Chapter 6

Models for Highway Cost Allocation

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Additional information is available at the end of the chapter

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1. Introduction

Historically, *equity* has been one of the most important principles applied to formulate tax policy. It has been considered when raising revenues and allocating funds for maintenance, capital improvements, operating programs, and services to the public. The problem of determining how the total cost of a shared facility or service should be divided fairly and rationally is common both in public and private enterprises. The theory of cooperative games is widely used for allocating these costs. Examples of this include but are not limited to public utilities providing telephone services, electricity, water, and transport; public works projects designed to serve different constituencies; access fees or user charges for airports, highways, bridges or waterways; internal accounting rules to allocate overhead costs in private companies [1-4].

The purpose of a *Highway Cost Allocation* (HCA) study is to determine the fair share that each class of road user (vehicle class) should pay for the construction, maintenance, operation, improvement, and related costs of highways, roads, bridges, and streets in a highway network, such as those managed by state Departments of Transportation in the U.S.A. Particular emphasis should be placed on criteria and methods for allocating costs among vehicle classes using a common highway facility (road or bridge, for example) in a just, equitable, fair, and reasonable manner. Cost allocation is ultimately concerned with fairness. Through a comparison of revenues (user fees paid) and cost responsibilities, this study will estimate current equity and recommend alternatives to bring about a closer match between payments and cost responsibilities for each vehicle class.

A significant objective of HCA studies is to analyze highway-related costs attributable to different highway users as a basis for evaluating the equity and efficiency of user charges. Ideally, the costs incurred by the various user groups should be in proportion to the damage they contribute to the highway system. The cost of supporting a highway infrastructure may be deemed fair if there is an equitable distribution of costs and revenues among the various groups of highway users. With this assumption, equity is achieved when each group's

percentage of total assigned costs is equal to the percentage of the revenues contributed by that group. This chapter focuses exclusively on highway cost allocation, specifically the allocation of pavement and bridge costs.

Highway users are concerned about the fairness of road-use charges and demand that these be allocated equitably among the various vehicle classes occasioning the total cost. Although the word *equity* conveys the general intent of any cost allocation procedure, there are many possible ways to formulate a cost allocation objective to measure equity. In general, there exists no perfect cost allocation method. This is why there is a rich menu of cost allocation methods each intended to reflect the problem-specific logical, historical, political, economic, as well as mathematical analysis.

Costs associated with highway construction, maintenance, and operation can be divided into several categories. Because the impact of different vehicle classes on the costs is different, each of the cost categories should be allocated among the various user groups or vehicle classes in a different manner. These cost categories are:

a. Costs associated with new pavement construction.
b. Costs associated with pavement maintenance, rehabilitation, and reconstruction.
c. Costs associated with new bridge construction.
d. Costs associated with bridge maintenance, rehabilitation, and reconstruction.
e. Costs associated with system enhancement.
f. Other highway-related costs.

In addition to this Introduction, this chapter is organized according to six additional sections. Section 2 briefly describes several traditional and non-traditional procedures for highway cost allocation and outlines some important properties of game-theory-based procedures. Section 3 presents a conceptual framework for conducting a highway cost allocation study for a transportation agency, such as a U.S. state Department of Transportation. Section 4 discusses the application of the *nucleolus method* in highway cost allocation combining it with the concept of statistical cost effect to determine a unique solution from multiple optimal solutions. Section 5 describes a new procedure for allocating highway costs having one component due to pavement thickness and another one due to traffic capacity (measured in terms of lanes). Section 6 develops a procedure for bridge cost allocation that integrates both game theory concepts and the traditional incremental approach. Two numerical examples are designed to illustrate the proposed procedures.

2. Highway cost allocation procedures and properties

2.1. Traditional HCA Methods

During the last three decades, several methods have been developed for the purpose of allocating the total cost of a transportation facility among all the vehicle classes using it. Most procedures that can be used to achieve this goal can be grouped as either *incremental* or *proportional* allocation procedures, or a combination of these two. The proportional and incremental methods have been used by the Federal Highway Administration [5][6] and by

several state Departments of Transportation. In the **Incremental Method** a highway facility is initially designed to accommodate only the vehicles with lowest axle weight, and then it is sequentially redesigned as the additional vehicle classes are included in increasing order of axle weights. As the process of adding vehicle classes continues, after each inclusion the marginal or incremental cost is charged to the most recently included class. This method satisfies two of the three fundamental properties: completeness and marginality, sometimes marginality but this not guaranteed. Furthermore, this method is not *consistent* because the cost allocated to each vehicle class depends on the *order* in which vehicle classes are included in the analysis. As the name suggests, the **Proportional Method** distributes costs proportionally among vehicle classes according to a specified measure. The cost allocator could be vehicle-miles of travel (VMTs), 18,000 lb. *equivalent single-axle loads* (ESALs), or some other measure. While this procedure may not satisfy marginality and rationality, it does satisfy the completeness principle.

2.2. Non-traditional HCA methods

Several non-traditional allocation methods have been developed based on concepts from the theory of cooperative games by Neumann and Morgenstern [7].The application of *non-atomic game theory* to cost allocation was proposed by Castaño-Pardo and Garcia-Diaz [4]. This approach is different from the analysis of the game in which entire vehicle classes are considered as players; instead, each vehicle passage is considered as a player. Such a game obviously has a large number of players, and the decisions of a single player are irrelevant to the total outcome of the game. The value of this non-atomic game is utilized to find the solution to the problem of pavement cost allocation.

The **Generalized Method** is based on concepts from the theory of cooperative games [7], and was proposed for conducting highway cost allocation by Villarreal and Garcia-Diaz [8]. The method satisfies completeness, marginality, and rationality because these principles are forcibly satisfied due to constraints in its mathematical formulation. In essence the method guarantees that every vehicle class will be allocated a lower cost in the *grand coalition* (consisting of all vehicle classes), as compared to any other *smaller coalition* (one with fewer vehicle classes than the grand coalition). This method is known in the game theory literature as the *Nucleolus Method*. Its conditions are considered of primary importance in a large number of applications (as in public utility pricing, for example). The solution procedure is actually an application of *Linear Programming* (*LP*). Sometimes the linear programming solution may not be unique and then there is the need to introduce a tie-breaker rule.

