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## Introduction of the Ratio of the Hardness to the Reduced Elastic Modulus for Abrasion

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#### 1. Introduction

Modeling the wear rate is a complex process. The several possibilities of chemical, physical and mechanical changes at the interface are the most probable reasons for this [1]. In this manner, it is reasonable to consider the wear rate as a stochastic process [2, 3], and indeed this approach was taken into account by Archard [4], when he formulated his well-known model. Since then, the majority of available models are based on his proposition, independent on the characteristics of mechanical system. Considering a sharp contact, both Torrance [5] and Yi-Ling & Zi-Shan [6] for sliding and rolling abrasion, respectively, modified Archard's equation based on elastic effects, and the ratio of the hardness (H) to the Young's modulus (E) was the main parameter of the models. In a tribological system with dissimilar materials, for example a ceramic abrading a metal, one material can experience the yielding and the other the brittle failure. This difference in the mechanical behaviors can be decisive for the final performance to wear.

The use of Young's modulus to model the wear resistance was applied for coatings [7]; it appeared in other modifications of Archard's equation [8], and even in empirical relationships between the wear rate and the mechanical properties [9]. Eventually, in all cited cases, the elastic modulus was not the reduced one  $(E_r)$ , as will be treated here.

A selection of parameters involving hardness and elastic moduli can be summarized (Table 1) [9-11]. All of them have physical meanings that possess some interest for abrasion resistance of materials. The last parameter presented in Table 1,  $H^3/E^2$ , is proportional to the load that defines the transition between elastic to plastic contact in a ball-on-plane system, applying the analytical solutions provided by Hertz in Contact Mechanics [12], where the reduced modulus is already taken into account.



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Parameter	Physical meaning (taking into account a rigid-plastic material)
H/E	Deformation relative to yielding [9]
(H/E) <sup>2</sup>	Transition on mechanical contact – elastic to plastic [9]
H <sup>2</sup> /2E	Modulus of resilience [9]
H/E <sub>r</sub> <sup>2</sup>	Resistance to the plastic indentation [10]
H <sup>3</sup> /E <sup>2</sup>	Resistance to the plastic indentation [11]

**Table 1.** Parameters based on the hardness and elastic moduli, used as indicators of abrasion resistance and their physical meanings

Using some of the abovementioned solutions, a case study will be presented. For two tribological pairs with known wear coefficients, the ratio of hardness to reduced modulus works well than the single property of the worn material (E). The expectation regarding the  $H/E_r$  ratio is confirmed also by other aspects used to characterize abrasion, especially the cutting efficiency.

#### 2. Modelling abrasion with E/H ratio

In 1980 Torrance [5] published a model for abrasive wear rate based on the elastic recovery after scratching, supposing that the abrasive particle has a conical geometry. This choice is suitable, because there is an analytical model to describe the changes at the recovered surface [13]. Some years later, another paper [6] adopted the same physical basis but here the application occurred to systems where the abrasive particles roll, instead of slide. The key similarities and differences of both manuscripts will be discussed below.

First of all, it is important to see the main definition presented in [13], because it was the basis for the referred models. This reference presents an equation that relates the reduced modulus with the amount of elastic recovery, h<sub>e</sub> (indicated in Figure 1), considering a conical indenter:



In Equation 1 the term  $E_r$  is the reduced modulus, defined as:

$$\frac{1}{E_r} = \frac{1 - v_i^2}{E_i} + \frac{1 - v^2}{E}$$
(2)

where,

 $E_r$  is the reduced modulus;

E<sub>i</sub> is the Young's modulus of conical indenter;

 $v_i$  is the Poisson's ratio of conical indenter;

E is the Young's modulus of tested material, and;

 $\nu$  is the Poisson's ratio of tested material.



**Figure 1.** Elastic recovery during an indentation process. Symbology - a: indentation radius,  $h_e$ : elastic recovery;  $h_P$ : final depth; h: the maximum depth;  $h_c$ : contact depth and  $h_s$ : deflected depth. Adapted from ISO 14577-1 [14]

A modern definition for the term  $E/(1-v^2)$  can be found in ISO/FDIS 14577-1 standard [14], and it is called as 'indentation modulus', using the symbol  $E_{IT}$ . Exactly this term was used by references [5] and [6]. In this way, the mechanical properties of abrasive particle were discarded in both cases. It is notable that this aspect has not been ruled out by Stilwell and Tabor [13] in 1961.

