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White Light Reconstructed Holograms

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1. Introduction

I would like to begin the chapter with two quotations, I came across during my study, I like very much and I am certain, they literally express holography, its properties, its beauty and its role today, at the beginning either of the 21st century or the third millenium, we have an opportunity to be eye witnesses.

“Holography is the only visual recording and playback process that can record our 3D world on a two-dimensional recording medium and playback the original object or scene to the unaided eyes, as a 3D image. The image demonstrates complete parallax and depth-of-field. The image floats in space either behind, in front, or straddling the recording medium.” – is the first one. It is hard to find its origin, since it can be found at the beginnings of many papers dealing with holography.

“Since its first commercial usage in the 70’s the demand has only increased with each passing year. Holography has found its applications in almost all industrial sectors including commercial and residential applications. Next years might bring us to see a new world of holograms in every aspect. The use of holograms is the representation of a new visual language in communication and we are moving into the age of light as the media of the future. Holography will soon be an integral part of the light age of information and communications.” [1]

Well, people today come across the term *hologram* and can meet holograms on banknotes, various cards and products. A holographic technology comes along with advertisements, promotions,... People specialized in photonics [2], an interdisciplinary field dealing with utilizing photons, may know that holography is closely related to a special kind of light – laser light, which possesses a special property – *coherence*, i.e. all the light waves coming from a laser have the same properties. It is very important, because there are two basic physical light wave phenomena, enabling *holography*.

To record, i.e. to create a hologram, the phenomenon of *two-wave interference of light* is applied. The term *hologram* tells us that all the information transported by light wave is record-

ed. What does it mean – *all the information*? Let us repeat some basic terms [3]. Modelling light as a light wave $A(r, t)$ depending on space (r) and time (t) means, that there are two main parameters – the *amplitude* A_0 of a wave and its *phase* Φ

$$A = A_0 \cos \Phi(r, t) = A_0 \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \Phi_0) \quad (1)$$

where $\mathbf{r}(x, y, z)$ is a displacement vector determining a point in a space and $\mathbf{k}(k_x, k_y, k_z)$ denotes for a *wave vector*, determining direction of wave propagation. Its absolute value $k = 2\pi/\lambda$ defines the *wave number*. Φ_0 is the *initial phase* of the wave. The distance between the two neighbouring amplitude peaks of the same kind is *wavelength* λ [m]. ω [rad.s⁻¹], is *angular frequency*, which is related to the linear *frequency*, ν [s⁻¹], by the formula $\omega = 2\pi\nu$. It lasts $T = \lambda/c$ seconds to pass the path λ at the speed c . Such a time interval is called *period* and $T = 1/\nu$.

A *wave front* is another useful term for us, else. It is the surface upon which the wave has equal phase. It is perpendicular to the direction of propagation, i.e. to the wave vector \mathbf{k} .

For example, considering a wave propagating in the $+z$ direction, the wave vector \mathbf{k} is parallel to the z -axis everywhere. Because of that, wave fronts are parallel planes, perpendicular to the z -axis. Such a wave is known as a *plane wave*. When the $\mathbf{k}(k_x, k_y, k_z)$ direction is general, the phase in a point determined by a displacement vector $\mathbf{r}(x, y, z)$ includes the scalar product of $\mathbf{k} \cdot \mathbf{r}$.

Let us mention also a *spherical wave*

$$A(\mathbf{r}, t) = \frac{A_0}{r} \cos(\omega t - k \cdot r + \varphi_0) \quad (2)$$

which is irradiated from a point light source in homogeneous medium. In such a case wave fronts are centrally symmetrical spheres, so it is enough to consider only radial coordinate r of *spherical* ones. Moreover, \mathbf{k} and \mathbf{r} are parallel, so $\mathbf{k} \cdot \mathbf{r} = kr$. Increasing distance from the source the surface of the sphere increases and amplitude A_0 decreases proportionally to $1/r$.

It is more convenient to represent the light wave expression in a complex form

$$A = A_0 e^{i\Phi} = A_0 \cos \Phi + i A_0 \sin \Phi \quad (3)$$

As for light wave recording, one has to realize a special property of light waves. The instantaneous amplitude A , which varies with both, time and space, cannot be measured experimentally in a direct way. The frequency of the *light wave* is too high for any known physical mechanism to reply to the changes of the instantaneous amplitude A . Any known detector or recording medium replies only to the incident energy. When denoting energy transferred by a wave as E , it can be got as square of the amplitude, i.e. $E = (A_0)^2 = A \cdot A^*$, when using the complex representation, $*$ -symbol represents the complex conjugate quantity.

The value known as *intensity* I of light, is proportional to the energy per unit of surface and unit of time. It is very important to realise that the *time averaged light intensity* is a measurable value, only. Because of that it is said that both, light detection and light recording are *quadratic*.

It is just *holography*, which provides us with a method allowing to record both, the amplitude and phase information despite quadratic recording. Naturally, it is impossible when there is only one light wave. *Dennis Gabor* realized, that there is included the phase information in intensity interference pattern of two waves and it can be utilized in a new – *holographic* method of recording [4].

When two light waves meet and are able to keep a constant phase difference at any point for a proper time interval (long enough to make a record), the interference pattern, i.e. a space redistribution of the resulted energy, can be recorded. Such a situation can come truth only for two coherent waves. Just because of that boom of practical holography started after laser had been invented [5].

To reconstruct, i.e. to see what information is hidden in a hologram, another basic physical phenomenon is applied – *diffraction of light*. There are various types of holograms. Some of them can be reconstructed only by laser light. However, there is also a group of holograms, which can be reconstructed also using common sunlight or another source of *white light*, in which, all the visible spectrum is included. Naturally, that group takes the greatest interest among public and this chapter is going to deal just with such holograms, named here *white light reconstructed holograms* (WLRH).

In the beginning, the basic properties of diffraction of light by a periodic structure are going to be noticed and correlation between diffraction and holographic reconstruction shown. Attention is concentrated especially to the white light diffraction. Understanding of the principles will help the reader to understand problems arising during white light reconstruction of a hologram and also give a hint how to proceed when recording a hologram to avoid such problems. Attention is going to be given to Denisjuk's holograms, image holograms and Benton's rainbow holograms.

The last part of the chapter is focused on some applications of WLRH, especially public ones. To mention also a scientific application, one of our former works, dealing with determination of index of refraction radial profile of a fibre is described briefly.

2. How to record a WLRH

The simple fact that there are two groups of holograms, one of them reconstructed only by coherent light, another one by common white light, tells us that the secret must be hidden in the hologram recording process. To reveal the secret of recording, it would be usable to understand phenomenon of light diffraction, which is the physical principle of hologram reconstruction.

2.1. Diffraction – the physical principle of hologram reconstruction

Firstly - it is said that any deviation from rectilinear propagation of light that cannot be explained because of reflection or refraction is included into diffraction. Such a deviation can be met when light either passes through or reflects from a structure, i.e. a space distribution either of the transmittance, or the refractive index.

Telling more precisely - diffraction is the spreading of waves from a wave-front limited in extent, occurring either when a part of the wave-front is removed by an obstacle, or when all, but a part of the wave-front is removed by an aperture or a stop. The Fraunhofer theory of diffraction, which is interesting for us from the point of view of hologram reconstruction, is concerned with the angular spread of light leaving an aperture of arbitrary shape and size [3].

Secondly – remember that a hologram is, in fact, a two-wave interference pattern recorded, i.e. a kind of structure of intensity distribution. It depends on the type of recording medium and related light-matter interaction mechanism, which of its optical properties distribution follows the interference pattern intensity distribution.

And thirdly – when reconstructing a hologram, it has to be illuminated with light that either passes through or reflects from the hologram. Now, it seems to be obvious to put the equal sign between reconstruction and diffraction.

An introduction to phenomenon of light diffraction and related basic relations can be found in any basic book of optics/photonics, e.g. [3] and even in [6]. Let us show briefly some basic results, related to white light reconstructed holograms.

For the readers, taking a deeper interest - exact solutions of diffraction problems are given by solving Maxwell's equations. However, well-known Kirchhoff's scalar theory gives very good results if period of diffraction structure does not approach a wavelengths size and amplitude vector does not leave a plane.

