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ANGEL: A Simplified Hybrid Metaheuristic for Structural Optimization

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1. Introduction

The weight minimization of the shallow truss structures is a challenging but sometimes frustrating engineering optimization problem. Theoretically, the optimal design searching process can be formulated as an implicit nonlinear mixed integer optimization problem with a huge number of variables. The flexibility of the shallow truss structures might cause different types of structural instability. According to the nonlinear behavior of the resulted light-weight truss structures, a special treatment is required in order to tackle the "hidden" global stability problems during the optimization process. Therefore, we have to replace the traditional "design variables → response variables" like approach with a more time-consuming "design variables → response functions" like approach, where the response functions describe the structural response history of the loading process up to the maximal load intensity without constraint violation.

In this study, a higher order path-following method [1] is embedded into a hybrid heuristic optimization method in order to tackle the structural stability constraints within the truss optimization. The proposed path-following method is based on the perturbation technique of the stability theory and a non-linear modification of the classical linear homotopy method.

The nonlinear function of the total potential energy for conservative systems can be expressed in terms of nodal displacements and the load parameter. The equilibrium equations are given from the principle of stationary value of total potential energy. The stability investigation is based on the eigenvalue computation of the Hessian matrix. In each step of the path-following process, we get information about the displacement, stresses, local, and global stability of the structure.

With the help of the higher-order predictor-corrector algorithm, we are able to follow the load-response path and detect the hidden bifurcation points along the path in time. During the optimization process, the optimal design is characterized by the maximal load intensity factor along the equilibrium path. Consequently, all the structural constraints are controlled by a fitness function in terms of the maximal feasible load intensity factor. Because the function evaluation is very expensive (for example, we have to call a professional system like ANSYS to carry out an "eigenvalue buckling analysis") we have to select the appropriate population-based metaheuristic frame very carefully. In everyday language, a population-based metaheuristic means a good tale usually inspired by the nature, a set of operators, which describes the daily life of the population, and a set of rules which controls the life or death of individuals. In the heuristic frame developing process we applied a "minimal art" like approach to reach the "good quality solution within reasonable time" goal. According to our approach, we decreased the number of operators and tunable-parameters, and simplified the significant operators and rules coming from different tales as much as possible.

In this chapter we present the result, which is a simple but very efficient hybrid metaheuristic for truss weight minimization with continuous and discrete design variables, and global and local stability constraints.

The presented "supernatural" ANGEL method [2-6] combines ant colony optimization (AN), genetic algorithm (GE) and gradient-based local search (L) strategy. In the algorithm, AN and GE search alternately and cooperatively in the design space. The powerful L algorithm, which is based on the local linearization of the constraint set, is applied to yield a more feasible or less unfeasible solution, when AN or GE obtains a solution.

The highly nonlinear and non-convex large-span and large-scale shallow truss examples with continuous and discrete design variables and response curves show that ANGEL may be more efficient and robust than the conventional gradient based deterministic or the traditional population based heuristic (metaheuristic) methods in solving explicit (implicit) optimization problems. ANGEL produces highly competitive results [16-18] in significantly shorter run-times than the previously described pure approaches.

The benefit of synergy can be demonstrated by standard statistical tests. To the best of our knowledge, no such work has been done in the literature for truss weight minimization with response curves. The reason is simple: the question of the global stability loss (the collapse of the structure as a whole) was not investigated very carefully in the truss optimization literature so far, according to a popular but totally misleading "assumption" of the truss optimization community that the local stability loss (local buckling) always precedes the global stability loss (the collapse), therefore the time-consuming investigation of the global stability is meaningless (see in Hanahara and Tada [20]).

2. Structural optimization

Generally, the traditional implicit "design variables → response variables" weight minimization problem with continuous and discrete design variables can be written as follows:

$$W(Z) \rightarrow \min \tag{1}$$

$$G_j(Z) \in \left[\underline{G}_i, \overline{G}_i\right], j \in \{1, 2, \ldots, M\} \tag{2}$$

$$X_i \in \left[\underline{X}_i, \overline{X}_i\right], i \in \{1, 2, \ldots, N\} \tag{3}$$

$$Y_g \in \{C_1, C_2, \ldots, C_C\}, g \in \{1, 2, \ldots, G\} \tag{4}$$

where $W(X)$ is the weight of the structure, G_j, $j \in \{1, 2, \ldots, M\}$ are the implicit response variables, and $Z = \{X = \{X_1, X_2 \ldots, X_N\}, Y = \{Y_1, Y_2 \ldots, Y_G\}\}$ is the set of continuous and discrete design variable sets.

The investigated new "design variables → response functions" weight minimization approach can be described as follows:

$$W(Z) \rightarrow \min \tag{5}$$

$$G_j(Z, \lambda) \in \left[\underline{G}_i, \overline{G}_i\right], j \in \{1, 2, \ldots, M\}, 0 \leq \lambda \leq 1 \tag{6}$$

$$X_i \in \left[\underline{X}_i, \overline{X}_i\right], i \in \{1, 2, \ldots, N\} \tag{7}$$

$$Y_g \in \{C_1, C_2, \ldots, C_C\}, g \in \{1, 2, \ldots, G\} \tag{8}$$

where $\lambda = \lambda(Z)$ the load intensity factor and constraint $0 \leq \lambda \leq 1$ means that loading process reached the maximal load intensity level without constraint violation.

In the path-following algorithm (details of the nonlinear structural investigation see in [1]), a design is represented by the set of $\{W, \lambda, Z, \Phi\}$, where W is the weight of the structure, λ is the maximal load intensity factor without constraint violation, and $Z = \{X, Y\}$ is the set of design variables. In our study, we used a problem-specific fitness function $\Phi = \Phi(Z)$ $(0 \leq \Phi \leq 2)$ which is defined as following:

$$\Phi = \begin{cases} 2 - \dfrac{W - W^L}{W^U - W^L} & \lambda = 1 \\ & \mathit{if} \\ \lambda & \lambda < 1 \end{cases} \tag{9}$$

where W^L $\left(W^U\right)$ is the minimal (maximal) weight of the structure, according to the given design space.

Our feasibility-oriented fitness function is based on the following set of criteria:

- Any feasible solution is preferred to any infeasible solution,

- Between two feasible solutions, the one having a smaller weight is preferred,

- Between two infeasible solutions, the one having a larger load intensity factor is preferred.

The minimal weight design problem can be formulated in terms of member cross-sections (a member cross-section may be a continuous variable or discrete value taken from a given catalogue) and nodal point shifts (to modify the shape), and may be constrained by the allowable nodal-point displacements, element stresses and the global stability requirement which simple means a non-singular Hessian on the load-response path.