A great difference between the Torrance's paper [5] and the Yi-Ling and Zi-Shan one [6] is with respect to the volume of wear. In the former, it was taken as directly proportional to  $h^2$ , and for the latter, related to  $h_p^2$ , following the symbology of Figure 1. The latter can be considered as more appropriate because it takes into account the elastic effects at a worn surface, so that the final formulation provided by [6] will be presented. Thus, an equation for wear rate, Q (m<sup>3</sup>/m), can be written as:

$$Q = C \frac{W}{H} K_P \tag{3}$$

where,

W is the applied load;

H is the hardness of worn material;

C is a constant and;

 $K_P$  will be called here as partial wear coefficient, based on elastic effects during indentation, defined as  $(1 + k \times H / E)^2$ , being k another constant.

In order to differentiate the use of elastic modulus to the reduced one, another symbology will be considered for the latter case, where  $K_{P}$ ' is introduced:

where, 
$$K_{p}' = \left(1 + k \times H / E_{r}\right)^{2}$$
(4)

k is a constant.

To define the constant C, Yi-Ling and Zi-Shan [6] made use of nine constants to fit the experimental results. The constant k of term  $K_P$  varies with many variables of tribosystem. On the other hand, although Torrance [5] has not make use of constants to fit experimental results obtained by others [15-17], he chose a material as reference (a steel of 401 HV hardness), and all cases were then compared with this material (Equation 5). In this way, he only needed to calculate the constant k (=10), finding a very interesting result, as can be seen in Figure 2.



$$\beta_{i}^{*} = \frac{H}{H_{ref}} \frac{\left(1 + 10H_{ref} / E_{ref}\right)}{\left(1 + 10H / E\right)}$$
(5)

**Figure 2.** Relative wear resistance ( $\beta_i$ ) as a function of relative partial wear coefficient ( $\beta_i^*$ ), as defined in [5]. Experimental points derived from [15-17]

Figure 2 shows a linear relationship between the relative wear resistance  $\beta_i$  and the relative partial wear coefficient, defined in [4] as  $\beta_i^*$  (Equation 5). The experimental points used to build this curve included pure metals and heat-treated steels. These groups of materials do not present the same behavior when they are abraded by hard particles (Figure 3).

In other words, a pure metal with similar hardness of heat-treated steel presents a higher abrasion resistance. This implies that steels present a different slope on a wear resistance curve when it is put as a function of hardness. Using the ratio of the hardness to the Young's modulus, Torrance [5] put in the same curve these referred groups of materials, which present, a priori, different behaviors. This kind of result was also described in reference [19], but based on a different approach. This research made use of the abrasion factor ( $f_{ab}$ ) definition (Equation 6 and Figure 4 [20]) to describe the wear resistance. It is a parameter related with the cutting efficiency, i.e., when this factor is equal to unity only the micro-cutting would be the wear mechanism, while for values correspondent to zero no removal of material would be detected. The experimental results obtained in [19] are shown in Figure 5.



**Figure 3.** Schematic representation of the wear resistance (Q' = 1/Q) produced by hard particles as a function of hardness, considering pure metals and heat-treated steels. Adapted from reference [18]

$$f_{ab} = \frac{A2 - A1}{A2} \tag{6}$$

where,

A1 is the cross section area relative to pileup produced by a single scratch and;

A2 is the cross section area relative to the groove produced by a single scratch.



Figure 4. Abrasion factor definition (areas A1 and A2 defined in Equation 6). Real profile adapted from reference [20]



**Figure 5.** Abrasive wear resistance (Q') produced by flint particles as a function of wear debris hardness-to- $f_{ab}$  ratio. Adapted from reference [19], which defines Q' as a non dimensional parameter

Again, in Figure 5 both pure metals and steels are put in the same curve, showing that the hardness alone, in some cases, is not a complete descriptor of the wear resistance. The similarity between the results presented in Figure 2 and Figure 5 opens a possibility to the abrasion factor to be described as a function of the hardness-to-elastic modulus ratio.

Although in reference [19] the elastic effects have not been incorporated to the model, there are results in the literature relating the pileup formation (Figure 6, [21]) with the mechanical properties [22]. In this case, it is possible to consider that the pileup formation (h<sub>c</sub>/h) works for

static (hardness test) and kinetic cases (scratch test), being the higher the pileup, the smaller the cutting efficiency.



