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Chapter 30

Pressurization of a PKN Fracture in a Permeable Rock During Injection of a Low Viscosity Fluid

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Additional information is available at the end of the chapter

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Abstract

The aim of the present work is to investigate injection of a low-viscosity fluid into a pre-existing fracture within a linear elastic, permeable rock, as may occur in waterflooding and supercritical CO2 injection. In conventional hydraulic fracturing, high viscosity and cake building properties of injected fluid limit diffusion to a 1-D boundary layer incasing the crack. In the case of injection of low viscosity fluid into a fracture, diffusion will take place over wider range of scales, from 1-D to 2-D, thus, necessitating a new approach. In addition, the dissipation of energy associated with fracturing of the rock dominates the energy expended to flow a low viscosity fluid into the crack channel. As a result, the rock fracture toughness is an important parameter in evaluating the propagation driven by a low-viscosity fluid. We consider a pre-existing, un-propped, stationary Perkins, Kern and Nordgren's (PKN) fracture into which a low viscosity fluid is injected under a constant flow rate. The fundamental solution to the auxiliary problem of a step pressure increase in a fracture [1] is used to formulate and solve the convolution integral equation governing the transient crack pressurization under the assumption of negligible viscous dissipation. The propagation criterion for a PKN crack [2] is then used to evaluate the onset of propagation. The obtained solution for transient pressurization of a stationary crack provides initial conditions to the fracture propagation problem.

1. Introduction

The problem of injection of a low-viscosity fluid into a pre-existing fracture may arise in several rock engineering areas, such as, injection of liquid waste (e.g., supercritical CO2) into deep geological formations for storage [3,4,5], waterflooding process to increase recovery from an oil reservoir [6], and control of possible leaks from pre-existing fractures around

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radioactive and nuclear wastes storage sites [7]. These fractures could be either of natural origin or man-made (e.g., hydraulic fractures used to stimulate production from a now depleted reservoir chosen for waste storage).

This paper attempts to study injection of a low viscosity fluid into a pre-existing un-propped fracture of the Perkins and Kern and Nordgren (PKN) geometry within a linearly elastic, permeable rock. In the classical PKN model, the fracture length is much larger than the fracture height [8] with the latter confined to a permeable (reservoir) layer sandwiched between two impermeable (cap) rock layers. This assumption allows to model a vertical fracture cross-section as a pressurized Griffith (plane strain) crack.

Until recently, in part due to the lack of a reliable fracture breakdown criterion for a PKN fracture, studies of the PKN fracture propagation have been bounded to the limiting regime corresponding to the dominance of the viscous dissipation in the fluid flow in the crack channel, i.e. when the rock toughness can be neglected [9, 10]. This particular dissipation regime is favored when a high viscosity fracturing fluid and/or high injection rates are used, or at late stages of fracture growth (long fractures). Moreover, for sufficiently large time, the history of injection prior to the onset of the viscosity dominated regime may have minor impacts on the modeling of the classical PKN fracture.

In unconventional hydraulic fracturing (injection of a low-viscosity fluid), on one hand, the dissipation of energy to extend the fracture in the rock may not be negligible compared to the viscous dissipation. On the other hand, the injection history prior to the onset of propagation may not be neglected. With this in mind, we investigate fluid injection into a stationary, pre-existing fracture up to the onset of the propagation, which is defined by the recently introduced propagation criteria for a PKN fracture [2]. The corresponding transient pressurization and leak-off history prior to the breakdown will provide initial conditions for the problem of a propagating PKN fracture in the toughness dominated regime, to be addressed elsewhere.

Contrary to conventional hydraulic fracture where high viscosity and cake-building properties of injected fluid limit the leak-off to a 1-D boundary layer incasing the crack, the low viscosity fluid allows for diffusion over a wider range of scales from 1-D to 2-D. Although, several investigations looked at the propagation of a fracture driven by a low viscosity fluid, when fluid diffusion is fully two-dimensional [11, 12, 13], the study of injection into a stationary, pre-existing fracture has not yet received due attention. One of the foci of this study is to identify solutions corresponding to the limiting cases of the small and large injection time (1-D and fully-developed 2-D diffusion, respectively), and the solution in the intermediate regime corresponding to the evolution between the two limiting cases.

