Chapter 4

Boundary-Layer Flow in a Porous Medium of a Nanofluid Past a Vertical Cone

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1. Introduction

The natural convection flow over a surface embedded in saturated porous media is encountered in many engineering problems such as the design of pebble-bed nuclear reactors, ceramic processing, crude oil drilling, geothermal energy conversion, use of fibrous material in the thermal insulation of buildings, catalytic reactors and compact heat exchangers, heat transfer from storage of agricultural products which generate heat as a result of metabolism, petroleum reservoirs, storage of nuclear wastes, etc.

The derivation of the empirical equations which govern the flow and heat transfer in a porous medium has been discussed in [1-5]. The natural convection on vertical surfaces in porous media has been studied used Darcy's law by a number of authors [6–20]. Boundary layer analysis of natural convection over a cone has been investigated by Yih [21-24]. Murthy and Singh [25] obtained the similarity solution for non-Darcy mixed convection about an isothermal vertical cone with fixed apex half angle, pointing downwards in a fluid saturated porous medium with uniform free stream velocity, but a semi-similar solution of an unsteady mixed convection flow over a rotating cone in a rotating viscous fluid has been obtained Roy and Anilkumar [26]. The laminar steady nonsimilar natural convection flow of gases over an isothermal vertical cone has been investigated by Takhar et al. [27]. The development of unsteady mixed convection flow of an incompressible laminar viscous fluid over a vertical cone has been investigated by Singh and Roy [28] when the fluid in the external stream is set into motion impulsively, and at the same time the surface temperature is suddenly changed from its ambient temperature. An analysis has been carried out by Kumari and Nath [29] to study the non-Darcy natural convention flow of Newtonian fluids on a vertical cone embedded in a saturated porous medium with power-law variation of the wall temperature/concentration or heat/mass flux and suction/injection. Cheng [30-34] focused on the problem of natural convection from a vertical cone in a porous medium with mixed thermal boundary conditions, Soret and Dufour effects and with variable viscosity.

The conventional heat transfer fluids including oil, water and ethylene glycol etc. are poor heat transfer fluids, since the thermal conductivity of these fluids play an important role on the heat transfer coefficient between the heat transfer medium and the heat transfer surface. An innovative technique for improving heat transfer by using ultra fine solid particles in the fluids has been used extensively during the last several years. Choi [35] introduced the term nanofluid refers to these kinds of fluids by suspending nanoparticles in the base fluid. Khanafer et al. [36] investigated the heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. The convective boundary-layer flow over vertical plate, stretching sheet and moving surface studied by numerous studies and in the review papers Buongiorno [37], Daungthongsuk and Wongwises [38], Oztop [39], Nield and Kuznetsov [40,41], Ahmad and Pop [42], Khan and Pop [43], Kuznetsov and Nield [44,45] and Bachok et al. [46].

From literature survey the base aim of this work is to study the free convection boundary-layer flow past a vertical cone embedded in a porous medium filled with a nanofluid, the basic fluid being a non-Newtonian fluid by using similarity transformations. The reduced coupled ordinary differential equations are solved numerically. The effects of the parameters governing the problem are studied and discussed.

2. Mathematical formulation of the problem

Consider the problem of natural convection about a downward-pointing vertical cone of half angle γ embedded in a porous medium saturated with a non-Newtonian power-law nanofluid. The origin of the coordinate system is placed at the vertex of the full cone, with x being the coordinate along the surface of the cone measured from the origin and y being the coordinate perpendicular to the conical surface Fig (1). The temperature of the porous medium on the surface of the cone is kept at constant temperature T_w, and the ambient porous medium temperature is held at constant temperature T_∞.

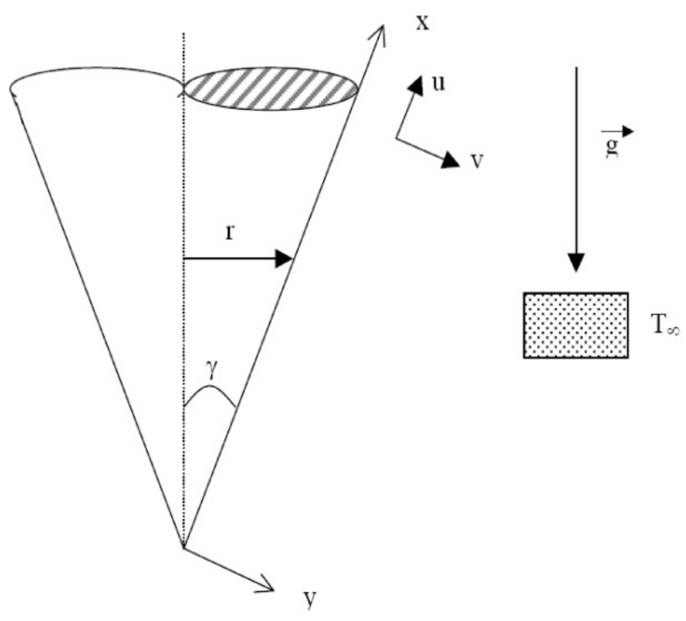

Figure 1. A schematic diagram of the physical model.

