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# Model-Based Adaptive Tracking Control of Linear Time-Varying System with Uncertainties

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Additional information is available at the end of the chapter

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## 1. Introduction

This primary purpose of this research is concerned with adaptive tracking control of a nonlinear system [6, 9]. Particularly, time-varying control approach has been designed for tracking of the system with application to a nonlinear dynamic model [1]. Furthermore, the time-varying system is further complicated by parametric uncertainty or disturbances such as external forces, continuous or discrete noise where the parameters are unknown. Over the past several years, trajectory tracking issue as a high-level control of a nonlinear system has been received a wide attention from control community. Hence, the discussion here is principally devoted to model-based adaptive trajectory tracking control algorithm of linear time-varying (LTV) systems in the presence of uncertainty [4, 5].

A system undergoing slow time variation in comparison to its time constants can usually be considered to be linear time invariant (LTI) and thus, slow time-variation is often ignored in dealing with systems in practice. An example of this is the aging and wearing of electronic and mechanical components, which happens on a scale of years, and thus does not result in any behavior qualitatively different from that observed in a time invariant system on a day-to-day basis. There are many well developed techniques for dealing with the response of linear time invariant systems such as Laplace and Fourier transforms, but not applicable to linear time varying or nonlinear systems, nor feasible to implement for complicated real-world systems. In addition, time-varying system may be difficult to satisfy global controllability or to show whether the time-varying system is even stable or not, due to difficulties in computing or finding solution. Unlike LTI systems, linear time varying systems may behave more like nonlinear systems [1, 2, 3]. In general all systems are time-varying in principle and a large number of systems arising in practice are time-varying. Time variation is a result of system

parameters changing as a function of time [5], such as aerodynamic coefficients of aircrafts, hydrodynamic terms in marine vessels, circuit parameters in electronic circuits, and mechanical parameters in machinery. Thus, we characterize systems as time-varying if the parameter variation is happening on time scales close to that of the system dynamics. Time variation also occurs as a result of linearizing a nonlinear system about a family of operating points and/or about a time-varying trajectory for developing control system. However, due to the desire to achieve better accuracy and quality in a wide range of applications [11], there have been increasing interests to include the effects of time-variation [12] while designing controllers or observers at the time analyzing and/or applying to such systems.

In this work, tracking error system is formulated based on its model-reference system which has a reference input and the nonlinear dynamic model of the inverted pendulum. We found a solution of the tracking nonlinear system after developing its linear time varying systems. For the development of subsequent control approach, the error system is linearized about given desired trajectory using a perturbation approach and produced a linear time-varying tracking error equation [3] with system matrices,  $A(t)$  and/or  $B(t)$ . At this time the controllability of this time-varying system only shows that the system is stable in an instant time or about a trajectory which can be locally controllable or stabilized. Then, a novelty of this research is that a controllability grammian matrix is found to be a necessary and sufficient conditions of the global controllability and the inverse of the grammian matrix exists, which is nonsingular, and is used for the designing the control input of the closed-loop system. In this research, a complicated solution of state transition matrix is obtained based on Taylor series expansion, categorized into feasible forms based on the system and the shape of matrix. The control input of the tracking system is designed from the state transition matrix and the grammian matrix, which makes the system globally controllable, and the control input of the actual system is redesigned via the tracking controller while compensating for the uncertainty as disturbances, which also yields the system globally stable. This chapter consists of as follows: a time-varying system is briefly described relative to a time-invariant system and a non-homogeneous system is introduced for linear time-varying system for the development of the solution which is state transition matrix in Section II followed by Introduction. Then a cart-pole nonlinear dynamic model where the system parameters are unknown is developed for the application of a proposed control algorithm and expressed into a state space form. For the trajectory tracking control, error signals are formulated from desired model-based reference system. Based on the analysis of the developed time-varying error system, the solution of the system, state transition matrix, is derived in a series form and then a special form of the matrix is obtained for the second-order error differential equation, which is used for the grammian matrix and the closed-loop controller. The control system is also developed to reject disturbances via a projection-based adaptive control approach and update laws for the parameter update in Section III. Numerical simulation results with analyses demonstrate the validity of the proposed system. This approach can be extended to other nonlinear time-varying dynamic systems such as aerial-, marine, or ground vehicles.

