Information Capacity of Quantum Transfer Channels and Thermodynamic Analogies

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1. Introduction

We will begin with a simple type of *stationary stochastic*[1] systems of quantum physics using them within a frame of the **Shannon** Information Theory and Thermodynamics but starting with their *algebraic representation*. Based on this algebraic description a model of information transmission in those systems by defining the Shannon information will be stated in terms of variable about the system state. Measuring on these system is then defined as a spectral decomposition of measured quantities - *operators*. The information capacity formulas, now of the *narrow-band* nature, are derived consequently, for the simple system governed by the **Bose–Einstein** (B–E) Law [bosonic (photonic) channel] and that one governed by the **Fermi-Dirac** (F–D) Law [fermionic (electron) channel]. *The not-zero value for the average input energy needed for information transmission existence in F–D systems* is stated [11, 12].

Further the *wide–band* information capacity formulas for B–E and F–D case are stated. Also the original *thermodynamic* capacity derivation for the wide–band photonic channel as it was stated by **Lebedev–Levitin** in 1966 is revised. This revision is motivated by apparent relationship between the B–E (photonic) wide–band information capacity and the *heat efficiency* for a certain *heat cycle*, being further considered as the demonstrating model for processes of information transfer in the original wide–band photonic channel. The information characteristics of a *model reverse* heat cycle and, by this model are analyzed, the information arrangement of which is set up to be most analogous to the structure of the photonic channel considered, we see the necessity of returning the transfer medium (the channel itself) to its initial state as a condition for a *sustain, repeatable* transfer. It is not regarded in [12, 30] where a single information transfer act only is considered. Or the return is

[1] We deal with such a system which is taking on at time $t = 0, 1, \dots$ states $\boldsymbol{\theta}_t$ from a state space $\boldsymbol{\Theta}$. If for any t_0 the relative frequencies I_B of events $B \subset \boldsymbol{\Theta}$ is valid that $\frac{1}{T} \sum_{t=t_0+1}^{t_0+T} I_B(\boldsymbol{\theta}_t)$ tends for $T \to \infty$ to probabilities $p_{t_0}(B)$ we speak about a *stochastic system*. If these probabilities do not depend on the beginning t_0, a *stationary stochastic system* is spoken about.

regarded, but by opening the whole transfer chain for covering these *return energy needs from its environment* not counting them in - *not within the transfer chain only* as we do now. *The result is the corrected capacity formula for wide–band photonic (B–E) channels being used for an information transfer organized cyclicaly.*

2. Information transfer channel

An *information, transfer* channel $\mathcal{K}$ is defined as an arranged *tri–partite* structure [5]

$$\mathcal{K} \stackrel{\text{Def}}{=} [X,\ \varepsilon,\ Y] \text{ where } X \stackrel{\text{Def}}{=} [A,\ p_X(\cdot)],\ Y \stackrel{\text{Def}}{=} [B,\ p_Y(\cdot)] \text{ and } \tag{1}$$

- X is an *input* stochastic quantity, a *source* of *input* messages $a \in A^+ \stackrel{\triangle}{=} \mathcal{X}$, a *transceiver*,
- Y is an *output* stochastic quantity, a *source* of *output* messages $b \in B^+ \stackrel{\triangle}{=} \mathcal{Y}$, a *receiver*,
- output messages $b \in \mathcal{Y}$ are *stochastic* dependent on input messages $a \in \mathcal{X}$ and they are received by the receiver of messages, Y,
- ε is the *maximal* probability of an error in the transfer of any symbol $x \in A$ in an input message $a \in \mathcal{X}$
[the maximal probability of erroneously receiving $y \in B$ (inappropriate for x) in an output message $b \in \mathcal{Y}$],
- A denotes a finite *alphabet* of elements x of the source of input messages,
- B denotes a finite alphabet of elements y of the source of output messages,
- $p_X(\cdot)$ is the *probability distribution* of evidence of any symbol $x \in A$ in an input message,
- $p_Y(\cdot)$ is the probability distribution of evidence of any symbol $y \in B$ in an output message.

The structure $(X,\ \mathcal{K},\ Y)$ or $(\mathcal{X},\ \mathcal{K},\ \mathcal{Y})$ is termed a *transfer (Shannon) chain*. The symbols $H(X)$ and $H(Y)$ respectively denote the *input information (Shannon) entropy* and the *output information (Shannon) entropy* of channel $\mathcal{K}$, *discrete* for this while,

$$H(X) \stackrel{\text{Def}}{=} -\sum_{x \in A} p_X(x)\ \ln p_X(x),\quad H(Y) \stackrel{\text{Def}}{=} -\sum_{y \in B} p_Y(y)\ \ln p_Y(y) \tag{2}$$

The symbol $H(X|Y)$ denotes the *loss entropy* and the symbol $H(Y|X)$ denotes the *noise entropy* of channel $\mathcal{K}$. These entropies are defined as follows,

$$H(X|Y) \stackrel{\text{Def}}{=} -\sum_A \sum_B p_{X,Y}(x,y)\ \ln p_{X|Y}(x|y),\quad H(Y|X) \stackrel{\text{Def}}{=} -\sum_A \sum_B p_{X,Y}(x,y) \ln p_{Y|X}(y|x) \tag{3}$$

where the symbol $p_{\cdot|\cdot}(\cdot|\cdot)$ denotes the *condition* and the symbol $p_{\cdot,\cdot}(\cdot,\cdot)$ denotes the *simultaneous* probabilities. For *mutual (transferred) usable* information, *transinformation* $T(X;Y)$ or $T(Y;X)$ is valid that

$$T(X;Y) = H(X) - H(X|Y) \text{ and } T(Y;X) = H(Y) - H(Y|X) \tag{4}$$

From (2) and (3), together with the definitions of $p_{\cdot|\cdot}(\cdot|\cdot)$ and $p_{\cdot,\cdot}(\cdot,\cdot)$, is provable prove that the transinformation is symmetric. Then the *equation of entropy (information) conservation* is valid

$$H(X) - H(X|Y) = H(Y) - H(Y|X) \tag{5}$$

The *information capacity* of the channel $\mathcal{K}$ (both discrete an continuous) is defined by the equation

$$C \stackrel{\text{Def}}{=} \sup I(X;Y) \tag{6}$$

over all possible probability distributions $q(\cdot)$, $p(\cdot)$.It is the maximum (supremum) of the medium value of the usable amount of information about the input message x within the *output* message y.

Remark: For continuous distributions (densities) $p_{[\cdot]}(\cdot)$, $p_{[\cdot|\cdot]}(\cdot|\cdot)$ on intervals $\mathcal{X}$, $\mathcal{Y} \in \mathbb{R}$, $x \in \mathcal{X}, y \in \mathcal{Y}$ is

$$H(X) = -\int_{\mathcal{X}} p_X(x) \ln p_X \, \mathrm{d}x, \tag{7}$$

$$H(Y) = -\int_{\mathcal{Y}} p_Y(y) \ln p_Y(y) \, \mathrm{d}y \tag{8}$$

$$H(X|Y) = -\int_{\mathcal{X}} \int_{\mathcal{Y}} p_{X,Y}(x,y) \ln p_{X|Y}(x|y) \, \mathrm{d}x\mathrm{d}y,$$

$$H(Y|X) = -\int_{\mathcal{X}} \int_{\mathcal{Y}} p_{X,Y}(x,y) \ln p(y|x) \, \mathrm{d}x\mathrm{d}y$$

Equations (4), (5 are valid for both the quantities $H(\cdot)$ and $H(\cdot|\cdot)$, as well as for their respective changes $\Delta H(\cdot)[= H(\cdot)]$ and $\Delta H(\cdot|\cdot)[= H(\cdot|\cdot)]$.

3. Representation of physical transfer channels

The most simple way of description of *stationary* physical systems is an *eucleidian* space Ψ of their states expressed as *linear operators*.[2] This way enables the *mathematical* formulation of the term (*physical*) *state* and, generally, the term (*physical*) *quantity*.

Physical quantities $\boldsymbol{\alpha}$, associated with a physical system $\boldsymbol{\Psi}$ represented by the Eucleidian space Ψ are expressed by *symmetric* operators from the linear space $L(\Psi)$ of operators on Ψ, $\boldsymbol{\alpha} \in \mathbf{A} \subset L(\Psi)$ [7]. The supposition is that any physical quantity can achieve only those *real* values α which are the *eigenvalues* of the associated *symmetric operator* $\boldsymbol{\alpha}$ (symmetric *matrix* $[\alpha_{i,j}]_{n,n}$, $n = \dim \Psi$). They are elements of the *spectrum* $\mathbf{S}(\boldsymbol{\alpha})$ of the *operator* $\boldsymbol{\alpha}$. The eigenvalues $\alpha \in \mathbf{S}(\boldsymbol{\alpha}) \subset \mathbb{R}$ of the quantity $\boldsymbol{\alpha}$ being measured on the system $\boldsymbol{\Psi} \cong \Psi$ depend on the (inner) states $\boldsymbol{\theta}$ of this system $\boldsymbol{\Psi}$.

- The *pure states* of the system $\boldsymbol{\Psi} \cong \Psi$ are represented by *eigenvectors* $\psi \in \Psi$. It is valid that the *scalar project* $(\psi, \psi) = 1$; in *quantum physics* they are called *normalized wave functions*.
- The *mixed states* are nonnegative quantities $\boldsymbol{\theta} \in \mathbf{A}$; their *trace* [of an *square* matrix (operator) $\boldsymbol{\alpha}$] is defined

$$\mathrm{Tr}(\boldsymbol{\alpha}) \stackrel{\text{Def}}{=} \sum_{i=1}^{n} \alpha_{i,i} \quad \text{and, for } \boldsymbol{\alpha} \equiv \boldsymbol{\theta} \text{ is valid that } \mathrm{Tr}(\boldsymbol{\theta}) = 1. \tag{9}$$

The *symmetric projector* $\boldsymbol{\pi}\{\psi\} = \boldsymbol{\pi}[\Psi(\{\psi\})]$ (orthogonal) on the one–dimensional *subspace* $\Psi(\{\psi\})$ of the space Ψ is nonnegative quantity for which $\mathrm{Tr}(\boldsymbol{\pi}\{\psi\}) = 1$ is valid. The projector $\boldsymbol{\pi}\{\psi\}$ *represents* [on the set of quantities $\mathbf{A} \subset L(\Psi)$] the pure state ψ of the system $\boldsymbol{\Psi}$. Thus, an arbitrary state of the system $\boldsymbol{\Psi}$ can be defined as a nonnegative quantity $\boldsymbol{\theta} \in \boldsymbol{\Theta} \subset \mathbf{A}$ for which $\mathrm{Tr}(\boldsymbol{\theta}) = 1$ is valid. For the *pure state* $\boldsymbol{\theta}$ is then valid that $\boldsymbol{\theta}^2 = \boldsymbol{\theta}$ and the *state space* $\boldsymbol{\Theta}$ of the system $\boldsymbol{\Psi}$ is defined as the set of all states $\boldsymbol{\theta}$ of the system $\boldsymbol{\Psi}$.

[2] The motivation is the *axiomatic theory of algebraic representation of physical systems* [7].

3.1. Probabilities and information on physical systems

3.1.0.1. Theorem:

For any state $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ the pure states $\boldsymbol{\theta}_i = \boldsymbol{\pi}\{\psi_i\}$ and numbers $q(i|\boldsymbol{\theta}) \geq 0$ exists, that

$$q(i|\boldsymbol{\theta}) \geq 0, \ \boldsymbol{\theta} = \sum_{i=1}^{n} q(i|\boldsymbol{\theta})\, \boldsymbol{\theta}_i \quad \text{where} \quad \sum_{i=1}^{n} q(i|\boldsymbol{\theta}) = 1 \tag{10}$$

3.1.0.2. Proof:

Let $D(\boldsymbol{\theta}) = \{D_\theta : \theta \in \mathbf{S}(\boldsymbol{\theta})\}$ is a *disjoint decomposition* of the set $\{1, 2, ..., n\}$ of indexes of the base $\{\psi_1, \psi_2, ..., \psi_n\}$ of Ψ. The set $\{\psi_i : i \in D_\theta\}$ is an orthogonal basis of the *eigenspace* $\Psi(\theta|\boldsymbol{\theta}) \subset \Psi$ of the operator $\boldsymbol{\theta}$ [for its eigenvalue $\theta \in \mathbf{S}(\boldsymbol{\theta})$]. Then

$$\text{card}\, D_\theta = \dim \Psi(\theta|\boldsymbol{\theta}) \geq 1 \ \text{ and } \ \boldsymbol{\pi}[\Psi(\theta|\boldsymbol{\theta})] = \sum_{i \in D_\theta} \boldsymbol{\pi}\{\psi_i\}, \ \ \forall \theta \in \mathbf{S}(\boldsymbol{\theta}) \tag{11}$$

Let $q(i|\boldsymbol{\theta}) = \theta$ is taken for all $i \in D_\theta$, $\theta \in \mathbf{S}(\boldsymbol{\theta})$. By the *spectral decomposition theorem* [9],

$$\boldsymbol{\theta} = \sum_{\theta \in \mathbf{S}(\boldsymbol{\theta})} \theta\, \boldsymbol{\pi}_\theta = \sum_{\theta \in \mathbf{S}(\boldsymbol{\theta})} \theta \boldsymbol{\pi}[\Psi(\theta|\boldsymbol{\theta})] = \sum_{\theta \in \mathbf{S}(\boldsymbol{\theta})} \theta \sum_{i \in D_\theta} \boldsymbol{\pi}\{\psi_i\} = \sum_{i=1}^{n} q(i|\boldsymbol{\theta})\, \boldsymbol{\pi}\{\psi_i\} \tag{12}$$

$$\text{Tr}(\boldsymbol{\theta}) = \sum_{i=1}^{n} q(i|\boldsymbol{\theta}) \cdot \text{Tr}(\boldsymbol{\pi}\{\psi_i\}) = \sum_{i=1}^{n} q(i|\boldsymbol{\theta}) = 1$$

The symbol $q(\cdot|\boldsymbol{\theta})$ denotes the *probability distribution* into *pure, canonic components* $\boldsymbol{\theta}_i$ of $\boldsymbol{\theta}$ is called:

- *canonic distribution (q-distribution)* of the state $\boldsymbol{\theta} \in \boldsymbol{\Theta}$.[3]

Further the two distribution defined on spectras of $\boldsymbol{\alpha} \in \mathbf{A}$ and $\boldsymbol{\theta} \in \boldsymbol{\Theta} \subset \mathbf{A}$ will be dealt:

- *dimensional distribution (d-distribution)* of the state $\boldsymbol{\theta} \in \boldsymbol{\Theta}$[4]

$$d(\theta|\boldsymbol{\theta}) \overset{\text{Def}}{=} \frac{\dim \Psi(\theta|\boldsymbol{\theta})}{\dim \Psi} = \frac{\dim \Psi(\theta|\boldsymbol{\theta})}{n}, \quad \theta \in \mathbf{S}(\boldsymbol{\theta}) \tag{13}$$

- *distribution of measuring (p-distribution)* of the quantity $\boldsymbol{\alpha} \in \mathbf{A}$ in the state $\boldsymbol{\theta} \in \boldsymbol{\Theta}$,

$$p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}) \overset{\text{Def}}{=} \text{Tr}(\boldsymbol{\theta}\boldsymbol{\pi}_\alpha), \quad \alpha \in \mathbf{S}(\boldsymbol{\alpha}) \tag{14}$$

where $\{\boldsymbol{\pi}_\alpha : \alpha \in \mathbf{S}(\boldsymbol{\alpha})\}$ is the *spectral decomposition* of the *unit operator* $\mathbf{1}$. Due the nonnegativity of $\boldsymbol{\theta}\boldsymbol{\pi}_\alpha$ is $p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}) = \text{Tr}(\boldsymbol{\alpha}\boldsymbol{\pi}_\alpha) \geq 0$. By spectral decomposition of $\mathbf{1}$ and by definition of the *trace* $\text{Tr}(\cdot)$ is

$$\sum_{\alpha \in \mathbf{S}(\boldsymbol{\alpha})} p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}) = \text{Tr}\left(\boldsymbol{\theta} \sum_{\alpha \in \mathbf{S}(\boldsymbol{\alpha})} \boldsymbol{\pi}_\alpha\right) = \text{Tr}(\boldsymbol{\theta}\mathbf{1}) = 1 \tag{15}$$

Thus the relation (14) defines the probability distribution on the spectrum of the operator $\mathbf{S}(\boldsymbol{\alpha})$.

[3] Or, the *system spectral distribution* SSD [11].

[4] Or, the *system distribution* of the system *spectral dimension* SDSD [11].

- The special case of the *p-distribution* is that *for measuring values of* $\boldsymbol{\alpha}\equiv\boldsymbol{\theta}$, $\boldsymbol{\theta}\in\boldsymbol{\Theta}$,

$$p(\theta|\boldsymbol{\theta}|\boldsymbol{\theta}) = \mathrm{Tr}(\boldsymbol{\theta}\boldsymbol{\pi}_\theta) = \theta \dim \Psi(\theta|\boldsymbol{\theta}), \quad \theta \in \mathbf{S}(\boldsymbol{\theta}) \tag{16}$$

It is the case of *measuring the* $\boldsymbol{\theta}$ *itself*. Thus, by the equation (14), the term ***measuring*** **means just the spectral decomposition of the unit operator 1 within the spectral relation with** $\boldsymbol{\alpha}$ [4, 20]. The act of measuring of the quantity $\boldsymbol{\alpha}\in\mathbf{A}$ in (the system) state $\boldsymbol{\theta}$ gives the value α from the spectrum $\mathbf{S}(\boldsymbol{\alpha})$ with the probability $p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}) = \mathrm{Tr}(\boldsymbol{\theta}\boldsymbol{\pi}_\alpha)$ given by the p-distribution,

$$\sum_{\mathcal{S}(\alpha)} p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}) = \sum_{\mathcal{S}(\alpha)} \mathrm{Tr}(\boldsymbol{\theta}\boldsymbol{\pi}_\alpha) = 1 \tag{17}$$

The ***measured*** **quantity** $\boldsymbol{\alpha}$ **in the state** $\boldsymbol{\theta}$ **is a *stochastic* quantity with its values [occuring with probabilities** $p(\cdot|\boldsymbol{\alpha}|\boldsymbol{\theta})$**] from its spectrum** $\mathbf{S}(\boldsymbol{\alpha})$. For its *mathematical expectation, medium value* is valid

$$E(\boldsymbol{\alpha}) = \sum_{\alpha\in\mathbf{S}(\boldsymbol{\alpha})} \alpha\,\mathrm{Tr}(\boldsymbol{\theta}\boldsymbol{\pi}_\alpha) = \mathrm{Tr}\left(\boldsymbol{\theta}\sum_{\alpha\in\mathbf{S}(\boldsymbol{\alpha})}\alpha\boldsymbol{\pi}_\alpha\right) = \mathrm{Tr}(\boldsymbol{\theta}\boldsymbol{\alpha}) = (\boldsymbol{\alpha}\psi,\psi) \tag{18}$$

Nevertheless, in the pure state $\boldsymbol{\theta} = \boldsymbol{\pi}\{\psi\}$ the values $i = \alpha \in \mathbf{S}(\boldsymbol{\alpha})$ are measured, $\mathrm{Tr}(\boldsymbol{\theta}_i\boldsymbol{\alpha}) = (\boldsymbol{\alpha}\psi_i,\psi_i) = \alpha_{ii}$.
Let $\boldsymbol{\Psi}$ is an *arbitrary* stationary physical system and $\boldsymbol{\theta}\in\boldsymbol{\Theta}\subset\mathbf{A}$ is its arbitrary state. The *physical entropy* $\mathcal{H}(\boldsymbol{\theta})$ of the system $\boldsymbol{\Psi}$ *in the state* $\boldsymbol{\theta}$ is defined by the equality

$$\mathcal{H}(\boldsymbol{\theta}) \stackrel{\mathrm{Def}}{=} -\mathrm{Tr}(\boldsymbol{\theta}\ln\boldsymbol{\theta}) \tag{19}$$

When $\{\boldsymbol{\pi}_\theta : \theta\in\mathbf{S}(\boldsymbol{\theta})\}$ is the decomposition of $\mathbf{1}$ spectral equivalent with $\boldsymbol{\theta}$, then it is valid that

$$\boldsymbol{\theta}\ln\boldsymbol{\theta} = \sum_{\theta\in\mathbf{S}(\boldsymbol{\theta})} \theta\ln\theta\,\boldsymbol{\pi}_\theta \ \text{ and }\ \mathcal{H}(\boldsymbol{\theta}) = -\sum_{\theta\in\mathbf{S}(\boldsymbol{\theta})} \theta\ln\theta\cdot\dim\Psi(\theta|\boldsymbol{\theta}) \tag{20}$$

3.1.0.3. Theorem:

For a physical system $\boldsymbol{\Psi}$ in any state $\boldsymbol{\theta}\in\boldsymbol{\Theta}$ is valid that

$$\begin{aligned}\mathcal{H}(\boldsymbol{\theta}) &= -\sum_{i=1}^{n} q(i|\boldsymbol{\theta})\cdot\ln q(i|\boldsymbol{\theta}) = H[q(\cdot|\boldsymbol{\theta})] = \ln n - \sum_{\theta\in\mathbf{S}(\boldsymbol{\theta})} p(\theta|\boldsymbol{\theta}|\boldsymbol{\theta})\cdot\ln\frac{p(\theta|\boldsymbol{\theta}|\boldsymbol{\theta})}{d(\theta|\boldsymbol{\theta})}\\ &= \ln n - I[p(\cdot|\boldsymbol{\theta}|\boldsymbol{\theta})\,\|\,d(\cdot|\boldsymbol{\theta})]\end{aligned} \tag{21}$$

where $H(\cdot)$ is the *Shannon entropy*, $I(\cdot\|\cdot)$ is the *information divergence*, $p(\cdot|\boldsymbol{\theta}|\boldsymbol{\theta})$ is the p-distribution for the state $\boldsymbol{\theta}$, $q(\cdot|\boldsymbol{\theta})$ is the q-distribution for the state $\boldsymbol{\theta}$ and $d(\cdot|\boldsymbol{\theta})$ is the d-distribution for the state $\boldsymbol{\theta}$, $n = \dim\Psi$.