The nanofluid properties are assumed to be constant except for density variations in the buoyancy force term. The thermo physical properties of the nanofluid are given in Table 1 (see Oztop and Abu-Nada [39]). Assuming that the thermal boundary layer is sufficiently thin compared with the local radius, the equations governing the problem of Darcy flow through a homogeneous porous medium saturated with power-law nanofluid near the vertical cone can be written in two-dimensional Cartesian coordinates (x,y) as:

$$\frac{\partial(r^m u)}{\partial x}+\frac{\partial(r^m v)}{\partial y}=0, \tag{1}$$

$$\frac{\partial u^n}{\partial y}=\frac{(\rho\beta)_{nf}Kg\cos\gamma}{\mu_{nf}}\frac{\partial T}{\partial y}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

Where u and v are the volume-averaged velocity components in the x and y directions, respectively, T is the volume-averaged temperature. n is the power-law viscosity index of the power-law nanofluid and g is the gravitational acceleration. $m=\gamma=0$ corresponds to flow over a vertical flat plate and $m=1$ corresponds to flow over a vertical cone. n is the viscosity index. For the case of $n=1$, the base fluid is Newtonian. We note that $n<1$ and $n>1$ represent pseudo-plastic fluid and dilatant fluid, respectively. Property ρ_{nf} and μ_{nf} are the density and effective viscosity of the nanofluid, and K is the modified permeability of the porous medium. Furthermore, α_{nf} and β_{nf} are the equivalent thermal diffusivity and the thermal expansion coefficient of the saturated porous medium, which are defined as (see Khanafer et al. [36]):

$$\begin{aligned}&\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,\quad \mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}},\quad \alpha_{nf}=\frac{k_{nf}}{(\rho C_p)_{nf}},\\ &(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s,\quad \frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+2\phi(k_f-k_s)}.\end{aligned} \tag{4}$$

Here ϕ is the solid volume fraction.

The associated boundary conditions of Eqs. (1)-(3) can be written as:

$$\begin{aligned}&v=0;\ T=T_w \quad \text{at}\ y=0;\\ &u=0;\ T\to T_\infty \quad \text{as}\ y\to\infty,\end{aligned} \tag{5}$$

where μ_f is the viscosity of the basic fluid, ρ_f and ρ_s are the densities of the pure fluid and nanoparticle, respectively, $(\rho C_p)_f$ and $(\rho C_p)_s$ are the specific heat parameters of the base fluid and nanoparticle, respectively, k_f and k_s are the thermal conductivities of the base fluid and nanoparticle, respectively. The local radius to a point in the boundary layer r can be represented by the local radius of the vertical cone $r = x\sin\gamma$.

	$\rho(kg/m^3)$	$C_p(J/kgK)$	$k(W/mK)$	$\beta\times10^5(K^{-1})$
Pure water	997.1	4179	0.613	21
Copper (Cu)	8933	385	401	1.67
Silver (Ag)	10500	235	429	1.89
Alumina (Al_2O_3)	3970	765	40	0.85
Titanium Oxide (TiO_2)	4250	686.2	8.9538	0.9

Table 1. Thermo-physical properties of water and nanoparticles [39].

By introducing the following non-dimensional variables:

$$\eta = \frac{y}{x} Ra_x^{1/2}, \qquad f(\eta) = \frac{\psi(x,y)}{\alpha_f r^m Ra_x^{1/2}}, \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}. \tag{6}$$

The continuity equation is automatically satisfied by defining a stream function $\psi(x,y)$ such that:

$$r^m u = \frac{\partial\psi}{\partial y} \text{ and } r^m v = -\frac{\partial\psi}{\partial x}. \tag{7}$$

where;

$$Ra_x = \left(\frac{x}{\alpha_f}\right)\left[\frac{Kg(\rho\beta)_f \cos\gamma (T_w - T_\infty)}{\mu_f}\right]^{1/n}. \tag{8}$$

Integration the momentum Eq. (2) we have:

$$\frac{\mu_{nf}}{\mu_f} u^n = \frac{(\rho\beta)_{nf} Kg\cos\gamma}{\mu_f}\left(T - T_\infty\right). \tag{9}$$