## 2. Linear time-varying system

A linear time-invariant system (LTI) is described as

$$\dot{x} = Ax(t), \quad \forall t \geq 0 \quad (1)$$

where the equilibrium point is at the origin and if  $\det(A) \neq 0$ , the fixed point is isolated and the stability of the origin depends on the location of the eigenvalues of the matrix  $A(\cdot) \in \mathfrak{R}^{n \times n}$  which is not a function of time. The solution of (1) with the initial state  $x(t_0)$  is given by

$$x(t) = \exp(At)x(t_0) \quad (2)$$

Another LTI state equations is given by

$$\dot{x} = Ax + Bu \quad (3)$$

where  $A$  and  $B$  are time-invariant. It is known that the solution of the equation (3) using an integrating factor yields  $A(\cdot) \in \mathfrak{R}^{n \times n}$  can be time-varying or time-invariant. The solution of (3) with the initial state  $x(t_0)$  is given by

$$x(t) = \Phi(t, t_0)x(t_0) + \int_{t_0}^t \Phi(t, \tau)Bu(\tau)d\tau \quad (4)$$

where this is a convolution control solution and the state transition matrix  $\Phi(t, t_0) = e^{A(t-t_0)}$  and  $\Phi(t, \tau) = e^{A(t-\tau)}$ . The solutions (2) and (4) make clear the importance of the matrix exponential  $\exp(At)$  and its eigenvalues. However, these techniques are not strictly valid for time-varying systems.

### 2.1. Homogeneous system

A time-varying system is described as

$$\dot{x} = A(t)x(t), \quad \forall t \geq 0 \quad (5)$$

where  $\dot{x}(t), x(t) \in \mathfrak{R}^n$ , and the matrix  $A(\cdot) \in \mathfrak{R}^{n \times n}$  is not a constant as a function of time; it is nonautonomous [6, 7]. The general solution of the (5) in  $n$ -dimensional linear vector space,  $\dot{x}_i(t) = A(t)x_i(t) \in \mathfrak{R}^n$ , is unique for the space on  $[t_0, t]$  in case  $A(t)$  is smooth where  $x_{i0}$  ( $i = 0, \dots, n$ ) is a basis set of  $n$  linearly independent initial condition. According to the linearly independent solutions, a system is defined as

$$\dot{X}(t) = A(t)X(t) \quad (6)$$

where  $X(t) = [x_1, \dots, x_n] \in \mathfrak{R}^{n \times n}$  is a matrix which has the linearly independence solutions which shows  $\dot{X}(t) = [A(t)x_1, \dots, A(t)x_n] \in \mathfrak{R}^{n \times n}$ . The general solution of (6) is given as

$$\dot{X}(t) = X(t_0)\Phi(t, \tau) \quad (7)$$

where  $X(t_0)$  is the matrix of the initial value of state,  $\Phi(t, \tau) \in \mathfrak{R}^{n \times n}$  is called the state transition matrix as known as a fundamental solution matrix associated with  $A(t)$ , having a form of exponential function.

## 2.2. Nonhomogeneous system

A linear time-varying system (LTV) is described as

$$\dot{x} = A(t)x(t) + B(t)u(t) \quad (8)$$

where  $B(t)u(t) \in \mathfrak{R}^n$ ,  $B(t) \in \mathfrak{R}^{n \times n}$ ,  $u(t) \in \mathfrak{R}^n$ ,  $B(t)u(t) \in \mathfrak{R}^n$ , in which  $B(t) \in \mathfrak{R}^{n \times n}$  can be input configuration matrix and  $u(t) \in \mathfrak{R}^n$  is the control input where  $n$  is the number of control inputs. Note that in case the control input is underactuated, then  $B(t) \in \mathfrak{R}^{n \times m}$  and  $u(t) \in \mathfrak{R}^m$  where  $n-m$  is the underactuation, or the number of underactuated inputs. For the controllability of time-varying systems given in (8), the state transition matrix (or known as fundamental solution matrix) is the overall solution and used to perform the function of integrating factor where the solution is derived from a linear independence on the columns of a matrix that was a function of  $\Phi(\cdot)$  and  $B(\cdot)$ .

## 2.3. Solution of the state transition matrix

The system is controllable if the controllability gramian (or grammian) matrix  $G_C \in \mathfrak{R}^{n \times n}$  below is nonsingular, *i.e.*, invertible for the necessary and sufficient condition

$$G_C = \int_{t_0}^t X(\tau)X^T(\tau)d\tau \quad (9)$$

where the rows of the matrix product  $X(\tau) = \Phi(t_0, \tau)B(\tau)$  are linearly independent in an interval. In order to prove the invertible exists, the control input  $u(t)$  of the system can be designed based on the gramian matrix as

$$u = -KB^T(\tau)\Phi^T(t_0, \tau)G_C^{-1}(t_0, \tau)x(t_0) \quad (10)$$

where  $K = \text{diag}\{k_1, \dots, k_l\} \in \mathfrak{R}^{n \times n}$  is a control input gain matrix and  $G_C^{-1}(t_0, \tau)$  is the inverse of (9). Convolution integral solution to determine the state at the end of the interval,  $x(t_1)$ , yields

$$x(t_1) = \Phi(t_1, t_0)x(t_0) + \int_{t_0}^{t_1} \Phi(t_1, \tau)B(\tau)u(\tau)d\tau \quad (11)$$

where the solution of linear time-varying system  $\Phi(t, \tau)$  is given by

$$x(t) = \Phi(t, t_0)x(t_0) + \int_{t_0}^t \Phi(t, \tau)B(\tau)u(\tau)d\tau \quad (12)$$