The **Shapley Value** [9] is the average marginal cost for a vehicle class considering all possible permutations of the vehicles in the grand coalition. For example, if there are three vehicle classes, represented by 1, 2, 3, the following permutations are possible: 123, 132, 213, 231, 321, and 312. If we calculate the marginal cost for each vehicle and the compute the average for the six permutations, this average marginal cost is known as the Shapley value. The Shapley value, primarily due to its simplicity and mathematical properties, is one of the most widely studied and used joint cost allocation solution concepts. It represents the

average marginal cost contribution each vehicle class *i* would make to the grand coalition if it were to form one vehicle class at a time. Thus the average or expected cost assessment is

$$x_i = \sum_{\substack{i \in S \\ S \subseteq N}} \frac{(|S|-1)!\,(|N|-|S|)!}{|N|!} C^i(S) \tag{1}$$

where $|S|$ and $|N|$ represent the cardinality of sets $\boldsymbol{S}$ and $\boldsymbol{N}$, $C^i(\boldsymbol{S})$ represents the marginal cost contribution of *i* relative to $\boldsymbol{S}$, which can readily be computed using $C^i(\boldsymbol{S}) = C(\boldsymbol{S})\text{-}C(\boldsymbol{S}\text{-}i)$ if $i \in \mathbf{S}$, and where the sum is computed over all subsets $\boldsymbol{S}$ containing vehicle class *i*. For example, for the cost game given by *C(1)=7, C(2)=8, C(3)=8, C(1,2)=10, C(1,3)=10, C(2,3)=15* and *C(1,2,3)=17*, the Shapley value allocation is calculated as shown below:

$$x_1 = \frac{0!2!}{3!}(7-0) + \frac{1!1!}{3!}(10-8) + \frac{1!1!}{3!}(10-8) + \frac{2!0!}{3!}(17-15) = \frac{11}{3} = 3.67$$

$$x_2 = \frac{0!2!}{3!}(8-0) + \frac{1!1!}{3!}(10-7) + \frac{1!1!}{3!}(15-8) + \frac{2!0!}{3!}(17-10) = \frac{20}{3} = 6.67$$

$$x_3 = \frac{0!2!}{3!}(8-0) + \frac{1!1!}{3!}(10-7) + \frac{1!1!}{3!}(15-8) + \frac{2!0!}{3!}(17-10) = \frac{11}{3} = 6.67$$

The **Aumann-Shapley Value** [10,11] is a procedure that considers two types of costs. The first cost is for ESALs (pavement thickness) and the second cost is for highway-lanes (traffic capacity). The total cost allocated to a vehicle class is the sum of these two costs. This procedure allows the consideration of the number of lanes as being variable and depending on the composition of the traffic using a highway. In particular, it addresses two seemingly conflicting objectives: lighter vehicles require less pavement thickness and more lanes while heavier vehicles require fewer lanes but thicker pavements. This method calculates a cost per ESAL and a cost per lane. Then it allocates the number of available lanes among the vehicle classes using the Shapley value (which is the average incremental number of lanes over all possible orderings of the vehicle classes). Since the ESALs are given as data, then the cost allocated to a vehicle class can be calculated as the sum of the *ESALs cost* plus the *lanes cost.*

2.3. Desirable HCA properties

In order to explain some desirable properties of Highway Cost Allocation (HCA) procedures we will consider a highway facility such as a pavement or a bridge. First, **completeness** is the property that highway costs (construction, rehabilitation, maintenance) are fully paid for by all participating vehicle classes. Second, **rationality** is the property that each vehicle class is guaranteed a lower cost by participating in the *grand coalition* (group consisting of all vehicle classes). The fundamental observation is that if a highway facility is designed for the grand coalition, the cost share of each vehicle class would be smaller than the share paid by the vehicle class in a smaller coalition for which an alternative facility can

be designed and for which the cost is available. **Marginality** means that each vehicle class should pay at least the incremental cost incurred by including it in the grand coalition. D**emand monotonicity** is a property that implies that the cost-share of a player does not decrease when the player increases its level of demand. **Additivity** means that the allocated costs can be divided into two corresponding components if a cost function can be divided into two distinct and independent cost components. The **dummy** property means that a cost allocation should be equal to zero for a player that does not contribute to any coalition. Some of these properties will be further addressed in Sections 4 and 5.

3. Overview of a highway cost allocation study

Figure 1 outlines a typical framework of a highway cost allocation study for a transportation agency, such as a State Department of Transportation. Instead of directly allocating a total cost at the state level, a more equitable approach is to divide the total cost on the basis of three *classification attributes* known as climatic region, highway system, and highway location. For each of these three attributes several choices must be identified. As an example, a state may be divided into one to four climatic regions depending on the climatic factors affecting pavement performance, the highways may be classified into at least two highway systems to include state and federal highways as a minimum, and the locations may be classified into at least two major classes to accommodate urban and rural highways.

For any *cost classification*, i.e. one choice of each region, highway system and location, the corresponding total cost to be allocated among vehicle classes is first calculated or estimated by dividing the state total among all classifications according to well-known cost allocators, such as vehicle miles of travel (VMTs) or vehicle loadings measured in terms of *18,000 lb*

