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Spectral Modeling and Numerical Simulation of Compressible Homogeneous Sheared Turbulence

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1. Introduction

This chapter is mainly in the area of the use of Rapid Distortion Theory (RDT) to clarify and to well better increase our understanding of the physics of the compressible turbulent flows. This theory is a computationally viable option for examining linear compressible flow physics in the absence of inertial effects. In this linear limit, the statistical evolution of incompressible homogeneous turbulence can be described completely in terms of closed spectral covariance equations (see Refs. (Hunt, 1990 & Savill, 1987) and references therein). Many papers in literature deal with homogeneous compressible turbulence and RDT solution (Cambon et al., 1993; Coleman & Mansour, 1991; Blaisdell et al., 1993, 1996; Durbin & Zeman, 1992; Jacquin et al., 1993; Livescu & Madnia, 2004; Riahi et al., 2007; Riahi, 2008; Riahi & Lili, 2011; Sarkar, 1995; Simone, 1995; Simone et al., 1997). These studies have yielded very valuable physical insight and closure model suggestions. In all the above works, the fluctuation equations are solved directly to infer turbulence physics. For the case of viscous compressible homogeneous shear flow in the RDT limit no analytical solutions are known. Simone et al. (1997) performed RDT simulations of homogeneous shear flow and showed that the role of the distortion Mach number, M_d , on the time variation of the turbulent kinetic energy is consistent with that found in the direct numerical simulation (DNS) results. In this chapter, numerical solutions to the RDT equations for the special case of mean shear is described completely by finding numerical solutions obtained by solving linear double point correlations equations. Numerical integration of these equations is carried out using a second-order simple and accurate scheme (Riahi & Lili, 2011). Indeed, this numerical method is proved more stable and faster than the previous one which use linear transfer matrix (Riahi et al., 2007 & Riahi, 2008) and allows in particular to obtain accurately the asymptotic behavior of the turbulence parameters (for large values of the non-dimensional times St) characteristic of equilibrium states. To perform this work, RDT code solving

linearized equations for compressible homogeneous shear flows is validated by comparing RDT results to those of direct numerical simulation (DNS) of Simone et al. (1997) and Sarkar (1995) for various values of initial gradient Mach number M_{g0} (Riahi & Lili, 2011).

A study of compressibility effects on structure and evolution of a sheared homogeneous turbulent flow is carried out using this theory (RDT). An analysis of the behavior of different terms appearing in the turbulent kinetic energy and the Reynolds stress equations permit to well identify compressibility effects which allow us to analyze performance of the compressible model of Fujiwara and Arakawa concerning the pressure-dilatation correlation (Riahi et al., 2007). The evaluation of this model stays in the field of RDT validity (Riahi & Lili, 2011).

Equilibrium states of homogeneous compressible turbulence subjected to rapid shear can be studied using rapid distortion theory (RDT) for large values of St ($St > 10$) in particular for large values of the initial gradient Mach number M_{g0} describing various regimes of flow. In fact, the study of the behavior of the non-dimensional turbulent kinetic energy $q^2(t)/q^2(0)$ (with $q^2 = \overline{u_i u_i}$) allows to check relevance of an incompressible regime for low values of initial gradient Mach number, of an intermediate regime for moderate values of M_{g0} and of a compressible regime for high values of M_{g0} (Riahi et al., 2007). The pressure-released regime is related to infinite values of M_{g0} (Riahi & Lili, 2011). The gradient Mach number M_g appears naturally when scaling the (linearized) RDT equations for homogeneous compressible turbulence (see equations (A.10), (A.11), (A.12) and (A.13) in appendix (Riahi et al., 2007)). This parameter can be viewed as the ratio of an acoustic time $\tau_a = \frac{l}{a}$ for a large eddy to the mean

flow time scale $\tau_d = \frac{1}{S} : M_g = \frac{\tau_a}{\tau_d} = \frac{Sl}{a}$, where l is an integral lengthscale and a is the mean sound

speed. The lengthscale of energetic structure is expressed by $l = \frac{q^3}{\varepsilon}$, where ε is the rate of

turbulent kinetic energy dissipation. Sarkar (1995) also used $\frac{Sl}{a}$ (which he referred to as 'gradient Mach number' M_g), to quantify compressibility effects for homogeneous shear flow. In the case of shear flow, l is chosen to be the integral lengthscale of the streamwise fluctuating velocity in the shearing direction x_2 . Another Mach number relevant to homogeneous shear flow and that characterize the effects of compressibility on turbulence is the turbulent Mach number $M_t = u/a$ based on a characteristic fluctuation velocity u and mean sound speed a .

2. RDT equations for compressible homogeneous turbulence

The flow to be considered is a homogeneous, compressible turbulent shear flow where we retain the same RDT equations adopted by Simone (1995), Simone et al. (1997), Riahi et al. (2007) and Riahi (2008). The linearized equations of continuity, momentum and entropy controlling the fluctuating of velocity u_i and pressure p lead to general RDT equations:

$$\left(\frac{\dot{p}}{\gamma \bar{P}} \right) = -u_{i,i}, \quad (1)$$

$$\dot{u}_i + u_j \frac{\partial \bar{U}_i}{\partial x_j} = -\frac{1}{\bar{\rho}} \frac{\partial p}{\partial x_i} + \frac{\nu}{3} \frac{\partial^2 u_j}{\partial x_i \partial x_j} + \frac{\nu}{3} \frac{\partial^2 u_i}{\partial x_j^2}, \quad (2)$$

where γ is the ratio of specific heats, $\bar{\rho}$ is the mean density and ν is the kinematic viscosity.

The dot superscript denotes a substantial derivative along the mean flow trajectories related to mean field velocity $\bar{U}_i$.

Equation (1) is derived from continuity equation

$$\left(\frac{\dot{\rho}}{\bar{\rho}} \right) = -u_{i,i} \quad (3)$$

associated to isentropic fluctuations hypothesis $\dot{s} = 0$ which is translated by

$$\left(\frac{\dot{p}}{\bar{P}} \right) = \gamma \left(\frac{\dot{\rho}}{\bar{\rho}} \right) \quad (4)$$

and leads to equation (1). So, the isentropic hypothesis $\dot{s} = 0$ simplifies governing equations by keeping the variables u_i and p and eliminating ρ . We can obviously discard the hypothesis $\dot{s} = 0$ by writing the (exact) energy equation. The linearized derived equation within the framework of RDT can be written as pressure fluctuation equation which contains some viscous terms (Livescu et al., 2004; see equation (8)). Such an equation is compatible with equation (2) which contains also viscous terms and represents the linearized equation of momentum. Concerning now the validity of isentropic hypothesis

$\dot{s} = 0$, Blaisdell et al. (1993) introduced a polytropic coefficient n defined by, $\frac{p}{P} = n \frac{\rho}{\bar{\rho}}$ $n = \gamma$

corresponding obviously to isentropic fluctuations. These authors performed direct numerical simulations (DNS) related to homogeneous compressible sheared turbulence with various initial conditions. They calculated temporal variation of an average polytropic

coefficient $n = \left(\frac{\overline{p^2 / \bar{P}^2}}{\overline{\rho^2 / \bar{\rho}^2}} \right)^{\frac{1}{2}}$. During the period of the turbulence establishment (for early

times), the evolution of n with St depends on initial conditions (and in particular of initial entropy fluctuations). However, for all of their simulations and for large values of St , n tends

toward an equilibrium value independent of initial conditions and slightly less than $\gamma = 1.4$. Blaisdell et al. (1993) concluded that thermodynamic variables “follow a nearly isentropic process” for large values of St and for compressible sheared turbulence. As a conclusion, we retain that with isentropic fluctuations hypothesis, we can obtain a good prediction of equilibrium states ($St \rightarrow \infty$) for compressible homogeneous sheared turbulence.

After these remarks, we consider the Fourier transform (denoted here by the symbol “ $\hat{}$ ”) of various terms in equations (1) and (2) which leads to the following equations expressed in the spectral space and in the case of turbulent shear flow:

$$\frac{\hat{p}}{\gamma P} = -\hat{u}_{i,i}, \quad (5)$$

$$\hat{u}_i + \lambda_{ij} \hat{u}_j + \frac{\nu}{3} k_i k_j \hat{u}_j + \frac{\nu}{3} k^2 \hat{u}_i = -Ik_i \frac{\hat{p}}{\rho}, \quad (6)$$

where λ_{ij} is the mean velocity gradient defined by:

$$\lambda_{ij} = S\delta_{i1}\delta_{j2} \quad (7)$$

and $I^2 = -1$.

