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Chapter 9

# Stochastic Observation Optimization on the Basis of the Generalized Probabilistic Criteria

Sergey V. Sokolov

Additional information is available at the end of the chapter

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## 1. Introduction

Till now the synthesis problem of the optimum control of the observation process has been considered and solved satisfactorily basically for the linear stochastic objects and observers by optimization of the *quadratic* criterion of quality expressed, as a rule, through the a posteriori dispersion matrix [1-4]. At the same time, the statement of the synthesis problem for the optimum observation control in a more general case assumes, first, a nonlinear character of the object and observer, and, second, the application of the non-quadratic criteria of quality, which, basically, can provide the potentially large estimation accuracy[3-6].

In connection with the fact that the solution of the given problem in such a statement generalizing the existing approaches, represents the obvious interest, we formulate it more particularly as follows.

## 2. Description of the task

Let the Markovian vector process $\xi_t$, described generally by the nonlinear stochastic differential equation in the symmetrized form

$$\dot{\xi}_t = f\left(\xi,t\right) + f_0\left(\xi,t\right)n_t, \qquad \xi\left(t_0\right) = \xi_0, \tag{1}$$

where $f$, $f_0$ are known $N$ – dimensional vector and $N \times M$ – dimensional matrix nonlinear functions;

$n_t$ is the white Gaussian normalized $M$ – dimensional vector - noise; be observed by means of the vector nonlinear observer of form:

$$Z = H\left(\xi,t\right) + W_t,$$

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where $Z - L \le N$ – dimensional vector of the output signals of the meter;

$h(\xi,t)$ – a known nonlinear $L$- dimension vector - function of observation;

$W_t$ – a white Gaussian $L$- dimension vector - noise of measurement with the zero average and the matrix of intensity $D_W$ .

The function of the a posteriori probability density (APD) of process $\rho(\xi,t) = \rho\left(\xi,t\middle|Z_\tau,\ \tau \in \left[t_0,t\right]\right)$ is described by the known integro-differential equation in partial derivatives (Stratonovich equation), the right-hand part of which explicitly depends on the observation function $h$:

$$\frac{\partial\rho(\xi,t)}{\partial t} = L\left\{\rho(\xi,t)\right\} + \left[Q - Q_0\right]\rho(\xi,t),$$

where $L\left\{\rho(\xi,t)\right\} = -div\left\{\left[f + \frac{1}{2}\frac{\partial f_0}{\partial\xi}\left(f_0^T\right)^{(V)}\right]\rho\right\} + \frac{1}{2}div\left\{\overline{div}\left[f_0 f_0^T \rho\right]\right\}$ – the Focker-Plank-operator,

$(A)^{(V)}$ is the operation for transforming the $n \times m$ matrix **A** into vector $(A)^{(V)}$ formed from its elements as follows:

$$A^{(V)} = \left|a_{11}a_{21}\ldots a_{m1}a_{12}a_{22}\ldots a_{m2}\ldots a_{1n}a_{2n}\ldots a_{mn}\right|^T,$$

$\overline{div}$ is the symbol for the operation of divergence of the matrix row,

$$Q = Q(\xi,t) = -\frac{1}{2}\left[Z - H(\xi,t)\right]^T D_W^{-1}\left[Z - H(\xi,t)\right],$$

$$Q_0 = \int_{-\infty}^{\infty} Q(\xi,t)\rho(\xi,t)d\xi.$$

As the main problem of the a posteriori analysis of the observable process $\xi_t$ is the obtaining of the maximum reliable information about it, then the synthesis problem of the optimum observer would be natural to formulate as the definition of the form of the functional dependence $h(\xi,t)$, providing the maximum of the a posteriori probability (MAP) of signal $\xi_t$ on the given interval of occurrence of its values $\xi_* = \left[\xi_{\min},\xi_{\max}\right]$ during the required interval of time $T = [t_0, t_k]$, i.e. in view of the positive definiteness $\rho(\xi,t)$

$$\max\left\{J = \int_T\int_{\xi_*}\rho(\xi,t)d\xi\,dt\right\}$$

or

$$\min\left\{-\int_{T}\int_{\xi_*}\rho(\xi,t)\,d\xi\,dt\right\}.$$

Generally instead of criterion MAP one can use, for example, the criterion of the minimum of the a posteriori entropy on interval $\xi_* = [\xi_{\min}, \xi_{\max}]$ or the criterion of the minimum of the integrated deviation of the a posteriori density from the density of the given form etc., that results in the need for representing the criterion of optimality ***J*** in the more generalized form:

$$J = \int_{T}\int_{\xi_*}\Phi\left[\rho(\xi,t)\right]d\xi\,dt,$$

where **Φ** is the known nonlinear function which takes into account generally the feasible analytical restrictions on the vector $\xi_t$;

$T = [t_0, t_k]$ is a time interval of optimization;

$\xi_*$ is some bounded set of the state parameters $\xi_t$.