**Figure 6.** Physical parameters extracted from a residual profile of a spherical indentation. Notation:  $a_c$  is the indentation radius at the contact, and *s* is the height associated to the indentation morphology. Pileup is associated to the  $h_c/h$  ratio. Adapted from reference [21]

In reference [22] an equation that correlates the pileup formation with the H/E ratio, based on results obtained in scratch tests conducted with a Berkovich indenter (Figure 7) can be found:

$$\frac{h_C}{h} = 0.41498 \ln \frac{E}{H} - 0.14224$$
 (7)

Important evidence was detected in [23] that the application of Equation (7) was dependent on the level of applied load.



**Figure 7.** Relation between the pileup ( $h_c/h$ ) and the ln(E/H), obtained after scratch tests with Berkovich indenter. Adapted from reference [22]

#### 3. Case study

Two tribological pairs were studied, also considered in a previous investigation [24]. Their wear rates are known: glass abrading a quenched and tempered (Q&T) 52100 steel [25], and alumina wearing a hard metal [26]. The mechanical properties, determined from instrumented indentation testing, used to calculate the partial wear coefficients,  $K_P$  and  $K_P'$  (Equations 3 and 4), are presented in Table 2. To calculate the reduced modulus (Equation 2), Poisson's ratios were extracted from [27].

As seen in the previous item, due to the similarity of Figures 2 and 5, the partial wear coefficient can be well related to the abrasion factor (cutting efficiency),  $f_{ab}$ . An indirect way to know the  $f_{ab}$  factor is found in reference [28], which defined it as the ratio between the wear coefficient K (Archard's definition) and the ploughing fraction of friction coefficient ( $\mu_p$ ). The cutting efficiency values, following this definition, are 0.106 for glass abrading bearing steel and 0.079 for alumina wearing hard metal, a difference of 34%. As the constant  $K_P'$  can vary along a broad range (Figure 8), we select a value so as to the difference is also 34%, and for this purpose it is 8.25.

Material	H, GPa	E, GPa	E <sub>r</sub> , GPa	H/E	H/E <sub>r</sub>
Soda-lime glass	4.07	69	- 53.24	0.080	0.103
Q&T steel	5.5	180*		0.023	0.076
Alumina	19.6	376.1	222 12	0.052	0.088
Hard metal (grade K)	11	480	- 222.45	0.023	0.049

**Table 2.** Mechanical properties of selected tribological pairs. \*Obs.: Q&T steel is a wire-drawing, which implies in a reduction of elastic modulus due to the work-hardening effect

An example of variation in  $K_{P}$  value with k factor is presented in Figure 8, for the tribological pairs considered in Table 2. Another pair was added (calcite-fluorite) in this figure, in order to help the discussion with their differences.



Figure 8. Variation of  $K_{P}'$  wear coefficient with factor k for three cases

The resulting values of partial wear coefficients, using 8.25 as k factor, are presented in Table 3.

Pair	$K_P$ (equation 3)	$K_{P}'$ (equation 4)	
Glass - Q&T steel	1.41	2.66	
Alumina – Hard metal	1.41	1.98	

Table 3 shows that the  $K_P$  values were similar for the studied pairs, while the  $K_P'$  did not follow this trend. The wear coefficients of tribological pairs (K values) determined using sliding abrasion tests were 0.014 and 0.008, for glass against steel [25] and alumina against hard metal [26], respectively. Visibly this difference in wear rates is only reflected by the  $K_P'$  values, which is a direct result of the ratio of hardness to the reduced modulus.

Another observation for the  $K_P$  values is that the similarity presented in Table 3 is not affected by the variation of the k factor. On the contrary,  $K_P'$  is strongly dependent on this factor, as can be seen in Figure 8.

One can ask about the good agreement with experimental data found in [5] and [6] when they used only the Young's modulus in wear model. The key point for that is the superiority of abrasive hardness in relation to the worn surface. All experimental results in these cases were performed using abrasives much harder than the tested materials, such that the mechanical properties of them can be considered as unaltered along the tests. An investigation [29] showed significant changes in glass particles when they abraded steel surfaces, even for non heat-treated ones. The same thing was demonstrated in [30] for different abrasives wearing WC-Co thermal sprayed coatings. When the abrasive particle is relatively soft to the abraded material, their mechanical properties play a key role during the wear process, and an extensive discussion on it can be found in [31].