Substituting variables (6) into Eqs. (1)–(5) with Eq. (9), we obtain the following system of ordinary differential equations:

$$\frac{1}{(1-\phi)^{2.5}}\left(f'\right)^n = \left[1 - \phi + \phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\right]\theta, \tag{10}$$

$$\frac{k_{nf}/k_f}{\left[1-\phi+\phi\frac{\left(\rho C_p\right)_s}{\left(\rho C_p\right)_f}\right]}\theta''+\left(m+\frac{1}{2}\right)f\theta'=0, \tag{11}$$

along with the boundary conditions:

$$\begin{aligned} &f(0)=0,\ \theta(0)=1,\\ &f'(\infty)=0,\ \theta(\infty)=1. \end{aligned} \tag{12}$$

where primes denote differentiation with respect to η , the quantity of practical interest, in this chapter is the Nusselt number Nu_x which is defined in the form:

$$Nu_x=\frac{hx}{k_m}=\frac{-\left.\frac{\partial T}{\partial y}\right|_{y=0}x}{T_w-T_\infty}=-Ra_x{}^{1/2}\theta'(0). \tag{13}$$

where h denotes the local heat transfer coefficient.

3. Results and discussion

In this study we have presented similarity reductions for the effect of a nanoparticle volume fraction on the free convection flow of nanofluids over a vertical cone via similarity transformations. The numerical solutions of the resulted similarity reductions are obtained for the original variables which are shown in Eqs. (10) and (11) along with the boundary conditions (12) by using the implicit finite-difference method. The physical quantity of interest here is the Nusselt number Nu_x and it is obtained and shown in Eqs. (13) and (14). The distributions of the velocity $f'(\eta)$, the temperature $\theta(\eta)$ from Eqs.(10) and (11) and the Nusselt number in the case of Cu-water and Ag-water are shown in Figs. 2–8. The computations are carried for various values of the nanoparticles volume fraction for different types of nanoparticles, when the base fluid is water. Nanoparticles volume fraction ϕ is varied from 0 to 0.3. The nanoparticles used in the study are from Copper (Cu), Silver (Ag), Alumina (Al_2O_3) and Titanium oxide (TiO_2).

In order to verify the accuracy of the present method, we have compared our results with those of Yih [22] for the rate of heat transfer $\theta'(0)$ in the absence of the nanoparticles $\left(\phi=0\right)$. The comparisons in all the above cases are found to be in excellent agreement, as shown in Table 2. It is clear that as a geometry shape parameter m increases, the local Nusselt number increases. While Table 3 depict the heat transfer rate $\theta'(0)$ for various values of nanoparticles volume fraction ϕ for different types of nanoparticles when the base fluid is water. Figs. 2 and 3 show the effects of the nanoparticle volume fraction φ on the velocity distribution in the case of Cu-water when $\phi=0,0.05,0.1,0.15,0.2,0.3$. It is noted that the velocity along the cone increases with the nanoparticle volume fraction in both of the two cases (i.e. Cu-water and Ag-water), moreover the velocity distribution in the case of Ag-water is larger than that for Cu-water. We can show that the change of the velocity distribution when we use different types of nanoparticles from Fig. 4, which depict the Ag-nanoparticles are the highest when the base fluid is water and when $\phi = 0.1$. Thus the presence of the nanoparticles volume fraction increases the momentum boundary layer thickness.

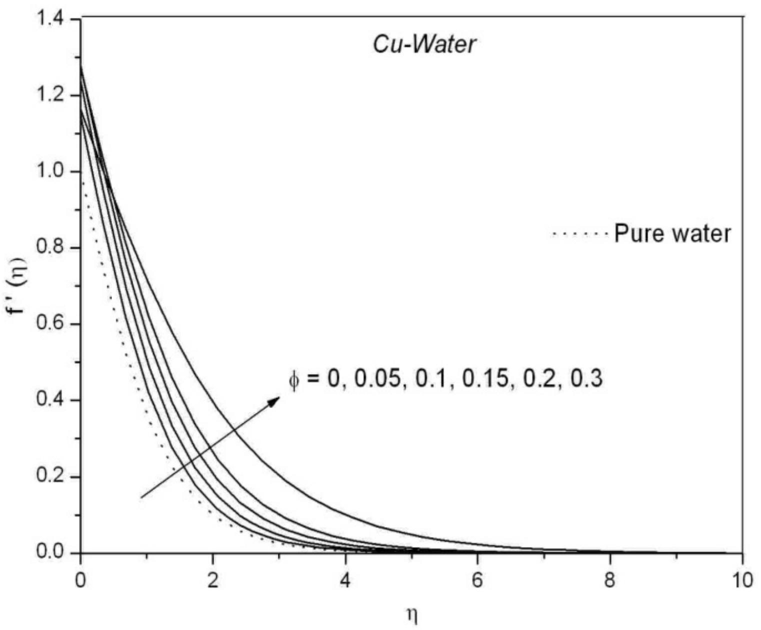

Figure 2. Effects of the nanoparticle volume fraction ϕ on velocity distribution $f'(\eta)$ in the case of Cu-Water.

n	Vertical plate		Vertical cone	
	Yih [22]	Present method	Yih [22]	Present method
0.5	0.3766	0.3768	0.6522	0.6524
0.8	0.4237	0.4238	0.7339	0.7340
1.0	0.4437	0.4437	0.7686	0.7686
1.5	0.4753	0.4752	0.8233	0.8233
2.0	0.4938	0.4938	0.8552	0.8552

Table 2. Comparison of results for the reduced Nusselt number $-\theta'(0)$ for vertical plate $(\lambda = 0)$ and vertical cone $(\lambda = 1)$ when $\phi = 0$.

