Chapter 7

Optical Wave Propagation in Kerr Media

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Additional information is available at the end of the chapter

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1. Introduction

Optical wave propagation and interaction are important effects usable in designing and implementing various photonic devices ranging from passive splitters to active switches to light amplifiers. The material aspects are crucial as strong effects are desirable for efficient and robust devices. Electronics has its silicon that is an amazing rather universal material that makes it possible to implement microelectronics chips of unthinkable performance and functionalities. Photonics does not have such a common material, and therefore one has to choose suitable material system for a given application. However, with silicon being the best technologically mastered material, attempts have been made to employ it also in the implementation of photonic functions. Examples include electro-optic modulators and, of course, high speed photodetectors.

Recently we have investigated physical effects in silicon that are usable for photonic functionalities[1]. Since silicon is a cubic material, it does not possess the classical electro-optic effect exploited in other material systems (e.g. GaAs, InP) for high-speed switching and modulation. On the other hand, as basically all materials, silicon does possess the third-order nonlinear effect, originally known as the Kerr effect, discovered by J. Kerr in 1875[2]. This is one of the most interesting phenomena for potential exploitation. The two facts, i.e. universality of silicon and the existence of a nonlinear effect in it, led to our thorough exploration of the possibilities that Kerr effect[3] in silicon can offer in terms of potential future photonic devices[4]. As expected, the theoretical, numerical and design studies have been dominated by the optical wave propagation issues[5]. The results are general enough to apply to a wide range of materials that do not possess the classical linear electro-optic effect.

This chapter describes the original results obtained from those studies. Optical wave propagation in a nonlinear medium with a Kerr-type nonlinearity (third-order susceptibility) is analyzed theoretically. New features are found where not only waves and their polarization interactions are present as a result of the nonlinearity of the medium, but also an interplay between the optical and electrical Kerr effects contributes to the resulting functionality. Several novel wave propagation effects are discovered and discussed. They include cross-polarized wave conversion, optical multistability, nonlinear tunability of periodic structures, ultra-fast electro-optic switching, and a new photorefractive effect. Possible applications of these functionalities are addressed as well. The fine physical and mathematical details of our unified treatment are well reviewed in[6].

2. Electro-optic effects

Electro-optic effects are reviewed[7] pertaining to cubic (e.g. silicon) and isotropic (e.g. glass) materials; therefore, the well-studied and widely exploited (e.g. lithium niobate, gallium arsenide or indium phosphide) linear electro-optic phenomenon is not discussed here.

2.1. Electro-absorption

The Franz-Keldysh effect in semiconductors alters the absorption spectrum of a material. The effect is due to field-induced tunneling between valence and conduction band states. The electric field affects the overlap of electron and hole wavefunctions, which leads to increased absorption at energies lower than the bandgap. This electro-optic effect is thus normally referred to as electro-absorption. The associated electro-refraction effect is coupled via the Kramer-Kronig relation. Both effects depend on the applied electric field, the wavelength, and the carrier density.

Electro-absorption has been used in switches and modulators in various materials including III-V semiconductors. Electro-refraction is quite weak; for example in an undoped silicon at the telecommunication wavelength of $1.55\ \mu m$, where it is caused mostly by indirect gap electro-absorption, the value of the refractive index change is[8] $\Delta n = 1.5 \times 10^{-6}$ at $V_{app} = 10\ V/\mu m$. The effect is polarization dependent and it is a factor of two stronger when the optical field is parallel to the applied field. It is a pure electric-field effect and as such, its speed is high (sub-picosecond range) and determined by the tunneling speed between the conduction and valence bands.

2.2. Quantum-confined Stark effect

The quantum-confined Stark effect is the similar phenomenon occurring in semiconductor quantum-well structures. In quantum wells when close to the exciton resonances, absorption changes and Kramer-Kronig-related refractive index changes behave in a Kerr-like fashion while the medium response is enhanced due to the electron-hole confinement. The quantum well confinement increases the overlap of electron and hole wave-functions while the applied electric field reduces this overlap. This results in a corresponding reduction in optical absorption. The direct change in the light intensity resulting from this electro-absorption effect has been used in efficient bulk as well waveguide modulators. Waveguide modulators achieve better performance overall due to the confinement of light. Device details are beyond the scope of this chapter and can be found in literature[9].

2.3. Free-carrier plasma dispersion effect

Injection of charge carriers into an undoped material or removal of free carriers from a doped material, changes the refractive index (generally optical properties, e.g. absorption). Generally, three carrier effects are involved: free-carrier absorption, Burstein-Moss band-filling (shifting the absorption spectrum to shorter wavelengths), and Coulombic interaction of carriers with impurities (shifting the absorption spectrum to longer wavelengths).

The refractive index increases when carriers are depleted from a doped material and it decreases when they are injected into an undoped material. This is the largest effect compared to the electro-refraction and the Kerr effects (see below). It is polarization independent, but generally the slowest of all effects. In the injection case, the switch-off time is limited by minority carrier lifetime (tens of picoseconds at best due to recombination). In the depletion mode, the response time is determined by carrier sweep-out (picoseconds at best due to carrier drift over a finite distance of a sample or a device).

A change in refractive index is always accompanied by a change in absorption, therefore a trade-off is required when utilizing this effect for applications. The residual linear loss is usually negligible at the telecommunications wavelength; the two-photon absorption is normally a concern[10]. Successful devices have been implemented using this effect in combination with a Mach-Zehnder waveguide configuration, one example being a reverse-biased pn junction (silicon)[11], the other being a forward-biased pn junction (indium phosphide)[12], and the third one being a MOS capacitor (fully compatible with standard CMOS)[13]. The designs exploited the free-carrier plasma dispersion effect in efficient ways.

2.4. Kerr effect

This effect is a pure electric field phenomenon and it is of interest in this work. It is a quadratic electro-optic effect caused by displacement of bound electrons under the influence of an external electric field. It is basically a nonlinear polarization generated in a medium, which results in changes of its refractive index. It exists in crystals, glasses, gasses, basically all materials including the isotropic ones, i.e. also in the cubic silicon and silicon nanocrystals. It is one of the several different phenomena (e.g. self-focusing, soliton generation, four-wave mixing, phase conjugation, etc.) associated with the third-order nonlinearity in a given material, usually described by the third-order susceptibility, $\chi^{(3)}$.