Let the plane (x_0, y_0) is the plane of a structure and diffraction is observed in the plane (x, y) . To find resulting amplitude A_p in $P(x, y)$, amplitudes of spherical waves from all the point sources in the plane of the structure have to be summed (Fig. 1). The idea is expressed by Kirchhoff's diffraction integral

$$A_p(x, y, z) \sim \iint_S \frac{A_0(x_0, y_0)}{r_2} \exp\{-ikr_2\} dx_0 dy_0 \quad (4)$$

where the distance $r_2 = [(x - x_0)^2 + (y - y_0)^2 + z^2]^{1/2}$ and S is size of the illuminated structure. The experimentally observable interference pattern is given by

$$I(x, y, z) \sim A_p(x, y, z) \cdot [A_p(x, y, z)]^*.$$

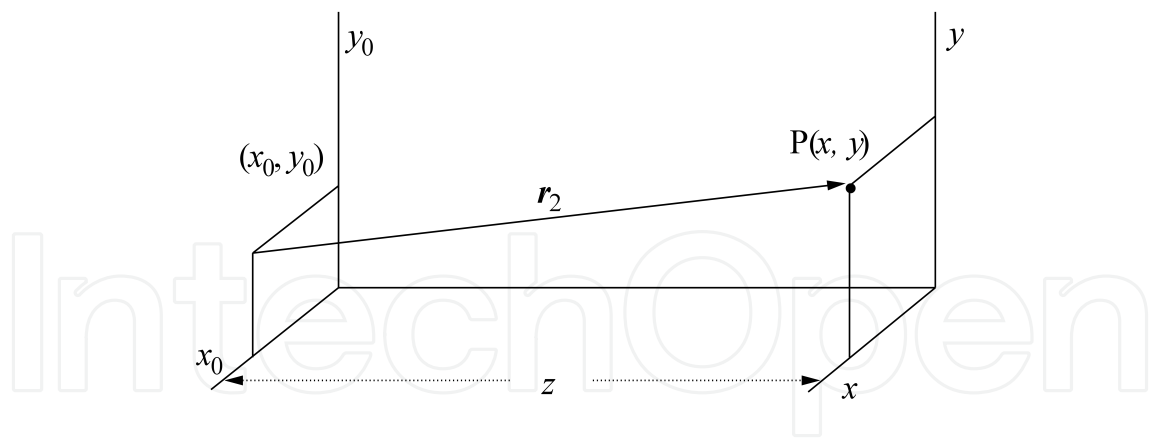


Figure 1. To the principle to solve scalar diffraction problems

To calculate the integral (4), two approximations for r_2 are used – Fresnel's approximation and Fraunhofer's ones. For the purposes of this chapter, let us mention Fraunhofer's approximation: $x_0, y_0 \ll z$, i.e. next mathematical approximation is used to express the phase

$$(x - x_0)^2 \rightarrow x^2 - 2xx_0 \quad \text{and} \quad (y - y_0)^2 \rightarrow y^2 - 2yy_0$$

and integral (4) turns into

$$A_P(x, y, z) = \frac{i}{\lambda z} \exp \left\{ -ik \left(z + \frac{x^2 + y^2}{2z} \right) \right\} \iint_S A_0(x_0, y_0, 0) \frac{\exp \{ -ik(xx_0 + yy_0) \}}{z} dx_0 dy_0 \quad (5)$$

A holographic record can be regarded as a record of a general structure. The light diffraction theory (5) gives us a detail description of light diffraction at a regular plane grating. It is widely used, especially in spectroscopy (looking for various wavelengths).

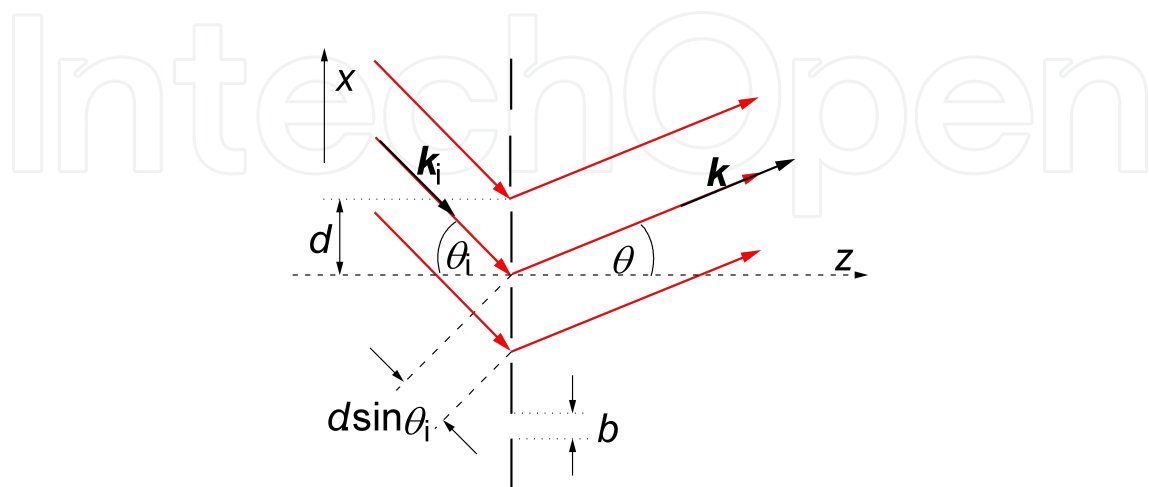


Figure 2. Diffraction by a plane grating (d – grating interval, b – slit width)

The grating is an ensemble of single equal slits, parallel to each other and having the same distance between each other.

There are two parameters, which define the grating – the *slit width* (b) and the *grating interval* (d) – distance between centres of any two adjacent slits (Fig. 2.). Symbols θ_i and θ denote angles of incident and diffracted waves respectively.

Diffraction at such a grating is, in fact, the interference of many “diffractions” by single slits. The number of interfering “diffractions” depends on the number N of illuminated slits. A detailed calculating of (5) results in the angular intensity distribution

$$I \sim A_p \cdot (A_p)^*$$

and it's normalized value can be get in the form

$$I_R = I_{R1} \cdot I_{R2} = \frac{I(\theta, \lambda, b, d)}{b I_0} = \left\{ \frac{\sin \left[\left(kb/2 \right) \sin \theta \right]}{\left(kb/2 \right) \sin \theta} \right\}^2 \left\{ \frac{\sin \left[\left(Nkd/2 \right) (\sin \theta + \sin \theta_i) \right]}{\sin \left[\left(kd/2 \right) (\sin \theta + \sin \theta_i) \right]} \right\}^2 \quad (6)$$

The angle θ appeared because of the Fraunhofer's approximation, where $x/z \sim \sin \theta$. In fact, relation (6) expresses N -wave interference (I_{R2}) modulated by diffraction at a single slit (I_{R1}). Practically usable is especially the condition for interference maxima

$$\frac{kd}{2} (\sin \theta + \sin \theta_i) = m\pi, \quad |m| = 0, 1, 2, 3, \dots \quad (7)$$

where $k = 2\pi n/\lambda_0$, n – refractive index of the space of diffraction, λ_0 – wavelength in vacuum and m – order of diffraction. It is obvious that real values of m can be achieved only when grating constant $d > \lambda_0/n = \lambda$. For simplicity, only one-dimensional (x) structure distribution was considered.

Study in the region of X-rays contributed to the phenomenon of diffraction. The short wavelengths of X-rays are not well suited for diffraction by optical gratings. They are, however, conveniently close to the spacing of atoms in crystal lattices, which therefore provide excellent three-dimensional diffraction gratings for X-rays. It was shown, that the diffraction pattern is intimately connected with the arrangement and spacing of atoms within a crystal, so that X-rays can be used for determining the lattice structure [3].

It was Max von Laue who first suggested that a crystal might behave towards a beam of X-rays rather as does a ruled diffraction grating to ordinary light. It is interesting, that at the time it was not certain either crystals really were such regular arrangements, or X-rays were short-wavelength electromagnetic radiation [3]. W. L. Bragg proved the idea in 1912.

