A Probabilistic Model of Rainfall-Induced Shallow Landslides

Ali Talebi

Faculty of Natural Resources, Yazd University, Yazd, Iran

Abstract

Shallow land sliding is a stochastic process, and understanding what controls the return period is crucial for risk assessment. In this paper, we present the new probabilistic model to describe the long-term evolution of colluvial deposits through a probabilistic soil mass balance at a point. Further building blocks of the model are: an infinite-slope stability analysis; a more realistic description of hollow hydrology (hillslope storage Boussinesq model, HSB); and a statistical model relating intensity, duration, and frequency of extreme precipitation. Long term analysis of shallow landslides by the presented model illustrates that all hollows show a quite different behavior from the stability view point. In hollows with more convergence, landslide occurrence is limited by the supply of deposits (supply limited regime) or rainfall events (event limited regime) while hollows with low convergence degree are unconditionally stable regardless of the soil thickness or rainfall intensity. Overall, our results show that in addition to the effect of slope angle, plan shape (convergence degree) also controls the subsurface flow and this process affects the probability distribution of landslide occurrence in different hollows.

Keywords: probabilistic model, shallow landslides, complex hollows

1. Introduction

Recently D'Odorico and Fagherazzi [1] have presented a probabilistic model of rainfall-triggered shallow landslides in hollows and showed that landslide frequency is linked to the rainfall intensity-duration-frequency characteristics of the region. They developed a stochastic model that computes the temporal evolution of regolith thickness in a hollow and hollow hydrologic response to rainfall based on a steady-state kinematic wave model for subsurface flow. In this research, we will use some elements of this model (stochastic soil mass balance) to simulate the soil production (colluvial deposit) and soil erosion (landslide) in time for hollows with complex shapes. Although our model is similar to that presented by D'Odorico and Fagherazzi [1] in that it is a probabilistic model of rainfall-induced shallow landslides, there is an important difference. Convergent plan shapes or concave profile curvatures cause the kinematic wave model to perform relatively poorly even in steep slopes (Hilberts et al., [2]). Troch et al. [3] observed that hillslope plan shape rather than mean bedrock slope angle determines the validity of the kinematic wave approximation to describe the subsurface flow process along complex hillslopes. Therefore, incorporating a more realistic description of hollow hydrology in the stochastic landslide model is needed, as hollows are generally convergent and hollows with more convergence have more potential for landslide occurrence.

To relax the KW assumptions, in this paper we substitute the linearized steady-state HSB model in the work of *D'Odorico and Fagherazzi* [1] for complex hollows (hollows with different length, slope angle and convergence degree). In fact, using an exponential width function, hollows with different convergence degree is presented and then for each hollow the critical soil depth, the minimum value of landslide-triggering saturated depth and the minimum rainfall intensity needed to trigger a landslide along hollow length are computed. Moreover, the temporal evolution of colluvium thickness is studied through a stochastic soil mass balance. Therefore, by considering the soil production function and hydrology condition in the different hollows, stability of each hollow is analyzed by the infinite slope stability method. Finally, the generalized model helps to investigate the relation between rainfall characteristics (intensity and duration), water table depth and slope stability of colluvial deposits in complex hollows.

2. Model formulation

2.1. Hollow geometry

We consider only hollows with moderate to steep slopes and shallow, permeable soils overlying a straight bedrock where subsurface storm flow is the dominant flow mechanism. Shallow soils are most prone to rain-induced landslides. It is assumed that the plan shape of the hollow can be described using an exponential width function:

$$w(x') = w_0 e^{ax'} \Rightarrow A(x') = \frac{w_0}{a}\left(e^{aL'} - e^{ax'}\right) \tag{1}$$

where w is the hollow width (deposits) along the X' direction, X' is the distance from the outlet of hollow parallel to bedrock), w_0 is the hollow width at the outlet, A is the hollow area, L' is the hollow length and a is a plan shape parameter. Allowing this plan shape parameter to assume either a positive, zero, or negative value, one can define several basic geometric relief forms: $a>0$ for convergent, $a<0$ for divergent and $a=0$ for parallel shapes. As hollows are generally convergent, we will assume a wide range of positive numbers for convergent hollows.

