Chapter 11

Determination of the Herd Size

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Additional information is available at the end of the chapter

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1. Introduction

Scientists can work out breeds to be kept for greatest milk yield. Such breeds can differ from area to area and breed to breed. Akarro (1995; 2009) developed a simulation model to identify breeds to be bred for greatest milk yield in selected well managed farms in Tanzania. Having found out the breed to keep for the greatest milk yield, the problem that follows is to work out the herd size or the stocking rate for a particular farm. Also a stocking rate should render a profitability to the farm, otherwise there is no need of carrying out the enterprise of keeping cows. Indeed a simulation model of the form presented in Fig 1 can be used, but the problem is that most of the animal activities can not be quantified neither can they be approximated by the well known probability distributions. Furthermore, even for the approximation of distribution of animal activities, there is a problem of quantifying certain parameters on the quality and quantity of forage on offer and feeds in general. In view of this, a linear programming model can be developed as a proxy to determine the herd size or the stocking rate so that an enlightenment on ways to organize the farm for a profitable farm operation can be achieved. This is done for one farm only which is believed to have the necessary data input for the development of the linear model. The farm selected is Uyole. This method can be adapted to similar farms in the world which operate on the basis of 'zero' grazing.

Zero grazing is hereby defined as

1.1. The Uyole Agricultural Centre (UAC)

The Uyole Agricultural Centre (UAC) is of particular importance in this respect. The predominant dairy breed at Uyole is the Friesian / Zebu cross. Natural pastures around Uyole have been observed to produce around 2500 kg. dry matter (DM/ha/year). Considering a 400 kg cow requiring 8.5 kg. DM/day, then this cow needs 3100 kg DM/year. One hectare cannot therefore, maintain such a cow (Kifaro & Akarro, 1987). It was therefore decided that in agricultural high potential areas of Rungwe district (this is the area surrounding Uyole) more productive pasture species were required. Sensing this, the Uyole Agricultural Centre established a pasture and forage research programme. It commenced its work in 1970 with the aim to improve the phytomass and quality of pastures. Initial work involved fertilization of natural pastures, introductions and testing of grass/legume mixtures, special purpose pastures and short term crops like oats, lupine, maize, and fodder sugar beets.

2. The Linear Programming (L.P.) model

Simulation of the cow activities and feeding regimes as shown in Fig. 1 would probably be a more appropriate method for establishing the stocking rate.

However, given the intricacies of implementing this simulation especially with reference to management policies of a particular farm, a linear programming (L.P.) method is suggested as a plausible alternative. Here L.P. is defined as a mathematical structure, involving particular mathematical assumptions that can be solved using a standard solution technique, called the *simplex method.* It is the purpose of this section to formulate a mathematical model that would enlighten us on the type of pastures to be grown, when and what supplementation level is required and the number of dairy animals to keep for the farm to be profitable. It was assumed that a known section of the farm was to be developed entirely to dairy enterprise and the problem was to find how to organize this farm so that its annual net profit would be maximized. An L.P. is suggested. The herd size was kept constant throughout the year. In formulating the L.P. there arose a need of identifying major constraints to dairy cow needs as discussed below.

2.1. Nutritional (energy, protein, minerals and vitamins) requirements of a cow

2.1.1. Energy requirements

Organic nutrients obtained from different sources of feed available to an animal are used for a variety of purposes, including maintenance of body functions, the construction of body tissues, the synthesis of milk, and the conversion to mechanical energy used for walking and other work. All these diverse functions require the transfer of considerable quantities of energy, so that in most situations when the energy requirement of the animals' different needs are met, it may be assumed that animal's non-energy requirements (protein-minerals and vitamins) are also met. Hence, the nutritive value of different feeds can be expressed by their energy content or by their ability to supply energy with high coefficient of conversion into usable energy for the different body functions. The gross energy contents of different forages are very similar at about 18 MJ/kg (Hunt, 1966). A portion of this energy is lost as faeces while the remaining digestible energy (DE) proportional to the digestibility (d) of the consumed feed is converted into metabolized energy (ME) after additional losses of about 19% of DE as urine and methane (Armstrong, 1964; MAFF, 1975).

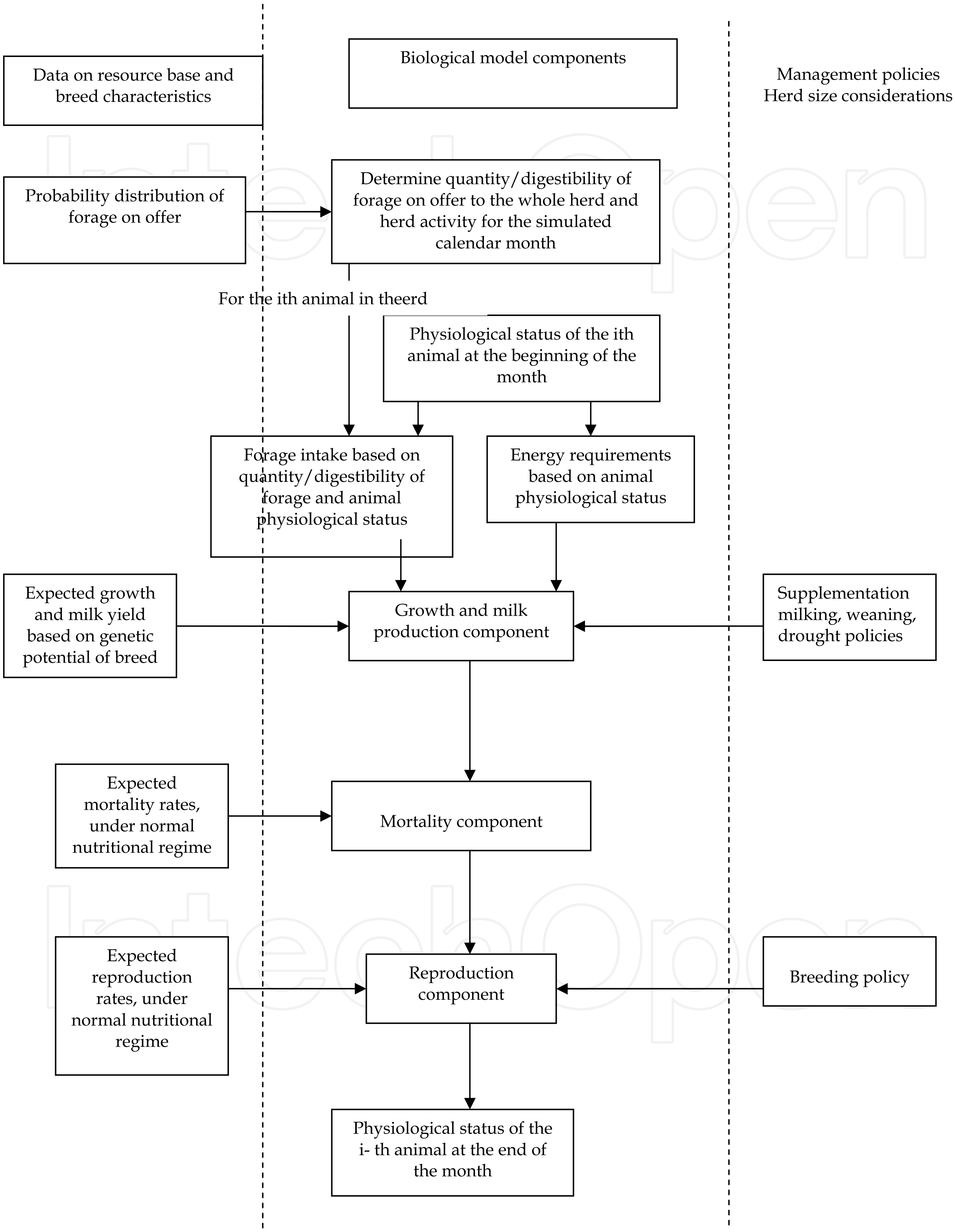

Figure 1. Components of a Dairy Cow Operational System : Source: Modified from Konandreas and Anderson (1982)

