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Analysis of Fractal Dimension of the Wind Speed and Its Relationships with Turbulent and Stability Parameters

Manuel Tijera, Gregorio Maqueda, Carlos Yagüe and José L. Cano

1. Introduction

The atmospheric fluxes in the boundary layer at large Reynolds numbers are assumed to be a superposition of periodic perturbations and non-periodic behavior that can obey an irregular state and variable motion that is referred as turbulence. Turbulence can be observed in time series of meteorological variables (wind velocity, for example). The analysis of these series presents a self-similarity structure, [1]. So, the wind velocity can be seen as a fractal magnitude. Fractal Dimension (FD) is an artifice that shows in some way the complexity of the time series and the variability degree of the physical magnitude including. Fractal dimensions will is correlated with the characteristic parameters of the turbulence. The non-integer values of the FD are assigned to time series which exhibit a self–similarity geometry and which show that structure on all length scales. In general, turbulent flows allowed us to recognize the coexistence of structure and randomness, they are a set of solution that are not unique or depend sensitively on initial conditions [2]. The structures of these flows are related with fractal geometry.

A structure or time series is said to be self–similar if it can be broken down into arbitrarily small pieces, each of which is a small replica of the entire structure. There is a way measure this degree of complexity by means of FD. The concept of dimension is no easy to understand probably to determine what dimension means and which properties have been one of the big challenges in Mathematics. In addition, mathematicians have come up with some tens of different notions of dimension: topological dimension, Hausdorf dimension, fractal dimension, box-counting dimension, capacity dimension, information dimension, Euclidean dimension, and more [3, 4]. They are all related. We focus in the fractal dimension of these series by means of dividing its structure onto a grid with size L, and counting the number of grid boxes which contain some of the structure of series, N. This number will be depending on the size L. To

obtain the FD it is needed to represent in a diagram the logarithm of N against the logarithm of the reciprocal of L (log (N(L)) versus log (1/L)). Then, the slope of the better linear fit or the linear regression between them corresponds to the searched fractal dimension or box-counting dimension. This dimension proposes a systematic measurement of degree of similarity or complexity of wind series. Various works have approached the problem of calculating the dimension associated with time series [5, 6].

In this Chapter it is going to study the Fractal Dimension (FD) of u and w fluctuations of time series of velocity recorded close to the ground, it is to say, into the low Planetary Boundary Layer, more specifically into the Surface Layer, closer to the ground and where the dynamic influence is predominant. The main aim is to seek physical quantities included in turbulent flows that correlate better with FD. Nevertheless, thermal, dynamical and combined parameters will be explored to complete the Chapter. The central idea is that the kind of stability controls the turbulent fluxes and it is showing in the values of the FD also. Differences of potential temperatures between two levels make the Surface Layer or the Low PBL unstable or stable depending on its sign. Shears in the wind produce dynamical instability making it easier the mix of properties, physical variables and mass. In neutral conditions mixture is completed and turbulence is well developed. The values of Fractal Dimension must be in accordance with the turbulent level.

It has been calculated the fractal dimension, d (Kolmogorov capacity or box-counting dimension) of the time series of the velocity component fluctuations u' and w' ($u' = u - U$, $w' = w - W$) with U the horizontal mean velocity, both in the physical space (velocity-time) [2]. It has been studied the time evolution of the fractal dimension of the u' and w' components (horizontal and vertical) of wind velocity series during several days and three levels above the ground (5.8 m, 13.5 m, 32 m).

This study is focused on the simplest boundary layer kind, over a flat surface. So, we could assume that the flow to be horizontally homogeneous. Its statistical properties are independent of horizontal position; they vary only with height and time [1, 7, 8]. The experimental data have been taken in a flat terrain with short uniform vegetation. It allows us to take on this approximation of horizontal homogeneity and on this context we focus this study on variation of fractal dimension of the horizontal and vertical components of the velocity of flow turbulent in the diurnal cycle versus to a variety of turbulent parameters: difference of potential temperatures in the layers 50-0.22 m and 32 – 5.8 m, Turbulent Kinetic Energy, friction velocity and Bulk Richardson number.

It has been observed that there is a possible correlation between the fractal dimension and different turbulent parameters, both from dynamical and thermal origin: turbulent kinetic energy, friction velocity, difference of temperature between the extreme of the layer studied close to the surface ($\Delta T_{50-0.22m}$). Finally, it has been analysed the behaviour of fractal dimension versus stability evaluated from the Richardson number.

The knowledge of turbulence and its relationships with fractal dimension and some turbulent parameters within the Planetary Boundary Layer (PBL) can help us understand how the atmosphere works.

2. Data

The data analysed was recorded in the experimental campaign SABLES-98 at the Research Centre for the Lower Atmosphere (CIBA in the Spanish acronyms), located in Valladolid province (Spain). SABLES-98 was an extensive campaign of measures in the PBL with a large number of participant teams and took place from 10th to 27th September 1998 [9]. The experimental site is located around 30km NW from Valladolid city, in the northern Iberian Plateau, on a region known as Montes Torozos, which forms a high plain of nearly 200 km^2 elevated above the plateau. The surrounding terrain is quite flat and homogeneous, consisting mainly on different crop fields and some scattered bushes. The Duero river flows along the SE border of the high plain, and two small river valleys, which may act as drainage channels in stable conditions, extends from the lower SW region of the plateau.

The main instrumentation available at CIBA is installed on the 100m meteorological tower, and includes sonic and cup anemometers, platinum resistance thermometers, wind vanes, humidity sensors, etc. Other instruments are spread in minor masts and ground based in order observes the main meteorological variables of the PBL. In this work we study five minute series from the sonic anemometers (20 Hz) installed at 5.8 (~ 6), 13.5 (~13) and 32m were used to evaluate the fractal dimension and turbulent parameters. These series have been obtained once we have carried out the necessary transformation to get the mean wind series in short intervals, namely 5 minutes, to ensure the consistent properties of turbulence [10].

We focus this study in a period of eight consecutive days (from 14 to 21 September 1998) in which have been analyzed every records of the velocity fluctuations. The synoptic conditions were controlled by a high pressure system which produces thermal convection during the daily hours and from moderate to strong stable stratification during the nights. The evolution of wind speed, Bulk Richardson number (Rib), Turbulent Kinetic Energy (TKE), friction velocity (u_*), potential temperature difference between 32m and 5.8m, and the temperature difference near surface but in an deeper layer (50-0.22 m, named $invT_{50-0.22}$, in reference to thermal inversion), with fractal dimension of velocity u and w component fluctuations at three levels above the ground (5.8 m, 13.5 m, 32 m) are analysed.