In the Fourier space, velocity field is decomposed on solenoidal and dilatational contributions by adopting a local reference mark of Craya (Cambon et al., 1993):

$$\hat{u}_i(\vec{k}, t) = \hat{\phi}^1(\vec{k}, t) e_i^1(\vec{k}) + \hat{\phi}^2(\vec{k}, t) e_i^2(\vec{k}) + \hat{\phi}^3(\vec{k}, t) e_i^3(\vec{k}), \quad (8)$$

where $\hat{\phi}^1(\vec{k}, t)$ and $\hat{\phi}^2(\vec{k}, t)$ are the solenoidal contributions and $\hat{\phi}^3(\vec{k}, t)$ is the dilatational contribution.

By separating solenoidal and dilatational contributions, we introduce the spectrums of doubles correlations:

$$\Phi_{ij}(\vec{k}, t) = \frac{1}{2} \left[\hat{\phi}^{i*}(\vec{k}, t) \hat{\phi}^j(\vec{k}, t) + \hat{\phi}^i(\vec{k}, t) \hat{\phi}^{j*}(\vec{k}, t) \right] \quad i, j = 1..4, \quad (9)$$

where $\hat{\phi}^4(\vec{k}, t)$ is related to the pressure and takes the following form:

$$\hat{\phi}^4(\vec{k}, t) = I \frac{\hat{p}}{\rho a} \quad (10)$$

We then write evolution equations of these doubles correlations:

$$\frac{d\Phi_{11}}{dt} = -2\nu k^2 \Phi_{11} - 2 \frac{Sk_3}{k} \Phi_{12} + 2 \frac{Sk_2 k_3}{kk'} \Phi_{13}, \quad (11)$$

$$\frac{d\Phi_{12}}{dt} = (-2\nu k^2 + \frac{Sk_1 k_2}{k^2})\Phi_{12} - \frac{Sk_1}{k'}\Phi_{13} - \frac{Sk_3}{k}\Phi_{22} + \frac{Sk_2 k_3}{kk'}\Phi_{23}, \quad (12)$$

$$\frac{d\Phi_{13}}{dt} = 2\frac{Sk_1 k'}{k^2}\Phi_{12} - (\frac{7}{3}\nu k^2 + \frac{Sk_1 k_2}{k^2})\Phi_{13} - ak\Phi_{14} - \frac{Sk_3}{k}\Phi_{23} + \frac{Sk_2 k_3}{kk'}\Phi_{33}, \quad (13)$$

$$\frac{d\Phi_{14}}{dt} = ak\Phi_{13} - \nu k^2\Phi_{14} - \frac{Sk_3}{k}\Phi_{24} + \frac{Sk_2 k_3}{kk'}\Phi_{34}, \quad (14)$$

$$\frac{d\Phi_{22}}{dt} = (-2\nu k^2 + 2\frac{Sk_1 k_2}{k^2})\Phi_{22} - 2\frac{Sk_1}{k'}\Phi_{23}, \quad (15)$$

$$\frac{d\Phi_{23}}{dt} = 2\frac{Sk_1 k'}{k^2}\Phi_{22} - \frac{7}{3}\nu k^2\Phi_{23} - ak\Phi_{24} - \frac{Sk_1}{k'}\Phi_{33}, \quad (16)$$

$$\frac{d\Phi_{24}}{dt} = ak\Phi_{23} + (-\nu k^2 + \frac{Sk_1 k_2}{k^2})\Phi_{24} - \frac{Sk_1}{k'}\Phi_{34}, \quad (17)$$

$$\frac{d\Phi_{33}}{dt} = 4\frac{Sk_1 k'}{k^2}\Phi_{23} - 2(\frac{4}{3}\nu k^2 + \frac{Sk_1 k_2}{k^2})\Phi_{33} - 2ak\Phi_{34}, \quad (18)$$

$$\frac{d\Phi_{34}}{dt} = 2\frac{Sk_1 k'}{k^2}\Phi_{24} + ak\Phi_{33} - (\frac{4}{3}\nu k^2 + \frac{Sk_1 k_2}{k^2})\Phi_{34} - ak\Phi_{44}, \quad (19)$$

$$\frac{d\Phi_{44}}{dt} = 2ak\Phi_{34}, \quad (20)$$

where $k' = \sqrt{k_1^2 + k_3^2}$ and k_1, k_2, k_3 are the components of the wave vector $\vec{k}$.

Numerical integration of these equations ((11)-(20)) is carried out using a simple second-order accurate scheme:

$$f(t + \Delta t) = f(t) + \Delta t f'(t) + \Delta t^2 f''(t)/2, \quad (21)$$

where the derivatives $f'(t)$ and $f''(t)$ are expressed exactly from evolution equations (11)-(20) and Δt is the time-step size.

3. RDTcode validation

3.1. Introduction

In this section, results are presented and used to verify the validity of the RDT code. For this, tests of this code have been performed by comparing RDT results with those of direct numerical simulation (DNS) of Simone et al. (1997) for various values of initial gradient Mach number M_{g0} which describe different regimes of the flow (see below). Comparisons

concern the turbulent kinetic energy $q^2(t)/q^2(0)$ [Figs. 1, 2], the non-dimensional production term $\frac{2P}{Sq^2} = -2b_{12}$ [Figs. 3, 4] and the turbulent kinetic energy growth rate $\Lambda = \frac{1}{Sq^2} \frac{dq^2}{dt}$ [Figs. 5, 6]. In the same way, comparisons deal with the solenoidal and the dilatational b_{12} anisotropy tensor components [Figs. 7, 8] (Riahi & Lili, 2011). We note that $b_{12} = \frac{\overline{u_1 u_2}}{q^2}$ is the relevant component of the Reynolds stress anisotropy tensor. Tavoularis & Corrsin (1981) have shown that in the incompressible homogeneous shear flow b_{12} tends toward an equilibrium value which is independent of initial conditions.

Concerning the existence of the evolution zones where shocks develop, we neither observe any anomaly in physic parameters (no problem of realisability), nor any instability in the calculation course. In addition, it is important to specify that our code was not conceived to take explicitly into account the existence of shocks. However, we think that in the applications presented, the flow is not probably contaminated by the appearance of shocks.

3.2. Initial parameters

Intrinsic parameters that characterize the flow include the initial turbulent Mach number $M_{t0} = \frac{q_0}{a}$ (recall that $\frac{1}{2}q_0^2$ is the initial turbulent kinetic energy and a is the mean speed of sound), the initial gradient Mach number $M_{g0} = M_{t0} S \frac{q_0^2}{\varepsilon_0}$ (S is the constant mean shear and ε_0 the initial total rate of turbulent kinetic energy dissipation) and the initial turbulent Reynolds number $R_{et0} = \frac{q_0^4}{\nu \varepsilon_0}$. The ratio of the gradient Mach number to the turbulent Mach number $r = S \frac{q_0^2}{\varepsilon_0}$ characterizes the rapidity of the shear.

Table 1 lists ten simulations cases labelled $A_1, A_2 \dots A_{10}$ corresponding to different values of the initial gradient Mach number M_{g0} . In these simulations, M_{g0} increases respectively in cases A_1 ($M_{g0} = 2.7$) to A_{10} ($M_{g0} = 66.7$) by keeping the same initial value of the turbulent Mach number M_{t0} and the turbulent Reynolds number R_{et0} .

In these cases, the initial turbulent kinetic energy spectrum is similar to that used by Simone (1995),

$$E(K, t_0) = K^4 \exp(-2 \frac{K^2}{\kappa_{pic}^2}),$$

where κ_{pic} denotes the wave number corresponding to the peak of the power spectrum and K is the initial wave number.

Case	M_{t0}	M_{g0}	r_0	Re_{t0}
A ₁	0.25	2.7	10.7	296
A ₂	0.25	4	16	296
A ₃	0.25	8.3	33.1	296
A ₄	0.25	12	48	296
A ₅	0.25	16.5	66.2	296
A ₆	0.25	24	96.1	296
A ₇	0.25	32	128.1	296
A ₈	0.25	42.7	170.8	296
A ₉	0.25	53.4	213.5	296
A ₁₀	0.25	66.7	266.9	296

Table 1. Initial parameters.

3.3. Results and discussion

All Figures 1-8 validate RDT approach and numerical code for low values of the non-dimensional times St (until 3.5). Indeed, comparisons carried out between our RDT results (Riahi & Lili, 2011) and DNS results of Simone et al. (1997) show that the linear analysis predicted correctly – as well qualitatively as quantitatively – the behavior of turbulence in particular for small values of St .