2.1.2. Energy requirements for maintenance

Maintenance can be defined as the state of the animal in which there is neither a net gain nor loss of nutrients (Kay, 1976). Maintenance requirements are estimates of the amount of nutrients required to achieve such an equilibrium. One component of the energy requirements for maintenance is referred to as basal metabolism and is proportional to the body size of the animal. The second component of the energy requirements for maintenance is related to the level of the animals activity and can be expressed approximately by live weight and the daily distance walked. Thus following Blaxter (1969) and Webster (1978), total net energy requirements for maintenance can be obtained from the relationship:

$$E_m = 0.376\, W^{.73} + 0.0021 WD \tag{1}$$

Where E_m= net energy requirements for maintenance (MJ/day).
W= live weight (kg)
D= distance walked (km/day).

The efficiency with which metabolizable energy is used for maintenance is a function of metabolizability of the consumed forage (see for example Blaxter, 1974; Van Es, 1976; Ministry of Agriculture Fisheries and Food (MAFF); 1975; Pigden et. al; 1979).

According to MAFF when distance walked is negligible, equation (2.1) reduces to

$$E_m = 8.3 + 0.091\, W. \left(\text{MAFF, 1975}\right) \tag{2}$$

Where E_m= net energy requirements for maintenance.
= metabolizable energy for maintenance (MJ/day).
W= body weight

2.1.3. Energy requirements for lactation

Net energy requirements for lactation are approximately proportional to the quantity of milk produced (ILCA, 1978), and this is given by

$$E_L = e_L M \tag{3}$$

Where E_L= net energy requirements for lactation (MJ/day).
e_L= energy content of milk.
M= milk yield (kg/day).

The energy content of milk is approximately given by the relationship (MAFF, 1975):

$$e_L = 0.03886\, BF + 0.0205\, SNF - 0.0236 \tag{4}$$

where BF= Butterfat content (g/kg).
SNF= solids not fat content (g/kg).

2.1.4. Protein requirements

Protein is an essential nutrient for animals. This nutrient however cannot be synthesized in sufficient quantities by animals to meet their requirements. Fortunately it is synthesized by plants and stored in plant cells. Through this means, a source of protein is provided for use by ruminants. An animal's requirement for protein is based on the protein stored in its body; its products such as milk, eggs, or wool; the products of conception, and the metabolic losses in faeces, endogenous losses in urine and by other losses (hair, skin, hoofs, etc.). To maintain an animal in protein equilibrium, these losses must be off set. The sum of these becomes the protein requirement for that animal. Protein requirements can be determined through nitrogen balance studies. In these studies, healthy adult animals are fed an adequate amount of energy and other nutrients in diets that contain different levels of protein. The minimum protein intake that will support nitrogen equilibrium is the maintenance requirement. The protein requirement for lactation is easily calculated by determining the amount of protein present in the milk and multiplying this by 1.25 (Kearl, 1982). Dairy animals seem to adapt very well to a wide range of protein intakes without any ill effects. The protein contained in milk, however, represents a direct loss of protein by the body and obviously this must be replaced.

2.1.5. Protein requirements for maintenance

The Digestible Protein (DP) maintenance requirements have been quite well defined. Orskov (1976) stated that the rate of protein deposition by young ruminants is appropriately expressed as the nitrogen retained per unit of energy digested, and that the retention of protein per unit of energy digested increases with the level of feeding and decreases as the animal matures. Balch (1976) suggested that at any given intake of protein, the response of the animal may vary greatly depending on the intake of energy. Poppe & Gabel (1977), after reviewing the literature concerning protein requirements for cattle, cited a DP requirement of 3g per kilogram of live weight W raised to power .75 for maintenance based on a digestible organic matter (DOM) fermentation rate of 60%.

Nehring (1970) suggested a value of 2.57g of DP per kilogram of live weight raised to the power .75 as the maintenance requirement of cattle weighing 400 to 800 kg. Sen et. al; (1978) whose data are used as the feeding standard in India, recommended 2.84g DP per kilogram of $W^{.75}$ for zebu cross bred cattle and buffaloes.

Additional information is needed to substantiate these results, but on the basis of a wide range of values found in the literature and those suggested as standards to be used in several countries an average value of

$$2.86\text{g DP per kilogram of } W^{0.75} \tag{5}$$

Where W is the live weight in kg has been used in estimating the DP maintenance requirement. This is the value used in Kearl (1982) which is also used in the formulation of feed values in food stuffs.

2.1.6. Protein requirement for lactation

Many studies have been done to determine the amount of Digestible Protein (DP) required to produce one kilogram (kg) of milk. Generally, the recommended amounts of DP per kilogram of milk have been correlated with the fat content of the milk. Nehring (1970) proposed a DP requirement of 50 to 80 g of DP for milk containing butterfat content from 3 to 6 percent. Ranjhan et. al; (1977) suggested 4.17 g of DP per kilogram of milk. Patle & Mudgal (1976) agreed with Ranjham et.al; (1977).. The National Research Council (1971) recommends a DP requirement of 42 to 60 g per kilogram of milk containing 2.5 to 6% fat. The Ministry of Agriculture Fisheries and Food (MAFF, 1979) noted a DP requirement of 48 to 63 g of DP per kilogram of milk containing 3.6 to 4.9% butterfat.

The MAFF (1979) values are the ones used in our estimates because they are regarded as standards in the formulation of feed values.

2.1.7. Estimation of nutritive values

On the average, a Friesian cow or a Friesian cross weighs 450 kg and produces milk whose composition is 3.6 percent butterfat (BF) and 8.6 percent solids not fat (SNF) at UAC (Myoya, 1980). Using results (2.2), (2.3), (2.4), (2.5) and (2.6), energy and protein requirements per dairy cow can be calculated on monthly basis for the available milk yields as shown in Table 2.1. The monthly yield figures were obtained from Uyole Agricultural Centre (UAC).

Month	Monthly Yield in Kg	Net Energy Lactation MJ/Month	Metabolizable Energy Required MJ/Month	Protein Required (DCP Kg per Month)
November	206	1018	2496	18.26
December	270	1334	2812	21.33
January	316	1561	3039	23.54
February	319	1576	3054	23.68
March	376	1858	3336	26.42
April	343	1695	3173	24.83
May	462	2283	3761	30.55
June	389	1922	3400	27.04
July	355	1754	3232	25.41
August	315	1556	3034	23.49
September	346	1709	3187	24.98
October	407	2011	3489	27.91

Table 1. Monthly Nutrition Requirements per Dairy Cow

Using equation (2.2), the Metabolizable Energy (ME) requirements for maintenance is 1478 Metabolizable Energy in Megajoules (MEMJ) per month. Using equation (2.4) the energy content e_L of milk is 4.94 megajoules (MJ) per kilogram. Using equation (2.3). net energy for lactation E_L is obtained. This is column 3 of Table 2.1. Using result (2.5), the Digestible Crude Protein (DCP) for maintenance is 279g per day. The value given by MAFF (1979) is 275g per day for a dairy cow of the same weight. DCP for maintenance required in a month is therefore 8.37 kg.