3. Methodology

The turbulent flows show very high irregularity for wind velocity time series. They present a self–similarity structure, it is to say, that for different scales the structure of the variables remains similar, as it is shown in Fig. 1. This property of the turbulent flows is related with the Fractal Dimension, since the irregularity is a common characteristic. Their non-integer values can help us analyze how the irregularity of the sign is, as well as of its geometry. As the bigger the values of FD, the more irregularity and random is the flow.
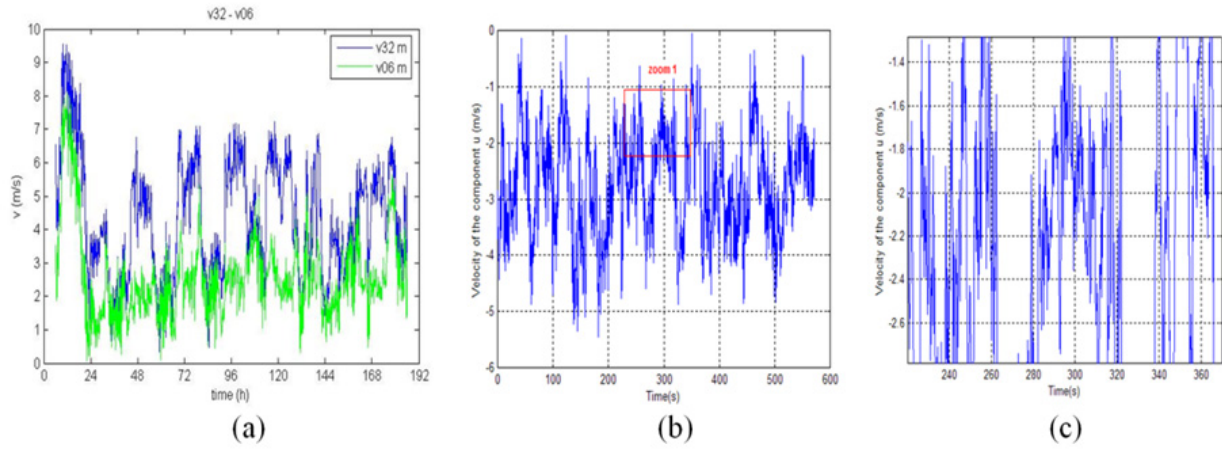
Figure 1. a) Variation of wind speed versus time from 14 to 21 September 1998, at 5.8 m and 32 m. b) If we zoom red window, in figure c), it is observed that the structure of this flow is similar over all scales of magnification.

In this section we describe the methodology applied to calculate the Fractal Dimension d (Kolmogorov capacity or box-counting dimension) [11].The more precise definition of the fractal dimension are in Hausdorff's work, become later known as Hausdorff dimension [12]. This dimension is not practical in the sense that it is very difficult to compute even in elementary examples and nearly impossible to estimate in practical applications. The box-counting dimension simplifies this problem, being an approximation of the Hausdorff dimension and is calculated approximately by

$$d = \lim_{L \to 0} \frac{\log N(L)}{\log \left(\frac{1}{L}\right)} \tag{1}$$

$N(L)$ is the number of the boxes of side L necessary to cover the different points that have been registered in the physical space (velocity-time) [13]. As $L \to 0$ then $N(L)$ increases, N meets the following relation:

$$N(L) \cong kL^{-d} \tag{2}$$

$$\log N(L) = \log k - d \log L \tag{3}$$

By means of least – square fitting of representation of log $N(L)$ versus log L, it has been obtained of the straight line regression given by equation (3), as is shown in Fig 2. The fractal dimension d will come given by the slope of this equation.

4. Stability of stratification and turbulence

The origin of turbulence cannot be easily determined, but it is know that both the dynamic and the thermal effects contribute strongly to turbulence by producing a breakdown of streamlined flow in a previously nonturbulent movement. The dynamical effects are represented by wind shear production and the thermal ones make differences of density in the fluid giving rise to hydrostatic phenomena and buoyance.
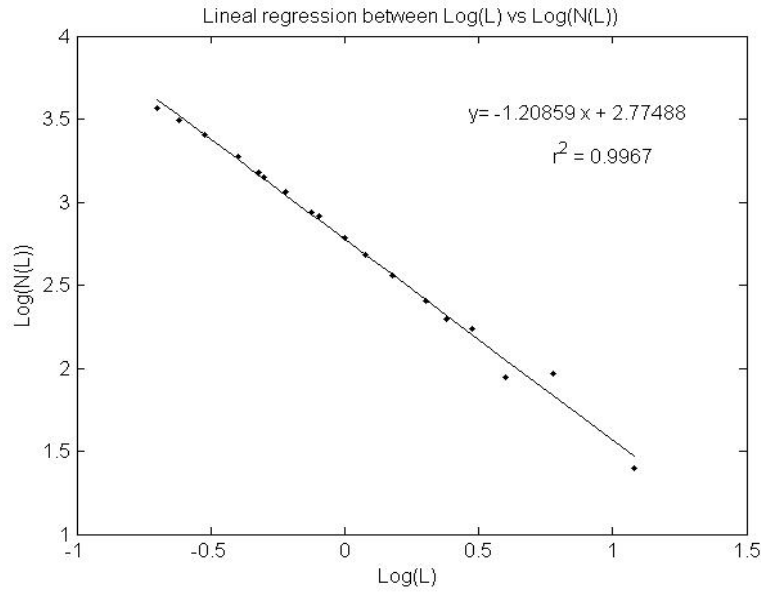
Figure 2. Example of linear regression between number of boxes not empty and length side of the box. The slope (*d*) is the fractal dimension, *d* =1.21 ± 0.02 for the example case.

The Richardson number is a parameter that includes both dynamic and thermal effects to measure the degree of stratification stability in the low Atmosphere. The static stability parameter, $s = (g / \theta)(\partial\theta / (\partial z))$, only takes in mind the buoyancy, i.e., the thermal effects. Nevertheless, a ratio between *s* and squared wind shear $(\partial V /\partial z)^{-2}$, gives a nondimensional product more appropriate for to calculate the stability. This ratio is known as gradient Richardson number. For the most practical cases a needed numerical approximation will be introduced below.

In the low Planetary Boundary Layer the atmosphere responds to changes in stratification stability brought about by the heating and cooling of the ground. We search the behaviour or relation between the Fractal Dimension and a parameter to establish a proper measure of stability in the surface layer. One of the most widely used indicator of stability close to the ground in atmospheric studies is the Bulk Richardson number Ri_B, a nondimensional parameter representing the ration of the rate of production or destruction of turbulence by buoyancy to that by wind shear strain is caused by mechanical forces in the atmosphere:

$$Ri_B = \frac{g}{\bar{\theta}}\frac{\Delta\bar{\theta}\Delta z}{(\Delta\bar{u})^2} \tag{4}$$

where *g* is the gravity acceleration and $\bar{\theta}$ the average potential temperature a the reference level, the term $\frac{g}{\theta}$ is referred to as the buoyancy parameter. Ri_B is positive for stable stratification, negative for unstable stratification and approximate to zero for neutral stratification, [10, 14, 15]. The way to calculate this number is following:

1. Calculate the mean potential temperatures at height $z = 32$ m, and the close surface $z = 5.8$ m, namely $\overline{\theta_{32}}$ and $\overline{\theta_{5.8}}$ respectively. Being $\Delta\overline{\theta} = \overline{\theta_{32}} - \overline{\theta_{5.8}}$.

2. Obtain $\overline{u_z}$ the module mean wind velocity at height $z = 32$ m and $z = 5.8$ m, denoted $\overline{u_{32}}$ and $\overline{u_{5.8}}$ respectively, where $\Delta\overline{u} = \overline{u_{32}} - \overline{u_{5.8}}$.

Once obtained the values of $\Delta\overline{\theta}$, $\Delta\overline{u}$ and Δz by means of the Eq. 4 calculate the Bulk Richardson number in a layer 32m to 5.8m. The mean properties of the flow in this layer as the wind speed and temperature experience their sharpest gradients. In order to know the influence of the kind of stratification over de fractal structure of the flow in the PBL, we going to analyse the behaviour of FD and its possible changes versus the parameter RiB, as the better parameter of stability obtained from the available data.

5. Results

In this section we present the variation of the Fractal Dimension of the *u'*, horizontal, and *w'*, vertical, components of the velocity fluctuations along the time at the three heights of the study: 5.8 m, 13.5 m and 32 m. We observe that these variations in the three heights are similar. The daily cycle during the period of study is clearly shown in Fig. 3. No significant different values are observed in levels, but a light increasing seems outstanding in diurnal time at the lower level (red line corresponds to 5.8m above the ground). Two components, *u'* and *w'*, present no differences in the time evolution.