In the final forming of structure of the criterion of optimality $J$ it is necessary to take into account the limited opportunities of the practical realization of the function of observation $h(\xi,t)$, as well, that results, in its turn, in the additional restriction on the choice of functional dependence $h(\xi,t)$. The formalization of the given restriction, for example, in the form of the requirement of the minimization of the integrated deviation of function $H$ from the given form $H_0$ on interval $\xi_*$ during time interval $T$ allows to write down analytically the form of the minimized criterion $J$ as follows:

$$J = \int_{T}\int_{\xi_*}\Phi\left[\rho(\xi,t)\right]d\xi\,dt + \int_{T}\int_{\xi_*}\left[H(\xi,t)-H_0(\xi,t)\right]^T\left[H(\xi,t)-H_0(\xi,t)\right]d\xi\,dt = \int_{T}W_*(t)\,dt. \quad (2)$$

Thus, the final statement of the synthesis problem of the optimum observer in view of the above mentioned reasoning consists in defining function $h(\xi,t)$, giving the minimum to functional (2).

## 3. Synthesis of observations optimal control

Function APD, included in it, is described explicitly by the integro-differential Stratonovich equation with the right-hand part dependent on $h(\xi,t)$. The analysis of the experience of the instrument realization of the meters shows, that their synthesis consists, in essence, in defining the parameters of some functional series, approximating the output characteristic of the device projected with the given degree of accuracy. As such a series one uses, as a rule, the final expansion of the nonlinear components of vector $h(\xi,t)$ in some given system of the multidimensional functions: power, orthogonal etc.

Having designated vector of the multidimensional functions as $\left|\psi_1 ... \psi_S\right|^T = \psi$, we present the approximation of vector $h(\xi,t)$ as

$$H(\xi,t) = \left(E \otimes \psi^T\right) h = \psi_E h, \tag{3}$$

$$h = \left|h_{11} ... h_{1S} h_{21} ... h_{2S} ... h_{N1} ... h_{NS}\right|^T,$$

where $h_i(\xi,t) = \sum_{j=1}^{S} h_{ij}(t)\psi_j(\xi)$ is the $i$-th component of vector $h$, the factors of which define the concrete technical characteristics of the device,

$\otimes$ is the symbol of the Kronecker product.

For the subsequent analytical synthesis of optimum vector - function $h(\xi,t)$ in form of (3) we rewrite the equation of the APD $\rho(\xi,t)$ in the appropriate form

$$\frac{\partial \rho}{\partial t} = L[\rho] + h^T H_1[\rho] - h^T H_2[\rho] h, \tag{4}$$

where

$$H_1[\rho] = \left[\psi_E^T - \int_{\xi_*} \psi_E^T(\xi)\rho(\xi,t)d\xi\right] D_W^{-1} Z\rho(\xi,t),$$

$$H_2[\rho] = \frac{\rho(\xi,t)}{2}\left[\psi_E^T D_W^{-1}\psi_E - \int_{\xi_*} \psi_E^T(\xi) D_W^{-1}\psi_E(\xi)\rho(\xi,t)d\xi\right].$$

The constructions carried out the problem of search of optimum vector $h(\xi,t)$ is reduced to the synthesis of the optimum in-the- sense -of-(2) control $h$ of the process with the distributed parameters described by Stratonovich equation (in view of representing vector $H_0(\xi,t)$ in the form similar to (3)

$$H_0(\xi,t) = \psi_E h_0).$$

The optimum control of process $\rho(\xi,t)$ will be searched in the class of the limited piecewise-continuous functions with the values from the open area $H_*$. For its construction we use the method of the dynamic programming, according to which the problem is reduced to the minimization of the known functional [1]

$$\min_{h \in H_*}\left\{\frac{dV}{dt} + W_*\right\} = 0 \tag{5}$$

under the final condition $V(t_k) = 0$ with respect to the optimum functional $V = V(\rho,t)$, parametrically dependent on $t \in [t_0, t_k]$ and determined on the set of functions satisfying (4).