Using expression (2.6) the DCP allowances for milk production per kg for a Friesian cow with Butter fat percentage (BF%) of 3.6% is 48g (MAFF, 1979).

Column 5 in Table 2.1 is obtained by multiplying 48 by milk yield in kilograms plus DCP for maintenance which is 8.37.

2.1.8. Energy and protein supply

The main energy and protein source of dairy cows is obtained from the bulky food eaten by the cow. The bulky foods can either be grown on a farm or be purchased. At Uyole, land is scarce and the nutritive value of natural pastures and DM yield is low. Subsequent research involved evaluation of improved pasture and legume species. These included Rhodes grass (*Chloris gayana)*, Napier grass (*Pennisetum purpureum*) *Desmodium spp*.; Nandi setaria, Lucerne, oats, Lupins etc. fertilizer application, cutting frequencies and grass/legume mixture.

Invariably, fertilizer application improved the quality and quantity of the production of feeds/ha but also the cost of production was increased due to input costs.

2.1.9. Fertilizer efficiency

An increase in nitrogen application leads to increase in Dry matter yield per hectare as can be seen from Table 2.2. However it is desired not to apply infinite amounts of fertilizer but to apply the amounts that will give the maximum yield (kg DM) per unit of fertilizer applied. Such amount will be termed 'efficient' fertilizer applications. Such quantities will be used in our model for the input costs in pasture/crop production. Efficient fertilizer applications were worked out as yield increase per amount of fertilizer applied. Data for yields and fertilizer applications were obtained from UAC. These are presented in Table 2.2

Nitrogen kg/ha/year	Rhodes grass yield kg DM/ha	Fertilizer efficiency kg Dm/kg N	Nandi setaria yield kg DM/ha	Efficiency kg DM /kg N
0	3700		2370	
60	6020	39	5000	44
120	8720	42	7820	45
240	14120	43	14820	52
380	17860	39	18160	44
480	21630	37	21460	40

Table 2. Dry Matter Yield (kg/ha) of Rhodes Grass and Nandi setaria under Six levels of Nitrogen (Mean of Three Years): Source: Myoya (1980).

The higher yield in Rhodes grass without nitrogen and with rates up to 120kg N/ha suggest that Rhodes grass requires less nitrogen than Nandi setaria. At higher nitrogen rates, the difference disappears.

2.2. Forage supply at UAC

Due to climatic variations at Uyole, certain types of crops are available only in particular months or periods. During the wet season one expects surplus fodder which is not the case in the dry season. Table 2.3 shows the forage/feed supply sequence of some of the animal food stuffs grown at Uyole during the year.

Crop	Season						
	January-February	March-April	May	June	July-August	September-November	December
X_1 - Natural pasture	V	V	V	X	X	X	X
X_2 - Rhodes grass pasture	V	V	V	V	X	X	V
X_3 – Rhodes/Desmodium pasture	V	V	V	V	X	X	V
X_4 – Napier grass green feed	X	V	V	V	V	V	V
X_5 – Lupins green feed	X	V	V	V	V	X	V
X_6 – Napier /Desmodium green feed	X	V	V	V	V	V	X
X_7 – Oats green feed	X	V	V	V	X	X	V
X_8 – Rhodes grass hay	X	X	X	X	X	V	X
X_9 – Maize silage	X	V	V	V	V	V	V
X_{10} – Rhodes grass silage	X	X	X	X	V	V	V
X_{11} – Rhodes/Desmodium silage	X	X	X	X	V	V	V
X_{12} – Napier grass silage	X	X	X	X	V	V	X
X_{13}- Oats silage	X	X	X	X	V	X	X
X_{14}- Lupins silage	X	X	X	X	V	V	V
X_{15} – Napier/Desmodium silage	X	X	X	X	V	V	V

Key: V Available
X Not available

Table 3. Availability of Crops

Hay and silage could also be grown and these can be fed at any time during the year although they are usually fed during the dry season.

Thus, the year could be divided into seven periods, in each of which a different combination of crops or grazing output was available as shown in Table 2.3.

2.3. Estimation of Metabolizable Energy (ME) and Digestible Crude Protein (DCP) in feed stuffs

At UAC only a few feed stuffs have had their ME and DCP estimates done. In this case the figures available from the literature are assumed to be similar for the same feed stuffs where ME and DCP estimates are lacking. These are used because even in the formulation of quantity of food required, the estimates found in the literature, especially MAFF (1979), are used (Kurwijila, 1991). Table 2.4 gives Metabolizable Energy in Megajoules (MEMJ), Crude Protein Percentages and Crude Protein Digestibility Coefficient percentages estimates based on one kilogram of Dry matter (Mbwile, et. al; 1981; Gohl, 1981; Bredon, 1963; MAFF, 1979).

Feed	CP %	CPDC %	MEMJ/Kg DM
Natural Pastures	9.9	69.5	9.2
Rhodes Grass	7.5	62.0	8.7
Rhodes/Desmodium	11.4	52.1	7.9
Green Feed (forage)			
Lupins	15.5	73.4	10.3
Oats	13.4	76.0	10.5
Napier Grass ***	15.3	77	10.4
Napier/Desmodium	26.5	85	12.1
Silages			
Rhodes Grass	6.0	44.7	7.7
Rhodes/Desmodium	7.2	35.3	7.2
Oats	8.0	57.9	8.4
Maize	5.7	48.6	8.9
Napier Grass	16.0	64	8.8
Napier/Desmodium	16.0	64	8.8
Hay			
Rhodes Grass**	8.5	46	8.4
Rhodes Desmodium*	10.1	57	9.0

* Approximated the same as grass moderate digestibility silage.
** Approximated the same as grass with high digestibility.
*** Approximated the same as pasture grass, set stocking, close grazing.
Note: CP% means Crude Protein percentage, CPDC% means Crude Protein Digestibility Coefficient, ME MJ/kg DM means metabolizable energy in mega joules per kg of dry matter.

Table 4. Metabolizable Energy, Protein Content and Digestibility of Some of the Common Feeds at UAC

2.4. Nutrient value for the purchased concentrates

Supplementary feeding by purchased concentrates is usually done to the milking cows in order to increase their milk output. This is done throughout the year. These concentrates are in the form of energy feeds and protein feeds. Their nutrient values are given in Table 2.5.

Type	CP %	CPDC %	MEMJ/Kg DM
Energy Feeds			
Maize meal	10.6	86	14.2
Maize bran	9.6	65	12.5
Rice Polishing	14.9	87	15.5
Protein Feeds			
Cotton seed case (undecorticated)	23.1	77	8.5
Cotton seed case (decorticated)	41.7	72	10.8
Sunflower cake (undecorticated)	20.6	90	9.5
Sunflower cake (decorticated)	31.0	75	11.9
Lupin grain	33.7	81	14

Table 5. Metabolizable Energy, Protein Content and Digestibility of the Concentrates

The yields of different crops and grasses for various fertilizer application levels are shown in Table 2.6 and 2.7. The total ME and DCP on the basis of ha can be estimated (see Table 2.8). The total ME and DCP for the purchased concentrates is estimated on the basis of tonnage (see Table 2.9)

Crop	Fertilizer Applied kg N/ha	DM Yield in kg/ha	Nitrogen Efficiency kg DM/kg N
Natural Pasture	0	3000	
	80	5500	31*
	160	7500	28
	320	12000	28
Rhodes Pasture	60	1410	24*
	120	2083	17
Rhodes Hay	60	3115	52*
	120	3515	29
Rhodes Silage	60	3455	58*
	120	5855	49
Napier Grass Silage	80	4720	59*
	160	7670	48
	320	11370	36
Napier Grass Greenfield	80	4490	56*
	160	7280	46
	320	15220	48

* The most efficient fertilizer application yield per kg. of dry matter.