As is shown in the Fig. 3, it is observed that the variation interval values of the fractal dimension range between 1.30 and nearly to 1.00. During the diurnal hours the fractal dimension is bigger than at night. A subtle question that concerns us is to what owes this. A possible explanation is that fractal dimension is related with atmospheric stability and, by the same reason, with the turbulence. It is well known that intensity of the turbulence grows as solar radiation increase, producing instability close the ground. In other hand, it is observed that Fractal Dimension is lightly inferior for stable stratification. We shall come back to this matter forward. In the nights a strong stability atmospheric usually exists, so the fractal dimension is usually smaller than during the diurnal hours.

5.1. Potential temperature and fractal dimension

This section concerns with the exam of the relation between the potential temperature differences and the Fractal Dimension. Potential temperature is a very useful variable in the Planetary Boundary Layer that can be replace the observed temperature in the vertical thermal structure, since an air parcel rises or goes down adiabatically at potential temperature constant. A vertical profile of potential temperature uniform represents a neutral stratification or it is called as adiabatic atmosphere.

Next, we show the variation of potential temperature at heights $z = 32$ m and $z = 5.8$ m along the time in Fig. 4. The features shown in this figure need some comment. The potential temperature at height $z = 5.8$ m is bigger than to 32 m in the nights. It is a characteristic of

the nocturnal cooling that produces inversion. During the noon the potential temperatures decrease slightly with height. It is corresponding to instable conditions in the surface layer and we have a possible mixture of both mechanical and convective turbulence [16]. However, the evolution of potential temperature from minimum value until maximum is identical for the two levels in every day studied. It is the consequence of neutral situation with efficient mix during the morning.
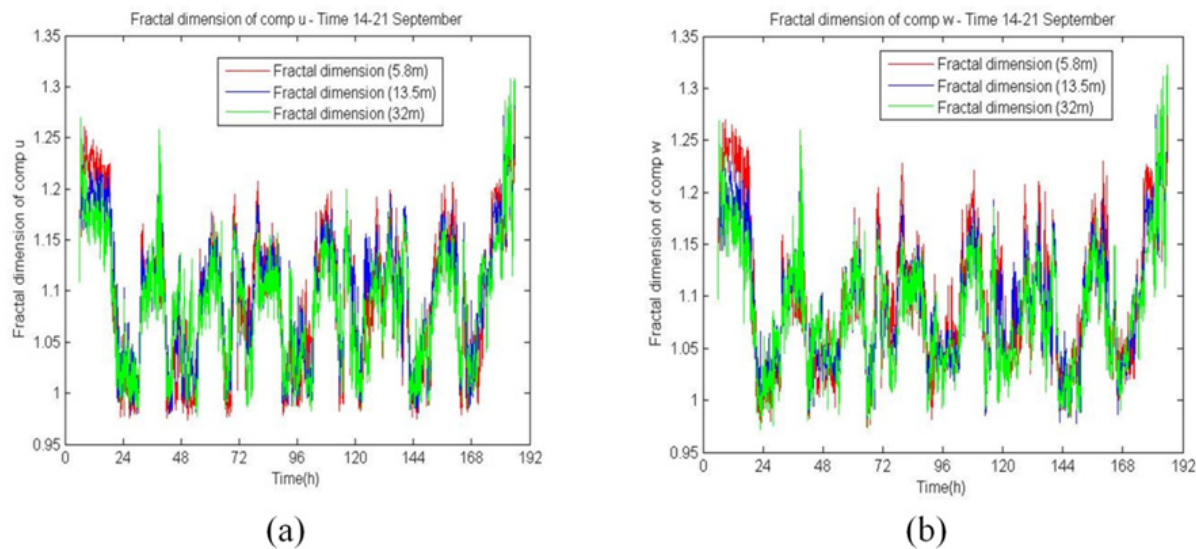

Figure 3. The variation of the Fractal Dimension (FD) *versus* time present an analogous behavior for the *u'* and *w'* component fluctuations ((a) and (b), respectively) at the 3 levels, showing the influence of the diurnal cycle.

In the Figure 4, it is clear to observe the diurnal cycle, observing a strong thermal inversion during the nights that is starting after noon and increasing uniformly to reach the maximum value. The main differences between potential temperatures are observed under inversion condition as a result of the separation among layers because the strong stratification.

In the Fig. 5, it is shown the fractal dimension at three heights and the difference of potential temperature between 50 and 0.22 m. In this study we will name it as thermal inversion because positive values correspond to actually inversion, in terms of potential temperature (invT$_{50-0.22}$). The layer 0.22 to 50 m is more extensive than the stratum defined by the levels of the sonic anemometers covering a deep part of the PBL and likely the whole Surface Layer. Besides the instruments to measure the invT are others different of the sonic anemometers which can to give a temperature by the speed of the sound measured also in they. This has an evident benefit in the results.

It is observed that fractal dimension correlates in opposed way with the difference of potential temperatures in the layer between 50 and 0.22 m. The more stable conditions are coincident with the bigger values of fractal dimension. In strong thermal inversions the fractal dimensions are lower. Although this good correlation is observed in the figure, we can justify it by mean of a least–squares fitting between the data in the different temporal intervals corresponding to the variables analyzed.
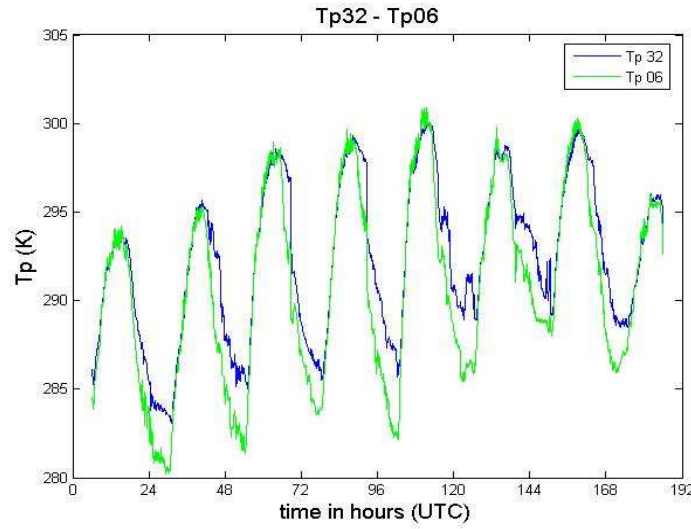
Figure 4. Time series of potential temperature at 32 and 5.8 m along the complete period of study.
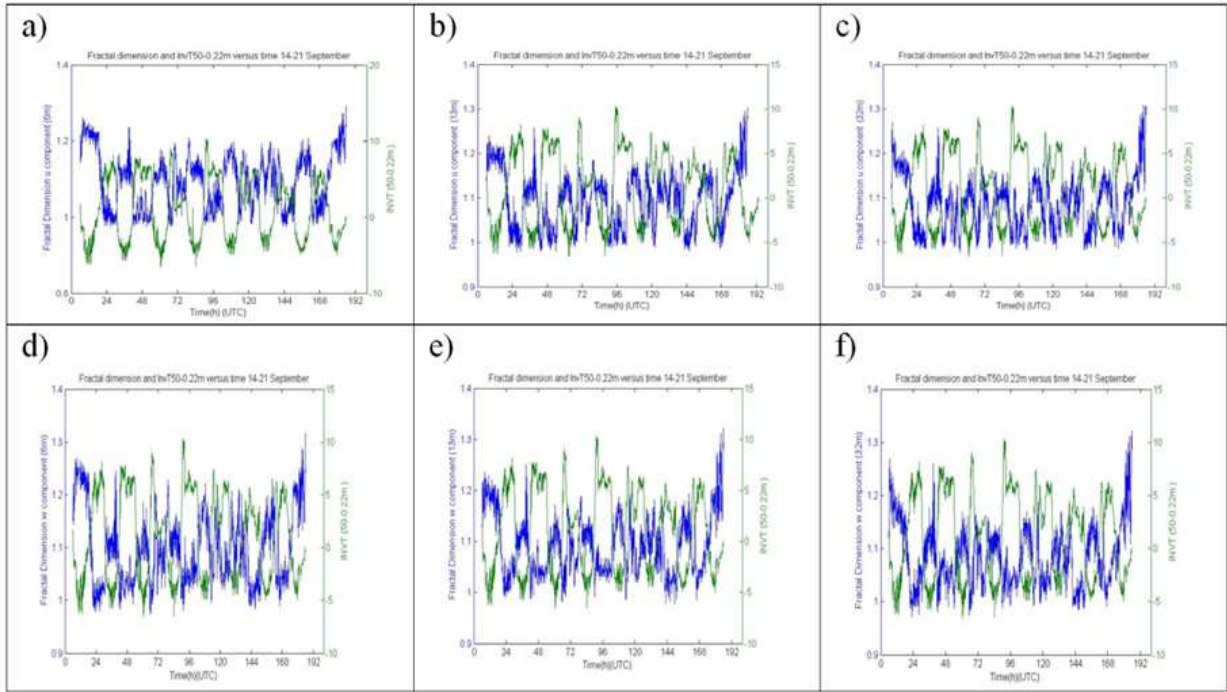