The expression (11) yields by factoring  $\Phi(t_1, t_0)$  of the left side as

$$x(t_1) = \Phi(t_1, t_0) \left[ x(t_0) - G_C G_C^{-1} x(t_0) \right] = 0 \quad (13)$$

where (9) was used and this implies the control input,  $u(t)$  in (10), drives the system to reach the zero state, in which  $K$  should be the identity matrix. Now the system is controllable and shows that the controllability gramian is invertible.

### 3. Application to nonlinear inverted pendulum system

#### 3.1. Dynamic model

A continuous nonlinear time-varying system is given as a combined model based on the inverted pendulum [1] expressed by the second-order differential equation by

$$\ddot{\theta} = \frac{g}{\hat{L}} \sin \theta + u \quad (14)$$

where  $\theta(t)$  is the angle of the pole of the inverted pendulum which is subjected to the external force  $u(t) \in \mathbb{R}^1$ ,  $g$  is the gravitational force, and  $L$  is the combined parameter term given by

$$\hat{L} = \frac{\hat{J} + \hat{m}\hat{l}^2}{\hat{m}\hat{l}},$$

where  $\hat{L}$  is an unknown lumped parameter, in which  $\hat{J}$  is the inertia of the pole,  $\hat{m}$  is the mass of the pole, and  $\hat{l}$  is the length of the pole. The system can be expressed into a state space model in order to analyze as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{g}{\hat{L}} \sin x_1 + u \end{aligned} \quad (15)$$

where  $u(t)$  is the actual control input of the inverted pendulum to be designed later. Let  $\dot{x}_{2d}(t)$  be the desired model-based reference system as follows

$$\begin{aligned} \dot{x}_{1d} &= x_{2d} \\ \dot{x}_{2d} &= -k_1 x_{2d} + \frac{g}{\hat{L}} \sin x_{1d} + u_r \end{aligned} \quad (16)$$

where the first term  $k_1 x_{2d}(t)$  in the right side of the second row equation is added because the response of the second equation can be divergent due to the positive reference input  $u_r$ , in which  $k_1$  is a positive constant.

#### 3.2. Error formulation

Then, the error equation can be derived from the subtraction between the desired and the actual system as

$$\dot{e}_1 \equiv x_{2d} - x_2 = e_2 \quad (17)$$

and subtracting  $\dot{x}_2(t)$  from  $\dot{x}_{2d}(t)$  and substituting the second equation of (16) yields

$$\dot{e}_2 \equiv \dot{x}_{2d} - \dot{x}_2 = \frac{g}{L}(\sin x_{1d} - \sin x_1) + u_r - k_1 x_{2d} - u. \quad (18)$$

Let the error,  $e_1(t)$ , assumed to be small. Then,  $e_1(t)$  produces

$$e_1 = x_{1d} - x_1 \equiv \varepsilon_0 x_{1d}, \quad (19)$$

which results in  $x_1 = (1 - \varepsilon_0)x_{1d}$  where  $\varepsilon_0$  is positive constant. Substituting this  $x_1(t)$  into the parenthesis term for  $\sin x_1(t)$  in (18) and using the sum of sines yields

$$\sin x_{1d} - (\sin x_{1d} \cos \varepsilon_0 x_{1d} - \cos x_{1d} \sin \varepsilon_0 x_{1d}) = e_1 \cos x_{1d}. \quad (20)$$

where  $\cos \varepsilon_0 x_{1d} = 1$  and  $\sin \varepsilon_0 x_{1d} = \varepsilon_0 x_{1d} = e_1$ . Hence, (18) yields

$$\dot{e}_2 = \frac{g}{L} \cos x_{1d} e_1 + u_r - k_1 x_{2d} - u \quad (21)$$

where the parameter  $L$  is unknown. Putting (17) and (21) together into a matrix yields

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{g}{L} \cos x_{1d} & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \bar{u} \quad (22)$$

where

$$\bar{u} \equiv u_r - k_1 x_{2d} - u. \quad (23)$$

### 3.3. Solution of the linear time-varying system

The solution of linear time-varying error system for (22) is given by

$$e(t) = \Phi(t, t_0)e(t_0) + \int_{t_0}^t \Phi(t, \tau)Bu(\tau)d\tau \quad (24)$$