ϕ	Cu	Ag	Al_2O_3	TiO_2
0.05	0.7423	0.7704	0.6604	0.6725
0.1	0.6931	0.7330	0.5642	0.5852
0.15	0.6301	0.6732	0.4780	0.5057
0.2	0.5591	0.6002	0.4006	0.4331
0.3	0.4052	0.4357	0.2673	0.3062

Table 3. Values of $-\theta'(0)$ for various values of ϕ when $n = 1$.

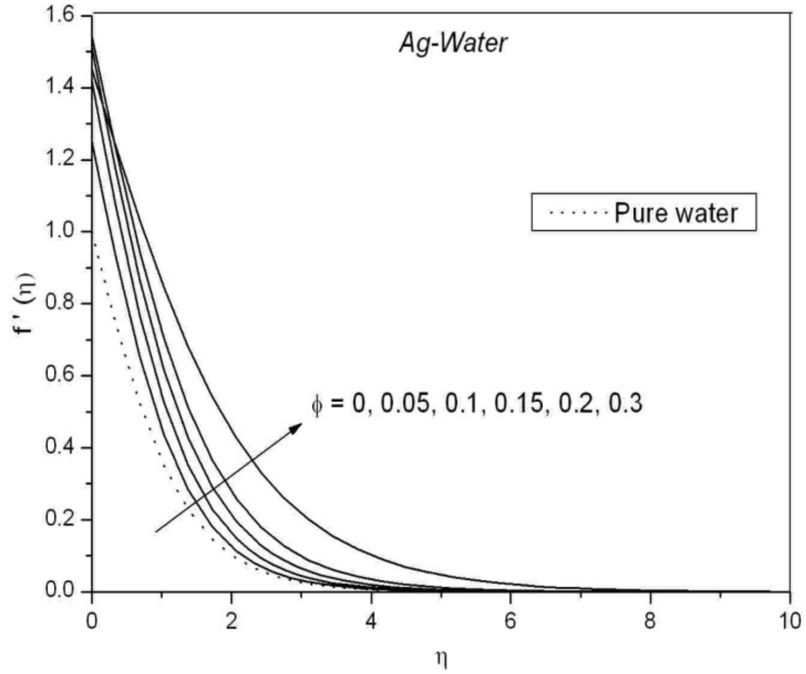

Figure 3. Effects of the nanoparticle volume fraction ϕ on velocity distribution $f'(\eta)$ in the case of Ag-Water.

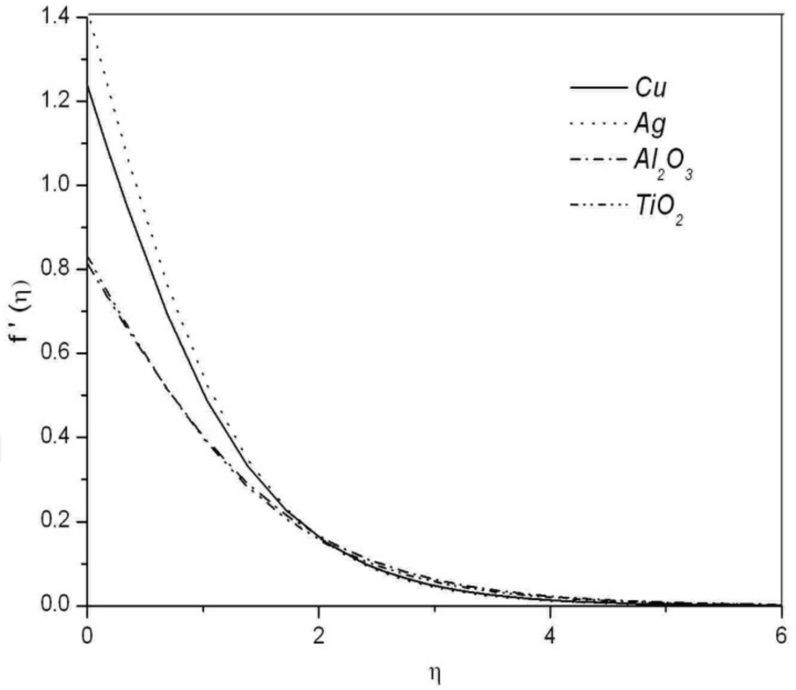

Figure 4. Velocity profiles $f'(\eta)$ for different types of nanofluids when $\phi = 0.1$.