Figure 5. Fractal dimension of component fluctuations u' and w' and the thermal inversion in layer between 0.22 and 50 m versus time at three heights: z=5.8 m, z=13 m and z=32 m

As it is observed in the Fig. 6, we show the linear regression in two temporal intervals corresponding to the first 24 hours -from 06 UTC of first day to 06 UTC of the second day- and at night interval from 18 UTC to 06 UTC. The values obtained of the correlation coefficient are very good, in the first case R = -0.925, in the second one R = -0.707. It is observed a better correlation during 24 hours than the night hours because the variation of the invT and FD are wider along all day. The fractal dimension is bigger during the day time than during the night, outstanding a strong inversion at night hours. We could achieve similar results in others temporal intervals of the two variables analysed in the

period study. These results are obtained for fluctuations of *w'* component of velocity at height *z* = 5.8 m. Similar results are obtained for other two heights *z* =13 m and *z* = 32 m, also for fluctuations of *u'* component, along of the mean wind direction at three heights studied.

In order to extent the study to all days of the experimental campaign it has been analysed the relation between the Fractal Dimension and the difference of potential temperatures using averages of FD in defined intervals of invT. Figure 7 presents the cloud of the points for all values measured in 5 minutes interval for FD and InvT with the average and the error bars based on standard deviation. Both *u'* as *w'* have similar behaviour respect the variation of invT. In these cases FD is calculate for the lowest level, i. e. 5.8 m (6m labelled in the figure). An analogous shape of the cloud of points and identical results can be obtained if the potential temperature differences when the stratum 32-5.8m has been used. It is not shown in this Chapter because repetitive.
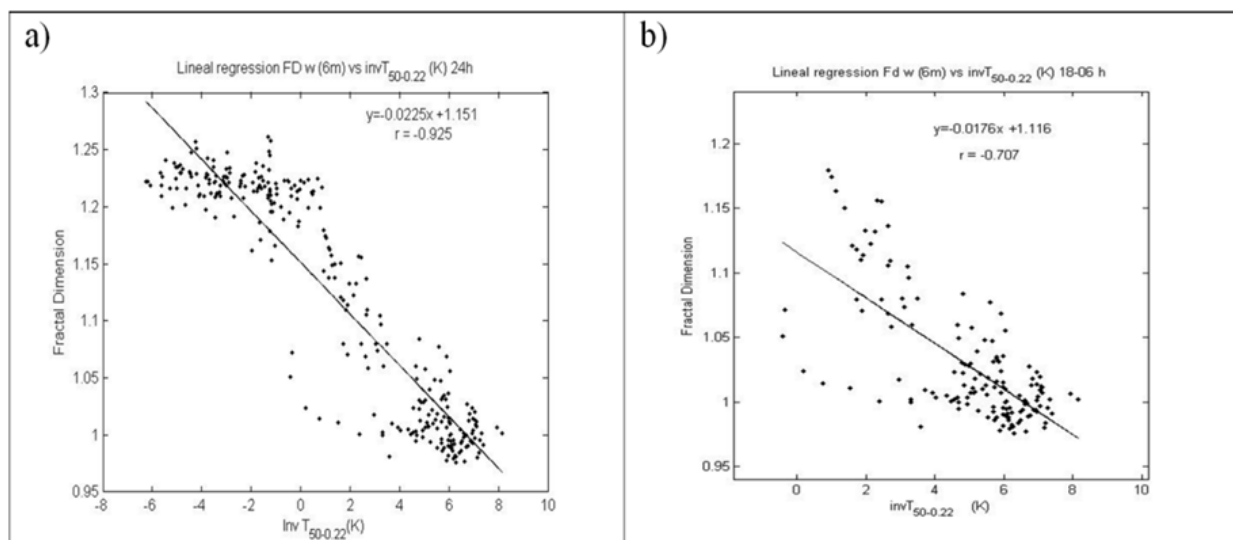

Figure 6. As it is observed the fit is very good in the temporal intervals with r satisfactory, it is showed the linear regression between the fractal dimension and la difference of the temperature between two levels in the periods of 24 h and 18-06h ((a) and (b) respectively).

It is observed that in two components a similar behaviour. Fractal Dimension have bigger values in unstable and near neutral conditions (negative and close to zero invT), reaching fractal dimension average near 1.15 value for the interval from -1 to 1 K of temperature differences. For strong inversion (> 1 K) the fractal dimension becomes smaller, around 1.05. The presented figures and values can be extent to FD of *u'* and *w'* fluctuations at others two heights *z* = 13 m and *z* = 32 m with similar results.

Finally, the good relation between FD and invT has been valued by mean a linear regression shown in Figure 8, where it has been used the average value of the scatter plot in Figure 7. It is notable the negative slope (- 0.029 –in the corresponding units -) of the regression and the high correlation coefficient, R = -0.93.
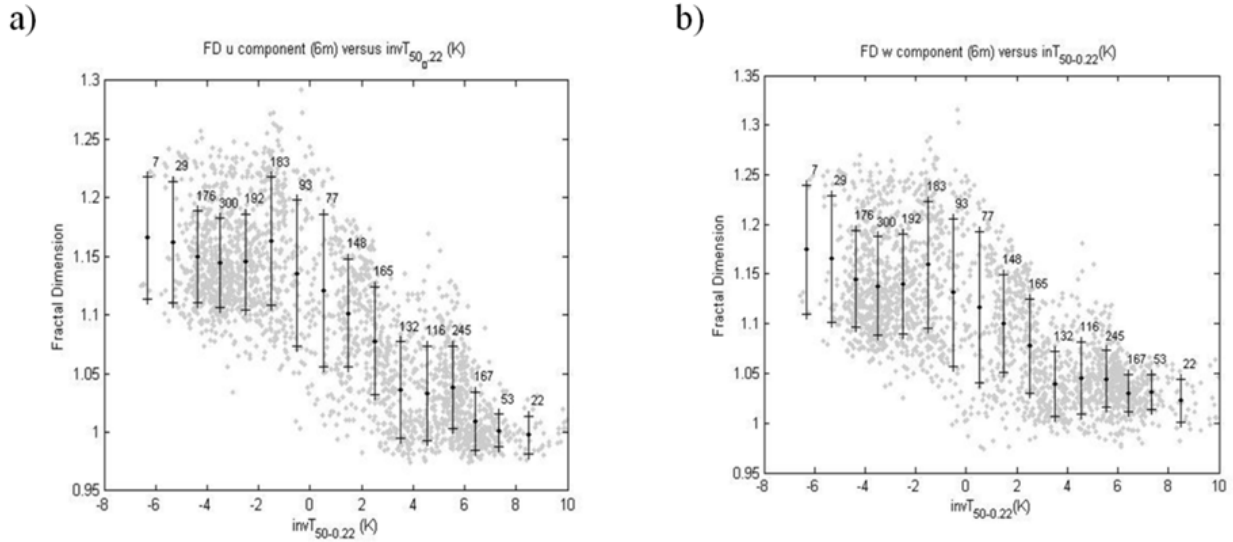
Figure 7. Scatter diagram of FD versus InvT$_{50-0.22}$. It is observed a similar behavior of the fractal dimension with the inversion of temperature, which is evaluated by difference of potential temperature at 50 – 0.22m layer. Average points and error bars are shown for a better understanding.
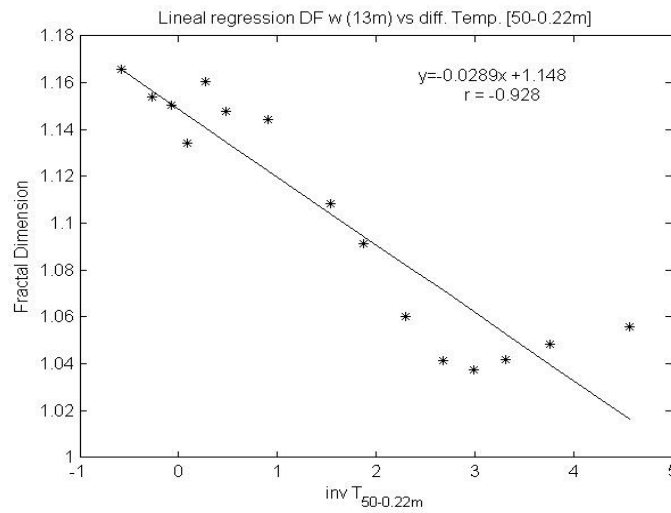