As the purpose of this study is to investigate the effect of hollow geometry and hydrology on landslide probability, we employ the subsurface flow similarity parameter for complex hollows proposed by *Berne et al.* [4] . This dimensionless parameter, the hillslope Péclet number, is defined for subsurface flow as the ratio between the characteristic diffusive time and the characteristic advective time, taken from the middle of the hillslope:

$$Pe = \left(\frac{L'}{2pD'}\right)\tan \mathrm{b} - \left(\frac{aL'}{2}\right) \tag{2}$$

where p is a linearization parameter, $D'$is the soil depth and β is the bedrock slope angle. As can be seen, Pe is a function of three independent dimensionless groups: $L'/(2pD')$, tan βand ; $aL'/2$; $L'/(2pD')$ represents the ratio of the half length and the average depth of the aquifer (related to the hollow hydrology), and tan β and $aL'/2$ define the hollow geometry.

2.2. Hollow stability Hollow stability

In this study the slope stability model is based on a Mohr-Coulomb failure law applied to an infinite planar slope. The failure condition can be expressed as (e.g. *Montgometry and Dietrich* [5], 1994; *D'Odorico and Fagherazzi*, [1]):

$$g_{sat} D' \sin b \;= c_t + \left(g_{sat} D' \cos b \;- g_w\, h' \cos b j \right) \tan \tag{3}$$

where γ_{sat} and γ_w are the specific weights of saturated soil and water respectively, β is the bedrock slope angle, ϕ is the soil repose angle, c_t is the soil cohesion and h′ is the saturated water depth, with both h′ and D′ (deposit thickness) being measured perpendicularly to the bedrock.

By solving Equation (3) for h′ the minimum value of landslide-triggering saturated depth (h_{cr}) can be obtained as (*D'Odorico and Fagherazzi*, [1]):

$$h_{cr} = \frac{g_{sat}}{g_w} D' \left(1 - \frac{\tan b}{\tan j} \right) + \frac{c_t}{g_w \tan j \;\; \cos b} \tag{4}$$

When the soil depth (D′) is equal to h_{cr}, the critical soil depth or immunity depth (D_{cr}) is given as follows

$$D_{cr} = \frac{c_t}{g_w \tan j \;\; \cos b + g_{sat} \cos b \left(\tan b - \tan j \right)} \tag{5}$$

In the case of relatively steep slopes (β>ϕ), h_{cr} decreases linearly (i.e. stability decreases) with an increase of soil depth D′ (see Equation (4)). The soil depth D_{max} for which shallow landsliding can occur without saturated throughflow (corresponding to h_{cr}=0) is (*Iida*, [6]):

$$D_{max} = \frac{c_t}{g_{sat} \cos b \left(\tan b - \tan j \;\; \right)} \tag{6}$$

2.3. Hollow Hydrology

Hillslope hydrological response has traditionally been studied by means of hydraulic groundwater theory (*Troch et al.*, [3]). In many regions, groundwater flow is the main source of streamflow between rainfall events. The basic macroscopic equation describing the movement of water in the soil is known as the three-dimensional Richards' equation.

Troch et al. [3]) reformulated the continuity and Darcy equations in terms of storage along the hillslope, which leads to the hillslope storage Boussinesq (HSB) equation for subsurface flow in hillslopes. Extending Brutsaert's [7]) analysis, they linearized this equation as:

$$\frac{\partial S'}{\partial t} = K \frac{\partial^2 S'}{\partial x'^2} + U \frac{\partial S'}{\partial x'} + \; Nw \tag{7}$$

with $K = \frac{k_s pD' \cos b}{f}$ and $U = \frac{k_s \sin b}{f} - aK$ where S′ is the subsurface saturated storage, N is the recharge to the ground water table, k_s is the saturated hydraulic conductivity and f is the drainable porosity (note that the value of p is determined iteratively as pD′ should be equal to the average water table height $\int_0^{L'} S'(x')dx'/(Af)$ where A is the hollow drainage area).