This paper is organized as follows. In section 2 we define and formulate the problem. In Section 3 we first revisit the problem of a step pressure increase in a crack [1], which we then use to formulate and solve the problem of transient pressurization of a crack due to a constant rate of fluid injection. The criterion of PKN propagation [2] is used to evaluate the onset of the fracture propagation. We illustrate the results of this study by considering a case study in which the transient pressurization and the breakdown of pre-existing fractures of different lengths are evaluated for a water injection project.

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2. Mathematical formulation

2.1. Problem definition

We consider a pre-existing, un-propped (zero opening) crack of length 2ℓ and height h within a linearly elastic, permeable rock characterized by the plane strain modulus $E^{'}$ and toughness K_{Ic} (Figure 1). The crack is aligned perpendicular to the minimum in-situ stress σ_{min} and is loaded internally by fluid pressure p_f, generated by the fluid injection at the crack center at a constant rate Q_o. The following assumptions are used in this work. 1) The crack height is small compared to the length, such that the deformation field in any vertical cross-section that is not immediately close to the crack edges ($x = \pm\ell$) is approximately plane-strain, and the fluid pressure is equilibrated within a vertical crack cross-section (the PKN assumptions). 2) The minimum in-situ stress σ_{min} and the initial reservoir pore pressure p_0 are uniform along the crack. 3) Initial reservoir pore pressure p_0 is approximately equal to the minimum in-situ stress σ_{min}, allowing the crack to open immediately upon the start of the injection; or alternatively, time t_0 from the onset of injection that is required to pressurize the initially closed crack ($p_0 < \sigma_{min}$) to the point of incipient opening ($p_f(t_0) = \sigma_{min}$) is small compared to the timescale of interest (e.g., the time to the onset of the fracture propagation). 4) Injected fluid is of a low viscosity (and/or the rate of injection is slow), such that the viscous pressure drop in the crack is negligible, or, in other words, the fluid pressure is uniform in the crack. 5) The crack is confined between two impermeable layers, which, together with the assumption of pressure equilibrium within a vertical crack cross-section, suggests a 2-D fluid diffusion within the permeable rock layer. 6) The injected and reservoir fluid have similar rheological properties.

2.2. Governing equations

2.2.1. Elasticity equation

The elasticity equation

$$w(x,z) = \frac{4\left(p_f(x) - \sigma_{min}\right)}{E'}\sqrt{\frac{h^2}{4} - z^2}, \tag{1}$$

is used to relate the opening of a PKN fracture w to the net pressure $p_f - \sigma_{min}$, which is assumed to be equilibrated in a vertical cross-section of the crack, $\partial p_f/\partial z = 0$, [14]. The opening of PKN fracture at mid height ($z = 0$) is

$$w(x) = \frac{2h}{E'}\left(p_f(x) - \sigma_{min}\right). \tag{2}$$

For the particular case of uniformly pressurized fracture, the fracture volume can be evaluated using the elasticity equation (1) as