We have to mention, that in our study we investigated only truss structures therefore the applied structural model was a large deflection truss model without simplifications. To avoid any type of stability loss even a structural collapse, a path-following approach was used to compute the structural response.

The applied measure of design infeasibility was defined as the maximal load intensity factor subject to all of the structural constraints. Naturally, ANGEL which is presented in the next section can be used in the traditional "design variables → response variables" approach and may be easily adopted for other types of optimization problems including the traditional explicit function minimization problems.

3. ANGEL

First, we have to note, that ANGEL as a name of a combined population-based metaheuristic for the resource-constrained project scheduling problem was introduced by Tseng and Chen [15]. We use this name in a different context with a different content. Our ANGEL algorithm, according to the systematic simplification, is based only three operators: random selection (RS), random perturbation (RP), and random combination (RC). In ANGEL the traditional mutation operator was replaced by the local search procedure (L) as a "locally best" form of mutation. That is, rather than introducing small random perturbations into the off-

spring solution, a gradient-based local search is applied to improve the solution until a local optimum is reached. The first result of our systematic simplification work is trivial: hard to imagine a population-based heuristic without an RS operator. The RS operator is in a special position in the heuristic community therefore the population-based heuristic literature is full with many general and problem-specific selection mechanisms (a good overview can be found in the work of Sivaraj and Ravichandran [13]). When we imagine the population as a matrix in which the rows are individuals and the columns are variables and the fitness function values of the individuals form a corresponding column vector, then very easy to identify the two basic selection possibilities: the column-wise (AN like) and the row-wise (GE like) selection mechanisms (see Figure 1). In Figure 1 we used a grey-scale to show the fitness of individuals (the lighter the grey color the better the individual) and we assumed that the individuals are ordered according to their fitness values. To demonstrate the possibilities we presented two similar cases (two parents (P2) → one child (C1)).

The AN mechanism selects at least one "more or less good parent" from each column step by step and after that applies the RP or RC operator procedure for each selected variable or variable set independently to get a child, from which L try to make a "better child".

The GE mechanism selects at least one "more or less good parent" in exactly one step for each case. In other words, GE selects at least one complete row. After that the algorithm repeats the previous steps to generate the child by applying the RP or RC operator for each variable or variable set of the selected parent or parents and after that L try to improve the quality of the child to get a "locally best" child.

In AN approach, by definition, the RS means a set of randomly selected "more or less good" element or elements according to the tale-dependent fitness function. This approach always imitates a "route" independently from its reality. When we imagine a bee flying from flower to flower or a salesperson travelling from city to city, the reality of the abstraction is trivial. But when we have to solve an optimal truss design problem minimizing its total weight on the set of element cross-sections as design variables, subject to the displacement and stress constraints, the local and global stability requirements and load conditions and imagine the construction as a whole, then the "from cross-section to cross-section" route may be totally meaningless and misleading abstraction. We may become the slave of the tale, which may yield a "brutal-force-search" like efficiency, because in our case the function evaluation is very expensive and time-consuming according to the implicit dependency between the design variables (element cross-sections) and the response variables (for example: global stability loss).

According to the optimal structural design problem, it is very easy to imagine the GE selection strategy, in which we select randomly at least one "more or less good design" and after that, according to the other operators of the tale, we try to make a better one (less unfeasible or lighter feasible) by RP or RC.

Easy to imagine, that the combination of the two selection mechanisms may increase the variability of the searching process as a synergism. The two selection mechanisms are very

general: from single-parent to multi-parents they are able to manage every case using only the RP and RC operators. In this study, "tradition is a tradition" we used the generally accepted operator types. Namely we used the AN-P1-C1 and GE-P2-C1 operators alternately and cooperatively using only the RP, RC, and L operators, which are invariant to the selection direction.
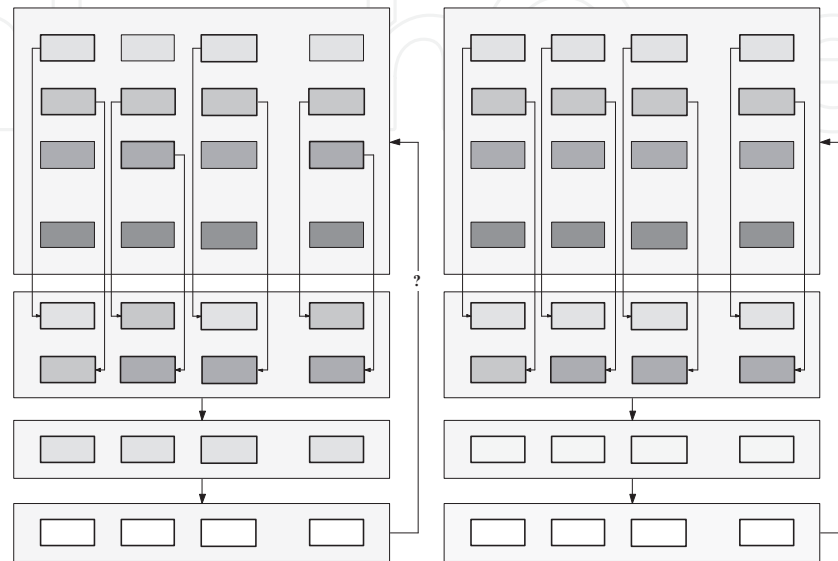

Figure 1. AN-P2-C1 + L and GE-P2-C1 + L

In the ANGEL developing process, we tried to simplify the three operators (RS, RP, and RC) and decrease the number of tunable-parameters, namely the size of the problem-specific "golden-number" set, as much as possible, to minimize the time requirement of the so-called "preliminary investigation". In our case, the preliminary investigation may be an "experimental design and analysis" like problem in the problem with terrible large computational cost which yields only 'good' problem-specific golden-number-set after several "try-and-error" iterations.

