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# Forced Convective Heat Transfer and Fluid Flow Characteristics in Curved Ducts

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# 1. Introduction

Curved fluid flow passages are common in most technological systems involving fluid transport, heat exchange and thermal power generation. Some examples are: compact heat exchangers, steam boilers, gas turbines blades, rocket engine nozzle cooling and refrigeration. Such flows are subjected to centrifugal forces arising from continuous change in flow direction and, exhibit unique fluid and thermal characteristics that are vastly different to those within straight passages.

The centrifugal action induced by duct curvature imparts two key effects on the fluid flow. It produces a lateral fluid movement directed from inner duct wall towards the outer wall in the axial flow, thus causing a spiralling fluid motion through the duct. This lateral fluid movement is manifested by large counter-rotating pairs of vortices appearing in the duct cross section and is referred to as secondary flow. The centrifugal action also forms a radial fluid pressure gradient positively biased towards the outer duct wall. The lateral fluid circulation takes place adversely to the radial pressure field and is dampened by the viscous effects. The combined actions of the positive radial pressure gradient and the viscous forces lead to the formation of a stagnant fluid region near the outer wall. Beyond a certain critical axial flow rate, the radial pressure gradient would exceed the equilibrium condition in the stagnant fluid region at outer wall and triggers a localised flow circulation that forms additional pairs of vortices. This flow condition is known as Dean Instability [1] and the additional vortices are called Dean Vortices.

In his pioneering work, Dean [1] proposed the dimensionless Dean Number  $K = \left[\frac{D_h}{R}\right]^{\frac{1}{2}}$  Re for characterising the secondary flow behaviour. Moffat [2] and Eustice [3] have



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experimentally observed and verified the critical velocity requirements for Dean Instability, while Baylis [4] and Humphery et al. [5] affirmed the use of Dean Number in designating secondary flow behaviour. However, subsequent studies by Cheng et al. [6], Ghia and Shokhey [7], and Sugiyama et al, [8] showed that the duct aspect ratio and curvature ratio also significantly influence Dean Instability in curved rectangular ducts.

For rectangular ducts, Chandratilleke et al. [9,10,11] reported an extensive parametric study, capturing the profound influence from duct aspect ratio, curvature ratio and wall heat flux on the flow behaviour. Using an approach based on the stream function, they developed a two-dimensional model simplified by dynamic similarity assumption in axial direction. This simulation showed very good correlation to the data from their own experimental work [10,11]. The locations of Dean vortex formation within the flow was qualitatively identified through the use of intersecting stream function contours of zero potential. Comprehensive results were presented within the Dean Number range of  $25 \le K \le 500$ , aspect ratio at  $1 \le Ar \le 8$  and Grashof number at  $12.5 \le Gr \le 12500$ . It was identified that the onset of Dean vortices are produced in ducts of high aspect ratio. The wall heat flux radically changed the flow patterns and showed a tendency to suppress Dean vortex formation. Subsequently, Yanase [12] and Fellouah et al. [13,14] confirmed and validated the findings by Chandratilleke et al. [11].

The stream function approach is clearly limited to two-dimensional flows and does not accurately applied for real flow situations. In developing improved models, Guo et al. [15] considered laminar incompressible flow and formulated a three-dimensional simulation to explore the interactive behaviour of geometrical and flow parameters on heat transfer and pressure drop. Using flow entropy for hydrothermal assessment, they reported the influence from Reynolds number and curvature ratio on the flow profile and Nusselt number. For curved rectangular ducts, Ko et al. [16] proposed to split the overall flow entropy into individual contributions from heat transfer and fluid viscous friction. The relative magnitudes of these components were appraised to determine the heat or viscous-biased irreversibility in the flow domain. This approach was adopted for achieving thermal optimisation in ducts through minimised overall entropy, and illustrated for enhancing forced convection in curved ducts with longitudinal fins under laminar and turbulent conditions [17,18,19]. However, the entropy approach did not warrant precise identification of Dean Instability.

Fellouah et al. [13,14] also have presented a useful three-dimensional model for rectangular curved ducts of aspect ratio within  $0.5 \le \text{Ar} \le 12$  and curvature ratio at  $5.5 \le \gamma \le 20$  and. Considering both water and air as working fluids, their results were was experimentally validated using a semi-circular duct. These tests also provided visualisation data on vortex formation at several Dean Numbers. For detecting Dean Instability, they suggested the use of the radial gradient of the axial fluid velocity. In this, a limiting value for the radial gradient of axial velocity is arbitrarily assigned as a triggering threshold for Dean Instability. This approach represents an early attempt to incorporate Dean vortex detection

to a simulation process rather than a trail-and-error method as previously practised. In spite of the perceived benefits in computation, this approach cannot be rationalised because the radial gradient of axial velocity is remotely linked with vortex generation. This short fall is reflected when the model of Fellouah et al. [13] failed to detect Dean Instability that was clearly observed in some reported cases [10,11]. With improved capabilities, Chandratilleke et al. [20,21] have suggested other approaches that will be discussed later.