3.1.0.4. Proof:

The relations in (21) follows from (20) and from definition (10) of the distribution $q(\cdot|\boldsymbol{\theta})$. From definitions (13) and (16) of the other two distributions follows that [12, 38]

$$\begin{aligned}I[p(\cdot|\boldsymbol{\theta}|\boldsymbol{\theta})\,\|\,d(\cdot|\boldsymbol{\theta})] &= \sum_{\theta\in\mathbf{S}(\boldsymbol{\theta})} \theta\dim\Psi(\theta|\boldsymbol{\theta})\cdot\ln\frac{n\theta\cdot\dim\Psi(\theta|\boldsymbol{\theta})}{\dim\Psi(\theta|\boldsymbol{\theta})}\\ &= \sum_{\theta\in\mathbf{S}(\boldsymbol{\theta})} \theta\dim\Psi(\theta|\boldsymbol{\theta})\cdot\ln n - \mathcal{H}(\boldsymbol{\theta}) = \ln n - \mathcal{H}(\boldsymbol{\theta})\end{aligned} \tag{22}$$

Due to the quantities X, Y, $X|Y$, $Y|X$ describing the information transfer are in our algebraic description denoted as follows, $X \triangleq \boldsymbol{\theta},\ Y \triangleq \boldsymbol{\alpha}$ or $Y \triangleq (\boldsymbol{\alpha}\|\boldsymbol{\theta})$, $(X|Y) \triangleq (\boldsymbol{\theta}|\boldsymbol{\alpha}),\ (Y|X) \triangleq (\boldsymbol{\alpha}|\boldsymbol{\theta})$. The laws of information transfer are writable in this way too:

$$C = \sup_{\boldsymbol{\alpha},\boldsymbol{\theta}} I(\boldsymbol{\alpha};\boldsymbol{\theta}), \quad I(\cdot||\cdot) \equiv T(\cdot;\cdot) \tag{23}$$

$$I(\boldsymbol{\theta};\boldsymbol{\alpha}) = I(\boldsymbol{\alpha};\boldsymbol{\theta}) = \mathcal{H}(\boldsymbol{\theta}) - H(\boldsymbol{\theta}|\boldsymbol{\alpha}) = H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) - H(\boldsymbol{\alpha}|\boldsymbol{\theta}) = \mathcal{H}(\boldsymbol{\theta}) + H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) - H(\boldsymbol{\theta},\boldsymbol{\alpha})$$

$$H(\boldsymbol{\theta},\boldsymbol{\alpha}) = H(\boldsymbol{\alpha},\boldsymbol{\theta}) = \mathcal{H}(\boldsymbol{\theta}) + H(\boldsymbol{\alpha}|\boldsymbol{\theta}) = H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) + H(\boldsymbol{\theta}|\boldsymbol{\alpha}) \tag{24}$$

4. Narrow-band quantum transfer channels

Let the symmetric operator $\boldsymbol{\varepsilon}$ of energy of quantum particle is considered, the spectrum of which eigenvalues ε_i is $\mathbf{S}(\boldsymbol{\varepsilon})$. Now the *equidistant* energy levels are supposed. In a pure state $\boldsymbol{\theta}_i$ of the measured (observed) system $\boldsymbol{\Psi}$ the eigenvalue $\varepsilon_i = i{\cdot}\varepsilon,\ \varepsilon > 0$. Further, the *output* quantity $\boldsymbol{\alpha}$ of the *observed* system $\boldsymbol{\Psi}$ is supposed (the system is *cell of the phase space B–E or F–D*) with the spectrum of eigenvalues $\mathbf{S}(\boldsymbol{\alpha}) = \{\alpha_0,\ \alpha_1,\ ...,\ \}$ being measured with probability distribution $\Pr(\cdot) = \{p(0),\ p(1),\ p(2),\ ...\}$

$$p(\alpha_k|\boldsymbol{\alpha}|\boldsymbol{\theta}_i) = \begin{cases} p(k-i) \text{ pro } k \geq i \\ 0 \qquad\qquad\ k < i \end{cases} \tag{25}$$

Such a situation arises when a particle with energy ε_i is *excited by an impact* from the output environment. The jump of energy level of the impacted particle is from ε_i up to $\varepsilon_{i+j},\ i+j=k$. The output ε_{i+j} for the excited particle is measured (it is the value on the output of the channel $\mathcal{K} \cong \boldsymbol{\Psi}$. This transition j occurs with the probability distribution

$$\Pr(j),\ j \in \{0,\ 1,\ 2,\ ...\} \tag{26}$$

Let be considered the *narrow–band* systems (with one *constant* level of a particle energy) $\boldsymbol{\Psi}$ of B–E or F–D type [27] (denoted further by $\boldsymbol{\Psi}_{\mathrm{B-E},\varepsilon}$, $\boldsymbol{\Psi}_{\mathrm{F-D},\varepsilon}$).

- In the B–E system, *bosonic*, e.g. the *photonic gas* the B–E distribution is valid

$$\Pr(j) = (1-p)\cdot p^j, \quad j \in \{0,\ 1,\ ...\},\ p \in (0,1),\ \ p^{-\frac{\varepsilon}{\mathrm{k}\Theta}} \tag{27}$$

- In the F–D system, *fermionic*, e.g. *electron gas* the F–D distribution is valid

$$\Pr(j) = \frac{p^j}{1+p}, \quad j \in \{0,1\},\ p \in (0,1),\ \ p^{-\frac{\varepsilon}{\mathrm{k}\Theta}} \tag{28}$$

where parameter p is variable with *absolute temperature* $\Theta > 0$; k is the **Boltzman** constant.

Also a collision with a bundle of j particles with constant energies ε of each and absorbing the energy $j \cdot \varepsilon$ of the bundle is considerable. E.g., by $\boldsymbol{\Psi}$ (e.g. $\boldsymbol{\Psi}_{\mathrm{B-E},\varepsilon}$ is the photonic gas) the monochromatic impulses with amplitudes $i \in \mathcal{S}$ are transferred, nevertheless generated from the environment of the same type but at the temperature T_W, $T_W > T_0$ where T_0 is the temperature of the transfer system $\boldsymbol{\Psi} \cong \mathcal{K}$ (the *noise* temperature).

It is supposed that both pure states $\boldsymbol{\theta}_i$ in the place where the input message is being *coded* - on the *input of the channel* $\boldsymbol{\Psi} \cong \mathcal{K}$ and, also, the measurable values of the quantity $\boldsymbol{\alpha}$ being observed on the place where the *output* message is *decoded* - on the *output of the channel* $\mathbf{\Psi} \cong \mathcal{K}$ are arrangable in such a way that in a given i-th pure state $\boldsymbol{\theta}_i$ of the system $\mathbf{\Psi} \cong \mathcal{K}$ only the values $\alpha_k \in \mathcal{S}(\boldsymbol{\alpha})$, $k = i + j$ are measurable and, that the probability of measuring the k-th value is $\mathrm{Pr}(j) = Pr(k - i)$. This probability distribution describes the *additive noise* in the given channel $\mathbf{\Psi} \cong \mathcal{K}$. Just it is this noise which creates observed values from the output spectrum $\mathcal{S}(\boldsymbol{\alpha}) = \{\alpha_i, \alpha_{i+1}, ...\}$ being the selecting space of the stochastic quantity $\boldsymbol{\alpha}$.

The pure states with energy level $\varepsilon_i = i \cdot \varepsilon$ are achievable by sending i particles with energy ε of each. When the environment, through which these particles are going, generates a bundle of j particles with probability $\mathrm{Pr}(j)$ then, with the same probability the energy $\varepsilon_{i+j} = k{\cdot}\varepsilon$ is decoded on the output.

It is supposed, also, the infinite number of states $\boldsymbol{\theta}_i$, infinite spectrum $\mathbf{S}(\boldsymbol{\alpha})$ of the measured quantity $\boldsymbol{\alpha}$, then $\mathbf{S}(\boldsymbol{\alpha}) = \{0,\ 1,\ 2,\ ...\}$, $\mathbf{S}(\boldsymbol{\alpha}) = D(\boldsymbol{\theta})$; $[\mathbf{S}(\boldsymbol{\alpha}) \stackrel{\triangle}{=} \mathbf{S},\ \alpha_k \stackrel{\triangle}{=} \alpha]$.

The *narrow–band, memory-less (quantum) channel, additive* (with additive noise) operating on the energy level $\varepsilon \in \mathcal{S}(\boldsymbol{\varepsilon})$ is defined by the tri–partite structure (1),

$$\mathcal{K}_\varepsilon = \{[\mathcal{S}, q(i|\boldsymbol{\theta})],\ p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}_i),\ [\mathcal{S},\ p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta})]\}. \tag{29}$$

4.1. Capacity of Bose–Einstein narrow–band channel

Let now $\boldsymbol{\theta}_i \equiv i,\ i = 0, 1,\ ...$ are pure states of a system $\mathbf{\Psi}_{\mathrm{B-E},\varepsilon} \cong \mathcal{K}$ and let $\boldsymbol{\alpha}$ is output quantity taking on in the state $\boldsymbol{\theta}_i$ values $\alpha \in \mathbf{S}$ with probabilities

$$p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}_i) = (1 - p)\, p^{\alpha - i} \tag{30}$$

Thus the distribution $p(\cdot|\boldsymbol{\alpha}|\boldsymbol{\theta}_i)$ is determined by the *forced* (inner–input) state $\boldsymbol{\theta}_i = \boldsymbol{\pi}\{\psi_i\}$ representing the coded input energy at the value $\varepsilon_i = i \cdot \varepsilon \in \mathbf{S}(\boldsymbol{\varepsilon})$, $\varepsilon = \mathrm{const}$. For the *medium value* $\mathcal{W}$ of the input i is valid

$$\mathcal{W} = \sum_{i=0}^{\infty} i \cdot q(i|\boldsymbol{\theta}) = E(\boldsymbol{\theta}), \quad W = \varepsilon \cdot \mathcal{W} \tag{31}$$

The quantity W is the medium value of the energy coding the input signal i.

For the medium value of the number of particles $j = \alpha - i \geq 0$ with B–E statistics is valid

$$\sum_{j=0}^{\infty} j \cdot (1 - p)\, p^j = (1 - p) \cdot p \cdot \sum_{j=0}^{\infty} j\, p^{j-1} = (1 - p) \cdot p \cdot \frac{\mathrm{d}}{\mathrm{d}p}\left[\frac{1}{1 - p}\right] = \frac{p}{1 - p}$$

The quantity $E(\boldsymbol{\alpha})$ is the medium value of the output quantity $\boldsymbol{\alpha}$ and

$$E(\boldsymbol{\alpha}) = \sum_{\alpha \in \mathbf{S}(\boldsymbol{\alpha})} \alpha \cdot p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}) \tag{32}$$

where $p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta})$ is the probability of measuring the eigenvalue $\alpha = k$ of the output variable $\boldsymbol{\alpha}$. This probability is defined by the state $\boldsymbol{\theta} = \sum_{i=0}^{n} q(i|\boldsymbol{\theta})\, \boldsymbol{\theta}_i$ of the system $\mathbf{\Psi}_{\mathrm{B-E},\varepsilon} \cong \mathcal{K}$

$$p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}) = \sum_{i=0}^{n} q(i|\boldsymbol{\theta}) \cdot p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}_i) = (1 - p) \cdot \sum_{i=0}^{n} q(i|\boldsymbol{\theta}) \cdot p^{\alpha - i} \quad [= \mathrm{Tr}(\boldsymbol{\theta}\boldsymbol{\pi}_\alpha)] \tag{33}$$

From the differential equation with the *condition* for $\alpha = 0$

$$p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}) = p(\alpha-1|\boldsymbol{\alpha}|\boldsymbol{\theta}) \cdot p + (1-p) \cdot q(\alpha|\boldsymbol{\theta}),\ \forall \alpha \geq 1; \quad p(0|\boldsymbol{\alpha}|\boldsymbol{\theta}) = (1-p) \cdot q(0|\boldsymbol{\theta}) \tag{34}$$

follows, for the medium value $E(\boldsymbol{\alpha})$ of the output stochastic variable $\boldsymbol{\alpha}$, that

$$\begin{aligned} E(\boldsymbol{\alpha}) &= \sum_{\alpha\geq 1} \alpha \cdot p(\alpha-1|\boldsymbol{\alpha}|\boldsymbol{\theta}) \cdot p + \sum_{\alpha\geq 1} \alpha \cdot (1-p) \cdot q(\alpha|\boldsymbol{\theta}) \\ &= \sum_{\alpha\geq 1} \alpha \cdot p(\alpha-1|\boldsymbol{\alpha}|\boldsymbol{\theta}) \cdot p + \mathcal{W} \cdot (1-p) = p \cdot E(\boldsymbol{\alpha}) + p \cdot \sum_{\alpha\geq 1} p(\alpha-1|\boldsymbol{\alpha}|\boldsymbol{\theta}) + \mathcal{W} \cdot (1-p) \end{aligned} \tag{35}$$

$$E(\boldsymbol{\alpha}) \ \cdot \ (1-p) = p + \mathcal{W} \cdot (1-p) \longrightarrow E(\boldsymbol{\alpha}) = \frac{p}{1+p} + \mathcal{W}, \quad \mathcal{W} = E(\boldsymbol{\theta}) > 0, \quad \boldsymbol{\theta} \in \boldsymbol{\Theta}_0 \tag{36}$$

The quantity $H(\boldsymbol{\alpha}\|\boldsymbol{\theta}_i)$ is the p-entropy of measuring $\boldsymbol{\alpha}$ for the input $i \in \mathbf{S}$ being represented by the pure state $\boldsymbol{\theta}_i$ of the system $\boldsymbol{\Psi}_{\mathrm{B-E},\varepsilon}$

$$\begin{aligned} -H(\boldsymbol{\alpha}\|\boldsymbol{\theta}_i) &= \sum_{j\in\mathbf{S}} (1-p)\, p^j \cdot \ln[(1-p)\, p^j] \\ &= (1-p) \cdot \ln(1-p) \cdot \sum_j p^j - (1-p) \cdot p \cdot \ln p \cdot \sum_j j\, p^{j-1} \\ &= \ln(1-p) - (1-p) \cdot p \cdot \ln p \cdot \frac{\mathrm{d}}{\mathrm{d}p}\left[\frac{1}{1-p}\right] = -\frac{h(p)}{1-p},\ \forall i \in \mathbf{S} \end{aligned} \tag{37}$$

where $h(p) \stackrel{\triangle}{=} -(1-p) \cdot \ln(1-p) - p \cdot \ln p$ is the Shannon entropy of Bernoulli distribution $\{p,\ 1-p\}$.

The quantity $H(\boldsymbol{\alpha}|\boldsymbol{\theta})$ is the conditional Shannon entropy of the stochastic quantity $\boldsymbol{\alpha}$ in the state $\boldsymbol{\theta}$ of the system $\boldsymbol{\Psi}_{\mathrm{B-E},\varepsilon}$, not depending on the $\boldsymbol{\theta}$ (the *noise* entropy)

$$H(\boldsymbol{\alpha}|\boldsymbol{\theta}) = \sum_{i=0}^{n} q(i|\boldsymbol{\theta}) \cdot H(\boldsymbol{\alpha}\|\boldsymbol{\theta}_i) = \sum_{i=0}^{n} q(i|\boldsymbol{\theta}) \cdot \frac{h(p)}{1-p} = \frac{h(p)}{1-p} \tag{38}$$

For capacity $C_{\mathrm{B-E}''}$ of the channel $\mathcal{K} \cong \boldsymbol{\Psi}_{\mathrm{B-E}}$ is, following the capacity definition, valid

$$C_{\mathrm{B-E}''} = \sup_{\boldsymbol{\theta}\in\boldsymbol{\Theta}_0} H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) - H(\boldsymbol{\alpha}|\boldsymbol{\theta}) = \sup_{\boldsymbol{\theta}\in\boldsymbol{\Theta}_0} H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) - \frac{h(p)}{1-p} \tag{39}$$

where the set $\boldsymbol{\Theta}_0 = \{\boldsymbol{\theta} \in \boldsymbol{\Theta},\ E(\boldsymbol{\theta}) = \mathcal{W} \geq 0\}$ represents the *coding procedure* of the input $i \in \mathbf{S}$. The quantity $H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) = H(p(\cdot|\boldsymbol{\alpha}|\boldsymbol{\theta}))$ is the p-entropy of the output quantity $\boldsymbol{\alpha}$. Its supremum is determined by the **Lagrange** multipliers method:

$$H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) = -\sum_{\alpha\in\mathbf{S}(\boldsymbol{\alpha})} p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}) \cdot \ln p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}) = -\sum_{\alpha\in\mathbf{S}(\boldsymbol{\alpha})} p_\alpha \cdot \ln p_\alpha, \quad p_\alpha \stackrel{\triangle}{=} p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}) \tag{40}$$

The conditions for determinating of the *bound* extreme are

$$\sum_{\alpha\in\mathbf{S}(\boldsymbol{\alpha})} p_\alpha = 1, \quad \sum_{\alpha\in\mathbf{S}(\boldsymbol{\alpha})} \alpha \cdot p_\alpha = E(\boldsymbol{\alpha}) = \text{const.} \tag{41}$$

The Lagrange function

$$L = -\sum_{\alpha} p_{\alpha} \cdot \ln p_{\alpha} - \lambda_1 \cdot \sum_{\alpha} p_{\alpha} + \lambda_1 - \lambda_2 \cdot \sum_{\alpha} \alpha \cdot p_{\alpha} + \lambda_2 E(\boldsymbol{\alpha}) \tag{42}$$

which gives the condition for the extreme, $\dfrac{\partial L}{\partial p_{\alpha}} = -\ln p_{\alpha} - 1 - \lambda_1 - \lambda_2 \cdot \alpha = 0$, yielding in

$$p_{\alpha} = e^{-1-\lambda_1} \cdot e^{-\lambda_2 \alpha} = p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}) \quad \text{and further, in } \sum_{\alpha} p_{\alpha} = \sum_{\alpha} \frac{e^{-1-\lambda_1}}{e^{\lambda_2 \alpha}} = \frac{e^{-1-\lambda_1}}{1-e^{-\lambda_2}} = 1 \tag{43}$$

Then for the medium value $E(\boldsymbol{\alpha})$ the following result is obtained

$$\begin{aligned} E(\boldsymbol{\alpha}) &= \sum_{\alpha} \alpha \cdot p_{\alpha} = \sum_{\alpha} \alpha \cdot e^{-1-\lambda_1} \cdot e^{-\lambda_2 \alpha} = -e^{-1-\lambda_1} \cdot \frac{\partial}{\partial \lambda_2} \sum_{\alpha} e^{-\lambda_2 \alpha} \\ &= -e^{-1-\lambda_1} \cdot \frac{\partial}{\partial \lambda_2} \left[\frac{1}{1-e^{-\lambda_2}} \right] = \frac{e^{-1-\lambda_1}}{(1-e^{-\lambda_2})^2} \cdot e^{-\lambda_2} = \frac{e^{-\lambda_2}}{1-e^{-\lambda_2}} \end{aligned} \tag{44}$$

By (35), (36) and for the parametr $p = e^{-\frac{\varepsilon}{k\Theta}} =$ const. ($\varepsilon =$ const. $\Theta =$ const.) $E(\boldsymbol{\alpha})$ is a function of $\mathcal{W}$ only. For $\lambda_2 = \dfrac{\varepsilon}{kT_W}$ is $e^{-\lambda_2} = p(\mathcal{W})$ and $E(\boldsymbol{\alpha})$ is the medium value for $\boldsymbol{\alpha}$ with the *geometric* probability distribution

$$p(\cdot) = p(\cdot|\boldsymbol{\alpha}|\boldsymbol{\theta}) = [1 - p(\mathcal{W})] \cdot p(\mathcal{W})^{\alpha}, \ \ \alpha \in \mathbf{S}(\boldsymbol{\alpha}) \tag{45}$$

depending only on $\dfrac{\varepsilon}{kT_W}$ or, on the absolute temperature T_W respectively. Thus for $E(\boldsymbol{\alpha})$ is valid that

$$E(\boldsymbol{\alpha}) = \frac{p(\mathcal{W})}{1 - p(\mathcal{W})}, \ \ p(\mathcal{W}) = e^{-\frac{\varepsilon}{kT_W}} \tag{46}$$