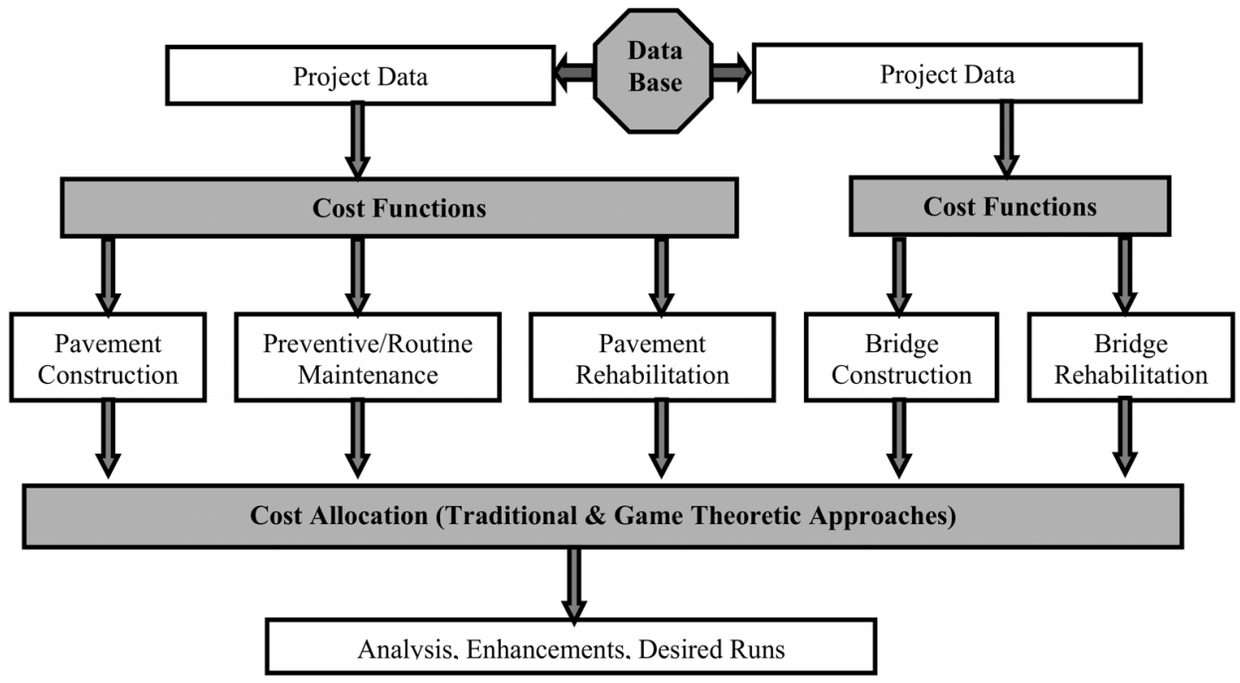

Figure 1. Framework for HCA Study.

Equivalent Single-Axle Load applications (ESALs). To divide the cost for any *cost classification* among vehicle classes, we need to find a cost function, known in game theory as the *characteristic function*, that provides a cost in $/mile for any specified number of ESALs. The characteristic function can be determined by statistical regression analysis using data on expenditures and traffic volumes extracted from several representative highway projects. The characteristic function allows the use of game-theoretic procedures that require costs estimates for coalitions or groups of vehicle classes. In particular, the Shapley value, Generalized Method, and A-S value methods require the use of a characteristic function.

3.1. Vehicle classes

Vehicle classes are viewed as players in a cooperative game. The object of a highway cost location procedure is to fairly divide the construction, rehabilitation or maintenance cost of a transportation facility, such as a highway or a bridge, among these users or players. The following vehicle classes are typically included in highway cost allocation studies:

1. Motorcycles
2. Passenger cars
3. Other Two-Axle, Four-Tire Single Unit Vehicles
4. Buses
5. Two-Axle, Six-Tire, Single-Unit Trucks
6. Three-Axle Single-Unit Trucks
7. Four or More Axle Single-Unit Trucks
8. Four or Fewer Axle Single-Trailer Trucks
9. Five-Axle Single-Trailer Trucks
10. Six or More Axle Single-Trailer Trucks
11. Five or fewer Axle Multi-Trailer Trucks
12. Six-Axle Multi-Trailer Trucks
13. Seven or More Axle Multi-Trailer Trucks

3.2. Database description

The database includes the information of traffic levels and costs of relevant pavement maintenance or rehabilitation projects for different data classifications. Typically the database has data for all classifications formed with the following attributes:

1. *Climactic Regions.* Since the performance of a pavement is affected by climatic conditions, it is customary to divide a large geographic area into smaller homogeneous climatic regions.
2. *Highway Systems.* In a number of studies two to three highway systems are included when defining the scope of the study. In a number of U.S. states at least Interstate Highways, US highways, and State highways/roads are included.
3. *Highway Locations.* There are two primary types of locations considered in a number of studies: urban and rural areas.

For each of the resulting classifications or combinations of climactic region, highway system, and location, at least three (or four) projects are extracted from the database and used to estimate cost relationships (characteristic functions) that can be used to estimate costs for different levels of ESALs. Typically traffic data available will include the following:

a. Annual Average Daily Traffic (AADT) and Equivalent Single-Axle Loads (ESALs).
b. The distribution of vehicles on the road (proportion of passenger cars, single-axle trucks, etc.)

In order to generate data for a more detailed level of classification, the following information can be used:

a. Vehicle Miles Traveled (VMT)
b. Required number of lanes for various combinations of vehicle classes

Since each treated or constructed pavement has a specific service life and all the vehicles traveled in its service life should pay the maintenance or construction cost, the *Equivalent Annual Cost* (*EAC*) of the project in its service life is calculated and used as the cost of that specific project. *EAC* is the cost per year of owning and operating an asset over its entire lifespan. This cost is calculated for the following highway work activities:

1. *Pavement maintenance*: typically both routine and preventive maintenance activities are included in this cost component. Routine maintenance activities are needed to repair cracks of different types, fill pot holes and correct other signs of pavement distress. Preventive maintenance is done mostly applying thin seal coats, micro surfacing, fog sealing, chip sealing, etc.
2. *Pavement rehabilitation*: pavement rehabilitation activities include conventional hot mixed asphalt overlay with or without milling. Generally, thicker overlays will be used for high traffic level roads and thus the cost will be also higher.
3. *Pavement construction*: new pavement construction includes the subgrade, base layer and surface layer.