The flowchart of the proposed simplified heuristic ANGEL method is presented in Figure 2.The main procedure of the proposed hybrid metaheuristic follows the repetition of these two steps:

1.   AN with LS and

2.   GE with LS.

According to the systematic simplification, the hybrid algorithm is based only three operators:

1.   random selection (AN+GE),

2.   random perturbation (AN), and

3.   random combination (GE).
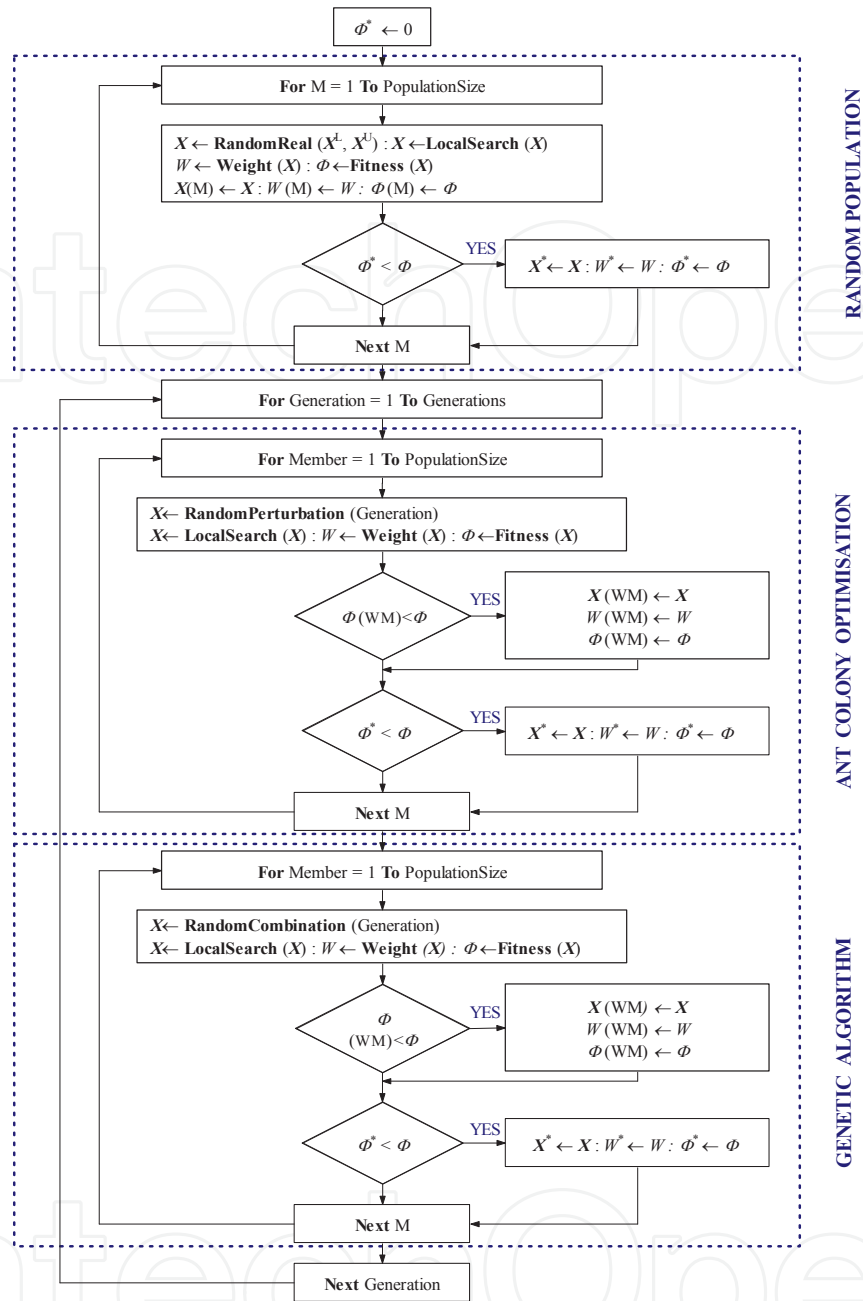
Figure 2. Flowchart of ANGEL

In the presented form, the population-based ANGEL has only three "tunable" parameters $\{PS, NG, MI\}$, where PS is the size of the population, NG is the number of generations, MI is the maximal number of gradient-based local search iterations ($0 \leq MI \leq 100$), and an additional parameter pair $\{\bar{S}, \underline{S}\}$ which defines a exponentially decreasing multiplier in the function of generation $gen, gen \in \{1, 2, \ldots, NG\}$:

$$S(gen) = \bar{S} * \exp \left(\log \left(\frac{\underline{S}}{\bar{S}} \right) * \frac{gen - 1}{NG - 1} \right) \tag{10}$$

The parameter pair $\left\{ \bar{S}, \underline{S} \right\}$, which controls the smooth transition from diversity to intensity, can be kept "frozen" in the algorithm:

$$\left\{ \bar{S}, \underline{S} \right\} = \left\{ 1.0,\ 0.01 \right\} \tag{11}$$

which means, that ANGEL is practically a "tuning-free" algorithm.

The monotonically decreasing standard deviation function for each continuous design variable can be defined in the following way:

$$S_i^{gen} = S(gen) * \left(\bar{X}_i - \underline{X}_i \right), gen \in \left\{ 1, 2, \ldots, NG \right\}, i \in \left\{ 1, 2, \ldots, N \right\} \tag{12}$$

In our approach, the case of the discrete design variables can be managed in a similar way. The only difference is that we replace the value set with the equivalent index set and carry out all the operations on the index set.

The main procedure of the proposed meta-heuristic method follows the repetition of these two steps:

1. AN with L and

2. GE with L.

In other words, meta-heuristic ANGEL firstly generates an initial population, after that, in an iterative process AN and GE search alternately and cooperatively on the current design set. The initial population is a totally random set. The random perturbation and random combination procedures which are based on the normal distribution, call therandom selection function which uses the discrete inverse method, to select a "more or less good" design (GE) or a set of "more or less good" design variable values from the current population. The higher the fitness values of a design (a design variable value) the higher the chance is that it will be selected by the function (see Figure 3).

The random perturbation procedure uses the continuous inverse method to generate a new solution from the old one (see Figure 4). The random combination procedure generates an offspring solution from the selected mother and father solutions (see Figure 5). The offspring solution is generated from the combined distribution, where the combined distribution is the weighted sum of the parent's distributions. The two procedures are controlled by the standard deviation, which is decreasing exponentially from generation to generation.

In our algorithm an offspring will not necessarily be the member of the current population, and a parent will not necessarily die after mating. The reason is straightforward, because our algorithm uses very simple rule without explicit pheromone evaporation handling: If the current design is better than the worst solution of the current population than the worst one will be replaced by the better one.