$$V_{crack} = \frac{\pi h^2 \ell}{E'}\left(p_f - \sigma_{min}\right). \tag{3}$$

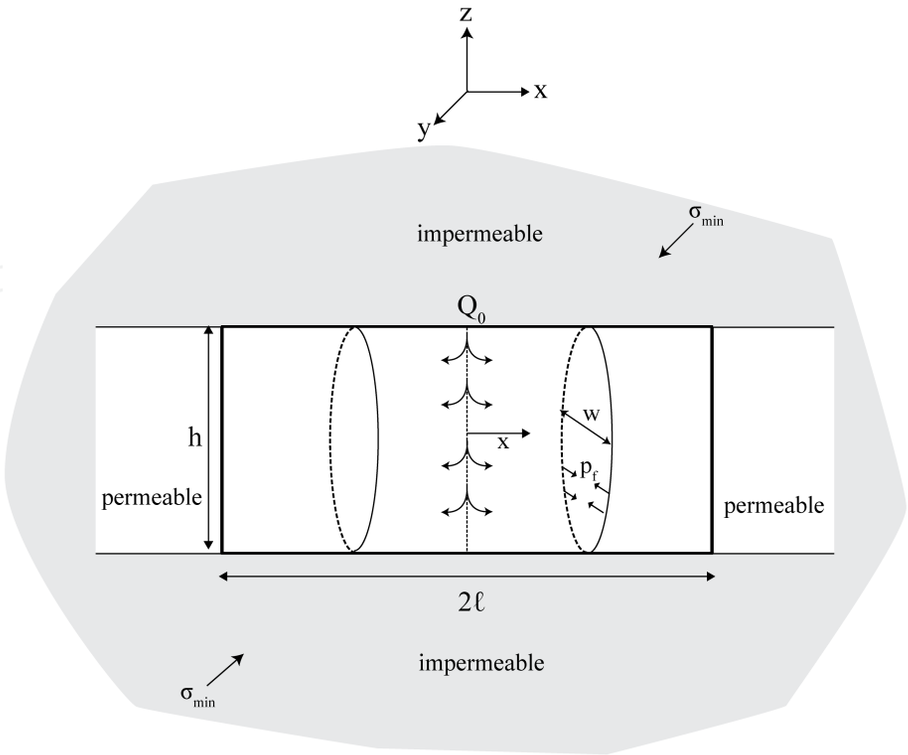


Figure 1. A pre-existing PKN fracture with length 2ℓ and height h

2.2.2. Fluid continuity

Local fluid continuity

Following [15, 16], lubrication equation can be used to describe the flow of an incompressible fluid in a crack (Figure 1) as follows

$$\frac{\partial w}{\partial t} + \bar{g}(x,t) = \frac{1}{12\mu}\frac{\partial}{\partial x}\left(w^3\frac{\partial p_f}{\partial x}\right), \quad \bar{g}(x,t) = 2g(x,t) \quad (t>0, |x|<\ell), \tag{4}$$

where $\bar{g}(x,t)$ is the fluid leak-off rate at the crack walls and μ is the viscosity of the injected fluid [17].

Global fluid continuity

The global volume balance of the fluid injected into the fracture is given by:

$$V_{inject} = V_{crack} + V_{leak}, \tag{5}$$

in which V_{inject} indicates the cumulative volume of the fluid injected into the fracture and V_{leak} is the cumulative leak-off volume.

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2.2.3. Propagation condition

The stress intensity factor $K_I = \sqrt{GE'}$ associated with the energy release rate G at the propagating PKN fracture edge is given by $K_I = (p_f(\ell) - \sigma_{min})\sqrt{\pi h/4}$ [2]. The criterion for the propagation of a PKN fracture in mobile equilibrium ($K_I = K_{Ic}$) can therefore be expressed as

$$p_f(\ell) - \sigma_{min} = \frac{2K_{Ic}}{\sqrt{\pi h}}. \tag{6}$$

2.2.4. Diffusivity equation and boundary integral representation

The Green's function method can be used to solve an inhomogenous differential equation subjected to boundary conditions. For the fluid flow through the porous media, the diffusivity equation is given by [18]:

$$\frac{\partial p}{\partial t} - \alpha \nabla^2 p = \frac{\dot{\gamma}}{S}, \tag{7}$$

where $\dot{\gamma}$ is the fluid source density (the rate of unit volume of injected fluid in a unit volume of material), $S = \phi c_t$ and $\alpha = k/\mu\phi c_t$ are fluid storage and diffusivity coefficients, respectively, expressed in terms of the formation permeability k, formation bulk compressibility c_t, and porosity ϕ. Due to the presence of the impermeable cap rock boundaries at $z = \pm h/2$ and pressure equilibrium in a vertical cross section , the diffusion problem is two dimensional (2-D). The general 2-D boundary integral for the pressure perturbation due to a distribution of instantaneous sources $g(x,t)$ $[L/T]$ along a crack $y = 0$, $|x| \leq \ell$ is given by [19]

$$p_f(x,t) - p_0 = \int_0^t \int_{-\ell}^{\ell} \frac{g(x',t')}{4\pi S\alpha(t-t')} \exp\left(-\frac{(x-x')^2}{4\alpha(t-t')}\right) dx'dt'. \tag{8}$$