Most experimental and numerical analyses on curved ducts have been performed on rectangular duct geometries. By virtue of shape, such ducts have less wall interference on secondary vortex formation, making it relatively easier for numerical modelling and convenient for experimentation including flow visualisation. Ducts with elliptical and circular cross sections have received less research attention in spite of being a common geometry used in most technological systems. The curved duct flow behaviour in such geometries remains relatively unexplored.

Dong and Ebadian [22], and Silva et al. [23] have numerically simulated the flow through elliptical curved channels and observed that the stagnation regions are vastly different to those in rectangular ducts. Unlike in rectangular ducts, Dean Instability was seen to originate within the flow rather than at the outer duct wall. Papadopoulos and Hatzikonstantinou [24] considered elliptical curved ducts with internal fins and have numerically investigated the effects of fin height on friction factor and heat transfer. They concluded that the appearance of secondary vortices next to the concave wall would make the friction factor dependent on both fin and duct heights. Andrade et al. [25] reported a study with a finite element numerical model and discussed the influence from temperature-dependent viscosity on heat transfer and velocity profile for fully-developed forced convection in elliptical curved tube. They considered both cooling and heating cases wherein the Nusselt Number was found to be lower for cases of variable viscosity than constant properties. This was attributed to the increased viscosity at the cooler inner duct wall dampening the secondary flow and the vortex formation.

The above review of published literature indicates that the available modelling methods have yet to develop for realistic representation of complex secondary flow behaviour and improved predictability for Dean Instability. Parametric influences of duct geometry and flow variables remain unexplored and poorly understood with no decisive approach for defining the onset of Dean vortices and their locations. Significantly improving these shortfalls, this chapter describes an advanced three-dimensional numerical simulation methodology based on helicity, which is congruent with the vortex motion of secondary flow. Facilitating much compliant tracking of vortex flow paths, the model uses a curvilinear mesh that is more effective in capturing the intricate details of vortices and flexibly applied to both rectangular and elliptical ducts. Two intuitive approaches for detecting the onset of Dean Instability are proposed and examined in the study. An extensive parametric investigation is presented with physical interpretation of results for improved understanding of flow behaviour. A thermal optimisation scheme based on flow irreversibilities is developed for curved ducts.

# 2. Nomenclature

Ar	Aspect Ratio= a/b
а	Height of cross section (mm)
b	Width of cross section (mm)
D <sub>h</sub>	Hydraulic diameter = $\frac{ab}{\sqrt{\frac{(a+b)}{2}}}$ (mm)
F <sub>c</sub>	Centrifugal force (N)
¢	Gravity (m/s <sup>2</sup> )
8	
*	Helicity (m/s <sup>2</sup> )
Н	Dimensionless Helicity = $\frac{\Pi D_h}{U_{in}^2}$
î,j,k	Unit vectors in x,y,z directions
Κ	Dean number = $\left(\frac{D_h}{R}\right)^{\frac{1}{2}}$ Re
р	Static pressure (Pa)
<i>p</i> *	Dimensionless static pressure = $\frac{p}{\frac{1}{2}\rho U_{in}^2}$
R	Z Radius of curved channel (m)
Re	Reynolds number = $\frac{U_{in}D_h}{v}$
$\widehat{S}$	Coordinate along duct cross section for defining secondary flow direction
$U_{in}$	Velocity at duct inlet (m/s)
<i>u,v,w</i>	Velocities component (m/s)
$u^*, v^*, w^*$	Dimensionless velocity = $\frac{u, v, w}{U_{in}}$
V <sub>r</sub>	Axial velocity (m/s)
<i>x</i> , <i>y</i> , <i>z</i>	Coordinates (m)
$x^*, y^*, z^*$ Dimensi	onless coordinates= $\frac{x, y, z}{D_h}$
Greek symbols	
γ	Curvature ratio= R/b
$\theta$	Angular position of cross section (deg)
Φ	Dissipation function
V	Kinematic Viscosity (m <sup>2</sup> /s)
μ	Dynamic viscosity (Ns/m <sup>2</sup> )
ρ	Density (kg/m <sup>3</sup> )
, <i>0</i>	

# 3. Model description and numerical analysis

Fig. 1 shows the rectangular and elliptical duct geometries used for the three-dimensional model developed in the current study. It also indicates the key geometrical parameters of duct height *a*, duct width *b* and the duct radius of curvature *R*. The geometrical model consists of a semi-circular curved duct test section fitted with a straight inlet passage to ensure fully-developed flow at entry to the curved duct and an outlet passage for smooth flow exit.