However, it is difficult to find the state transition matrix of (22) since the system has a function of time in the  $A(t)$  and coupled. However, the fact that (17) and (21) can be considered as decoupled between two equations helps to find the state transition matrix,  $\Phi(t, \tau)$ . The solution of the first equation in (22), i.e., (17), yields

$$e_1 = e^{t-\tau} e_{20} \quad (25)$$

where  $e_2(\tau) = e_{20}$ . Substituting (25) for  $e_1(t)$  into the second differential equation of (22) produces

$$\dot{e}_2 = f(t)e^{t-\tau} e_{20} + \bar{u} \quad (26)$$

where  $f(t) = \frac{g}{\hat{L}} \cos x_{1d}$  and the equation can be easily decoupled from the (25).

Thus, the state transition matrix,  $\Phi(t, \tau)$ , yields

$$\Phi(t, \tau) = \exp\left[\int_{\tau}^t f(v)e^{t-\tau} dv\right] \quad (27)$$

where it is identified that  $A(t)$  is a scalar form,  $f(t)e^{t-\tau}$ . In this case the gramian matrix is defined by utilizing (27) as

$$G_C(t_0, t_1) = \int_{t_0}^t \Phi(\tau, t_0) B B^T \Phi^T(\tau, t_0) d\tau, \quad (28)$$

where this gramian matrix is positive definite and nonsingular, whose inverse exists and satisfies the sufficient and necessary condition of the controllability due to the time-varying system. Applying (27) to (24) for solving (26) yields

$$e_2(t) = \exp\left[\int_{\tau}^t f(v)e^{(v-\tau)} dv\right] e_{20} + \int_{\tau}^t \exp\left[\int_{v'}^t f(v')e_2^{(v'-\tau)} dv'\right] B \bar{u}(v) dv \quad (29)$$

where  $e_2(t_0) = e_{20}$ . Then the open-loop control input for the second equation of (22) using a controllability gramian term is

$$\bar{u} = -K B^T \Phi^T(\tau, t_0) G_C^{-1}(t_0, \tau) e_2(\tau) \quad (31)$$

where  $B=1$  and  $K$  is the control input gain constant, and  $u(t)$  in (29) will be designed in the next. From the definition of  $\bar{u}(t)$  in (23), the control input  $u(t)$  is designed in the presence of the parametric uncertainty as

$$u = u_1 + \frac{g}{\hat{L}} \cos x_{1d} e_1 + \frac{1}{\gamma_e} \tilde{\Theta} \hat{\Theta} \quad (32)$$

where the first term,  $u_1(t)$ , is designed for subsequent control development as

$$u_1 = u_r - k_1 x_2 + k_2 e_1, \quad (33)$$

the second term,  $\hat{L}$ , in (32) is the estimated parameter term of (14) and the following adaptation laws are used for the parameter estimator,  $\hat{\Theta}(t)$ , while compensating the parametric uncertainty.

### 3.4. Adaptation laws for parameter update

Substituting (32) for  $u(t)$  into  $\bar{u}(t)$  and rearranging yields

$$\dot{e}_2 = Y_d \tilde{\Theta} e_1 + \tilde{u} \quad (34)$$

where  $Y_d(t) = g \cos x_{1d}$ ,  $\tilde{\Theta} = \frac{1}{L} - \frac{1}{\hat{L}}$ , and

$$\tilde{u} = -k_1 e_2 - k_2 e_1 - \frac{1}{\gamma_e} \dot{\tilde{\Theta}} \hat{\Theta}, \quad (35)$$

in which  $\gamma_e$  is constant gain value. Rearranging yields

$$\dot{e}_2 = \tilde{\Theta}(Y_d e_1 - \frac{1}{\gamma_e} \dot{\hat{\Theta}}) - k_1 e_2 - k_2 e_1 \quad (36)$$

Then, the adaptation law is designed as

$$\dot{\hat{\Theta}} = \gamma_e Y_d e_1 \quad (37)$$

Hence, the final error system utilized (37) results in

$$\dot{e}_2 + k_1 e_2 + k_2 e_1 = 0 \quad (38)$$

The following is assumed to define the upper and lower bounds of each unknown parameters

$$\underline{\hat{\Theta}} \leq \hat{\Theta} \leq \bar{\hat{\Theta}} \quad (39)$$

where  $\hat{\Theta}$  is the estimated constant parameters,  $\underline{\hat{\Theta}}$ ,  $\bar{\hat{\Theta}}$  are unknown lower and upper bounds of the estimated parameters as shown in system parameters, respectively, which will be set to the amount of percentage of their true values.  $\dot{\hat{\Theta}}(t)$  vector is designed to update using a projection-based algorithm as