The susceptibility $\chi^{(3)}$ is, generally, dispersive. Depending on the frequency region, it describes the nonlinear response in a phenomenological way, which means that it includes combination of all physical effects that contribute to the response in that particular frequency range and usually on different time scales. Normally, the resonant effects are the strongest and thus may dominate the behavior and the values of $\chi^{(3)}$. On the other hand, in a lossless medium and/or far away from any resonant frequencies (absorption lines) of a material, the dispersion of $\chi^{(3)}$ is insignificant and the response is instantaneous; thus $\chi^{(3)}$ can be considered dispersion-less. This was shown using the classical anharmonic oscillator model[14], whereby, basically, the electro-optic Kerr effect (DC Kerr effect) is a quasi-static limit of the optical one (AC Kerr effect).

The AC Kerr effect is responsible for what is generally known as all-optical effects (e.g. self-phase and cross-phase modulation, four-wave mixing). The corresponding refractive index change, Δn, is a linear function of light intensity, I:

$$\Delta n_{opt} = n_{NL} I = 3\,\chi^{(3)}\, Z_0 / 4(n_L)^2, \tag{1}$$

where n_{NL} is the nonlinear refractive index coefficient, n_L is the linear refractive index, and Z_0 is the free space impedance. The external-electric-field-controlled refractive index change is a result of the DC Kerr effect and can be described by:

$$\Delta n_{ext} = 3\,\chi^{(3)}/8\; n_L = n_L\, n_{NL}/2\; Z_0. \tag{2}$$

The pure Kerr effect is very fast, well in the sub-picosecond range. It is polarization dependent; there is a factor of one-third involved whereby parallel optical and electrical fields display a stronger interaction. The Kerr effect makes an isotropic material behave as a uniaxial crystal once the voltage is applied, with an optical axis being in the direction of the external field. The Kerr effect depends on the bandgap energy, thus it is much stronger in, for example, semiconductors than in silica glass. At wavelengths far enough away from the band-edge the effect may be considered as a pure Kerr effect in a moreless lossless medium, although multi-photon absorption might have to be considered in some cases.

In order to avoid large absorption losses required to obtain enhanced resonant nonlinearities, it is preferable to propagate waves at around a half-gap wavelength, as successfully exploited in the past in the III-V semiconductor technology[15]. At that wavelength range in semiconductors, the nonlinear refractive index, arising from the real part of the third-order susceptibility, is still relatively large to be usable. At the same time the two-photon absorption that contributes to the imaginary part of the third-order susceptibility, is relatively low to obtain reasonable propagation lengths. The real and imaginary parts are related by causality expressed by the well-known Kramers-Kronig relation.

Since the Kerr effect is bound-electron related, it is very fast and usually dominant. The refractive index change is positive. However, for higher intensities, the nonlinear two-photon absorption will start generate more free carriers as intensities increase. The free-carrier refraction is negative and slow; therefore, it is desirable to avoid such ranges of optical intensities that may lead to large nonlinear losses of propagating waves. A strong and low-loss interaction of propagating waves is the key to designing and developing efficient and robust optoelectronic devices such as switches or modulators.

The fundamental problem with the third-order nonlinearity is that the effect is very weak in most materials. The promise of the development of new materials that fall under the umbrella of nanotechnology is quite attractive. Materials are being developed on the nanometer scale, thus promising a potential to open a new world of scalability and integration. Reducing the size of the optical material structures to a nanoscale leads to significant (orders of magnitudes) enhancements of the third-order optical susceptibility due to the confinement effect. Combined with optical waveguide enhancing effects (e.g.

photonic crystals or high-contrast slot waveguides), a much stronger nonlinear interaction of propagating waves and modes is achieved. It is this promise that led us to the studies of nonlinear wave propagation in the Kerr media, with integrated optoelectronics and nanophotonics being the main area for potential applications.

3. Optical wave propagation in nonlinear media

The interaction of a light wave with a propagating medium and additionally with an applied external electric field is described by the nonlinear wave equation. When considering the general vector nonlinear wave equation, the polarization components are mutually coupled nonlinearly thus yielding coupled differential equations. Finding solutions to such a nonlinear system, which possess physical meaning, is a challenging problem even with today's available powerful computing technology. In order to obtain at least an approximate analytical solution that would offer an insight into the complexity of the problem, simplifying assumptions have to be made[3].

In a third-order nonlinear medium, the relationship between the nonlinear polarization and the electric field vector of an optical wave is governed by a fourth-ranked susceptibility tensor, $\chi^{(3)}$. For materials of interest here (cubic, isotropic), the tensor is much simpler having most of its components zero. The wave propagation can then be simplified to the point that after neglecting second-order coupling between the polarization components, nonlinear wave equations can be solved as individual scalar equations (Helmholz equations). Such a scalar equation can be solved approximately for some situations, the most known being the spatial soliton in optical fibers[16]. For this approximate case specifically, the equation is in literature incorrectly called the nonlinear Schrödinger equation due to its similar form. However, Schrödinger himself did not derive any nonlinear equation. In 1925 he formulated a linear motion equation for a free particle[17].

In the Cartesian coordinates *(x, y, z)* with *z* being the propagation direction, the optical wave field components solution can be written in a form:

$$\begin{aligned} E_x &= e_x(0)\, e^{j\omega t} e^{-jk_0(n_L + n_{NL} I + n_{EXT} E_{ext}^2) z} \\ E_{y,z} &= e_{y,z}(0)\, e^{j\omega t} e^{-jk_0(n_L + n_{NL} I + \frac{1}{3} n_{EXT} E_{ext}^2) z}, \end{aligned} \tag{3}$$

where $e_x(0)$ and $e_{y,z}(0)$ are the field's initial amplitudes, $k_0 = \omega/c$ is the free-space wavevector, c is the speed of light in vacuum, n_L is the linear refractive index, n_{NL} is the nonlinear index coefficient that describes the Kerr-like all-optical effects, n_{EXT} is what we call the nonlinear electrical index coefficient that describes the Kerr electrical effect. It is related to the original Kerr constant, K, by $n_{EXT} = 2\pi K/k_0$. With I being the intensity of the propagating optical wave, the second term in the exponents of Eq. (1) represents known all-optical effects of self-phase and cross-phase modulation. The third term in the exponents is a new addition to the mutual wave interaction control via an applied external electric field. Obviously, both effects now being incorporated into the overall interaction of the propagating waves, interplay between the all-optical and electro-optic phenomena significantly affects the propagation properties.