Table 6. Approximate Nitrogen Efficiency for some of the Crops where Different levels of Fertilizer are applied at UAC

Dry matter yield and fertilizer applied figures were obtained from the UAC.

Type	Use	DM Yield in kg/ha	Fertilizer Applied kgN/ha/ (or P/ha)
Natural Pastures	Pasture	5500	80
Rhodes Grass	Silage	3455	60
	Hay	3115	60
	Pasture	1410	60
Rhodes/Desmodium	Pasture	2100	0
	Silage	2100	0
Napier Grass	Silage	4720	80
	Green Feed	4490	80
Napier/Desmodium	Silage	4000	0
	Green feed	4000	0
Maize	Silage	10000	100N 20P[1]
Oats	Green Forage	2500	50N 20P[1]
	Silage	2500	
Lupins	Silage	6000	40P[1]
	Green Forage	500	

[1] Phosphate was included.

Table 7. Approximate Yield of Different Crops and Grasses (Feeds) for the Most Efficient Fertilizer Levels

The figures for dry matter yield and fertilizer applied were obtained from Uyole Agricultural Centre (UAC).

Metabolizable energy and DCP in Table 2.8 were obtained by multiplying dry matter yield in Table 2.7 by MEMJ/kg DM CP% and CPDC% in Table 2.4 respectively..

2.5. Fertilizer use and pasture production costs

The primary inputs involved in crop production are fertilizer, labour and cost of seeds in certain types of crops.

By using nitrogen, the carrying capacity of the land is increased which directly affects the cost of production. The profitability of applying nitrogen depends on the relationship between the cost of inputs and the value of realized output in the form of livestock and livestock products.

Based on records from production at the UAC, the following costs (Table 2.10) are incurred in pasture production – seed, cultivation and planting costs distributed over five years (the leys/grown pasture assumed life time) and fertilizer application and harvesting costs for three yearly harvest.

Type	Fertilizer Level N kg	Metabolizable Energy values in MEMJ/ha	DCP kg/ha
Pasture			
Natural Pastures	80	50600	382.3
Rhodes Grass	60	12267	65.6
Rhodes/Desmodium	-	16590	124.7
Green Feed (Forage)			
Lupins	20P	57500	568.9
Oats	25N 10P	26250	254.6
Napier Grass	80N	46696	529
Napier/Desmodium	-	48400	901
Hay			
Rhodes Grass	60N	28035	174
Silages			
Rhodes Grass	60N	26604	92.7
Rhodes/Desmodium	-	15120	53.4
Maize	100N 20P	89000	277
Oats	25N 10P	21000	115.8
Lupins*	20P	85200	1650
Napier Grass	40N	41536	483
Napier/Desmodium		35200	409.6

* Assumed same as lupin grain.

Table 8. Estimated Total Metabolizable Energy (ME) and Digestible Crude Protein (DCP) of some of the Commonly Grown Feeds per hectare at UAC

Type	ME/ton	DCP/ton
Energy Feeds		
Maize Meal	14200	91.2
Maize Bran	12500	62.4
Rice Polishing	15000	129.6
Protein feeds		
Cotton Seed Cake (undecorticated)	8500	178
Cotton Seed Cake (decorticated)	10800	296.6
Sunflower Cake (undecortimated)	9500	185.4
Sunflower Cake (decorticated)	11900	232.5
Lupin Grain	14200	273

Note: Digestible Crude Protein (DCP) is a measure of the useful protein potential of the feed and has been calculated from the crude protein content and the crude protein digestibility of the feed. The nutritive value of each feed has been expressed in terms of its Metabolizable energy and the Digestible Crude Protein (DCP).
ME and DCP in Table 2.9 were obtained by multiplying 1000 by MEMJ/kg Dm, CP% and CPDC% in Table 2.5 respectively..

Table 9. Estimated Total Metabolizable Energy (ME) and Digestible Crude Protein (DCP) of the Purchased Concentrates per ton at UAC

Input Operation	Units per ha	Cost/unit	Total cost per ha T shs	Cost per Year T shs
Rhodes Grass Seed	5 kg	40	200	40
Desmodium Seed	5 kg	80	400	80
Napier Grass Seed (Assumed for Nandi setaria)	7 kg	40	280	56
Cultivation	1.8 hrs	165	297	223
Fertilizer Application	0.8 hrs	165	132	528
Harvesting	2.0 hrs	225	450	1350
Interest on Working Capital				100

Table 10. Pasture Production Input Cost at UAC. Source: Myoya ., 1980, p. 41.

The production costs are based on the following:

1. That the harvesting costs per hectare are those obtained at UAC farm where the rates of 100-150 kg. N/ha per year are used.
2. That with yield increase due to the increase in nitrogen application, more dry matter per hectare are handled and as such the harvesting costs for 120 kg. N/ha will be taken as 100%. For 0 kg. N/ha as 50%, for 60 kg. N/ha as 75%, for 80 kg. N/ha as 80%, for 160 kg. N/ha as 120%, for 240 kg. N/ha as 150%, for 320 kg. N/ha as 180% and for 480 kg N/ha as 200%.
3. The production costs for silages and hay are assumed to be 50% higher than those of the corresponding grass or crop. Taking into account the harvesting costs of 450 T shs. For 120 kg. N/ha, phosphorous fertilizer cost 8.5 T shs. and nitrogen cost 6.5 T shs. The following were the prices for common feeds at UAC (Table 2.11). The prices for supplementary feeds which are usually bought, are given on tonnage basis.
4. The official currency of Tanzania is Tanzanian shillings hereby abbreviated as T shs.

Note that at the time of this research, 100 T shs was approximately equivalent to 1 U.S. $. Thus the estimated cost of production of various feeds is depicted in Table 2.11 below.

Feed	Fertilizer Applied	Cost (T shs)
Pasture		
Natural Pasture (per ha)	80N	1048
Rhodes Grass (per ha)	80N	2353
Rhodes/Desmodium (per ha)	-	2032
Green Feed		
Napier Grass (per ha)	80N	2566
Napier/Desmodium (per ha)	-	1653
Lupins (per ha)	20P	3544
Oats (per ha)	25N 10P	3661
Hay		
Rhodes Grass (per ha)		3530

Silage		
Maize (per ha)		5820
Lupins (per ha)		5316
Oats (per ha)		5492
Napier Grass (per ha)		3849
Napier/Desmodium (per ha)		2480
Rhodes Grass (per ha)		3530
Rhodes/Desmodium		3048
Purchased Foods Cost in Tshs (per ton)		
Energy Feeds		
Maize Meal		10000
Maize bran		4000
Rice Polishing		12000
Protein Feeds		
Cotton Seed Cake (undecorticated)		6000
Cotton seed Cake (decorticated)		6000
Cotton Seed Cake (undecorticated)		6000
Sunflower Cade (undecorticated)		6000
Lupin Grain		8000

Table 11. Costs of Feeds per Hectare or per ton Depending on the Nature of Feed (grown or purchased) for most Efficient Fertilizer Applications.

3.

3.1. Setting up the Linear Programming (L.P.) model

All together 23 different feeds were available at UAC under the land utilisation programme. The 23 different feeds include concentrates and minerals which are fed according to milk production. We denote the acreage of the different crop types for the most 'efficient' fertilizer application by X_j in hectares for the grown crops and by Y_j in tons for the purchased feeds.