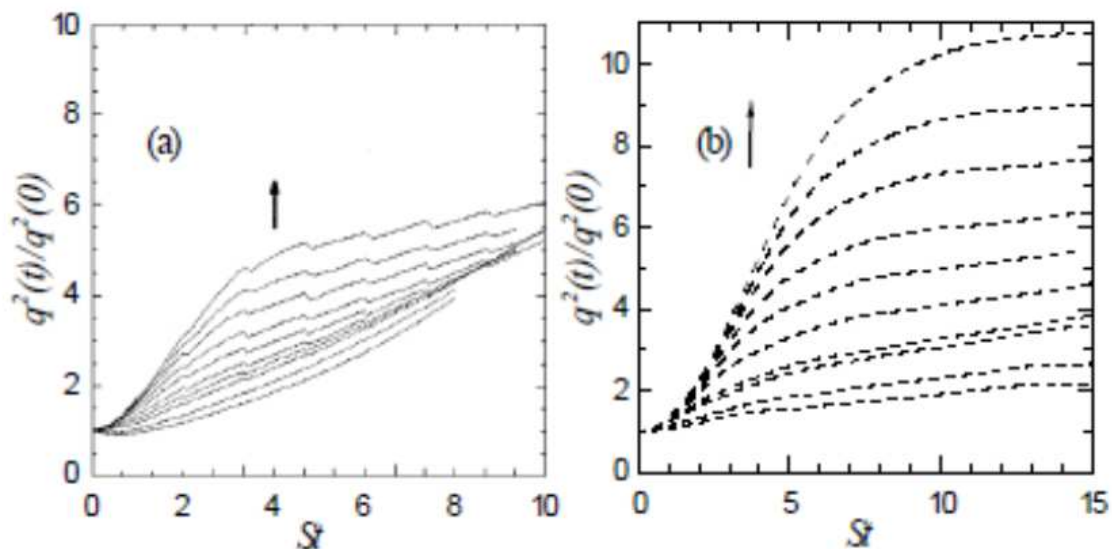


Figure 1. Histories of the turbulent kinetic energy for pure-shear flow: (a) DNS results of Simone et al. (1997) and (b) our RDT results. Initial gradient Mach number M_{g0} ranges from 2.7 to 67 for both DNS and RDT; arrows show trend with increasing M_{g0} .

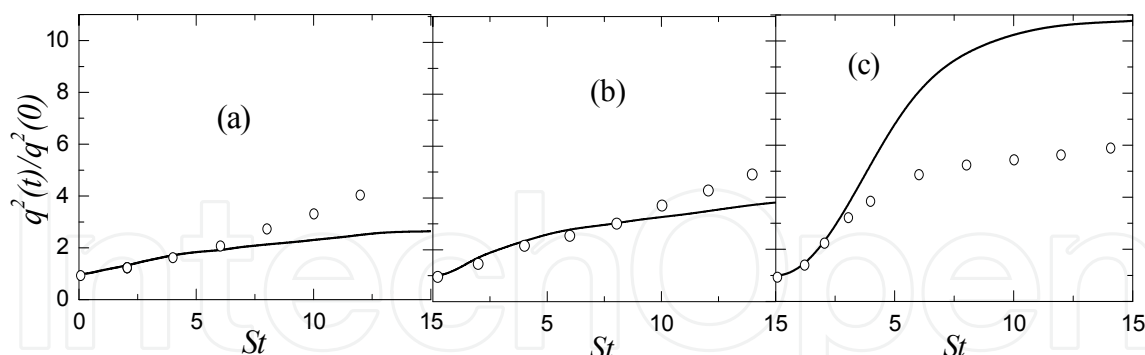


Figure 2. Evolution of the turbulent kinetic energy (a) case A2 ($M_{g0} = 4$), (b) case A4 ($M_{g0} = 12$) and (c) case A10 ($M_{g0} = 66.7$). — : RDT results, o : DNS results of Simone et al. (1997).

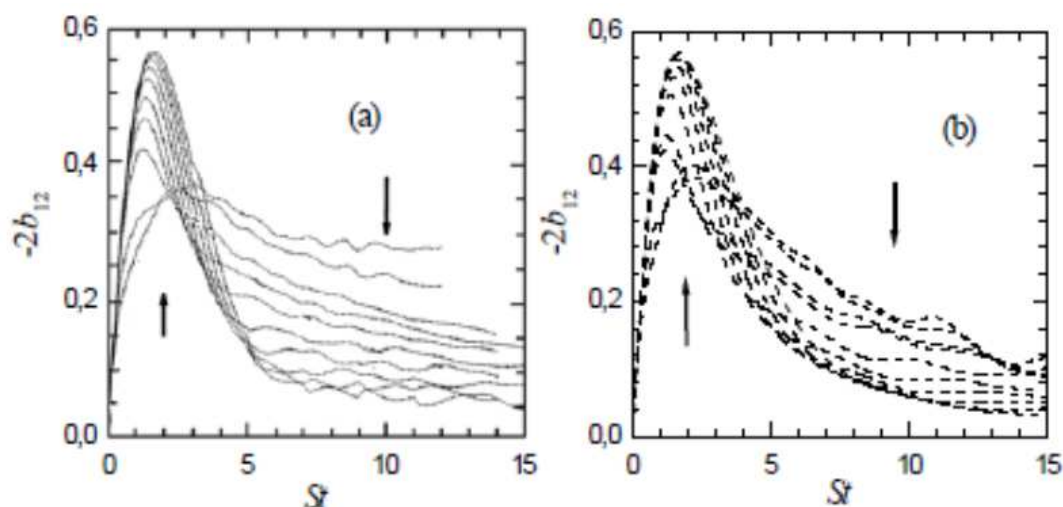


Figure 3. Histories of the non-dimensional production term $-2b_{12}$ for pure-shear flow: (a) DNS results of Simone et al. (1997) and (b) our RDT results. Initial gradient Mach number M_{g0} ranges from 2.7 to 67 for both DNS and RDT; arrows show trend with increasing M_{g0} .

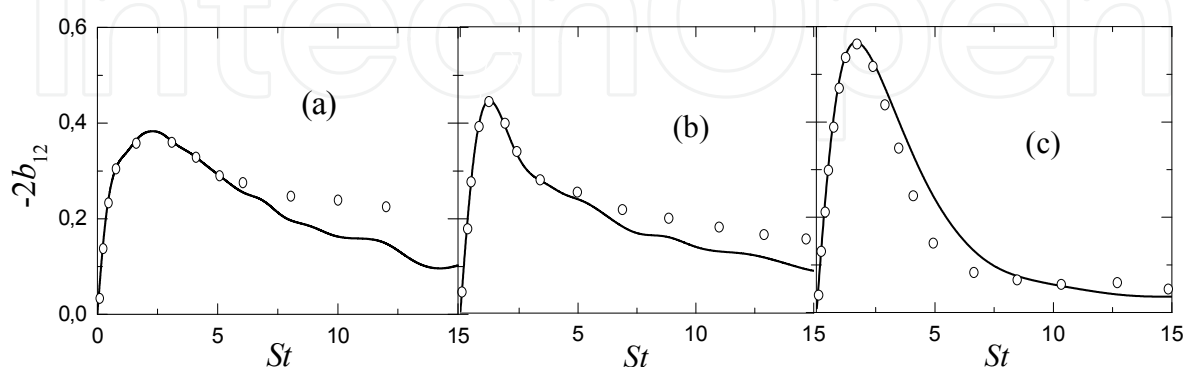


Figure 4. Evolution of the non-dimensional production term (a) case A2 ($M_{g0} = 4$), (b) case A4 ($M_{g0} = 12$) and (c) case A10 ($M_{g0} = 66.7$). — : RDT results, o : DNS results of Simone et al. (1997).