For the processes, described by the linear equations in partial derivative, and criteria of the form of the above-stated ones, functional $V$ is found in the form of the integrated quadratic form [1], therefore in this case we have:

$$V = \int_{\xi_*} v(\xi,t)\rho^2(\xi,t)d\xi.$$

Calculating derivative $\frac{dV}{dt}$

$$\frac{dV}{dt} = \int_{\xi_*}\left(\frac{dv}{dt}\rho^2 + 2v\rho\frac{d\rho}{dt}\right)d\xi = \int_{\xi_*}\left(\frac{dv}{dt}\rho^2 + 2v\rho L[\rho] + 2v\rho h^T H_1[\rho] - 2v\rho h^T H_2[\rho]h\right)d\xi,$$

the functional equation for $v$ is obtained in the following form:

$$\min_{h\in H_*}\int_{\xi_*}\left(\frac{dv}{dt}\rho^2 + 2v\rho L[\rho] + 2v\rho\left(h^T H_1[\rho] - h^T H_2[\rho]h\right) + \Phi[\rho]\right)d\xi +$$

$$+\left(h-h_0\right)^T\int_{\xi_*}\psi_E^T(\xi)\psi_E(\xi)d\xi\left(h-h_0\right) = 0,$$

whence we have optimum vector $h_{opt}$:

$$h_{opt} = \left\{\int_{\xi_*}\left[\psi_E^T(\xi)\psi_E(\xi) - v\rho\left(H_2 + H_2^T\right)\right]d\xi\right\}^{-1}\int_{\xi_*}\left(\psi_E^T(\xi)\psi_E(\xi)h_0 - v\rho H_1\right)d\xi =$$

$$= B(v,\rho)\int_{\xi_*}\left(\psi_1(\xi)h_0 - v\rho H_1\right)d\xi.$$

Using condition $\left\{\frac{dV}{dt} + W_*\right\}_{h=h_{opt}} = 0$, for $v(\xi,t)$ we have the following equation:

$$\frac{dv}{dt} = -2v\rho^{-1}L[\rho] - \rho^{-2}\left(h_0^T\psi_{1\xi}B^T - \int_{\xi_*}v\rho H_1^T d\xi\, B^T\right)\left(2v\rho H_1 - \psi_1 h_0\right) +$$

$$+\rho^{-2}h_0^T\psi_1\left(B\psi_{1\xi}h_0 - B\int_{\xi_*}v\rho H_1 d\xi\right) + \rho^{-2}\left(h_0^T\psi_{1\xi}B^T - \int_{\xi_*}v\rho H_1^T d\xi\, B^T\right)\times$$

$$\times\left(2v\rho H_2-\psi_1\right)\left(B\psi_{1\xi}h_0-B\int_{\xi_*}v\rho H_1 d\xi\right)-\rho^{-2}h_0^T\psi_1 h_0-\rho^{-2}\Phi\left[\rho\right], \tag{6}$$

where

$$\psi_{1\xi}=\int_{\xi_*}\psi_1\left(\xi\right)d\xi,$$

which is connected with the equation of the APD, having after substitution into it expression $h_{opt}$ the following form:

$$\frac{d\rho}{dt}=L\left[\rho\right]+\left(h_0^T\psi_{1\xi}B^T-\int_{\xi_*}v\rho H_1^T d\xi\, B^T\right)H_1-$$

$$-\left(h_0^T\psi_{1\xi}B^T-\int_{\xi_*}v\rho H_1^T d\xi\, B^T\right)H_2\left(B\psi_{1\xi}h_0-B\int_{\xi_*}v\rho H_1 d\xi\right). \tag{7}$$

## 4. Observations suboptimal control

The solution of the obtained equations (6), (7) exhausts completely the problem stated, allowing to generate the required optimum vector - function *h* of form (3). On the other hand, the solution problem of system (6), (7) is the point-to-point boundary-value problem for integrating the system of the integro-differential equations in partial derivatives, general methods of the exact analytical solution of which , as it is known, does not exist now. Not considering the numerous approximated methods of the solution of the given problem oriented on the trade-off of accuracy against volume of the computing expenses, then as one of the solution methods for this problem we use the method based on the expansion of function **v,** ***p*** in series by some system of the orthonormal functions of the vector argument :

$$V\left(\xi,t\right)=\sum_{\mu}\alpha_\mu\left(t\right)\phi_\mu\left(\xi\right)=\phi^T\alpha\,,$$

$$\rho\left(\xi,t\right)=\sum_{\mu}\beta_\mu\left(t\right)\phi_\mu\left(\xi\right)=\phi^T\beta\,,$$

where μ is the index running a set of values from (0,...,0) to (*M*,...,*M*) [2];
φ is the vector of the orthonormal functions of argument ξ;
α, β are vectors of factors of the appropriate expansions.
In this case the solution is reduced to the solution of the point-to-point boundary-value problem for integrating the system of the following equations, already ordinary ones:

$$\dot{\beta} = \int_{\xi_*} \phi L\left[\phi^T\beta\right]d\xi + \int_{\xi_*} \phi\left[h_0\psi_{1\xi}B^T\left(\alpha,\beta,\phi\right) - \int_{\xi_*}\phi^T\alpha\phi^T\beta H_1^T\left(\phi^T\beta\right)d\xi\, B^T\left(\alpha,\beta,\phi\right)\right]H_1\left(\phi^T\beta\right)d\xi -$$

$$-\int_{\xi_*}\phi\left[h_0^T\psi_{1\xi}B^T\left(\alpha,\beta,\phi\right) - \int_{\xi_*}\phi^T\alpha\phi^T\beta H_1^T\left(\phi^T\beta\right)d\xi\, B^T\left(\alpha,\beta,\phi\right)\right]H_2\left(\phi^T\beta\right) -$$

$$-\left(B\left(\alpha,\beta,\phi\right)\psi_{1\xi}h_0 - B\left(\alpha,\beta,\phi\right)\int_{\xi_*}\phi^T\alpha\phi^T\beta H_1\left(\phi^T\beta\right)d\xi\right)d\xi,$$

$$\dot{\alpha} = \int_{\xi_*}\phi^T\left\{-2\phi^T\alpha\left(\phi^T\beta\right)^{-1}L\left[\phi^T\beta\right] - \left(\phi^T\beta\right)^{-2}\left[h_0^T\psi_{1\xi}B^T\left(\alpha,\beta,\phi\right) -\right.\right.$$

$$\left.\left. - \int_{\xi_*}\phi^T\alpha\phi^T\beta H_1^T\left(\phi^T\beta\right)d\xi\, B^T\left(\alpha,\beta,\phi\right)\right]\left(2\phi^T\alpha\phi^T\beta H_1\left(\phi^T\beta\right) - \psi_1 h_0\right) +\right.$$

$$+\left(\phi^T\beta\right)^{-2}h_0^T\psi_1\left(B\left(\alpha,\beta,\phi\right)\psi_{1\xi}h_0 - B\left(\alpha,\beta,\phi\right)\int_{\xi_*}\phi^T\alpha\phi^T\beta H_1\left(\phi^T\beta\right)d\xi\right) + \qquad (8)$$

$$+\left(\phi^T\beta\right)^{-2}\left[h_0^T\psi_{1\xi}B^T\left(\alpha,\beta,\phi\right) - \int_{\xi_*}\phi^T\alpha\phi^T\beta H_1^T\left(\phi^T\beta\right)d\xi\, B^T\left(\alpha,\beta,\phi\right)\right]\times$$

$$\times\left(2\phi^T\alpha\phi^T\beta H_2\left(\phi^T\beta\right) - \psi_1\right)\left(B\left(\alpha,\beta,\phi\right)\psi_{1\xi}h_0 -\right.$$

$$\left.-B\left(\alpha,\beta,\phi\right)\int_{\xi_*}\phi^T\alpha\phi^T\beta H_1\left(\phi^T\beta\right)d\xi\right) - \left(\phi^T\beta\right)^{-2}h_0^T\psi_1 h_0 - \left(\phi^T\beta\right)^{-2}\Phi\left[\phi^T\beta\right]\Big\}d\xi$$

under boundary value conditions $\alpha\left(T\right) = 0,\ \beta\left(t_0\right) = \beta_0$, where the values of the components are defined from the expansion of function $\rho\left(\xi, t_0\right) = \rho_0$.

From the point of view of the practical realization the integration of system (8) under the boundary-value conditions appears to be more simple than integration (6), (7), but from the point of view of organization of the estimation process in the real time it is still hindered: first, the volume of the necessary temporary and computing expenses is great, secondly the feasibility of the adjustment of the vector of factors h in the real time of arrival of the signal of measurement $Z$ - is excluded, the prior simulation of realizations $Z$ appears to be necessary (in this case in the course of the instrument realization, as a rule, one fails to maintain the precisely given values $h$ all the same). Thus, the use of the approximated methods of the problem solution (8) is quite proved in this case, then as one of which we consider the method of the invariant imbedding [3], used above and providing the required approximated solution in the real time.