3. Transient pressurization due to fluid injection

In this section, we study transient pressurization due to the injection of a fluid at a constant rate of flow into a pre-existing and stationary fracture. In order to facilitate the solution to this problem, we first revisit the fundamental solution to an auxiliary problem of a step pressure increase in crack [1] and introduce a new result for the large time asymptote of this problem. This fundamental solution is then used to formulate and solve a convolution integral equation governing the solution for the transient pressurization.

3.1. Auxiliary problem: step pressure increase

Consider a fracture subjected to a step pressure increase of magnitude p_*,

$$p(x,t) - p_0 = p_* H(t), \quad (|x| < \ell) \tag{9}$$

where $H(t)$ is a Heaviside function. To facilitate solution of (8) with (9), we rewrite it in the normalized form

$$1 = \frac{1}{2} \int_0^{\tau} \int_{-1}^{1} \psi(\xi', \tau') \exp\left(-\frac{(\xi - \xi')^2}{\tau - \tau'} \right) \frac{d\xi' d\tau'}{\tau - \tau'}, \tag{10}$$

where the nondimensional time (τ), coordinate (ξ), leak-off rate (ψ), and cumulative leak-off volume (Φ) are defined as

$$\tau = \frac{t}{t_*}, \quad \xi = \frac{x}{\ell}, \quad \psi(\xi,\tau) = \frac{\ell g(x,t)}{2\pi\alpha S p_*}, \quad \Phi(\tau) = \frac{2V_{leak}(t)}{\pi \ell^2 h S p_*}, \tag{11}$$

and $t_* = \ell^2/4\alpha$ is diffusion timescale. After applying Laplace transform, (10) becomes:

$$1/s = \int_{-1}^{1} \psi(\xi', s) K_0\left(2\sqrt{s}|\xi - \xi'| \right) d\xi', \tag{12}$$

where K_0 is the modified Bessel function of the second kind, s is the Laplace transform parameter and $\psi(\xi, s)$ is the Laplace image of $\psi(\xi, \tau)$.

Before integral convolution equation (12) is treated numerically, it is useful to consider its asymptotics for short and long injection times.

During the injection process when the characteristic lengthscale for fluid diffusion $\sqrt{\alpha t}$ is small compared to the crack size ℓ, or in terms of the normalize time, $\tau \ll 1$, the fluid diffusion pattern is approximately 1-D and the normalized leak-off rate is given by [1]:

$$\psi(\tau) = \frac{2}{\pi^{3/2}\sqrt{\tau}} \quad (\tau \ll 1). \tag{13}$$

As the injection time increases, the 1-D fluid diffusion pattern is no longer valid and a 2-D fluid diffusion pattern must be considered. We can show that for long enough injection times the Laplace image of the fluid leak-off rate is given by:

$$\psi(\xi, s) = \left(-\frac{\pi}{2}\sqrt{1-\xi^2}\, s \left[\ln(s/4) + 2\gamma \right] \right)^{-1} \quad (\tau \gg 1), \tag{14}$$

where $\gamma = 0.5772$ is the Euler's constant. The approximate image of (14) in actual time domain is:

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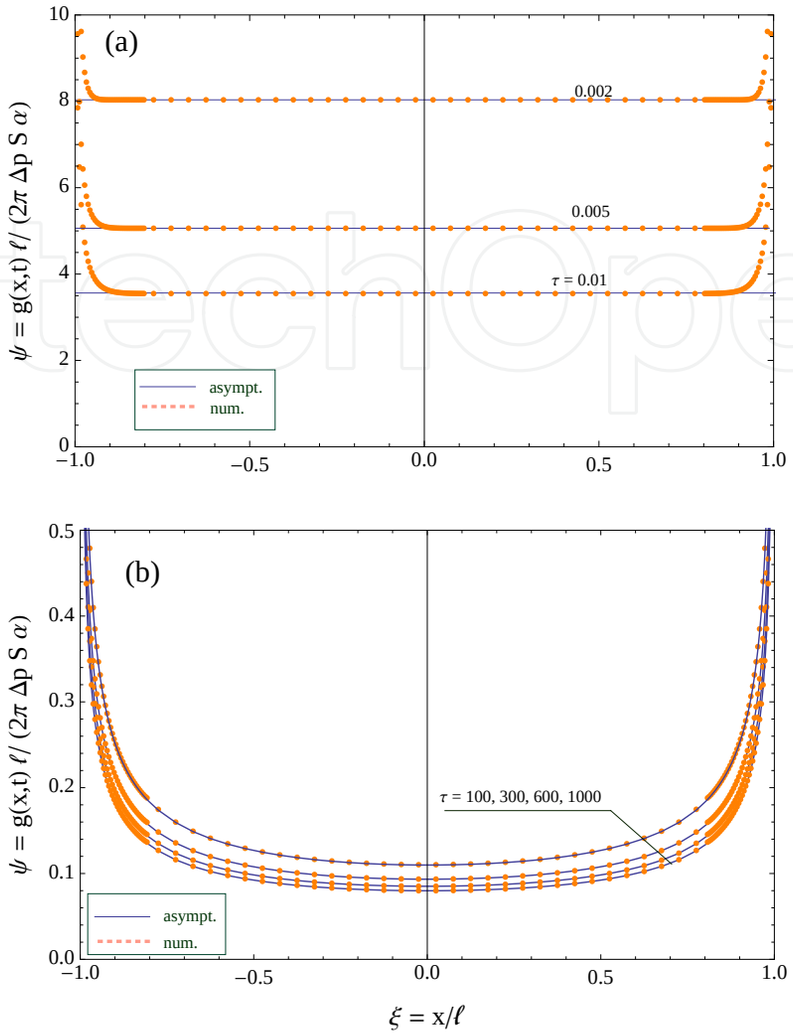


Figure 2. Comparison of the numerical solution for the normalized leak-off rate in the auxiliary problem (step pressure increase along the crack) with the small time (a) and large time (b) asymptotes.

$$\psi(\xi,\tau) \approx \left(\frac{\pi}{2}\sqrt{1-\xi^2}\left(\ln\left(\omega^2\tau\right)-2\gamma\right)\right)^{-1} \quad (\tau \gg 1), \tag{15}$$

where $\omega = 2.67$.

Following [1] we solve (12) numerically for the Laplace image $\psi(\xi,s)$ ($-1 \le \xi \le 1$ and $10^{-9} \le s \le 10^{9}$) using $N = 110$ discretization nodes along the fracture, and then apply inverse numerical Laplace transform (Stehfest algorithm [20] with six terms) to tabulate the solution for the normalized leak-off rate $\psi(\xi,\tau)$. This solution is contrasted to the small and large time asymptotes in Figure 2.

The normalized cumulative leak-off from a fracture subjected to a step pressure increase can be obtained by integrating from fluid leak-off rate with respect to time and space and is given by [1] :

$$\Phi(\tau) = \int_0^{\tau} \int_{-1}^{1} \psi\left(\xi', \tau'\right) d\xi' d\tau', \tag{16}$$

3.2. Transient pressurization problem

Assuming a uniform pressure along the crack channel (the viscous pressure drop in the crack is negligible), and "instantaneous" pressure build-up at the beginning of injection from the initial pore pressure value p_0 to the value $p_f = \sigma_{min}$ corresponding to the incipient crack opening, the cumulative leak-off volume V_{leak} can be obtained by the applying the Duhamel's theorem [1, 19]

$$V_{leak} = \nu(t)(\sigma_{min} - p_0) + \int_{0^+}^{t} \nu(t - t') \frac{dp_f}{dt'} dt', \tag{17}$$

where $\nu(t) = \frac{\pi}{2} \ell^2 h S \Phi(t/t_*)$ is the cumulative leak-off volume of the fracture subjected to a unit step pressure increase, as discussed in the previous section.