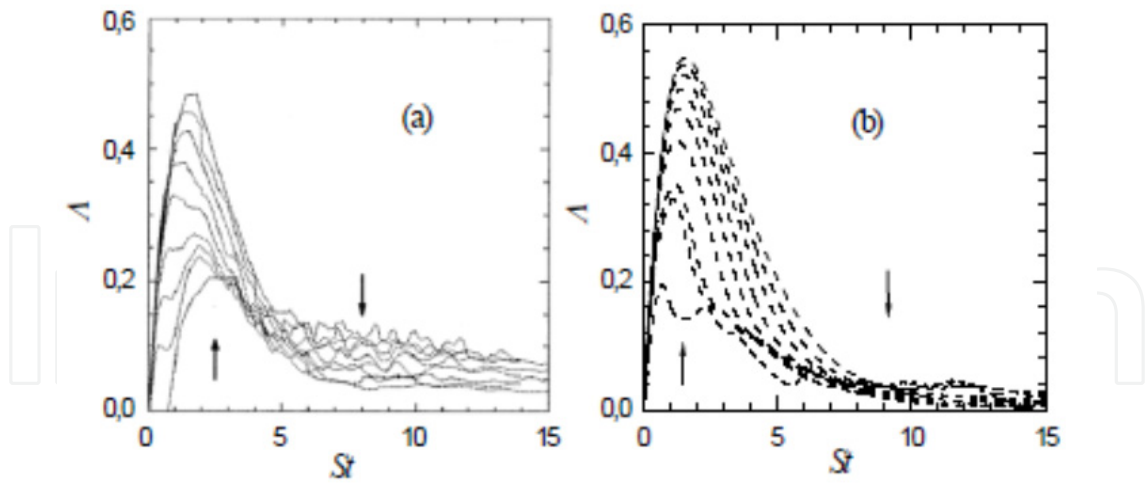


Figure 5. Histories of the temporal energy growth rate for pure-shear flow: (a) DNS results of Simone et al. (1997) and (b) our RDT results. Initial gradient Mach number M_{g0} ranges from 2.7 to 67 for both DNS and RDT; arrows show trend with increasing M_{g0} .

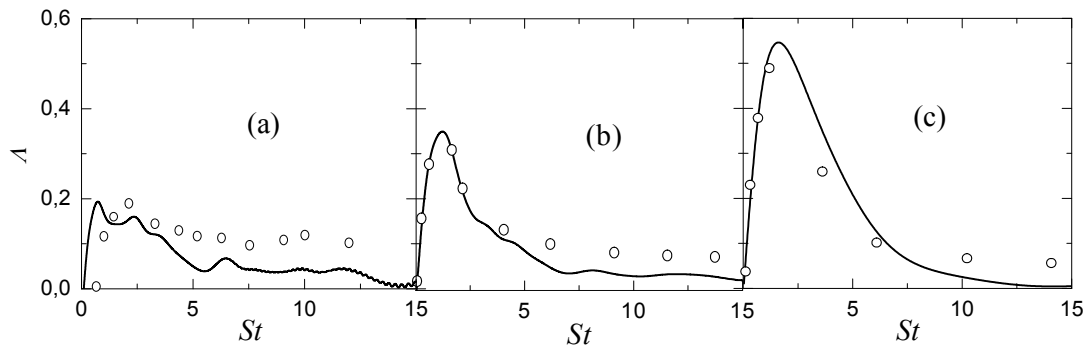


Figure 6. Evolution of the turbulent kinetic energy growth rate (a) case A2 ($M_{g0} = 4$), (b) case A4 ($M_{g0} = 12$) and (c) case A10 ($M_{g0} = 66.7$). — : RDT results, o : DNS results of Simone et al. (1997).

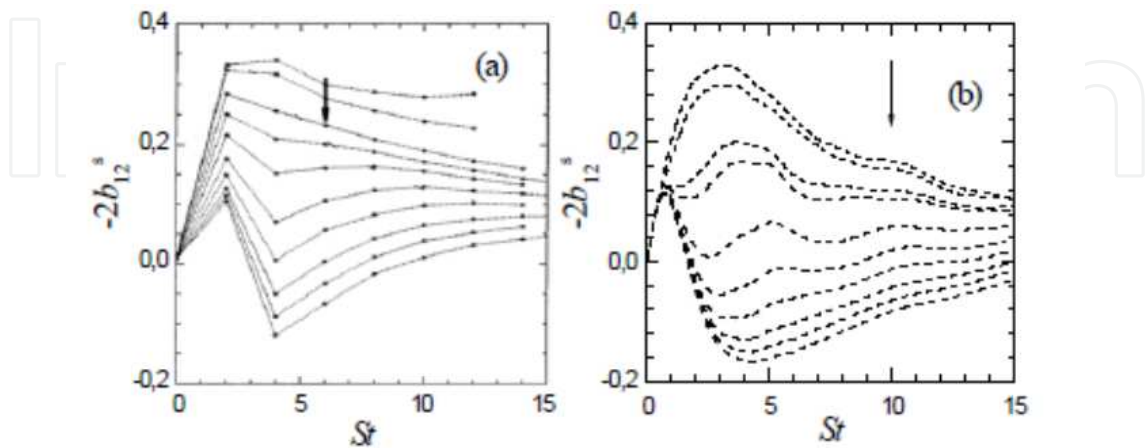


Figure 7. Histories of the solenoidal b_{12} component anisotropy tensor for pure-shear: (a) DNS results of Simone et al. (1997) and (b) our RDT results. Initial gradient Mach number M_{g0} ranges from 2.7 to 67 for both DNS and RDT; arrows show trend with increasing M_{g0} .

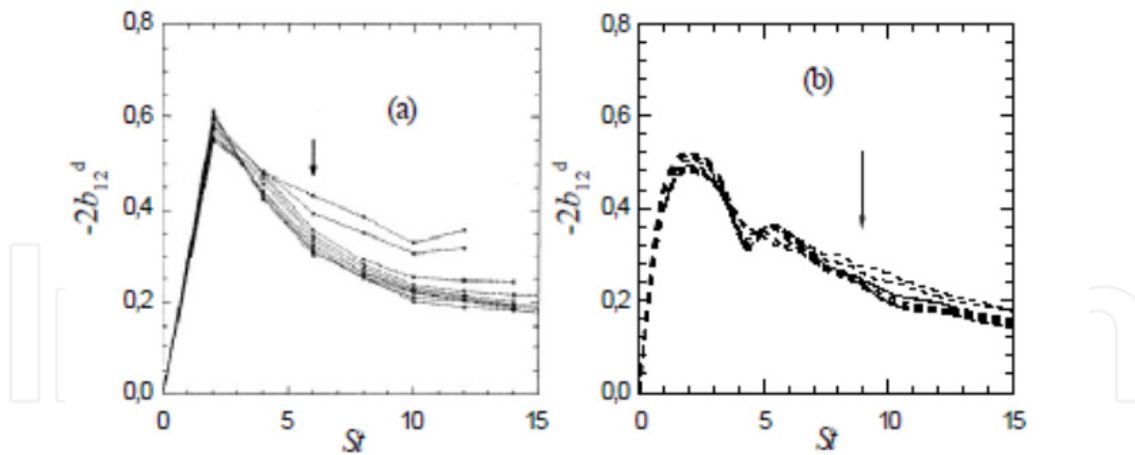


Figure 8. Histories of the dilatational b_{12} component anisotropy tensor for pure-shear flow: (a) DNS results of Simone et al. (1997) and (b) our RDT results. Initial gradient Mach number M_{g0} ranges from 2.7 to 67 for both DNS and RDT; arrows show trend with increasing M_{g0} .

In the compressible regime ($M_{t0} = 0.25$ and $M_{g0} = 66.7$), RDT and DNS equilibrium values of the non-dimensional production $-2b_{12}$ of Simone et al. (1997) are very close [Fig. 4(c)]. As one can remark from this figure that for large values of St ($St > 10$), there is a small difference between RDT and DNS results of Simone et al. (1997). Thus, in this compressible regime, RDT predicts correctly equilibrium behavior of the turbulence (asymptotic behavior ($St \rightarrow \infty$)) relatively to the non-dimensional production $-2b_{12}$ (strongly compressible regime $M_{g0} = 66.7 \gg 1$ and $r_0 \gg 1$) which is a rapid distortion regime compatible with the framework of RDT. The intermediate regime (case A₄, $M_{g0} = 12$) and especially the incompressible regime, for which non-linear effects are preponderant, are poorly estimated by RDT for large values of St . Moreover, Simone et al. (1997) indicate, as general conclusion that compressibility effects on homogeneous sheared turbulence, concerning particularly b_{12} and the non-dimensional production, which are generally associated with non linear phenomena, can be explained in terms of RDT. So, non linear and dissipative effects (through the non linear cascade) are not significant for predicting equilibrium states for strongly compressible regime (Simone et al., 1997). Consequently, we can justify the resort to RDT in order to determine equilibrium states in the compressible regime.

In Figures 9-11, we present evolutions of b_{12} (which is linked to production), b_{11} and b_{22} components anisotropy tensor in the incompressible ($M_{g0} = 2.2$) and intermediate regimes ($M_{g0} = 13.2$). In the incompressible regime, as illustrated in Figures 9(a), 10(a), 11(a), we confirm that RDT validity is limited for small values of St ; considerable differences appear between RDT and DNS results of Sarkar (1995) beyond. With RDT, it is not possible to predict evolution of sheared turbulence for large values of St . Consequently, we can't predict, by means of RDT, equilibrium states of sheared turbulence in the incompressible regime. We note that in the intermediate regime [Figs. 9(b), 10(b), 11(b)], the behavior of DNS results of Sarkar (1995) concerning b_{12} , b_{11} and b_{22} seems to be compatible with RDT results for large values of St . In the intermediate zone of St (between small and large values of St) differences are appreciable.