The second-order correction[16] to the solutions above yields more complex field expressions that, however, provide interesting insight into the waves/components interaction:

$$e_x = e^{-j\frac{k_0}{n_L}\chi\frac{3}{8}\{|e_x|^2+|e_y|^2+E_{ext}^2-\frac{1}{3}|e_y|^2[1-sinc(2\phi z)]\}z} \times e^{\frac{k_0}{n_L}\chi\frac{1}{8}|e_y|^2\cdot\frac{1-cos(2\phi z)}{2\phi}} \tag{4}$$

In Eq. (2), only the transversal x-component is shown; the transversal y-component is symmetrically identical. The parameter $\Phi = k_0\, \chi^{(3)}(E_{ext})^2/4n_L$ and is a determining factor in the overall propagation interaction; when $\Phi = 0$, only all-optical effects of self-phase and cross-phase modulation remain with the waves amplitude being constant. Eq. (2) clearly indicates periodic exchange of power between both components along the propagation path, as the second multiplicative term is an amplitude rather than a phase term.

The power densities of both components, p_x and p_y, with the total power being p_T, can then be found as:

$$\begin{aligned} p_x &= \frac{1}{f}\, ln[1 + 2\, sinh(f\, p_T\ /\ 2)\, p_x^N\, e^{f\, p_T/2}] \\ p_y &= \frac{1}{f}\, ln[1 - 2\, sinh(f\, p_T\ /\ 2)\, p_y^N\, e^{-f\, p_T/2}]. \end{aligned} \tag{5}$$

The function $f = Z_0\ [1\text{-}cos(2\Phi z)]/n_L(E_{ext})^2$ ($Z_0 = 377\ \Omega$) is the key variable controlling the interaction. As can be seen form Eq. (3), the power exchange is controlled by both, the optical power in the interacting waves (all-optical effect) and the applied external electric field (electrical Kerr effect). It should be pointed out that the nonlinear susceptibility of various materials is not a unique quantity as several physical effects contribute to a material response on different time scales[18]. Also, the third-order susceptibility possesses normally real and imaginary parts that correspond to the nonlinear phase and loss, respectively. A summary of nonlinear parameters of a number of materials relevant to the optical wave propagation issues is given in[19].

4. Re-configurable all-optical switching

The power exchange described by Eq. (3) represents basically cross-polarized wave conversion controlled optically as well as electrically. The optical control is obtained via the optical Kerr effect whereby the intensity of the wave changes the refractive index of the material. In silicon nanocrystal, for example, $\Delta n = 2\ x\ 10^{-6}$ at an intensity of $10^6\ W/cm^2$. The price paid for this all-optical control is absorption that leads to the total power loss. Linear absorption is usually negligible since it is desirable that the waves propagate in the transparent region of a given material. The nonlinear absorption however can and does play a negative role due to two-photon or even three-photon absorption. The more common two-photon absorption coefficient causes the nonlinear absorption increase with a square of the intensity thus becoming detrimental at higher wave power densities such as those in optical waveguides and optical fibers.

The electrical control is achieved via the electrical Kerr effect whereby bound electrons in the material are displaced by an electric field, which leads to changes in refractive index with the square of the voltage. In silicon nanocrystal, for example, $\Delta n = 4.2 \times 10^{-5}$ at an electric field of $10\ V/\mu m$. The attractiveness in exploiting this electro-optic control is in its extremely high speed in the subpicosecond range. An example of electrically controlled periodic power exchange in silicon nanocrystal[20] is shown in figure 1. The applied electric field values are $E_{ext(1)} = 0.8\ V/\mu m$, $E_{ext(2)} = 2\ V/\mu m$, $E_{ext(1)} = 3\ V/\mu m$, respectively. The total optical power is $0.11\ W/cm^2$.

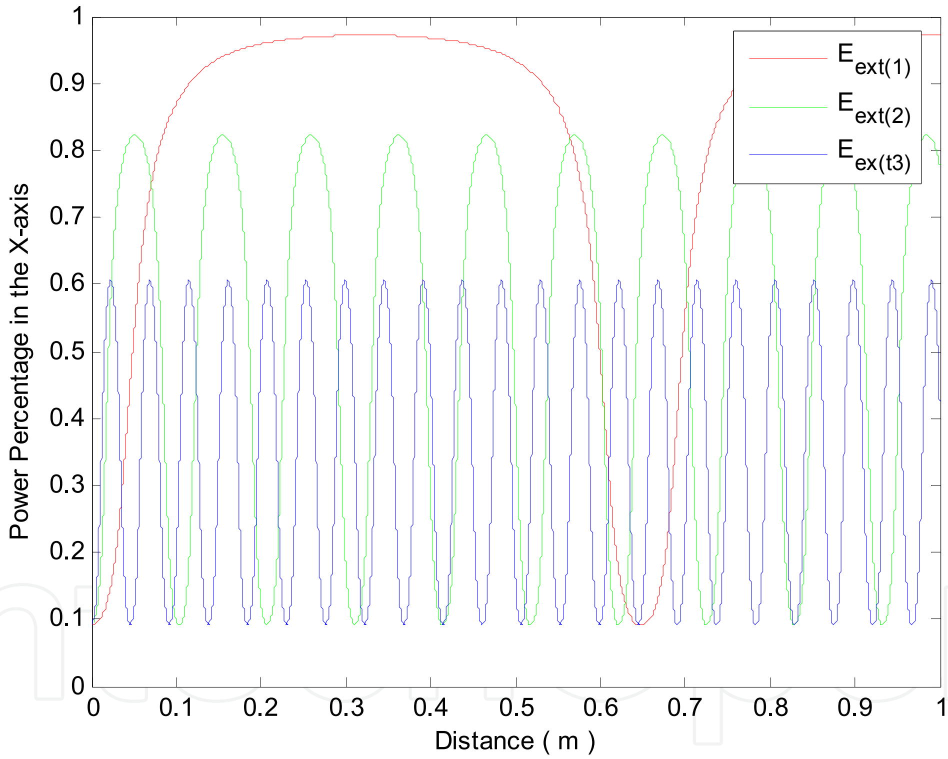

Figure 1. Electrically controlled cross-polarized wave-conversion power exchange

5. Electrically controlled optical multistability

Waves propagating in a resonator filled with an optical Kerr medium that is also subject to an external electric field establish nonlinear behaviour characterized by hysteresis. The shape and the size of the hysteresis transfer function of such a nonlinear optical resonator[21] are controlled by the electric field as well as by the power inside the resonator. Interplay between self-modulation optical and quadratic electro-optic effects is conveniently described using the concept of an effective nonlinear refractive index[22], $n_{eff} = n_L + 3\,\chi^{(3)}\,(E_{ext})^2 + 3\,\chi^{(3)}\,(E_{opt})^2/4$. The transfer function of the resonator is written as:

$$F(\gamma) = \frac{1}{1 + \frac{4R}{T^2}\sin^2(\gamma)}. \tag{6}$$

It is controlled by the phase parameter, γ, which is a function of the intensity, the electric field, and the wavelength. Figure 2 is an example of the nonlinear behaviour of the input-output characteristic as it depends on the external electric field. A material with $\chi^{(3)} = -9 \times 10^{-16}\ cm^2/V^2$ (silicon) was used and $E_{ext\,1,2,3,4,5} = 0.5, 0.8, 7, 8, 9\ V/\mu m$, respectively.