As the application of the given method assumes the specifying of all the components of the required approximately estimated vector in the differential form, then for the realization of the feasibility of the synthesis of vector $h$ through the given method in the real time we introduce a dummy variable v, allowing to take into account from here on expression $h_{\text{opt}}$ as the differential equation

$$\dot{v} = h_{\text{opt}}\left(\phi^T\alpha, \phi^T\beta\right),$$

forming with equations (8) a unified system. The application of the method of the invariant imbedding results in this case in the following system of equations:

$$\begin{vmatrix}\dot{v}_0\\ \dot{\beta}_0\end{vmatrix} = \left|\int_{\xi_*}^{h_0} \phi\left[\phi^T\beta_0\right] + h_0^T\left(H_1\left[\phi^T\beta_0\right] - H_2\left[\phi^T\beta\right]h_0\right)d\xi\right| - D\int_{\xi_*}\phi\left(\phi^T\beta_0\right)^{-2}\Phi\left[\phi^T\beta_0\right]d\xi,$$

$$\dot{D} = 2\int_{\xi_*}\phi\left(\frac{\partial}{\partial\beta_0}\left\{L\left[\phi^T\beta_0\right]\right\} + h_0^T\frac{\partial H_1}{\partial\beta_0}\left[\phi^T\beta_0\right] - h_0^T\frac{\partial H_2}{\partial\beta_0}\left[\phi^T\beta_0\right]h_0\right)d\xi\, D +$$

$$+2D\int_{\xi_*}\phi\left(\phi^T\beta_0\right)^{-1}\left(\phi^T L\left[\phi^T\beta_0\right] - h_0^T\left(\phi^T\otimes H_2\left[\phi^T\beta_0\right]\right)h_0\right)d\xi -$$

$$-\int_{\xi_*}\phi\left(H_1^T\left[\phi^T\beta_0\right] - 2h_0^T H_2\left[\phi^T\beta_0\right]\right)d\xi\,\mu +$$

$$+2D\int_{\xi_*}\phi\left(\phi^T\beta_0\right)^{-2}\left(2\left(\phi^T\beta_0\right)^{-1}\phi^T\Phi\left[\phi^T\beta_0\right] - \frac{\partial}{\partial\beta_0}\Phi\left[\phi^T\beta_0\right]\right)d\xi\, D,$$

$$\mu = 2\psi_{1\xi}^{-1}\int_{\xi_*}\left(\phi^T\beta_0\right)\left[\left(\phi^T\otimes H_2\right)h_0 - \frac{1}{2}H_1\phi^T\right]d\xi.$$

By virtue of the fact that matrix $D$ in the method of the invariant imbedding plays the role of the weight matrix at the deviation of the vector of the approximated solution from the optimum one, in this case for variables $\beta_{i0}$ the appropriate components $D$ characterize the degree of their deviation from the factors of expansion of the true APD (components $D_0$ - are deviations of the parameters at the initial moment). The essential advantage of the approach considered, despite the formation of the approximated solution, is the feasibility of the synthesis of the optimum observation function in the real time, i.e. in the course of arrival of the measuring information.

## 5. Example

For the illustration of the feasibility of the practical use of the suggested method the numerical simulation of the process of forming vector $h = \left|h_1 h_2\right|^T$ of factors of the observer

$Z = h_1\xi + h_2\xi^2 + W_t$ for target $\dot{\xi} = -\xi^3 + n_t$ was carried out the normalized Gaussian white noises of the target and meter. As the criterion of optimization the criterion of the maximum of the a posteriori probability of the existence of the observable process on interval $\xi_*$ = = [-2.5, 2.5] was chosen that provided the additional restriction in the form of the requirement of the minimal deviation of vector *h* from the given vector $h_0 = \left|0.95,\ 0.3\right|^T$ that allows to write down the minimized functional as

$$J = -\int_T \int_{\xi_*} \rho(\xi, t)\, d\xi\, dt + \int_T \left(h - h_0\right)^T D_H \left(h - h_0\right) dt,$$

where

$$D_H = \int_{\xi_*} \begin{vmatrix} \xi \\ \xi^2 \end{vmatrix} \begin{vmatrix} \xi & \xi^2 \end{vmatrix} d\xi = \begin{vmatrix} 10.4 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 39.1 \end{vmatrix}, \quad T = \left|0;\ 600\right|.$$

In this case the equation of the APD has the form

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial \xi}\left(\xi^3 \rho\right) + \frac{1}{2}\frac{\partial^2 \rho}{\partial \xi^2} + h^T H_1 - h^T H_2 h,$$

where

$$H_1 = \left( \begin{vmatrix} \xi \\ \xi^2 \end{vmatrix} - \int_{\xi_*} \begin{vmatrix} \xi \\ \xi^2 \end{vmatrix} \rho(\xi, t)\, d\xi \right) Z\rho,$$

$$H_2 = \frac{\rho}{2}\left( \begin{vmatrix} \xi^2 & \xi^3 \\ \xi^3 & \xi^4 \end{vmatrix} - \int_{\xi_*} \begin{vmatrix} \xi^2 & \xi^3 \\ \xi^3 & \xi^4 \end{vmatrix} \rho(\xi, t)\, d\xi \right).$$