According to the definition of the storage S′, the mean groundwater table height (over the hill-slope width) is:

$$\overline{h}'(x') = \frac{S'(x')}{fw(x')} = \frac{Ne^{-ax'}}{af}\left[\frac{e^{aL'}}{U}\left(1 - e^{\frac{U}{K}x'}\right) + \frac{1}{(Ka+U)}\left(e^{\frac{U}{K}x'} - e^{ax'}\right)\right] \tag{8}$$

Again, for parallel hillslopes this reduces to:

Now, we can obtain the maximum groundwater table depth in each hillslope (which is critical for landslide occurrence):

$$\overline{h}'(x'_m) = \frac{N}{fa(aK+U)}\left\{e^{aL'}\left[1 + \frac{U}{aK}\left(1 - e^{-aL'}\right)\right]^{-\frac{aK}{U}} - 1\right\} \tag{9}$$

Equating $\overline{h'}(x'_m)$ and h_{cr}, the critical rainfall intensity for triggering landslides (R_{cr}) can now be calculated as:

$$R_{cr} = \frac{h_{cr} fa\,(aK+U)}{\left\{e^{aL'}\left[1 + \frac{U}{aK}\left(1 - e^{-aL'}\right)\right]^{-\frac{aK}{U}} - 1\right\}} \tag{10}$$

3. Results and discussion

Figure 1 shows long term simulations of deposit thickness evolution in the four hollows (from top to bottom) and illustrates how shallow landsliding occurs when the soil thickness (D′) ranges between D_{cr} and D_{max}. The left figure shows the time series of deposit thickness for the HSB model. As can be seen, this figure1 (left) indicates how, as a function of the hollow geometry from steep slopes (top) to gentle slopes (bottom), the landslide probability is changed as well. For instance in hollow (where T_r>>T_{im}), landslides never occur and the system can be termed "unconditionally-stable".

Figure 1 (right) illustrates the probability distribution of colluvium thickness when a landslide occurs as simulated by the HSB model (right column) (D_{slide}). This histogram shows that the different hollows have different distributions of scar depth. As can be seen, the probability of distribution of D_{slide} is concentrated close to the immunity depth (D_{cr}) for the supply-limited case, whereas it is concentrated at significantly larger depths for the event-limited cases.

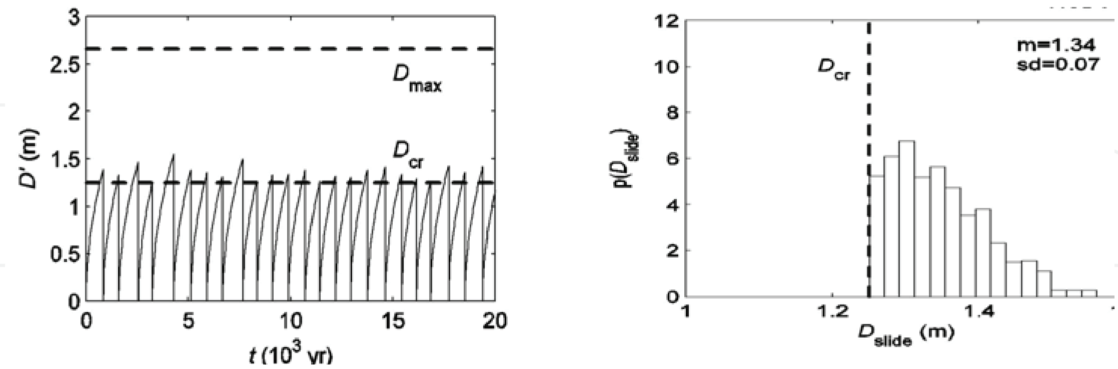

Figure 1. Long term simulation of deposit thickness (left) and Probability distribution of scar depth (colluvium thickness when a landslide occurs) for hollows (right).

Figures 2 (left) and (right) indicate how the probability distributions of the interarrival of the landslide-producing rain events (T_{slide}) and the corresponding rainfall intensities (R_{slide}) vary for the different hollows. These results show that in hollow (which has less convergence and a larger area), T_{slide} is close to T_{im} (supply limited regime). , while in other hollows (which has more convergence and a smaller area), TS_{lide} moves in the direction of T_r (event limited regime).