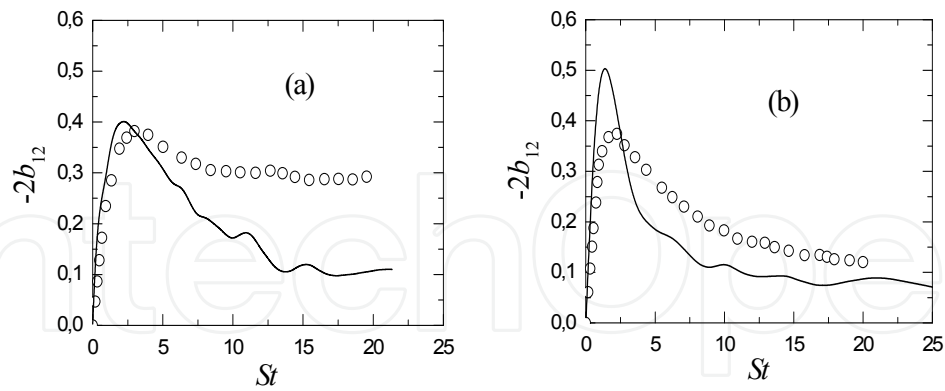


Figure 9. Evolution of the non-dimensional production term with (a): $M_{g0} = 2.2$ and (b): $M_{g0} = 13.2$. — : RDT results, o : DNS results of Sarkar (1995).

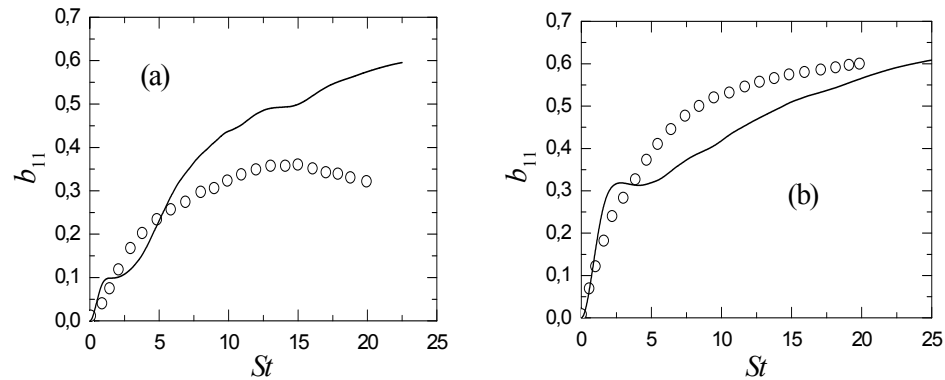


Figure 10. Evolution of b_{11} component anisotropy tensor with (a): $M_{g0} = 2.2$ and (b): $M_{g0} = 13.2$. — : RDT results, o : DNS results of Sarkar (1995).

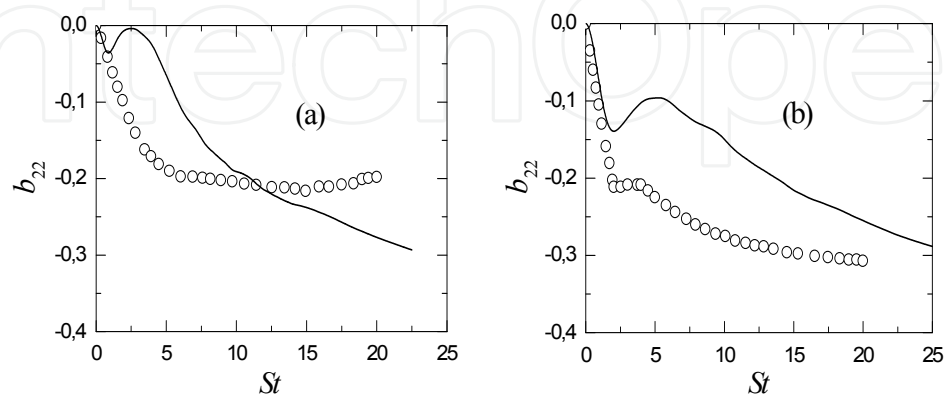


Figure 11. Evolution of b_{22} component anisotropy tensor with (a): $M_{g0} = 2.2$ and (b): $M_{g0} = 13.2$. — : RDT results, o : DNS results of Sarkar (1995).

In conclusion, RDT is valid for small values of the non-dimensional times St ($St < 3.5$). RDT is also valid for large values of St ($St > 10$) in particular for large values of M_{g0} . This essential feature justifies the resort to RDT in order to critically study equilibrium states in the compressible regime.

4. Various regimes of flow

The various curves of Figure 1(b) permit to determine different regimes of the flow (Riahi & Lili, 2011). This figure shows that there is an increase in the turbulent kinetic energy when M_{g0} increases for various cases considered $A_1, \dots, A_{10}$. In addition, when initial gradient Mach number increases, we observe a break of slope which is accentuated when the value of M_{g0} becomes more significant. It appears from this figure that the turbulent kinetic energy varies quasi-linearly in cases A_1 and A_2 , where M_{g0} takes respectively values 2.7 and 4. These two values of M_{g0} correspond to the incompressible regime. A weak amplification of the turbulent kinetic energy shows that cases A_4 ($M_{g0} = 12$) and A_{10} ($M_{g0} = 66.7$) correspond to the intermediate regime. From $M_{g0} = 16.5$, the regime becomes compressible. Indeed, cases $A_5, \dots, A_{10}$ show a significant amplification of the total kinetic energy more and more marked when the initial gradient Mach number M_{g0} increases.

These different regimes permit to better understanding compressibility effects on structure of homogeneous sheared turbulence and to analyze the causes of turbulence structure modification generated by compressibility.

Several explanations were proposed these last years to analyze causes of the stabilising effect of compressibility which are still not cleared up.

5. Compressibility effects on structure of homogeneous sheared turbulence

5.1. Introduction

The study of compressibility effects on the turbulent homogeneous shear flow behavior made these last years the objective of several researches as mentioned in the works of Blaisdell et al. (1993 & 1996) and Sarkar et al. (1991 & 1992). DNS developed by Sarkar (1995) show that the temporal growth rate of the turbulent kinetic energy is extensively influenced by compressibility. Simone et al. (1997) identified the new coupling term in the quasi-isentropic RDT equations, which was responsible for long-term stabilization, comparing incompressible and compressible RDT. They concluded that the modification of the compressible turbulence structure is due to linear processes.

The prediction of compressible turbulent flows by rapid distortion theory provides useful results which can be used to clarify the physics of the compressible turbulent flows and to study compressibility effects on structure of homogeneous sheared turbulence. This study is important for the analysis of various physical mechanisms which characteristic the turbulence. Thus, physical comprehension of compressibility effects on turbulence leads it

possible to better predict compressible turbulent flows and to improve existing turbulence models. The Helmholtz decomposition of the velocity field in solenoidal and dilatational parts reveals two additional terms in the turbulent kinetic energy budget. Several studies of the behavior of the different terms present in this budget and the Reynolds stress equations show the role of the explicit compressible terms. From Simone et al. (1997) and Sarkar (1995), for the case in which the rate of shear is much larger than the rate of non-linear interactions of the turbulence, amplification of turbulent kinetic energy by the mean shear caused by compressibility is due to the implicit pressure-strain correlations effects and to the anisotropy of the Reynolds stress tensor. These authors also recommend to take into account correctly the explicit dilatational terms such as the pressure-dilatation correlation and the dilatational dissipation. In contrast, the role of explicit terms was over-estimated by Zeman (1990) and Sarkar et al. (1991). These last authors show that both those terms have a dissipative contribution in shear flow, leading to the reduced growth of the turbulent kinetic energy.

The study of the budget behaviors of the turbulent kinetic energy and the Reynolds stress anisotropy by RDT enables to better understand and explain compressibility effects on structure and evolution of a sheared homogeneous turbulence.