3.1.1. The objective function

The objective of the model is to determine the herd size that would maximize the net profit at UAC.

3.2. The objective function coefficient

3.2.1. Milk output and its revenue

According to the annual livestock report of 1984-85 of UAC, Gross income was 3.5 million Tshs. (90% was from dairy, excluding butter processing and cream). Gross income from dairy was 3.15 million T shs. The variable costs of production were 1.575 million T shs. Gross profit was therefore 1.575 million T shs. According to the same report the average number of cows was 100. Therefore the profit per cow was 15,750 T shs per annum.

3.2.2. Milk production input costs

In order to find the optimum herd size, it is important that the inputs i.e. crops involved in dairy production are included in the programme. As already seen earlier, various costs of production could be worked out.

The objective function is therefore to Maximize the Net Profit. Denote the different acreage for the grown crops types by X where j = 1, 2, …. 15 are grown crops in hectares, Y_j for the purchased concentrates in tons where $j \geq 17$ and by Z_j for the herd size when j = 16 where,

For grown crops	
X_1	hectares of Natural grass pasture.
X_2	hectares of Rhodes pasture.
X_3	hectares of Rhodes/Desmodium pasture.
X_4	hectares of Napier grass green feed.
X_5	hectares of Lupins green feed.
X_6	hectares of Napier/Desmodium green feed.
X_7	hectares of Oats green feed.
X_8	hectares of Rhodes grass hay.
X_9	hectares of Maize silage.
X_{10}	hectares of Rhodes grass silage.
X_{11}	hectares of Rhodes/desmodium silage.
X_{12}	hectares of Napier grass silage.
X_{13}	hectares of Oasts silage.
X_{14}	hectares of Lupins silage.
X_{15}	Hectares of Napier/Desmodium silage.
For Purchased Feeds	
Y_{17}	tons Maize meal.
Y_{18}	tons of Maize bran.
Y_{19}	tons of Rice polishing.
Y_{20}	tons of Lupin grain.
Y_{21}	tons of Cotton seed cake (undecorticated).
Y_{22}	tons of Cotton seed cake (decorticated).
Y_{23}	tons of Sunflower cake (undecorticated).
Y_{24}	tons of Sunflower cake (decorticated).
Z_{16}	the Herd size or the Stocking rate.

Table 12.

Using the cost values in Table 2.12, the objective function will be to

Maximize

$- 15750 \quad Z_{16} - 1048X_{1} - 2353X_{2} - 2032X_{3}$
$- 2566X_{4} - 3544X_{5} - 1653X_{6} - 3661X_{7} - 3530X_{8} - 5820X_{9} - 3530X_{10}$
$- 3048X_{11} - 3840X_{12} - 5492X_{13} - 5316X_{14} - 2480X_{15} - 10000Y_{17}$
$- 4000Y_{18} - 12000Y_{19} - 8000Y_{20} - 6000Y_{21} - 6000Y_{22} - 6000Y_{23} - 6000Y_{24}$

At UAC, the objective is formulated on the basis of one type of breed only since there is only one breed at UAC for dairy production. In situations where multiple breeds are involved, a multiple objective function can be formulated in line with modified costs and profits accordingly.

The Constraints

3.2.3. Land constraint

Let the total acreage available be A. The acreage constraint ensures that the amount of land available for the growth of various crops is not exceeded.

$$\sum_{j=1}^{15} X_j \leq A$$

where A is the total acreage in hectares
X_j is the acreage for different crop types j in hectares.

In the case of UAC, A is 790 hectares.

3.2.4. Maintenance energy requirement constraint

As a cow needs a minimum quantity of bulky food in its diet, it was decided that at least sufficient energy to supply maintenance requirements should come from food of this type, and should be grown on the farm.

Suppose crop j supplies a_j kg of energy (MJME) per ha. If one cow requires E_m MJME for maintenance, then

$$\sum_{j=1}^{15} a_j X_j \geq E_m Z_{16}$$

3.2.5. Total energy requirement constraint

Suppose crop j supplies a_j megajoules of metabolizable energy per hectare and suppose the purchased concentrates do supply b_j megajoules of metabolizable energy per ton. If one cow requires E_l metabolizable energy for maintenance and lactation then.

$$\sum_{j=1}^{15} a_j X_j + \sum_{j=17}^{24} b_j Y_j \geq E_l Z_{16}$$

3.2.6. Maintenance protein requirement constraint

Suppose crop j supplies p_j kg. of digestible crude protein per hectare. If one cow requires q kg. of Digestible Crude Protein for maintenance then.

$$\sum_{j=1}^{15} p_j X_j \geq q Z_{16}$$

3.2.7. Total protein requirement constraint

Suppose crop j supplies p_j kg. of digestible crude protein per hectare and suppose the purchased concentrated do supply r_j kg. of digestible crude protein per ton. If one cow requires t kg. of digestible Crude Protein for maintenance and lactation

Then $\sum_{j=1}^{15} p_j X_j + \sum_{j=17}^{24} r_j Y_j \geq t Z_{16}$

3.2.8. Space constraint

Let the space needed for a cow on the average be s m 2. If the available space has a total area of h square metres then this particular farm can accommodate a maximum of M = h/s animals. Thus

$$Z_{16} \leq M$$

At UAC s = 6 m 2 area needed by one cow. H = 24,000 m 2 is area of the available shelter at UAC. Then M = h/s = 4000. The number of animals that can be 'accommodated'. Therefore

$$Z_{16} \leq 4000$$

Three L.P. models were run with different assumptions for each model. In the first model, the model was run for grown crops only. In the second model the imposed restriction was that maize should not be grown for the purpose of feeding animals (with an intuitive idea that maize should be for humans only). This was removed from the programme in the usual way by making its cost of production exorbitantly high. The problem was unchanged except for the coefficient C9 which was changed from 5820 to 99999. In the third model, the model was run for grown crops and purchased concentrates. The third model was feasible and gave the maximum profit. Thus the third model was adopted for our study. The solutions to the third model is presented in section 3. Together with the solution, post optimality analysis i.e. how sensitive is the optimal solution - and the appropriate interpretation are given for the this model.

We shall use the cost values in Table 2.12 and the net profit of 15,750 Tshs. per cow as calculated in section 3.2.1 for the objective function coefficients. The feed values in Table 2.9

and 2.10 will be used as the coefficients of the left hand side of crop and purchased constraints respectively while feed requirement values in Table 2.1 will be used as the coefficient of the Z on right hand sides of the constraints for the available feed supply periods as shown in Table 2.4. Our linear programming problem involving all the feeds (grown foods and purchased concentrates) is presented as follows:

Maximize

$15750 Z_{16} - 1048X_1 - 2353X_2 - 2032X_3 - 2566X_4 - 3544X_5 - 1653X_6 - 3661X_7$
$- 3530 X_8 - 5820X_9 - 3530X_{10} - 3048X_{11} - 3840X_{12} - 5492X_{13} - 5316X_{14}$
$- 2480X_{15} - 10000Y_{17} - 4000Y_{18} - 12000Y_{19} - 8000Y_{20} - 6000Y_{21} - 6000Y_{22}$
$- 6000Y_{23} - 6000Y_{24}$

Subject to

$$\sum_{j=1}^{15} X_j \le 790 \text{ land constraint} \tag{6}$$

$$Z_{16} \le 4000 \text{ Fencing space constraint} \tag{7}$$

Total energy requirement constraint in January and February

$$\begin{aligned} &50600X_1 + 12267X_2 + 16590X_3 + 14200Y_{17} + 12500Y_{18} = 15000Y_{19} + \\ &+ 14200Y_{20} + 8500Y_{21} + 10800Y_{22} + 9500Y_{23} + 11900Y_{24} \ge 6073 Z_{16} \end{aligned} \tag{8}$$