Figure 1. Geometry of rectangular and elliptical curved duct models

The analysis focuses on semi-circular curved ducts having fixed width b of 10 mm and constant radius of curvature R of 125 mm. The duct aspect ratio Ar is changed by varying duct height a. The working fluid air is assumed to be an incompressible Newtonian fluid with temperature-dependent fluid properties. The air enters at an inlet temperature of 300 K and flows steadily through the passage under laminar flow conditions. A uniform heat flux is applied on the outer wall (rectangular duct) or outer half of duct periphery (elliptical

duct), while all the other walls being adiabatic. Constant velocity condition is applied to the inlet of the straight duct preceding the curved duct. The flow exit is taken to be a pressure outlet. The duct walls are assumed to have no slip boundary condition.

 $\nabla . (\vec{V}) = 0$ 

The numerical model solves the following governing equations:

Time-averaged continuity equation:

Momentum and Energy conservation equations:

$$\vec{V}.\nabla(\rho\vec{V}) = -\nabla p + \mu\nabla^{2}\vec{V} + \rho_{m}\vec{g} + \vec{F}_{c} \quad (a)$$
  
$$\vec{V}.\nabla(\rho c_{n}T) = k\nabla^{2}T + S_{T} \quad (b)$$

The magnitude of the centrifugal body force term in radial direction is given by,

$$F_c = \rho \frac{V_r^2}{r} = \rho \frac{(u^2 + w^2)}{\sqrt{x^2 + z^2}}$$
(3)

(1)

Considering the position and alignment of curved part of geometry, centrifugal source term in Cartesian coordinate system is obtained from,

$$\vec{F}_{c} = \rho \frac{1 + sign(-z)}{2} \frac{(u^{2} + w^{2})}{x^{2} + z^{2}} (x\hat{i} + z\hat{k})$$
(4)

In Eq. 4, a Sign Function is used to ensure the centrifugal source term is applied only on the curved side of the geometry (i.e.  $z \le 0$ ). For obtaining the dimensionless parameters, the characteristics length, velocity and pressure are chosen to be  $D_h$ ,  $U_{in}$ ,  $\frac{1}{2}\rho U_{in}^2$ , respectively.

The thermally-induced buoyancy is included in the model by relating the density  $\rho_m$  in Eq. 2(a) to local fluid temperature. For this, a sixth-order polynomial as given by Eq. 5 is developed, where the coefficients are obtained using the published data. This approach is necessary because the linearity of the Boussinesq approximation caused some discrepancy and is found to be inadequate for evaluating the wall pressure gradient.

$$\rho(T) = 10^{-15}T^6 - 3 \times 10^{-12}T^5 + 3 \times 10^{-9}T^4 - 2 \times 10^{-6}T^3 + 6 \times 10^{-4}T^4 - 0.1008T + 9.3618$$
(5)

In capturing the helix-like fluid motion of secondary flow, this three-dimensional model incorporates a helicity function, which is defined by Eq. 6. The helicity is non-dimensionalised using the reference helicity based on hydraulic diameter, as given by Eq. 7.

$$H = \vec{V}.\vec{\omega} = u(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}) + v(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}) + w(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})$$
(6)

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$$H \approx \frac{U_{in}^2}{D_h} \Longrightarrow H^* = H(\frac{D_h}{U_{in}^2}) \tag{7}$$

For identifying the onset of Dean Instability, the model proposes two separate criteria. The first criterion is based on helicity, which is computed from Eq. 7. The second criterion uses the outer duct wall pressure gradient profile for which the non-dimensional wall pressure gradient is obtained from Eq. 8. A sign convention is incorporated in Eq. 8 to designate the opposite rotational directions of the vortices in upper and lower half of duct cross section, following the selection of coordinate  $\vec{S}$  along the outer wall of duct cross section, as depicted in Fig. 1(b).

$$\frac{dp^{*}}{d\bar{S}} = \begin{cases}
-\frac{D_{h}}{\frac{1}{2}\rho U^{2}_{in}}\frac{dp}{ds} & \text{at upper half of cross section} \\
\frac{D_{h}}{\frac{1}{2}\rho U^{2}_{in}}\frac{dp}{ds} & \text{at lower half of cross section}
\end{cases}$$
(8)

For the forced convective heat transfer in the duct, the local Nusselt Number is defined as  $Nu = \frac{hD}{k}$  where the heat transfer coefficient h, is determined by considering the grid cell temperature difference between the heated wall and the adjacent fluid cell. The average  $\int NudA$ 

Nusselt number is obtained from the surface integral,  $\overline{N}u = \frac{\int NudA}{A}$ .