It can be derived in a simply way taking into account the result for plane grating and the influence of a thickness h of the grating (Fig. 3). Let us consider angles of incidence θ_i and

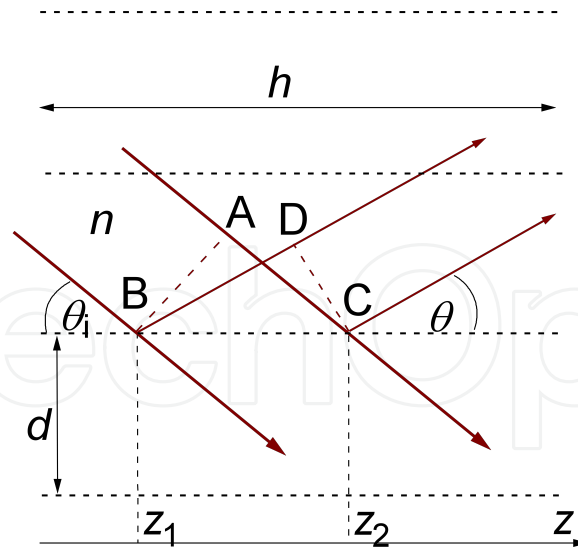


Figure 3. Diffraction by a volume grating (h – thickness, n – index of refraction)

angle of refraction θ . To get the intensity maximum in passing light, the condition (7) has to be fulfilled, i.e.

$$nd(\sin \theta + \sin \theta_i) = m\lambda_0 \quad (8)$$

Moreover, because of the thickness of the grating, all the waves irradiated along the z -axis should be in phase in direction of the angle θ , i.e.

$$(AC - BD)n_0 = n_0(z_2 - z_1) [\cos \theta_i - \cos \theta] = m\lambda_0, \text{ where } |m| = 0, 1, 2, \dots \quad (9)$$

The condition has to be satisfied for all the z coordinates. It is possible only for $\theta = \pm \theta_i$ that leads to the well known Bragg's law for X-ray reflection from a crystal

$$2nd \sin \theta_i = m\lambda_0, \quad |m| = 0, 1, 2, \dots \quad (10)$$

used also to describe diffraction at optical volume gratings.

Concluding this paragraph, let us try to describe the diffraction phenomenon using holographic terms. Two coherent plane waves

$$w_1 \sim \exp(i\varphi_1), \quad w_2 \sim \exp(i\varphi_2) \quad (11)$$

interfere (Fig.4a). The interference pattern that will work as a grating can be expressed in the form, which corresponds with the amplitude transmission t of produced grating

$$t = (w_1 + w_2)(w_1 + w_2)^* = |w_1|^2 + |w_2|^2 + w_1 w_2^* + w_1^* w_2 \quad (12)$$

Let us suppose the grating to be very thin and illuminated by a plane wave w_0 in the z axis direction (Fig. 4b). The phases of all the waves can be expressed as

$$\varphi_j = kx \sin \phi_j, \quad j = 0, 1, 2 \quad (13)$$

The light wave w passing the grating consists of three parts

$$w = w_0 t = w_0 (|w_1|^2 + |w_2|^2) + w_0 w_1 w_2^* + w_0 w_1^* w = w_A + w_B + w_C \quad (14)$$

The phases of three waves in (14) are

$$\varphi_A = kx \sin \phi_0, \quad \varphi_B = kx (\sin \phi_0 + \sin \phi_1 - \sin \phi_2), \quad \varphi_C = kx (\sin \phi_0 - \sin \phi_1 + \sin \phi_2) \quad (15)$$

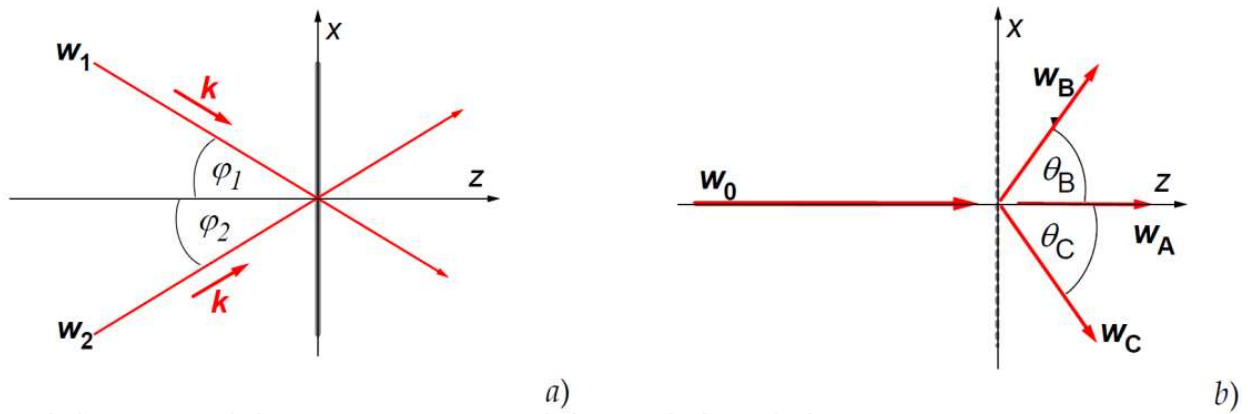


Figure 4. Diffraction as holography. a - 2-wave interference produces a grating; b - diffraction at the grating

Supposing $\varphi_0 = 0$, $\varphi_1 = \alpha/2$ and $\varphi_2 = -\alpha/2$ one can obtain phases

$$\varphi_A = 0, \quad \varphi_B = 2kx \sin(\alpha/2), \quad \varphi_C = 2kx \sin(-\alpha/2) \quad (16)$$

Let us compare (16) to the light diffraction at a plane grating with the grating interval d . The grating interval in our case is

$$d = \lambda / 2 \sin(\alpha/2) \quad (17)$$

Taking into account $\theta_i = 0$, the condition (7) for intensity maximum turns into

$$\sin \theta = m.2 \sin(\alpha / 2), \quad |m| = 0, 1, 2, 3, \dots \quad (18)$$

The relation (18) gives the same angles of propagation $\sin \theta_A = 0$, $\sin \theta_B = 2 \sin(\alpha/2)$, and $\sin \theta_C = 2 \sin(-\alpha/2)$, as relations (16).

2.2. How to avoid problems arising during white-light reconstruction

Now we possess all the knowledge to realize what kind of problems can be met when reconstructing a hologram using white light and even to find a way to avoid them. There are two important results that had been derived above – relation (7) expressing the conditions for existence of a diffracted wave of the order m at a plane grating that can be rewritten into:

$$dn(\sin \theta + \sin \theta_i) = m\lambda_0, \quad |m| = 0, 1, 2, 3, \dots \quad (19)$$

The second one is the same kind of relation, valid for a volume grating - relation (10)

$$2nd \sin \theta = 2nd \sin \theta_i = m\lambda_0, \quad |m| = 0, 1, 2, \dots \quad (20)$$

Let us consider white light reconstruction of a *plane hologram*, which deals with the relation (19). It is obvious that when using waves with various wavelengths λ_j , they will be diffracted at various angles θ_j . When considering reconstruction as a diffraction, $m = 1$. The angle θ of diffraction depends on the wavelength λ_{0j} and various angles $\theta_j(\lambda_{0j})$ will cause various positions of the reconstructed object (Fig. 5.) Naturally, it results in “a blurred” reconstructed object both, horizontally and vertically. However, looking at Fig. 5, it could have occurred to us that if the distance between the object and hologram would have been smaller a less blurred reconstruction could be obtained (image holograms).

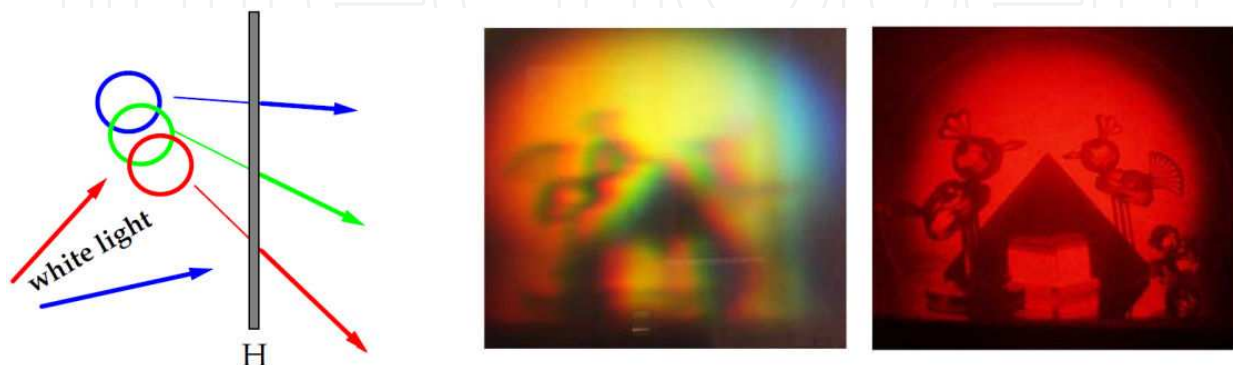


Figure 5. White light reconstruction – the scheme and white light & HeNe laser reconstruction

Moreover, not only a colour blur appears. It is also a *space blur* present. Both, lens and hologram create an image, so hologram is often compared to a lens. So-called *hologram formula*

$$\frac{1}{R_2^{(r,v)}} + \frac{1}{R_1} = \frac{1}{R_1} \left(1 + \frac{\lambda_2}{\lambda_1} \right) - \frac{\lambda_2/\lambda_1}{L_1} \pm \frac{1}{L_2} = \frac{1}{f_H^{(r,v)}} \quad (21)$$

similar to the *lens formula* can be derived, too. A hologram also can be characterised by its *focal length* f_H . It depends on the wavelength of light when either recording (λ_1), or reconstructing (λ_2), object distance from the hologram (R_1) and distance of the source of both, reference (L_1) and reconstructing (L_2) beam from the hologram [7]. Paraxial approximation is supposed, all the distances are measured perpendicularly to the plane of hologram and indices (signs) r ($\pm \rightarrow -$), v ($\pm \rightarrow +$) in (21) are related to *real* and *virtual* reconstruction.