Equation (5) can be expressed in the case of fluid injection at a constant rate Q_0 as

$$Q_0 t = \frac{\pi h^2 \ell}{E'} (p_f(t) - \sigma_{min}) + \nu(t)(\sigma_{min} - p_0) + \int_{0^+}^{t} \nu(t - t') \frac{dp_f}{dt'} dt', \tag{18}$$

where expression (3) for V_{crack} was used. Let us now define a characteristic pressure perturbation $p_* = Q_0/(\pi h S \alpha)$, which is then used to scale the net pressure and the initial effective stress

$$\Pi = \frac{p_f - \sigma_{min}}{p_*}, \qquad \Sigma_0 = \frac{\sigma_{min} - p_0}{p_*} \tag{19}$$

respectively. Using normalized parameters (11) and (19), we convert (18) to the nondimensional form:

$$\frac{\tau}{4} = \eta \Pi(\tau) + \frac{1}{2} \Sigma_0 \Phi(\tau) + \frac{1}{2} \int_{0^+}^{\tau} \Phi(\tau - \tau') \frac{d\Pi}{d\tau'} d\tau', \qquad \left(\eta = \frac{h}{\ell S E'}\right) \tag{20}$$

where $\Phi(\tau)$ is the normalized cumulative leak-off rate in the auxiliary problem, (16), and η is a scaled crack height-to-length ratio. Applying the Laplace transform to (20) yields the solution for the Laplace image of the normalized pressure in the crack:

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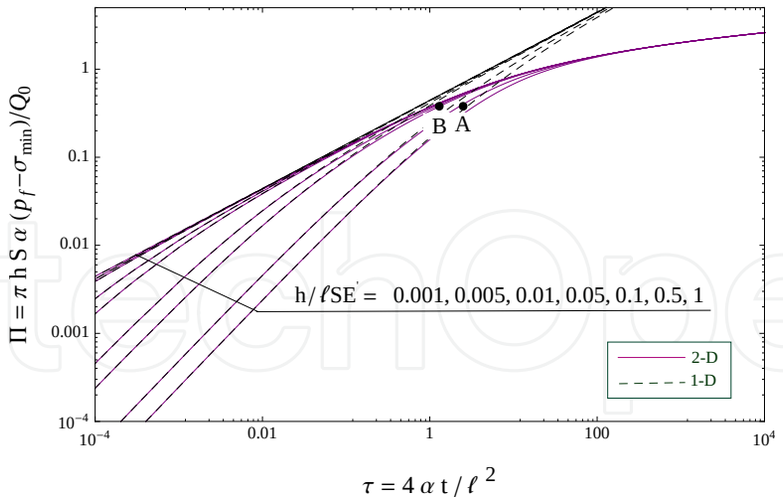


Figure 3. Evolution of the fluid net-pressure during a constant-rate injection into a stationary crack within an abnormally-pressurized reservoir $p_0 \approx \sigma_{min}$ for various values of the crack height-to-length ratio $h/\ell SE'$.

$$\Pi(s) = \frac{1}{2s^2}\frac{1 - 2\Sigma_0 s^2 \Phi(s)}{2\eta + s\Phi(s)}, \tag{21}$$

in which $\Phi(s)$ is the Laplace image of $\Phi(\tau)$. This solution is then numerically inverted to the time domain using the Stehfest algorithm [20].