$$\dot{\hat{\Theta}} = Proj\{\gamma_e Y_d e_1, \hat{\Theta}\} \quad (40)$$

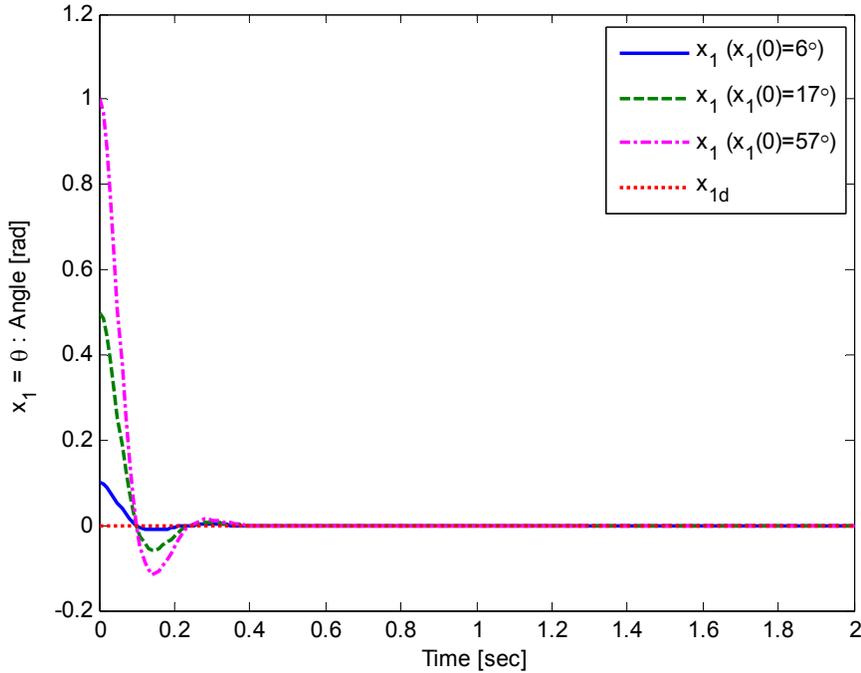
where  $Proj\{\cdot\}$  is the projection operator [8] and each parameter is adaptively updated using adaptation laws for online estimation of unknown parameter as follows:

$$Proj\{\dot{\hat{\Theta}}\} = \begin{cases} \gamma_e Y_d e_1 & \text{if } \hat{\Theta} > \underline{\hat{\Theta}} \text{ and } \hat{\Theta} < \bar{\hat{\Theta}} \\ \gamma_e Y_d e_1 & \text{if } \hat{\Theta} = \bar{\hat{\Theta}} \text{ and if } \gamma W \leq 0 \\ \gamma_e Y_d e_1 & \text{if } \hat{\Theta} = \underline{\hat{\Theta}} \text{ and if } \gamma W > 0 \\ 0 & \text{elsewhere} \end{cases} \quad (41)$$

It is straightforward to make a conclusion that the above adaptive control approach is applied to (36) and then the parenthesis term in (36) will be going to zero, resulting in (38) if both are perfectly canceled, which yields globally stable tracking result.