In fact, the relation (21) defines a space blur (various R_{2j}) because of various wavelengths λ_{0j} . Remembering of various horizontal and vertical resolution of a human eye, it gives us another hint – to try to limit a possibility of the reconstruction *in whole* (Benton's holograms).

Now, let us take an interest in *volume holograms*, where results obtained for diffraction at a volume grating were obtained. Ideally there are only two diffraction grating order numbers relevant – $m = 0, 1$. Intensity of higher diffraction grating orders diminish to zero because of mutual interference and the condition (20) turns into

$$2nd \sin \theta_j = 2nd \sin \theta_{ij} = \lambda_{0j} \quad (22)$$

It is a very interesting and for WLRH important result. According to the relation (22) there is an unambiguous relation among the grating period d , wavelength λ_{0j} and angle $\theta_j = \theta_{ij}$. That means – when reconstructing by white light there is the only special angle for every wavelength. This way, the *volume hologram* picks from the white-light spectrum different reconstruction angle for every wavelength, i.e. prepares time coherent light for reconstruction (Denisyuk's hologram). It depends on the method of hologram recording if the reconstructed object is observed either in single colours or colourfully.

2.3. Denisyuk's holograms

Let us proceed following the history of holography and start with Denisyuk's holograms. In 1962, Yuri Denisyuk combined holography with 1908 Nobel Laureate Gabriel Lippmann's work in natural color photography [8]. Denisyuk's approach produced a white-light reflection hologram which, for the first time, could be viewed in light from an ordinary incandescent light bulb [9].

To explain the principle briefly, at least: Isaac Newton found out colours produced by a very thin layer due to interference of light. Colours in butterflies are the result of interference phenomena, too. And in 1886, when the photography was still struggling to transfer the col-

ours of nature to adequate tonal values in black and white, the French physicist Gabriel Lippmann conceived a method to record and reproduce colour images directly through the wavelengths from the lighted object. He introduced a photographic colour process that demanded no colorants, dyes, or pigments, based on light waves interference principles, too.

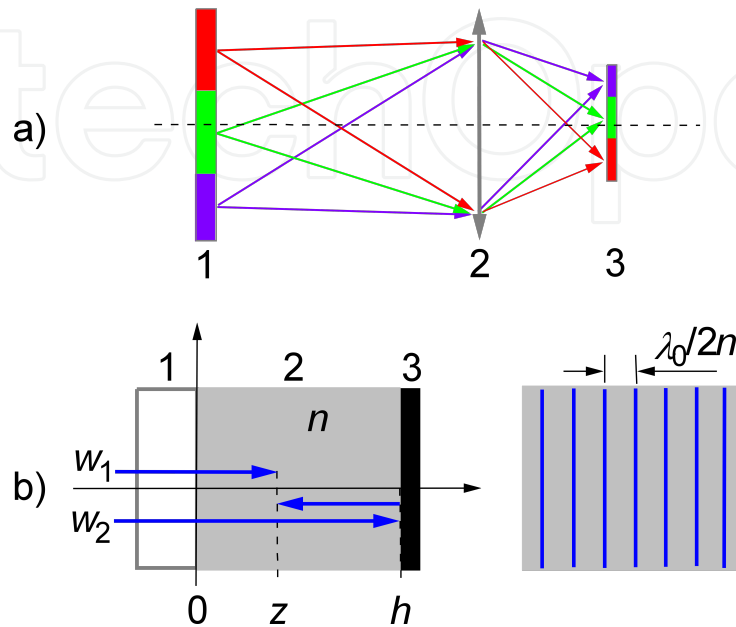


Figure 6. Principle of Lippmann's colour photography. a - colour photography (1-object, 2-lens, 3-image); b - two-wave interference (w_1 and w_2 -interfering waves, 1-photographic glass plate, 2-photographic emulsion, 3-mercury mirror, $n(h)$ -refractive index (thickness) of the emulsion).

He used a photographic emulsion between a photographic glass plate and a mercury mirror (Fig. 6a). The glass plate and emulsion are nearly transparent. Light waves coming from the object reflect from the HG-mirror (w_1) and interfere with coming light waves (w_2) – Fig. 6b. Since the Hg index of refraction prevails the emulsion index of refraction n , standing waves are created [3]. The interference pattern consists of *nodes*-surfaces and *loops*-surfaces, i.e. dark and bright interference surfaces that are recorded in the fine grain emulsion. Fig. 6b demonstrates a simple section of the interference surfaces to evaluate their spacing $\lambda_0/2n$, which depends on the wavelength λ_0 .

It can be derived in a very simply way, when realizing the phase difference $\Delta\Phi(z)$ between two plane waves meeting at the coordinate z

$$\Delta\varphi(z) = \frac{2\pi}{\lambda_0} n [h + (h - z) - z] = \frac{2\pi}{\lambda_0} 2n(h - z) \quad (23)$$

If $\Delta\Phi(z_m) = 2m\pi$, where $m = 0, 1, 2, \dots$ interference maximum is located at z_m . It follows for the spacing between two adjacent maxima the value $\Delta z = z_{m+1} - z_m = \lambda_0/2n$. For simplicity, to point the principle, only, the angle between interfering plane waves was taken to be π .

This way the processed photographic emulsion became a layer where beside an image also volume gratings with various grating intervals, related to the local wavelength, were recorded.

Taking into account the former paragraph (2.2), it is obvious that after the photograph is illuminated by white light, the image in its original colours can be observed because of diffraction of white light at such a structure.

It is hardly imaginable that in the late 1800s such an advanced photographic technique was already conceived and realized. The resemblance with volume holography, published in 1962 by Yuri Denisyuk, is striking. Lippmann photography actually is the ancestor of volume reflection holography or Denisyuk holography, which is sometimes also referred to as Denisyuk-Lippmann-Bragg holography.

Yu. Denisyuk, inspired by the ingenious Lippmann's colour photography technique did extensive theoretical analysis and experimented with mercury lamp sources since 1958 and even enlisted colleagues to develop a special thick, high-resolution, and relatively sensitive photographic emulsion to record the wave pattern in depth [10].

Later, realizing the potential of wave-front reconstruction, devised a different approach to record a hologram. A laser beam illuminated an object through a photographic plate (reference beam) interfered with light reflected from the object (object beam) to produce a hologram in the layer of the photographic emulsion (Fig. 7a).

Reconstruction can be performed by sunlight or another white-light source lighting the hologram. If the direction is the same as when recording, the Bragg's diffractive reflection provides us with a monochromatic reconstructed image of the object in the wavelength used when recording the hologram (e.g. Fig. 7b). Fig. 7c demonstrates a possible two-beam experimental set-up.

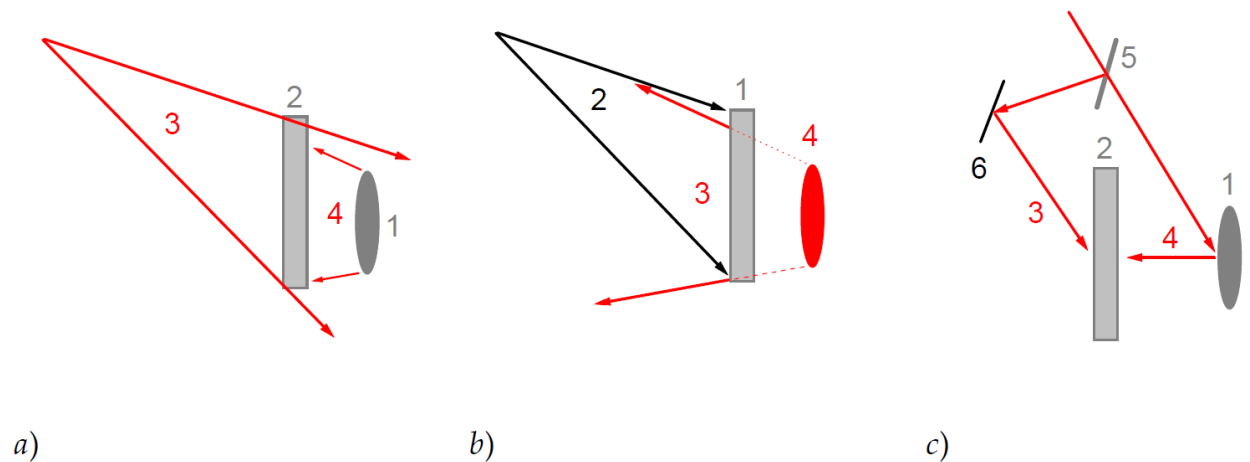


Figure 7. Denisyuk's 1-beam holographical set-up. *a* – recording (1-object, 2-recording medium, 3-reference beam, 4-object beam), *b* – reconstruction (1-hologram, 2-reconstructing beam, 3-reconstructed wave, 4-holographic image), *c* – 2-beam set-up (1-object, 2-recording medium, 3-reference beam, 4-object beam, 5-beam splitter, 6-mirror).