From (35) and (46) is visible that $p(\mathcal{W})$ or T_W respectively is the *only one* root of the equation

$$\frac{p(\mathcal{W})}{1-p(\mathcal{W})} = \frac{p}{1-p} + \mathcal{W}, \text{resp. } \frac{e^{-\frac{\varepsilon}{kT_W}}}{1-e^{-\frac{\varepsilon}{kT_W}}} = \frac{e^{-\frac{\varepsilon}{kT_0}}}{1-e^{-\frac{\varepsilon}{kT_0}}} + \mathcal{W} \tag{47}$$

From (34) and (45) follows that for state $\boldsymbol{\theta} \in \boldsymbol{\Theta}_0$ or, for the q-distribution $q(\cdot|\boldsymbol{\theta})$ respectively, in which the value $H(\boldsymbol{\alpha}\|\boldsymbol{\theta})$ is maximal [that state in which $\boldsymbol{\alpha}$ achieves the distribution $q(\cdot|\boldsymbol{\theta}) = p(\cdot)$], is valid that

$$q(\alpha|\boldsymbol{\theta}) = \frac{1-p(\mathcal{W})}{1-p}, \ \alpha = 0 \quad \text{and} \quad q(\alpha|\boldsymbol{\theta}) = \frac{1-p(\mathcal{W})}{1-p} \left[p(\mathcal{W}) - p \right] \cdot p(\mathcal{W})^{\alpha-1}, \ \alpha > 0 \tag{48}$$

For the *effective temperatutre* T_W *of coding input messages* the distribution (45) supremizes (maximizes) the p-entropy $H(\boldsymbol{\alpha}\|\boldsymbol{\theta})$ of $\boldsymbol{\alpha}$ and, by using (37) with $p(\mathcal{W})$, is gained that

$$\sup_{\boldsymbol{\theta} \in \boldsymbol{\Theta}_0} H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) = \frac{h[p(\mathcal{W})]}{1-p(\mathcal{W})} \tag{49}$$

From (39) and (49) follows [12, 37] the capacity $C_{\mathrm{B-E},\varepsilon}$ of the narrow–band channel $\mathcal{K} \cong \boldsymbol{\Psi}_{\mathrm{B-E}}$

$$C_{\mathrm{B-E},\varepsilon} = \frac{h[p(\mathcal{W})]}{1-p(\mathcal{W})} - \frac{h(p)}{1-p} \tag{50}$$

By (35), (36) and (46) the medium value $\mathcal{W}$ of the input message $i \in \mathbf{S}$ is derived,

$$\mathcal{W} = \frac{p(\mathcal{W})}{1 - p(\mathcal{W})} - \frac{p}{1-p} \tag{51}$$

By (31) the condition for the *minimal* average energy W_{Krit} needed for coding the input message is

$$\mathcal{W} \geq 0 \quad \text{resp.} \quad W = \varepsilon \cdot \mathcal{W} \geq 0, \quad \varepsilon \in \mathbf{S}(\boldsymbol{\varepsilon}) \tag{52}$$

$$W \geq W_{Krit}, \quad W_{Krit} = 0 \tag{53}$$

The relations (35), (36), (46) and (52), (53) yield in

$$E(\boldsymbol{\alpha}) = \frac{p(\mathcal{W})}{1 - p(\mathcal{W})} \geq \frac{p}{1-p} \quad \text{and, then} \quad p(\mathcal{W}) \geq p,\ 1 - p(\mathcal{W}) \leq 1 - p \tag{54}$$

From (47) and (54) follows that for the defined direction of the signal (messaage) transmission at the temperature T_W of its sending and decoding is valid that

$$\frac{p(\mathcal{W})}{1 - p(\mathcal{W})} > \frac{p}{1-p},\ \ p(\mathcal{W}) > p,\ \mathcal{W} > 0 \tag{55}$$

$$p(\mathcal{W}) = e^{-\frac{\varepsilon}{kT_W}} \geq e^{-\frac{\varepsilon}{kT_0}} = p \quad \text{and thus} \quad T_W \geq T_0 \tag{56}$$

4.2. Capacity of Fermi–Dirac narrow–band channel

Let is now considered, in the same way as it was in the B–E system, the pure states $\boldsymbol{\theta}_i \equiv i$ of the system $\boldsymbol{\Psi}$ which are coding the input messages $i = 0, 1, \ldots$ and the output stochastic quantity $\boldsymbol{\alpha}$ having its selecting space $\mathbf{S}$. On the spectrum $\mathbf{S}$ probabilities of realizations $\alpha \in \mathbf{S}$ are defined,

$$p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}_i) = \frac{p^{\alpha - i}}{1+p}, \quad p \in (0,1),\ i = 0, 1, \ldots \tag{57}$$

expressing the additive stochastic transformation of an input i into the output α for wich is valid $\alpha = i$ or $\alpha = i + 1$.[5] The uniform energy level $\varepsilon =$ const. of particles is considered.

The quantity W is the mathematical expectation of the energy coding the input signal

$$W = \varepsilon \cdot \mathcal{W}, \quad \mathcal{W} = \sum_{i \in \mathbf{S}} i \cdot q(i|\boldsymbol{\theta}) = E(\boldsymbol{\theta}) \tag{58}$$

The medium value of a stochastic quantity with the F–D statistic is given by

$$\sum_{j \in \{0,1\}} j \cdot \frac{p^j}{1+p} = \frac{p}{1+p} \tag{59}$$

The quantity $E(\boldsymbol{\alpha})$ is the medium value of the output quantity $\boldsymbol{\alpha}$,

$$E(\boldsymbol{\alpha}) = \sum_{\alpha \in \mathbf{S}(\boldsymbol{\alpha})} \alpha \cdot p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}) \tag{60}$$

[5] In accordance with ***Pauli*** *excluding principle* (valid for *fermions*) and a given energetic level $\varepsilon \in \mathbf{S}(\boldsymbol{\varepsilon})$.

where $p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta})$ is probability of realization of $\alpha \in \mathbf{S}$ in the state $\boldsymbol{\theta}$ of the system $\boldsymbol{\Psi} \equiv \boldsymbol{\Psi}_{\mathrm{F-D},\varepsilon} \cong \mathcal{K}$,

$$\boldsymbol{\theta} = \sum_{i \in \mathbf{S}} q(i|\boldsymbol{\theta})\,\boldsymbol{\theta}_i, \quad p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}) = \sum_{i=0}^{n} q(i|\boldsymbol{\theta}) \cdot p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}_i) = \frac{1}{1+p} \cdot \sum_{i=0}^{n} q(i|\boldsymbol{\theta}) \cdot p^{\alpha-i} \tag{61}$$

From the differential equation

$$p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}) = \frac{q(\alpha-1|\boldsymbol{\theta}) \cdot p + q(\alpha|\boldsymbol{\theta})}{1+p}, \quad \alpha \geq 1 \tag{62}$$

with the condition for

$$\alpha = 0, \quad p(0|\boldsymbol{\alpha}|\boldsymbol{\theta}) = \frac{1}{1+p} \cdot q(0|\boldsymbol{\theta})$$

follows that for the medium value $E(\boldsymbol{\alpha})$ of the output stochastic variable $\boldsymbol{\alpha}$ is valid that

$$\begin{aligned} E(\boldsymbol{\alpha}) &= \frac{p}{1+p} \cdot \sum_{\alpha \geq 1} \alpha \cdot q(\alpha-1|\boldsymbol{\theta}) + \frac{1}{1+p} \cdot \sum_{\alpha \geq 1} \alpha \cdot q(\alpha|\boldsymbol{\theta}) \\ &= \frac{p}{1+p} \cdot \sum_{\alpha \geq 1} (\alpha-1) \cdot q(\alpha-1|\boldsymbol{\theta}) + \frac{p}{1+p} \cdot \sum_{\alpha \geq 1} q(\alpha-1|\boldsymbol{\theta}) + \frac{1}{1+p} \cdot \mathcal{W} \\ &= \frac{p}{1+p} \cdot \mathcal{W} + \frac{p}{1+p} + \frac{1}{1+p} \cdot \mathcal{W} \quad \longrightarrow \quad E(\boldsymbol{\alpha}) = \frac{p}{1+p} + \mathcal{W}, \ \ \mathcal{W} = E(\boldsymbol{\theta}), \ \ \boldsymbol{\theta} \in \boldsymbol{\Theta}_0 \end{aligned} \tag{63}$$

The quantity $H(\boldsymbol{\alpha}\|\boldsymbol{\theta}_i)$ is the p-entropy of measuring $\boldsymbol{\alpha}$ for the input $i \in \mathbf{S}$ being represented by the pure state $\boldsymbol{\theta}_i$ of the system $\boldsymbol{\Psi}_{\mathrm{F-D},\varepsilon}$

$$\begin{aligned} H(\boldsymbol{\alpha}\|\boldsymbol{\theta}_i) &= -\sum_{j=0}^{1} \frac{p^j}{1+p} \cdot \ln \frac{p^j}{1+p} = -\frac{1}{1+p} \cdot \ln \frac{1}{1+p} - \frac{p}{1+p} \cdot \ln \frac{p}{1+p} \\ &= -\left(1 - \frac{p}{1+p}\right) \cdot \ln \left(1 - \frac{p}{1+p}\right) - \frac{p}{1+p} \cdot \ln \frac{p}{1+p} = h\left(\frac{p}{1+p}\right), \ \ \forall i \in \mathbf{S} \end{aligned} \tag{64}$$

The quantity $H(\boldsymbol{\alpha}|\boldsymbol{\theta})$ is the conditional (the *noise*) Shannon entropy of the stochastic quantity $\boldsymbol{\alpha}$ in the system state $\boldsymbol{\theta}$, but, independent on this $\boldsymbol{\theta}$,

$$H(\boldsymbol{\alpha}|\boldsymbol{\theta}) = \sum_{i=0}^{n} q(i|\boldsymbol{\theta}) \cdot H(\boldsymbol{\alpha}\|\boldsymbol{\theta}_i) = \sum_{i} q(i|\boldsymbol{\theta}) \cdot h\left(\frac{p}{1+p}\right) = h\left(\frac{p}{1+p}\right) \tag{65}$$

For capacity $C_{\mathrm{F-D},\varepsilon}$ of the channel $\mathcal{K} \cong \boldsymbol{\Psi}_{\mathrm{F-D},\varepsilon}$ is, by the capacity definition in (23)-(24), valid that

$$C_{\mathrm{F-D}''} = \sup_{\boldsymbol{\theta} \in \boldsymbol{\Theta}_0} H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) - H(\boldsymbol{\alpha}|\boldsymbol{\theta}) = \sup_{\boldsymbol{\theta} \in \boldsymbol{\Theta}_0} H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) - h\left(\frac{p}{1+p}\right) \tag{66}$$

where the set $\boldsymbol{\Theta}_0 = \{\boldsymbol{\theta} \in \boldsymbol{\Theta} : E(\boldsymbol{\theta}) = \mathcal{W} > 0\}$ represents the coding procedure.

The quantity $H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) = H(p(\cdot|\boldsymbol{\alpha}|\boldsymbol{\theta}))$ is the p-entropy of the stochastic quantity $\boldsymbol{\alpha}$ in the state $\boldsymbol{\theta}$ of the system $\boldsymbol{\Psi}_{\mathrm{F-D},\varepsilon}$. Its supremum is determined by the **Lagrange** multipliers method in the same way as in B–E case and with the same results for the probility distribution $p(\cdot|\boldsymbol{\alpha}|\boldsymbol{\theta})$ (geometric) and the medium value $E(\boldsymbol{\alpha})$

$$p(\cdot) = p(\cdot|\boldsymbol{\alpha}|\boldsymbol{\theta}) = [1 - p(\mathcal{W})] \cdot p(\mathcal{W})^{\alpha}, \ \ \alpha \in \mathbf{S}(\boldsymbol{\alpha}) \tag{67}$$
$$E(\boldsymbol{\alpha}) = \frac{p(\mathcal{W})}{1 - p(\mathcal{W})}, \ \ p(\mathcal{W}) = e^{-\frac{\varepsilon}{\mathrm{k}T_W}}$$

Again , the value $E(\boldsymbol{\alpha})$ depends on $\frac{\varepsilon}{\mathrm{k}T_W}$, or on absolute temperature T_W respectively, only. By using $E(\boldsymbol{\alpha})$ in (63) it is seen that $p(\mathcal{W})$ or T_W respectively is the only one root of the equation [12, 30]

$$\frac{p(\mathcal{W})}{1 - p(\mathcal{W})} = \frac{p}{1+p} + \mathcal{W}, \ \ \text{resp.} \ \ \frac{e^{-\frac{\varepsilon}{\mathrm{k}T_W}}}{1 - e^{-\frac{\varepsilon}{\mathrm{k}T_W}}} = \frac{e^{-\frac{\varepsilon}{\mathrm{k}T_0}}}{1 + e^{-\frac{\varepsilon}{\mathrm{k}T_0}}} + \mathcal{W} \tag{68}$$

For the q-distribution $q(\cdot|\boldsymbol{\theta}) = p(\cdot)$ of states $\boldsymbol{\theta} \in \boldsymbol{\Theta}_0$, for which the relation (62) and (67) is gained, follows that

$$\frac{q(\alpha - 1|\boldsymbol{\theta}) \cdot p + q(\alpha|\boldsymbol{\theta})}{1+p} = [1 - p(\mathcal{W})] \cdot p(\mathcal{W})^{\alpha}, \quad \alpha \in \mathbf{S} \ \text{ with conditions} \tag{69}$$
$$q(0|\boldsymbol{\theta}) = (1+p) \cdot [1 - p(\mathcal{W})] \ \ \text{and} \ \ q(1|\boldsymbol{\theta}) = (1+p) \cdot [1 - p(\mathcal{W})] \cdot [p(\mathcal{W}) - p];$$
$$q(\alpha|\boldsymbol{\theta}) = (1+p) \cdot [1 - p(\mathcal{W})] \cdot \left[\left(\sum_{i=0}^{\alpha-1} (-1)^i \cdot p(\mathcal{W})^{\alpha - i} \cdot p^i \right) + (-1)^{\alpha} \cdot p^{\alpha} \right], \ \alpha > 1$$

For the *effective temperatutre T_W of coding the input messages* the distribution (67) supremises (maximizes) the p-entropy $H(\boldsymbol{\alpha}\|\boldsymbol{\theta})$ of $\boldsymbol{\alpha}$ is valid, in the same way as in (49), that

$$\sup_{\boldsymbol{\theta} \in \boldsymbol{\Theta}_0} H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) = \frac{h[p(\mathcal{W})]}{1 - p(\mathcal{W})} \tag{70}$$

By using (70) in (66) the formula for the $C_{\mathrm{F-D},\varepsilon}$ capacity [12, 37] is gained

$$C_{\mathrm{F-D},\varepsilon} = \frac{h[p(\mathcal{W})]}{1 - p(\mathcal{W})} - h\left(\frac{p}{1+p}\right) \tag{71}$$

The medium value $\mathcal{W}$ of the input $i = 0, 1, 2, \ldots$ is limited by a minimal not-zero and positive 'bottom' value $\mathcal{W}_{Krit}$. From (58), (63) and (68) follows

$$E(\boldsymbol{\alpha}) = \frac{p(\mathcal{W})}{1 - p(\mathcal{W})} \geq \frac{p}{1-p}, \ \ p(\mathcal{W}) = e^{-\frac{\varepsilon}{\mathrm{k}T_W}} \ \geq \ e^{-\frac{\varepsilon}{\mathrm{k}T_0}} = p \ \ \text{and thus} \ \ T_W \geq T_0 \tag{72}$$

$$\mathcal{W} = \frac{p(\mathcal{W})}{1 - p(\mathcal{W})} - \frac{p}{1+p} \geq 0, \ \ \mathcal{W} \ \geq \ \frac{2p^2}{1-p^2} = \mathcal{W}_{Krit}, \text{resp.} \ W = \varepsilon \cdot \mathcal{W} \ \geq \ \varepsilon \cdot \frac{2e^{-2\frac{\varepsilon}{\mathrm{k}T_0}}}{1 - e^{-2\frac{\varepsilon}{\mathrm{k}T_0}}} \tag{73}$$

For the average coding energy W, when the channel $C_{\mathrm{F-D},\varepsilon}$ acts on a uniform energetic level ε, is

$$W \ \geq \ W_{Krit} \ = \ \frac{2p^2}{1-p^2} \tag{74}$$

For the F–D channel is then possible speak about the ***effect of the not-zero capacity when the difference between the coding temperatures T_W and the noise temperature T_0* is zero**.[6] This phennomenon is, by necessity, *given by properties of cells of the F–D phase space*.

[6] Not *not-zero capacity* for zero input power as was stated in [37]. The (74) also repares small missprint in [11].

5. Wide–band quantum transfer channels

Till now the narrow–band variant of an information transfer channel $\mathcal{K}_\varepsilon$, $\varepsilon \in \mathbf{S}(\boldsymbol{\varepsilon})$, card $\mathbf{S}(\boldsymbol{\varepsilon}) = 1$ has been dealt. Let is now considered the symmetric operator of energy $\boldsymbol{\varepsilon}$ of a particle, having the spectrum of eigenavalues

$$\mathbf{S}(\boldsymbol{\varepsilon}) = \left\{0,\ \frac{h}{\tau},\ \frac{2h}{\tau},\ \dots,\ \frac{nh}{\tau},\ \dots\right\} = \left\{\frac{rh}{\tau}\right\}_{r=0,\,1,\,\dots,\,n},\ \text{card}\,\mathbf{S}(\boldsymbol{\varepsilon}) = n+1 \tag{75}$$

where $\tau > 0$ denotes the time length of the input signal and h denotes **Planck** constant. The *multi–band physical transfer channel* **K***, memory-less, with additive noise* is defend by the (arranged) set of narrow–band, *independent* components $\mathcal{K}_\varepsilon$, $\varepsilon \in \mathbf{S}(\boldsymbol{\varepsilon})$,

$$\mathbf{K} = \underset{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}{\times} \mathcal{K}_\varepsilon = \underset{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}{\times} \left\{\boldsymbol{i}_\varepsilon,\ p(\alpha_\varepsilon|\boldsymbol{\alpha}_\varepsilon|\boldsymbol{\theta}_{\varepsilon,i_\varepsilon}),\ \boldsymbol{\alpha}_\varepsilon\right\} = \left\{\boldsymbol{i},\ p(\overline{\alpha}|\boldsymbol{\alpha}|\boldsymbol{\theta}_{\overline{i}}),\ \boldsymbol{\alpha}\right\} \tag{76}$$

$$\boldsymbol{i} = \underset{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}{\times} \boldsymbol{i}_\varepsilon = \underset{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}{\times} [\mathbf{S},\ q_\varepsilon(i_\varepsilon|\boldsymbol{\theta}_\varepsilon)],\ \ i_\varepsilon \in \mathbf{S} = \{0,\ 1,\ 2,\ \dots\}$$

$$\boldsymbol{\alpha} = \times_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})} \boldsymbol{\alpha}_\varepsilon = \underset{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}{\times} [\mathbf{S},\ p(\alpha_\varepsilon|\boldsymbol{\alpha}_\varepsilon|\boldsymbol{\theta}_\varepsilon)],\ \ \alpha_\varepsilon \in \mathbf{S} = \{0,\ 1\ ,2,\ \dots\}$$

Due the independency of narrow–band components $\mathcal{K}_\varepsilon$ the vector quantities $\boldsymbol{i}_\varepsilon$, $\boldsymbol{\alpha}_\varepsilon$, $\boldsymbol{\theta}_\varepsilon$, $\boldsymbol{j}_\varepsilon$ are independent stochastic quantities too.

The simultaneous *q-distribution* of the input vector of i_ε and the simultaneous *p-distribution* of measuring the output vector of values α_ε (of the individual narrow–band components $\mathcal{K}_\varepsilon$) are

$$\prod_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})} q_\varepsilon(i_\varepsilon|\boldsymbol{\theta}_\varepsilon) = q(\overline{i}|\boldsymbol{\theta}), \qquad \boldsymbol{\theta} = \underset{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}{\times} \boldsymbol{\theta}_\varepsilon, \quad \overline{i} \in \mathbf{S}(\boldsymbol{i}) \tag{77}$$

$$\prod_{\varepsilon\in\mathbf{S}(\varepsilon)} p(\alpha_\varepsilon|\boldsymbol{\alpha}_\varepsilon|\boldsymbol{\theta}_\varepsilon) = p(\overline{\alpha}|\boldsymbol{\alpha}|\boldsymbol{\theta}), \quad \boldsymbol{\theta} = \underset{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}{\times} \boldsymbol{\theta}_\varepsilon, \quad \overline{\alpha} \in \mathbf{S}(\boldsymbol{\alpha}) \tag{78}$$

The system of quantities $\boldsymbol{\theta}_\varepsilon$ (the set of states of the narrow–band components $\mathcal{K}_\varepsilon$) is the state $\boldsymbol{\theta}$ of the multi–band channel **K** in which the (*canonic*) *q-distribution* of the system **K** is defined. Values i', j', α'

$$\alpha' = j' + i';\ \ \alpha' = \sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})} \alpha_\varepsilon,\ j' = \sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})} j_\varepsilon,\ i' = \sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})} i_\varepsilon;\ j_\varepsilon = \alpha_\varepsilon - i_\varepsilon \geq 0,\ \forall\varepsilon \in \mathbf{S}(\boldsymbol{\varepsilon}),\ \text{card}\mathbf{S}_\varepsilon > 1 \tag{79}$$

are the numbers of the input, output and additive (noise) particles of the *multi–band* channel **K**. In this channel the stochastic transformation of the input i' into the output α' is performed, being determined by additive stochastic transformations of the input i_ε into the output α_ε in individual narrow–band components $\mathcal{K}_\varepsilon$.