The optimum vector *h* is defined from expression $h_{\text{opt}}$ as

$$h_{\text{opt}} = \left[ D_H - \int_{\xi_*} V\rho^2 \left( \begin{vmatrix} \xi^2 & \xi^3 \\ \xi^3 & \xi^4 \end{vmatrix} - \int_{\xi_*} \begin{vmatrix} \xi^2 & \xi^3 \\ \xi^3 & \xi^4 \end{vmatrix} \rho(\xi, t)\, d\xi \right) d\xi \right]^{-1} \times$$

$$\times \left( D_H h_0 - Z \int_{\xi_*} V\rho^2 \left[ \begin{vmatrix} \xi \\ \xi^2 \end{vmatrix} - \int_{\xi_*} \begin{vmatrix} \xi \\ \xi^2 \end{vmatrix} \rho(\xi, t)\, d\xi \right] d\xi \right).$$

Using the Fourier expansion up to the 3-rd order for the approximated representation of functions *V*, ρ

$$V(\xi,t)=\frac{1}{2}\alpha_0+\sum_{k=1}^{2}\alpha_{1k}\cos k\omega_0\xi+\alpha_{2k}\sin k\omega_0\xi\,,$$

$$\rho(\xi,t)=\sum_{k=1}^{2}\beta_{1k}\cos k\omega_0\xi+\beta_{2k}\sin k\omega_0\xi\,,$$

$$\omega_0=\frac{2\pi}{5},$$

(then for $v\rho^2$ the following representation holds true

$$v\rho^2(\xi,t)=\gamma_0+\sum_{k=1}^{6}\gamma_{1k}\cos k\omega_0\xi+\gamma_{2k}\sin k\omega_0\xi\,,$$

$\gamma_0,\gamma_{ik}=\gamma(\alpha,\beta)$ are functions linearly dependent on factors $\alpha_{ik}$ and quadratically - on $\beta_{ik}$) and introducing designations

$$\zeta_C(k,n)=\int_{\xi_*}\xi^n\cos k\omega_0\xi\,d\xi=2\sum_{i=1,3}^{n}i!C_i^n\frac{(2.5)^{n-i}}{(k\omega_0)^{i+1}}\sin\left(k\pi+\frac{i\pi}{2}\right),\quad n=2;4;$$

$$\zeta_S(k,m)=\int_{\xi_*}\xi^m\sin k\omega_0\xi\,d\xi=-2\sum_{i=0,2}^{m}i!C_i^m\frac{(2.5)^{m-i}}{(k\omega_0)^{i+1}}\cos\left(k\pi+\frac{i\pi}{2}\right),\quad m=1;3;$$

vector $h_{\text{opt}}$ of the factors of the observer we write down as follows:

$$h_{\text{opt}}=\left(D_H+\begin{vmatrix}\gamma_0\left(5\sum_{k=1}^{2}\beta_{1k}\zeta_C(k,2)-10,4\right)-\sum_{k=1}^{6}\gamma_{1k}\zeta_C(k,2) & \cdots\\ \vdots & \\ 5\gamma_0\sum_{k=1}^{2}\beta_{2k}\zeta_S(k,3)-\sum_{k=1}^{6}\gamma_{2k}\zeta_S(k,3) & \cdots\end{vmatrix}\right.$$

$$\left.\begin{vmatrix}\cdots & 5\gamma_0\sum_{k=1}^{2}\beta_{2k}\zeta_S(k,3)-\sum_{k=1}^{6}\gamma_{2k}\zeta_S(k,3)\\ & \vdots\\ \cdots & \gamma_0\left(5\sum_{k=1}^{2}\beta_{1k}\zeta_C(k,4)-39,1\right)-\sum_{k=1}^{6}\gamma_{1k}\zeta_C(k,4)\end{vmatrix}\right)^{-1}\times$$

$$\times\left(D_Hh_0-Z\begin{vmatrix}\sum_{k=1}^{6}\gamma_{2k}\zeta_S(k,1)-5\gamma_0\sum_{k=1}^{2}\beta_{2k}\zeta_S(k,1)\\ \vdots\\ \sum_{k=1}^{6}\gamma_{1k}\zeta_C(k,2)-5\gamma_0\sum_{k=1}^{2}\beta_{1k}\zeta_C(k,2)\end{vmatrix}\right)=h(\alpha,\beta).$$