Figs. 5 and 6 are presented to show the effect of the volume fraction of nanoparticles Cu and Ag respectively, on temperature distribution. These figures illustrate the streamline for different values of ϕ , when the volume fraction of the nanoparticles increases from 0 to 0.3, the thermal boundary layer is increased. This agrees with the physical behavior, when the volume of copper and silver nanoparticles increases the thermal conductivity increases, and then the thermal boundary layer thickness increases. Moreover Fig. 7 displays the behavior of the different types of nanoparticles on temperature distribution when $\phi = 0.1$. The figure showed that by using different types of nanofluid as the values of the temperature change and the Ag-nanoparticles are the lower distribution.

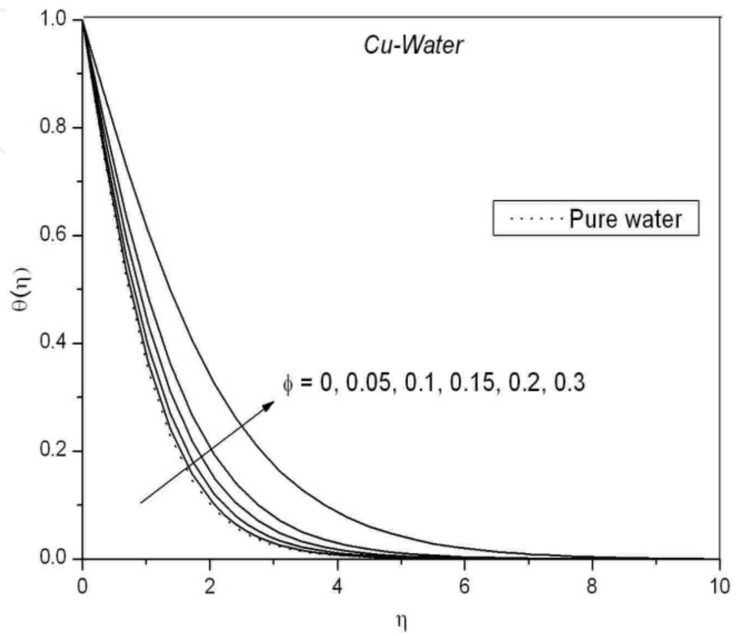

Figure 5. Effects of the nanoparticle volume fraction ϕ on temperature distribution $\theta(\eta)$ in the case of Cu-Water.

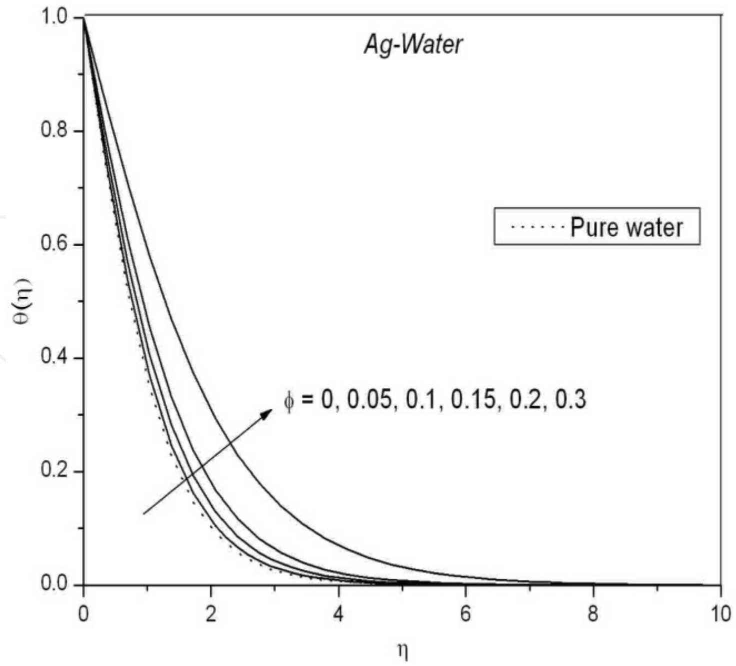

Figure 6. Effects of the nanoparticle volume fraction ϕ on temperature distribution $\theta(\eta)$ in the case of Ag-Water.

