and enhanced nonlinear optics in a four-level scheme, *Journal of Optics B: Quantum and Semiclassical Optics* 2002; 4: S372-5.
- [15] Paspalakis E, Kylstra N J, Knight P L. Propagation and nonlinear generation dynamics in a coherently prepared four-level system. *Physical Review A* 2002; 65: 053808 1-8.
- [16] Raczynski A, Rzepecka M, Zaremba J, Zielińska-Kaniasty S. Polariton picture of light propagation and storing in a tripod system. *Optics Communications* 2006; 260: 73 -80.
- [17] Raczynski A, Zaremba J, Zielińska-Kaniasty S. Beam splitting and Hong-Ou-Mandel interference for stored light. *Physical Review A* 2007; 75: 013810 1-7.
- [18] Wang T, Koštrun M, Yelin S F. Multiple beam splitter for single photons. *Physical Review A* 2004; 70: 053822 1-5.
- [19] Hong C K, Ou Z Y, Mandel L. Measurement of subpicosecond time intervals between two photons by interference. *Physical Review Letters* 1987; 59(18): 2044-6.
- [20] Wang H, Li S, Xu Z, Zhao X, Zhang L, Li J, Wu Y, Xie C, Peng K, Xiao M. Quantum interference of stored dual-channel spin-wave excitations in a single tripod system. *Physical Review A* 2011; 83: 043815 1-6.
- [21] Cerboneschi E, Arimondo E. Transparency and dressing for optical pulse pairs through a double- $\Lambda$  absorbing medium. *Physical Review A* 1995; 52: R1823-6.

- [22] Cerboneschi E, Arimondo E. Propagation and amplitude correlation of pairs of intense pulses interacting with a double- $\Lambda$  system. *Physical Review. A* 1996; 54: 5400-9.
- [23] Raczynski A and Zaremba J, Controlled light storage in a double lambda system. *Optics Communications* 2002; 209: 149-54.
- [24] Raczynski A, Zaremba J and Zielińska-Kaniasty S. Electromagnetically induced transparency and storing of a pair of pulses of light. *Physical Review A* 2004; 69: 043801 1-5.
- [25] Li Z, Xu L and Wang K. The dark-state polaritons of a double  $\Lambda$  atomic ensemble. *Physics Letters A* 2005; 346: 269-74.
- [26] Milonni P W, Furuya K, and Chiao R Y. Quantum theory of superluminal pulse propagation. *Optics Express* 2001; 8: 59-65.
- [27] Dogariu A, Kuzmich A and Wang L J. Transparent anomalous dispersion and superluminal light-pulse propagation at a negative group velocity. *Physical Review A* 2001; 63: 053806 1-12.
- [28] Steinberg A M, Chiao R Y. Dispersionless highly superluminal propagation in a medium with a gain doublet. *Physical Review A* 1994; 49: 2071-5.
- [29] Ghulghazaryan R, Malakyan Y P. Superluminal optical pulse propagation in nonlinear coherent media. *Physical Review A* 2003; 67: 063806 1-9.
- [30] Kang H, Hernandez G, Zhu Y. Superluminal and slow light propagation in cold atoms. *Physical review A* 2004; 70: 011801R 1-4.
- [31] Diener G. Energy transport in dispersive media and superluminal group velocities. *Physics. Letters* 1997; 235: 118-24.
- [32] Diener G. Superluminal group velocity and information transfer. *Physics Letters A* 1996; 223: 327-31.
- [33] Friedler I, Kurizki G, Pertosyan D. Deterministic quantum logic with photons via optically induced photonic band gaps. *Physical Review A* 2005; 71: 023803 1-8.
- [34] Wu J H, Raczynski A, Zaremba J, Zielińska-Kaniasty S, Artoni M and La Rocca G C. Tunable photonic metamaterials. *Journal of Modern Optics* 2009; 56(6): 768-83.