Figure 8. Linear regression of the FD against invT, using the average values from the cloud of points in Fig 7. Negative slope and the high correlation coefficient show a good correlation.

5.2. Turbulent Kinetic Energy and fractal dimension

The Turbulent Kinetic Energy (TKE) correspond to the quantity of energy associated to the movement of the turbulent flow and it is evaluated from the variances of the components of velocity: $\overline{u'^2}$, $\overline{v'^2}$ and $\overline{w'^2}$. TKE is given by Eq. 5, in terms of energy per unit mass [15].

$$TKE = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right) \tag{5}$$

In this Section we will examine the relationship between TKE and Fractal Dimension of the fluctuations u' and w'. Before incoming to study with certain details of this relationship, it

would be interesting to relate the TKE to different kind of atmospheric stability. The budgets of TKE in the instable, stable and neutral conditions near the surface are next summarized.

In the stable case, the production of the TKE by shear is not sufficient to balance the dissipation energy at all levels and production by buoyancy is not happening. So, turbulence decay and TKE decrease along the time. In neutral conditions near the surface in agreement with the observations suggest that exists a near balance between shear production and viscous dissipation [14, 17]. In this case, a strong shear wind is normally present in the low atmosphere. In the unstable layer, convective case, vertical gradients of w are bigger than the vertical gradients of u and v, the TKE production is mainly due to buoyancy and transport from other levels. Turbulence is supported by thermal effects in the case of instability [17].
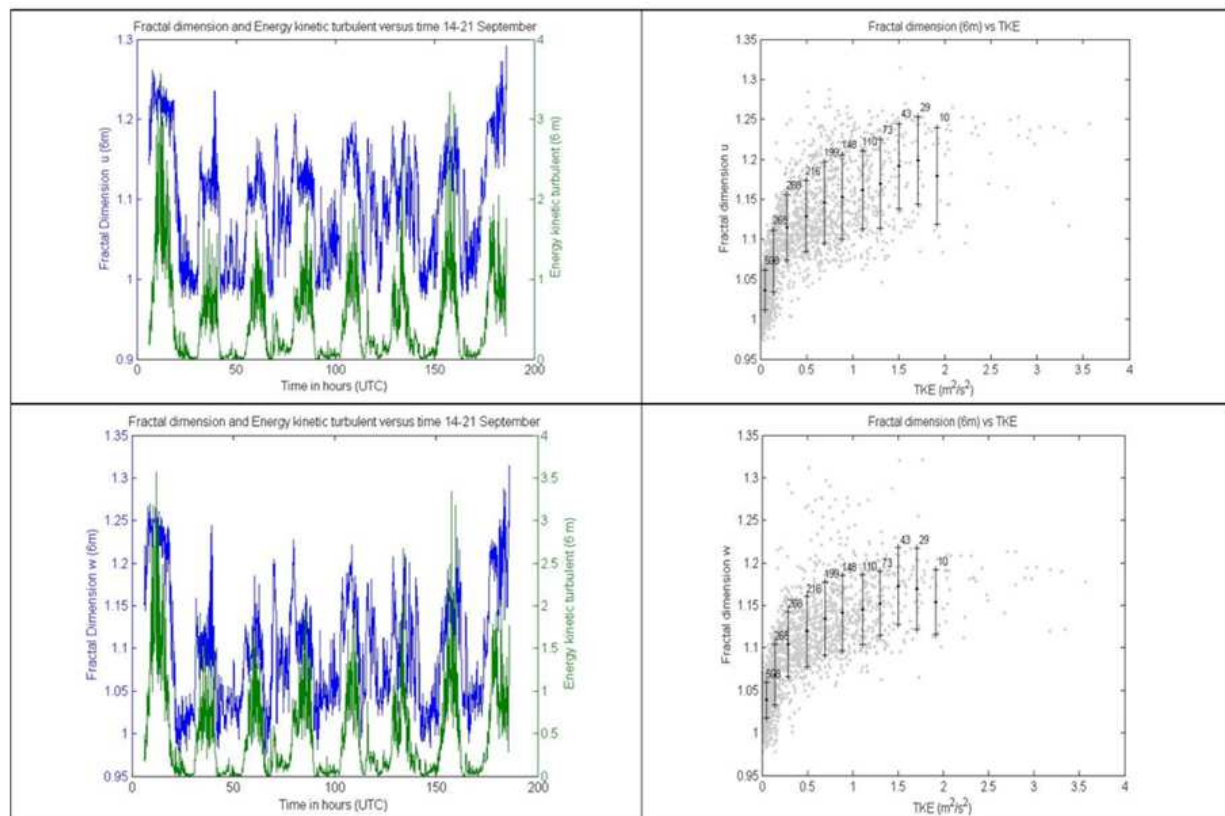

Figure 9. Variation of Fractal dimension and Energy kinetic turbulent versus time for u (upper-left panel) and w (lower-left) component of wind velocity at height z = 5.8 m. Scattered diagram of the fractal dimension and TKE with averages values and error bars (right panels).

As results it has found that an strong correlation between the dynamic magnitude, the Turbulent Kinetic Energy, and the Fractal Dimension exists. Figs. 9 and 10, in the left panels, show the evolution of the FD and TKE along the time for all days studied at two levels (5.8m Fig 9 and 13m in Fig 10). Diurnal cycle is also observed like in the study of invT in previous sections. This first result is in agreement with the normal variation of the turbulence between the day time and night time. Values of TKE are similar at two levels analyzed which may be interpreted as

turbulence is development into the stratum in a uniform way. Right panels, in the Figs. 9 and 10, is showing the scatter plot of FD and TKE values for all data used. Averaged of FD for arbitrary intervals of TKE together its error bars are also represented. It is observed a continuous increasing of the Fractal Dimension when the TKE grows, both for the u' and w' fluctuations. The maxima values of the kinetic energy are 2 m^{2}s^{-2}, in average, at heights z =5.8 m, z =13 m and z = 32m (do not shown) corresponds to maxima values of mean FD. Differences may be observed for FD among the figures. For u' components at 5.8 m present the highest values of FD (around 1.20 ± 0.05) while in the remainder cases w' in two levels and u' at 13m the maximum of FD are nearly to 1.15. The relation studied does not seem linear, but FD increases very fast for small values of TKE and it has a light increasing of FD beyond of 1 m^2/s^2.