Realizations of the stochastic systems $\boldsymbol{i}$, $\boldsymbol{\alpha}$, $\boldsymbol{\theta}$, $\boldsymbol{j}$ are the *vectors (sequences)* $\overline{i}$, $\overline{\alpha}$, $\overline{\theta}$, $\overline{j}$

$$\overline{i} = (i_\varepsilon)_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}, \quad \overline{\alpha} = (\alpha_\varepsilon)_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}, \quad \overline{\theta} = (\theta_\varepsilon)_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}, \quad \overline{j} = (j_\varepsilon)_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})};\ \overline{i}, \overline{\alpha}, \overline{j} \in \underset{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}{\times} \mathbf{S}, \tag{80}$$

$$i_\varepsilon,\ \alpha_\varepsilon,\ j_\varepsilon \in \mathbf{S},\ \ \theta_\varepsilon \in \mathbf{S}(\boldsymbol{\theta}_\varepsilon), \quad \boldsymbol{\theta}_\varepsilon = \sum_{i_\varepsilon\in\mathbf{S}} \theta_\varepsilon\, \boldsymbol{\theta}_{\varepsilon,i_\varepsilon}, \quad \overline{\theta} \in \mathbf{S}(\boldsymbol{\theta})$$

For the probability of the additive stochastic transformation (77), (78) of input $\bar{i}$ into the output $\bar{\alpha}$ is valid

$$\prod_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})} p(\alpha_\varepsilon|\boldsymbol{\alpha}_\varepsilon|\boldsymbol{\theta}_{\varepsilon,i_\varepsilon}) = p(\bar{\alpha}|\boldsymbol{\alpha}|\boldsymbol{\theta}_{\bar{i}}),\ \ \bar{\alpha} = \bar{j} + \bar{i},\ \ \bar{i} \in \mathbf{S}(\boldsymbol{i}),\ \ \bar{j} \in \mathbf{S}(\boldsymbol{j}),\ \ \bar{\alpha} \in \mathbf{S}(\boldsymbol{\alpha}) \tag{81}$$

The symbol $\boldsymbol{\theta}_{\varepsilon,i_\varepsilon}$ denotes the pure state coding the input $i_\varepsilon \in \mathbf{S}$ of a narrow–band component $\mathcal{K}_\varepsilon$ and the state $\boldsymbol{\theta}_{\bar{i}} = \times_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}\boldsymbol{\theta}_{\varepsilon,i_\varepsilon}$ codes the input $\bar{i}$ for which

$$q(\bar{i}|\boldsymbol{\theta}) = q_{\overline{\theta}} = \prod_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})} q_\varepsilon(\theta_\varepsilon) \tag{82}$$

For the multi–band channel $\mathbf{K}$ the following quantities are defined:[7]

- the *p-entropy of the output* $\boldsymbol{\alpha}$

$$\begin{aligned} H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) &= \sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})} H(\boldsymbol{\alpha}_\varepsilon\|\boldsymbol{\theta}_\varepsilon) = -\sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}\sum_{\alpha_\varepsilon\in\mathbf{S}} p(\alpha_\varepsilon|\boldsymbol{\alpha}_\varepsilon|\boldsymbol{\theta}_\varepsilon)\cdot\ln p(\alpha_\varepsilon|\boldsymbol{\alpha}_\varepsilon|\boldsymbol{\theta}_\varepsilon) \\ &\le \sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})} \sup_{\boldsymbol{\theta}_\varepsilon} H(\boldsymbol{\alpha}_\varepsilon\|\boldsymbol{\theta}_\varepsilon) \end{aligned} \tag{83}$$

for which, following the output narrow–band B–E and F–D components $\mathcal{K}_\varepsilon \in \mathbf{K}$, is valid that

$$\sup_{\boldsymbol{\theta}\in\overline{\boldsymbol{\Theta}}_0} H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) = \sup_{\boldsymbol{\theta}\in\overline{\boldsymbol{\Theta}}_0}\sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})} H(\boldsymbol{\alpha}_\varepsilon\|\boldsymbol{\theta}_\varepsilon) = \sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}\sup_{\boldsymbol{\theta}_\varepsilon} H(\boldsymbol{\alpha}_\varepsilon\|\boldsymbol{\theta}_\varepsilon) = \sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}\frac{h[p_\varepsilon(\mathcal{W})]}{1-p_\varepsilon(\mathcal{W})} \tag{84}$$

- the conditional *noise entropy* (entropy of the *multi–band B–E | F–D noise*)

$$H(\boldsymbol{\alpha}|\boldsymbol{\theta}) = \sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})} H(\boldsymbol{\alpha}_\varepsilon|\boldsymbol{\theta}_\varepsilon) = \sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}\sum_{i\in\mathbf{S}} q(i|\boldsymbol{\theta}_{\varepsilon,i})\cdot H(\boldsymbol{\alpha}_\varepsilon\|\boldsymbol{\theta}_{\varepsilon,i}) = \sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}\left[\frac{h(p_\varepsilon)}{1-p_\varepsilon}\,\middle|\, h\left(\frac{p_\varepsilon}{1+p_\varepsilon}\right)\right] \tag{85}$$

where $p_\varepsilon(W) = e^{-\frac{\varepsilon}{kT_{0W}}},\ p_\varepsilon = e^{-\frac{\varepsilon}{kT_0}},\ T_W \ge T_0 > 0$ and $h(p) = -p\ln p - (1-p)\ln(1-p)$.

- the *transinformation* $T(\boldsymbol{\alpha};\boldsymbol{\theta})$ and the *information capacity* $C(\mathbf{K})$,

$$\begin{aligned} C(\mathbf{K}) &= \sup_{\boldsymbol{\theta}\in\overline{\boldsymbol{\Theta}}_0} T(\boldsymbol{\alpha};\boldsymbol{\theta}) = \sup_{\boldsymbol{\theta}\in\overline{\boldsymbol{\Theta}}_0} H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) - H(\boldsymbol{\alpha}|\boldsymbol{\theta}) \\ &= \sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}\frac{h[p_\varepsilon(\mathcal{W})]}{1-p_\varepsilon(\mathcal{W})} - \sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}\left[\frac{h(p_\varepsilon)}{1-p_\varepsilon}\,\middle|\, h\left(\frac{p_\varepsilon}{1+p_\varepsilon}\right)\right] \end{aligned} \tag{86}$$

The set $\overline{\boldsymbol{\Theta}}_0 = \underset{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}{\times}\{\boldsymbol{\theta}_\varepsilon \in \Theta_\varepsilon;\ E(\boldsymbol{\theta}_\varepsilon) = W_\varepsilon \ge 0\}$ represents a coding procedure of the input $\bar{i}$ of the $\mathbf{K}$ into $\boldsymbol{\theta}_{\bar{i}}$, [by transforming each input i_ε into pure state $\boldsymbol{\theta}_{\varepsilon,i_\varepsilon}$, $\forall\varepsilon \in \mathbf{S}(\boldsymbol{\varepsilon})$].

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[7] Using the *chain rule* for simultaneous probabilities it is found that for information entropy of an independent stochastic system $\overrightarrow{X} = (X_1, X_2, ..., X_n)$ is valid that $H(\overrightarrow{X}) = \sum_i H(X_i|X_1,...X_{i-1}) = \sum_i H(X_i)$. Thus the physical entropy $\mathcal{H}(\boldsymbol{\theta})$ of independent stochastic system, $\boldsymbol{\theta} = \{\boldsymbol{\theta}_\varepsilon\}_\varepsilon$, is the sum of $H_\varepsilon[q(\cdot|\boldsymbol{\theta}_\varepsilon)]$ over $\varepsilon \in \mathbf{S}_\varepsilon$ too.

5.1. Transfer channels with continuous energy spectrum

Let a spectrum of energy with the finite cardinality $n+1$ and a finite time interval $\tau > 0$ are considered

$$\mathbf{S}(\boldsymbol{\varepsilon}) = \{\varepsilon_r\}_{r=0,\,1,\,\ldots,\,n} = \left\{\frac{rh}{\tau}\right\}_{r=0,\,1,\,\ldots,\,n}, \quad \Delta\varepsilon = \frac{h}{\tau},\ \varepsilon_r = \frac{rh}{\tau} = r\cdot\Delta\varepsilon, \tag{87}$$

$$\operatorname{card}\mathbf{S}(\boldsymbol{\varepsilon}) = \frac{\varepsilon_n}{\Delta\varepsilon} = n+1,\ n\cdot\Delta\varepsilon = n\cdot\frac{h}{\tau} = \varepsilon_n,\ \frac{\tau}{n} = \frac{h}{\varepsilon_n} = \text{const}.$$

For a transfer channel with the continuous spectrum of energies of particles and with the *band–width* equal to card $\mathbf{S}(\boldsymbol{\varepsilon}) = \frac{\varepsilon_n}{h}$, is valid that

$$\lim_{\tau\to\infty}\varepsilon_r = \lim_{\tau\to\infty}\frac{rh}{\tau} = \lim_{\tau\to\infty} r\cdot\Delta\varepsilon \stackrel{\triangle}{=} r\,\mathrm{d}\,\varepsilon \ \text{ resp. } \lim_{\tau\to\infty}\frac{1}{\tau} = \frac{\mathrm{d}\varepsilon}{h}, \quad \mathbf{S}(\boldsymbol{\varepsilon}) = \langle 0, \varepsilon_n) \tag{88}$$

But the infinite wide–band and infinite number of particles ($\tau \longrightarrow \infty,\ n \longrightarrow \infty$) will be dealt with. Then

$$\mathbf{S}(\boldsymbol{\varepsilon}) = \{\varepsilon_r\}_{r=0,\,1,\,\ldots} = \left\{\frac{rh}{\tau}\right\}_{r=0,\,1,\,\ldots} = \lim_{\tau\to\infty}\varepsilon_r = \lim_{\tau\to\infty}\frac{rh}{\tau} = \lim_{\varepsilon\to 0} r\,\Delta\varepsilon = r\,\mathrm{d}\,\varepsilon \tag{89}$$

and thus the *wide–band spectrum* $\mathbf{S}(\boldsymbol{\varepsilon})$ *of energies* is

$$\lim_{\tau\to\infty}\frac{1}{\tau} = \frac{\mathrm{d}\varepsilon}{h}, \quad \mathbf{S}(\boldsymbol{\varepsilon}) = \langle 0, \infty) \tag{90}$$

With the denotation $\alpha_\varepsilon \stackrel{\triangle}{=} \alpha,\ \ i \stackrel{\triangle}{=} i_\varepsilon,\ \ j \stackrel{\triangle}{=} j_\varepsilon,\ \ i_\varepsilon,\ j_\varepsilon \in \mathbf{S},\ \alpha_\varepsilon \in \mathbf{S}, \varepsilon \in \mathbf{S}(\boldsymbol{\varepsilon})$ For the p-entropy of the output $\boldsymbol{\alpha}$ of the wide–band transfer channel $\mathbf{K}_{\mathrm{B-E|F-D}}$ is valid that

$$\begin{aligned} H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) &= \lim_{\tau\to\infty}\frac{1}{\tau}\sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})} H(\boldsymbol{\alpha}_\varepsilon\|\boldsymbol{\theta}_\varepsilon) = \lim_{\tau\to\infty} -\frac{1}{\tau}\sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}\sum_{\alpha_\varepsilon\in\mathbf{S}} p(\alpha_\varepsilon|\boldsymbol{\alpha}_\varepsilon|\boldsymbol{\theta}_\varepsilon)\cdot\ln p(\alpha_\varepsilon|\boldsymbol{\alpha}_\varepsilon|\boldsymbol{\theta}_\varepsilon) \quad (91)\\ &= -\frac{1}{h}\int_0^\infty\left[\sum_{\alpha\in\mathbf{S}} p(\alpha|\boldsymbol{\alpha}_\varepsilon|\boldsymbol{\theta}_\varepsilon)\cdot\ln p(\alpha|\boldsymbol{\alpha}_\varepsilon|\boldsymbol{\theta}_\varepsilon)\right]\mathrm{d}\varepsilon \\ \sup_{\boldsymbol{\theta}} H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) &= \lim_{\tau\to\infty}\frac{1}{\tau}\sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}\sup_{\boldsymbol{\theta}_\varepsilon} H(\boldsymbol{\alpha}_\varepsilon\|\boldsymbol{\theta}_\varepsilon) = \frac{1}{h}\int_0^\infty \frac{h[p_\varepsilon(\mathcal{W})]}{1-p_\varepsilon(\mathcal{W})}\,\mathrm{d}\varepsilon \end{aligned}$$

For conditional entropy of the wide–band transfer channel $\mathbf{K}_{\mathrm{B-E|F-D}}$ [entropy of the wide–band noise independent on the system ($\mathbf{K}_{\mathrm{B-E|F-D}}$) state $\boldsymbol{\theta}$] and for its information capacity is valid, by (85) and (86)

$$\begin{aligned} H(\boldsymbol{\alpha}|\boldsymbol{\theta}) &= \lim_{\tau\to\infty}\frac{1}{\tau}\sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})} H(\boldsymbol{\alpha}_\varepsilon|\boldsymbol{\theta}_\varepsilon) = \lim_{\tau\to\infty}\frac{1}{\tau}\sum_{\varepsilon\in\mathbf{S}(\boldsymbol{\varepsilon})}\sum_{i\in\mathbf{S}} q(i|\boldsymbol{\theta}_{\varepsilon,i})\cdot H(\boldsymbol{\alpha}_\varepsilon\|\boldsymbol{\theta}_{\varepsilon,i}) \quad (92)\\ &= \frac{1}{h}\int_0^\infty H(\boldsymbol{\alpha}_\varepsilon|\boldsymbol{\theta}_\varepsilon)\,\mathrm{d}\varepsilon = \frac{1}{h}\int_0^\infty\left[\frac{h(p_\varepsilon)}{1-p_\varepsilon}\middle| h\left(\frac{p_\varepsilon}{1+p_\varepsilon}\right)\right]\mathrm{d}\varepsilon \end{aligned}$$

$$\mathrm{C}(\mathbf{K}_{\mathrm{B-E|F-D}}) = = \frac{1}{h}\int_0^\infty \frac{h[p_\varepsilon(\mathcal{W})]}{1-p_\varepsilon(\mathcal{W})}\mathrm{d}\varepsilon - \frac{1}{h}\int_0^\infty\left[\frac{h(p_\varepsilon)}{1-p_\varepsilon}\middle| h\left(\frac{p_\varepsilon}{1+p_\varepsilon}\right)\right]\mathrm{d}\varepsilon \tag{93}$$

By (52) and (74) the average number of particles on the input of a narrow–band component $\mathcal{K}_\varepsilon$ is

$$\mathcal{W}_\varepsilon = \sum_{i \in \mathbf{S}} i \cdot q(i|\boldsymbol{\theta}_\varepsilon) \geq \left[0 \,\middle|\, \frac{2p_\varepsilon^2}{1 - p_\varepsilon^2} \right] \tag{94}$$

and then for the *whole average number of input particles* $\mathcal{W}'$ of the wide–band transfer channel $\mathbf{K}$ is obtained

$$\mathcal{W}' = \frac{1}{h} \int_0^\infty \mathcal{W}_\varepsilon \, \mathrm{d}\varepsilon, \;\; W = \frac{1}{h} \int_0^\infty \varepsilon \, \mathcal{W}_\varepsilon \, \mathrm{d}\varepsilon \tag{95}$$

where W is *whole input energy* and T_W is the *effective coding temperature* being supposingly at the value T_{W_ε}, $T_W = T_{W_\varepsilon}$, $\forall \varepsilon \in \mathbf{S}(\boldsymbol{\varepsilon})$.

5.2. Bose–Einstein wide–band channel capacity

By derivations (86) and (92), (93) is valid that [12]

$$C(\mathbf{K}_{\mathrm{B-E}}) = \frac{1}{h} \int_0^\infty \frac{h[p_\varepsilon(\mathcal{W})]}{1 - p_\varepsilon(\mathcal{W})} \, \mathrm{d}\varepsilon - \frac{1}{h} \int_0^\infty \frac{h(p_\varepsilon)}{1 - p_\varepsilon} \, \mathrm{d}\varepsilon \tag{96}$$

For the first or, for the second integral respectively, obviously is valid

$$\frac{1}{h} \int_0^\infty \frac{h[p_\varepsilon(\mathcal{W})]}{1 - p_\varepsilon(W)} \, \mathrm{d}\varepsilon = \frac{\pi^2 \mathrm{k} T_W}{3h} \quad \text{resp.} \quad \frac{1}{h} \int_0^\infty \frac{h(p_\varepsilon)}{1 - p_\varepsilon} \, \mathrm{d}\varepsilon = \frac{\pi^2 \mathrm{k} T_0}{3h} \tag{97}$$

Then, for the capacity of the wide–band B–E transfer channel $\mathbf{K}_{\mathrm{B-E}}$ is valid

$$C(\mathbf{K}_{\mathrm{B-E}}) = \frac{\pi^2 \mathrm{k}}{3h} (T_W - T_0) = \frac{\pi^2 \mathrm{k} T_W}{3h} \cdot \frac{T_W - T_0}{T_W} \triangleq \frac{\pi^2 \mathrm{k} T_W}{3h} \cdot \eta_{\max}, \; T_W \geq T_0 \tag{98}$$

and for the *whole average output energy* is valid

$$\lim_{\tau \to \infty} \frac{1}{\tau} \sum_{\varepsilon \in \mathbf{S}(\boldsymbol{\varepsilon})} \varepsilon \frac{p_\varepsilon(\mathcal{W})}{1 - p_\varepsilon(\mathcal{W})} = \frac{1}{h} \int_0^\infty \varepsilon \frac{p_\varepsilon(\mathcal{W})}{1 - p_\varepsilon(\mathcal{W})} \, \mathrm{d}\varepsilon = -\frac{\mathrm{k}^2 T_W^2}{h} \int_0^1 \frac{\ln(1 - t)}{t} \, \mathrm{d}t = \frac{\pi^2 \mathrm{k}^2 T_W^2}{6h} \tag{99}$$

For the *whole average energy* of the B–E *noise* must be valid

$$\lim_{\tau \to \infty} \frac{1}{\tau} \sum_{\varepsilon \in \mathbf{S}(\boldsymbol{\varepsilon})} \varepsilon \frac{p_\varepsilon}{1 - p_\varepsilon} = \frac{1}{h} \int_0^\infty \varepsilon \frac{p_\varepsilon}{1 - p_\varepsilon} \, \mathrm{d}\varepsilon = \frac{\pi^2 \mathrm{k}^2 {T_0}^2}{6h} \tag{100}$$

From the relations (79) among the energies of the output α', of the noise j' and the input i',

$$\frac{\pi^2 \mathrm{k}^2 T_W^2}{6h} = \frac{\pi^2 \mathrm{k}^2 {T_0}^2}{6h} + W \tag{101}$$

the effective coding temperature T_W is derivable, $T_W = T_0 \cdot \sqrt{1 + \frac{6hW}{\pi^2 \mathrm{k}^2 {T_0}^2}}$. Using it in (98) gives

$$C(\mathbf{K}_{\mathrm{B-E}}) = \frac{\pi^2 \mathrm{k} T_0}{3h} \left(\sqrt{1 + \frac{6hW}{\pi^2 \mathrm{k}^2 {T_0}^2}} - 1 \right) \tag{102}$$

For $T_0 \to 0$ the *quantum aproximation* of $C(\mathbf{K}_{\mathrm{B-E}})$, independent on the heat noise energy (deminishes whith temperture's aiming to absolute 0° K)

$$\lim_{T_0 \to 0} C(\mathbf{K}_{\mathrm{B-E}}) = \lim_{T_0 \to 0} \left(\sqrt{\frac{\pi^4 \mathrm{k}^2 {T_0}^2}{9h^2} + \pi^2 \frac{2W}{3h}} - \frac{\pi^2 \mathrm{k} T_0}{3h} \right) = \pi \sqrt{\frac{2W}{3h}} \tag{103}$$

The *classical approximation* of $C(\mathbf{K}_{\mathrm{B-E}})$ is gaind for temperatures $T_0 \gg 0$ ($T_0 \to \infty$ respectively). It is near to value $\frac{W}{\mathrm{k}T_0}$, the Shannon capacity of the wide–band **Gaussian** channel with the whole noise energy $\mathrm{k}T_0$ and with the whole average input energy W. For T_0 from (101), great enough, is gained that[8]

$$C(\mathbf{K}_{\mathrm{B-E}}) \doteq \frac{\pi^2 \mathrm{k} T_0}{3h} \left(\frac{3hW}{\pi^2 \mathrm{k}^2 {T_0}^2} \right) = \frac{W}{\mathrm{k}T_0} \tag{104}$$

5.3. Fermi–Dirac wide–band channel capacity

By derivations (86) and (92), (93) is valid that [12]

$$C(\mathbf{K}_{\mathrm{F-D}}) = \frac{1}{h} \int_0^\infty \frac{h[p_\varepsilon(\mathcal{W})]}{1 - p_\varepsilon(\mathcal{W})} \,\mathrm{d}\varepsilon - \frac{1}{h} \int_0^\infty h\left(\frac{p_\varepsilon}{1 + p_\varepsilon} \right) \mathrm{d}\varepsilon = \frac{\pi^2 \mathrm{k} T_W}{3h} - \frac{1}{h} \int_0^\infty h\left(\frac{p_\varepsilon}{1 + p_\varepsilon} \right) \mathrm{d}\varepsilon \tag{105}$$

For the second integral obviously is valid

$$\frac{1}{h} \int_0^\infty h\left(\frac{p_\varepsilon}{1 + p_\varepsilon} \right) \mathrm{d}\varepsilon = \frac{\pi^2 \mathrm{k} T_0}{6h} \tag{106}$$

By figuring (105) the capacity of the wide–band F–D channel $\mathbf{K}_{\mathrm{F-D}}$ is gained,

$$C(\mathbf{K}_{\mathrm{F-D}}) = \frac{\pi^2 \mathrm{k}}{3h} \left(T_W - \frac{T_0}{2} \right) \tag{107}$$

and for $T_W > T_0$is writable

$$C(\mathbf{K}_{\mathrm{F-D}}) = C(\mathbf{K}_{\mathrm{B-E}}) \cdot \frac{2T_W - T_0}{2T_W - 2T_0} \tag{108}$$

For the *whole average output energy* is valid the same as for the B–E case,

$$\frac{1}{h} \int_0^\infty \varepsilon \, \frac{p_\varepsilon(\mathcal{W})}{1 - p_\varepsilon(\mathcal{W})} \,\mathrm{d}\varepsilon = \frac{\pi^2 \mathrm{k}^2 T_W^2}{6h} \tag{109}$$

For the *whole average F–D wide–band noise* energy is being derived

$$\begin{aligned} \lim_{\tau \to \infty} \frac{1}{\tau} \sum_{\varepsilon \in \mathbf{S}(\varepsilon)} \varepsilon \frac{p_\varepsilon}{1 + p_\varepsilon} &= \frac{1}{h} \int_0^\infty \varepsilon \frac{e^{-\frac{\varepsilon}{\mathrm{k}T_0}}}{1 + e^{-\frac{\varepsilon}{\mathrm{k}T_0}}} \,\mathrm{d}\varepsilon = \frac{\mathrm{k}^2 {T_0}^2}{h} \int_0^\infty x \, \frac{e^{-x}}{1 + e^{-x}} \,\mathrm{d}x \\ &= -\frac{\mathrm{k}^2 {T_0}^2}{h} \int_0^1 \frac{\ln t}{t + 1} \,\mathrm{d}t = \frac{\pi^2 \mathrm{k}^2 {T_0}^2}{12h} \end{aligned} \tag{110}$$

[8] For $|x| < 1$, $\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \ldots \doteq 1 + \frac{1}{2}x$ where $x = \dfrac{6hW}{\pi^2 \mathrm{k}^2 {T_0}^2} < 1$.