However, as mentioned above, such a reconstruction is a monochromatic one, only (Fig. 8). What about colour holograms?

To produce colourful hologram, one needs three lasers generating on three basic wavelengths (red, green, blue) to record such a hologram. Each of the waves creates own interference structure (Fig. 9). After illuminating such a hologram with white light, each structure helps to reconstruct the object wave in related wavelength. The reconstruction can be seen in three colours and thanks to sophisticated activity of our brain as colourful.



Figure 8. Yu. Denisyuk reconstructed from a hologram and alive [11]

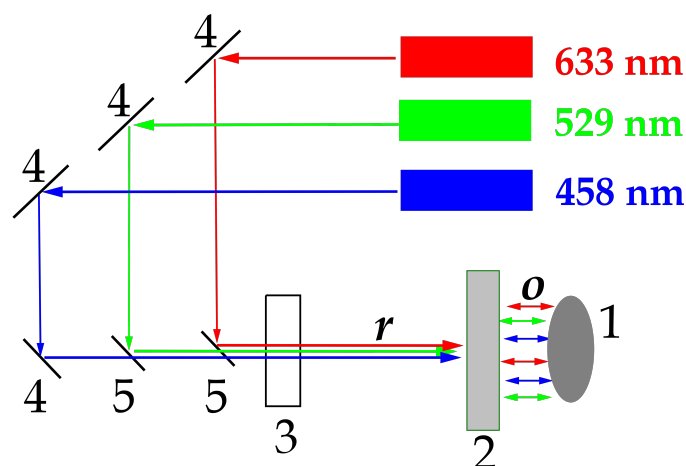


Figure 9. A colourful hologram recording (1-object, 2-recording medium, 3-optics to spread light, 4-mirror, 5-semi-transparent mirrors)

Yu. N. Denisyuk presented his technique as a generalized form of Lippmann photography, or as a colour-dependent optical element. This technique, using reflection holography and the white-light reconstruction technique, seems to be the most promising one as regards the actual recording of colour holograms [12].

2.4. Image hologram

Writing on white-light reconstructed holograms it is necessary to take into account that depending on the relation between the thickness h of the recording medium and the average spacing \bar{d} of the recorded holographic structure fringes, there are defined two types of holograms – *thin* and *thick*, i.e. *volume* ones. They are characterized by a parameter Q

$$Q = \frac{2\pi\lambda_0 h}{n\bar{d}^2} \quad (24)$$

introduced in [13]. $Q < 1$ corresponds to thin gratings, while $Q > 1$ corresponds to volume gratings. Moreover the objects around us can be defined as either 2D or 3D objects.

It is obvious that Denisyuk's method of holography is applicable to any object and the recording media suitable to create a volume hologram. Moreover, it represents a reflection holography, i.e. the reconstructed wave seems like reflection of the reconstructing wave from the hologram. When recording, the reference beam and the object beam come from opposite sides of the recording medium. Because of that, when reconstructing, observer and the source of the reconstructing wave are on the same side of the hologram. The next two paragraphs are dealing with so-called *transmission thin holograms*.

Let us remember of the physical principle of reconstruction of a hologram. It is diffraction of either transmitted or reflected incident, i.e. reconstructing light. Due to that, the white-light reconstruction of transmission thin holograms brings a possible colour and space *blur* of the reconstructed image (Fig. 5). The illustrative Fig.5 gives us a hint to decrease the object – hologram distance when recording the hologram as much as possible to avoid the colour blur. It would be the best to decrease the distance to zero. However, the reference beam must not be blocked.

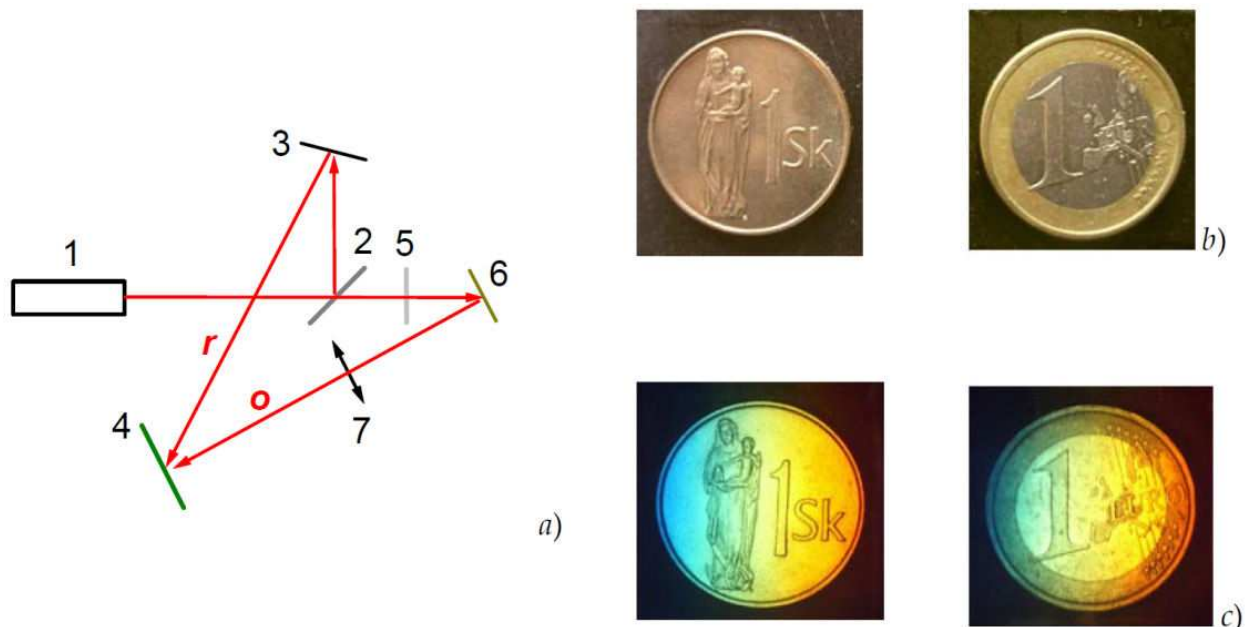


Figure 10. Image holography. a – experimental set-up (1-laser & collimator, 2-beam splitter, 3-mirror, 4-hologram, 5-ground glass, 6-object, 7-projecting lens; b - objects, c - reconstruction

There is a simple possibility to put the recorded object (Fig. 10b) even “into” the recording medium and not to restrict the reference wave while recording the hologram – to *project* it

there by a projecting lens (Fig. 10a). When reconstructing in white light the reconstruction is observed in various (rainbow) colours from various directions at the same place (Fig. 10c.).

When the image coincides with the hologram plane, it is perfectly sharp. Naturally, projection of a 3D object has 3D properties, too. In practice, display holograms up to around 2 cm in depth can be acceptable. This type of hologram is called an *open-aperture* hologram [14]. However, such a simple method is used mostly for approximately “2D” objects like medals, coins, photos, and so on.

A more detailed estimation of the image blur can be found in [15]. It can be shown that if the source used to reconstruct the hologram is located at the same position as the reference source used to record it and has very nearly the same wavelength, the blur Δx_{R2} of the reconstructed image along the x axis for a source size Δx_{L2} is

$$\Delta x_{R2} = (R_1 / L_2) \Delta x_{L2} \quad (25)$$

Similarly, if the source used to reconstruct the hologram has a mean wavelength λ_2 very nearly equal to λ_1 , the wavelength used to record the hologram, and a spectral bandwidth $\Delta \lambda_2$, the transverse image blur Δx_{R2} due to the finite spectral bandwidth of the source of reconstructing wave, located at x_{L2} can be shown to be

$$|\Delta x_{R2}| = (x_{L2} / L_2) R_1 (\Delta \lambda_2 / \lambda_2) \quad (26)$$

The image blur increases with the depth of the image and the interbeam angle.