5.2. The turbulent kinetic energy equation

In the case of homogeneous turbulence, the turbulent kinetic energy is written (Simone, 1995) as:

$$\frac{d}{dt} \left(\frac{q^2}{2} \right) = P - \varepsilon_s - \varepsilon_d + \Pi_d, \quad (22)$$

in which $P = -\overline{Su_1u_2}$ is the rate of production by the mean flow and $\Pi_d = \overline{pu_{i,i}}$ the pressure-dilatation correlation. $\varepsilon_s = \nu \overline{\omega_i \omega_i}$ and $\varepsilon_d = \frac{4}{3} \nu \overline{u_{i,i} u_{i,i}}$ are respectively the solenoidal and the dilatational parts of the turbulent dissipation rate given that ω_i is the fluctuating vorticity and $u_{i,i}$ denotes the fluctuating divergence of velocity. ε_s represents the turbulent dissipation arising from the traditional energy cascade which is solenoidal, ε_d represents the turbulent dissipation arising from dilatational regimes. Note that the last two terms on the right-hand side in equation (22) do not appear when the flow is incompressible. The explicit/energetic approach is embodied in the modeling of ε_d and Π_d done by Zeman (1990), Sarkar et al. (1991) and others.

5.3. Results

Figures 12(a), (b), (c) show the budget of the turbulent kinetic energy for three values of initial gradient Mach number (respectively 1, 12 and 66.7) which describe the various regimes of the flow (Riahi & Lili, 2011). It will be shown from Figures 12(b), (c) that the

production P and the pressure-dilatation Π_d intervene significantly in this budget. In the compressible regime [Fig. 12(c)], the production is dominating and the pressure-dilatation Π_d remains with relatively low values (practically null until $St = 1$). Figure 12(a) shows the results for $M_{g0} = 1$ and $M_{t0} = 0.1$ (case A0). In this case, it is still the production which is dominating. The explicitly compressible terms which are the dilatational dissipation rate ε_d and the pressure-dilatation Π_d are not negligible in spite of the small value of the initial gradient Mach number M_{g0} . Consequently, we can affirm that “the incompressible behavior” cannot be obtained in the borderline case of low values of M_{g0} ; that’s why we preferred to give the results for $M_{g0} = 1$ and not for $M_{g0} = 2.7$ and 4 with an aim of better approaching the “incompressible limit”.

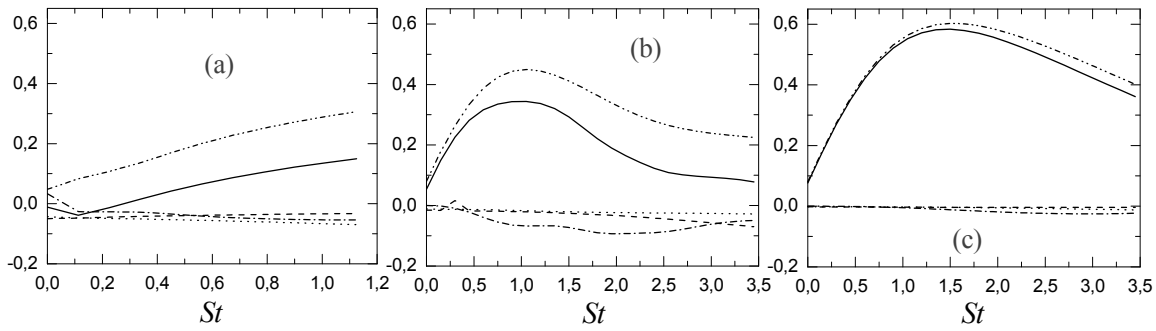


Figure 12. The turbulent kinetic energy budget (a) case A0 ($M_{g0} = 1$), (b) case A4 ($M_{g0} = 12$) and (c) case

A10 ($M_{g0} = 66.7$). — : $\frac{d}{dt}(\frac{q^2}{2}) = P - \varepsilon_s - \varepsilon_d + \Pi_d$, --- : rate of the turbulent production (P), ... : rate of the solenoidal dissipation ($-\varepsilon_s$), -.- : rate of the dilatational dissipation ($-\varepsilon_d$), -.- : pressure-dilatation correlation term (Π_d).

In conclusion, compressibility affects more the turbulent production term which represents the dominating parameter in the turbulent kinetic energy budget. In the compressible regime, the production becomes preponderant. This property is already observed in the analysis of the budget of the Reynolds stress equations related to $\overline{u_1^2}$ and $\overline{u_1 u_2}$ (Riahi et al., 2007).

In Figures 13(a), (b), (c), (d) are represented the different components of the anisotropy tensor b_{ij} . These figures show obviously a remarkable property concerning the anisotropy. In fact, it appears clearly that the anisotropy increases with M_{g0} i.e. with compressibility and that it is responsible of the behavior of $\overline{u_1^2}$ which becomes dominating compared to $\overline{u_2^2}$ and $\overline{u_3^2}$ in the compressible case ($M_{g0} = 66.7$). As an indication, $\frac{\overline{u_1^2}}{\overline{u_2^2}} = 12.62$ and $\frac{\overline{u_1^2}}{\overline{u_3^2}} = 13.11$ for $St = 3.5$.

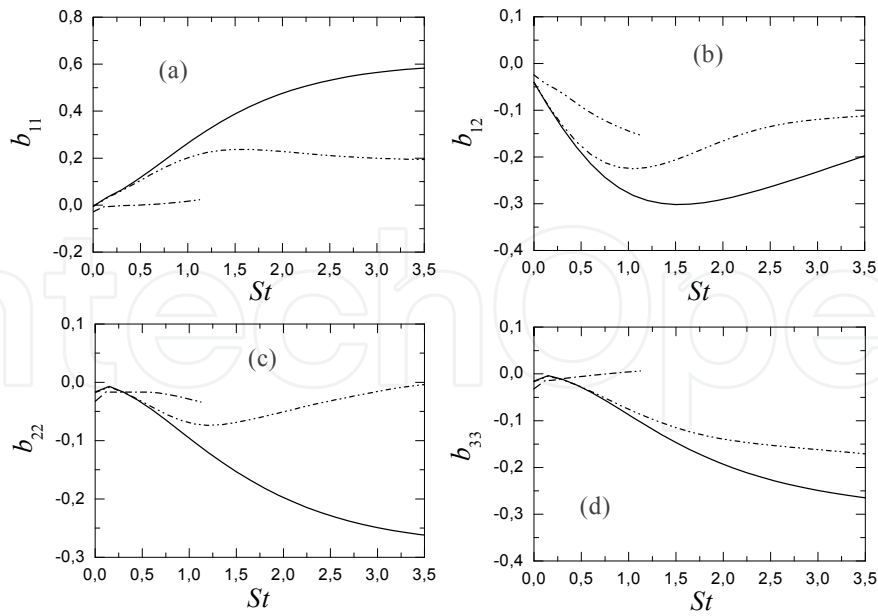


Figure 13. Components evolution of the anisotropy tensor (a) b_{11} , (b) b_{12} , (c) b_{22} and (d) b_{33} .
 - - - : case A0 ($M_{g0} = 1$), — · · : case A4 ($M_{g0} = 12$), — : case A10 ($M_{g0} = 66.7$).

6. Equilibrium states of homogeneous compressible turbulence

RDT is also used to determine equilibrium states in the compressible regime (Riahi & Lili, 2011) in particular for large values of St ($St > 10$) and M_{g0} . Evolution of the relevant component of Reynolds stress anisotropy tensor b_{12} is presented in Figure 14 for different values of the initial gradient Mach number M_{g0} (66.7, 200, 500 and 1000). Numerical results show that, for large values of St , b_{12} is independent of the initial turbulent Mach number M_{t0} as shown in Figure 14(a) ($M_{t0} = 0.25$) and in Figure 14(b) ($M_{t0} = 0.6$). As one can remark also from these results that b_{12} reaches its stationary value all the more quickly as M_{g0} is large ($M_{g0} = 1000$). At this stage, we study equilibrium states corresponding to $M_{g0} = 1000$, value which we assimilate to M_{g0} arbitrarily large (M_{g0} infinity). We present now some properties relative to these equilibrium states corresponding to pressure-released regime. The case corresponding to pressure-released limit has been discussed by Cambon et al. (1993), Simone et al. (1997) and Livescu et al. (2004). We begin by presenting equilibrium values of Reynolds stress anisotropy components: $(b_{11})_{\infty} = 0.66$, $(b_{22})_{\infty} = (b_{33})_{\infty} = -0.33$ and $(b_{12})_{\infty} = 0$. These values confirm the independence of the anisotropy tensor with M_{t0} in the pressure-released regime; calculation gives effectively these values for $M_{t0} = 0.25, 0.4, 0.5$ and 0.6 . Pressure-released state corresponds thus to a particular state of one-component turbulence in the direction of the shear (x_1 direction): $(\overline{u_1^2})_{\infty} = q^2$, $(\overline{u_2^2})_{\infty} = 0$ and $(\overline{u_3^2})_{\infty} = 0$.