Evolution of the normalized pressure Π during the transient pressurization of a crack is shown in Figure 3 for the case of an abnormally pressurized reservoir $\Sigma_0 \approx 0$ ($p_0 \approx \sigma_{min}$) and for various values of the scaled crack height-to-length ratio $\eta = h/\ell SE'$. (The 1-D diffusion solution to the same problem is shown by dashed lines for comparison).

With the solution for the normalized pressure in hand, the onset of the fracture propagation can be determined from the normalized form of (6):

$$\Pi = \Pi_B \quad \text{with} \quad \Pi_B = \frac{2\sqrt{\pi h} S\alpha K_{Ic}}{Q_0}, \tag{22}$$

where Π_B is the normalized breakdown pressure.

Example. Water injection project

Consider an example of the fracture breakdown calculations for a water injection project in a sandstone formation [21] characterized by porosity $\phi = 0.1$, permeability $k = 10.132$ md, pre-existing fracture height $h = 30.48$ m (assumed to span the height of the sandstone layer), minimum in-situ stress $\sigma_{min} = 28.8$ MPa, bulk rock compressibility $c_t = 5.35 \times 10^{-10}$ Pa^{-1}, fluid viscosity $\mu = 1$ cp, rock toughness $K_{Ic} = 1$ MPa $m^{1/2}$, plane strain modulus $E' = 9.3$ GPa. The reported injection rate was $Q_0 = 0.00052$ m^3/s. The calculated values are $S = 5.35 \times 10^{-11}$ Pa^{-1} (storage parameter), $\alpha = 0.19$ m^2/s (diffusivity coefficient), $p_* = Q_0/(\pi h S\alpha) = 0.537$ MPa (characteristic pressure perturbation). The normalized breakdown

net-pressure, (22), is $\Pi_B = 0.38$ (or, in dimensional terms, $(p_f - \sigma_{min})_B = p_* \Pi_B = 0.204$ MPa). We chose two arbitrary fracture half-lengths $\ell = 100$ m $(h/\ell SE^{'} = 0.61)$ and $\ell = 1000$ m $(h/\ell SE^{'} = 0.061)$ to estimate the onset of fracture propagation from Figure 3 to be at $\tau = 2.5$ (point A) and $\tau = 1.22$ (point B), respectively. The corresponding dimensional breakdown times are 9 hrs ($\ell = 100$ m) and 19 days ($\ell = 1000$ m).

4. Conclusions

Important applications of injection of a low viscosity fluid into a pre-existing fracture, such as waterflooding and supercritical CO2 injection in geological sequestration, necessitate comprehensive studies of mechanical and hydraulically properties of fractures from the beginning of injection until the onset of fracture propagation. In this study, we considered a low viscosity fluid injection into a pre-existing, un-propped crack of a PKN geometry. We focus on the case of a critically-overpressured reservoir and initially closed (un-propped) crack. The extension of this work to propped cracks and more general reservoir conditions are reported elsewhere.

The analysis assumes negligible viscous dissipation during injection of a low viscosity fluid at a sufficiently slow injection rate [22], and, as a result, approximately uniform pressure distribution in the crack. Furthermore, the poroelastic effects are also neglected in this study. To outline the validity of the latter assumption, we can show that the later stages of transient pressurization (the so-called leak-off dominated regime when the injection time $\gg$ diffusion timescale $\ell^2/4\alpha$) with and without poroelasticity effects are identical. However, the generated poroelastic backstress which tends to close the fracture may cause a delay in the initiation of crack propagation when compared to the case where poroelastic effects are neglected. In addition, for certain ranges of fracture and fluid properties and field operating condition the backstress may become large enough to prevent the fracture from propagation indefinitely.

We evaluated the evolution of the fluid pressure inside the fracture during the transient pressurization by considering 2-D fluid diffusion from the fracture into the surrounding porous rock. As the fracture is pressurized, the condition for the onset of its propagation (breakdown condition) is eventually reached. We quantified how the fracture breakdown condition depends upon the rock and fluid properties, the in-situ stress and the fluid injection rate. The history of the transient pressurization prior to breakdown can be used to provide the initial conditions for the fracture propagation problem.