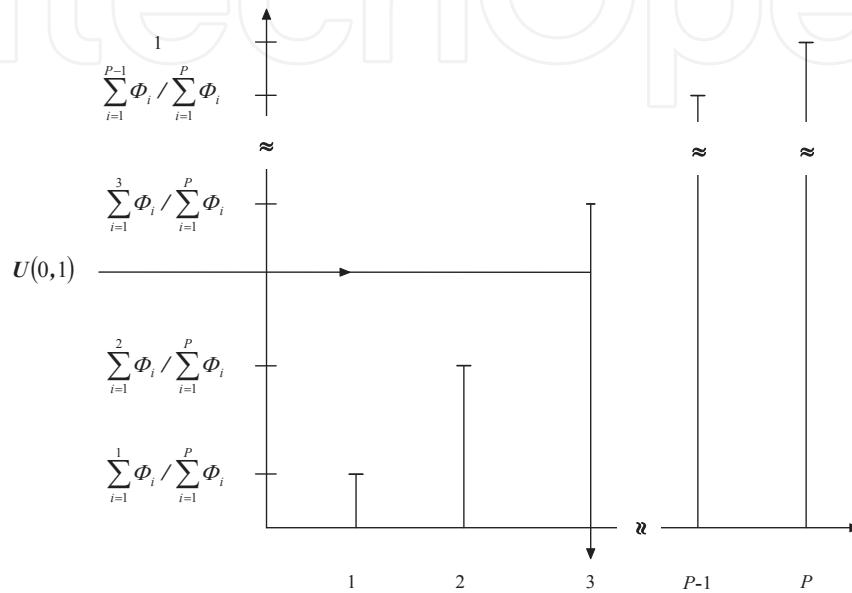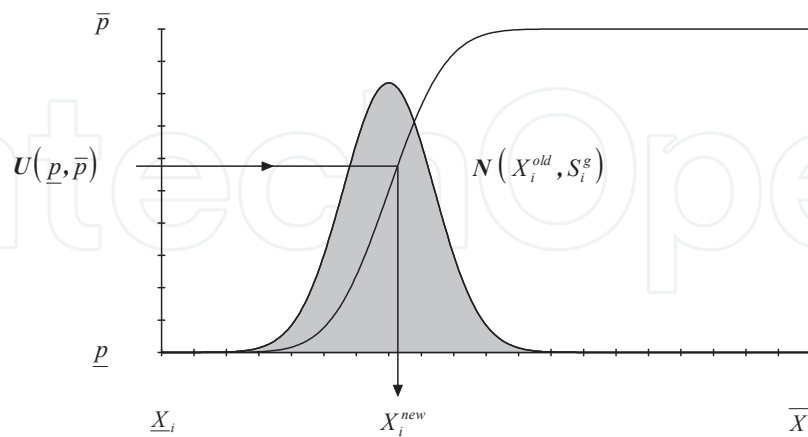**Figure 3.** Random selection

Figure 4. Random perturbation

In this work, without loss of generality, we only deal with the two fundamental cases when the design variables are only element (element-group) cross-section areas. In the continuous

case a cross-section area may be any value from a given interval and in the discrete case a cross-section area has to be taken from a discrete catalogue. Additionally, also without loss of generality, it is assumed that we are interested only in the local and global stability investigation without displacement constraints. We assume that the allowed maximal positive (stretching) stress defined by a constant, and the allowed minimal negative (compressive) stress is constrained by a local buckling function, which is a function of the material properties, the element length, and the element cross-sectional area. The global stability investigation is based on the load-eigenvalue curves. From the global stability point of view a truss design is feasible, when during the loading process each load-eigenvalue curve remains in the positive segments up to the end of the process.

$$\pi^{mother} = \frac{\varphi^{mother}}{\varphi^{mother} + \varphi^{father}} \qquad \pi^{father} = \frac{\varphi^{father}}{\varphi^{mother} + \varphi^{father}}$$

$$\pi^{mother} \, G\left(X_i^{mother}, S_i^g \right) \; + \; \pi^{father} \, G\left(X_i^{father}, S_i^g \right)$$
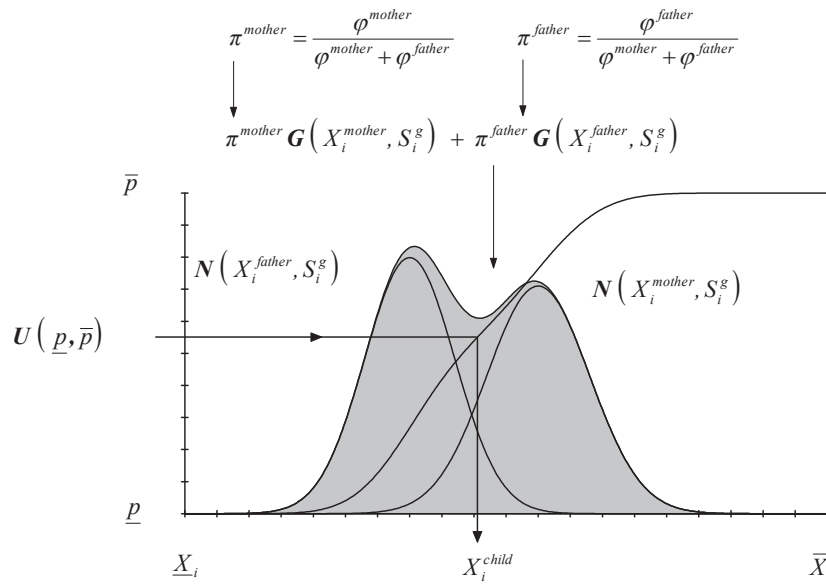
Figure 5. Random combination

In the pure continuous case (when only the cross-section range is fixed) the iterative local search procedure (L) alternates two approaches according to the current feasibility indicator value.

If the current design is feasible, namely:$\lambda = 1$, then it solves the following linear programming problem to get a lighter but feasible design allowing only a limited weight decrement in each iteration (see Lamberti and Pappalettere [11]):

$$\Delta W \left(\Delta X_1, \; ...., \Delta X_i, ..., \Delta X_N \right) \rightarrow \max \tag{13}$$

$$G_j(X) + \sum_{i=1}^{N} \frac{\partial G_j(X)}{\partial X_i} * \Delta X_i \in \left[\underline{G}_j, \bar{G}_j \right], j \in \{ 1, 2, ..., M \} \tag{14}$$

$$\Delta X_i \in \left[\Delta \underline{X}_i, \Delta \overline{X}_i \right], i \in \left\{ 1,2,\dots,N \right\} \tag{15}$$

$$0 \le \Delta W \le \Delta \overline{W} \tag{16}$$

If the current design is infeasible, namely:$\lambda 1$, the local search procedure tries to get a less infeasible solution allowing only a limited weight increment (decrement) in each iteration (see Lamberti and Pappalettere [11]):

$$\sum_{i=1}^{M} \left(\Delta \underline{G}_i + \Delta \overline{G}_i \right) \rightarrow \min \tag{17}$$

$$G_j(X) + \sum_{i=1}^{N} \frac{\partial G_j(X)}{\partial X_i} * \Delta X_i \in \left[\underline{G}_j - \Delta \underline{G}_j, \overline{G}_j + \Delta \overline{G}_j \right], j \in \left\{ 1,2,\dots,M \right\} \tag{18}$$

$$\Delta X_i \in \left[\Delta \underline{X}_i, \Delta \overline{X}_i \right], i \in \left\{ 1,2,\dots,N \right\} \tag{19}$$

$$\Delta W \in \left[\Delta \underline{W}, \Delta \overline{W} \right] \tag{20}$$

In the pure discrete case (when the cross-sections are taken from a catalogue) we have two possibilities to develop a local search procedure.