Another interesting property of the pressure-released regime is the quadratic increase of $\overline{u_1^2}$ as shown in Figure 15 related to $M_{g0} = 10^6$ and $M_{t0} = 0.25$ (Riahi & Lili, 2011). This property has been established by Livescu et al. (2004) as analytical RDT solution in the pressure-released limit.

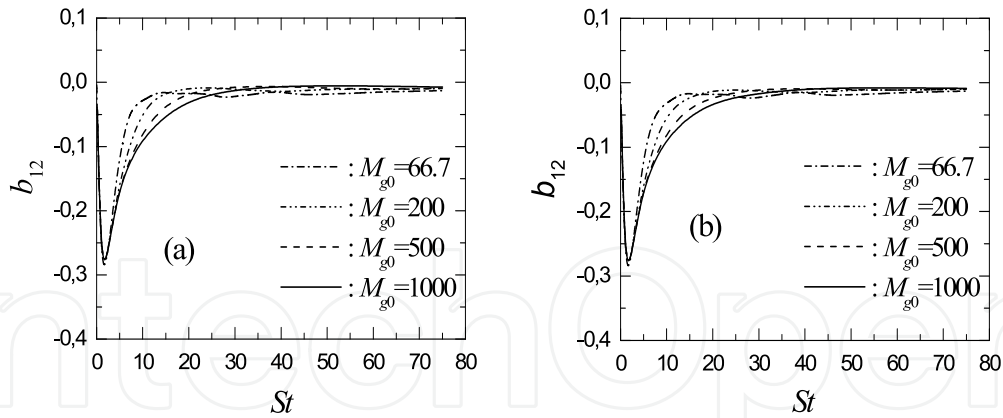


Figure 14. Evolution of the relevant component of the Reynolds stress anisotropy tensor b_{12} for various values of initial gradient Mach number M_{g0} with (a): $M_{t0} = 0.25$ and (b): $M_{t0} = 0.6$.

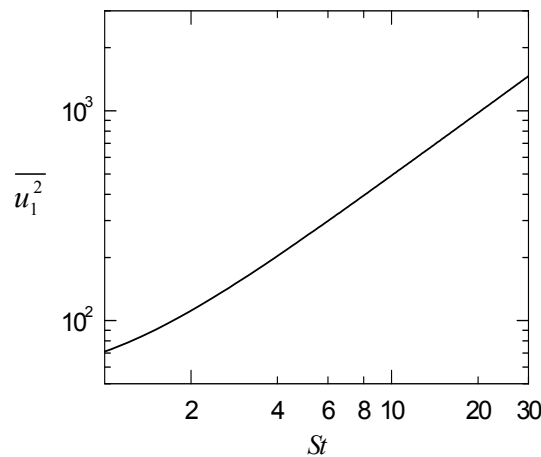


Figure 15. Quadratic evolution of $\overline{u_1^2}$.

Table 2 lists equilibrium values of the normalized dissipation rate due to dilatation $\left(\frac{\varepsilon_d}{\varepsilon_s + \varepsilon_d}\right)_\infty$, the normalized pressure-dilatation correlation $\left(\frac{\overline{pd}}{Sq^2}\right)_\infty$ and the normalized pressure variance $\left(\frac{\overline{p^2}}{\overline{\rho}^2 q^4}\right)_\infty$ for various values of initial turbulent Mach number M_{t0} (Riahi & Lili, 2011). From this table, we deduce the dependence of all these parameters with M_{t0} .

Figure 16 shows variation of the equilibrium normalized pressure variance $\left(\frac{\overline{p^2}}{\overline{\rho}^2 q^4}\right)_\infty$ with various initial turbulent Mach number ($M_{t0} = 0.25, 0.4, 0.5$ and 0.6 ($M_{t0} = 0.6$ is relatively high initial turbulent Mach number)) (Riahi & Lili, 2011). From this figure, it can be seen that this parameter can be written as $\left(\frac{\overline{p^2}}{\overline{\rho}^2 q^4}\right)_\infty = (M_{t0})^\alpha$ where $\alpha = 5.8$.

Equilibrium parameters \ M_{t0}	0.25	0.4	0.5	0.6
$\left(\frac{\varepsilon_d}{\varepsilon_s + \varepsilon_d} \right)_\infty$	0.0410	0.0277	0.0230	0.0197
$\left(\frac{\overline{pd}}{Sq^2} \right)_\infty$	0.000387	0.000311	0.000280	0.000257
$\left(\frac{\overline{p^2}}{\overline{\rho}^2 q^4} \right)_\infty$	0.00937	0.00278	0.00156	0.000973

Table 2. Equilibrium values of the normalized dissipation rate due to dilatation, the normalized pressure-dilatation and the normalized pressure variance for various initial turbulent Mach number M_{t0} .

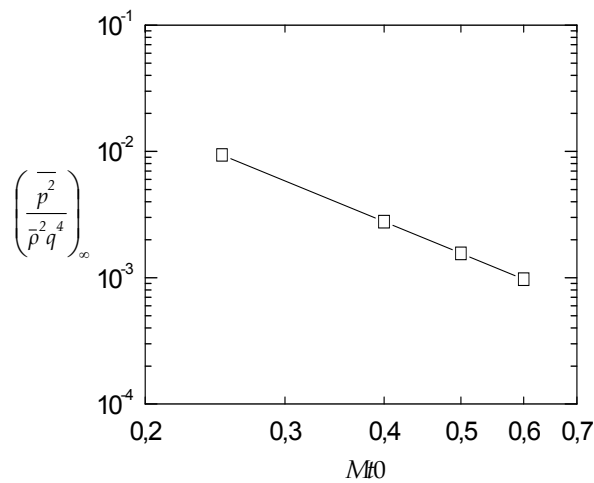


Figure 16. Variation of the equilibrium normalized pressure variance with various initial turbulent Mach number ($M_{t0} = 0.25, 0.4, 0.5$ and 0.6).

7. Turbulence model to be tested

7.1. Introduction

RDT is an efficient and accurate method for testing linear turbulence models. The Fujiwara and Arakawa model (1995) has been proposed in literature for pressure-dilatation correlation studying compressible turbulence. In order to verify its capacity to describe the homogeneous compressible flow, since it was established to this objective, three cases of the initial gradient Mach number ($M_{g0} = 1, 12$ and 66.7) are considered. The evaluation of this model, in the different regimes of flow, stays in the field of RDT validity (Riahi & Lili, 2011). Its detailed linear form is provided below.

7.2. Fujiwara and Arakawa model

The model suggested by Fujiwara and Arakawa (1995) for the pressure-dilatation correlation takes into account the compressible part (dilatational) of the kinetic energy (q_d^2). Linear part of this model has the following form:

$$\Pi_d = C_1 \bar{\rho} q^2 b_{ij} \sqrt{\frac{q_d^2}{q^2}} \frac{\partial U_i}{\partial x_j},$$

where $C_1 = 0.3$ and ε is the turbulent dissipation rate. b_{ij} is the Reynolds anisotropy tensor.

7.3. Results

Comparison between RDT and Fujiwara and Arakawa (1995) model results concerning the pressure-dilatation correlation term Π_d are represented in Figures 17(a), (b), (c). As one can remark from these results, that the contribution of Π_d obtained by this model is negligible. Except in the case of the intermediate regime ($M_{g0} = 12$), the Fujiwara and Arakawa model does not represent an appreciable discrepancies with the RDT results.

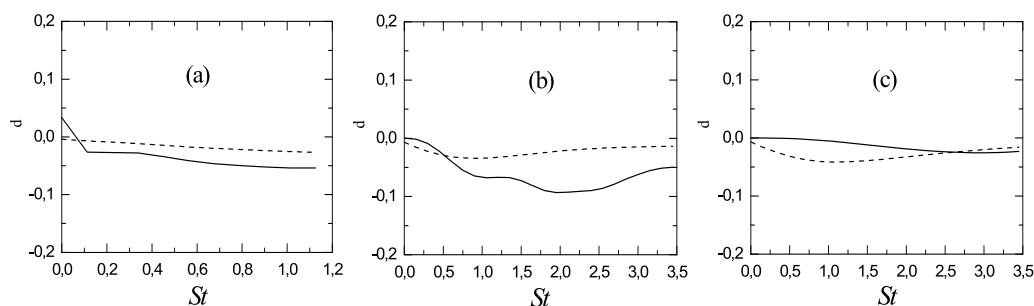


Figure 17. Evolution of the pressure dilatation correlation term Π_d (a) case A0 ($M_{g0} = 1$), (b) case A4 ($M_{g0} = 12$) and (c) case A10 ($M_{g0} = 66.7$). — : RDT results, - - - : Fujiwara and Arakawa model.