Maintenance energy requirement constraint in January and February

$$50600X_1 + 12267X_2 + 16590X_3 \ge 2956 Z_{16} \tag{9}$$

Total protein requirement constraint in January and February

$$\begin{aligned} &382.3 X_1 + 65.6X_2 + 124.7X_3 + 91.2Y_{17} + 62.8Y_{18} 129.6 Y_{19} + \\ &+273Y_{20} + 178Y_{21} + 296.6Y_{22} + 185.4Y_{23} + 232.5Y_{24} \ge 47.22 Z_{16} \end{aligned} \tag{10}$$

Maintenance protein requirement constraint in January and February

$$382.3 X_1 + 65.6X_2 + 124.7X_3 \ge 16.74 Z_{16} \tag{11}$$

Total energy requirement constraint in December

$$\begin{aligned} &50600X_1 + 12267X_2 + 16590X_3 + 46696X_4 + 48400X_6 + 28035X_8 + 89000X_9 + \\ &+ 26604X_{10} + 15120X_{11} + 41536X_{12} + 35200X_{15} + 14200Y_{17} 12500Y_{18} + \\ &+15000Y_{19} + 14200Y_{20} + 85000Y_{21} = 10800Y_{22} + 9500Y23 + 11900Y24 \ge \\ &\ge 2812 Z_{16} \end{aligned} \tag{12}$$

Maintenance energy requirement constraint in December

$$50600X_1 + 12267X_2 + 16590X_3 + 46696X_4 + 48400X_6 + 28035X_8 + \\ + 89000X_9 + 26604X_{10} + 15120X_{11} + 41536X_{12} + 35200X_{15} \geq 1478\, Z_{16} \quad (13)$$

Total protein requirement constraint in December

$$382.3X_1 + 65.6X_2 + 124.7X_3 + 529X_4 + 901X_6 + 174X_8 + 277X_8 + \\ + 92.7X_{10} + 53.4X_{11} + 483X_{12} + 409.6X_{15} + 91.2Y_{17} + 62.4Y_{18} + 129.6Y_{19} + \\ + 273Y_{20} + 178Y_{21} + 296.6Y_{22} + 185.4Y_{23} + 232.5Y_{24} \geq 21.33\, Z_{16} \quad (14)$$

Maintenance protein requirement in December

$$382.3X_1 + 65.6X_2 + 124.7X_3 + 529X_4 + 901X_6 + \\ +174X_8 + 277X_8 + 92.7X_{10} + 53.4X_{11} + 483X_{12} + 409.6X_{15} \geq 8.37\, Z_{16} \quad (15)$$

Total energy requirement constraint in March and April

$$50600X_1 + 12267X_2 + 16590X_3 + 46696X_4 + 57500X_5 + 48400X_6 + 26250X_{11} + \\ +1400Y_{17} + 12500Y_{18} + 15000Y_{19} + 14200Y_{20} + 8500Y_{21} + 10800Y_{22} + 95500Y_{23} + \\ +11900Y_{24} \geq 6509\, Z_{16} \quad (16)$$

Maintenance energy requirement constraint in March and April

$$50600X_1 + 12267X_2 + 16590X_3 + 46696X_4 + 57500X_5 + 48400X_6 + 26250X_7 \geq 2956\, z_{16} \quad (17)$$

Total protein requirement constraint in March and April

$$382.3X_1 + 65.6X_2 124.7X_3 + 529X_4 + 569X_6 + 901X_6 + 256X_7 + \\ + 91.2Y_{17} + 62.4Y_{18} + 129.6Y_{19} 273Y_{20} + 178Y_{21} + 296.6Y_{22} + 185.4Y_{23} + \\ + 232.5Y_{24} \geq 51.25\, Z_{16} \quad (18)$$

Maintenance protein requirement in March and April

$$382.3X_1 + 65.6X_2 124.7X_3 + 529X_4 + 569X_6 + 901X_6 + 256X_7 \geq 16.74\, Z_{16} \quad (19)$$

Total energy requirement constraint in May

$$50600X_1 + 12267X_2 + 16590X_3 + 46696X_4 + 57500X_5 + \\ + 48400X_6 + 26250X_7 + 89000X_9 + 14200Y_{17} + 12500Y_{18} + \\ + 15000Y_{19} + 14200y_{20} + 85500y_{21} + 10800Y_{22} + 9500Y_{23} + 11900Y_{24} \geq 3761\, Z_{16} \quad (20)$$

Maintenance energy requirement constraint in May

$$50600X_1 + 12267X_2 + 16590X_3 + 46696X_4 + \\ + 57500X_5 + 48400X_6 + 26250X_7 + 89000X_9 \geq 1478\, Z_{16} \quad (21)$$

Total protein requirement constraint in May

$$\begin{aligned}&382.3X_1 + 65.6.X_2 + 124.7X_3 + 529X_4 + 569X_4 + 901X_6 + 256X_7 + \\&+ 91.2Y_{17} + 62.4Y_{18} + 129.6Y_{19} + 273Y_{20} + 178Y_{21} + 296.6Y_{22} + 185.4Y_{23} + \\&+ 232.5Y_{24} \geq 30.55\ Z_{16}\end{aligned} \tag{22}$$

Maintenance protein requirement constraint in May

$$\begin{aligned}&382.3X_1 + 65.6.X_2 + 124.7X_3 + 529X_4 + 569X_4 + 901X_6 + \\&+ 256X_7 + 277X_9 \geq 8.37\ Z_{16}\end{aligned} \tag{23}$$

Total energy requirement constraint in June

$$\begin{aligned}&46696X_4 + 57500X_5 + 48400X_6 + 26250X_7 + \\&+ 89000X_9 + 14200Y_{17} + 12500Y_{18} + 15000Y_{19} + \\&+ 14200Y_{20} + 8500Y_{21} + 10800Y_{22} + 9500Y_{23} + \\&+ 11900Y_{24} \geq 3400\ Z_{16}\end{aligned} \tag{24}$$

Maintenance energy requirement constraint in June

$$46696X_4 + 57500X_5 + 48400X_6 + 26250X_7 + 89000X_9 \geq 1478\ Z_{16} \tag{25}$$

Total protein requirement constraint in June

$$\begin{aligned}&529X_4 + 569X_5 + 901X_6 + 256X_7 + 277X_9 9.2Y_{17} + \\&+ 62.4Y_{18} + 129.6Y_{19} + 273Y_{20} + 178Y_{21} + 296.6Y_{22} + \\&+ 185.4Y_{23} + 232.5Y_{24} \geq 27.04\ Z_{16}\end{aligned} \tag{26}$$

Maintenance protein requirement constraint in June

$$529X_4 + 569X_5 + 901X_6 + 277X_9 \geq 8.37\ Z_{16} \tag{27}$$

Total energy requirement constraint in July and August

$$\begin{aligned}&46696X_4 + 57500X_5 + 48400X_6 + 89000X_9 + 26604X_{10} + 15120X_{11} + \\&+ 41536X_{12} + 21000X_{13} + 14200Y_{17} + 12500Y_{18} + 15000Y_{19} + 14200Y_{20} + \\&+ 14200Y_{20} + 8500Y_{21} + 10800Y_{22} + 9500Y_{23} + 11900Y_{24} + 85200X_{14} + \\&+ 35200X_1 \geq 6266\ Z_{16}\end{aligned} \tag{28}$$