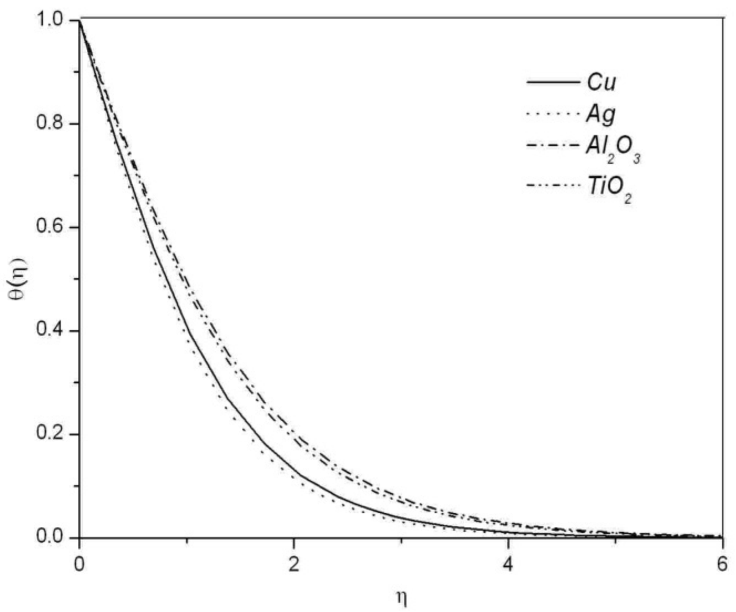

Figure 7. Temperature profiles $\theta(\eta)$ for different types of nanofluids when $\phi = 0.1$.

Fig. 8 shows the variation of the reduced Nusselt number with the nanoparticles volume fraction ϕ for the selected types of the nanoparticles. It is clear that the heat transfer rates decrease with the increase in the nanoparticles volume fraction ϕ . The change in the reduced Nusselt number is found to be lower for higher values of the parameter ϕ . It is observed that the reduced Nusselt number is higher in the case of Ag-nanoparticles and next Cu-nanoparticles, TiO_2-nanoparticles and Al_2O_3-nanoparticles. Also, the Fig. 8 and Table 3 show that the values of $\theta'(0)$ change with nanofluid changes, namely we can say that the shear stress and heat transfer rate change by taking different types of nanofluid. Furthermore this depicts that the nanofluids will be very important materials in the heating and cooling processes.

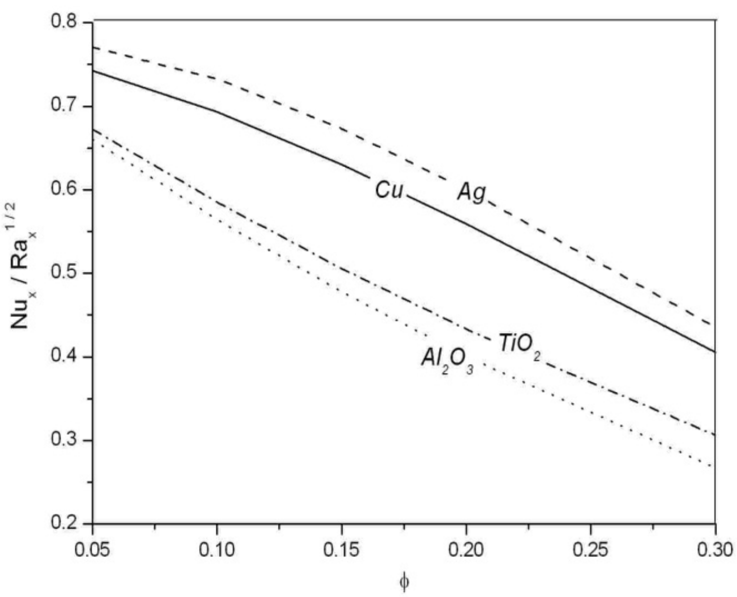

Figure 8. Effects of the nanoparticle volume fraction ϕ on dimensionless heat transfer rates.

4. Conclusions

The problem of the steady free convection boundary layer flow past a vertical cone embedded in porous medium filled with a non-Newtonian nanofluid has been studied and the special case when the base fluid is water has been considered. The effects of the solid volume fraction ϕ on the flow and heat transfer characteristics are determined for four types of nanofluids: Copper (Cu), Silver (Ag), Alumina (Al_2O_3) and Titanium oxide (TiO_2). It has been shown, as expected, that increasing of the values of the nanoparticles volume fraction lead to an increase of the velocity and the temperature profiles and to an decrease of the Nusselt number for the values of the parameter ϕ. It has been found that the Ag-nanoparticles proved to have the highest cooling performance and Alumina-nanoparticles enhanced to have highest heating performance for this problem.

Nomenclature

C_p	specific heat at constant temperature
f	dimensionless stream function
g	acceleration due to gravity
h	local heat transfer coefficient
K	permeability coefficient of the porous medium
k	thermal conductivity
m	geometry shape parameter
Nu_x	reduced Nusselt number
n	viscosity index, $n \geq 0$
Ra_x	modified Rayleigh number
r	local radius of the cone
T	temperature
T_w	temperature at the surface of the cone
T_∞	ambient temperature attained as $y \to \infty$
u, v	Darcian velocity components in x - and y -directions
x, y	Cartesian coordinates

Greek symbols

α	thermal diffusivity
β	volumetric expansion coefficient
γ	half angle of the cone
η	similarity variable
θ	dimensionless temperature
μ	effective viscosity