The model formulates a thermal optimisation scheme using flow irreversibility as a criterion. For this, the overall entropy generation is split in to the components of irreversibility contributed by the wall heat transfer, including thermally-induced buoyancy effects and, that due to fluid flow friction compounded by secondary fluid motion. As such, the components of entropy generation from heat transfer ( $S_T$ ) and that from fluid friction ( $S_p$ ) within the solution domain are evaluated using the expressions 9 (a), (b) and (c).

$$S_{T} = \frac{k}{T^{2}} \left[ \left( \frac{\partial T}{\partial x} \right)^{2} + \left( \frac{\partial T}{\partial y} \right)^{2} + \left( \frac{\partial T}{\partial z} \right)^{2} \right]$$
(a)  
$$S_{P} = \frac{\mu}{T} \Phi$$
(b)

(9)

where

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 \tag{c}$$

The entropy terms are generalised as volumetric-averaged values using,

$$S_T''' = \frac{\int S_T dv}{dv}$$
(a)  

$$S_p''' = \frac{\int S_p dv}{dv}$$
(b)  

$$S_g''' = S_T''' + S_p'''$$
(c)

Based on the above volumetric entropy generation terms, Bejan number is defined as,

$$Be = \frac{S_T''}{S_g'''} \tag{11}$$

Bejan Number reflects the relative domination of flow irreversibility by heat transfer with respect to the overall irreversibility [26]. In this, magnitude 1.0 for Bejan Number indicates the entropy generation totally dominated by heat transfer while 0 refers to fluid friction causing all of the flow irreversibility. Thus for curved ducts, the Bejan number performs as a visual map signifying the relative strength of thermal effects within the fluid domain interacted by the secondary flow and Dean vortices. This approach is then utilised for thermally optimising the forced convection process in curved ducts.

Incorporating above governing equations, a finite volume-based CFD model is formulated with the commercial package FLUENT where SIMPLE algorithm is used for pressure-velocity coupling. The momentum and energy equations are discretised by first and second order schemes, respectively. Since the model considers both buoyancy and centrifugal source terms, pressure discretisation is performed with body force-weighted approach. The stability of the solution is monitored through continuity, velocity, energy and dimensionless helicty where the convergence is achieved with values not higher than 10<sup>-5</sup>. Grid independency is carefully checked paying a special attention to boundary layer modelling at the outer wall.

The simulation is performed to obtain the profiles of velocity, helicity, temperature and Bejan Number at curved duct for a range of flow rates giving Dean Number within 80 to 1600. The wall heat flux is varied up to  $1000 \text{ W/m}^2$ . The results showing some signs of Dean vortices are further refined by re-running the simulations with closer steps of K to determine the exact point of instability and the critical Dean number. This procedure is repeated for all aspect ratios, flow rates and heat fluxes. Forced convection is evaluated by the local and average Nusselt numbers at the duct wall.

### 3.1. Mesh generation, grid sensitivity and model validation

In secondary flow simulations, the solution convergence is critically dependant on the grid selection because of the intricate flow patterns and intense flow gradients. For rectangular ducts, a fully structured mesh would be adequate. However, due to the geometrical shape, elliptical ducts tend to have much extreme flow field gradients and would require a more stringent grid arrangement.

In rectangular ducts a progressively reducing mesh is used with a much finer mesh near the outer wall where the onset of Dean Instability is anticipated. This approach has not been attempted in previous studies [11-13, 18-19] wherein it was argued that a mesh size less than 1 mm would not improve the accuracy, but only increase the computational time. The mesh refinement of the present analysis clearly identifies that a finer mesh near the wall is critical for detecting the onset of Dean vortices as accurately as possible. For testing grid dependency, the current study uses five mesh schemes indicated in Table 1. In this, columns A, B and C represent the number of grids over duct width, height and length, respectively, while the column D indicates the progressive reduction of grid size over duct width towards the outer wall.

	1			
	Number of Grids			
Scheme	A	В	С	D
Mesh1	26	51	305	1
Mesh2	31	64	305	1
Mesh3	43	84	305	1
Mesh4	50	98	305	1
Mesh5	26	51	305	1.05

**Table 1.** Grid selection and mesh schemes for rectangular ducts



Figure 2. Grid dependancy test for rectangular duct using velocity derivative in y-direction at exit

#### K=130, Ar=2

For rectangular ducts, Fig. 2 illustrates the grid dependency test conducted using the velocity derivative in y-direction at the duct outer wall. The Schemes 4 and 5 having mesh size less than 1 mm show better suitability than the other three schemes. However, the Scheme 5 is chosen as the optimum because of its slightly larger cell volume arising from

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progressively varied mesh size. This approach remarkably improved the vortex capturing ability in the solution domain without excessive computational demand. As such, the present study performed all computations with Scheme 5 of mesh size less than 1 mm, achieving much higher accuracy than any previously reported work.