From the relation (79) among the whole output, input, and noise energy,

$$\frac{\pi^2 \mathrm{k}^2 T_W^2}{6h} = \frac{\pi^2 \mathrm{k}^2 {T_0}^2}{12h} + W \tag{111}$$

follows the effective coding temperature $T_W = T_0 \cdot \sqrt{\frac{1}{2} + \frac{6hW}{\pi^2 \mathrm{k}^2 {T_0}^2}}$. Using it in (107) the result is [24]

$$C(\mathbf{K}_{\mathrm{F-D}}) = \frac{\pi^2 \mathrm{k} T_0}{3h} \left(\sqrt{\frac{1}{2} + \frac{6hW}{\pi^2 \mathrm{k}^2 {T_0}^2}} - \frac{1}{2} \right) \tag{112}$$

For $T_0 \to 0$ the *quantum approximation* capacity $C(\mathbf{K}_{\mathrm{F-D}})$, independent on heat noise energy $\mathrm{k}T_0$ is gaind (the same as in the B–E case (103),

$$\lim_{T_0 \to 0} C(\mathbf{K}_{\mathrm{F-D}}) = \lim_{T_0 \to 0} \left(\sqrt{\frac{\pi^4 \mathrm{k}^2 {T_0}^2}{9h^2} \cdot \frac{1}{2} + \pi^2 \frac{2W}{3h}} - \frac{\pi^2 \mathrm{k} T_0}{3h} \cdot \frac{1}{2} \right) = \pi \sqrt{\frac{2W}{3h}} \tag{113}$$

The *classical approximation* of the capacity $C(\mathbf{K}_{\mathrm{F-D}})$ is gained for $T_0 \gg 0$[9]

$$\begin{aligned} C(\mathbf{K}_{\mathrm{F-D}}) &= \frac{\pi^2 \mathrm{k} T_0}{3h} \left[\frac{1}{\sqrt{2}} \sqrt{1 + \frac{12hW}{\pi^2 \mathrm{k}^2 {T_0}^2}} - \frac{1}{2} \right] \doteq \frac{\pi^2 \mathrm{k} T_0}{3h} \left[\frac{1}{\sqrt{2}} \left(1 + \frac{6hW}{\pi^2 \mathrm{k}^2 {T_0}^2} \right) - \frac{1}{2} \right] \\ &= \frac{\pi^2 \mathrm{k} T_0}{6h} \left(\sqrt{2} - 1 \right) + \sqrt{2} \frac{W}{\mathrm{k} T_0} \quad \left[\underset{T_0 \to \infty}{\longrightarrow} \frac{\pi^2 \mathrm{k} T_0}{6h} \left(\sqrt{2} - 1 \right),\ W = \text{const.} \geq W_{crit} \right] \end{aligned} \tag{114}$$

By (74) the condition for the medium value of the input particles of a narrow–band component $\mathcal{K}_\varepsilon$, $\varepsilon \in \mathbf{S}(\varepsilon)$, of the channel $\mathbf{K}_{\mathrm{F-D}}$ is valid, $\mathcal{W}_\varepsilon \geq \frac{2p_\varepsilon^2}{1 - p_\varepsilon^2}$, from which the condition for the whole input energy of the wide–band channel $\mathbf{K}_{\mathrm{F-D}}$ follows. By (95) it is gaind, for $T_W \geq T_0 > 0$, that[10]

$$W \geq \lim_{\tau \to \infty} \frac{1}{\tau} \sum_{\varepsilon \in \mathbf{S}(\varepsilon)} \varepsilon \mathcal{W}_\varepsilon \geq \lim_{\tau \to \infty} \frac{1}{\tau} \sum_{\varepsilon \in \mathbf{S}(\varepsilon)} \varepsilon \frac{2p_\varepsilon^2}{1 - p_\varepsilon^2} \doteq \frac{2}{h} \int_0^\infty \varepsilon \frac{e^{-2\frac{\varepsilon}{\mathrm{k}T_0}}}{1 - e^{2\frac{\varepsilon}{\mathrm{k}T_0}}} \mathrm{d}\varepsilon = \frac{\pi^2 \mathrm{k}^2 {T_0}^2}{12h} = W_{crit} > 0 \tag{115}$$

6. Physical information transfer and thermodynamics

Whether the considered information transfers are narrow–band or wide–band, their algebraic-information description remains the same. So let be considered an arbitrary *stationary physical system* $\boldsymbol{\Psi}$ of these two band–types as usable for information transfer.

Let a system state $\boldsymbol{\theta}' = \sum_{i=1}^{n} q(i|\boldsymbol{\theta}')\, \boldsymbol{\pi}\{\psi_i'\} \in \boldsymbol{\Theta}$ of the system $\boldsymbol{\Psi}$ is the *successor (follower, equivocant)* of the system state $\boldsymbol{\theta} = \sum_{i=1}^{n} q(i|\boldsymbol{\theta})\, \boldsymbol{\pi}\{\psi_i\} \in \boldsymbol{\Theta}$, $\boldsymbol{\theta} \longrightarrow \boldsymbol{\theta}'$ is written. The

[9] For $\sqrt{1+x} \doteq 1 + \frac{1}{2}x$ when $|x| < 1$; $x = \frac{12hW}{\pi^2 \mathrm{k}^2 {T_0}^2}$.

[10] If, in the special case of F–D channel, it is considered that the value W given by the number of electrons as the average energy of the *modulating current* entering into a wire, over a *time unit*, then it is the average power on the electric resistor $R = 1\Omega$ too.

distribution $q(i|\boldsymbol{\theta}') = \sum_{j=1}^{n} p(i|j)\, q(j|\boldsymbol{\theta}),\ p(i|j) = (\psi_i', \psi_j)^2 = u_{ji}^2$ and $\boldsymbol{\theta} \longrightarrow \boldsymbol{\theta}'$, ensures existence of the *transformation matrix* $[u_{i,j}]$ of a *base* of the space $\Psi = \{\Psi\}_{i=1}^{n}$ into the base $\{\Psi'\}_{i=1}^{n}$.[11]

From the relation $\boldsymbol{\theta} \longrightarrow \boldsymbol{\theta}'$ also is visible that it is *reflexive* and *transitive* relation between states and, thus, it defines (a partial) *arrangment* on the set space $\boldsymbol{\Theta}$. The *terminal, maximal* state for this arrangement is the equilibrial state $\boldsymbol{\theta}^{+}$ of the system $\boldsymbol{\Psi}$: it is the successor of an arbitrary system state, including itself.

The *statistic, Shannon, information)* entropy $H(\cdot)$ is a generalization of the physical entropy$\mathcal{H}(\boldsymbol{\theta})$. The quantity I-divergence $I(\cdot\|\cdot)$ is, by (21), a generalization of the physical quantity $I(p\|d) = \mathcal{H}(\boldsymbol{\theta}^{+}) - \mathcal{H}(\boldsymbol{\theta})$ where the state

$$\boldsymbol{\theta}^{+} = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{\theta}_i \in \boldsymbol{\Theta} \quad \text{for} \quad \boldsymbol{\theta}_i = \boldsymbol{\pi}\{\psi_i\}, \quad i = 1,\ 2,\ ...,\ n \tag{116}$$

is the *equlibrial state* of the system $\boldsymbol{\Psi}$. The probability distribution into the *canonic components* $\boldsymbol{\theta}_i$ of $\boldsymbol{\theta}^{+}$ is *uniform* and thus

$$\mathcal{H}(\boldsymbol{\theta}^{+}) = \ln n = \ln \dim (\Psi) \tag{117}$$

Information divergence $I(p\|d) \geq\ 0$ expresses the *distance* of the two probability distributions $q(\cdot|\boldsymbol{\theta})$ and $q(\cdot|\boldsymbol{\theta}^{+})$ of states (stochastic quantities)

$$\boldsymbol{\theta} = [\mathbf{S}, q(\cdot|\boldsymbol{\theta})] \quad \text{and} \quad \boldsymbol{\theta}^{+} = [\mathbf{S}, q(\cdot|\boldsymbol{\theta}^{+})] \tag{118}$$

In the physical sense the divergence $I(p\|d)$ is a *measure of a not-equilibriality* of the state $\boldsymbol{\theta}$ of the physical (let say a thermodynamic) system $\boldsymbol{\Psi}$. Is maximized in the initial (starting), not-equilibrium state of the (time) evolution of the $\boldsymbol{\Psi}$. It is clear that $I(p\|d) \equiv T(\boldsymbol{\alpha};\boldsymbol{\theta})$

6.0.0.5. $\mathcal{H}$-Theorem, II. *Second Principle of Thermodynamics*:

Let for states $\boldsymbol{\theta}$, $\boldsymbol{\theta}' \in \boldsymbol{\Theta}$ of the system $\boldsymbol{\Psi}$ is valid that $\boldsymbol{\theta} \rightarrow \boldsymbol{\theta}'$. Then

$$\mathcal{H}(\boldsymbol{\theta}') \geq \mathcal{H}(\boldsymbol{\theta}) \tag{119}$$

and the equality arises for $\boldsymbol{\theta} = \boldsymbol{\theta}'$ only [12, 38].

6.0.0.6. Proof:

(a) For a *strictly convex* function $f(u) = u \cdot \ln u$ the ***Jensen** inequality* is valid [23]

$$f\left[\sum_{j=1}^{n} p(i|j)\, q(j|\boldsymbol{\theta})\right] \leq \sum_{j=1}^{n} p(i|j)\, f[q(j|\boldsymbol{\theta})], \quad i = 1,\ 2\ ,\ ...,\ n \tag{120}$$

$$\sum_{i=1}^{n} f\left[\sum_{j=1}^{n} p(i|j)\, q(j|\boldsymbol{\theta})\right] \leq \sum_{i=1}^{n} p(i|j) \sum_{j=1}^{n} f[q(j|\boldsymbol{\theta})]$$

$$= \sum_{j=1}^{n} f[q(j|\boldsymbol{\theta})] = \sum_{j=1}^{n} q(j|\boldsymbol{\theta}) \ln q(j|\boldsymbol{\theta}) = -H[q(\cdot|\boldsymbol{\theta})] = -\mathcal{H}(\boldsymbol{\theta}) \quad \text{due to} \quad \sum_{i=1}^{n} p(i|j) = 1$$

[11] It is the matrix of the *unitary operator* $\mathbf{u}(t)$ expressing the time evolution of the system $\boldsymbol{\Psi}$.

and for distributions $q(i|\boldsymbol{\theta}')$ is valid $\mathcal{H}(\boldsymbol{\theta}') \geq \mathcal{H}(\boldsymbol{\theta})$:

$$\sum_{i=1}^{n} f\left(\sum_{j=1}^{n} p(i|j)\, q(j|\boldsymbol{\theta})\right) = \sum_{i=1}^{n} q(i|\boldsymbol{\theta}') \ln q(i|\boldsymbol{\theta}') = -\mathcal{H}(\boldsymbol{\theta}') \ \ [\leq -\mathcal{H}(\boldsymbol{\theta})]$$

(b) The equality in (119) arises if and only if the index permutation $[i(1),\ i(2),\ ...,\ i(n)]$ exists that $p(i|j) = \delta[i|i(j)],\ j = 1,\ 2,\ ...,\ n;$ then $q[i(j)|\boldsymbol{\theta}'] = q(j|\boldsymbol{\theta}),\ j = 1,\ 2,\ ....$
Let a fixed j is given. Then, when $0 = p(i|j) = (\psi_i', \psi_j)^2,\ i \neq i(j)$, the orthogonality is valid

$$\Psi(\psi_j|\boldsymbol{\psi}_j) \perp \left[\bigoplus_{i \neq i(j)} \Psi(\psi_i'|\boldsymbol{\psi}_i')\right], \quad \boldsymbol{\psi}_j = \boldsymbol{\pi}\{\psi_j\} = \boldsymbol{\theta}_j, \quad \boldsymbol{\psi}_i' = \boldsymbol{\pi}\{\psi_i'\} = \boldsymbol{\theta}_i' \tag{121}$$

and, consequently, $\psi_j \in \Psi[\psi_{i(j)}'|\boldsymbol{\psi}_{i(j)}']$, $p[i(j)|j] = (\psi_{i(j)}', \psi_j)^2 = 1$. It results in $\psi_j = \psi_{i(j)}'$. This prooves that the equality $\mathcal{H}(\boldsymbol{\theta}') = \mathcal{H}(\boldsymbol{\theta})$ implies the equality $q(j|\boldsymbol{\theta})\, \boldsymbol{\pi}\{\psi_j\} = q[i(j)|\boldsymbol{\theta}]\, \boldsymbol{\pi}\{\psi_{i(j)}'\}$ and $\boldsymbol{\theta} = \boldsymbol{\theta}'$.

$\mathcal{H}$-theorem says, that a **reversible transition is not possible between any two different states** $\boldsymbol{\theta} \neq \boldsymbol{\theta}'$. From the inequality (119) also follows that any state $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ of the system $\boldsymbol{\Psi}$ is the successor of itself, $\boldsymbol{\theta} \to \boldsymbol{\theta}$ and, that any **reversibility** of the **relation $\boldsymbol{\theta} \to \boldsymbol{\theta}'$ (the transition $\boldsymbol{\theta}' \to \boldsymbol{\theta}$) is not possible within the system only, it is not possible without openning this system $\boldsymbol{\Psi}$**. The difference

$$\mathcal{H}(\boldsymbol{\theta}^+) - \mathcal{H}(\boldsymbol{\theta}) = \max_{\boldsymbol{\theta}' \in \boldsymbol{\Theta}} \mathcal{H}(\boldsymbol{\theta}') - \mathcal{H}(\boldsymbol{\theta}) = H\left[q(\cdot|\boldsymbol{\theta}^+)\right] - H\left[q(\cdot|\boldsymbol{\theta})\right] \tag{122}$$

reppresents the information-theoretical expressing of the ***Brillouin*** *(maximal) entropy defect* ΔH (the Brillouin *negentropic information principle* [2, 30]). For the state $\boldsymbol{\theta}^+$ is valid that $\boldsymbol{\theta} \to \boldsymbol{\theta}^+, \forall \boldsymbol{\theta} \in \boldsymbol{\Theta}$. It is also called the *terminal* state or the (*atractor* of the time evolution) of the system $\boldsymbol{\Psi}$.[12]

6.0.0.7. Gibbs Theorem:

For all $\boldsymbol{\theta},\ \tilde{\boldsymbol{\theta}} \in \boldsymbol{\Theta}$ of the system $\boldsymbol{\Psi}$ is valid

$$\mathcal{H}(\boldsymbol{\theta}) \leq -\mathrm{Tr}(\boldsymbol{\theta} \ln \tilde{\boldsymbol{\theta}}) \tag{123}$$

and the equality arises only for $\boldsymbol{\theta} = \tilde{\boldsymbol{\theta}}$ [38].

6.0.0.8. Proof:

Let for $\boldsymbol{\theta},\ \tilde{\boldsymbol{\theta}} \in \boldsymbol{\Theta}$ is valid that $\boldsymbol{\theta} = \sum_{i=1}^{n} q(i|\boldsymbol{\theta})\, \boldsymbol{\pi}\{\psi_i\},\ \tilde{\boldsymbol{\theta}} = \sum_{i=1}^{n} q(i|\tilde{\boldsymbol{\theta}})\, \boldsymbol{\pi}\{\psi_i'\}$ and let the operators $\boldsymbol{\alpha},\ \boldsymbol{\theta}$ are *commuting* $\boldsymbol{\alpha\theta} = \boldsymbol{\theta\alpha},\ D(\boldsymbol{\alpha}) = \{D_\alpha : \alpha \in \mathbf{S}(\boldsymbol{\alpha})\},\ D(\boldsymbol{\theta}) = \{D_\theta : \theta \in \mathbf{S}(\boldsymbol{\theta})\}$, are their spectral decompositions. Let be the state $\boldsymbol{\theta}'$ the successor of $\boldsymbol{\theta},\ \boldsymbol{\theta} \to \boldsymbol{\theta}'$ and relations $p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}) = \sum_{i \in D_\alpha} q(i|\boldsymbol{\theta}') = p(\alpha|\boldsymbol{\alpha}|\boldsymbol{\theta}')$ and $\mathcal{H}(\boldsymbol{\theta}') \geq \mathcal{H}(\boldsymbol{\theta})$ are valid. For the matrix (θ_{ij}) of the

[12] In this sense, the physical entropy $\mathcal{H}(\boldsymbol{\theta})$ (19), (21) determines the direction of the *thermodynamic time arrow* [2], $\dfrac{\mathcal{H}(\boldsymbol{\theta}') - \mathcal{H}(\boldsymbol{\theta})}{\Delta t} = \dfrac{\partial \mathcal{H}}{\partial t} \geq 0, \quad \Delta t = t_{\theta'} - t_\theta > 0$. The equality occurs in the *equlibrial (stationary)* state $\boldsymbol{\theta}^+$ of the system $\boldsymbol{\Psi}$ and its environment.

operator $\boldsymbol{\theta}$ in the base $\{\psi_1', \psi_2', \ldots, \psi_n'\}$ is obtained that $\theta_{ij} = \sum_{k=1}^{n} u_{ki}\, u_{kj}\, q(k|\boldsymbol{\theta})$ and thus for operators $\ln\tilde{\boldsymbol{\theta}}$ and $\mathrm{Tr}(\boldsymbol{\theta}\ln\tilde{\boldsymbol{\theta}})$ is valid that

$$\ln\tilde{\boldsymbol{\theta}} = \sum_{i=1}^{n} \ln q(i|\tilde{\boldsymbol{\theta}})\, \pi\{\psi_i'\} \text{ and } -\mathrm{Tr}(\boldsymbol{\theta}\ln\tilde{\boldsymbol{\theta}}) = -\sum_{i=1}^{n}\left[\sum_{k=1}^{n} u_{ki}^2\, q(k|\boldsymbol{\theta})\right]\ln q(i|\tilde{\boldsymbol{\theta}}) = -\sum_{i=1}^{n} q(i|\boldsymbol{\theta}')\ln q(i|\tilde{\boldsymbol{\theta}}) \tag{124}$$

For the information divergence of the distributions $q(\cdot|\boldsymbol{\theta}')$, $q(\cdot|\tilde{\boldsymbol{\theta}})$ and the entropy $\mathcal{H}(\boldsymbol{\theta}')$ is valid that

$$I[q(\cdot|\boldsymbol{\theta}')\|q(\cdot|\tilde{\boldsymbol{\theta}})] = \sum_{i=1}^{n} q(i|\boldsymbol{\theta}')\ln\frac{q(i|\boldsymbol{\theta}')}{q(i|\tilde{\boldsymbol{\theta}})} \geq 0, \quad -\mathcal{H}(\boldsymbol{\theta}') \geq \sum_{i=1}^{n} q(i|\boldsymbol{\theta}')\ln q(i|\tilde{\boldsymbol{\theta}}). \tag{125}$$

By (119) for $\boldsymbol{\theta} \to \boldsymbol{\theta}'$ is writable that $\mathcal{H}(\boldsymbol{\theta}) \leq \mathcal{H}(\boldsymbol{\theta}') \leq -\mathrm{Tr}(\boldsymbol{\theta}\ln\tilde{\boldsymbol{\theta}})$. By (123) $-\mathrm{Tr}(\boldsymbol{\theta}'\ln\tilde{\boldsymbol{\theta}}) \geq \mathcal{H}(\boldsymbol{\theta}') \geq \mathcal{H}(\boldsymbol{\theta})$ are valid; the first equality is for

$$I[q(\cdot|\boldsymbol{\theta}')\|q(\cdot|\tilde{\boldsymbol{\theta}})] = 0,\ \mathcal{H}(\boldsymbol{\theta}') = \mathcal{H}(\tilde{\boldsymbol{\theta}}),\ q(i|\boldsymbol{\theta}') = q(i|\tilde{\boldsymbol{\theta}}),\quad i = 1, 2, \ldots, n,\ \boldsymbol{\theta}' = \tilde{\boldsymbol{\theta}} \tag{126}$$

the second equality is for $\boldsymbol{\theta}' = \boldsymbol{\theta}$. The **Gibbs theorem expresses, in the deductive (matematical-logical) way, the phenomenon of Gibbs paradox**.[13]

From formulas (47), (55), (56) and (68), (72), (73) for the narrow-band B–E and F–D capacities follows that

$$e^{-\frac{\varepsilon}{kT_W}} \cdot e^{\frac{\varepsilon}{kT_0}} \geq 1,\ e^{\frac{\varepsilon}{kT_0}\left(\frac{T_W - T_0}{T_W}\right)} \geq e^0;\ \varepsilon > 0,\ T_0 > 0 \longrightarrow T_W \geq T_0 \longrightarrow \frac{T_W - T_0}{T_W} \triangleq \eta_{\max} \geq 0 \tag{127}$$

and it is seen that the quantity temperature is decisive for studied information transfers. The last relation envokes, inevitably, such an opinion, that these transfers are able be modeled by a *direct* reversible **Carnot** cycle with efficiency $\eta_{max} \in (0, 1\rangle)$. Conditions leading to $C_{[\cdot|\cdot]} < 0$ mean, in such a *direct* thermodynamic model, that its efficiency should be $\eta_{\max} < 0$. This is the contradiction with the *Equivalence Principle of Thermodynamics* [19]; expresses only that the transfer is running in the opposite direction (as for temperatures).