It follows from (25) and (26) that if the interbeam angle and the depth of the object are small, it is possible to use an extended white-light source to view the image.

2.5. Benton's holograms

When a real 3D object is recorded the well-known Benton's method [16], which allows us to get rid of vertical blur of the reconstructed image, has to be used. The Benton (rainbow) hologram is a transfer transmission hologram, which reconstructs a bright, sharp, monochromatic image when illuminated with white light [17]. Benton holograms are produced by means of an optical technique that sacrifices the vertical parallax of the holographic image in favour of a sharp monochromatic reconstruction by a white light point source. In other words - the physical basis of this method is to reduce the amount of information on the hologram. The vertical parallax is eliminated. The method relies on the fact that human beings have two eyes in horizontal position, i.e. people are less sensitive to vertical parallax.

In fact, it is a hologram of a hologram. The first (master) hologram H_1 of the object O is produced (Fig. 11a). The object and reference waves are directed around the horizontal plane.

Then, H_1 is masked with a narrow horizontal slit S and wave 1, conjugated to the reference wave 2 from Fig. 11a is used to reconstruct the free part of H_1 . The conjugate wave [6] of the

original object wave is reconstructed and the real holographic image HI serves as an object to record the second hologram H_2 (the part *a*) in Fig. 11b). The part *b*) of Fig. 11b demonstrates the view to the process of recording H_2 from the side. The 3D object, represented by reconstructed HI straddles the plane of medium to record H_2 and a converging reference wave 3 is now inclined in the vertical plane.

When viewing such a hologram in the same monochromatic light as the recording one, the holographic image from H_2 that straddles the plane of H_2 is reconstructed. However, while recording H_2 the width of the slit S limited the vertical parallax of the object and it will be limited when observing the reconstruction, too. That means, there is a small interval of angles in vertical direction that allows observing the reconstruction of H_2 , i.e. the perspective information in vertical axis is lost. The horizontal parallax is much more wider, given by the width of H_1 determined by the length of the slit S .

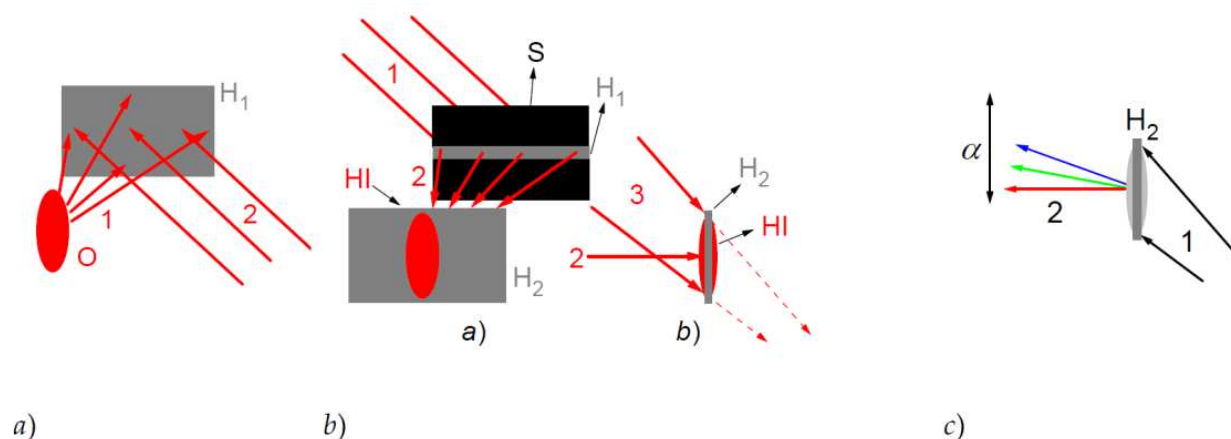


Figure 11. Principle of Benton's rainbow hologram. *a* – object O , hologram H_1 , object wave 1, reference wave 2; *b* – hologram H_1 , slit S , reconstructing wave 1, reconstructed wave 2 as a new object wave, holographic image HI , reference wave 3, hologram H_2 ; *c* – hologram H_2 , reconstructing white light 1, colour reconstructed waves 2 related to various angles α .

It seems like a kind of limitation, but remember – when recording H_2 , the reference wave 3 was inclined in the vertical plane. This way, the hologram H_2 , which is a set of arbitrary gratings with horizontal fringes in principle, became a dispersion element for vertical angle variations. That provides the hologram H_2 with colour dispersion. With a white-light source, located approximately in the convergence point of 3 (Fig. 11c), the reconstruction is dispersed in the vertical plane to form a continuous spectrum (hence the term "rainbow"). An observer whose eyes are positioned at any part of this spectrum then sees a sharp, three-dimensional image of the object in the corresponding colour. In [18] is presented an optical system that permits both steps of the recording process to be carried out with a minimum of adjustment.

The image reconstructed from a rainbow hologram is free from speckle, because it is illuminated with incoherent light. However, it is not free from blur. A more detailed analysis of rainbow holograms can be found in [19].

One cause of image blur is the finite wavelength spread in the image. The maximum wavelength spread observed when the rainbow hologram is illuminated with white light can be estimated as

$$(\Delta\lambda)_{\max} \sim \lambda / \sin \theta / (D + b) / L \quad (27)$$

where θ is the angle between object and reference waves when recording H_2 , L is the distance between H_1 and H_2 , D is eye pupil diameter and b is the slit S width. The eyes of the person viewing the image are supposed to be placed in the position, where the image of the slit is formed. The relation (27) leads to the image blur due to wavelength spread of about

$$\delta_{\Delta\lambda} \sim z_0 (D + b) / L \quad (28)$$

where z_0 represents the H_2 -to-image distance generally, when the image does not straddle the H_2 plane. With $z_0 \rightarrow 0$ the image blur (28) also goes to zero.

Another cause of image blur is the finite size of the source used to illuminate the hologram. If the source has the angular subtense Ω_s , as viewed from the hologram, the resultant image blur is approximately

$$\delta_s \sim z_0 \Omega_s \quad (29)$$

A final cause of image blur is diffraction at the slit

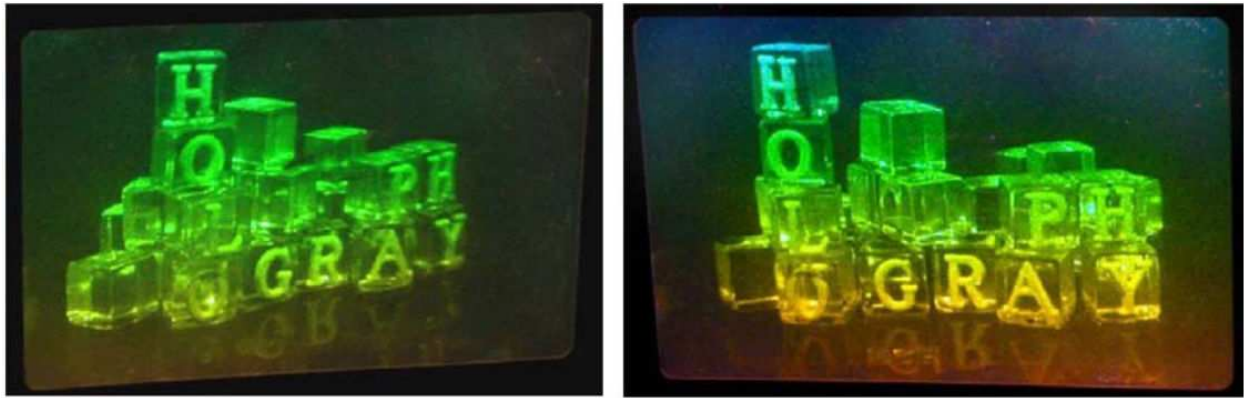


Figure 12. White light reconstruction of Benton's hologram "Holography Blocks" (made of tens of 1 inch transparent acrylic cubes) photographed from two directions to demonstrate its 3D property [20].

$$\delta_{diff} \sim 2\lambda (z_0 + L) / b \quad (30)$$

However, this can be neglected unless the width of the slit is very small.