4. Generalized method

Let N be the set (grand coalition) of all vehicle classes using a highway. Let $C(N)$ be the cost per mile of this highway (construction, rehabilitation or maintenance). Furthermore, let R_i the cost paid by vehicle class $i \in N$. The *completeness* property can be formulated as

$$\sum_{i \in N} R_i = C(N) \tag{2}$$

Now, let us consider a subset (coalition) of vehicle classes, $S \in N$, and let $C(S)$ be the cost per mile of a highway designed specifically to accommodate only the vehicle classes in S. The *rationality* property can be formulated as

$$\sum_{i \in S} R_i \leq C(S) \quad \text{for all} \quad S \subset N \tag{3}$$

Furthermore, the *marginality* property implies that

$$\sum_{i \in S} R_i \geq C(N) - C(N - S) \quad \text{for all} \quad S \subset N \tag{4}$$

It can be proved that if the completeness property (2) is held then the rationality and marginality properties (3) and (4) are equivalent. From (3) it is concluded that the savings enjoyed by a coalition *S* when joining the grand coalition are given by

$$C(S) - \sum_{i \in S} R_i \tag{5}$$

To maximize these savings, we maximize *t*, where

$$C(S) - \sum_{i \in S} R_i \geq t$$

which can be rewritten as

$$\sum_{i \in S} R_i \leq C(S) - t \tag{6}$$

As an illustration, for $\boldsymbol{N}$ = {1,2,3}, the *LP* model for the Generalized Method is formulated in (7)-(15).

$$\text{Maximize } t \tag{7}$$

Subject to

$$R_1 \leq C_1 - t \tag{8}$$

$$R_2 \leq C_2 - t \tag{9}$$

$$R_3 \leq C_3 - t \tag{10}$$

$$R_1 + R_2 \leq C_{12} - t \tag{11}$$

$$R_1 + R_3 \leq C_{13} - t \tag{12}$$

$$R_2 + R_3 \leq C_{23} - t \tag{13}$$

$$R_1 + R_2 + R_3 = C_{123} \tag{14}$$

$$R_1, R_2, R_3, t \geq 0 \tag{15}$$

Constraints (8)-(10) correspond to highways (pavements) designed to accommodate single-vehicle-class coalitions. Constraints (11)-(13) correspond to two-vehicle-class coalitions. Constraint (14) corresponds to the grand coalition. Each coalition has a level of traffic

loadings, measured in ESALs, for a specified design period (typically 20 years). The highway cost per mile should be strictly increasing as the number of ESALs increases. Under this assumption, Constraints (8)-(14) define a feasible region called the *core of the game* when $t=0$. If W_1 and W_2 are measured in ESALS then the core exists if C_1, C_2, C_3, C_{12}, C_{13}, C_{23} and C_{123} satisfy the following condition

$$C(W_1 + W_2) \leq C(W_1) + C(W_2) \tag{16}$$

It can be proved that a typical non-decreasing cost function satisfying (16) is the one represented in Figure 2. In this figure, W is the total number of standard loads (ESALs) for the grand coalition and $C(W)$ is the cost to be allocated. In a number of highway cost allocation studies functions like the one shown in this figure are found using regression analysis from cost data for a set of highway projects available in the database of the study.

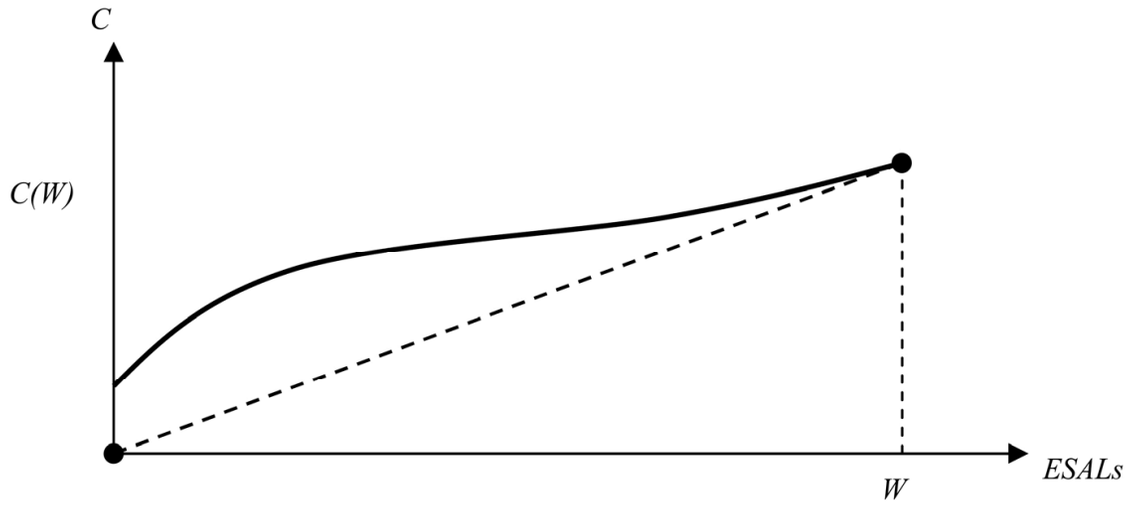

Figure 2. Cost function.

Figure 3(a) shows the feasible region for the above formulation when $t = 0$. Figure 3(b) shows the effect of increasing the value of the variable t. It is noted in this figure that as the value of t increases, the feasible region gets smaller, becoming either a point or a line when t reaches its maximum value. A solution represented by one point indicates a unique solution. The line represents infinitely many optimal solutions, a case already indicated in Section 2.

When the model formulated in (7)-(15) has infinitely many optimal solutions an additional condition must be considered to select a unique solution. The solution procedure can, therefore, be divided into two phases, with the second one needed only to break the tie among multiple solutions in the first phase.

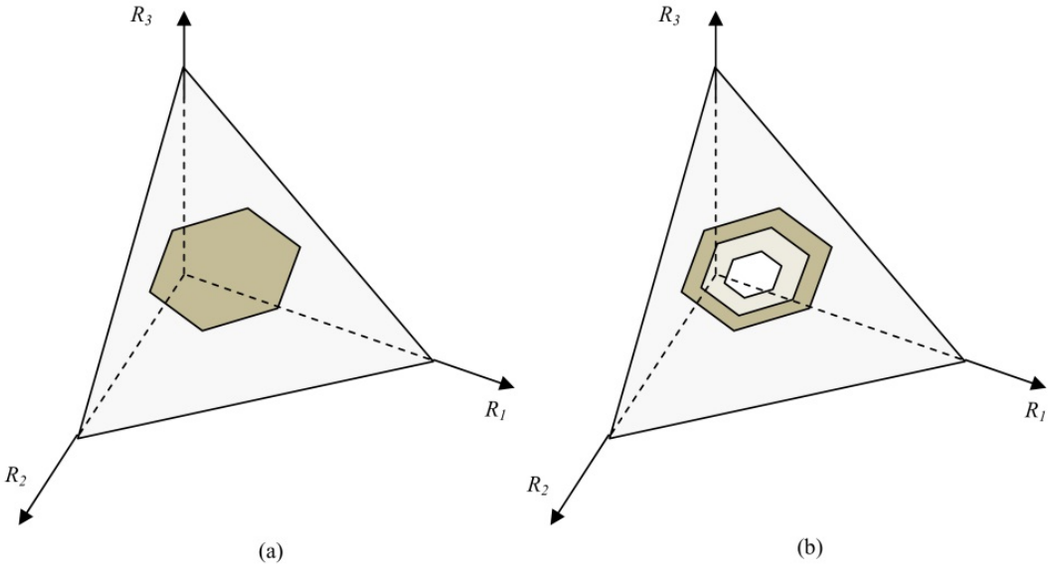

Figure 3. Feasible region.