ρ_f	density of the fluid
ρ_s	nanoparticles mass density
$\left(\rho C_p\right)_{nf}$	heat capacitance of the nanofluid
$\left(\rho C_p\right)_f$	heat capacity of the fluid
$\left(\rho C_p\right)_s$	effective heat capacity of the nanoparticles material
ϕ	nanoparticles volume fraction
ψ	stream function

Subscripts

f	fluid fraction
nf	nanofluid fraction
s	solid fraction
w	condition at the wall
∞	stream function condition at the infinity

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5. References

[1] Cheng P (1978) Heat transfer in geothermal systems. *Adv. Heat Transfer* 14: 1–105.
[2] Nield DA, Bejan A (1999) Convection in Porous Media. second ed., *Springer: New York*.
[3] Vafai K (2000) Handbook of Porous Media. *Marcel Dekker: New York*.
[4] Pop I, Ingham DB (2001) Convective Heat Transfer: Mathematical and Computational Modelling of Viscous Fluids and Porous Media. *Pergamon Press*: *Oxford* .
[5] Ingham DB, Pop I (2002) Transport Phenomena in Porous Media. *Pergamon Press: Oxford*
[6] Cheng P, Minkowycz WJ (1977) Free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from a dike. *J. Geophys. Res.* 82(14): 2040–2044.

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[7] Gorla RSR, Tornabene R (1988) Free convection from a vertical plate with nonuniform surface heat flux and embedded in a porous medium. *Transp. Porous Media* 3 : 95–106.

[8] Bakier AY, Mansour MA, Gorla RSR, Ebiana AB (1997) Nonsimilar solutions for free convection from a vertical plate in porous media. *Heat Mass Transfer* 33: 145–148.

[9] Gorla RSR, Mansour MA, Abdel-Gaied SM (1999) Natural convection from a vertical plate in a porous medium using Brinkman's model. *Transp. Porous Media* 36 : 357–371.

[10] Mulolani I, Rahman M (2000) Similarity analysis for natural convection from a vertical plate with distributed wall concentration. *Int. J. Math. math. Sci.* 23(5) : 319 – 334.

[11] Jumah RY, Mujumdar AS (2000) Free convection heat and mass transfer of non-Newtonian power law fluids with yield stress from a vertical plate in saturated porous media. *Int. Comm. Heat Mass Transfer* 27 : 485–494.

[12] Groşan T, Pop I (2001) Free convection over a vertical flat plate with a variable wall temperature and internal heat generation in a porous medium saturated with a non-Newtonian fluid. *Technische Mechanik* 21(4) : 313–318.

[13] Kumaran V, Pop I (2006) Steady free convection boundary layer over a vertical flat plate embedded in a porous medium filled with water at 4°C. *Int. J. Heat Mass Transfer* 49 : 3240–3252.

[14] Magyari E, Pop I, Keller B (2006) Unsteady free convection along an infinite vertical flat plate embedded in a stably stratified fluid-saturated porous medium. *Transp. Porous Media* 62 : 233–249.

[15] Cheng C-Y (2006) Natural convection heat and mass transfer of non-Newtonian power law fluids with yield stress in porous media from a vertical plate with variable wall heat and mass fluxes. *Int. Comm. Heat Mass Transfer* 33 : 1156–1164.

[16] Chamkha AJ, Al-Mudhaf AF, Pop I (2006) Effect of heat generation or absorption on thermophoretic free convection boundary layer from a vertical flat plate embedded in a porous medium. *Int. Comm. Heat Mass Transfer* 33 : 1096–1102.

[17] Magyari E, Pop I, Postelnicu A (2007) Effect of the source term on steady free convection boundary layer flows over an vertical plate in a porous medium. Part I. *Transp. Porous Media* 67 : 49–67.

[18] Nield DA, Kuznetsov AV (2008) Natural convection about a vertical plate embedded in a bidisperse porous medium. *Int. J. Heat Mass Transfer* 51 : 1658–1664.

[19] Mahdy A, Hady FM (2009) Effect of thermophoretic particle deposition in non-Newtonian free convection flow over a vertical plate with magnetic field effect. *J. Non-Newtonian Fluid Mech.* 161: 37–41.

[20] Ibrahim FS, Hady FM, Abdel-Gaied SM, Eid MR (2010) Influence of chemical reaction on heat and mass transfer of non-Newtonian fluid with yield stress by free convection from vertical surface in porous medium considering Soret effect. *Appl. Math. Mech. -Engl. Ed.* 31(6): 675–684.

[21] Yih KA (1997) The effect of uniform lateral mass flux effect on free convection about a vertical cone embedded in a saturated porous medium. *Int. Comm. Heat Mass Transfer* 24(8): 1195–1205.

[22] Yih KA (1998) Uniform lateral mass flux effect on natural convection of non-Newtonian fluids over a cone in porous media. *Int. Comm. Heat Mass Transfer* 25(7): 959–968.