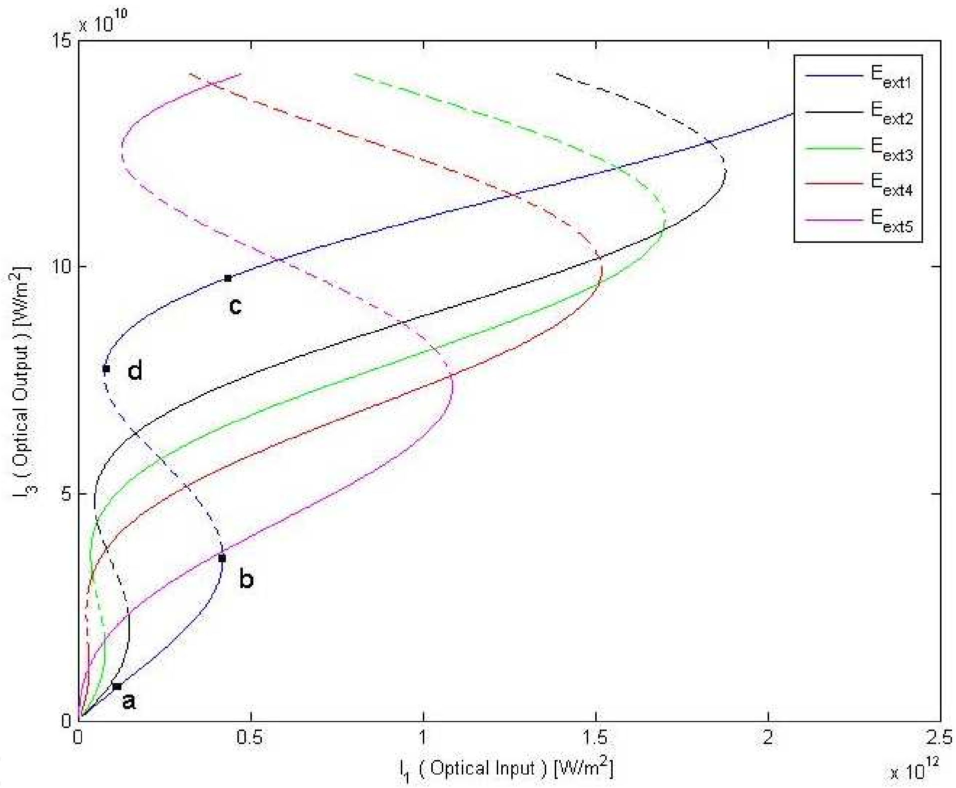

Figure 2. Input-output characteristic of optical nonlinear resonator for different external electric fields

The dashed lines are the unstable optical outputs for given dc electric fields that cause the multistabilities. The electric field controls the required optical input for bistable switching as well as the corresponding output values. Some outputs, which are unstable, become stable, and vice versa, for different biases. This is a tuning property that combined with the previously discussed switching/power-exchange behavior suggests electrically controlled tunable reconfigurability. One can construct a phase diagram of all possible stable optical outputs for a given input as a function of the external electric field [23].

The analysis concludes that the dependence of the system output evolution on the external electric field exhibits a hysteresis-like character as much as with respect to the optical input/output intensity levels. For example, the value of the optical output, at a certain electric field, depends on the history of that field. Such an optically stored electric hysteresis control is a novel feature that can be potentially utilized in the future applications. For example, this hysteresis effect can be used to store an electrical signal (information) optically since the optical system remembers and stores the action of the past electrical signal behavior.

6. Electrically tunable Bragg grating

The rich dispersion properties of Bragg gratings offer many interesting wave propagation features when a nonlinear Kerr medium is incorporated into a grating structure. The key feature is the well-known dispersion property of Bragg gratings whereby their dispersive response is very strong when the operating wavelength is sufficiently close to the Bragg resonance[24] and even if the refractive index changes are very small. An electronically tunable Bragg grating can be constructed based on the third-order nonlinearity discussed here. The nonlinear wave propagation characteristics become interesting and attractive for potential applications.

A homogenous electrical field is known to control, via the Kerr quadratic electro-optic effect, the average refractive index and the birefringence[25]. An inhomogeneous (spatially profiled) electric field bias has been proposed to mediate a linear electro-optic waveguide, by which an effective electro-optic grating is induced[26]. We proposed a novel scheme in which a spatially modulated electric field is applied to a Kerr-nonlinear periodic structure[27]. It was found that several phenomena, including the modulation instability gain, the amplitude (and the width) of the gap soliton, and the band gap, can be efficiently electrically controlled, as long as a proper spatial profile of the electric field is formed.

This can be explained by noting that the electrical spatial profile needs to be designed to act as an extension of the linear perturbation of the periodic structure. An illustrative example of an electrical field bias that has a quasi square-wave shape, which is further modulated by a slow profile, was studied. It was found that, besides the functionality of the periodic part in inducing/controlling all above mentioned phenomena, the slow part of the spatial electrical bias was able to manipulate the linear and nonlinear switching parameters of the band gap. The action of this inhomogeneous electric field is facilitated via the Kerr quadratic electro-optic effect such that the structure's coupling coefficient as well as the average refractive index are controlled.