As for B–E channel; for the supposition $W < 0$ the inequalities $T_W < T_0$ and $p(\mathcal{W}) < p$ would be gained which is the *contradiction* with (35), (36) and (47). It would be such a situation with the *information is transferred in a different direction and under a different operation mode*. Our sustaining on the meaning about the original organization of the transfer, for $T_W > T_0$, then leads to the contradiction mentioned above saying only that we are convinced mistakenly about the actual direction of the information transfer. In the case $T_W = T_0$ for the capacity $C_{\mathrm{B-E}''}$ from (50) is valid that $C_{\mathrm{B-E}''} = 0$. Then $W = W_{Krit}\ [= 0]$ for $p(\mathcal{W}) = p$.

As for F–D channel; for the supposition $\mathcal{W} < \dfrac{2p^2}{1-p^2}$ $T_W < T_0$ and $p(\mathcal{W}) < p$ is gained which is the contradiction with (68). For $T_W = T_0$ is for $C_{\mathrm{F-D}}$ from (71) valid

[13] Derived by the information-thermodynamic way together with the *I.* and *II. Thermodynamic Principle* and with the *Equivalence Principle of Thermodynamics in [16, 17, 19].*

$C_{\mathrm{F-D}} = \dfrac{h(p)}{1-p} + \dfrac{p}{1+p} \cdot \ln p - \ln(1+p)$.[14]

Let be noticed yet the relations between the wide–band B–E and F–D capacities and the model heat efficiency η_{max}. For the B–E capacity (98) is gained that

$$C(\mathbf{K}_{\mathrm{B-E}}) = \frac{\pi^2 \mathrm{k} T_W}{3h}\left(\frac{T_W - T_0}{T_W}\right) = \frac{\pi^2 \mathrm{k} T_W}{3h}\,\eta_{\max} \underset{\eta_{\max}\to 1}{\longrightarrow} \frac{\pi^2 \mathrm{k} T_W}{3h} = C^{\max}(\mathbf{K}_{\mathrm{B-E}}) \quad (128)$$

$$C^{\max}(\mathbf{K}_{\mathrm{B-E}}) = \sup_{\boldsymbol{\theta}} H(\boldsymbol{\alpha}\|\boldsymbol{\theta}) = H(\boldsymbol{i}) = \mathcal{H}(\boldsymbol{\theta})$$

$$C(\mathbf{K}_{\mathrm{B-E}}) > 0,\ T_W > T_0, \quad C(\mathbf{K}_{\mathrm{B-E}}) \underset{\substack{T_W \to T_0 \\ (\eta_{\max}\to 0)}}{\longrightarrow} 0,\ T_W \longrightarrow T_0$$

It is the information capacity for such a direct Carnot cycle where $H(X) = \dfrac{\pi^2 \mathrm{k} T_W}{3h} = C^{\max}(\mathbf{K}_{\mathrm{B-E}})$.

For the wide-band F–D capacity from (105) is valid

$$C(\mathbf{K}_{\mathrm{F-D}}) = \frac{\pi^2 \mathrm{k} T_W}{3h} - \frac{\pi^2 \mathrm{k} T_0}{6h},\ T_W \geq T_0 \ \text{ and for }\ T_W > T_0, \quad (129)$$

$$C(\mathbf{K}_{\mathrm{F-D}}) = \frac{\pi^2 \mathrm{k} T_W}{3h} \cdot \frac{2T_W - T_0}{2T_W} = \frac{\pi^2 \mathrm{k} T_W}{3h} \cdot \frac{2T_W - T_0}{2(T_W - T_0)} \cdot \eta_{\max}$$

$$= C(\mathbf{K}_{\mathrm{B-E}}) \cdot \frac{2T_W - T_0}{2(T_W - T_0)}$$

Due to $1 - \eta_{\max} = \dfrac{T_0}{T_W}$ is valid $T_0 = T_W(1 - \eta_{\max})$ and also $C(\mathbf{K}_{\mathrm{F-D}}) = \dfrac{\pi^2 \mathrm{k} T_W}{6h} \cdot (1 + \eta_{\max})$. Then,

$$C\ (\mathbf{K}_{\mathrm{F-D}}) = \underset{\eta_{\max}\to 1}{\longrightarrow} \frac{\pi^2 \mathrm{k} T_W}{3h}, \quad (130)$$

$$C\ (\mathbf{K}_{\mathrm{F-D}}) \underset{\substack{T_W \to T_0 \\ (\eta_{\max}\to 0)}}{\longrightarrow} \frac{1}{2} H(\boldsymbol{i}) = \frac{1}{2}\,\mathcal{H}(\boldsymbol{\theta}) = \frac{\pi^2 \mathrm{k} T_W}{6h}$$

$$C\ (\mathbf{K}_{\mathrm{F-D}}) \in \left\langle \frac{\pi^2 \mathrm{kT_W}}{6\mathrm{h}}, \frac{\pi^2 \mathrm{kT_W}}{3\mathrm{h}} \right\rangle = \left\langle \frac{1}{2}\,\mathcal{H}(\boldsymbol{\theta}), \mathcal{H}(\boldsymbol{\theta}) \right\rangle$$

Again the phenomenon of the not-zero capacity is seen here when the difference between the coding temperature T_W and the noise temperature T_0 is zero. Capacities $C(\mathbf{K}_{\mathrm{F-D}}) \geq 0$ are, surely, considerable for $T_W \in \langle \frac{T_0}{2}, T_0 \rangle$ and being given by the property of the F–D phase space cells. Capacities $C(\mathbf{K}_{\mathrm{B-E}}) < 0$ and $C(\mathbf{K}_{\mathrm{F-D}}) < 0$ are without sense for the given direction of information transfer.

[14] Nevertheless the capacity $C_{\mathrm{F-D}}$ for this case $W < W_{\mathrm{Krit}}$ is set in [12, 13]. *Similar* results as this one and (74) are gained for the **Maxwell–Boltzman** (M–B) system in [13].

Nevertheless, it will be shown that all these processes themselves are not organized cyclically 'by themselves'.

Further the relation between the information transfer in a wide–band B–E (photonic) channel *organized in a cyclical way* and a relevant (reverse) heat cycle will be dealt with. But, firstly, the way in which the capacity formula for an information transfer system of photons was derived in [30] will be reviewed.

6.1. Thermodynamic derivation of wide–band photonic capacity

A transfer channel is now created by the electromagnetic radiation of a system $\mathcal{L} \cong \mathbf{K}_{\mathrm{L-L}}$ of photons being emitted from an *absolute black body* at temperature T_0 and within a frequency bandwidth of $\Delta\nu = R^+$, where ν is the frequency. Then the energy of such radiation is the *energy* of *noise*. A source of *input messages, signals* transmits monochromatic electromagnetic impulses (numbers a_i of photons) into this environment with an average *input energy* W. This source is defined by an alphabet of input messages, signals $\{a_i\}_{i=1}^n$, with a probability distribution $p_i = p(a_i)$, $i = 1, 2, \ldots, n$.[15] The *output (whole, received)* signal is created by *additive* superposition of the input signal and the noise signal. The input signal a_i, within a frequency ν, is represented by the *occupation* number $m = m(\nu)$, which equates to the number of photons of an input field with an energy level $\varepsilon(\nu) = h\nu$. The output signal is represented by the occupation number $l = l(\nu)$. The noise signal, created by the number of photons emitted by absolute black body radiation at temperature T_0, is represented by the occupation number $n = n(\nu)$. The medium values of these quantities (*spectral* densities of the input, noise and output photonic stream) are denoted as $\overline{m}$, $\overline{n}$ and $\overline{l}$. In accordance with the ***Planck** radiation* law, the spectral density $\bar{r}$ of a photonic stream of absolute black body radiation at temperature Θ and within frequency ν, is given by the ***Planck** distribution*,

$$\bar{r}(\nu) = \frac{p(\nu,\Theta)}{1-p(\nu,\Theta)}, \quad \overline{n}(\nu) = \frac{p(\nu,T_0)}{1-p(\nu,T_0)}, \quad \bar{l}(\nu) = \frac{p(\nu,T_W)}{1-p(\nu,T_W)}, \quad p(\nu,\Theta) = e^{-\dfrac{h\nu}{k\Theta}} \tag{131}$$

Thus, for the average energy P of radiation at temperature Θ within the bandwidth $\Delta\nu = R^+$ is gained that

$$P(\Theta) = \int_0^\infty \bar{\varepsilon}(\nu,\Theta)\mathrm{d}\nu = \frac{\pi^2 \mathrm{k}^2 \Theta^2}{6\hbar} \ \text{ where } \ \bar{\varepsilon}(\nu,\Theta) = \bar{r}(\nu)h\nu \ \text{ and } \ \frac{\mathrm{d}P(\Theta)}{\mathrm{d}\Theta} = \frac{\pi^2 \mathrm{k}^2 \Theta}{3\hbar}. \tag{132}$$

Then, for the average noise energy P_1 at temperature T_0, and for the average output energy P_2 at temperature T_W, both of which occur within the bandwidth $\Delta\nu = R^+$ is valid that

$$P_1(T_0) = \frac{\pi^2 \mathrm{k}^2 {T_0}^2}{6\hbar}, \ \ P_2(T_W) = \frac{\pi^2 \mathrm{k}^2 {T_W}^2}{6\hbar}. \tag{133}$$

The entropy H of radiation at temperature ϑ is derived from ***Clausius** definition* of *heat entropy* and thus

$$H = \int_0^\vartheta \frac{1}{\mathrm{k}\Theta} \frac{\mathrm{d}P(\Theta)}{\mathrm{d}\Theta} \mathrm{d}\Theta = \frac{\pi^2 \mathrm{k}\vartheta}{3\hbar} = \frac{2P(\vartheta)}{\mathrm{k}\vartheta} \tag{134}$$

[15] To distinguish between two frequencies mutually deferring at an infinitesimally small $\mathrm{d}\nu$ is needed, in accordance with ***Heisenberg** uncertainty principle*, a time interval spanning the infinite length of time, $\Delta t \longrightarrow \infty$; analog of the *thermodynamic stationarity*.

Thus, for the entropy H_1 of the noise signal and for the entropy H_2 of the ouptut signal on the channel $\mathcal{L}$ is

$$H_1 = \frac{\pi^2 \mathrm{k} T_0}{3\hbar} = \frac{2P_1}{\mathrm{k}T_0}, \quad H_2 = \frac{\pi^2 \mathrm{k} T_W}{3\hbar} = \frac{2P_2}{\mathrm{k}T_W} \tag{135}$$

The information capacity C_{T_W,T_0} of the wide-band photonic transfer channel $\mathcal{L}$ is given by the maximal *entropy defect* [2, 30]) by

$$C_{T_0,T_W} = H_2 - H_1 = \int_{T_0}^{T_W} \frac{1}{\mathrm{k}\Theta} \frac{\mathrm{d}P(\Theta)}{\mathrm{d}\Theta} \mathrm{d}\Theta = \frac{\pi^2 \mathrm{k}}{3\hbar} \int_{T_0}^{T_W} \mathrm{d}\Theta = \frac{\pi^2 \mathrm{k}}{3\hbar} \cdot (T_W - T_0)\,. \tag{136}$$

For $P_2 = P_1 + W$, where W is the average energy of the input signal is then valid that

$$\frac{\pi^2 \mathrm{k}^2 {T_W}^2}{6h} = \frac{\pi^2 \mathrm{k}^2 {T_0}^2}{6h} + W \quad \longrightarrow \quad T_W = T_0 \cdot \sqrt{1 + \frac{6h \cdot W}{\pi^2 \mathrm{k}^2 {T_0}^2}} \tag{137}$$

Then, in accordance with (102), (103), (104) [30]

$$C_{T_0,W}(\mathbf{K}_{\mathrm{L-L}}) = \frac{\pi^2 \mathrm{k} T_0}{3h} \cdot \left(\sqrt{1 + \frac{6h \cdot W}{\pi^2 \mathrm{k}^2 {T_0}^2}} - 1 \right) \tag{138}$$

7. Reverse heat cycle and transfer channel

A *reverse* and reversible Carnot cycle $\mathcal{O}_{rrev}$ starts with the *isothermal expansion* at temperature T_0 (the *diathermic* contact [31] is made between the system $\mathcal{L}$ and the cooler $\mathcal{B}$) when $\mathcal{L}$ is receiving the *pumped out, transferred* heat ΔQ_0 from the $\mathcal{B}$. During the *isothermal compression*, when the temperature of both the system $\mathcal{L}$ and the heater $\mathcal{A}$ is at the same value T_W, $T_W > T_0 > 0$, the *output* heat ΔQ_W is being delivered to the $\mathcal{A}$

$$\Delta Q_W = \Delta Q_0 + \Delta A \tag{139}$$

where ΔA is the *input* mechanical energy (work) delivered into $\mathcal{L}$ during this isothermal compression. It follows from [2, 8, 28] that when an average amount of information ΔI is being *recorded, transmitted, computed*, etc. at temperature Θ, there is a need for the average energy $\Delta W \geq k \cdot \Theta \cdot \Delta I$; at this case $\Delta W \triangleq \Delta A$. Thus $\mathcal{O}_{rrev}$ is considerable as a *thermodynamic model* of information transfer process in the channel $\mathcal{K} \cong \mathcal{L}$ [14]. The following values are *changes* of the information entropies defined on $\mathcal{K}$[16]:

$$H(Y) \triangleq \frac{\Delta Q_W}{\mathrm{k}T_W} \;\; output \; (\triangleq \Delta I), \;\; H(X) \triangleq \frac{\Delta A}{\mathrm{k}T_W} \;\; input, \;\; H(Y|X) \triangleq \frac{\Delta Q_0}{\mathrm{k}T_W} \;\; noise \tag{140}$$

where k is Boltzman constant. The information transfer in $\mathcal{K} \cong \mathcal{L}$ is *without losses* caused by the *friction, noise heat* ($\Delta Q_{0x} = 0$) and thus $H(X|Y) = 0$.

By assuming that for the changes (140) and $H(X|Y) = 0$ the channel equation (4), (5) and (23) is valid The result is

$$\begin{aligned} T(X;Y) &= \frac{\Delta A}{\mathrm{k}T_W} - 0 = \frac{\Delta Q_W}{\mathrm{k}T_W} \cdot \eta_{max} = H(X) \\ T(Y;X) &= \frac{\Delta Q_0 + \Delta A}{\mathrm{k}T_W} - \frac{\Delta Q_0}{\mathrm{k}T_W} = \frac{\Delta A}{\mathrm{k}T_W} = H(X). \end{aligned} \tag{141}$$

[16] In *information* units *Hartley, nat, bit*; $H(\cdot) = \Delta H(\cdot)$, $H(\cdot|\cdot) = \Delta H(\cdot|\cdot)$.

But the other *information arrangement, description* of a revese Carnot cycle will be used further, given by

$$\Delta Q_0 \sim H(X),\ \ \Delta Q_W \sim H(Y)\ \text{ and }\ \Delta A \sim H(Y|X),\ \ H(X|Y) = 0 \tag{142}$$

In a general (reversible) *discrete* heat cycle $\mathcal{O}$ (with temperatures of its heat reservoires changing in a discrete way) considered as a model of the information transfer process in an transfer channel $\mathbf{K} \cong \mathcal{L}$ [17, 19] is, for the *elementary* changes $H(\Theta_k) \cdot \eta_{[max_k]}$ of information entropies of $\mathcal{L}$, valid that[17]

$$H(\Theta_k) \cdot \eta_{[max_k]} \triangleq \frac{\Delta Q(\Theta_k)}{\mathrm{k}\Theta_k} \cdot \eta_{[max_k]},\ \ k = 1,\ 2,\ ...,\ n \tag{143}$$

where $n \geq 2$ is the maximal number of its *elementary* Carnot cycles $\mathcal{O}_k$.[18] The change of heat of the system $\mathcal{L}$ at temperatures Θ_k is $\Delta Q(\Theta_k)$.

In a general (reversible) *continuous* cycle $\mathcal{O}$ [with temperatures changing continuously, $n \longrightarrow \infty$ in the previous discrete system, at Θ will be $\Delta Q(\Theta)$] considered as an information transfer process in a transfer channel $\mathbf{K} \cong \mathcal{L}$ is valid that

$$\mathrm{d}H(\Theta) \triangleq \frac{\delta Q(\Theta)}{\mathrm{k}\Theta} = \frac{\frac{\partial Q(\Theta)}{\partial \Theta}\mathrm{d}\Theta}{\mathrm{k}\Theta}\ \text{and}\ H(\Theta) = \int_0^{\Theta} \frac{\delta Q(\theta)}{\mathrm{k}\theta}\mathrm{d}(\theta);\ \ \Delta Q(\Theta) = \int_0^{\Theta} \delta Q(\theta)\mathrm{d}\theta \tag{144}$$

For the whole cycle $\mathcal{O}_{rrev}$, $T_W > T_0 > 0$, let be $H(X|Y) = 0$ and then

$$\begin{aligned}
H(X) &= \frac{S(T_W)}{\mathrm{k}} - \frac{S(T_0)}{\mathrm{k}} = \oint_{\mathcal{O}_{rrev}} \frac{\delta A(\Theta)}{\mathrm{k}\Theta} = \int_{T_0}^{T_W} \mathrm{d}H(\Theta) = \frac{2\Delta Q_W}{\mathrm{k}{T_W}^2} \cdot (T_W - T_0) \qquad (145)\\
H(Y) &= \frac{S(T_W)}{\mathrm{k}} = \int_0^{T_W} \frac{\delta Q_W(\Theta_W)}{\mathrm{k}\Theta_W} = \int_0^{T_W} \mathrm{d}H(\Theta) = \frac{2\Delta Q_W}{\mathrm{k}T_W} \quad [H(X) = H(Y) \cdot \eta_{\max}]\\
H(Y|X) &= H(Y) - H(X) = \int_0^{T_W} \mathrm{d}H(\Theta) - \int_{T_0}^{T_W} \mathrm{d}H(\Theta) = \int_0^{T_0} \mathrm{d}H(\Theta)\\
&= \int_0^{T_0} \frac{\partial Q_W(\Theta)}{\mathrm{k}\Theta} = \frac{S(T_0)}{\mathrm{k}} = \frac{2\Delta Q_W}{\mathrm{k}{T_W}^2} \cdot T_0 = \frac{2\Delta Q_W}{\mathrm{k}T_W} \cdot \frac{T_0}{T_W} = H(X) \cdot \beta\\
T(Y;X) &= H(Y) - H(Y|X) = \int_0^{T_W} \mathrm{d}H(\Theta) - \int_0^{T_0} \mathrm{d}H(\Theta) = \int_{T_0}^{T_W} \mathrm{d}H(\Theta)\\
&= \oint_{\mathcal{O}_{rrev}} \frac{\delta A(\Theta)}{\mathrm{k}\Theta} = \frac{2\Delta Q_W}{\mathrm{k}T_W} \cdot \eta_{\max} = H(Y) \cdot \eta_{\max} = H(X) = \Delta I
\end{aligned}$$

Obviously, $T(X;Y) = H(X) - H(X|Y) = T(Y|X)$. Further it is obvious that $T(X;Y)$ is the capacity C_{T_W,T_0} of the channel $\mathbf{K} \cong \mathcal{L}$ too,

$$\begin{aligned}
C_{T_W,T_0} &= T(X;Y) = H(X) = \oint_{\mathcal{O}_{rrev}} \frac{\delta A(\Theta)}{\mathrm{k}\Theta} = \int_{T_0}^{T_W} \frac{\delta Q_W(\Theta)}{\mathrm{k}\Theta} \qquad (146)\\
&= \frac{2\Delta Q_W}{\mathrm{k}{T_W}^2} \cdot (T_W - T_0),\ \ C^{\max}_{T_W,T_0} = H(Y)
\end{aligned}$$

[17] In reality for the least elementary heat change $\delta Q = \hbar\nu$ is right where $\hbar = \dfrac{h}{2\pi}$ and h is ***Planck*** *constant*.