Maintenance energy requirement constraint in July and August

$$\begin{aligned}&46696X_4 + 57500X_5 + 48400X_6 + 89000X_9 + 26604X_{10} + \\&+ 15120X_{11} + 41536X_{12} + 21000X_{13} + 85200X_{14} + 35200X_{15} \geq \\&\geq 2956\ Z_{16}\end{aligned} \tag{29}$$

Total protein requirement constraint in July and August

$$
\begin{aligned}
&529X_4 + 569X_5 + 901X_6 + 277X_9 + 92.7X_{10} + 91.2Y_{17} + \\
&+ 62.4Y_{18} + 129.6Y_{19} + 273Y_{20} + 178Y_{21} + 296.6Y_{22} + \\
&+ 185.4Y_{23} + 232.5Y_{24} + 53.4X_{11} + 483X_{12} + 115.8X_{13} + 1650X_{14} + \\
&+ 409.6X_{15} \geq 48.9\, Z_{16}
\end{aligned} \tag{30}
$$

Maintenance protein requirement constraint in July and August

$$
\begin{aligned}
&529X_4 + 569X_5 + 901X_X + 277X_9 + 92.7X_{10} + \\
&+ 53.4X_{11} + 483X_{12} + 115.8X_{13} + 1650X_{14} + \\
&+ 409.6X_{15} \geq 16.74\, Z_{16}
\end{aligned} \tag{31}
$$

Total energy requirement constraint in September, October and November

$$
\begin{aligned}
&46696X_4 + 48400X_6 + OX7 + 28035X_9 + 26604X_{10} + \\
&+ 15120X_{11} 41536X_{12} + 35200X_{15} + 14200Y_{17} + 12500Y_{18} + \\
&+ 15000Y_{19} + 14200Y_{20} + 8500Y_{21} + 10800Y_{22} + 9500Y_{23} + \\
&+ 11900Y_{24} \geq 9172\, Z_{16}
\end{aligned} \tag{32}
$$

Maintenance energy requirement constraint in September, October and November

$$
\begin{aligned}
&46696X_4 + 48400X_6 + 28035X_8 + 89000X9 + \\
&+ 26604X_{10} + 15120X_{11} + 41536X_{12} + 35200X_{15} \geq 4434\, Z_{16}
\end{aligned} \tag{33}
$$

Total protein requirement constraint in September, October and November

$$
\begin{aligned}
&529X_4 + 901X_6 + 174X_8 + 277X_9 + 92.7X_{17} + \\
&+ 91.2Y_{17} + 62.4Y_{18} + 129.6Y_{19} + 273Y_{20} + 178Y_{21} + \\
&+ 296.6Y_{22} + 185.4Y_{23} + 232.5Y_{24} 53.4X_{11} + 484X_{12} + \\
&+ 409.6X_{15} \geq 71.15\, Z_{16}
\end{aligned} \tag{34}
$$

Maintenance protein requirement constraint in September, October and November

$$
\begin{aligned}
&529X_4 + 901X_6 + 174X_8 + 277X_9 + 92.7X_{10} + \\
&+ 53.4X_{11} + 483X_{12} + 409.6X_{15} \geq 25.11\, Z_{16}
\end{aligned} \tag{35}
$$

ALL X's, Y's and Z ≥ 0.

3.3. The stocking rate or the herd size model

The L.P. problem for UAC was run using the OR software by Dennis and Dennis (1991).

One could run any number of models with different assumptions for each model. In our case, three LP models were considered with different assumptions for each model. The assumptions considered were running the LP with all grown crops included, concentrates excluded, running the LP with maize and concentrates excluded and running the LP with all grown crops and concentrates included. The purpose of doing this was to find out what combination of foods would give the maximum profit. This and the previous linear programming problems were run on an IBM PC using the OR software by Dennis and Dennis (1991). Here OR (Operations Research) is defined as the systematic application of quantitative methods, techniques, and tools to the analysis of problems involving the operation of the system. The aim is the evaluation of probable consequences of decision choices, usually under conditions requiring the allocation of scarce resources –funds, manpower, time, or raw materials (Daellenbach & George, 1978). The computer output tables are presented in Table 3.1. The simplex tableau for the grown crops and purchased concentrates is discussed.

Inclusion of concentrates results into a big profit of T shs. 58,752,345.70. The option of giving concentrates to cows has a significant impact on profit maximization at UAC as shown in Table 3.1 We recommend this model.

The following results were obtained after 15 iterations of the Simplex Algorithm.

Variable	Quantity	Variable	Quantity	Variable	Quantity
				S_{17}	300499.771
X_1	361.234	S_8	49054442.076	S_{18}	389219.771
X_6	184.378	S_9	328539.771	S_{19}	23088000
X_9	244.388	S_{10}	389219.771	S_{20}	30776000
Y_{22}	85.545	S_{11}	7179873.951	S_{21}	176440
Y_{18}	407.174	S_{12}	21391873.951	S_{22}	251120
Z_{16}	4000	S_{13}	150004.182	S_{23}	11624000
S_4	12468000	S_{14}	288044.182	S_{24}	24864000
S_6	121920	S_{15}	39922442.076	S_{25}	89000
S_7	43718442.076	S_{16}	49054442.076	S_{26}	217640
				S_{28}	18952000
				S_{30}	184160

Table 13. The Final Simplex Tableau displaying all the Feeds (Grown and Purchased Concentrates).

Optimal Profit = T.shs 58,752,345.717. S here refers to slacks.

Results show that land to be allocated for natural pastures is 361.234 hectares, Napier/Desmodium 184.378 hectares and maize production 244.388 hectares. Total land used for their production is therefore 790 hectares i.e. the whole land available is utilized. Concentrate supplementation is 85.545 tons of cotton seed cake (decorticated) and 407.174 tons of maize bran. Fencing land for the cows is fully utilized. Herd size is 4000 cows. Since whatever available land and fencing has been utilized under this programme and profit has been maximized, it was deemed reasonable to adapt this model.

4.

4.1. Sensitivity analysis

4.1.1. Abundant and scarce resources

Associated with every LP; there is a corresponding optimization problem called the Dual Problem. The original problem is called the primal problem. The purpose of the dual in our case is to identify scarce and abundant resources and as such give recommendations if any. Dual values represent quite precisely the per unit increase in the objective function which would follow from an increase in the availability of the corresponding factors or resources.

It should be obvious, first of all, that an increased availability of a factor which is not fully used will only leave more of it unused and add nothing to the objective function and such a constraint has zero dual value – it is a free good. (Note that a good is free, not because it is not used, but because there is more available than is required. Air and water are the classical cases of free goods which would be very far from free if their availability were restricted).

To summarise, we can assert that if Y_k represents the per unit increase in revenue from an increase in the availability of the k th factors, then a change in availability of Δ_k will lead to a change in revenue of $Y_k\Delta_k$.

It is obvious that an increased availability of a factor which is fully utilized can add considerably to the value of the objective function. Land constraint is fully utilized and its dual value is positive (8003.55). Similarly fencing space for cows (constraint 2) is fully utilized, its dual value is positive (13107.383). So an increase in the land for crops and an increase in the space for the animals can still add considerably to the revenue of the enterprise by rearing more cows. Thus, per unit increase in the land acreage would increase the objective function by 8003.55 whereas per unit increase in the fencing space would increase the objective function by 13107.385 with all other coefficients in the problem remaining the same.