The effective average refractive index, $\bar{n}_{eff}$, and the effective coupling coefficient, κ_{eff}, of the nonlinear grating were derived[27] showing as they depend on the character of the applied electric field, including its period and shape. The bandwidth of the main reflectivity peak of the grating, $\Delta\omega_{gap}$, was found to be approximately:

$$\Delta\omega_{gap} = \frac{2c}{\bar{n}_{eff}}\left|\kappa_{eff}\right|. \qquad (7)$$

It can be seen from Eq. (5) that the band gap width is electrically controlled through the effective coupling coefficient κ_{eff} and the effective average refractive index $\bar{n}_{eff}$. The controllability through the coupling coefficient is more significant. The reason is that the perturbation in the refractive index, which causes the coupling dynamics, is much smaller than the linear refractive index and thus it is more sensitive to a small change in the refractive index that is induced by the electric field. For example, for a fiber Bragg grating at $\lambda = 1550\ nm$, with $n_{NL} = 2.6 \times 10^{-16}\ cm^2/W$ and grating linear refractive index variations of $n_1 = 10^{-4}$, the periodically shaped applied electric field will initiate the band gap width change of *287.5 MHz* for a field value of $1\ V/\mu m$. As a comparison, if the applied field is spatially constant, the bandwidth of the band gap will change by only *15 kHz* for the same $1\ V/\mu m$. The average refractive index is not as sensitive to the external electric field as the grating coupling coefficient is.

We note that intensive optical excitations can detune themselves (totally or partially) out from a band gap of a periodic medium[28]. As the external electric field also controls the periodic structure dispersion properties, one possesses a Bragg grating band gap that has a dually controllable reflection/transmission characteristic. Using a silicon nanocrystal material[29], figure 3 shows a full simulation of a Bragg grating in a waveguide subjected to an electric field. Both the optical and electrical controls are demonstrated.

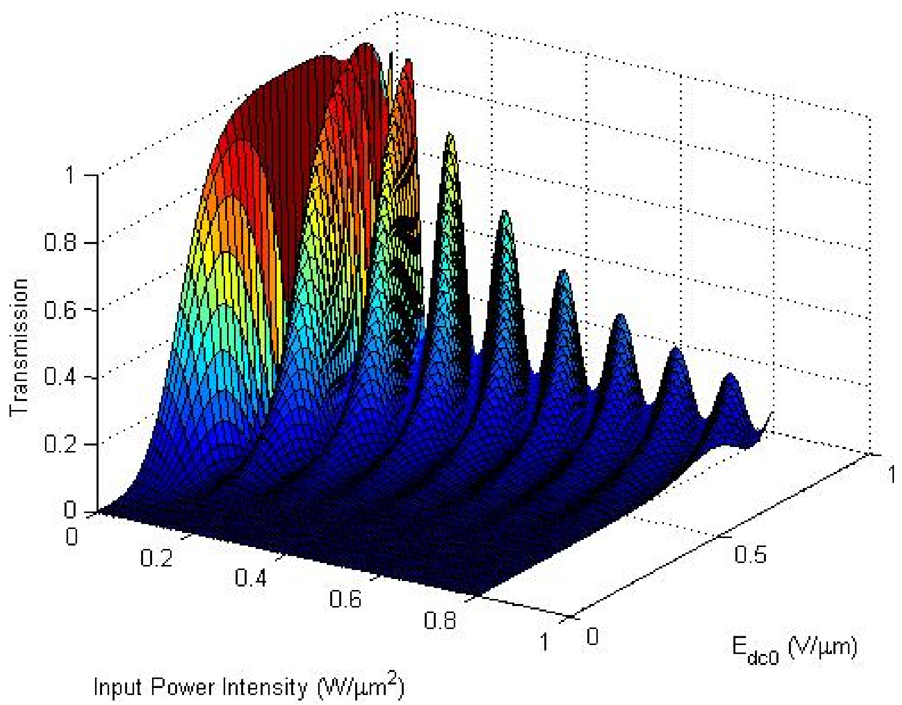

Figure 3. The transmitivity of 5-mm nonlinear waveguide with grating, as function of external electric field and input optical power density; waveguide filled with silicon nanocrystal

7. Kerr switch

The Kerr nonlinearity is usually weak in most materials, although some new materials being developed with nanotechnologies offer promise of significantly enhanced third-order nonlinear coefficients. Silicon nanocrystals can serve as a good example of such a promising material with a stronger nonlinearity of at least one order of magnitude larger than in the bulk counterpart[19]. In order to obtain strong interaction of propagating waves, it is not only the material's nonlinearity that is important. The interaction can be drastically enhanced when optical waves are confined to within a small interaction volume, possibly on the order of less than the wavelength of the waves. Photonic crystal structures, nanoresonators or slot optical waveguides offer such enhanced light confinement.

A combination of all nonlinear interaction enhancing approaches is demonstrated in a Kerr switch design[7] where a ring nanoresonator structure coupled with a slot waveguide filled with silicon nanocrystal is used. For a nominal design wavelength of $\lambda = 1.55\ \mu m$ and the resonator's length of $38\ \mu m$, the free spectral range of *15.5 nm* was obtained with the linewidth of *0.043 nm*. The transmission characteristics are shown in figure 4 for zero refractive index change and for a *10 dB* extinction ratio, respectively.

This shift in figure 4 requires an effective index change of $\Delta n_{eff} = 1.9 \times 10^{-5}$, which in turn calls for the material index change of $\Delta n_{Si\text{-}nc} = 3.8 \times 10^{-5}$. Taking an experimental value for silicon nanocrystal as $\chi^{(3)} = 2 \times 10^{-14}\ cm^2/V^2$ (29% of Si in SiO_2), the refractive index change for a voltage of *1V/100 nm* is $\Delta n_{Si\text{-}nc} = 4.2 \times 10^{-5}$. The nonlinear loss coefficient is $\beta_2 = 70\ cm/GW$. Taking the power inside the waveguide as *4 mW* and with the cross-section being $4 \times 10^{-10}\ cm^2$, the nonlinear absorption per one round trip is less than *0.03 dB*. This is less than in[30], where the loss is due to free carriers. The Kerr effect is as fast as sub-picoseconds. The practical speed of the switch is limited by the capacitance of the electrical contacts onto the resonator/waveguide configuration. For realistic parameters chosen[7], the capacitance is approximately *0.01pF*, which with $50\ \Omega$ yields a theoretical bandwidth of *300 GHz*.