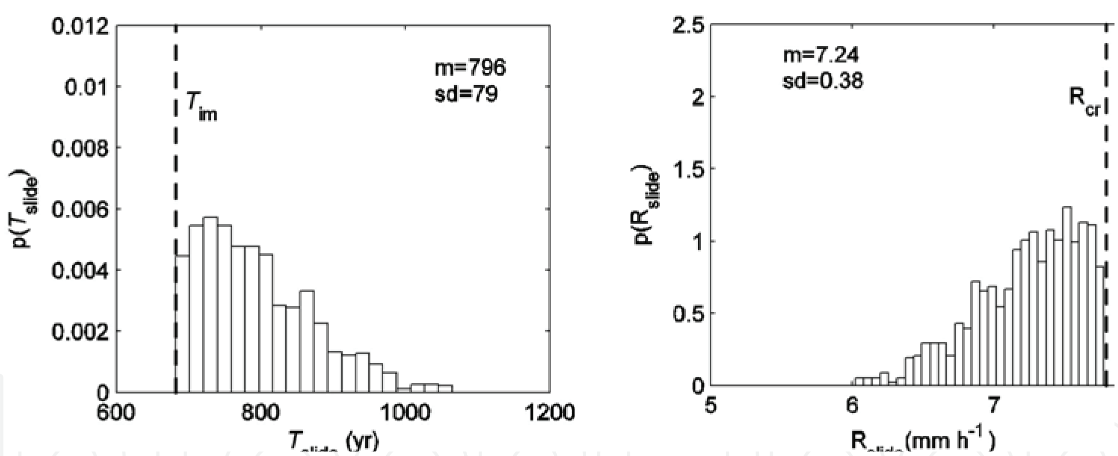

Figure 2. Probability distribution of landslide return period (left) and probability distribution of the landslide triggering rainfall intensity for hollows.

4. Conclusions

The following conclusions can be drawn from our rainfall-induced landslide stability analysis in response to deposit thickness evolution in complex hollows:

(i) Although shallow landslides in hollows are mainly triggered by high rainfall intensities, deposit thickness also plays an important role in stability.

(ii) With other site variables constant, shallow landslides usually occur when the soil depth (deposits thickness) is between D_{cr} and D_{max}.

(iii) Given a deposit thickness, for each hollow there exists a critical rainfall intensity leading to the highest water table and subsequent landslide occurrence.

(iv) In general, when convergence degree of hollows increases, the time period between land slides (T_{slide}) decreases. This means that hollows with more convergence degree are generally more susceptible to landsliding.

(v) In addition to the effect of slope angle, plan shape also controls the subsurface flow and this process affects the probability distribution of landslide occurrence in complex hollows and should be considered in hollow stability analysis.

5. References

[1] D'Odorico, P., and S. Fagherazzi (2003), A probabilistic model of rainfall-triggered shallow landslides in hollows: A long-term analysis, Water Resour. Res., 39: 1262. doi: 10.1029/2002WR001595.

[2] Hilberts, A., E. Van Loon, P. A. Troch and C. Paniconi (2004), The hillslope-storage Boussinesq model for non-constant bedrock slope, J. Hydrol., 291, 160-173.

[3] Troch, P. A., C. Paniconi and E. Van Loon (2003), Hillslope-storage Boussinesq model for subsurface flow and variable source areas along complex hillslopes: 1-Formulation and characteristic response, Water Resour. Res., 39(11):1316, doi: 10.1029/2002WR001728.

[4] Berne, A., R. Uijlenhoet and P. A. Troch (2005), Similarity analysis of subsurface flow response of hillslopes with complex geometry, Water Resour. Res., 41, W09410, doi:10.1029/2004WR003629.

[5] Montgomery, D. R., and W. E. Dietrich (1994), A physically based model for the topographic control on shallow landsliding, Water Resour. Res., 30, 1153-1171.

[6] Iida, T. (1999), A stochastic hydro-geomorphological model for shallow landsliding due to rainstorm. CATENA, 34, 293-313.

[7] Brutsaert, W., (2005), Hydrology: An Introduction, Cambridge University Press.