Concluding this part, I would like to memory the inventor of this kind of holography, Benton S. A., and include the reconstruction of his *Holography Blocks*, the transmission rainbow hologram that he made of tens of 1 inch transparent acrylic cubes (1975). The pictures (Fig. 12) were photographed from his hologram from the left and from the right to demonstrate its 3D property by R. L. van Renesse and published in his monograph written in remembrance of Steve Benton after he passed away, in 2003 [20].

3. Some applications of WLRH

This paragraph consists of two parts. The first one mentions some situations briefly, at least, when anyone can meet white light reconstructed holograms. For those, taking an interest, there is great amount of Internet information accessible now, even pointing to details related to methods and technologies to produce WLRH.

Naturally, there are many scientific applications of WLRH, too. To solve the problem of commercial application can, also, be considered as a scientific application. I decided to use the second part to present short information, at least, on one of our former works, which was published in Slovak, only. It is focused on using the phase-shifting image holographic interferometry to study an optical fibre refractive index radial distribution. The image holography was used for us to be able to profit from possible magnification of the studied region.

3.1. Popular WLRH applications

When considering current applications of holographic technology enabling white light hologram reconstruction, i.e. displaying and viewing holograms using a common white light source, consumer products and advertising materials have to be mentioned firstly [1].

Security and product authentication seem to be the most popular growing areas for the use of holograms, especially of white-light reconstructed holograms. Why is it so?

Generally speaking, holograms can reconstruct one of two waves used to record them, when illuminated by the second one. That means optical reconstruction of 3D space from a 2D record. After four milestones in history of holography, represented by D. Gabor (father of holography), Yu. N. Denisyuk (invented volume reflection holography), E. N. Leith and J. Upatnieks (invented off-axis holography), and S. Benton (invented rainbow holography), the subsequent development of the micro-embossing technique allowed their mass replication [21].

For example a very popular variation became the *embossed hologram*. Such holograms are easily produced in mass quantity and at a very low cost. The holographic structure is recorded on a light sensitive medium (a photo-resistor), which can be processed by etching and a microscopic relief is created. By electro-deposition of a metal (nickel) on the relief a stamper is made and its surface relief is copied by impressing it onto another material (e.g. polyester base film, thermoplastic film).

It was soon realized that holograms might be used as security features on valuable documents and products and this way classical hologram became the first of a manifold of diffractive structures developed to thwart counterfeiting. Many of these diffractive structures can no longer be called holograms in the strict sense and some of them are no longer made by laser interference techniques, but created by advanced electron beam lithographic techniques [21].

These holograms, i.e. diffractive elements provide a powerful obstacle to counterfeiting. One can meet holograms either on various goods itself or in the packaging of products, on banknotes, various types of cards, and so on. For example - almost all credit cards carry a hologram, which is a good sign that security holography has proven to be very effective. There are various kinds of holographic labels and stickers [1].

Holographic labels are firmly affixed on the desired place of a product to verify its genuineness. Such labels cannot be copied, altered, either adapted or manufactured in an easy way. Moreover, hidden information can be embedded, visible at special circumstances, only. To increase the uniqueness of holographic labels, there are used another specific techniques, e.g. custom etching and overwriting of labels. To give some examples, holographic labels are used in various kinds of cards, artwork, banknotes, bank checks, product packaging for brand protection, alcohol, cosmetics, and so on.

Holographic stickers (HS) – most of them are self adhesive, providing authentication, security and protection against counterfeit, too. To increase their security level, various techniques are used. Let us mention some of them, like laser beam engraved dots (*dot-matrix HS*), double-exposed hologram of two objects from two directions (*flip-flop HS*) that displays two images from two different viewing angles), combinations of holograms, micro-information included, visible only by magnifier (*micro text/image HS*). Locally different thickness of the hologram layer gives reconstruction in various colours. A hidden text or image invisible to the naked eye but visible by means of a laser reader, serial numbers, overprinting left after the sticker is peeled off, are next examples of holographic stickers security increasing.

To minimise counterfeiting in holograms various methods during recording are used. It is possible to include hidden information or make the image so complicated that it is not worth to duplicate it, considering the time and money involved [1]. However, hidden information is of great value only if the cheater cannot find it or duplicate it. So, effective use of hidden information or any kind of complex images requires some sort of relatively simple and inexpensive reading device or decoding device.

Another possibility profits from using variable processing parameters, like randomly changed exposure, development time or other processing parameters to produce variable shrinkage all over the hologram. This results into a hologram whose colour varies from point to point and when using a monochromatic source - laser the brightness changes dramatically with the shrinkage of the film.

Holograms are not easy to counterfeit either if variable information like serial numbers, encoded personal information or dates are included or they are made of some special materials. Combined countermeasures can be another effective approach, too.

To show another ways of WLRH using, let us remember that in the beginning, holograms, a kind of "windows with a memory" with unfamiliar properties, were regarded as similar to daguerreotypes, photography predecessors [22]. Like early photography, holography was expected to develop technically and become cheaper, more capable and widespread. Where possible, people worked to realize these expectations of progress, especially for trade show displays. A pulse ruby laser had been developed to enable recording a human portrait in a darkened room to create static 3D human scenes for advertising. While such holograms attracted interest their display requirements (laser) made them too unwieldy to be sold or even displayed outside the laboratories.

Some artists, supported by scientists at first, began to take up holography. Inspired by the art and technology movement that was then exploring videotape, architecture, and other influences, they sought to make fine-art holograms. A second group - artisans intended to make holography an expressive medium for anyone.

However, the cost of the laser, needed for both, recording and viewing holograms, was a crucial constraint for artists and advertisers. Moreover, the monochromaticity of laser light provided portraits inferior to the panchromatic black-and-white films. For photographers then, holographic portraiture represented problems not progress. These limitations restricted holography to a narrow class of subjects and applications [22].

Denisyuk's holograms offered a solution of the most pressing problem for aesthetic and commercial users - the need for a laser to display the hologram. Then Benton's rainbow hologram also became widespread, which could be viewed in white light in a spectrum of colours.

As mentioned above, embossed holograms provided new audiences, manufactured by the millions on metal foil, they became ubiquitous in packaging, graphic arts, and security applications. Embossed holograms were inexpensive, reducing the cost of copies by a hundredfold. They could be mass-produced reliably by using a number of proprietary techniques. And they were chemically and mechanically stable, unlike most previous hologram materials that were susceptible to breakage, humidity, or aging [22]. Together, these technical advantages promoted the widespread application of embossed holograms.

On the other hand, their flexibility, particularly on magazine covers, caused colour shifts and image distortion. Moreover, the holograms were usually viewed in uncontrolled lighting, images could appear fuzzy or dim. In response to these limitations, their producers progressively simplified the imagery. This way embossed holograms promoted low-cost mass production but had relatively poor image quality. However, these characteristics were deemed to be a serious defect for imaging purposes. Fine-art holograms declined in popularity, with artists complaining that embossed holograms irreparably devalued the aesthetic attraction of the medium [22].

Despite all of that, holograms are used in advertisement to attract potential buyers. They can be met on magazine and book covers. Display holograms are widely used wherever an audience needs to be reached (e.g. at trade fairs, presentations). Holograms found their place in the entertainment industry (movies), became popular in the area of packaging and for

promotional purpose. This type of holography can be found as a part of some either pure or combined holographic artist works (in special galleries), too [22, 23].

3.2. An example of WLRH in science - Interferometric analysis of optical fibre profile

There is a special interest focused to optical fibres all over the world. They became of great importance as parts of many photonic devices and various methods are used to study their properties. Radial profile of fibre optical thickness, i.e. of its refractive index is one of the most important fibre characteristics. Many papers arise contemporary. Mostly they are based on classic interferometric microscopy of a fibre located in a wedge shaped layer of an immerse liquid. When applying optical imaging, methods are limited by the diffractive resolution $\lambda/2n \cdot \sin\theta$, where n is the refractive index of the object space, θ is the angle related entrance aperture of the objective. The limit follows from the Rayleigh criterion, applicable in the Fraunhofer approximation. Taking into account a conception of near-field optics it would be a better solution [12].

The experimental method elaborated in our paper is based on double-exposure phase-shifting interferometry. Moreover, the image holographic interferometry was used to profit from both the advantages - lower demands on optical quality of all the set-up elements and also the possibility of white-light reconstruction and magnifying the object size. Moreover, phase-shifting interferometry allows determining both, the size and direction of optical path changes. Fig. 13 demonstrates the experimental set-up. The laser beam is divided into collimated waves, object (o) and reference (p) ones. The object V is projected on the recording medium with a proper magnification and hologram H is recorded. To apply the phase-shifting interferometry the wedge K inclines the reference beam between two expositions. V represents two states of the object – either a glass cell filled with glycerine ($n_K = 1,4534 \pm 0.0005$) and the fibre embedded into it, or the glass cell filled with glycerine without the fibre.