4.1. Phase 1 of generalized method

$$Maximize\ t \tag{17}$$

Subject to

$$\sum_{i \in N} R_i = C(N) \tag{18}$$

$$\sum_{i \in S} R_i \leq C(S)\text{-}t \quad \text{for all} \quad S \subset N \tag{19}$$

$$R_i,\ t \geq 0\ for\ all\ i \in N \tag{20}$$

4.2. Phase 2 of generalized method

Villarreal-Cavazos and Garcia-Diaz [13] proposed to break the tie among multiple solutions using the concept of statistical *cost effect* of vehicle classes. This is defined as the difference in average cost between all coalitions *including* a given vehicle class and all coalitions *not including* the class. If E_i is the cost effect of vehicle class i, the *relative effect* is defined as

$$e_i = \frac{E_i}{\sum_{i \in N} E_i} \quad \text{for all} \quad i \in N$$

Also, the *relative cost allocated* to vehicle class i is defined as

$$r_i = \frac{R_i}{\sum_{i \in N} R_i} \quad \text{for all} \quad i \in N$$

A unique solution is obtained from the solution to the *non-linear* model formulated in (21)-(24).

Minimize

$$\sum_{i \in N} | r_i - e_i | \tag{21}$$

Subject to

$$\sum_{i \in N} R_i = C(N) \tag{22}$$

$$\sum_{i \in S} R_i \leq C(S) - t^* \quad \text{for all } S \subset N \tag{23}$$

$$R_i,\ t \geq 0 \ \textit{for all}\ i \in N \tag{24}$$

where t^* is the optimal value obtained for t in Phase 1. The model formulated in (21)-(24) can be *linearized* as shown in (25)-(30).

Minimize

$$\sum_{i \in N} (L_i + H_i) \tag{25}$$

Subject to

$$r_i + L_i - H_i = e_i \ \text{ for all } \ i \in N \tag{26}$$

$$\sum_{i \in S} R_i \leq C(S) - t^* \ \text{ for all } \ S \subset N \tag{27}$$

$$\sum_{i \in N} R_i = C(N) \tag{28}$$

$$(\sum_{i \in N} R_i) \times r_i = R_i \ \text{ for all } \ i \in N \tag{29}$$

$$R_i,\ t \geq 0 \ \textit{for all}\ i \in N \tag{30}$$

It is noted that by *LP* optimality conditions,

$$L_i = \begin{cases} e_i - r_i \ ; \ r_i < e_i, \\ 0 \ ; \ r_i \geq e_i, \end{cases} \tag{31}$$

$$H_i = \begin{cases} 0 \ ; \ r_i \leq e_i, \\ r_i - e_i \ ; \ r_i > e_i. \end{cases} \tag{32}$$

4.3. Statistical cost effects

A grand coalition consisting of the set of vehicle classes {1,2,3} is considered again to illustrate the calculation of the relative cost effects of the classes. First we regard each vehicle class as a *two-level factor*. The levels can be represented by the signs – and +, where – means that the vehicle class is not in a coalition and + indicates that it is in the coalition. Moreover, the number of level combinations for three two-level factors is equal to $2^3 = 8$. These eight combinations are listed in Table 1. Now, it is noted that combinations 2-8 represent the 7 coalitions that can be formed with the three vehicle classes being considered. The last column in the table shows the highway cost for each coalition. Combination 1 corresponds to an *empty coalition.* Its cost can be viewed as the environmental cost, that is, the cost needed to have a facility able to withstand the impact of climatic conditions alone, not considering the impact of vehicle loadings. In HCA studies this cost can be regarded as a specified fraction of C_{123}.

Combination	X_1	X_2	X_3	Cost
1	-	-	-	C_o
2	+	-	-	C_1
3	-	+	-	C_2
4	+	+	-	C_{12}
5	-	-	+	C_3
6	+	-	+	C_{13}
7	-	+	+	C_{23}
8	+	+	+	C_{123}

Table 1. Level combinations

The effect of factor X_i, for example, as previously indicated, is the difference in average cost between the coalitions including vehicle class i and those not including it. Based on this definition, the cost effects of the three vehicle classes are obtained as follows using the results shown in Table 1:

$$E_1 = \frac{C_1 + C_{12} + C_{13} + C_{123}}{4} - \frac{C_o + C_2 + C_3 + C_{23}}{4}$$

$$E_2 = \frac{C_2 + C_{12} + C_{23} + C_{123}}{4} - \frac{C_o + C_1 + C_3 + C_{13}}{4}$$

$$E_3 = \frac{C_3 + C_{13} + C_{23} + C_{123}}{4} - \frac{C_o + C_1 + C_2 + C_{12}}{4}$$

Once $E_1, E_2, \ldots, E_n$ are calculated their values are used to define the relative cost effects

$$e_i = \frac{E_i}{\sum_{i \in N} E_i} \quad \text{for all } i \in N$$

and we can formulate the tie-breaking constraints (26) needed in the second phase of the generalized method.