We can define a simple "thumb rule" used to improve the quality of the generated discrete solutions. The starting base of the thumb rule is a discrete solution given by applying the usual "rounding to the next catalogue value" rule. When the discrete solution is feasible (infeasible) then, in a cyclically repairable process, we try to decrease the cross-sectional areas step by step selecting always the "best" element (element group), where "best" means an element (element group) for which the element stress is minimal (maximal) in absolute value.

An improvement, namely a cross-sectional area decreasing (increasing) is accepted, when the starting design feasibility level is not decreased by the current modification. The process terminates, when no such an element exists. We have to emphasize that in the presented path following approach the design feasibility is measured by the maximal load intensity factor, and therefore, the designs satisfy the stress constraints up to the maximal load intensity factor computed by the applied path following method.

The other possibility would be a "locally exact" binary formulation. The proposed binary linear (or quadratic) programming (BLP or BQP) approach exploits the fact, that using a "state-of-the-art" solver the solution time of a local BLP (or BQP) problem is competitive with the solution time of the "thumb rule" heuristic.

Naturally, a local BLP (BQP) formulation can give better results, as a pure heuristic approach. Using a "dense" catalogue the problem can be managed as linear programming problem, when the catalogue is "sparse", we have to use a quadratic formulation to describe the possible stress changes accurately, in the function of the "local" catalogue values. The immediate predecessor (successor) of the current catalogue value defines the "local catalogue", for each element (element group), if such a value exists. According to the "local environment", an element (element group) can be described by at least three binary variables.

Naturally, using the standard trick (special ordered set (SOS) constraint management) of the operations research (OR), the formulation which has at least three binary variables, can be replaced by an equivalent formulation which has only at least two binary variables. Let g, $g \in \{1, 2, \ldots, G\}$ the member-group index and c, $c \in \{1, 2, \ldots, C\}$ the catalogue index, where G is the number of elements (member-groups) and C is the size of the discrete catalogue of possible cross-sectional areas: $\{C_1, C_2 \ldots, C_C\}$.

Let $\left\{ B_{g\,j}^i \mid j \in J^i \right\}$ be the set of the binary variables needed to describe the possible movement and A_g^i the cross-sectional area for element (member-group) g, $g \in \{1, 2, \ldots, G\}$ in iteration i, $\{i = 1, 2, \ldots, MI\}$. The "local catalogue" and the constraints connected to the local binary variables which describe the possible movements are presented in Figure 6-7. In iteration i, $\{i = 1, 2, \ldots, MI\}$ the local environment is defined by the result of the previous iteration.

In the local model exact analytical derivatives were used. To generate the symbolic derivatives, optimized to speed, Wolfram Mathematica 8.0 was used. Naturally, a linearized model can be replaced by a quadratic one, and the simplified assumption that the stress change of member-group g can be described by its cross-section change.

$$B_{g1}^{i-1} = 1 \qquad \bullet\!\!-\!\!\circ\!\!-\!\!-\!\!-\!\!-\!\!-\!\!\cdots\!\!-\!\!-\!\!-\!\!|$$
$$\qquad\qquad\quad C_1 \quad C_2$$

$$B_{gk}^{i-1} = 1 \qquad |\!\!-\!\!\circ\!\!-\!\!\bullet\!\!-\!\!\circ\!\!-\!\!\cdots\!\!-\!\!| \quad 1 < k < C$$
$$\qquad\qquad\quad C_{k-1}\; C_k\; C_{k+1}$$

$$B_{gC}^{i-1} = 1 \qquad |\!\!-\!\!-\!\!-\!\!-\!\!-\!\!\cdots\!\!-\!\!\circ\!\!-\!\!\bullet$$
$$\qquad\qquad\qquad\qquad\qquad C_{C-1}\; C_C$$
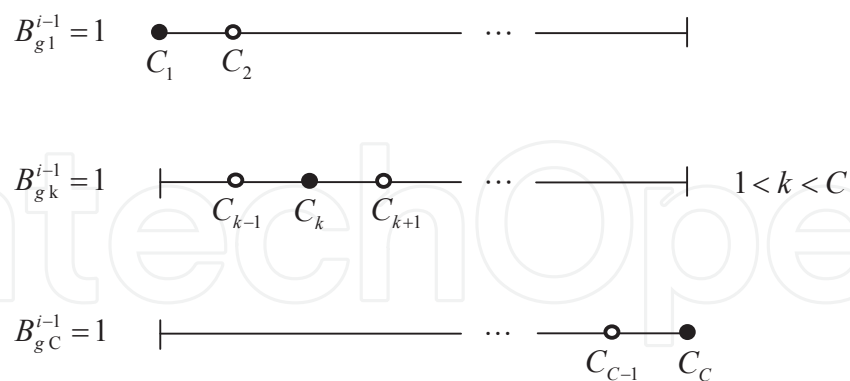
Figure 6. Local binary variables

The local search algorithm, in an iterative process, minimizes the weight increment (maximizes the weight decrement) needed to get a better (a lighter feasible or less unfeasible) solution. The OR formulation follows the conception of the "thumb rule", the "at least as good" quality requirement is managed by non-smoothed formulation, namely in the formulation the maximal constraint violation is constrained.

Naturally, the non-smooth $max()$ function can be replaced by an equivalent smooth formulation, by omitting the function and introducing additional constraints. In other words, when the starting base of an iteration is unfeasible ($\lambda 1$), than the local search algorithm generates a "mini-max" model, in which the maximal slack of the constraint set will be minimized according to the allowed maximal structural weight increase.

$$if \begin{cases} B_{g1}^{i-1}=1 & \begin{aligned} & B_{g1}^i + B_{g2}^i = 1 \\ & A_g^i = C_1 B_{g1}^i + C_2 B_{g2}^i \\ & S_g^i = S_g^{i-1} B_{g1}^i + \left(S_g^{i-1} + \frac{\partial S_g^{i-1}}{\partial A_g^{i-1}}(C_2 - C_1) \right) B_{g2}^i \end{aligned} \\[2em] B_{gk}^{i-1}=1 \quad then \quad & \begin{aligned} & B_{gk-1}^i + B_{gk}^i + B_{gk+1}^i = 1 \\ & A_g^i = C_{k-1} B_{gk-1}^i + C_k B_{gk}^i + C_{k+1} B_{gk+1}^i \\ & S_g^i = S_g^{i-1} B_{gk}^i + \left(S_g^{i-1} + \frac{\partial S_g^{i-1}}{\partial A_g^{i-1}}(C_{k-1} - C_k) \right) B_{gk-1}^i + \left(S_g^{i-1} + \frac{\partial S_g^{i-1}}{\partial A_g^{i-1}}(C_{k+1} - C_k) \right) B_{gk+1}^i \end{aligned} \\[2em] B_{gC}^{i-1}=1 & \begin{aligned} & B_{gC-1}^i + B_{gC}^i = 1 \\ & A_g^i = C_{C-1} B_{gC-1}^i + C_C B_{gC}^i \\ & S_g^i = S_g^{i-1} B_{gC}^i + \left(S_g^{i-1} + \frac{\partial S_g^{i-1}}{\partial A_g^{i-1}}(C_{C-1} - C_C) \right) B_{gC-1}^i \end{aligned} \end{cases}$$