[18] It is provable that the Carnot cycle itself is elenentary, *not dividible* [18].

7.1. Triangular heat cycle

Elementary change $\mathrm{d}\Theta$ of temperature Θ of the *environment* of the general continuous cycle $\mathcal{O}$ and thus of its working *medium* $\mathcal{L}$ (both are in the *diathermic contact at* Θ) causes the elementary reversible change of the heat $Q*(\Theta)$ *delivered, (radiated)* into $\mathcal{L}$, just about the value $\delta Q*(\Theta)$,

$$\delta Q*(\Theta) = \frac{\partial Q*(\Theta)}{\partial \Theta}\mathrm{d}\Theta, \quad Q*(\Theta_W) = \int_0^{\Theta_W} \frac{\partial Q*(\Theta)}{\partial \Theta}\mathrm{d}\Theta \tag{147}$$

The heat $Q*(\Theta_W)$ is the whole heat delivered (reversibly) into $\mathcal{L}$ (at the *end* temperature Θ_W). For the infinitezimal heat $\delta Q*(\Theta)$ delivered (reversibly) into $\mathcal{L}$ at temperature Θ and in accordance with the *Clausius* definition of heat entropy $S*_{\mathcal{L}}$ [22] is valid that

$$\delta Q*(\Theta) = \Theta \cdot \mathrm{d}S*_{\mathcal{L}}(\Theta), \quad \mathrm{d}S*_{\mathcal{L}}(\Theta) = \frac{\delta Q*(\Theta)}{\Theta} \tag{148}$$

For the whole change of entropy $\Delta S*_{\mathcal{L}}(\Theta_W)$, or for the entropy $S*_{\mathcal{L}}(\Theta_W)$ respectively, delivered into the medium $\mathcal{L}$ by its heating within the temperature interval $\langle 0, \Theta_W \rangle$, is valid that

$$\Delta S*_{\mathcal{L}}(\Theta_W) = \int_0^{\Theta_W} \mathrm{d}S*_{\mathcal{L}}(\Theta) = \int_0^{\Theta_W} \frac{\delta Q*(\Theta)}{\Theta} = \int_0^{\Theta_W} \frac{\frac{\partial Q*(\Theta)}{\partial \Theta}\mathrm{d}\Theta}{\Theta} = S*_{\mathcal{L}}(\Theta_W) \tag{149}$$

when $S*_{\mathcal{L}}(0) \overset{\text{Def}}{=} 0$ is set down. By (148) for the whole heat $Q*(\Theta_W)$ deliverd into $\mathcal{L}$ within the temperature interval $\Theta \in \langle 0, \Theta_W \rangle$ also is valid that

$$Q*(\Theta_W) = \int_0^{\Theta_W} \Theta \mathrm{d}S*_{\mathcal{L}}(\Theta) \tag{150}$$

Then, by *medium value* theorem[19] is valid that $\overline{\Theta_{(0,\Theta_W)}} = \dfrac{0+\Theta_W}{2} = \dfrac{\Theta_W}{2}$ and

$$Q*(\Theta_W) = \int_{S*_{\mathcal{L}}(0)}^{S*_{\mathcal{L}}(\Theta_W)} \Theta \mathrm{d}S*_{\mathcal{L}}(\Theta) = [S*_{\mathcal{L}}(\Theta_W) - S*_{\mathcal{L}}(0)] \cdot \overline{\Theta_{(0,\Theta_W)}} \tag{151}$$

For the extremal values T_0 a T_W of the cooler temperature Θ of $\mathcal{O}$ and by (151)

$$Q*_0 \overset{\triangle}{=} Q*(T_0) = \int_0^{T_0} \delta Q*_W(\Theta) \quad \text{and} \quad Q*_W \overset{\triangle}{=} Q*(T_W) = \int_0^{T_W} \delta Q*_W(\Theta) \tag{152}$$

$$Q*_0 = \int_{S*_{\mathcal{L}}(0)}^{S*_{\mathcal{L}}(T_0)} \Theta \mathrm{d}S*_{\mathcal{L}}(\Theta) = [S*_{\mathcal{L}}(T_0) - S*_{\mathcal{L}}(0)] \cdot \overline{\Theta_{(0,T_0)}}, \quad \overline{\Theta_{(0,T_0)}} = \frac{T_0}{2}$$

$$Q*_W = \int_{S*_{\mathcal{L}}(0)}^{S*_{\mathcal{L}}(T_W)} \Theta \mathrm{d}S*_{\mathcal{L}}(\Theta) = [S*_{\mathcal{L}}(T_W) - S*_{\mathcal{L}}(0)] \cdot \overline{\Theta_{(0,T_W)}}, \quad \overline{\Theta_{(0,T_W)}} = \frac{T_W}{2}$$

With $S*_{\mathcal{L}}(0) = 0$ for the (end) temperatures Θ, T_0, T_W of $\mathcal{L}$ and the relevant heats and their entropies is valid

$$Q*(\Theta) = S*_{\mathcal{L}}(\Theta) \cdot \frac{\Theta}{2} \quad \text{and then} \quad S*_{\mathcal{L}}(\Theta) = \frac{2Q*(\Theta)}{\Theta} \tag{153}$$

$$Q*_W = S*_{\mathcal{L}}(T_W) \cdot \frac{T_W}{2} \quad \text{and then} \quad S*_{\mathcal{L}}(T_W) = \frac{2Q*_W}{T_W}$$

$$Q*_0 = S*_{\mathcal{L}}(T_0) \cdot \frac{T_0}{2} \quad \text{and then} \quad S*_{\mathcal{L}}(T_0) = \frac{2Q*_0}{T_0}$$

[19] Of Integral Calculus.

For the change $\Delta S*_{\mathcal{L}}$ of the thermodynamic entropy $S*_{\mathcal{L}}$ of the system $\mathcal{L}$, at the temememperature Θ running through the interval $\langle T_0, T_W \rangle$, by gaining heat from its environment (the environment of the cycle $\mathcal{O}$), is valid

$$\Delta S*_{\mathcal{L}} = S*_{\mathcal{L}}(T_W) - S*_{\mathcal{L}}(T_0) = 2\left(\frac{Q*_W}{T_W} - \frac{Q*_0}{T_0}\right) = \int_{T_0}^{T_W} \frac{\delta Q*(\Theta)}{\Theta} \tag{154}$$

By (149) for the entropy $S*_{\mathcal{L}}(\Theta_W)$ of $\mathcal{L}$ at variable temperature $\Theta \in \langle 0, \Theta_W \rangle$, $\Theta_W \leq T_W$, is gained that

$$\begin{aligned} S*_{\mathcal{L}}(\Theta_W) &= \int_0^{\Theta_W} \left(\frac{\partial}{\partial\Theta}\left[\frac{S*_{\mathcal{L}}(\Theta)\cdot\Theta}{2}\right]\right)\frac{\mathrm{d}\Theta}{\Theta} = 2\cdot\frac{1}{2}\int_0^{\Theta_W} \mathrm{d}S*_L(\Theta) \\ &= \frac{1}{2}\int_0^{\Theta_W} S*'_{\mathcal{L}}(\Theta)\mathrm{d}\Theta + \frac{1}{2}\int_0^{\Theta_W} S*_{\mathcal{L}}(\Theta)\frac{\mathrm{d}\Theta}{\Theta} \end{aligned} \tag{155}$$

and then

$$S*_{\mathcal{L}}(\Theta_W) = \int_0^{\Theta_W} S*_{\mathcal{L}}(\Theta)\frac{\mathrm{d}\Theta}{\Theta} = \int_0^{\Theta_W} \mathrm{d}S*_{\mathcal{L}}(\Theta) \quad \left[= \frac{2Q*(\Theta_W)}{\Theta_W}\right]$$

and thus

$$S*_{\mathcal{L}}(\Theta)\frac{\mathrm{d}\Theta}{\Theta} = \mathrm{d}S*_{\mathcal{L}}(\Theta) \tag{156}$$

By the result of derivation (155)-(156) for an arbitrary temperature Θ of medium $\mathcal{L}$ is valid that[20]

$$S*_{\mathcal{L}}(\Theta) = l\cdot\Theta, \quad l = \frac{2Q*(\Theta)}{\Theta^2} \quad \longrightarrow \quad Q*(\Theta) = \lambda\cdot\Theta^2, \quad \lambda = \frac{l}{2} \tag{157}$$

Obviously, from (154) for $\Theta \in \langle T_0, T_W \rangle$ is derivable that

$$\Delta S*_{\mathcal{L}} = l\cdot(T_W - T_0) = l\cdot T_W\cdot(1-\beta), \quad \beta = \frac{T_0}{T_W} \tag{158}$$

Let such a reverse cycle is given that the medium $\mathcal{L}$ of which takes, through the elementary isothermal expansions at temperatures $\Theta \in \langle T_0, T_W \rangle$, the whole heat ΔQ_0

$$\Delta Q_0 = \int_{T_0}^{T_W} \delta Q*(\Theta) = \int_{T_0}^{T_W} l\Theta\mathrm{d}\Theta = \frac{l}{2}\cdot\left({T_W}^2 - T_0^2\right) = Q*_W - Q*_0 \tag{159}$$

or, with medium values

$$\begin{aligned} \Delta Q_0 &= \int_{T_0}^{T_W} \delta Q*(\Theta) = \int_{S*_{\mathcal{L}}(T_0)}^{S*_{\mathcal{L}}(T_W)} \Theta\mathrm{d}S*_{\mathcal{L}}(\Theta) \\ &= \overline{\Theta_{W_{(T_0,T_W)}}}\cdot[S*_{\mathcal{L}}(T_W) - S*_{\mathcal{L}}(T_0)] = \frac{T_W + T_0}{2}\cdot 2\left[\frac{Q*_W}{T_W} - \frac{Q*_0}{T_0}\right] \end{aligned}$$

and thus equivalently

$$\Delta Q_0 = (T_W + T_0)\cdot\lambda\cdot[T_W - T_0] = \lambda {T_W}^2\cdot(1-\beta^2), \quad \beta = \frac{T_0}{T_W}$$

For a reverse reversible Carnot cycle $\mathcal{O}'_{rrev}$, equivalent with the just considered general continuous heat cycle $\mathcal{O}$, drawing up the same heat ΔQ_0, consumpting the same mechanical

[20] If $\int \frac{f(x)}{x}\mathrm{d}x = \int \mathrm{d}f(x)$, or $\frac{\mathrm{d}f(x)}{f(x)} = \frac{\mathrm{d}x}{x}$, then $\ln|f(x)| = \ln|x| + \ln L$, $L > 0$, $\ln|f(x)| = \ln(L\cdot|x|)$, $f(x) = l\cdot x$, $l \in \mathbb{R}$.

work ΔA and giving, at its higher temperature t_W (the average temperature of the heater of our general cycle), the same heat ΔQ_W, is valid that $\Delta Q_0 = \Delta Q_W \cdot \gamma$ where $\gamma = \frac{\frac{T_0+T_W}{2}}{t_W}$ is the *transform ratio*.

Then

$$\Delta Q_W = \Delta Q_0 \cdot \frac{2t_W}{T_0+T_W} = \frac{l}{2} \cdot \left({T_W}^2 - {T_0}^2\right) \cdot \frac{2t_W}{T_0+T_W} = l \cdot t_W \cdot (T_W - T_0) \tag{160}$$

$$\Delta A = \Delta Q_W \cdot (1-\gamma) = l \cdot t_W \cdot (T_W - T_0) \cdot \left(1 - \frac{T_0+T_W}{2t_W}\right) \tag{161}$$

$$= \frac{l}{2} \cdot (T_W - T_0) \cdot (2t_W - T_0 - T_W)$$

For the elementary work $\delta A(\cdot,\cdot)$ corresponding with the heat $Q*(\Theta)$ pumped out (reversibly) from $\mathcal{L}$ at the (end, output) temperature Θ of $\mathcal{L}$ and for the entropy $S*_{\mathcal{L}}(\Theta)$ of the whole environment of $\mathcal{O}$ (including $\mathcal{L}$ with $\mathcal{O}$) is valid

$$S*_{\mathcal{L}}(\Theta)\frac{\mathrm{d}\Theta}{\Theta} = \frac{Q*(\Theta)\frac{\mathrm{d}\Theta}{\Theta}}{\Theta} \stackrel{\triangle}{=} \frac{\delta A(\Theta, \mathrm{d}\Theta)}{\Theta} = \mathrm{d}S*_{\mathcal{L}}(\Theta) = l \cdot \mathrm{d}\Theta \tag{162}$$

$$\delta A(\Theta, \mathrm{d}\Theta) = S*_{\mathcal{L}}(\Theta)\mathrm{d}\Theta = l \cdot \Theta \mathrm{d}\Theta \text{ and } \delta A(\mathrm{d}\Theta, \mathrm{d}\Theta) = l \cdot \mathrm{d}\Theta\mathrm{d}\Theta = \mathrm{d}S*_{\mathcal{L}}(\Theta)\mathrm{d}(\Theta) \stackrel{\triangle}{=} \delta A$$

$$\left(\int_{T_0}^{\Theta} l\mathrm{d}\theta\right)\mathrm{d}\Theta = l \cdot (\Theta - T_0)\mathrm{d}\Theta \stackrel{\triangle}{=} \delta A(\Theta, \mathrm{d}\Theta; T_0) \tag{163}$$

For the whole work $\Delta A(\Theta_W; T_0)$ consumpted by the general reverse cycle $\mathcal{O}$ between temperatures T_0 and Θ_W, being coverd by elementary cycles (162), is valid that

$$\Delta A(\Theta_W; T_0) = \int_{T_0}^{\Theta_W} \left[\int_{T_0}^{\Theta} \mathrm{d}S*_{\mathcal{L}}(\theta)\right] \mathrm{d}\Theta = \int_{T_0}^{\Theta_W} \left[\int_{T_0}^{\Theta} l\mathrm{d}\theta\right] \mathrm{d}\Theta = l \cdot \int_{T_0}^{\Theta_W} (\Theta - T_0)\mathrm{d}\Theta \tag{164}$$

$$= \frac{l}{2} \cdot ({\Theta_W}^2 - {T_0}^2) - l \cdot T_0(\Theta_W - T_0) = \frac{l}{2} \cdot {\Theta_W}^2 + \frac{l}{2} \cdot {T_0}^2 - \frac{2l}{2} \cdot T_0\Theta_W = \lambda \cdot (\Theta_W - T_0)^2$$

$$\stackrel{\triangle}{=} \oint_{\mathcal{O}_{(\Theta_W, T_0)}} \delta A = \lambda \cdot {T_W}^2 \cdot (1-\beta)^2, \text{ when it is valid that } \Theta_W = T_W,\ \beta = \frac{T_0}{T_W}$$

But, then for the results for ΔA in (160), (161) and (164) follows that

$$t_W = \frac{T_W + \Theta_W}{2} = T_W \text{ and then } t_W = T_W = \text{const.} \tag{165}$$

Thus our general cycle $\mathcal{O}$ is of a triangle shape, $\mathcal{O} \stackrel{\triangle}{=} \mathcal{O}_{rrev\triangle}$ with the *apexes*

$$[lT_0, T_0],\ [lT_W, T_W],\ [lT_0, T_W] \tag{166}$$

and its efficiency is $1 - \gamma = \frac{1-\beta}{2} = \frac{1}{2} \cdot \eta_{\max}$. Thus, the return to the *initial (starting) state* of the medium $\mathcal{L}$ is possible by using the oriented *abscissas* (in the $S - T$ diagram)

$$\overrightarrow{[lT_W, T_W], [lT_0, T_W]} \text{ and } \overrightarrow{[lT_0, T_W], [lT_0, T_0]} \tag{167}$$

For works $\delta A(\Theta, \mathrm{d}\Theta; T_0)$ of elementary Carnot cycles covering cycle $\mathcal{O}_{rrev\triangle}$ (166), the range of their working temperatures is $\mathrm{d}\Theta$ and, for the given heater temperature $\Theta \in \langle T_0, \Theta_W \rangle$ is, by (162)-(163) valid that

$$\begin{aligned}
\delta A(\Theta, \mathrm{d}\Theta; T_0) &= \Delta Q_W(\Theta) \cdot \frac{\mathrm{d}\Theta}{\Theta} = l \cdot \Theta \cdot (\Theta - T_0) \frac{\mathrm{d}\Theta}{\Theta} = l \cdot (\Theta - T_0)\mathrm{d}\Theta \\
&= [S*_{\mathcal{L}}(\Theta) - S*_{\mathcal{L}}(T_0)] \cdot \mathrm{d}\Theta \\
\Delta Q_W(\Theta) &= l\Theta\,(\Theta - T_0) \quad \text{and for } \gamma \text{ used in (160) is gained that} \\
\Delta Q_0(\Theta) &= \Delta Q_W(\Theta) \cdot \gamma(\Theta) = l \cdot \Theta \cdot (\Theta - T_0) \cdot \frac{\Theta + T_0}{2\Theta} = \lambda \cdot \left(\Theta^2 - {T_0}^2\right)
\end{aligned} \tag{168}$$

For the whole heats ΔQ_0 a ΔQ_W being changed mutually between the working medium $\mathcal{L}$ of the whole triangular cycle $\mathcal{O}_{rrev\triangle}$ and its environment (166), and for the work ΔA, in its equivalent Carnot cycle $\mathcal{O}'_{rrev}$ with working temperatures $\dfrac{T_0 + T_W}{2}$ and T_W, will be valid that[21]

$$\Delta Q_0 = \int_{T_0}^{T_W} \left[\int_0^{\Theta} l\mathrm{d}\theta\right] \mathrm{d}\Theta = \frac{l}{2} \cdot {T_W}^2 \cdot (1-\beta^2) \stackrel{\triangle}{=} W\left[= l \cdot (T_W - T_0) \cdot T_0 + \frac{l}{2} \cdot (T_W - T_0)^2\right] \tag{169}$$

$$\begin{aligned}
\Delta Q_W &= \int_{T_0}^{T_W} \left[\int_0^{T_W} l\mathrm{d}\theta\right] \mathrm{d}\Theta = l \cdot (T_W - T_0) \cdot T_W = l \cdot {T_W}^2 \cdot (1 - \beta) \\
\Delta A &= \frac{1}{2} \int_{T_0}^{T_W} \left[\int_{T_0}^{T_W} l\mathrm{d}\theta\right] \mathrm{d}\Theta = \cdot l{T_W}^2 \cdot (1 - \beta) - l \cdot {T_W}^2 \cdot (1 - \beta) \\
&= l \cdot {T_W}^2 \cdot (1 - \beta) \cdot [1 - \frac{l}{2}(1 + \beta)] = \frac{l}{2} \cdot {T_W}^2 \cdot (1 - \beta)^2 = \oint_{\mathcal{O}_{rrev\triangle}} \delta A
\end{aligned} \tag{170}$$

7.2. Capacity corections for wide–band photonic transfer channel

The average *output* energy $P_2(\Theta_W)$ of the message being received within interval $(0, T_W\rangle$ of the temperature Θ of the medium $\mathcal{L} \cong \mathbf{K}_{\mathrm{L-L}}$ from [30], when $0 < \Theta_0 \leq \Theta \leq \Theta_W$ and $\Theta_0 \leq T_0$ and $\Theta_W \leq T_W$ are valid, is given by the sum of the *input* average energy $W(\Theta_W, \Theta_0)$ and the average energy $P_1(\Theta_0)$ of the *additive noise*

$$P_2(\Theta_W) = P_1(\Theta_0) + W(\Theta_W, \Theta_0) \tag{171}$$

The output message bears the *whole average output information* $H_2(\Theta_W)$. By the medium value theorem is possible, for a certain maximal temperature $\Theta_W \leq T_W$ of the temperature $\Theta \in (0, \Theta_W\rangle$, consider that the receiving of the output message is performed at the *average (constant) temperature* $\overline{\Theta_W} = \dfrac{\Theta_W}{2}$. Then for the whole change of the output information entropy $\Delta H_2 \stackrel{\triangle}{=} H_2(\Theta_W)$ [the thermodynamic entropy $S*_{\mathcal{L}}(\Theta_W)$ in information units] is valid

$$H_2(\Theta_W) = \frac{P_2(\Theta_W)}{\mathrm{k}\overline{\Theta_W}} = \frac{P_1(\Theta_0) + W(\Theta_W, \Theta_0)}{\mathrm{k}\overline{\Theta_W}} \stackrel{\triangle}{=} H_1(\Theta_W, \Theta_0) + H[W(\Theta_W, \Theta_0)] \tag{172}$$

[21] Further it willbe layed down $\lambda = \dfrac{\pi^2 k^2}{6\hbar}$, $l = \dfrac{\pi^2 k^2}{3\hbar} = 2 \cdot \lambda$.