Figure 3. Grid arrangement and grid sensitivity analysis for elliptical curved duct at exit

For handling steeper flow gradients in elliptical ducts, the current analysis uses the different grid arrangement depicted in Fig. 3. This scheme divides the elliptical duct cross section into

five regions, as shown in Fig. 3(a) and, the rectangular cells are swept along the axial direction to obtain a fully hexagonal structured mesh. The grid distribution is determined by the parameters nw, nh, nb and na, which are respectively, the number of grids on duct width, duct height and duct periphery while *na* represents the number of layers along the axial direction. This formation ensures uniform grid residual throughout the solution domain. Figs. 3(b), (c) and (d) show the grid sensitivity analysis in terms of nw, nh and nb and na, where 36 permutations are considered. For elliptical duct simulation, the optimal grid selection is taken to be nw = 12, nh = 52, nb = 7 and na = 250.

In validating results for a rectangular duct, Figs. 4(a) and 4(b) show a comparison of the axial flow velocity in x and y directions from the current model with those from Ghia and Shokhey [7] and Fellouah et al. [14]. It is seen that both magnitudes and trends of axial velocity are very favourably matched confirming the integrity of the current numerical process. Similarly for an elliptical duct, Figs. 4(c) and 4(d) provide a comparison of the axial velocity profiles at the mid cross section planes from the current study with those from Dong and Ebadian [22] and Silva et al. [23]. A very good agreement is clearly evident, validating the numerical consistency of the present simulation.



Figure 4. Model validation using dimensionless axial velocity profile at curved duct exit

## 4. Results and discussion

# 4.1. Fluid flow characteristics and geometrical influence

Fig. 5 depicts typical dimensionless helicity profiles at the exit of both rectangular and elliptical curved ducts for several Dean numbers with no external wall heating. These patterns show unique flow features that are not present in straight ducts. Initially at low *K*, the flow profiles indicate two large counter-rotating vortices that are attributable to the centrifugal action from the duct curvature.



Figure 5. Dimensionless helicity contours in curved ducts for varied flow rates-no wall heating

In curved ducts, the centrifugal action is manifested as two key effects. It generates a positive radial pressure field directed towards the outer duct wall (left wall in Fig. 5). Within this positive (adverse) pressure field, the centrifugal force drives the fluid radially away from the inner wall towards the outer duct wall. This sets up a lateral fluid circulation called secondary flow, which is manifested as the formation of counter-rotating vortices observed in Fig 5 in the duct cross section.

Increased axial flow rate (larger *K*) makes the secondary fluid motion more vigorous to produce stronger vortices. The positive radial pressure field is also Higher flow rate also intensifies that imparts more adverse effect on the secondary fluid motion. This latter effect adversely acts on to slow it down assisted by fluid viscosity. Consequently near the outer duct wall, a stagnant flow region is formed and a state of fluid equilibrium is established. Above a certain critical value of *K*, the radial pressure field becomes far greater for fluid equilibrium within the stagnant region and a weak local fluid recirculation is triggered. This flow situation is referred to as the Dean Instability identified by the critical value of the Dean number. The local fluid circulation triggering flow instability is manifested as pairs of additional vortices called Dean Vortices. In Fig. 5, the onset of Dean Instability occurs within *K* = 80 to 180 for the rectangular duct while for the Dean vortices between *K* = 180 to 260. These Dean vortices are designated as the helicity contour corresponding to  $H^* = 0.01$ . Upon inception, Dean vortices gradually grow bigger with increasing *K*, as depicted.

It is noticed that in rectangular ducts, the Dean instability tends to occur at a lower *K* than in elliptical ducts of the same aspect ratio. This is because in rectangular ducts, the secondary vortex motion is less steeply deflected at the outer wall by the cross sectional geometry than in elliptical ducts allowing more freedom for fluid movement and Dean vortex formation.

## 4.2. Dean instability and detection of Dean vortices

In curved ducts, the appearance of Dean vortices is traditionally identified through tedious flow visualisation or by numerical trial-and-error approach. In the latter, simulations are repeatedly performed to gradually narrow down the range of K for determining the critical Dean number and the flow patterns within a chosen tolerance limit. This involves guesswork and significant computational time. Chandratilleke et.al. [11] successfully developed a criterion based on zero-potential stream function contours to identify the locations of Dean vortex generation. Whilst that approach is adequate for two-dimensional cases, it is not applicable for three-dimensional flow. The work of Fellouah et al. [13] used the radial gradient of the axial velocity as a measure of identifying the flow instability. Such technique is not justifiable because the axial velocity change in radial direction is not physically connected with the secondary vortex generation. This inadequacy is clearly reflected in the work of Fellouah et al. [13], where their simulation failed to detect Dean vortices for some basic flow conditions. For example around *K* = 180 for a rectangular duct, Fellouah et al. [13] and Silva et al. [23] did not detect Dean vortices while the current study clearly predicts such vortices.

As of now, a reliable technique for identifying Dean vortices is not available in literature, particularly for three-dimensional simulations. The formulation of a generalised approach is

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difficult because the inception of Dean vortices are influenced by strongly inter-dependant effects of duct geometry and the flow variables. In forming a technique for Dean vortex detection, the present study proposes two practical criteria that can be integrated into and directly performed within the computational process.