## 4. Numerical results

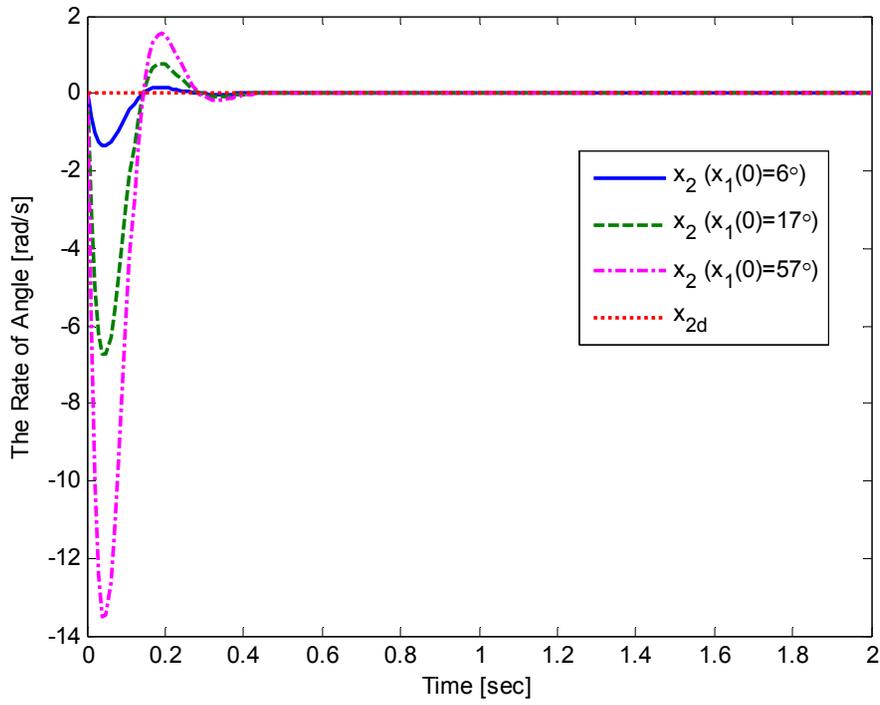
The initial condition of inverted pendulum angles is given as  $x_1(0)=[0.1, 0.5, 1.0]$  (rad)  $\cong$   $\cong$   $[6, 17, 57]$  (deg) as shown in Figure 1 where each actual angle of the pendulum track the desired angle  $x_{1d}=0$ , starting from its initial value. Note that the initial angular rate,  $x_2(0)$ , is zero. In Figure 2, the actual angular rate tracks the desired angular rate of the inverted pendulum. In Figure 3, their tracking angle errors are shown in the top plot and the error rates are shown in the bottom plot, where the errors and error rates are close to zero and thus the tracking system works well. The control inputs are in Figure 4; the control input shown in plot (a) is the designed control input in (32), which is used for the control input of the system dynamic model given in (15), the control input shown in plot (b) is the control solution given in (31) of the tracking error dynamics in (34), which enables the global stability, and finally the plot in (c) is proposed controller of this research, i.e., the closed-loop adaptive tracking control input designed in (35). Figure 5 is the estimate of the time varying parameter,  $\hat{L}$  of  $L$ , in which the simulation parameters such as mass ( $m$ ), length of the pole ( $l$ ), and inertia of the pole ( $J$ ) are combined together and the values used in simulation are as follows:  $m=0.127$  [kg],  $l=0.3$  [m],  $J=0.05$  [kgm<sup>2</sup>], and  $g=9.81$  [m/s<sup>2</sup>]. The percentage of the upper and lower bounds given in (39) is set to 100%. The nominal value of  $1/L$  is 0.612. Thus, the upper bound is 1.232 and lower bound is zero as shown in Figure 5 and the time-varying parameter estimate is varying within the bounds. The error dynamics,  $\dot{e}_2(t)$ , developed in the main body of this chapter are shown in Figure 6; the plot (a) is the second equation of (22) with the control input given in (31), the plot (b) is the output of the error dynamics given in (34) with the control input (35), and the plot in (c) is the final error dynamics given in (38). In Figure 7, those velocity errors with regard to the dynamics are shown. The reference velocity, error control gain constants, gain value, and control input gain matrix are



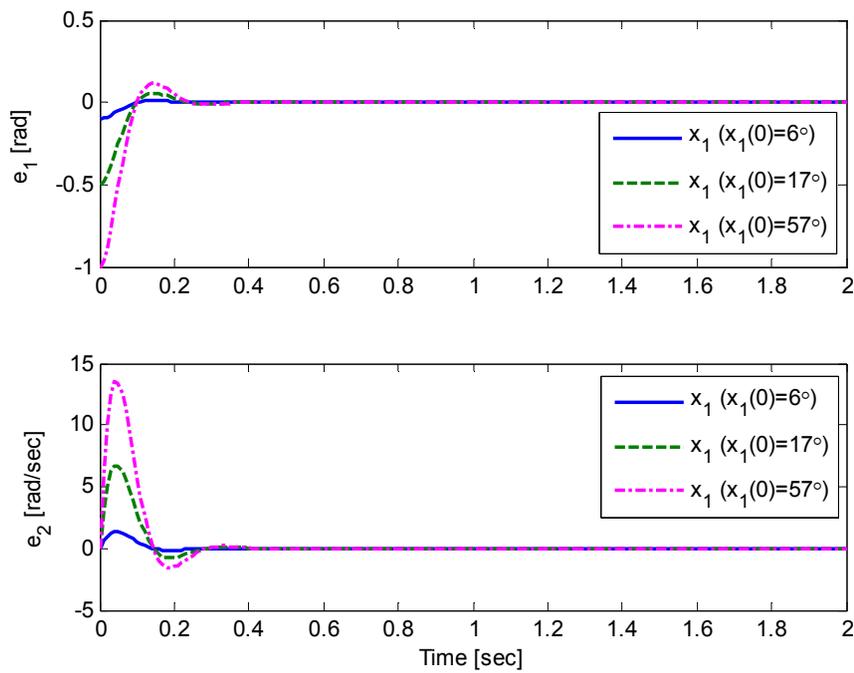
**Figure 1.** Tracking Angle ( $x_1(t)$ ,  $x_{1d}(t)$ )