5. Separation of pavement thickness and traffic capacity costs

The proposed approach [12] distributes traffic-related costs in a more fair way than any other method by considering both traffic loads and traffic capacity. Furthermore, the development of a new cost allocation methodology considering allows us to analyze the impact of traffic capacity costs. The two concepts used in the proposed methodology to allocate costs among vehicle classes, according to traffic load and capacity requirements, are known as the Shapley value and the Aumann-Shapley value. In essence the Aumann-Shapley value determines an *average cost per ESAL* and an *average cost per lane* (per mile). The Shapley value allocates the total number of lanes of a highway among the vehicle classes. With these results, it is then possible to calculate costs per mile for each vehicle class by adding the cost due to ESALs (pavement thickness) and the cost due to lanes (capacity).

Two types of players will be considered. $\mathbf{E} = \{1,2,\ldots,q_1\}$ and $\mathbf{L} = \{1,2,\ldots,q_2\}$ are sets of players of type 1 and type 2, respectively. Thus, $\mathbf{M} = \mathrm{E} \cup \mathbf{L}$ is the set of all players. Now, let $P(\mathbf{M})$ be the set of all subsets or *coalitions* formed with the elements of **M**. Furthermore, let **N** be the set of natural numbers and $\mathbf{R}^+$ be the set of positive real numbers. Let C: $P(\mathbf{M}) \rightarrow \mathbf{R}^+$ be a real-valued cost function known as the *characteristic function*. Finally, let $x(q_1,q_2;C)$ be allocated costs yielded by a cost allocation method, $x_1(q_1,q_2;C)$ be the cost allocated to player 1 and $x_2(q_1,q_2;C)$ be the allocated cost to player 2. With these conventions, four important definitions are given below.

5.1. Definitions

Definition 1

If $x_1(q_1,q_2;C) + x_2(q_1,q_2;C) = C(\mathbf{M})$, then the method x is called *complete*.

Definition 2

If $x(q_1,q_2;C_1+C_2) = x(q_1,q_2;C_1) + x(q_1,q_2;C_2)$, where C_1 and C_2 are non-decreasing cost functions, then the method x is called *additive*. If a cost function can be divided into two distinct and independent cost components, then the *allocated costs* can be divided into two corresponding components.

Definition 3

If $C(S) - C(S\setminus\{i\}) = 0$ for any $i \in S$, $i \in N$, and $S \subseteq N$, then $x_i(q_1,q_2;C) = 0$. In this case, the method x is called *dummy*. If any player does not contribute to any coalition, then the cost allocated to it is zero.

Definition 4

If $x_1(q_1,q_2;C) \geq x_1(q_1\text{-}1,q_2;C)$, then the cost allocation method x is called *demand monotonic* for any $q_1>2$. Similarly, if $x_2(q_1,q_2;C) \geq x_2(q_1,q_2\text{-}1;C)$, then the cost allocation method x is called

demand monotonic for any $q_2>2$. The cost-share of a player should not decrease when the player increases its demand.

Friedman [13] shows that the A-S value is *complete*, *additive*, and *dummy*. In addition, Friedman and Moulin [14] show that the A-S value does not satisfy the demand monotonicity property for general non-decreasing cost functions. Lee & Garcia-Diaz [15] show that demand monotonicity will be held in the following cases.

5.2. Pavement and capacity costs allocation

Assume the *log concave* cost function formulated in (33):

$$C(e,l) = l\left(a + b\, e^{\,r}\right) \tag{33}$$

where e is the number of *ESAL*s, $l \in \mathbf{N}$ is the number of lanes, $C(e,l)$ is the cost in dollars per lane-mile, and a, b, and r are non-negative parameters. For this function the following results can be proved:

a. Demand monotonicity for the number of lanes.
b. Demand monotonicity for the number of *ESAL*s $r \geq 0.32$.

In this chapter we use a *compact form* developed for the discrete A-S value [15]. This compact form allows the use of the A-S value in realistic applications with a large number of players, where the computational work becomes excessive without using the form. This section states some fundamental results regarding the demand monotonicity of the log concave characteristic function. The proposed approach [12] is composed of the following three steps.

Step 1. Traffic-related pavement cost separation

To separate traffic-related pavement costs into the costs for traffic load and the costs for traffic capacity, the discrete A-S value is used. Suppose that there are m types of players and q_i players of a type i. Further, let

$$Q = \sum_i q_i,\ T = \sum_i t_i,\ T' = \sum_i t'_i$$

and

$$t'_i = q_i - t_i.$$

There are two formulas for the discrete A-S value. A formula by Moulin [11] is shown in (34), where $i = 1,\ldots, m$:

$$x_i(q;C) = \frac{q_1!\ldots q_m!}{Q} \sum_{t\in[0,q]} \frac{T!}{t_1!\ldots t_m!} \frac{T'!}{t'_1!\ldots t'_m!} \left(\frac{t_i}{T} - \frac{t'_i}{T'}\right) C(t). \tag{34}$$

Another formula by Redekop [16] is given in (35):

$$x_i(q;C)=\sum_{\substack{t\in[0,q]\\ t_i>0}} q_i\times\frac{\binom{q_i-1}{t_i-1}\left(\prod_{j\neq i}\binom{q_j}{t_j}\right)}{T\binom{Q}{T}}[C(t)-C(t_1,t_2,...,t_i-1,...,t_m)]. \tag{35}$$

The cost per ESAL and the cost per lane are calculated by averaging since all the players of the same type are identical. Thus, the cost per ESAL (C_e) and the cost per lane (C_l) can be calculated as follows, where i = e or l:

$$C_i=\frac{x_i(q;C)}{q_i} \tag{36}$$

There are two types of players, namely, ESALs and lanes. Furthermore, let q_1 be the total number of players for ESALs, and q_2 be the total number of players for lanes. Then, the cost per lane and the cost per ESAL can be calculated from Redekop's formula as shown in (37) and (38).

$$C_l=\frac{q_1!(q_2-1)!}{(q_1+q_2)!}\sum_{t_1=0}^{q_1}\sum_{t_2=1}^{q_2}\frac{(t_1+t_2-1)!}{t_1!(t_2-1)!}\frac{(q_1-t_1+q_2-t_2)!}{(q_1-t_1)!(q_2-t_2)!}\{C(t_1,t_2)-C(t_1,t_2-1)\} \tag{37}$$

$$C_e=\frac{(q_1-1)!q_2!}{(q_1+q_2)!}\sum_{t_2=0}^{q_1}\sum_{t_1=1}^{q_2}\frac{(t_1+t_2-1)!}{(t_1-1)!t_2!}\frac{(q_1-t_1+q_2-t_2)!}{(q_1-t_1)!(q_2-t_2)!}\{C(t_1,t_2)-C(t_1-1,t_2)\} \tag{38}$$