The glycerine helped to reduce somehow the influence of the basic index of refraction n_0 of the fibre on the interference pattern. Accurately, the cell exit was projected on the plane of hologram. The object wave passes through the fibre perpendicularly to its axis.

The glass wedge K , passed through by the reference wave enabled phase-shifting interferometry. It was a very simple method to change the object-reference-beam angle in the order of the hundredths of a degree [24].

Two fibre specimen of the same kind from Slovak Academy of Sciences were used with radii $R_1 = 0.825$ mm and $R_2 = 0.55$ mm. The numerical aperture of them was 0.22 and relation between radii $R_j = 0.9R$ (Fig. 13).

The basic fibre index of refraction was $n_0 = 1,45718$ (at $\lambda = 632.8$ nm).

Interferograms (e.g. Fig. 14) were analysed supposing radial refractive index increment distribution $\Delta n(x, y)$ within the core radius R_j and the thickness of the glycerine layer d . The change of optical thickness Δl at coordinate y (Fig. 13) can be expressed in the form

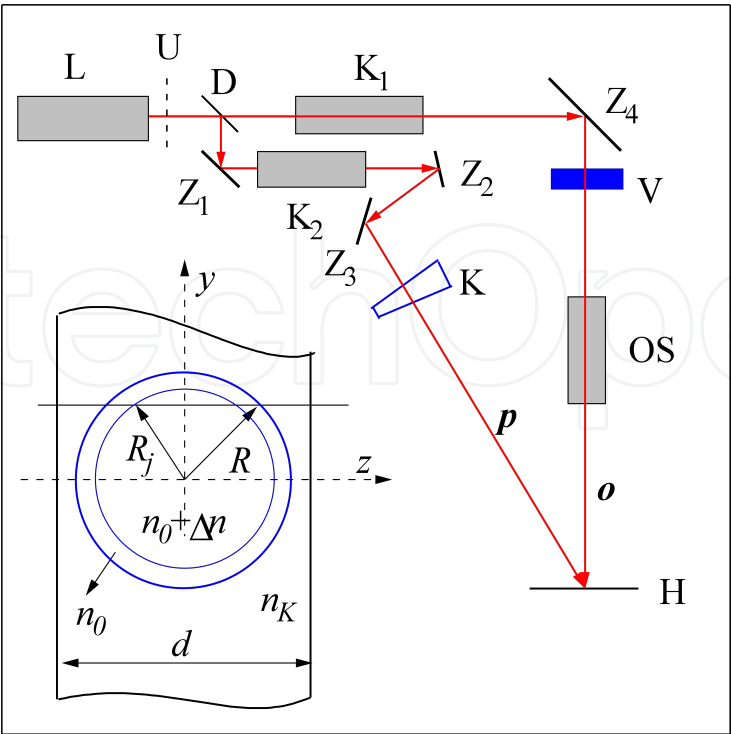
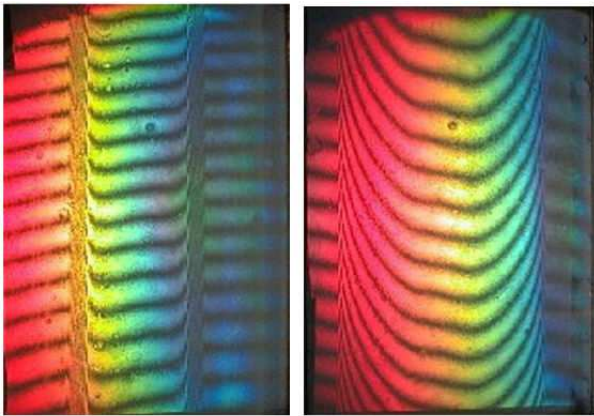


Figure 13. Experimental set-up. L–He-Ne laser, D–beam splitter, U–switch, K_i –collimators, Z_i –mirrors, K–wedge, V–the object, OS–objective, H–hologram, \mathbf{o} (\mathbf{p})–object (reference) beam. In left bottom corner - the scheme of the fibre cross-section in a cell.



$R_1 = 0.55 \text{ mm};$	$R_2 = 0.825 \text{ mm};$
$\Delta m_1 \sim 4$	$\Delta m_2 \sim 5.5$
$\Delta n_1 \sim 0.015$	$\Delta n_2 \sim 0.017$

Figure 14. Demonstration of interferograms.

$$\Delta l(y) = 2 \int_0^{R(y)} [n_0 - \Delta n(r) - n_K] dz + 2 \int_0^{R_j(y)} \Delta n dz \quad (31)$$

Supposing axially symmetrical refractive index distribution, it is convenient to introduce the substitution

$$z(y) = \sqrt{r^2 - y^2} \quad (32)$$

Interference maxims appear when Δl is equal to an integral number of wavelengths - $m\lambda$. Due to that the interference order m becomes a function of the coordinate y

$$m(y) = \Delta l(y) / \lambda \quad (33)$$

The shift Δm of the interference order was estimated from the interference pattern obtained (Fig. 14), which were digitalized and numerically analysed using the relation (33). Fig. 15 is to demonstrate analysis of one set of experimental data. It is necessary to realize that also the change $\Delta m_c(y)$ caused by the cylindrical shape of the fibre, denoted as y_0 in Fig. 15, is included in the experimentally obtained distribution of $m(y)$ expressed by y_m in Fig. 15. The distribution $\Delta m_c(y) = y_0$ was determined theoretically using the known fibre parameters.

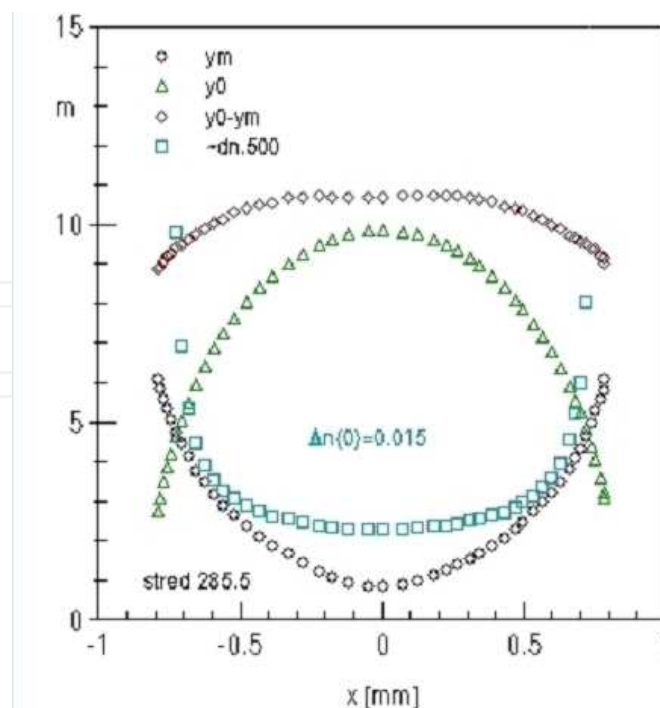


Figure 15. Analysis of experimental data

This way the producer's data, related to the original preform were approved (a stepped-index fibre with core radius $R_j = 0.9R$ and $\Delta n = 0.016$). The results obtained confirmed possibilities provided by holographic interferometry to analyse refractive index profile of an optical fibre in the region of sizes not demanding near-field optics concepts. The numerical analysis had been improved applying method of genetic algorithm and extended to a gradient-index fibre [25].

4. Conclusion

Today, holography seems to be a common method of optical information recording and especially - information advertising. Not only holograms can be met everywhere, moreover, terms, like "hologram" and "holographic" can be found in many fields of human life.

I suppose white-light reconstructed holograms to be of great interest especially for common public, without a special optical education. However, it is not possible to restrict their usage for public reasons, only. They are applicable as both, scientific and measuring tools, too.

Great amount of information is accessible by the Internet. However, it is important to read patiently, to find the truth. Due to that I tried to explain especially the basic principles to understand the matter.

To conclude, I would like to use another quotation from a very interesting paper, devoted to a historian's view of holography [22]: *„From a historian's point of view, then, holography represents a fascinating case of modern science and technology. It is a complex example of a surprisingly common but little noticed situation in modern science, in which a technical subject has created new communities and grown with them. Its evolution has been distinctly different from what most historians of science-and even holographers-might have expected, which can help us to better understand how modern sciences emerge, and how to more realistically chart their future trajectories. And because of the rich variety of communities that the subject has embraced, ranging from artists to defense contractors, its history is likely to be of enduring interest to broad audiences.“*

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