$$u_r = 0 [m/s], k_1 = 30, k_2 = 700, \gamma_e = 100, \text{ and } K = I_2$$

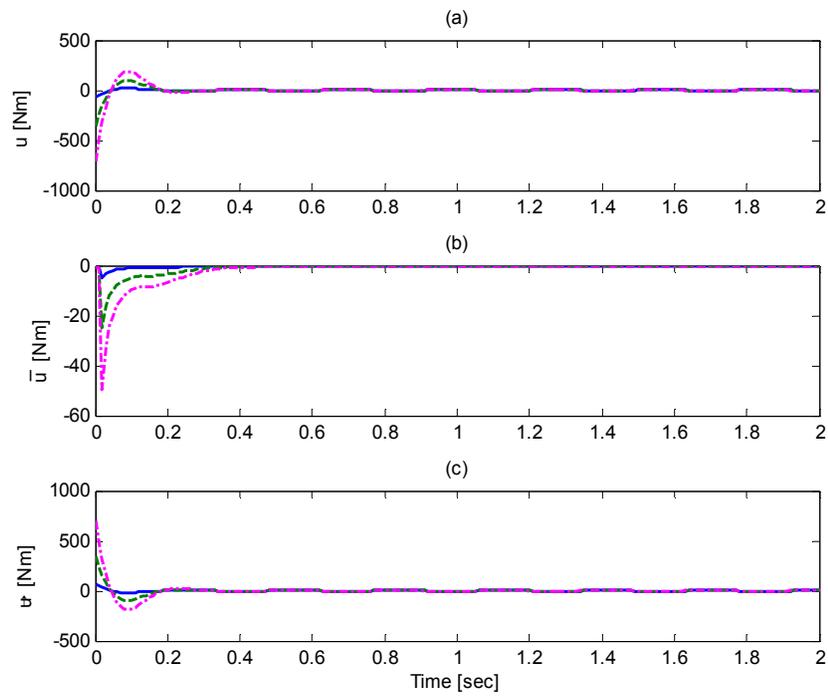
where  $I_2$  is  $2 \times 2$  identity matrix.



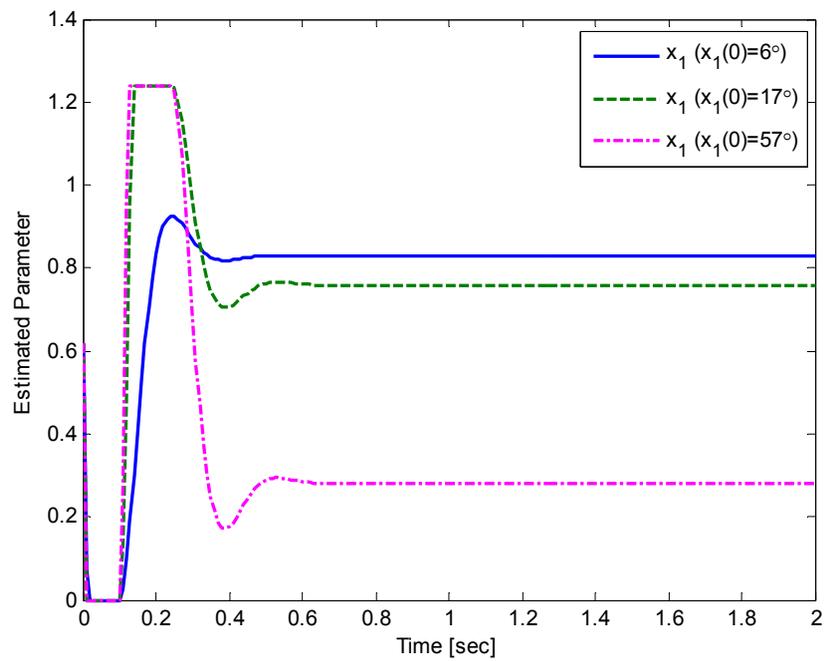
**Figure 2.** Tracking Angular Rate ( $x_2(t)$ ,  $x_{2d}(t)$ )