If the cost increment remains the same when t_1 (or t_2) is fixed and t_2 (or t_1) is increased by 1 the A-S value can be determined using the simplified compact form formulated in (39).

$$x_{t_2}(q,C)=\frac{q_2}{q_1+1}\sum_{t_1=0}^{q_1}C(t_1,1) \tag{39}$$

Step 2. Lane assignment

Since the A-S value satisfies the *completeness property*, the sum of costs for traffic capacity and traffic load for the grand coalition equals the total cost for that coalition. The sum of ESALs over all vehicle classes is equal to the number of ESALs for the grand coalition (q_1), but the sum of the lanes required for each vehicle class is greater than or equal to the lanes required for the grand coalition (q_2). Hence, to calculate cost responsibilities for each vehicle class, the number of lanes for the grand coalition should be assigned to the vehicle classes. The Shapley value will be used to determine the number of lanes assigned to vehicle class i (L_i). The i^{th} Shapley value for n players is determined using (1), with i = 1, …, n

$$L_i=\sum_{s=1}^{n}\frac{(s-1)!(n-s)!}{n!}\sum_{\substack{S\subseteq N:i\in S\\ |S|=s}}\left(F(S)-F(S-i)\right) \tag{40}$$

Step 3. Cost allocation

Costs are allocated to each vehicle class in proportion to the number of ESALs and the number of lanes, that is

$$x_i\left(E_i, L_i\right) = E_i C_e + L_i C_l \tag{41}$$

where

$x_i(E_i, L_i)$: Cost allocated to vehicle class i
E_i : ESALs for vehicle class i
C_e: Cost per ESAL
L_i : Number of lanes assigned to vehicle class i
C_l: Cost per lane

5.3. An example

The proposed approach is now illustrated using a simple example. Suppose that there are 3 vehicles: two automobiles (A), one pickup truck (P), and one 5-axle-trailer truck (T). Furthermore, there is 1 *base lane*, 2 additional lanes, and a total of 4 ESALs. These loads are divided into 1 ESAL for two automobiles, 1 ESALs for one pickup truck, and 2 ESALs for one 5 axle-trailer truck. The numbers of additional lanes required by each vehicle coalition are in shown in Table 2.

COALITION	{A}	{P}	{T}	{A,P}	{A,T}	{P,T}	{A,P,T}
Number of additional lanes	1	1	0	2	2	1	2

Table 2. Number of additional lanes required by each vehicle coalition

The cost in $/mile as a function of the number of ESALs and the number of lanes is assumed to be $C(e,l) = l\left(2 + 3\sqrt{e}\right)$. To calculate the A-S value for cost per ESAL (C_e) and cost per lane (C_l), Table 3 will be used. All possible 6!/2!4! = 15 inclusion sequences are shown in this table, where an E stands for one unit of ESALs and an L for one unit of lanes. The gray-colored column is for the *base lane*.

A *base lane* is first included in any possible sequence, and then either E or L is included. The average marginal costs, C_e and C_l, for including E or L in each sequence can be calculated from Table 3. The A-S values (C_e and C_l) can be also calculated by using the formulas shown in Step 1. The calculated values for C_e and C_l are 2.66 and 5.68, respectively.

To calculate number of lanes assigned to each vehicle class by the Shapley value, we first determine the total number of possible sequences as 3! = 6. The average marginal number of lanes, L_i, for including A, P, or T in each sequence is calculated from Table 4. The Shapley value for L_i can be also calculated by using formulas shown in Step 2.

Sequences	Inclusion Sequences						
1	L	E	E	E	E	L	L
2	L	E	E	E	L	E	L
3	L	E	E	E	L	L	E
4	L	E	E	L	E	E	L
5	L	E	E	L	E	L	E
6	L	E	E	L	L	E	E
7	L	E	L	E	E	E	L
8	L	E	L	E	E	L	E
9	L	E	L	E	L	E	E
10	L	E	L	L	E	E	E
11	L	L	E	E	E	L	E
12	L	L	E	E	E	E	L
13	L	L	E	E	L	E	E
14	L	L	E	L	E	E	E
15	L	L	L	E	E	E	E

Table 3. All possible inclusion sequences for the A-S value

Sequences	Including sequences			Marginal number of lanes		
1	A	P	T	1	1	0
2	A	T	P	1	1	0
3	P	A	T	1	1	0
4	P	T	A	1	0	1
5	T	A	P	0	2	0
6	T	P	A	0	1	1

Table 4. All possible inclusion sequences for the Shapley value

The Shapley values for the three vehicle classes are:

$$L_A = \frac{1}{6}(1+1+1+1+2+1) = 1.67$$

$$L_P = \frac{1}{6}(1+0+1+1+0+1) = 0.67$$

$$L_T = \frac{1}{6}(0+1+0+0+0+0) = 0.16\ .$$

The cost for the base lane is 2. This cost may be allocated proportionally by ESALs or, perhaps more appropriately, by vehicle miles of travel (VMT), since this cost is a non-load-related cost. Cost responsibilities for the three vehicle classes are shown in Table 5, where the base lane cost has been allocated proportionally according to ESALs.

Vehicle Classes	Load costs (E_iC_e)	Capacity costs (L_iC_l)	Costs for base lane (proportional)	Cost responsibilities
Automobile	1×2.66	1.17×5.68	2×0.50	10.30
Pickup truck	1×2.66	0.67×5.68	2×0.25	6.97
5-ax-trailer truck	2×2.66	0.16×5.68	2×0.25	6.73

Table 5. Cost responsibility calculation for each vehicle class

6. Separation of bridge construction and traffic capacity costs

A cost function is needed to estimate the bridge construction cost for the gross vehicle weight associated with any coalition of vehicle classes. This cost function can be developed by determining the cost of the bridge required by a coalition as a percentage of the cost of a *baseline bridge.* To accommodate all possible coalitions, the range of gross vehicle weight can be divided into an adequate number of intervals or categories. Results for nine categories of gross weight ranging from 5,000 lb to 108,000 lb are shown in Table 6. This table was built using a study by Moses [17] and the 1997 Federal Highway Cost allocation Study [6]. The table shows the required bridge cost for each gross vehicle weight category as a percentage of the cost of a baseline HS20 bridge which has a weight carrying capacity of 72,000 lb. The results for each gross vehicle weight category are the coordinates of one point of the bridge cost function.