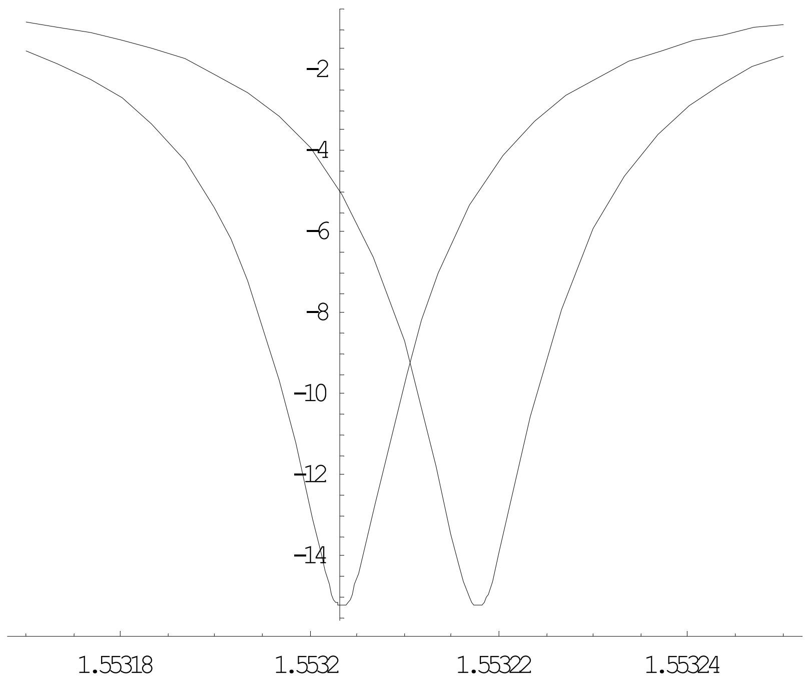

Figure 4. The spectra of Kerr switch.

8. Electrically induced birefringence

Polarization dynamics of ultra-short pulses propagating in an electrically biased silicon waveguide[31] showed interesting features related to wave propagation in nonlinear media. An external electric filed applied to a cubic material induces birefringence via the quadratic electro-optic effect (DC Kerr effect). Transmitted optical pulse polarization can thus be controlled by adjusting the magnitude of the external electric field. When studying propagation of waves in semiconductor materials, the free-carrier induced susceptibility needs to be accounted for by including the free-carrier index changes and the free-carrier absorption into the analysis[31, 32]. The birefringence coefficient as it depends on the applied external electric field can be written as:

$$\kappa_{eff} = [\Delta\beta + \varepsilon_0 k_0 c\, n_L n_{NL} (E_{ext})^2]/2, \qquad (8)$$

where $\Delta\beta$ is the material linear birefringence. Figure 5 illustrates the electrical-control effect by showing the polarization component transmission coefficient of a *6-mm* long waveguide as it changes with the input optical power of a *70-fs* long Gaussian pulse.

It should be pointed out that the quantities E_{ext1} and E_{ext2} are amplitudes of the electric field that has, generally, a certain profile along the propagation direction (along the waveguide). Shaping the electric field offers an additional control parameter. The temporal profile of the ultra-short pulse can be governed by a field profile properly designed[31]; an example of a profile is an exponential dependence such that:

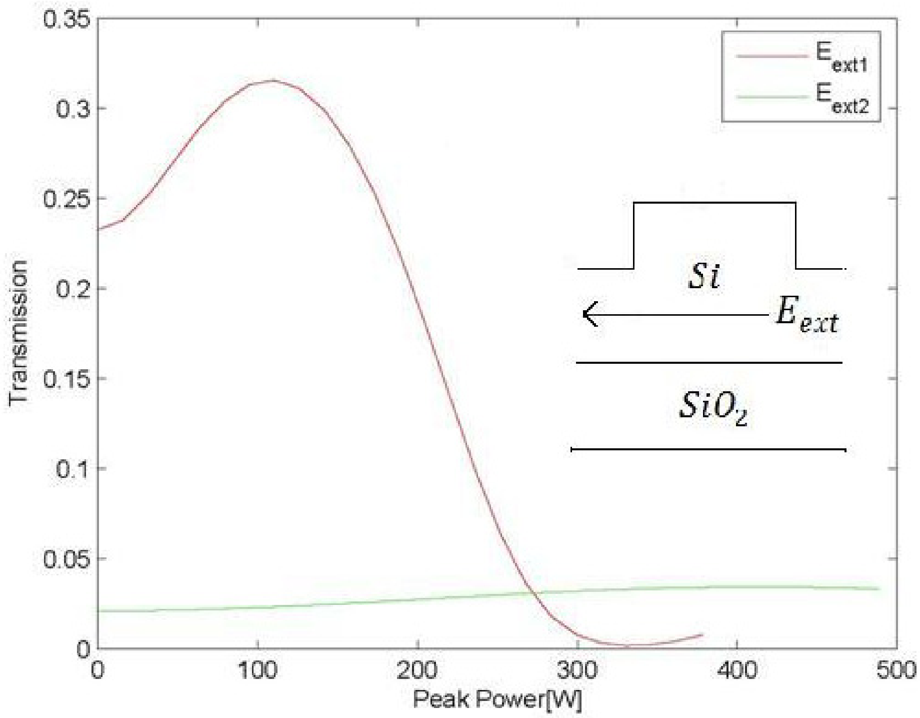

Figure 5. Fig. 5: Transmission coefficient of an electrically induced birefringence waveguide; E_{ext1}=12.5 V/μm, E_{ext2}=0 V/μm, $\Delta\beta$=2x10^{-5}k_0.

$$E_{ext}(z) = \left(\frac{\Delta\beta}{\varepsilon_0 k_0 n_L n_{NL}}\left(e^{\alpha(L-z)} - 1\right)\right)^{1/2}, \tag{9}$$

where L is the length of the electric field profiling along the propagation direction, and α is a design parameter determining the rate of decay of the exponential profile. Figure 6 illustrates the effect of a properly designed spatial profile of the control electric field for a Gaussian *70-fs* pulse traveling along a *2-cm* long waveguide. The azimuths of the pulse are chosen within the polarization instability regime. The field shape design parameter $\alpha = 57.5\ m^{-1}$.

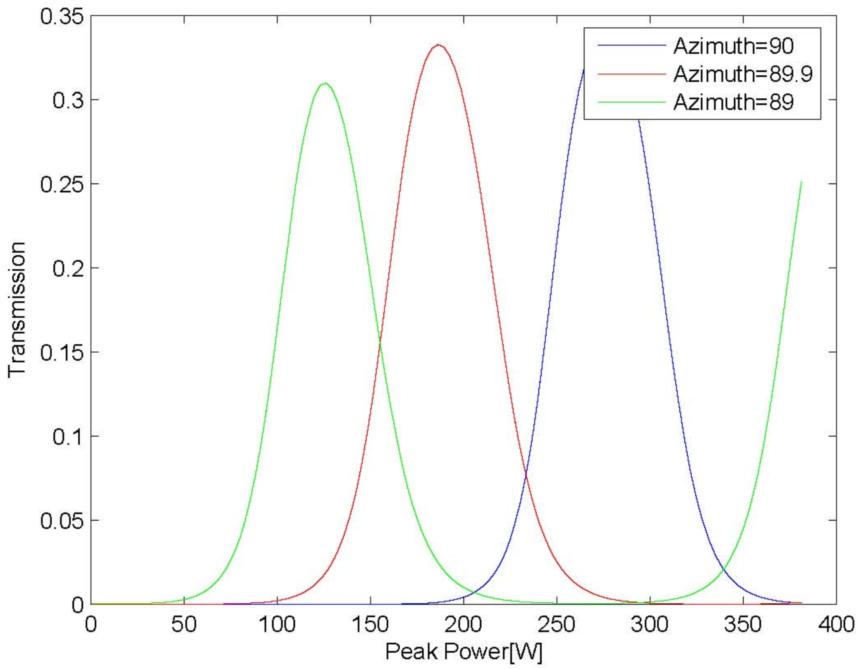

Figure 6. Transmission of Gaussian pulse through electric-field profiled nonlinear waveguide