In order to investigate the correlation between FD and TKE, we were tried a linear regression for two different time interval, at first, from 06 UTC to 24 UTC of the initial day, including the period of day time and early night. In this record the variation of FD is maximum. Figure 11a presents the scatter plot and the linear regression obtaining an R^2 coefficient of 0.698 that can be to consider satisfactory in spite of the shape of the cloud of points. The second period valued corresponds to the third day of the experiments from 48 UTC to 60 UTC, i.e., from midnight to noon. The scatter plot (Fig. 11b) presents similarity with the previous one, a cumulated group closes to the zero of TKE and there is spread points of higher values of FD for bigger TKE. The determination coefficient $R^2 = 0.689$ seems similar to the other period. Analogous results are obtained for u' component.
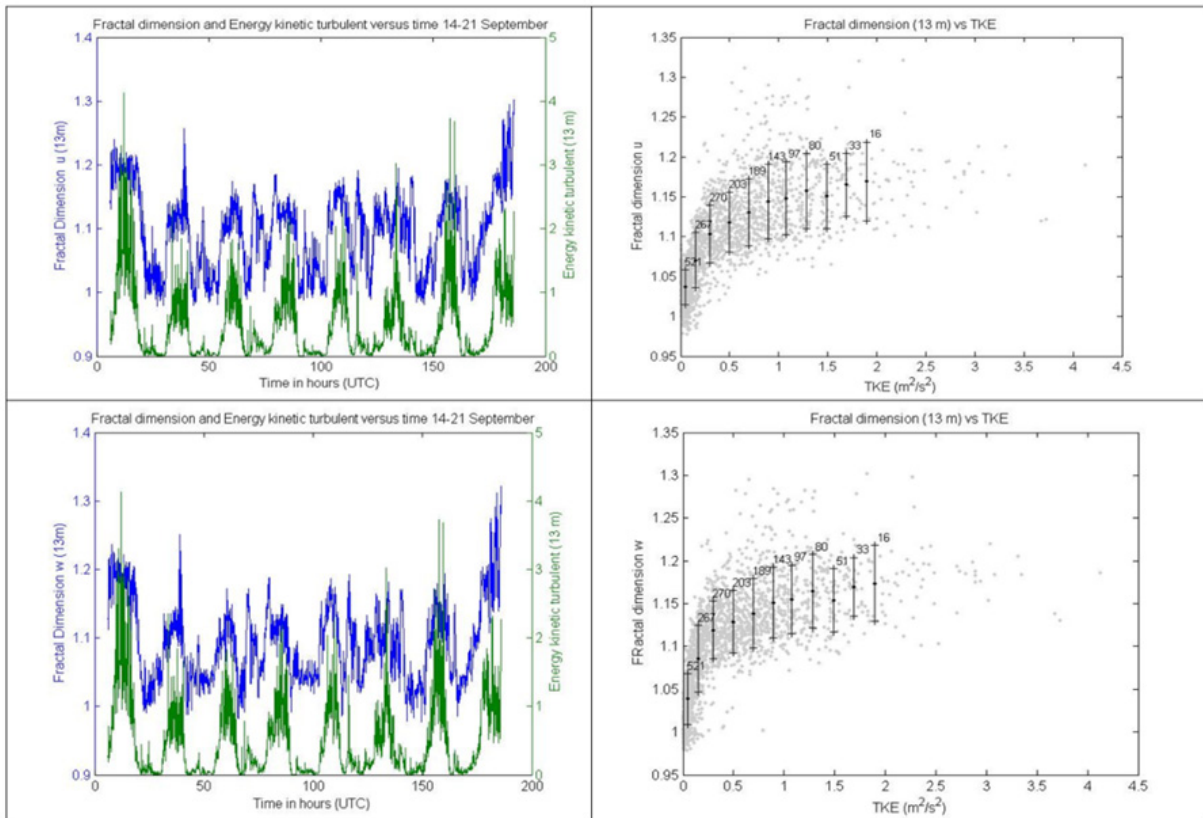

Figure 10. Variation of Fractal dimension and Energy kinetic turbulent versus time for u and w component of wind velocity at height z= 13 m. Diagram of fractal dimension and TKE with error bars

5.3. Friction velocity and fractal dimension

In the atmosphere the relevant turbulent velocity scale is the surface friction velocity u$_*$ which includes the vertical momentum flux related by Eq. 6.

$$u_* = \left[\left(\overline{-u'w'} \right)^2 + \left(\overline{-v'w'} \right)^2 \right]^{1/4}$$

(6)

Friction velocity is also a turbulent parameter that measures the shearing stress into the surface layer, $\tau = \tau_0 = \rho u_*^2$. It is considered constant in whole Surface Layer. So, it is an important dynamic property of the vertical structure of the lower atmosphere. Friction velocity can be obtained from vertical profile of average horizontal component of the wind but also by mean of Eq. 6, based in turbulent fluxes.
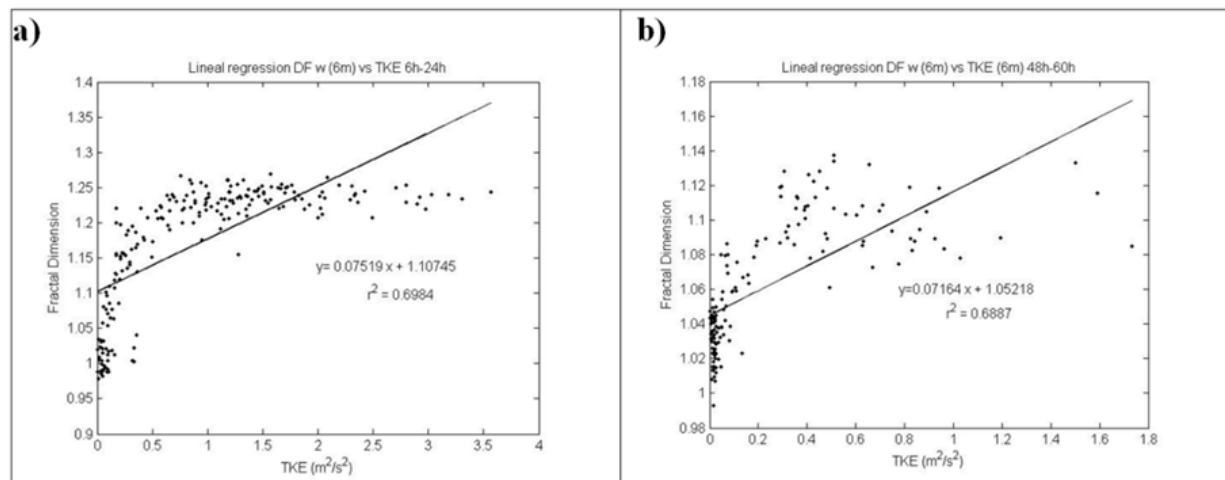

Figure 11. Linear regression between fractal dimension w component and TKE at height z=5.8 m. It is shown the correlation coefficient in the two temporal intervals analyzed.
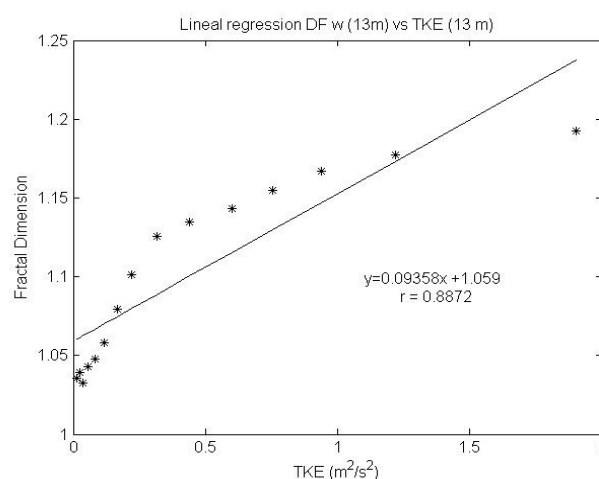

Figure 12. Average points according defined interval of TKE of Fractal Dimension of vertical component fluctuations at 13 m level. Coefficient of correlation, $R = 0.8872$, is shown together the right of regression.