Gross Vehicle Weight (1000 lb)	5	10	20	30	40	54	**72**	90	108
Bridge Cost Percentage	80.78	82.61	86.52	90.43	95.80	94.59	**100**	105	110

Table 6. Bridge cost percentages considering a baseline HS20 bridge

6.1. Bridge cost allocation procedure

The proposed model for the relationship between cost per lane-mile and the gross vehicle weight to be applied is formulated as

$$Y = l_i\ (a_i + b_i X) \tag{42}$$

where Y is the cost in dollars per lane-mile, l_i is the number of lanes of bridge type i, X is the gross vehicle weight in kips, and a_i and b_i are known parameters (to be estimated using regression analysis). Depending on the required number of lanes, more than one cost function can be formulated to determine accurate bridge construction cost estimates. A short-span structured bridge may be proper for a bridge with one lane in each direction, while a longer-span structured bridge may be so for a bridge with more lanes

The bridge construction cost allocation procedure is outlined below [18]. The procedure is essentially the same one developed in Section 5. In the case of bridges, however, there is an additional step (referred to as Step 2 below) to apply the incremental method of highway cost allocation.

Step 1. Traffic-related pavement cost separation

This step is identical Step 1 in the methodology described in Section 5 of this chapter.

Step 2. Traffic-load cost allocation

The cost per unit of weight (C_e) was obtained in *Step 1*. The traffic-load cost can be allocated to each weight group in vehicle class by using the incremental method, as indicated below:

a. The lightest vehicle group is first considered. The unit of weight (C_e) is allocated to each vehicle class in this group and all heavier groups according to average daily traffic (ADT).
b. The next light group is considered. The marginal cost equal to C_e is allocated to each vehicle class in this group and all heavier groups according to ADT.
c. If the heaviest group is considered, then go to d. Otherwise, continue to b.
d. If a vehicle class *i* has several weight groups, then sum up the cost for those weight groups.

Step 3. Lane assignment

Again, this procedure is identical to the Step 2 of the methodology described in Section 5.

Step 4. Cost allocation

This procedure is also identical to the Step 3 of the methodology described in Section 5.

6.2. An example

A simple hypothetical numerical example is presented in this section to illustrate and clarify the application of the proposed method. It is assumed that there are 3 vehicles: automobile {A}, pickup truck {P}, and 5-ax-trailer truck {T}. Also, it is assumed that 1 *base* lane is required. The number of *additional* lanes is the same in Table 1. The total vehicle weight is distributed along four intervals: 0-10 kips, 11-20 kips, and 21-30 kips. The percentages of total ADT due to vehicles of each class, for the given weight intervals, are: {A} belongs to the 0-10 kip interval with 65 % of ADT; {P} belongs to the 0-10 kip interval with 20 % of ADT and to the 11-20 kip interval with 5 percent of ADT; {T} belongs to the11-20 kips interval with 5 percent of ADT and to the 21-30 kip interval with 5 percent of ADT. The cost functions for this example are formulated below:

$$C(k,l) = l(1+2k) \quad l = 1$$

$$C(k,l) = l(2+3k) \quad l \geq 2$$

The following results are obtained in each step.

Step 1. Bridge construction cost separation

To calculate the A-S value for the cost per unit of weight (10 kips in this example) and the cost per lane the sequences shown in Table 7 can be used. It is noted that the total number of sequences is 5!/(3!2!) = 10. In Table 7 letter K represents one unit of weight (10 kips) and

letter L represents one unit of lanes. A gray-shaded column is used for the base lane. A base lane is first included in any possible sequence, and then either a K or an L is included. The average marginal costs (or the A-S value) C_k and C_l can be calculated by using Table 7. The calculated values are $C_k = 170/30 = 5.67$ and $C_l = 150/20 = 7.5$.

Sequence	Including Sequence						Marginal Cost					
1	L	K	K	K	L	L	1	2	2	2	15	11
2	L	K	K	L	K	L	1	2	2	11	6	11
3	L	K	K	L	L	K	1	2	2	11	8	9
4	L	K	L	K	K	L	1	2	7	6	6	11
5	L	K	L	K	L	K	1	2	7	6	8	9
6	L	K	L	L	K	K	1	2	7	5	9	9
7	L	L	K	K	K	L	1	3	6	6	6	11
8	L	L	K	K	L	K	1	3	6	6	8	9
9	L	L	K	L	K	K	1	3	6	5	9	9
10	L	L	L	K	K	K	1	3	2	9	9	9

Table 7. Sequences and marginal cost for calculation of A-S value

Step 2. Traffic-load cost allocation:

$$E_T: \quad 5.67 \times \frac{5+5}{65+20+5+5+5} + 5.67 \times \frac{5+5}{5+5+5} + 5.67 \times \frac{5}{5} = 10$$

$$E_A: \quad 5.67 \times \frac{65}{65+20+5+5+5} = 3.7$$

$$E_P: \quad 5.67 \times \frac{20+5}{65+20+5+5+5} + 5.67 \times \frac{5}{5+5+5} = 3.3$$

Step 3. Lane assignment:

See Table 3. L_A=1.17, L_P=0.67, L_T=0.16

Step 4. Cost allocation:

The value (cost) of parameter *a* for the base lane is 2. This cost is allocated proportionally by ADTs in this example. The total cost allocations for the three vehicle classes are shown in Table 8.

Vehicle Classes	Load costs (E_i)	Capacity costs (L_iC_l)	Costs for base lanes (proportional)	Cost responsibilities
Automobile	3.7	1.17×7.5	1×0.65	13.12
Pickup truck	3.3	0.67×7.5	1×0.25	8.58
5-ax-trailer truck	10	0.16×7.5	1×0.10	11.30

Table 8. Cost allocations for vehicle classes

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