## 4.2.1. Criterion 1 - Helicity threshold method

This criterion utilises dimensionless helicity  $H^*$  for tracking the onset of Dean Instability. It assigns the minimum threshold value for  $H^*$  by which the helicity contours are demarcated in the flow domain. These contours are designated as Dean vortices. This selection of  $H^*$  threshold essentially depends on the contour detection accuracy required, similar to defining the boundary layer thickness with a chosen velocity tolerance in traditional fluid flows.

Through exhaustive simulation runs, the current study has identified and proposes  $H^* = \pm$  0.01 to be an excellent choice that warrants precise and consistent detection of Dean vortices for all cases examined. A key advantage of this technique is that the detection precision can be varied to suit the required accuracy. Also, the technique can be readily integrated into the computational process to perform locally in the solution domain rather than as a cumbersome post-processing method. Hence, the determination of Dean vortices is more precise and less time consuming. The achievable precision is demonstrated below in Fig. 6, where the helicity contours obtained by using  $H^* = 0.01$  are shown just before and just after the onset of Dean vortices (indicated by arrows).

For the rectangular duct, the Dean vortices are absent in the flow for K = 95. However, when K is increased to 102, the helicity contours indicating Dean vortices are first detected. Similarly for the elliptical duct, the appearance of Dean vortices is detectable within K = 230 to 235. By adopting  $H^*$  threshold less than 0.01, this range can be further narrowed to improve precision of K value that corresponds to the onset of Dean Instability. The critical Dean numbers thus determined are: K = 100 for the rectangular duct and K = 234 for the elliptical duct. However, evidently such refinements to  $H^*$  would only incur a marginal benefit towards precision at the expense of significant increase in computational demand. Therefore,  $H^* = 0.01$  is concluded to be a very appropriate threshold for detecting Dean Instability. Previous methods never provided this degree of accuracy, flexibility or ability for vortex detection, signifying that this method is far superior to any reported approach including that by Ghia and Shokhey [7] and Fellouah et al. [13] using the axial velocity gradient.

### 4.2.2. Criterion 2 - Adverse wall pressure gradient method

This criterion is based on the unique features of the fluid pressure distribution along the outer duct wall. The gradient of this pressure profile shows inflection points that initially remain negative at low flow velocities and gradually shifts towards positive magnitudes as the flow rate is increased. Outer wall duct locations indicating negative-to-positive gradient change strongly correlates to the appearance of Dean vortices and forms the basis for a criterion to identify Dean Instability. These characteristics are illustrated in Fig. 7.



Figure 6. Detection of Dean vortex formation using helicity threshold method  $H^* = \pm 0.01$ 

For several Dean Numbers, Fig. 7 shows typical outer wall pressure gradient profiles evaluated from Eq. 8 for a rectangular duct in Fig. 7(a) and, for elliptical ducts in Figs. 7 (b) and (c). The variable *S* is the displacement coordinate along the outer duct wall boundary, as indicated in Fig. 1. At low values of *K* in Fig. 7(a), the pressure gradient remains negative over the entire outer wall. As *K* increases, the inflection points of the profile gradually shift to towards a positive magnitude, thus creating regions of adverse pressure gradients at the outer wall. The corresponding helicity contours indicate that these localities at the outer wall would develop flow reversal leading to the generation of Dean vortices. At *K* = 139, the inflection points have just become positive, representing the critical Dean Number and the onset of Dean Instability.

Showing similar trends, Fig. 7(b) illustrates the pressure gradient profile for an elliptical duct of aspect ratio 3. At K = 285, the entire pressure profile remains negative over the outer wall and the corresponding helicity patterns in Fig. 8(a) show Dean vortices are absent in the flow. At K = 291, the pressure gradient just changes from negative to positive at the duct centre and the first appearance (onset) of Dean vortices is noted in the flow profile of Fig. 8(a). For K = 297, the pressure gradient profile shows distinctly positive regions that well correlate to Dean vortex locations, as depicted.



Figure 7. Pressure gradient profiles along the outer wall of curved duct

Above observations conclusively indicate that the adverse (positive) pressure gradients at the outer duct wall could be effectively used for detecting the onset and location of Dean vortices. However, this approach has some drawback as discussed below.

Fig. 7(c) shows the outer wall pressure gradient for an elliptical duct of aspect ratio Ar = 1 (circular). This profile does not exhibit the negative-to-positive changeover in the pressure gradient as for elliptical ducts with Ar > 1 illustrated in Figs. 7(a) and 7(b), meaning that Dean vortices are not predicted according to the adverse pressure criterion. However to the contrary, Fig. 8(c) indeed shows the onset of Dean vortices at K = 92. This may seem a contradiction that is clarified below.