9. Photorefractive effect in silicon

The crystal symmetry in cubic or isotropic materials, e.g. silicon or glass, etc., can be broken via the third-order nonlinearity by applying an external electric field. This causes such materials behave as if they possessed the linear electro-optic effect. The photorefractive effect has been known since the early 1960's[33]; it was observed in many electro-optic crystals, including $LiNbO_3$, $GaAs$, InP, or $CdTe$[34]. The photorefractive effect is an automatically phase-matched nonlinear phenomenon, whereby interfering light modes can generate a spatially phase-shifted electric field in a host material. This spatially phase-shifted electric field, in turn, couples the interfering light modes in a phase-matched fashion.

It is therefore interesting to investigate the existence and properties of the photorefractive effect in materials without the natural linear electro-optic effect, and exploit the third-order nonlinearity to establish it[6]. The detailed and complex spatio-temporal nonlinear analysis of two contra-propagating waves, including the free carriers in a semiconductor, shows that the photorefractive effect can lead to gain or loss of one of the waves (signal or probe). The power exchange with the other wave (pump) is controlled by the polarity of the external electric field. This is electronically controlled unidirectional power transfer. It may also be considered as a parametric process.

Figure 7 shows an example of such power transfer in a silicon waveguide at $\lambda = 1.55$ μm with an effective cross-section of $0.3\ \mu m^2$, linear loss of $0.57\ cm^{-1}$, and an n-doping of 10^{19} cm^{-3}.

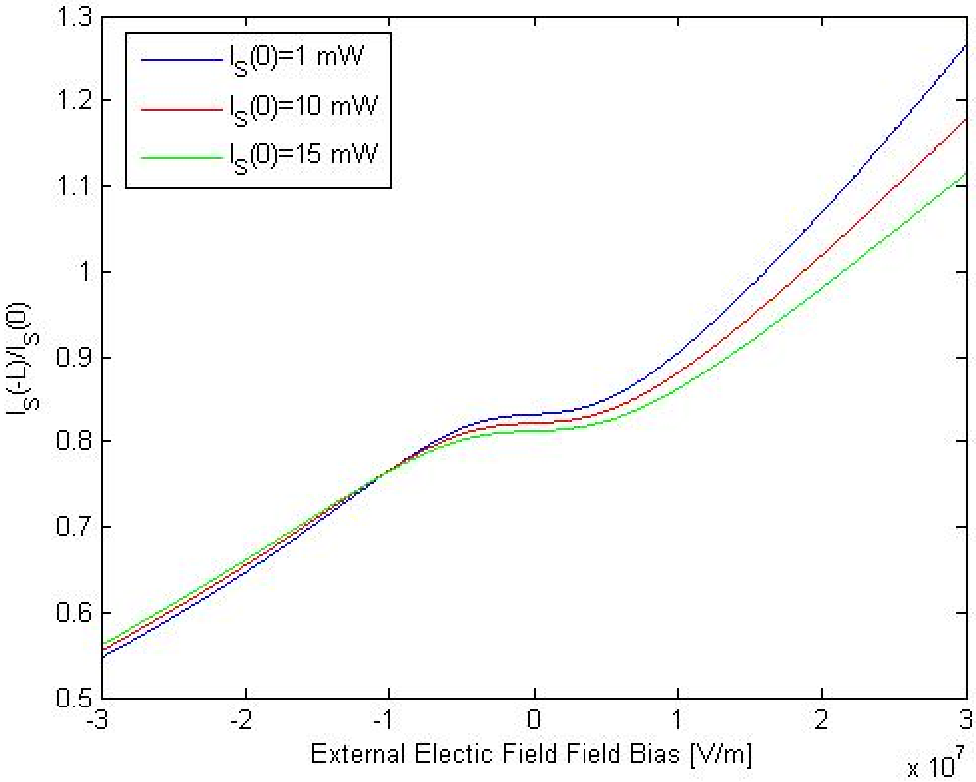

Figure 7. Net signal gain in electrically induced photorefractive silicon waveguide

As can be seen in the figure, the net signal gain can be achieved despite material losses. Also, as is expected in any photorefractive medium, the net signal gain increases with decreasing input signal power for a constant pump power.

If the frequency detuning, Ω, is different from zero, gain enhancement will be experienced at a certain frequency detuning, i.e. at which the amplitude of the photo-induced space charge electric field is maximized[6]. The net signal gain versus frequency detuning is exhibited in figure 8.