Under neutral conditions an extended model of vertical profiles of wind is the known logarithm profile: $U = \frac{u_*}{k} \ln\left(\frac{z}{z_0}\right)$, where k is the Von Karman constant ($k \approx 0.4$) and z_0 is called the roughness length depending on the terrain. In the case of this study a very high rate of measurement of three components of velocity is available; therefore vertical flux of momentum is utilized here.

In this section we study as the Fractal Dimension behaves versus the friction velocity. The friction velocity u_* presents maxima at day time closed to noon in every levels studied (5.8 and 13m) and minima at night. This maxima and minima are in concordance with the variation of FD in an analogous way to TKE treated in the last section (see Fig 13 a, c). The scatter plot for FD for w' component at the same levels and u_* is showing in Fig 13b and d; it indicates an acceptable correlation between them. FD increase according to growing of u_*. Again, a linear regression has been tried in order to quantify this correlation. Points in Fig 14 correspond to average values of FD of vertical components fluctuations against friction velocity. The positive slope and a good coefficient of correlation ($R = 0.961$) are outstanding results since they improve the ones in the TKE. This trend is observed in the u' and w' components at three heights studied.
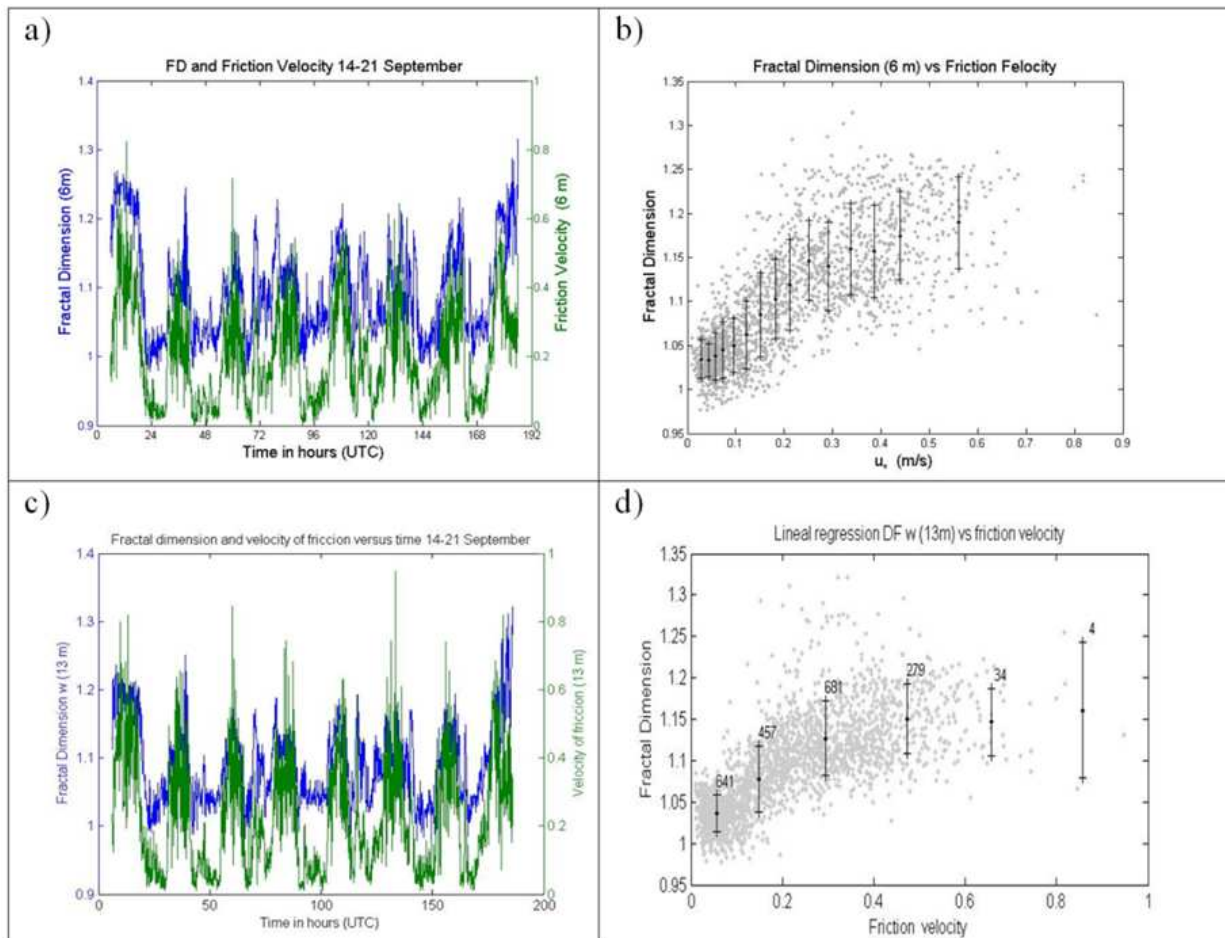

Figure 13. a) and c) Fractal dimension of fluctuation of w component at 5.8 m and 13 m (blue) and Friction Velocity at the same height (green). b and d) Fractal dimension versus Friction Velocity.
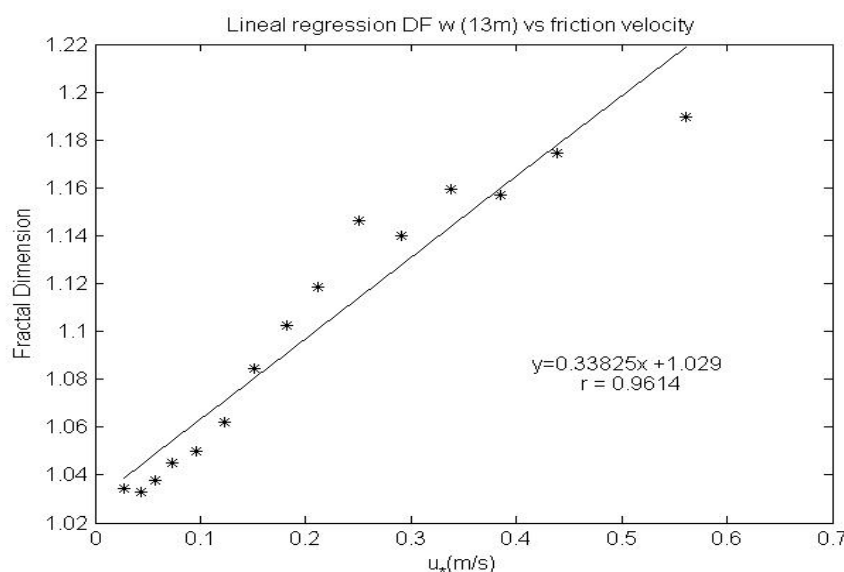
Figure 14. Linear regression of FD calculated for w' wind component versus friction velocity obtained from vertical fluxes of momentum (Eq. 6). The regression coefficients are presented ($R = 0.9614$).