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References

[1] E. Detournay and A.H-D. Cheng. Plane strain analysis of a stationary hydraulic fracture in a poroelastic medium. *Int. J. Solids Structures*, 27(13):1645–1662, 1991.

[2] D.I. Garagash. Fluid-driven crack tunneling in a layer. *to be submitted to ASME J. Appl. Mech.*, 2013.

[3] J.B. Osborne and D.I. Garagash. Using CO2 as a fracture fluid in tight reservoirs. Technical report, Petroleum Technology Research Centre, 2012.

[4] S. Bachu. Screening and ranking of sedimentary basins for sequestration of CO2 in geological media in response to climate change. *Environmental Geology*, 44(277–289), 2003.

[5] J. Bradshaw, S. Bachu, D. Bonijoly, R. Burruss, S. Holloway, N.P. Christensen, and O.M. Mathiassen. CO2 storage capacity estimation: issues and development. *Int. J. Greenhouse Gas Control*, 1(62-68), 2007.

[6] A. Settari and G.M. Warren. Stimulation and field analysis of waterflood induced fracturing. Society of Petroleum Engineers, 1994.

[7] K. Pruess, J. S. Y. Wang, and Y. W. Tsang. On thermohydrologic conditions near high-level nuclear wastes emplaced in partially saturated fractured tuff: 1. simultion studies with explicit consideration of fracture effects. *Water Resources Res.*, 26(6):1235–1248, 1990.

[8] T.K. Perkins and L.R. Kern. Widths of hydraulic fractures. *J. Pet. Tech., Trans. AIME*, 222:937–949, 1961.

[9] E. Detournay, A.H-D. Cheng, and J.D. McLennan. A poroelastic PKN hydraulic fracture model based on an explicit moving mesh algorithm. *ASME J. Energy Res. Tech.*, 112:224–230, 1990.

[10] Y. Kovalyshen and E. Detournay. A reexamination of the classical PKN model of hydraulic fracture. *Transport in Porous Media*, 81:317–339, January 2009.

[11] Y. N. Gordayev and A. F. Zazovsky. Self-similar solution for deep-penetrating hydraulic fracture propagation. *Transport in Porous Media*, 7:283–304, 1992.

[12] Y. N. Gordayev and V. M. Entov. The pressure distribution around a growing crack. *J. Appl. Maths Mechs.*, 61(6):1025–1029, 1997.

[13] Y. Kovalyshen. *Fluid-Driven Fracture in Poroelastic Medium*. PhD thesis, 2009.

[14] R.P. Nordgren. Propagation of vertical hydraulic fractures. *J. Pet. Tech.*, 253:306–314, 1972. (SPE 3009).

[15] D.A. Spence, P.W. Sharp, and D.L. Turcotte. Buoyancy-driven crack propagation: a mechanism for magma migration. *J. Fluid Mech.*, 174:135–153, 1987.

[16] J. R. Lister. Buoyancy-driven fluid fracture: The effects of material toughness and of low-viscosity precursors. *J. Fluid Mech.*, 210:263–280, 1990.

[17] G. K. Batchelor. *An Introduction to Fluid Dynamics*. Cambridge University Press, Cambridge UK, 1967.

[18] J. Bear. *Dynamics of Fluids in Porous Media*. Dover Publications, 1988.

[19] H. Carslaw and J. C. Jaeger. *Conduction of Heat in Solids*. Oxford University Press, 2nd edition, 1959.

[20] H. Stehfest. Numerical inversion of Laplace transform. *Communications of the Association for Computing Machinery (ACM)*, 13:47–49, January 1970.

[21] L.X. Nghiem, P.A. Forsyth Jr., and A. Behie. A fully implicit hydraulic fracture model. *Journal of Petroleum Technology*, 36(7):1191–1198, July 1984.

[22] D. Garagash and E. Detournay. An analysis of the influence of the pressurization rate on the borehole breakdown pressure. *Int. J. Solids Structures*, 34(24):3099–3118, 1997.

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