Figure 7. Local binary constraints

4. Numerical example

4.1. Sizing optimization with buckling constraints - 120-bar truss dome

In this paper, in order to demonstrate the proposed solution method a well-known space dome structure is presented as a simple sizing problem, where two basic sub problems, continuous and discrete optimization problems are distinguished.

Saka and Ülker [12], as a continuous optimization problem, have introduced first time the 120-bar example. The minimal weight design subjected to structural constraints imposed on the member stress and nodal displacements based on linear and non-linear analysis. Subsequently, Soh and Yang [14] have been analyzed the same structure to obtain the optimal design related to sizing and configuration variables Kaveh and Talatahari [7] presented a heuristic method where the particle swarm optimizer, ant colony strategy and harmony search are hybridized.Therefore, several techniques have been incorporated to handle the constraints. Similar to Lee and Geem [10], Kelesoglu and Ülker [9], only sizing variables are considered to minimize the structural weight. According to the complexity of the concerned problems, another method has been proposed by Kaveh and Talatahari [8], namely a hybrid big bang–big brunch (HBB–BC) algorithm.The comparisons of numerical results using the HBB–BC method with the results obtained by other heuristic approaches are performed to

demonstrate the robustness of the present algorithm. With respect to the big bang–big brunch (BB–BC) approach, HBB–BC has better solutions and standard deviations. In addition, HBB–BC has low computational time and high convergence speed compared to BB–BC. However, when the number of design variables increases the hybrid BB–BC shows better performance. The effects of nonlinear behavior to the optimal results have been investigated by Hadi and Alvani [19] and Lemonge and Barbosa [21].

The geometry and nodal coordinates are presented in Figure 8 and in Table 1. According to the structural symmetry, truss members are grouped into seven member-groups (see in Table 2). The truss is subjected to the given applied external loads in Table 3. The truss members as design variables are grouped into seven group variables (Table 4).

Figure 8. The layout of the 120-bar shallow truss dome

Nodes	X [m]	Y [m]	Z [m]
1	0.	0.	7.000
4	6.01108	3.4705	5.850
5	6.94100	0.	5.850
18	10.82532	6.2500	3.000

Nodes	X [m]	Y [m]	Z [m]
19	11.66266	3.1250	3.000
20	12.50000	0.	3.000
40	13.76114	7.9450	0.
41	15.89000	0.	0.

Table 1. The geometry of the 120-member truss dome

Node	1	2-13	14-37
Load [kN]	60	30	10

Table 2. The load condition of the 120-bar truss dome

Modulus of elasticity	**$E = 210000 / MPa$**
Material density	$\rho = 7850 / kg/m^3$
Stress constraints for tension	$\sigma_e^U = 140 / MPa$
Stress constraints for compression	$\sigma_e^L = -140 / MPa$

Table 3. Properties of the applied material

Groups	Truss members					
G_1	1-2	1-3	1-4	1-5	1-6	1-7
	1-8	1-9	1-10	1-11	1-12	1-13
G_2	2-3	3-4	4-5	5-6	6-7	7-8
	8-9	9-10	10-11	11-12	12-13	13-2
G_3	2-14	3-16	4-18	5-20	6-22	7-24
	8-26	9-28	10-30	11-32	12-34	13-36
G_4	2-15	3-17	4-19	5-21	6-23	7-25
	3-15	4-17	5-19	6-21	7-23	8-25
	8-27	9-29	10-31	11-33	12-35	13-37
	9-27	10-29	11-31	12-33	13-35	2-37
G_5	14-15	16-17	18-19	20-21	22-23	24-25
	15-16	17-18	19-20	21-22	23-24	25-26
	26-27	28-29	30-31	32-33	34-35	36-37
	27-28	29-30	31-32	33-34	35-36	37-14

Groups	Truss members					
G_6	14-38	16-39	18-40	20-41	22-42	24-43
	26-44	28-45	30-46	32-47	34-48	36-49
G_7	15-38	17-39	19-40	21-41	23-42	25-43
	15-39	17-40	19-41	21-42	23-43	25-44
	27-44	29-45	31-46	33-47	35-48	37-49
	27-45	29-46	31-47	33-48	35-49	37-38

Table 4. Groups of truss elements

Refer to the formerly presented papers (e.g. [16-18]), in this study, stainless steel tubular cross-sections are considered as design variables. According to the thin-wall pipe structural behavior, the following local stability constraints are proposed. The stress constraint for against of Euler-buckling or peripheral shell-like buckling is given in terms of the thickness ratio:

$$\sigma_e^E = \frac{\pi\,E}{4\,\mathrm{L}^2} \cdot \frac{0.5 - \alpha + \alpha^2}{\alpha\,(1-\alpha)} \cdot G_e \tag{21}$$

$$\sigma_e^B = KE\alpha \tag{22}$$

where $\alpha = T/D$ is the ratio of the wall-thickness and diameter of the applied G_e group elements. In the present study, since continuous and discrete design variables are considered as well we applied tubular cross sections with given $\alpha = 0.05$ thickness ratio. Cross sectional variables are changing from $G_{min} = 5.0\,cm^2$ up to $G_{max} = 50.0\,cm^2$. In this paper, only stress and buckling constraints are considered.

The obtained results for continuous problem using linear and non-linear structural model are compared are presented in Table 5 and in Table 6. Comparing with the results of continuous optimizations shows that GA based approach [19] gives a better minimum weight than the optimality criteria approach [12]. It is observed that further reduction is possible in the weight of the space truss considering the geometrically nonlinear analysis as compared to linear one.

Worthy of note, that the optimal design obtained by the proposed hybrid ANGEL seems much better than the results of previously presented compared methods. Remarkable in this study - using the formula (9)- that the related fitness value is $\Phi = 1.928$ i.e. very close to the defined maximal fitness value. In the resulted optimal design only one buckling constraint is active, namely in the member-group 6.