On the other hand, a small increase in the right-hand-side of an abundant resource constraint will only change the amount of slacks or surplus and will not affect the value of the objective function. Thus the shadow price for any non-binding constraint is zero.

The other constraints, for example constraint (4) and constraints (6) to (30) except constraints (27) and (29), are not so binding in our case since an increase in their availability will leave more of them unused and add nothing to the revenue and, as such, their dual values are zero.

Constraints (1), (2), (3), (5), (27), and (29) are binding in our case and as such their dual values are positive. They are therefore scarce resources. If we go back to the primal problem we will see that these aforementioned constraints have all their slack values equal to zero and their corresponding dual variables are positive.

Variable	Dual solution or shadow price	Constraint
S_1	8003.55	(1)
S_2	13107.385	(2)
S_3	.121	(3)
S_4	0	(4)
S_5	7.632	(5)
S_6	0	(6)
S_7	0	(7)
S_8	0	(8)
S_9	0	(9)
S_{10}	0	(10)
S_{11}	0	(11)
S_{12}	0	(12)
S_{13}	0	(13)
S_{14}	0	(14)
S_{15}	0	(15)
S_{16}	0	(16)
S_{17}	0	(17)
S_{18}	0	(18)
S_{19}	0	(19)
S_{20}	0	(20)
S_{21}	0	(21)
S_{22}	0	(22)
S_{23}	0	(23)
S_{24}	0	(24)
S_{25}	0	(25)
S_{26}	0	(26)
S_{27}	.146	(27)
S_{28}	0	(28)
S_{29}	2.851	(29)
S_{30}	0	(30)

Table 14. Dual values for the recommended programme (model three whereby grown crops and concentrates are included).

The scarce resources in our model are therefore land, fencing space, energy supply from January to February, protein supply from January to February, energy supply from September to November and protein supply from September to November. Energy and protein supplies are scarce from September to November because these are dry months in Mbeya Region and as such food is scarce during this period. Similarly the supply of food from January to February is not adequate in Mbeya Region.

As for the abundant resources these have dual values equal zero in their constraints. An abundant resource worth mentioning is energy supply from March to April. The slack of this constraint S_{11} has the value 7179873.953 in the primal. This slack is an indication of surplus food available during rainy season in Mbeya Region which is mostly pronounced in March and April.

4.1.2. The objective function coefficients

It is important for us to know, for example, for what ranges of prices of the inputs in the objective function is the solution still optimal. To do this we assume the coefficient matrix A and the right hand side constraints b are unchanged but the profits vector c is changed to $c+\lambda c$, where λ is any constant. The results are presented in Table 4.2.

Coefficient of Variables	Lower Limit	Original Value	Upper Limit
X_1	-9043.741	-1048	20358.836
X_2	NO LIMIT	-2553	6015.1
X_3	NO LIMIT	-2032	5040.77
X_4	NO LIMIT	-2566	-343.01
X_5	NO LIMIT	-3544	8003.55
X_6	-5666.778	-1653	672.409
X_7	NO LIMIT	-3661	8003.55
X_8	NO LIMIT	-3530	3401.85
X_9	-9099.061	-5820	2958.965
X_{10}	NO LIMIT	-10000	-4756.94
X_{11}	NO LIMIT	-3530	3843.18
X_{12}	NO LIMIT	-3048	5637.02
X_{13}	NO LIMIT	-3840	543.8
X_{14}	NO LIMIT	-5316	8003.55
Y_{19}	NO LIMIT	-1200	-5373.61
Y_{20}	NO LIMIT	-8000	-6662.69
Y_{21}	NO LIMIT	-6000	-4141.11
Y_{22}	-6529.272	-6000	-4058.013
Y_{23}	NO LIMIT	-6000	-4486.36
Y_{24}	NO LIMIT	-6000	-5622.5
Y_{18}	-5124.335	-4000	-2448.285
X_{15}	NO LIMIT	-2480	-1680.90
Z_{16}	2642.615	15750	NO LIMIT

Table 15. Sensitivity Analysis of Objective Function Coefficients

Of interest are the coefficients of the variables X_1, X_6, X_9, Y_{18}, Y_{22} and Z_{16}. the lower and upper limits within which the solution is still optimal are shown in Table 4.2.

For example, the solution is still optimal so long as -9043.741 < C_1< 20358.836 and so on. The cost (C_1) of natural pasture in the objective function is -1048 per hectare. As long as this cost lies between -9043-741 and 20358..836 the solution is still optimal so long as the other costs C_1's remain as they were in the primal.

4.2. The right-hand-side ranges

The right-hand-side ranges provide limits within which the objective coefficients of the dual problem are allowed to change without changing the solution. For changes outside the range the problem must be resolved to find the new optimal solution and the new dual price. We call the range over which the dual price is applicable the range of feasibility.

Assuming A and c are unchanged, b changes to b+χb where χ is any constant, the right-hand side ranges within which the objective function remains optimal are presented in table 4.3.

Constraint	Lower Limit	Original Value	Upper Limit
1	174.02	790	983.61
2	3212.67	4000	18158.99
3	-3640461.74	0	5212071.15
4	-12468000	0	NO LIMIT
5	-84149.1	0	21767.12
6	121920	0	NO LIMIT
7	43718442.08	0	NO LIMIT
8	4905442.08	0	NO LIMIT
9	328539.77	0	NO LIMIT
10	-389219.77	0	NO LIMIT
11	-7179873.95	0	NO LIMIT
12	-21391873.95	0	NO LIMIT
13	-150004.18	0	NO LIMIT
14	-288044.18	0	NO LIMIT
15	-39922442.08	0	NO LIMIT
16	-49054442.08	0	NO LIMIT
17	-300499.77	0	NO LIMIT
18	-389219.77	0	NO LIMIT
19	-23088000	0	NO LIMIT
20	-30776000	0	NO LIMIT

Constraint	Lower Limit	Original Value	Upper Limit
21	-176440	0	NO LIMIT
22	-251120	0	NO LIMIT
23	-11624000	0	NO LIMIT
24	-24864000	0	NO LIMIT
25	-89000	0	NO LIMIT
26	-217640	0	NO LIMIT
27	-25622891.49	0	NO LIMIT
28	-18952000	0	NO LIMIT
29	-281115.74	0	NO LIMIT
30	-184160	0	NO LIMIT

Table 16. Sensitivity Analysis of Right Hand Ranges

Of interest are constraints (1), (2), (3), (5), (27) and (29) i.e., land for cultivation, fencing space, energy and protein supply from January to February and energy and protein supply from September to November constraints. These are the binding constraints in our model. As shown in Table 4.3, the ranges of constraints (1) and (2) are all positive i.e. 174.04 < Land size < 983.61 and 3212.67 < fencing space < 18158.99 and so on. For example, land size could be increased up to 983.61 hectares so long as the A matrix and the objective function vector are unchanged. The solution would still be optimal. An increase of one hectare of land would increase the objective function by 8003.55 as provided for by the dual.

Changes in the right-hand side of the constraints show how the optimal solution and net profit would change if we could obtain additional land or fencing space.

5. Conclusion

The model has managed to ascertain the profitability of a dairy farm. Indeed, this form of argument can be useful in the management of dairy farms of similar traits elsewhere. The assumption here is that the herd size was kept constant throughout the year. Perhaps this is an oversimplification but it provides a starting point. There is a need of formulating Operational Research models for which the need for having a fixed herd size can be relaxed. As can be seen from the input parameters of the L.P., the values are probably not in line with dynamics of time and technological advancement of raring /keeping dairy cattle. Perhaps there is a need of updating the input parameters so that they can match with time from farm to farm.

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