For a duct with Ar = 1 (circular duct), Fig. 8(c) clearly shows that the Dean vortices appear in the centre of the flow, but not at the outer wall as with rectangular and elliptical ducts. Therefore, the outer wall pressure gradient has lesser bearing on the flow reversal associated with Dean vortex formation. As such, the pressure gradient criterion tends to over predict the critical Dean Number for instability and has diminished suitability for elliptical ducts with aspect ratio near unity. It would perform satisfactorily for ducts with aspect ratio Ar > 1 where Dean vortices are formed at outer duct wall. Upon this overarching scrutiny, the helicity threshold method can be regarded as a precise, reliable and universally applicable technique for detecting Dean vortices in curved ducts of any shape and aspect ratio.

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**Figure 8.** Dean vortex formation in elliptical ducts of different aspect ratios (*detected using*  $H^* = 0.01$ )

## 4.2.3. Effect of duct aspect ratio

Obtained by using the helicity threshold method in the current study, Fig. 9 illustrates the influence of duct aspect ratio on the critical flow requirement for triggering Dean Instability in both rectangular and elliptical ducts. For comparison, the figure also provides the critical Dean number from the experimental results of Chen et al. [6] for rectangular ducts.

It is noted that the critical Dean number initially increases with the aspect ratio up to a certain value and then falls away for higher *K*. This behaviour conforms to the trend shown

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in Fig. 8 for elliptical ducts and the observations reported in previous experimental and numerical work. For rectangular ducts, the current model under-predicts the critical requirement for Dean Instability compared to the experimental results of [6]. This is because, the helicity method identifies Dean vortices much early in the growth process, whilst in experiments, the vortices are visually observed after they have grown to a detectable size at a higher Dean number.



Figure 9. Effect of aspect ratio on critical Dean number

## 4.3. Thermal characteristics and forced convection

### 4.3.1. Effects of flow rate

The helicity contours and the fluid temperature fields in Fig. 10 illustrate the effect of outer wall heating in rectangular and elliptical ducts for two selected values of K. The comparison of helicity contours in Figs. 10 and 5 indicates the wall heating essentially sets up a convective fluid circulation that interacts with the secondary flow and significantly alters the flow characteristics. This fluid circulation is driven by the buoyancy forces resulting from thermally-induced density changes.

As evident in Fig. 10 for K =80, at low flow rates, the convective circulation dominates the flow behaviour with less influence from secondary vortices. When the flow rate is increased, the centrifugal action intensifies and more vigorous secondary flow develops in the duct. Consequently, the secondary vortices overcome the thermal buoyancy effects to become more dominant and determine the overall fluid behaviour, as illustrated in Fig. 10 for K = 380.

Due to the confined geometry, the fluid flow within elliptical ducts is generally more constrained and tends to have steeper fluid flow gradients than in rectangular ducts. This geometrical effect is more pronounced at low flow rate when the flow patterns are dominated by thermally-induced buoyancy. Fig. 10 with K = 80, clearly demonstrates that the elliptical duct has sharper velocity and temperature gradients at the outer wall

compared to the rectangular duct. Consequently, the elliptical duct exhibits a higher rate of forced convection than in the rectangular duct within the lower range of K in Fig. 11. For increased flow rate, the secondary vortices begin to dominate the flow behaviour. As shown in Fig. 10 for K = 380, the rectangular duct has steeper velocity and temperature gradients at the outer wall compared to elliptical duct. This gives rise to a higher forced convection in rectangular ducts compared to the elliptical duct when K approximately exceeds 350, as depicted in Fig. 11.



Figure 10. Outer wall heating effect on helicity and temperature profiles,  $q = 250 \text{ W/m}^2$ 



**Figure 11.** Fig. 11 Variation of average Nusselt number at outer wall with Dean number, Ar = 4, q = 100 W/m<sup>2</sup>



**Figure 12.** Variation of duct skin friction factor with Dean number, Ar = 4,  $q = 100 \text{ W/m}^2$ 

For elliptical and rectangular ducts, Fig. 12 shows the comparison of skin friction factor, which essentially depends on the velocity gradient and is related to the duct pressure drop. With inherently steep fluid profiles, the elliptical duct has slightly larger skin friction compared to rectangular ducts with no crossover in the entire flow range.

#### 4.3.2. Effects of outer wall heating

Comparison of helicity contours in Figs. 10 and 5 at K = 380 indicates that with outer wall heating, a tendency is developed to impede the formation process of Dean vortices in both types of ducts. This is instigated by the thermal buoyancy-driven convection that continually acts to displace the fluid layer at the outer wall, thus weakening the formation of the stagnant fluid region, where the Dean Instability would occur. Hence, the flow reversal

is undermined with diminished potential for Dean vortex generation. High wall heat fluxes would impart more adversity on the triggering flow conditions of Dean Instability. In Fig. 13, these wall heating effects are clearly evident as a gradual decline in Nusselt number with increased outer wall heat flux for both types of ducts.