[23] Yih KA (1999) Effect of radiation on natural convection about a truncated cone. *Int. J. Heat Mass Transfer* 42: 4299 – 4305.

[24] Yih KA (1999) Coupled heat and mass transfer by free convection over a truncated cone in porous media: VWT/VWC or VHF/VMF. *Acta Mech.* 137 : 83–97.

[25] Murthy PVSN, Singh P (2000) Thermal dispersion effects on non-Darcy convection over a cone. *Compu. Math. Applications* 40: 1433 – 1444.

[26] Roy S, Anilkumar D (2004) Unsteady mixed convection from a rotating cone in a rotating fluid due to the combined effects of thermal and mass diffusion. *Int. J. Heat Mass Transfer* 47:1673–1684.

[27] Takhar HS, Chamkha AJ, Nath G (2004) Effect of thermophysical quantities on the natural convection flow of gases over a vertical cone. *Int. J. Eng. Sci.* 42: 243–256.

[28] Singh PJ, Roy S (2007) Unsteady mixed convection flow over a vertical cone due to impulsive motion. *Int. J. Heat Mass Transfer* 50: 949–959.

[29] Kumari M, Nath G (2009) Natural convection from a vertical cone in a porous medium due to the combined effects of heat and mass diffusion with non-uniform wall temperature/concentration or heat/mass flux and suction/injection. *Int. J. Heat Mass Transfer* 52: 3064–3069.

[30] Cheng C-Y (2009) Natural convection heat transfer of non-Newtonian fluids in porous media from a vertical cone under mixed thermal boundary conditions. *Int. Comm. Heat Mass Transfer* 36: 693 – 697.

[31] Cheng C-Y (2009) Soret and Dufour effects on natural convection heat and mass transfer from a vertical cone in a porous medium, *Int. Comm. Heat Mass Transfer* 36 : 1020 – 1024.

[32] Cheng C-Y (2010) Soret and Dufour effects on heat and mass transfer by natural convection from a vertical truncated cone in a fluid-saturated porous medium with variable wall temperature and concentration. *Int. Comm. Heat Mass Transfer* 37 : 1031 – 1035.

[33] Cheng C-Y (2010) Soret and Dufour effects on heat and mass transfer by natural convection from a vertical truncated cone in a fluid-saturated porous medium with variable wall temperature and concentration. *Int. Comm. Heat Mass Transfer* 37 : 1031 – 1035.

[34] Cheng C-Y (2010) Nonsimilar boundary layer analysis of double-diffusive convection from a vertical truncated cone in a porous medium with variable viscosity. *Int. Comm. Heat Mass Transfer* 37: 1031 – 1035.

[35] Choi SUS (1995) Enhancing thermal conductivity of fluid with nanoparticles. developments and applications of non-Newtonian flow. *ASME FED 231/MD* 66 : 99–105.

[36] Khanafer K, Vafai K, Lightstone M (2003) Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. *Int. J. Heat Mass Transfer* 46 : 3639–3653.

[37] Buongiorno J (2006) Convective transport in nanofluids. *ASME J. Heat Transfer* 128 : 240–250.

[38] Daungthongsuk W, Wongwises S (2007) A critical review of convective heat transfer nanofluids. *Ren. Sustainable Energy Rev.* 11: 797–817.

[39] Oztop HF, Abu-Nada E (2008) Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. *Int. J. Heat Fluid Flow* 29: 1326–1336.

[40] Nield DA, Kuznetsov AV (2009) The Cheng–Minkowycz problem for natural convective boundary-layer flow in a porous medium saturated by a nanofluids. *Int. J. Heat Mass Transfer* 52: 5792–5795.

[41] Nield DA, Kuznetsov AV (2011) The Cheng–Minkowycz problem for the double-diffusive natural convective boundary-layer flow in a porous medium saturated by a nanofluids. *Int. J. Heat Mass Transfer* 54: 374–378.

[42] Ahmad S, Pop I (2010) Mixed convection boundary layer flow from a vertical flat plate embedded in a porous medium filled with nanofluids. *Int. Comm. Heat Mass Transfer* 37: 987 – 991.

[43] Khan WA, Pop I (2010) Boundary-layer flow of a nanofluid past a stretching sheet. *Int. J. Heat Mass Transfer* 53: 2477–2483.

[44] Kuznetsov AV, Nield DA (2010) Natural convective boundary-layer flow of a nanofluid past a vertical plate. *Int. J. Thermal Sci.* 49: 243 – 247.

[45] Kuznetsov AV, Nield DA (2010) Effect of local thermal non-equilibrium on the onset of convection in a porous medium layer saturated by a nanofluid. *Transp. Porous Media* 83: 425–436.

[46] Bachok N, Ishak A, Pop I (2010) Boundary-layer flow of nanofluids over a moving surface in a flowing fluid. *Int. J. Thermal Sci.* 49: 1663 – 1668.