In addition, in reference [24] these tribological pairs were separated using the difference in the plasticity index,  $\delta_{H'}$  defined in [32] (Equation 8), being the smallest difference related to the calcite-fluorite, and the largest for glass-steel pair within the materials analyzed in [24]. This aspect seems to be important again when one observes Figure 8. When the difference is insignificant, as in the case of calcite-fluorite, the variation of  $K_{P'}$  with k factor is minor, and on the other hand, for the case of the glass-steel pair, for a notable difference in the plasticity index, a great variation of  $K_{P'}$  with the k factor occurs.

$$\delta_H = 1 - 14.3 (1 - \nu - 2\nu^2) H \Big|_E \tag{8}$$

A brief discussion of the k factor is instructive. Following [6], this factor is especially associated to the particle geometry. As stated, a hypothetical  $K_{P'}$  of calcite-fluorite pair would be less affected by the variation on k factor. Therefore, one can imagine a small fragment of mineral

used in Mohs scale [33] and, considering the previous assertive, conclude that its geometry would not be important. Probably, this is not the case. The k factor should be understood in a broad manner, i.e., all variables of a system can alter its value. Thus, slight changes in tribological variables could bring higher alterations in the wear coefficient for the pair glass-steel than for the other described cases. At this moment, no experimental result is available to corroborate this hypothesis, but it is an interesting field to be explored further.

Finally, a discussion concerning the Equation (7) and the values presented in Table 2 is valuable. If the values of H/E ratio for steel and alumina were applied in Equation (7), the results would be similar. A similar height of pileup for these materials obviously is not a reasonable result, taking into account the experimental values obtained for wear and friction coefficients for them. Nevertheless, applying the reduced modulus in the place of E in that equation, one can find fewer tendencies to form pileup after abrasion of steel by glass, which means a higher cutting efficiency in this case, meeting with the values described before. Consequently, it is more a case of successful application of reduced modulus to predict changes at the mechanical contact.

#### 4. Conclusions and final remarks

The viability of the use the hardness-to-reduced modulus ratio to model the wear coefficient for abraded materials was demonstrated. Previous models were developed taking into account only the Young's modulus of worn surface, discarding the properties of abrasive material. These cases work only for pairs where the abrasive particle is harder than the abraded material and it was demonstrated that they fail when the abrasive hardness is relatively low.

In addition, other questions were discussed and they open some possibilities to carry out future research. First, the model of wear coefficient treated here involves higher requirements, because a constant is needed. This constant seems to affect more the wear coefficient of some pairs than the others, and the reason for that is not clear. Finally, an extensive work could be made exploring the relation between cutting efficiency (abrasion factor) and  $H/E_r$  ratio, computing a large variation of the applied load and the abrasive (indenter) properties. Probably, an investigation based on these aspects should supply answers to the improvement of a wear model containing the  $H/E_r$  ratio.

#### Nomenclature

A1 and A2 Cross section areas used in the definition of abrasion factor

a Indentation radius

- $a_c$  Indentation radius at the contact
- C Constant

- E Young's modulus
- E<sub>i</sub> Young's modulus of indenter
- E<sub>r</sub> Reduced modulus
- $E_{ref}$  Young's modulus of a reference material
- f<sub>ab</sub> Abrasion factor or cutting efficiency
- H Vickers hardness of the worn material
- $H_d$  Vickers hardness of wear debris
- $H_{ref}$  Vickers hardness of a reference material
- h Maximum depth at applied load
- h<sub>C</sub> Contact depth
- h<sub>e</sub> Elastic recovery
- h<sub>P</sub> Final depth
- h<sub>s</sub> Deflected depth
- K Wear coefficient
- $K_P$  Partial wear coefficient, defined as  $(1 + k \times H / E)^2$
- $K_{P'}$  Partial wear coefficient calculated with Er instead of E
- k Constant
- Q Wear rate
- Q' Wear resistance (= 1/Q)
- s Height associated to the indentation morphology
- W Applied load
- $\beta_i$  Relative wear resistance
- $\beta_i^*$  Relative partial wear coefficient
- $\delta_H$  Plasticity parameter
- $\mu_p$  Ploughing fraction of friction coefficient, taken as 0.2
- $\nu$  Poisson's ratio of the worn material
- $\nu_i$  Poisson's ratio of the indenter

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