Then the system of equations for the factors of expansion has the following form:

$$\dot{\beta}_{1(2)i} = \frac{1}{2}\sum_{K=1}^{2}\beta_{1(2)K}\left(3\left[\zeta_C(k-i,2)\pm\zeta_C(k+i,2)\right]\mp\right.$$

$$\left.\mp k\omega_0\left[\zeta_S(k+i,3)+\zeta_S(k-i,3)\right]\right)\mp\frac{(i\omega_0)^2}{2}\beta_{1(2)i}+$$

$$+Zh^T(\alpha,\beta)\begin{vmatrix}\sum_{K=1}^{2}\beta_{2(1)K}\frac{1}{2}\left[\zeta_S(k+i,1)+\zeta_S(k-i,1)\right]-\beta_{2K}\zeta_S(k,1)\beta_{1(2)i}\\ \vdots\\ \sum_{K=1}^{2}\beta_{1(2)K}\frac{1}{2}\left[\zeta_C(k-i,2)\pm\zeta_C(k+i,2)\right]-\beta_{1K}\zeta_C(k,2)\beta_{1(2)i}\end{vmatrix}-$$

$$-\frac{1}{2}h^T(\alpha,\beta)\begin{vmatrix}\sum_{K=1}^{2}\beta_{1(2)K}\frac{1}{2}\left[\zeta_C(k-i,2)\pm\zeta_C(k+i,2)\right]-\beta_{1K}\zeta_C(k,2)\beta_{1(2)i} & \cdots\\ \vdots & \\ \sum_{K=1}^{2}\beta_{2(1)K}\frac{1}{2}\left[\zeta_S(k+i,3)+\zeta_S(k-i,3)\right]-\beta_{2K}\zeta_S(k,3)\beta_{1(2)i} & \cdots\end{vmatrix}$$

$$\begin{vmatrix}\cdots & \sum_{K=1}^{2}\beta_{2(1)K}\frac{1}{2}\left[\zeta_S(k+i,3)+\zeta_S(k-i,3)\right]-\beta_{2K}\zeta_S(k,3)\beta_{1(2)i}\\ & \vdots\\ \cdots & \sum_{K=1}^{2}\beta_{1(2)K}\frac{1}{2}\left[\zeta_C(k-i,4)\pm\zeta_C(k+i,4)\right]-\beta_{1K}\zeta_C(k,4)\beta_{1(2)i}\end{vmatrix}h(\alpha,\beta),$$

$$\beta_{1i}(0)=10^{-2},\qquad \beta_{2i}(0)=0,\qquad i=1,2;$$

$$\dot{\alpha}_0=-2\left(3\sum_{K=1}^{2}\alpha_{1K}\zeta_C(k,2)+\alpha_{1K}\left[\chi_C(k,\beta)+\omega_C(k,\beta)\right]+\alpha_{2K}\left[\chi_S(k,\beta)+\omega_S(k,\beta)\right]\right)-$$

$$-2Zh^T(\alpha,\beta)\begin{vmatrix}\sum_{K=1}^{2}\alpha_{2K}\zeta_S(k,1)\\ \sum_{K=1}^{2}\alpha_{1K}\zeta_C(k,2)\end{vmatrix}-h^T(\alpha,\beta)\begin{vmatrix}\sum_{K=1}^{2}\alpha_{1K}\zeta_C(k,2) & \cdots & \sum_{K=1}^{2}\alpha_{2K}\zeta_S(k,3)\\ \vdots & & \vdots\\ \sum_{K=1}^{2}\alpha_{2K}\zeta_S(k,3) & \cdots & \sum_{K=1}^{2}\alpha_{1K}\zeta_C(k,4)\end{vmatrix}\times$$

$$\times h(\alpha,\beta)+\mu(\beta)-(h-h_0)^T\mu_1(\beta)(h-h_0),$$

$$\dot{\alpha}_{1(2)i} = -2\left\{\frac{3}{2}\sum_{K=1}^{2}\alpha_{1(2)K}\left[\zeta_C(k-i,2)\pm\zeta_C(k+i,2)\right]+\right.$$

$$\left.+\alpha_{1K}\left[\chi_{1(2)C}(k,i,\beta)+\omega_{1(2)C}(k,i,\beta)\right]+\alpha_{2K}\left[\chi_{1(2)S}(k,i,\beta)+\omega_{1(2)S}(k,i,\beta)\right]\right\}-$$

$$-2Zh^T(\alpha,\beta)\left(\begin{vmatrix}\sum_{K=1}^{2}\alpha_{2(1)K}\frac{1}{2}\left[\zeta_S(k+i,1)+\zeta_S(k-i,1)\right]\\ \sum_{K=1}^{2}\alpha_{1(2)K}\frac{1}{2}\left[\zeta_C(k-i,2)\pm\zeta_C(k+i,2)\right]\end{vmatrix}-\alpha_{1(2)i}\begin{vmatrix}\sum_{K=1}^{2}\beta_{2K}\zeta_S(k,1)\\ \sum_{K=1}^{2}\beta_{1K}\zeta_C(k,2)\end{vmatrix}\right)-$$