5.4. Bulk Richardson number and fractal dimension

In this last Sub-section of results we try to investigate the relation between Fractal Dimension and the atmospheric stability in the Planetary Boundary Layer. We utilize the dynamic and thermal parameter denoted Bulk Richardson number, Ri_B, discussed in Section 4, and presented as a numerical approximation of the gradient Richardson number, Ri. The Ri_B value enables us to judge the atmospheric stability or, better named, the stability of stratification in the lower atmosphere. When Ri is small with any sign, the air flow is a pure shear flow driven by dynamic forces and we can say that layer is neutrally stratified. If Ri is near zero corresponding to neutrally stable boundary layer, one in which parcels displaced up and down adiabatically maintain exactly the same density as the surrounding air and thus experience no net buoyancy forces.

When Ri is large the air flow is driven by buoyancy. Positive values of Ri correspond with stable conditions and buoyancy forces keep out vertical displacement and mixing become less active. At the contrary, negatives and large Ri is in instability, where the thermal effects are doing air move vertically and fluctuations of wind components happen. Under instability turbulence increase and mixing of atmospheric properties: momentum, energy and mass (concentration of the components) are more efficient.

Thus, we go to exam whether the parameter Fractal Dimension can be an appropriated index to classify the atmospheric stratification. As it is already explained the numerical approximation used here is the Ri_B (Eq. 4). The main difficulty in the use of this parameter is the fact of which is not robust, since small shear in the wind produce values extremes of Ri_B, both positive and negative. In order to avoid such situations it has been remove of the study cases with $Ri_B < -5$ (convective) and $Ri_B > 1.5$ (strong stability, hard inversion). The time series of Ri_B along the complete period of study is drawn in Figure 15a. Can be see how negative values of Ri_B are in day time and positive or near-zero are far away of the central day.

The Fractal Dimension *versus* Bulk Richardson number shows a different behavior depending on the kind of stability as is shown in the Fig. 15b. That is, in strong instability it is observed mean values of the Fractal Dimension almost constant, around 1.15, but increasing when we are going to $Ri_B = 0$ (near neutral stratification). In neutral conditions the Fractal Dimension is maximum and decreases quickly for stable conditions. This result agrees with the relationships found for the potential temperature differences in the Fig. 7 and the temperature differences in the layer 50 – 0.22 m (Fig. 5). This behavior is similar at three heights studied.
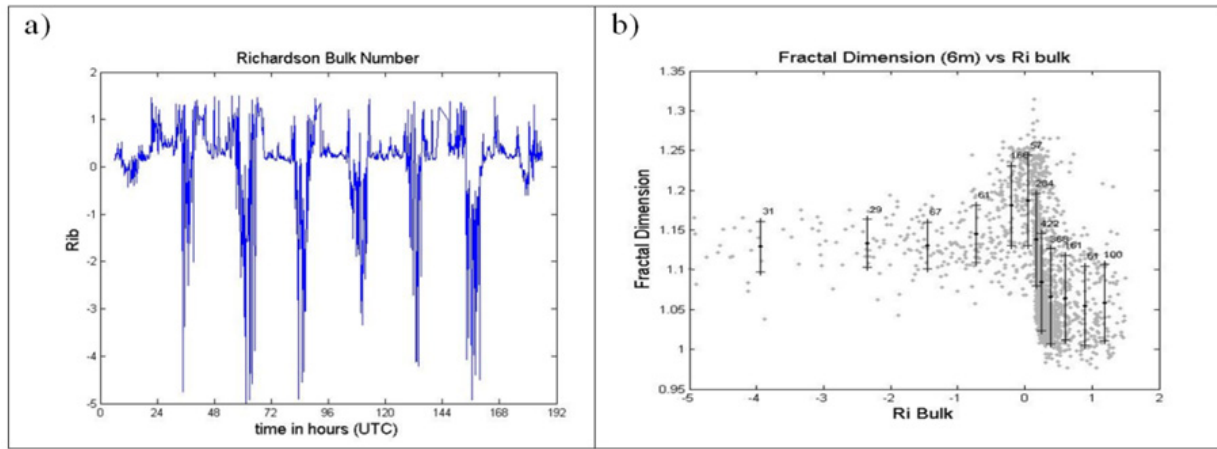

Figure 15. a) Bulk Richardson Number evaluated from the 32 - 5.8 m layer, along the whole period of study, where extreme values have been removed (-5< Ri_B< 1.5). b) Scatter plot of Fractal Dimension versus Bulk Richardson number; the maxima values of FD are in positive of Ri_B close to zero, and minima correspond to strong stability.
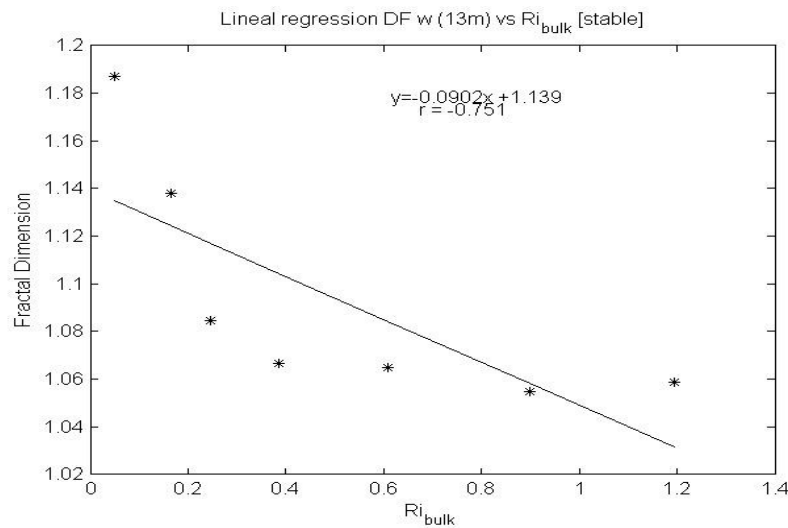

Figure 16. Linear regression of a set points, average FD in Ri_B intervals for the stable regime at height 13 m.

Behavior of FD according stability must be carry out separately in both kind of stability, as it was say before, instability give near constant values of Fractal Dimension except in quasi-neutrality, but in stability conditions, variation of FD it is clearly observed. In Fig. 16 it is shown the linear regression of mean values of the set points in stable stratification of the scatter plot of the Fig 13. As a result of that fit, can be conclude FD decrease as stability

increase, note the negative slope, but the coefficient of correlation is weaker than in dynamic analysis cases (r = 0.751).

Maxima values of FD in near-neutral stratification it is supported for the results in the study of TKE or friction velocity, where enhance of dynamical effects gave a rising of the Fractal Dimension.

6. Conclusions

This Chapter has treated about the influence of the atmospheric conditions, dynamics and thermal, over the fractal structure of the wind near the ground. The results that have been obtained in the work presented along the Chapter, lead to main following conclusions:

- An easy method for obtain the Fractal Dimension of velocity component fluctuation in the PBL has been carried out. It has been based in the Kolmogorov capacity or box-counting dimension concept.
- In the 5.8 - 32 m layer studied the Fractal Dimension is greater when the differences of potential temperature are negative (instable), reaching maxima values at differences near to zero (near-neutral stratification), and with positive values (stable stratum) the Fractal Dimension is lightly inferior. This result is according to behavior of the thermal inversion in the stratum 0.22 - 50 m.
- There exist an increasing of the Fractal Dimension there exists with the growing of the two dynamic magnitudes studied: Turbulent Kinetic Energy and friction velocity. The behavior of TKE and friction velocity are similar at three heights analyzed. The values of Fractal Dimension with these dynamics parameters are maxima at day time, close to noon, and minima at night according to the turbulence variation in the daily cycle.
- The Fractal Dimension is depending on kind of stratification. For negative values of Bulk Richardson number FD keeps approximately constant but in stability FD decrease quickly with Ri_B. An acceptable correlation between FD and Bulk Richardson Number has been observed for positives Ri_B. In the neutral conditions the Fractal Dimension reach its maximum.

Finally, it can conclude that dynamical origin of the turbulence has a more clear relation than the thermal origin with the fractal structure of the wind, but both are important.

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