Figure 13. Variation of average Nusselt number at outer wall with outer wall heat flux, Ar = 4, K = 250



Figure 14. Variation of average duct Skin Friction with outer wall heat flux, Ar = 4, K = 250

With high wall heat fluxes, thermally induced convective circulation becomes stronger, imparting additional resistance to the axial fluid motion. This effect is more pronounced in elliptical ducts than in rectangular ducts. Consequently, duct skin friction coefficient tends to be relatively higher for elliptical ducts while indicating some increase with the wall heat flux, as evident in Fig. 14.

### 4.4. Thermal optimisation

Fig. 15 shows the Bejan number (Be) distribution in curved duct cross sections. For both rectangular and elliptical ducts at lower *K*, the flow irreversibility is practically dominated by the entropy generation from wall heat transfer (Be  $\approx$  1 with a red cast) over the entire flow cross section. As K increases, Bejan Number contours gradually acquire magnitudes less than 1 indicated by the blue cast. This signifies that in curved ducts, the secondary flow provides favourable fluid mixing to transport the hot fluid away from the heated wall, which in turn will improve forced convection. Magnitudes of Be < 1 also signifies an increased contribution to the total irreversibility from viscous effects, which would negate the overall flow benefits. These effects are less pronounced in rectangular ducts than in elliptical ducts. The opposing thermal and hydrodynamic trends identify a potentially useful technique for thermal optimisation of fluid flow through heated curved ducts, as explained below.



**Figure 15.** Bejan Number contours for rectangular and elliptical curved ducts,  $q = 100 \text{ W/m}^2$ 

Fig. 16 illustrates the variation of the flow irreversibility components  $S_t$ ,  $S_p$  and  $S_g$  for rectangular and elliptical ducts with *K*, evaluated using Eq. 9 and 10. It is evident from the figure that, when *K* is increased,  $S_t$  steadily falls while  $S_p$  rapidly grows. Although this falling trend is similar to both types of ducts, the thermal irreversibility component  $S_t$  decays more gently in elliptical duct compared to the rectangular duct. This means, the elliptical duct is less efficient in transporting heat from the wall than the rectangular duct. On the other hand, the viscous irreversibility component  $S_p$  grows more rapidly in elliptical ducts, signifying that the elliptical shape causes higher frictional pressure loses.

The overall irreversibility  $S_{g}$ , which is the sum of  $S_t$  and  $S_p$ , is initially dominated by the thermal irreversibility component and steadily falls with K for both duct shapes. However, the falling gradient is steeper for the rectangular duct, which implies more effective heat transfer in the fluid domain with lesser adversity from viscous effects.

At a certain value of K,  $S_g$  reaches a minimum. This point of inflection is identified as practically the best possible "trade-off" between the highest achievable thermal benefits with the least viscous penalty for curved duct flows. It is noted that the magnitude of the lowest overall irreversibility is higher for elliptical ducts than for rectangular ducts, indicating rectangular ducts would thermally better perform.



Figure 16. Curved duct thermal optimisation using total entropy generation

# 5. Conclusions

This book chapter provides a broad overview of the reported findings from the published numerical and experimental studies on fluid flow in curved duct geometries. It then presents and discusses the latest contributions to this field realised through an extensive research programmes of the authors where new analytical techniques and methodologies have been discovered for clearer fundamental understanding on fluid and thermal behaviour in curved ducts.

Improving accuracy over previous methods, the current study formulates a new threedimensional simulation methodology based on helicity that realistically represents the secondary vortex structures in curved ducts of any shape. The model accurately identifies and predicts unique features of secondary flow and the associated forced convection in both rectangular and elliptical curved ducts. Conforming to the limited data in literature, the predicted results examine the effects of fluid flow rate, duct aspect ratio and heat flux over a wide practical range. The duct geometry and aspect ratio are found to have critical influence on the secondary flow characteristics, indicating a profound effect on Dean vortex formation, Dean Instability and forced convection. Elliptical ducts show marginally inferior thermal performance compared to rectangular ducts under identical flow conditions.

Overcoming a current analytical limitation, the study appraises two novel methods for detecting the onset of Dean Instability in curved ducts, ascertaining their technical feasibility. One such approach uses a defined helicity threshold while the other makes use of features of the outer wall pressure gradient. Both methods offer the process flexibility by integrating into and performed within a computational scheme, for fast, reliable and accurate detection of Dean vortex formation. However, the helicity threshold method is recognised to be more universally applied to all duct shapes and aspect ratios. The pressure gradient approach shows limited functionality for it over-predicts the critical Dean Number in curved ducts of aspect ratio near unity.

The study also presents a useful thermal optimisation technique for heated curved duct based on the Second Law irreversibilities. It identifies the elliptical ducts to have less favourable overall thermal and hydrodynamics characteristics than rectangular ducts.

# Author details

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