$$-h^T(\alpha,\beta)\left(\begin{vmatrix}\sum_{K=1}^{2}\alpha_{1(2)K}\frac{1}{2}\left[\zeta_C(k-i,2)\pm\zeta_C(k+i,2)\right] & \cdots\\ \vdots & \\ \sum_{K=1}^{2}\alpha_{2(1)K}\frac{1}{2}\left[\zeta_S(k+i,3)+\zeta_S(k-i,3)\right] & \cdots\end{vmatrix}\right.$$

$$\left.\begin{matrix}\cdots & \sum_{K=1}^{2}\alpha_{2(1)K}\frac{1}{2}\left[\zeta_S(k+i,3)+\zeta_S(k-i,3)\right]\\ & \vdots\\ \cdots & \sum_{K=1}^{2}\alpha_{1(2)K}\frac{1}{2}\left[\zeta_C(k-i,4)\pm\zeta_C(k+i,4)\right]\end{matrix}\right|-$$

$$-\alpha_{1(2)i}\left|\begin{matrix}\sum_{K=1}^{2}\beta_{1K}\zeta_C(k,2) & \cdots & \sum_{K=1}^{2}\beta_{2K}\zeta_S(k,3)\\ \vdots & & \vdots\\ \sum_{K=1}^{2}\beta_{2K}\zeta_S(k,3) & \cdots & \sum_{K=1}^{2}\beta_{1K}\zeta_C(k,4)\end{matrix}\right|h(\alpha,\beta)+$$

$$+\mu_{C(S)}(i,\beta)-(h-h_0)^T\mu_{1C(S)}(i,\beta)(h-h_0),$$

$$\alpha(t_K)=0,$$

where the expressions of factors χ, ω, μ (determined by the numerical integration in the course of solving) aren't given as complicated. In the reduced form the system obtained can be given as

$$\dot{\beta}=\Phi_\beta\left[\beta,h(\alpha,\beta)\right],$$

$$\dot{\alpha} = G_1(\alpha, h) + G_2(\beta),$$

$$G_2(\beta) = \left|\mu(\beta) : \mu_C(1,\beta) : \mu_C(2,\beta) : \mu_S(1,\beta) : \mu_S(2,\beta)\right|^T.$$

The approximated solving of the given boundary-value problem by the method of the invariant imbedding results in the required system of the equations allowing to carry out simultaneously the definition of vector $h_{opt}$ and formation of vector β in the real time:

$$\left|\begin{matrix}\dot{\nu}_0 \\ \dot{\beta}_0\end{matrix}\right| = \left|\begin{matrix}h_0 \\ \Phi_\beta(\beta_0, h_0)\end{matrix}\right| + DG_2(\beta_0),$$

$$\dot{D} = 2\frac{\partial \Phi_\beta}{\partial \beta}(\beta_0, h_0)D - \frac{\partial \Phi_\beta}{\partial \alpha}(\beta_0, \alpha)\Big|_{\alpha=0} + 2D\frac{\partial G_2}{\partial \beta}(\beta_0)D - D\frac{\partial G_1}{\partial \alpha}(\alpha, \beta_0, h_0)\Big|_{\alpha=0}.$$

The integration of the given system was made by the Runge-Kutta method on interval [0; 600] s. with the step equal to 0,05 s.

For the comparison of efficiency of the approach suggested with that of the existing ones the formation of the optimum- by-the- criterion-of-the-MAP estimation $\hat{\xi}$ by two ways was carried out: on the basis of the MAP - filter with the linear observer [4] and by defining the maximum of the function of the APD, approximated by series $\phi^T \beta_0$ (where $\beta_0$ is the solution of the last system of the estimation equations ), by means of the method of the random draft. The search of the maximum of the APD was carried out on the simulation interval [500; 600] s. for the estimations of vector $\beta_0$, taken with interval 1 s. The generated test sample of dimension 100 was the normalized Gaussian sequence.

The calculation of the estimation errors was made by comparing the current values of estimations with the target coordinate and subsequent defining of the average values of the errors on interval [500; 600] s. Upon terminating the simulation interval the value of the average error obtained in this way for the estimation equations [4], using the linear observer, has exceeded the average estimation error carried out by the technique suggested, using the information of the optimum observer, by the factor of ~ 1,52.

## Author details

Sergey V. Sokolov
*Rostov State University of Means of Communication, Russia*

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