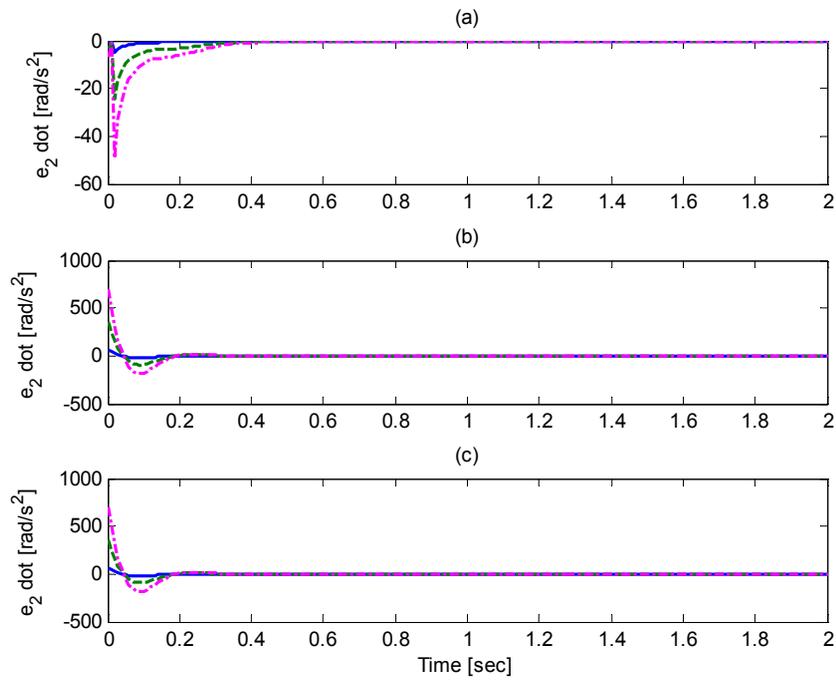
**Figure 3.** Angle and Angular Rate Errors : ( $e_1(t)$ ,  $e_2(t)$ )



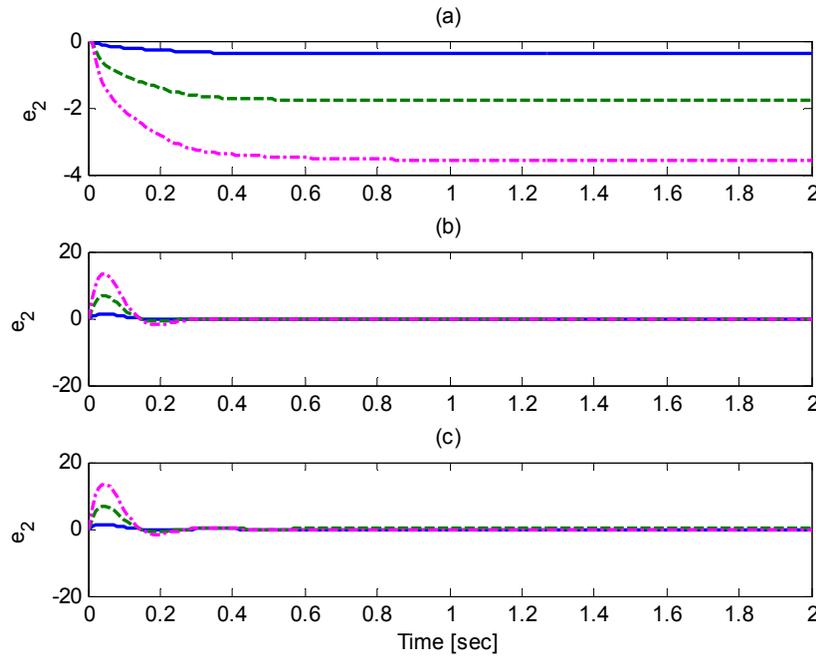
**Figure 4.** Control Inputs: (a)  $u(t)$  in (32), (b)  $\bar{u}(t)$  in (31), and (c)  $\tilde{u}(t)$  in (35)



**Figure 5.** Parameter Estimate ( $\hat{L}(t)$ )



**Figure 6.** Error Dynamics of the Pendulum (a)  $\dot{e}_2(t)$  in (22), (b)  $\dot{e}_2(t)$  in (34), and (c)  $\dot{e}_2(t)$  in (35)



**Figure 7.** Velocity Errors from (a)  $e_2(t)$  from (22), (b)  $e_2(t)$  in (34), and (c)  $e_2(t)$  in (35)

## 5. Conclusion

A tracking control of a model-based linear time-varying system is developed in application to the nonlinear inverted pendulum model. A novelty of this paper is that not only found a gramian matrix which is difficult to find or compute but also utilized to the linear time-varying tracking controller which satisfies the necessary and sufficient of the global stability of the system. Another is that the linear time-varying system is further complicated by parametric uncertainty where the combined parameters are unknown. The suggested adaptive control approach and update laws are applied for estimating the parameters while preserving the system to be stable and converging the tracking error close to zero. Numerical simulation results are demonstrated the validity of the proposed system.

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