By (153) is valid that $Q*(\Theta) = \lambda\Theta^2$ and $\delta Q*(\Theta) = l\Theta d\Theta$. Thus for $\Theta \in (0, T_0\rangle$ a $\Theta \in (0, T_W\rangle$ is valid

$$dH_{\Theta_W}(Y) = \frac{\delta Q*(\Theta_W)}{k\Theta_W} = \frac{l}{k}d\Theta_W,\ dH_{\Theta_W}(Y|X) = \frac{2l\Theta d\Theta}{k\Theta_W} \tag{173}$$

With $\Theta_W = T_W$ and $\Theta_0 = T_0$ and with the reducing temperature $\frac{T_W}{2}$ is possible to write

$$P_2 = W + P_1 \stackrel{\triangle}{=} Q*_W = \lambda T_W{}^2 \stackrel{\triangle}{=} Y \tag{174}$$

$$H_2 \stackrel{\triangle}{=} H(Y) = \int_0^{T_W} \frac{l}{k} d\Theta_W = \frac{l}{k} T_W = \frac{P_2}{k\frac{T_W}{2}} = \frac{2(W+P_1)}{kT_W} = \frac{2\lambda T_W{}^2}{kT_W} = \frac{lT_W}{k}$$

$$P_1 \stackrel{\triangle}{=} Q*_0 = \lambda T_0{}^2 \stackrel{\triangle}{=} Y|X$$

$$H_1 \stackrel{\triangle}{=} H(Y|X) = \int_0^{T_0} \frac{2l\Theta d\Theta}{kT_W} = \frac{l}{kT_W} \cdot T_0{}^2 = \frac{P_1}{k\frac{T_W}{2}} = \frac{2\lambda T_0{}^2}{kT_W} = \frac{lT_0{}^2}{kT_W}$$

$$W = Q*_W - Q*_0 \stackrel{\triangle}{=} X$$

$$H[W(T_W, T_0)] \stackrel{\triangle}{=} H(X) = \frac{W}{k\frac{T_W}{2}} = \frac{2\lambda}{kT_W} \cdot (T_W{}^2 - T_0{}^2) \tag{175}$$

By the channel equation (4), (5) and by equations (23)-(24) and also by definitions (174)-(175) and with the loss entropy $H(X|Y) = 0$ it must be valid for the transinformation $T(\cdot;\cdot)$ that

$$T(Y;X) = H(Y) - H(Y|X) = \frac{l}{k} \cdot (T_W - T_0) \cdot (1+\beta) = H(X) \tag{176}$$

$$T(X;Y) = H(X) - H(X|Y) = H(X) = T(X,Y) \text{ and by using } l = \frac{\pi^2 k^2}{3\hbar},\ \beta = \frac{T_0}{T_W},$$

$$T(Y;X) = \frac{\pi^2 k}{3h} \cdot T_W \cdot (1-\beta^2) = \frac{\pi^2 k}{3h} \cdot T_W \cdot (1-\beta) \cdot (1+\beta) = C_{T_0,W}(\mathbf{K}_{L-L}) \cdot (1+\beta)$$

For the given extremal temperatures T_0, T_W the value $T(X;Y)$ stated this way is the only one, and thus also, it is the information capacity $C'_{T_0,T_W}(W)$ of the channel $\mathbf{K}_{L-L}$ (the *first* correction)

$$C'_{T_0,T_W} = (W) = T(X;Y) = \frac{\pi^2 k T_0}{3h} \cdot \left(\sqrt{1 + \frac{6h \cdot W}{\pi^2 k^2 T_0{}^2}} - 1\right) \cdot (1+\beta) \tag{177}$$

The information capacity correction (177) of the wide–band photonic channel $\mathbf{K}_{L-L}$ [30], stated this way, is $(1+\beta)$-times *higher* than the formulas (102) and (138) say. The reason is in using two different information descriptions of the oriented abscissa $\overrightarrow{[l0,0],[lT_W,T_W]}$ in derivation (138) and (177) which abscissa $\overrightarrow{[l0,0],[lT_W,T_W]}$ is on one line in $S-T$ diagram and is composed from two oriented abscissas,

$$\overrightarrow{[l0,0],[lT_0,T_0]} \text{ and } \overrightarrow{[lT_0,T_0],[lT_W,T_W]} \tag{178}$$

The first abscissa represents the *phase of noise generation* and the second one the *phase of input signal generation*. The whole composed abscissa represents the *phase of whole output signal generation*.

For the **sustaining, in the sense repeatable, cyclical information transfer, the renewal of the initial or starting state of the transfer channel $\mathbf{K}_{\mathrm{L-L}} \cong \mathcal{L}$, after any *individual information transfer act* - the *sending input and receiving output message* has been accomplished, is needed**.
Nevertheless, in derivations of the formulas (102), (177) and (138) **this return of the physical medium $\mathcal{L}$**, after accomplishing any individual information transfer act, into the starting state **is either not considered**, **or, on the contrary, is considered, but by that the whole transfer chain is opened *to cover the energetic needs for this return transition from another, outer resources* than from those ones within the transfer chain itself**. In both these two cases the channel equation is fulfilled. This enables any individual act of information transfer be realized by *external and forced out, repeated starting* of each this individual transfer act.[22] [23]

If for the creation of a cycle the resources of the transfer chain are used only, the need for *another correction*, this time in (177) arises. To express it it will be used the *full cyclical thermodynamic analogy* $\mathbf{K}$ of $\mathbf{K}_{\mathrm{L-L}}$ used *cyclically*, $\mathbf{K'}_{\mathrm{L-L}}$. The information transfer will be modeled by the cyclical thermodynamic process $\mathcal{O}_{rrev\triangle}$ of reversible changes in the channel $\mathbf{K} \cong \mathbf{K'}_{\mathrm{L-L}} \cong \mathcal{L}$ and without opening the transfer chain. (Also $\mathbf{K} \cong \mathbf{K'}_{\mathrm{B-E}}$).

7.2.1. Return of transfer medium into initial state, second correction

Now the further correction for capacity formulas (102), (138) and (177) will be dealt with for that case that the *return* of the medium $\mathcal{L}$ into its initial, starting state is performed *within the transfer chain* only. It will be envisiged by a triangular reverse heat cycle $\mathcal{O}_{rrev\triangle}$ created by the oriented abscissas within the apexes in the $S-T$ diagram (166), $[lT_0, T_0]$, $[lT_W, T_W]$, $[lT_0, T_W]$. The abstract experiment from [30] will be now, formally and as an analogy, realized by this *reverse* and reversible heat cycle $\mathcal{O}_{rrev\triangle} \equiv \mathcal{O'}_{rrev}$, described informationaly, and thought as modeling information transfer process in a channel $\mathbf{K} \cong \mathbf{K'}_{\mathrm{L-L|B-E}} \cong \mathcal{L}$. Thus the denotation $\mathbf{K} \equiv \mathbf{K}_{\triangle}$ is usable. By (153)-(157) it will be

$$Q*(\Theta) = \frac{\pi^2 \mathrm{k}^2 \Theta^2}{6h} = \frac{l}{2} \cdot \Theta^2, \;\; Q*_W = \frac{l}{2} \cdot {T_W}^2, \;\; Q*_0 = \frac{l}{2} \cdot {T_0}^2 \tag{179}$$

The working temperature Θ_0 of cooling and Θ_W of heating are changing by (157),

$$\Theta_0 = \frac{1}{l} \cdot S*_{\mathcal{L}}(\Theta_0) \in \langle T_0, T_W \rangle \;\text{ and }\; \Theta_W = T_W = \text{const.} \tag{180}$$

and the heat entropy $S*_{\mathcal{L}}(\Theta)$ of the medium $\mathcal{L}$ is changing by (155)-(156),

$$\mathrm{d}S*_{\mathcal{L}} = \frac{\partial Q*(\Theta)}{\partial \Theta} \mathrm{d}\Theta \cdot \frac{1}{\Theta} \;\text{ and then }\; S*_{\mathcal{L}}(\Theta) = l \cdot \Theta = \frac{2Q*(\Theta)}{\Theta} \tag{181}$$

Using integral (149) it is possible to write that

$$Q*(\Theta) = \int_0^{\Theta} \delta Q*(\theta) = \int_0^{\Theta} \frac{\partial Q*(\theta)}{\partial \theta} \mathrm{d}\theta = \int_0^{\Theta} l\theta \mathrm{d}\theta \tag{182}$$

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[22] For these both cases is not possible to construct a construction-relevant heat cycles *described in a proper information way*.
[23] But the modeling by the direct cycle such as in (128) is possible for the *II. Principle of Thermodynamics* is valid in any case and giving the possibility of the cycle description.

For the whole heats ΔQ_0 and ΔQ_W being changed mutually between $\mathcal{L}$ with the cycle $\mathcal{O}_{rrev\triangle}$ and its environment and, for the whole work ΔA for the equivalent Carnot cycle $\mathcal{O}'_{rrev}$ with working temperatures $\frac{T_0 + T_W}{2}$ and T_W is valid, by (169)-(170), that

$$W = \Delta Q_0 = \frac{l}{2} \cdot {T_W}^2 \cdot (1 - \beta^2) \stackrel{\triangle}{=} X, \quad \Delta Q_W = l \cdot {T_W}^2 \cdot (1 - \beta) \stackrel{\triangle}{=} Y \tag{183}$$

$$\Delta Q_W - \Delta Q_0 = \Delta A = \frac{l}{2} \cdot {T_W}^2 \cdot (1 - \beta)^2 \stackrel{\triangle}{=} Y|X$$

For the whole work ΔA delivered into the cycle $\mathcal{O}_{rrev\triangle}$, at the temperature T_W, and the entropy $S*_{\mathcal{L}}$ of its working medium $\mathcal{L}$ is valid

$$\frac{\Delta A}{T_W} = \oint_{\mathcal{O}_{rrev\triangle}} \frac{\delta A}{T_W} = \int_{T_0}^{T_W} l(\Theta - T_0) \cdot \frac{\mathrm{d}\Theta}{T_W} \tag{184}$$

$$\oint_{\mathcal{O}_{rrev\triangle}} \frac{\delta A}{T_W} = \frac{1}{2} \int_{T_0}^{T_W} \left[\int_{T_0}^{T_W} \mathrm{d}S*_{\mathcal{L}}(\theta) \right] \frac{\mathrm{d}\Theta}{T_W} = \int_{T_0}^{T_W} \left[S*_{\mathcal{L}}(T_W) - S*_{\mathcal{L}}(T_0) \right] \frac{\mathrm{d}\Theta}{2T_W}$$

$$= \frac{l}{2T_W} \cdot (T_W - T_0)^2 = \frac{l}{2} \cdot T_W \cdot (1 - \beta)^2 = \frac{\Delta A}{T_W}$$

Following (4), (5) and (23) and the triangular shape of the cycle $\mathcal{O}_{rrev\triangle}$, the changes of information entropies by expressions (142), (169)-(170) are defined, valid for the equivalent $\mathcal{O}'_{rrev}$[24], see (142),

$$H(X) \stackrel{\mathrm{Def}}{=} \frac{\Delta Q_0}{\mathrm{k}T_W} = \int_{T_0}^{T_W} \left[\int_0^{\Theta} \frac{\delta Q*(\theta)}{\theta} \right] \frac{\mathrm{d}\Theta}{\mathrm{k}T_W} = \int_{T_0}^{T_W} \left[\int_0^{\Theta} l \mathrm{d}\grave{} \right] \frac{1}{\mathrm{k}T_W} \mathrm{d}\Theta \tag{185}$$

$$H(Y) \stackrel{\mathrm{Def}}{=} \frac{\Delta Q_W}{\mathrm{k}T_W} = \int_{T_0}^{T_W} \left[\int_0^{T_W} \frac{\delta Q*(\theta)}{\theta} \right] \frac{\mathrm{d}\Theta}{\mathrm{k}T_W} = \int_{T_0}^{T_W} \left[\int_0^{T_W} l \mathrm{d}\grave{} \right] \frac{1}{\mathrm{k}T_W} \mathrm{d}\Theta$$

$$H(Y|X) \stackrel{\mathrm{Def}}{=} \frac{\Delta A}{\mathrm{k}T_W} = \frac{1}{2} \int_{T_0}^{T_W} \left[\int_{T_0}^{T_W} \frac{\delta Q*(\theta)}{\theta} \right] \frac{\mathrm{d}\Theta}{\mathrm{k}T_W} = \frac{1}{2} \int_{T_0}^{T_W} \left[\int_{T_0}^{T_W} l \mathrm{d}\theta \right] \frac{1}{\mathrm{k}T_W} \mathrm{d}\Theta$$

$$T(Y;X) = H(Y) - H(Y|X) = \frac{l}{2\mathrm{k}T_W} \int_{T_0}^{T_W} \left[2 \int_0^{T_W} \mathrm{d}\theta - \int_{T_0}^{T_W} \mathrm{d}\theta \right] \mathrm{d}\Theta$$

and by figguring these formulas with $l = \frac{\pi^2 k}{3h}$ is gained that

$$H(X) = \frac{l}{2\mathrm{k}} \cdot T_W \cdot (1 - \beta^2) = \frac{\pi^2 \mathrm{k} T_W}{6h} \cdot (1 - \beta^2)$$

$$H(Y) = \frac{l}{\mathrm{k}} \cdot T_W \cdot (1 - \beta) = \frac{\pi^2 \mathrm{k} T_W}{3h} \cdot (1 - \beta)$$

$$H(Y|X) = \frac{l}{2\mathrm{k}} \cdot T_W \cdot (1 - \beta)^2 = \frac{\pi^2 \mathrm{k} T_W}{6h} \cdot (1 - \beta)^2 = \oint_{\mathcal{O}_{rrev}} \frac{\delta A}{\mathrm{k}T_W}$$

$$T(Y;X) = \frac{l}{\mathrm{k}} \cdot T_W \cdot (1 - \beta) - \frac{l}{2\mathrm{k}} \cdot T_W \cdot (1 - \beta)^2 = \frac{l}{2\mathrm{k}} \cdot T_W \cdot (1 - \beta) \cdot (2 - 1 + \beta)$$

$$= \frac{l}{2\mathrm{k}} \cdot T_W \cdot (1 - \beta^2) = \frac{\pi^2 \mathrm{k} T_W}{6h} \cdot (1 - \beta^2) = H(X) = T(X;Y)$$

$$H(X|Y) = 0$$

[24] In accordance with the input energy delivered and the extremal temperatures used in [30].

It is visible that the quantity $H(Y)$ $[= H(X) + H(Y|X)]$ is introduced correctly, for by (185) is valid that

$$H(Y) = \frac{l}{2\mathrm{k}} \cdot T_W \cdot (1-\beta)^2 + \frac{l}{2\mathrm{k}} \cdot T_W \cdot (1-\beta^2) = \frac{l}{2\mathrm{k}} \cdot T_W \cdot (1-\beta) \cdot (1-\beta+1+\beta) \quad (186)$$
$$= \frac{l}{\mathrm{k}} \cdot T_W \cdot (1-\beta)$$

For the transinformation and the information capacity of the transfer organized this way is valid (177),

$$T(X;Y) = \frac{1}{2} C'_{T_0,T_W}(W) \quad (187)$$

With the extremal temperatures T_0 and T_W the information capacity $C*_{T_0,T_W}(W)$ is given by

$$T(X;Y) = C*_{T_0,T_W}(W) \ \text{ and then } \ C^{\max} = \lim_{T_0 \longrightarrow T_W} C*_{T_0,T_W}(W) = H(Y) \quad (188)$$

From the difference $\Delta Q_0 \overset{\triangle}{=} W = Q*_0 - Q*_W$ (in $\mathcal{L}$) follows that the temperature $T_W = T_0 \cdot \sqrt{1 + \frac{6h \cdot W}{\pi^2 \mathrm{k}^2 {T_0}^2}}$.

Then, for the transinformation, in the same way as in (187), is now valid

$$T(X;Y) = \frac{\pi^2 k}{6h} \cdot \left(1-\beta^2\right) = \frac{\pi^2 k}{3h} \cdot (T_W - T_0) \cdot \frac{1+\beta}{2} = \frac{\pi^2 \mathrm{k} T_0}{3h} \cdot \left(\sqrt{1 + \frac{6h \cdot W}{\pi^2 \mathrm{k}^2 {T_0}^2}} - 1 \right) \cdot \frac{1+\beta}{2} \quad (189)$$

The transiformation $T(X;Y)$ is the capacity $C(\mathbf{K}_\triangle)$ and it is possible to write

$$T(X;Y) = C(\mathbf{K}_\triangle) = C*_{T_0,T_W}(W) = \frac{\pi^2 \mathrm{k} T_0}{6h T_W} \cdot \left(\sqrt{1 + \frac{6h \cdot W}{\pi^2 \mathrm{k}^2 {T_0}^2}} - 1 \right) \cdot (T_0 + T_W) \quad (190)$$
$$= C*_{T_W}(W) = \frac{W}{\mathrm{k} T_W} = C*_{T_0}(W) = \frac{W}{\mathrm{k} T_0 \cdot \sqrt{1 + \frac{6h \cdot W}{\pi^2 \mathrm{k}^2 {T_0}^2}}} \overset{\triangle}{=} C(\mathbf{K}'_{\mathrm{L-L|B-E}})$$

which value is $2\times$ *less* than (177) and $\frac{2}{1+\beta}\times$ *less* than (138).

For $T_0 \longrightarrow 0$ the *quantum approximation* $C(W)$ of the capacity $C*_{T_0,T_W}(W)$ is obtained, independent on the noise energy (the noise power deminishes near the abslute $0°\ K$)

$$C(W) = \lim_{T_0 \to 0} \left(\sqrt{\frac{\pi^4 \mathrm{k}^2 {T_0}^2}{6^2 \hbar^2} + \pi^2 \frac{W}{6h}} - \frac{\pi^2 \mathrm{k} T_0}{6h} \right) \cdot (1+\beta) = \pi \cdot \sqrt{\frac{W}{6h}} \quad (191)$$

The *classical aproximation* $C_{T_0}(W)$ of $C*_{T_0,T_W}(W)$ is gained for $T_0 \gg 0$. This value is near Shannon capacity of the wide–band *Gaussian* channel with noise energy $\mathrm{k}T_0$ and with the whole average input energy (energy) W; in the same way as in (104) is now gained

$$C_{T_0}(W) \doteq \frac{\pi^2 \mathrm{k} T_0}{6h} \left(\frac{3h \cdot W}{\pi^2 \mathrm{k}^2 {T_0}^2} \right) \cdot (1+\beta) = \frac{W}{2\mathrm{k} T_0} \cdot (1+\beta) \longrightarrow \frac{W}{\mathrm{k} T_0} \quad (192)$$

The mutual difference of results (189) and and (102), (138) [12, 30] is given by **the necessity of the returning the transfer medium, the channel $\mathbf{K}_{\triangle} \cong \mathbf{K'}_{\mathrm{L-L}|B-E} \cong \mathcal{L}$ into its initial state after each individual information transfer act has been accomplished and, by the relevant temperatutre reducing of the heat** ΔQ_0 [by T_W in (183)-(189)]. Thus, our thermodynamic cyclical model $\mathbf{K}_{\triangle} \cong \mathcal{O}_{rrev\triangle}$ for the repeatible information transfer through the channel $\mathbf{K'}_{\mathrm{L-L|B-E}}$ is of the information capacity (189), while in [12, 30] the information capacity of the *one-act* information transfer is stated.[25] By (189) the *whole energy costs* for the cyclical information transfer considered is countable.[26]

8. Conclusion

After each completed 'transmission of an input message and receipt of an output message' ('one-act' transfer) the transferring system must be reverted to its starting state, otherwise the constant (in the sense repeatable) flow of information could not exist. The author believes that either the opening of the chain was presupposed in the original derivation in [30], or that the return of transferring system to its starting state was not considered at all, it was not counted-in. In our derivations this needed state transition is considered be powered within the transfer chain itself, without its openning. Although our derivation of the information capacity for a cyclical case (using the cyclic thermodynamic model) results in a lower value than the original one it seems to be more exact and its result as more precise from the *theoretic point of view*, extending and not ceasing the previous, *original* result [12, 30] which *remains* of its *technology-drawing value*. Also it forces us in being aware and respecting of the *global costs* for (any) communication and its evaluation and, as such, it is of a *gnoseologic character*.

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[25] For one-act information transfer the choose between two information descriptions is possible which result in capacity (138) or (177).

[26] For the energy $T(X;Y)\cdot T_W$ on the output the energy $\frac{2}{1+\beta}\cdot T_W\times$ *greater* is needed on the input of the transfer channel which is in accordance with the *II. Principle of thermodynamics* for $\frac{2}{1+\beta} > 1$ is valid.

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