8. Conclusion

Rapid distortion theory (RDT) is a computationally viable option for examining linear compressible flow physics in the absence of inertial effects. Evolution of compressible homogeneous turbulence has been described completely by finding numerical solutions obtained by solving linear double correlations spectra evolution. Numerical integration of these equations has been carried out using a second-order simple and accurate scheme. This numerical method has proved more stable and faster and allows in particular to obtain accurately the asymptotic behavior of the turbulence parameters (for large values of St) characteristic of equilibrium states. In this chapter, RDT code developed by authors solves linearized equations for compressible homogeneous shear flows (Riahi & Lili, 2011). It has been validated by comparing RDT results with direct numerical simulation (DNS) of Simone et al. (1997) and Sarkar (1995) for various values of initial gradient Mach number M_{g0} which is the key parameter controlling the level of compressibility. The study of the behavior of the

non-dimensional turbulent kinetic energy $q^2(t)/q^2(0)$ permit to determine various regimes of flow. This study allows to check relevance of an incompressible regime for low values of initial gradient Mach number M_{g0} , of an intermediate regime for moderate values of M_{g0} and of a compressible regime for high values of M_{g0} . Agreement between RDT and DNS of Simone et al. (1997) and Sarkar (1995) is obtained for small values of the non-dimensional times St ($St < 3.5$). This agreement gives new insight into compressibility effects and reveals the extent to which linear processes are responsible for modifying the structure of compressible turbulence. The behavior analysis of the various terms presented in the turbulent kinetic energy budget shows that compressibility affects more the turbulent production which becomes preponderant in the compressible regime. This property is already observed in the analysis of the budget of the Reynolds stress equations related to $\overline{u_1^2}$ and $\overline{u_1 u_2}$. Another important property reveals that anisotropy increases with compressibility.

Agreement of RDT with DNS (Simone et al., 1997) found for large values of St ($St > 10$) in particular for large values of M_{g0} which allows to determine equilibrium states in the compressible regime. Evolution of the relevant component of the Reynolds stress anisotropy tensor b_{12} , for different values of initial gradient Mach number M_{g0} (66.7, 200, 500 and 1000) and for large values of St , shows that b_{12} is independent of the initial turbulent Mach number M_{t0} (which is a parameter characterizing the effects of compressibility). In addition, b_{12} becomes stationary all the more quickly as M_{g0} is large ($M_{g0} = 1000$). Considering equilibrium states associated to this value of M_{g0} as representative of equilibrium states related to infinite M_{g0} (pressure-released regime), equilibrium values of the Reynolds stress anisotropy components $(b_{11})_\infty$ and $(b_{22})_\infty$ are independent of the initial turbulent Mach number M_{t0} . In contrast, equilibrium values of the normalized dissipation rate due to dilatation, the normalized pressure-dilatation correlation and the normalized pressure variance are dependent of M_{t0} . Variation of the equilibrium normalized pressure variance with various initial turbulent Mach

number ($M_{t0} = 0.25, 0.4, 0.5$ and 0.6) can be written as $\left(\frac{\overline{p^2}}{\overline{\rho^2} q^4} \right)_\infty = (M_{t0})^\alpha$ where $\alpha = 5.8$.

In conclusion, after a critical analysis, we were able to justify that RDT permits to well identify compressibility effects in order to develop models taking them into account satisfactorily. The analysis of rapid distortion theory showed that it is possible to better understand the compressible turbulent flows. In addition, RDT is valid to predict asymptotic equilibrium states for compressible homogeneous sheared turbulence for large values of initial gradient Mach number (pressure-released regime). RDT is an efficient method for testing linear contribution of turbulence models.

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Abbreviations

RDT Rapid Distortion Theory
DNS Direct Numerical Simulation

Nomenclature

M_g, M_{g0}	gradient Mach number, initial gradient Mach number
M_t, M_{t0}	turbulent Mach number, initial turbulent Mach number
S	magnitude of the mean velocity gradient
u_i	velocity fluctuation
p	pressure fluctuation
$\bar{\rho}$	mean density
ν	kinematic viscosity
λ_{ij}	mean velocity gradient
a	mean sound speed
Δt	time-step size
$q^2/2$	turbulent kinetic energy
$b_{ij} = \frac{\overline{u_i u_j}}{q^2} - \frac{\delta_{ij}}{3}$	anisotropy tensor
ε_0	initial total (solenoidal and dilatational) dissipation rate of turbulent kinetic energy
R_{et0}	initial turbulent Reynolds number
r_0	initial rapidity of the shear
γ	ratio of specific heats

9. References

- Blaisdell, G.A.; Mansour, N.N. & Reynolds, W.C. (1993). Compressibility effects on the growth and structure of homogeneous turbulent shear flow. *Journal of Fluid Mechanics*, vol. 256, pp. 443-485.
- Blaisdell, G. A.; Coleman, G. N. & Mansour, N. N. (1996). Rapid distortion theory for compressible homogenous turbulence under isotropic mean strain. *Physics of Fluids*, vol. 8, pp. 2692-2705.

- Coleman, G. N. & Mansour, N. N. (1991). Modeling the rapid spherical compression of isotropic turbulence, *Physics of Fluids*, vol. A 3, pp. 2255-2259.
- Cambon, C.; Coleman, G.N. & Mansour, N.N. (1993). Rapid distortion analysis and direct simulation of compressible homogeneous turbulence at finite Mach number. *Journal of Fluid Mechanics*, vol. 257, pp. 641-665.
- Durbin, P.A. & Zeman, O. (1992). Rapid distortion theory for homogeneous compressed turbulence with application to modelling. *Journal of Fluid Mechanics*, vol. 242, pp. 349-370.
- Fujiwara, H. & Arakawa, C. (1995). Modeling the pressure-dilatation correlation in isotropic and homogeneous sheared turbulence. *Proc. 10th Symposium on Turbulent Shear Flows*, vol. 26, pp. 1-6.
- Hunt, J. C. R. & Carruthers, D. J. (1990). Rapid distortion theory and the problems of turbulence. *Journal of Fluid Mechanics*, vol. 212, pp. 497-532.
- Jacquín, L.; Cambon, C. & Blin, E. (1993). Turbulence amplification by a shock wave and rapid distortion theory. *Physics of Fluids*, vol. A 5, pp. 2539-2550.
- Livescu D. & Madnia, C. K. (2004). Small Scale Structure of Homogeneous Turbulent Shear Flow. *Physics of Fluids*, vol. 16, pp. 2864-2876.
- Riahi, M.; Chouchane, L. & Lili, T. (2007). A study of compressibility effects on structure of homogeneous sheared turbulence. *The European Physical Journal Applied Physics*, vol. 39, Issue: 1, pp. 67-75.
- Riahi, M. (2008). Modélisation et simulation d'une turbulence homogène compressible. *Thèse de Doctorat, Faculté des Sciences Mathématiques Physiques et Naturelles de Tunis, Tunisie*.
- Riahi, M.; & Lili, T. (2011). Equilibrium states of homogeneous sheared compressible turbulence, *AIP Advances*, vol. 1, Issue: 2, pp. 022117-022117-11, ISSN 2158-3226.
- Savill, A. M. (1987). Recent Developments in Rapid-Distortion Theory. *Annual Review of Fluid Mechanics*, vol. 19 pp. 531-573.
- Sarkar, S.; Erlebacher, G.; Hussaini, M. & Kreiss, H.O. (1991). The analysis and modelling of dilatational terms in compressible turbulence. *Journal of Fluid Mechanics*, vol. 227, pp. 473-493.
- Sarkar, S. (1992). The pressure-dilatation correlation in compressible flows, *Physics of Fluids*, vol. A 4, pp. 2674-2682.
- Sarkar, S. (1995). The stabilizing effect of compressibility in turbulent shear flow. *Journal of Fluid Mechanics*, vol. 282, pp. 163-186.
- Simone, A.; Coleman, G.N. & Cambon, C. (1997). The effect of compressibility on turbulent shear flow: a rapid-distortion-theory and direct-numerical-simulation study. *Journal of Fluid Mechanics*, vol. 330, (July 1996), pp. 307-338.
- Simone, A. (1995). Etude théorique et simulation numérique de la turbulence compressible en présence de cisaillement ou de variation de volume à grande échelle. *Thèse de Doctorat, Ecole Centrale de Lyon, France*.

Tavoularis, S. & Corrsin, S. (1981). Experiments in nearly homogeneous turbulent shear flow with a uniform mean gradient temperature. *Part I. Journal of Fluid Mechanics*, vol. 104, pp. 311-347.

Zeman, O. (1990). Dilatation dissipation: The concept and application in modeling compressible mixing layers. *Physics of Fluids*, vol. 2, pp. 178-188, ISSN 0899-8213.

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