In this paper for discrete optimization problem two types of catalogue values are distinguished, a sparse (case 1) and a dense (case 2) with the following cross sections:

Case 1: {5.0; 10.0; 15.0; 20.0; 25.0;…; 50.0}

Case 2: {5.0; 7.5; 10.0; 12.5; 15.0; 17.5; 20.0; 22.5; 25.0;…; 50.0}

We have to note that the related fitness value is $\Phi = 1.889$ (Case 1) and $\Phi = 1.922$ (Case 2), i.e. in case a sparse catalogue we obtained a bit worst fitness value than in case of dense catalogue values, but the difference is natural and both adjacent to the continuous one $\Phi = 1.928$.

Groups / cm2	Saka,Ulker* [12]		Hadi, Alvani* [19]	Proposed method
Linear	Non-linear	Non-linear	Non-linear	
G_1	16.66	17.50	10.85	12.968
G_2	44.89	45.56	38.70	8.282
G_3	24.89	25.45	35.40	13.325
G_4	9.66	8.44	5.23	7.964
G_5	21.93	22.30	27.37	8.316
G_6	16.59	15.96	15.30	7.776
G_7	11.74	3.90	3.90	7.990
W / kg	8511	7587	7158.6	4650.659

Table 5. The best results of the continuous problem (*Note: section shape is not available)

Using a state-of-the-art callable BLP (BQP) solver, for example: CPLEX 12.0, the time requirement of the improved local search is compatible with the time requirement of the traditional "thumb rule" like approach. However, the improved approach is more efficient, because it is able to modify more than one cross-sectional area in one iteration.

In the presented computational test, ANGEL was run with the following parameters:

- the population size was 100,
- the number of generations was 10, and
- the maximal number of local search iterations was 10.

We note, that the maximal number of iterations does not necessarily mean that the number of iterations always 10.

4.2. Sizing-shaping optimization with stability constraints -24-bar truss dome

This academic example has been analyzed by the author previously [17] to demonstrate the difficulties of the stability investigation. The layout and the initial data are presented in Figure 9 and Table 7-8. At the central node, the load is 0.5, while at nodes 2-7 it is 1.0 unit.

Groups / cm2	Hadi, Alvani* [19]		Proposed method (Case 1)	Proposed method (Case 2)
	Linear	Non-linear	Non-linear	Non-linear
G_1	15.00	12.30	10.0	10.0
G_2	46.70	46.70	10.0	10.0
G_3	27.00	27.00	15.0	12.5
G_4	7.05	5.33	10.0	7.5
G_5	27.60	24.70	5.0	5.0
G_6	11.10	17.80	10.0	10.0
G_7	1.82	1.53	10.0	7.5
W / kg	7264.6	7229.0	4979.681	4242.075

Table 6. The best results of the discrete problem(*Note: section shape is not available)The local search terminates when, according to the given "play-field", in the current step no improvement can be reached without affecting the maximal allowable weight increase or the maximal allowable constraint violation defined by the previous step.

Nodes	X [cm]	Y [cm]	Z [cm]
1	0	0	8.216
2	12.50	21.65063509	6.2.16
3	25.00	0	6.216
8	0	50.00	0
9	43.330127019	25.00	0

Table 7. Initial coordinates of 24-bar shallow space truss

The equilibrium path that involves in this case four critical points has been determined inside of the optimization process. First is a single bifurcation (λ_1=8.68), while the following two are double bifurcation points (λ_2=10.26;λ_3=15.67). The fourth is a simple limit point (λ_4=18.40).We have to note that only the fourth singular point is a simple limit point. With the help of this simple example easy to confirm the hazardous of the theories and methods which are able to tackle only snap-through phenomenon.

In this paper, a weight optimization is considered subjected to global stability constraints. The cross-sections as design variables are involved into three groups (Figure 9). The load intensity factor is changing from zero to one.

Using the proposed hybrid metaheuristic method, where the number of generations is 10 and the population size is 100, two optimization problems are considered.

Case 1:

In first case, a sizing optimization problem is solved for minimal volume optimization subjected to structural stability. The structure is loaded up to the maximal load intensity factor while the smallest eigenvalue becomes zero. The obtained best solution for the grouped design variables are the following: $A_1 = 1.000; A_2 = 1.321; A_1 = 1.119$. The optimal volume in this case is $V_{opt} = 773.127$.

Case 2:

In the second case, a sizing-shaping optimization problem is presented. The three sizing variables are extended with three shift variables namely the vertical position of all free joints ($Z_i; i = 1, 2, ..., 7$), and the horizontal position of the joints 2-7 ($R_j; j = 2, ..., 7$). In this case, the same proposed hybrid metaheuristic method has been applied, with the number of generations 10 and the population size 100. The obtained best solution is the following: $A_1 = 1.000$; $A_2 = 1.378; A_1 = 1.084; Z_1 = 7.685; Z_{2-7} = 6.121; R_{2-7} = 24.665$. The optimal volume is $V_{opt} = 765.699$ and the lowest eigenvalue is zero for three digits in the best solution.

Design variables	$A_i \in [1.00; 2.00]$ (cm²); $i \in \{1, 2, 3\}$	
Load cases	Nodes	Z
1	1	$-5.00\ kN$
	2, 3, 4, 5, 6, 7	$-10.00\ kN$
Material properties	Modulus of elasticity	$E = 10000\ kN\ /\ cm^2$

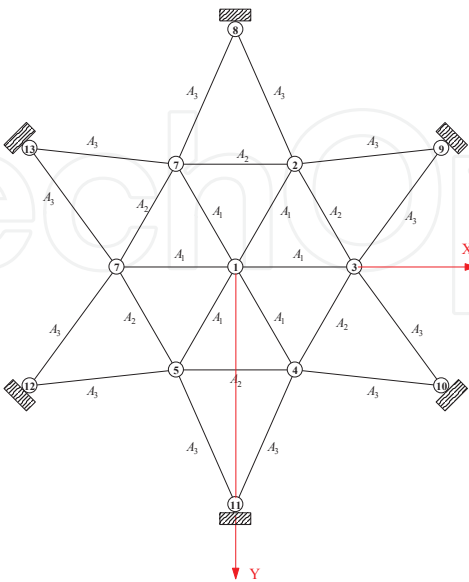**Table 8.** Initial data of 24-bar shallow space truss

Figure 9. The layout of the 24-bar truss dome

5. Conclusion

The weight minimization of the shallow truss structures is a challenging but sometimes frustrating engineering optimization problem. Theoretically, the optimal design searching process can be formulated as an implicit nonlinear mixed integer optimization problem with a huge number of variables. The flexibility of the shallow truss structures might causes different type of structural instability. According to the nonlinear behavior of the resulted lightweight truss structures, a special treatment is required in order to tackle the "hidden" global stability problems during the optimization process. Therefore, we have to replace the traditional "design variables → response variables" like approach with a more time-consuming "design variables → response functions" like approach, where the response functions describe the structural response history of the loading process up to the maximal load intensity without constraint violation.