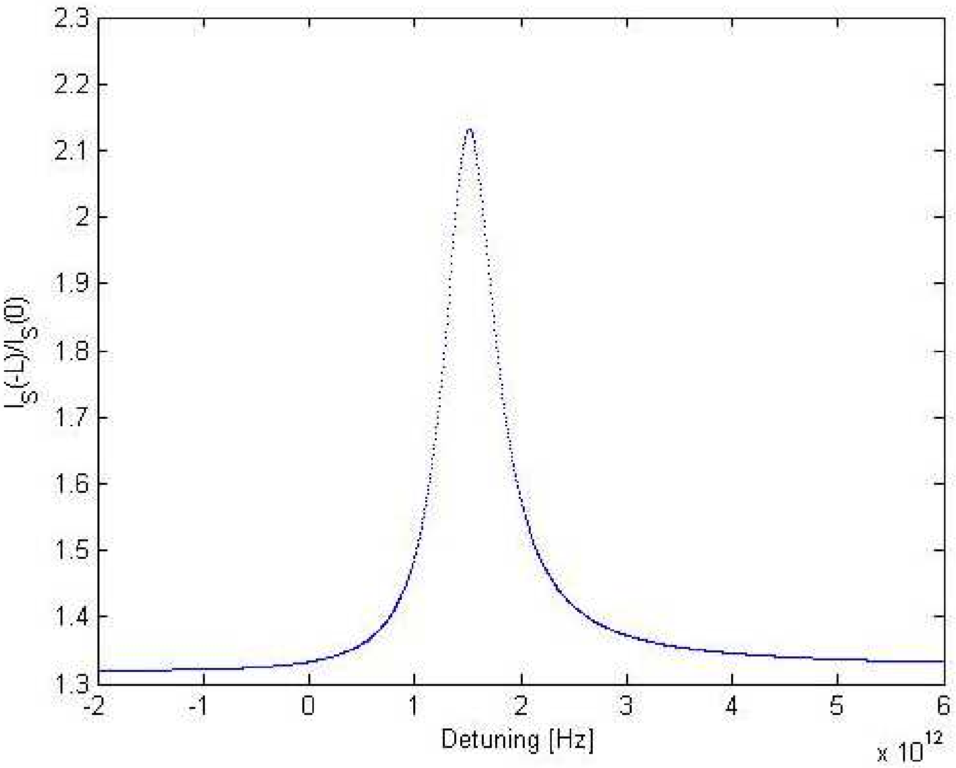

Figure 8. Net signal gain enhancement versus frequency detuning; $E_{ext} = 25\ V/\mu m$

10. Discussion

This work focused on investigating optical wave propagation properties and associated potential devices functionalities in Kerr-type media with applied external electric field. The assisting external fields induce a quadratic electro-optic effect in a centro-symmetric (cubic, isotropic) third-order nonlinear materials (e.g. glass, silicon, silicon nanocrystal). If the optical field of propagating waves is sufficiently intense, the all-optical effects start to appear as well. The interplay between these two effects (the Kerr electro-optic and all-optical effects) was the main focus with the goal to demonstrate phenomena potentially useful in the design of novel photonic devices. Although the few presented numerical examples are for silicon, silicon nanocrystal or silicon nanowires, chosen due to the silicon's attractiveness in its integreability with standard microelectronics technologies, the obtained results are applicable to a variety of other optical materials, including silica glass, GaAs bulk, CdTe bulk, GaAs and InP quantum wells, CdTe nanocrystal, CdS nanocrystal, poly (β-pinene), fullerene-containing polyurethane films, natural rubber, and many others.

Optical waves propagating in an electro-optic-Kerr-effect-induced birefringence medium were studied showing electrical and optical control of power exchange between their components. The birefringence is proportional to the square of an applied external electric field. The concept of an effective refractive index containing nonlinear optical as well as electrical dependencies was introduced to model Kerr nonlinear wave propagation behaviour when the material is subjected to external electric fields The wave properties suggest that one can exploit them in designing novel photonic devices, such as, for example, an optically controlled electro-optical switch or an electronically controlled all-optical modulator.

An optical Fabry–Pérot resonator filled with a Kerr nonlinear material and subjected to an external electric field was investigated. As expected, the optical input-output transfer function displays hysteresis that is controlled by the applied electric field. The stability analysis revealed electronically-tunable optical multi-stability of the resonator. This means that the state of the optical intensity implies a desired information for a given external electric field. Such a feature offers a new functionality whereby electrical information can be stored optically.

When a Bragg grating is made in a Kerr material, the electrically induced control of the spatial inhomogeneity is established. As a result, the modulation instability gain was found to be electrically controlled. This suggests a possible realization of an electrically controlled pulse generator. Also, the amplitude and the width of the gap soliton can be, in such a case, electrically adjustable, and thus a tunable soliton channel is possible. The external electric field having a spatial profile modulated by a slow varying profile makes it possible to control the nonlinear grating band gap electrically very efficiently compared to other known means, the reason being that by profiling the electric field one gains a direct control over the grating coupling coefficients.

Ultra-short optical pulses propagating in a nonlinear Kerr medium while an external electric field was applied, was studied. A silicon waveguide was considered. Several realistic effects were taken into account, including large linear loss, nonlinear anisotropy, two-photon absorption, and associated free carriers. It was shown that the silicon waveguide can be used as a practical platform for all-optical applications, including polarization switching and pulse shaping. A properly designed of the external electric spatial profile was shown to help achieve polarization instability regime, which is important for realizing sensitive polarization discriminating devices.

The new photorefractive effect in cubic materials (e.g. silicon nanocrystal) was established and investigated. As cubic materials do not possess the linear electro-optic effect, the photorefractive effect is not readily obtained. We demonstrated that a proper external electric field can assist in realizing the effect in such materials and structures, for example in silicon waveguides. Despite the linear and nonlinear losses, it was shown that a weak signal counter-propagating with a strong pump can experience a net gain. One may suggest that integrated photorefractive devices that are optically and electronically controlled can be designed based on this new phenomenon.

The four-wave mixing (FWM) phenomenon was not examined in this work. However, since it is a nonlinear process that takes place in a third-order nonlinear medium, resulting in the generation of a new optical wave with a new frequency (parametric process), it is realizable in the Kerr media considered here. As it is known, a readily efficient FWM process cannot be achieved because of the phase-mismatching dilemma. One way to achieve efficient FWM is to utilize a nonlinear periodic structure[35]. Based on the work presented here, a properly profiled external electric field could be employed to achieve a tunable quasi-phase matching operation. It is thus interesting to investigate the FWM phenomenon in the presence of an external electric field, as this is very important in realizing tunable devices for all-optical communications and processing.

As a conclusion, owing to its potential for integration with micro-electronics, the silicon-based technology is considered one of the most important means for photonic applications. The dual electro-optical and all-optical functionality studied here in materials of the same symmetry as silicon and its derivatives, resulting from the Kerr effect, offers a promise and a potential for realizing technologically compatible and implementation friendly electrically-controlled all-optical devices and/or optically controlled electro-optical devices that can be readily integrated onto a common material platform.

11. Conclusion

Optical wave propagation in a Kerr-type nonlinear medium has been analyzed theoretically and studied numerically. New features were found where not only waves and their polarization interactions are present as a result of the nonlinearity of the medium, but also an interplay between the optical and electrical Kerr effects contributes to the resulting functionality. Several novel wave propagation effects were discovered. They include cross-polarized wave conversion, optical multistability, nonlinear tunability of periodic structures, ultra-fast electro-optic switching, and a new photorefractive effect. Possible applications of such novel functionalities were discussed. Examples utilizing silicon as the common semiconductor material were given.

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