In this study, a higher order path-following method was embedded into a hybrid heuristic optimization frame in order to tackle the global structural stability constraints within the truss optimization. The proposed path-following method is based on the perturbation technique of the stability theory and a non-linear modification of the classical linear homotopy method.

In this chapter we presented a simple but very efficient hybrid metaheuristic for truss weight minimization with continuous and discrete design variables, and local and global stability constraints. The presented "supernatural" ANGEL method combines ant colony optimization (AN), genetic algorithm (GE) and gradient-based local search (L) strategy. In the algorithm, AN and GE search alternately and cooperatively in the design space. The powerful L algorithm, which is based on the local linearization of the constraint set, is applied to yield a more feasible or less unfeasible solution, when AN or GE obtains a solution.

The highly nonlinear and non-convex large-span and large-scale shallow truss examples with continuous and discrete design variables and non-linear response curves show that ANGEL may be more efficient and robust than the conventional gradient based deterministic or the traditional population based heuristic (metaheuristic) methods in solving explicit (implicit) optimization problems. ANGEL produces highly competitive and from engineering point of view safe and accurate results in significantly shorter run-times than the previously described pure approaches. The benefit of synergy was demonstrated by standard statistical tests. To the best of our knowledge, no such work has been done in the literature for truss weight minimization with non-linear response curves so far.

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References

[1] Csébfalvi, A. (1998). A nonlinear path-following method for computing the equilibrium curve of structures. *Annals of Operations Research*, 15-23, 10.1023/A:1018944804979.

[2] Csébfalvi, A. (2007). Angel method for discrete optimization problems. *Periodica Polytechnica Civil Eng*, 51/2, 37-46, 10.3311/pp.ci.2007-2.06.

[3] Csébfalvi, A. (2007). Optimal design of frame structures with semi-rigid joints. *Periodica Polytechnica Civil Eng*, 51/1, 9-15, 10.3311/pp.ci.2007-1.02.

[4] Csébfalvi, A. (2009). A hybrid meta-heuristic method for continuous engineering optimization. *Periodica Polytechnica, Ser Civ Eng*, 53(2), 93-100, 10.3311/pp.ci.2009-2.05.

[5] Csébfalvi, A. (2011). Multiple constrained sizing-shaping truss-optimization using ANGEL method. *Periodica Polytechnica Civil Engineering*, 55/1, 81-6, 10.3311/pp.ci. 2011-1.10.

[6] Csébfalvi, A. (2012). Kolmogorov- Smirnov Test to Tackle Fair Comparison of Heuristic Approaches in Structural Optimization. *Int. J. Optim. Civil Eng*, 2(1), 135-150.

[7] Kaveh, A., & Talatahari, S. (2009). Particle swarm optimizer, ant colony strategy and harmony search schemehybridized for optimization of truss structures. *Computers and Structures*, 87, 267-283.

[8] Kaveh, A., & Talatahari, S. (2009). Size optimization of space trusses using Big Bang-Big Crunch algorithm. *Computers and Structures*, 87, 1129-1140.

[9] Kelesoglu, O., & Ülker, M. (2005). Fuzzy optimization geometrical nonlinear space truss design. *Turkish Journal of Engineering &Environmental Sciences*, 29, 321-329.

[10] Lee, K. S., & Geem, Z. W. (2004). A new structural optimization method based on the harmony search algorithm. *Computers and Structures*, 82, 781-98.

[11] Lamberti, L., & Pappalettere, C. (2004). Improved sequential linear programming formulation for structural weight minimization. *Comput. Methods in Appl. Mech. Engrg.*, 193, 3493-3521.

[12] Saka, M. P., & Ülker, M. (1992). Optimum design of geometrically non-linear space trusses. *Computers and Structures*, 42, 289-299.

[13] Sivaraj, R., & Ravichandran, T. (2011). A review of selection methods in genetic algorithm. *International Journal of Engineering Science and Technology (IJEST)*, 0975-5462, 3(5), May.

[14] Soh, C. K., & Yang, J. (1996). Fuzzy controlled genetic algorithm search for shape optimization. *Journal of Computing in Civil Engineering, ASCE*, 10(2), 143-50.

[15] Tseng, L. Y., & Chen, S. C. (2006). A hybrid metaheuristic for the resource-constrained project scheduling problem. *European Journal of Operational Research*, 175, 707-721.

[16] Csébfalvi, A. (2003). Optimal design of space structures with stability constraints. *Bontempi F (ed) System-Based Vision for Strategic and Creative Design, Vols 1-3: 2nd International Conference on Structural and Construction Engineering, September 23-26, Rome, Italy, Leiden:Balkema Publishers*, 493-497, 9-05809-599-1.

[17] Csébfalvi, A. (2010). A Higher-Order Path-Following Method for Stability-Constrained Optimization of Geometrically Nonlinear Shallow Trusses. *O Allix, P Wriggers (ed) ECCM 2010, IV European Conference on Computational Mechanics, Palais des Congrès, Paris, France, May 16-21,2010: European Committee on Computational Solids*, 2010.05.16-2010.05.21. , 1-7.

[18] Csébfalvi, A. An Improved ANGEL Algorithm for the Optimal Design of Shallow Truss Structures with Discrete Size and Continuous Shape Variables and Stability Constraints. *Erik Lund (ed) 9th World Congress on Structural and Multidisciplinary Optimization: WCSMO-9. Shizuoka, Japan*, Shizuoka:paper ID: 100_1, 1-8.

[19] Hadi, M. N. S., & Alvani, K. S. (2003). Discrete Optimum Design of Geometrically Non-Linear Trusses using Genetic Algorithms, Seventh International Conference on The Application of Artificial Intelligence toCivil and Structural Engineering. *B.H.V. Topping (Ed.), Civil-Comp Press, Stirling, Scotland*, paper 37.

[20] Hanahara, K., & Tada, Y. Global Buckling Has to be Taken into Account for Optimal Design of Large-Scale Truss Structure. *Erik Lund (ed) 9th World Congress on Structural and Multidisciplinary Optimization: WCSMO-9. Shizuoka, Japan*, Shizuoka:paper ID: 142_1, 1-10.

[21] Lemonge, A. C. C., Barbosa, H. J. C., Fonseca, L. G., & Coutinho, A. L. G. A. (2010). A genetic algorithm for topology optimization of dome structures. *Helder C. Rodrigues (ed)2nd International Conference on Engineering Optimization, September 6-9, Lisbon, Portugal*, paper ID: 01284, 1-15.