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# Cartesian Impedance Control of Flexible Joint Robots: A Decoupling Approach

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## 1. Introduction

Whenever a robotic manipulator is supposed to get in contact with its environment, the achievement of a compliant behavior is relevant. This is a classical control problem for rigid body robots, which led to control approaches like impedance control, admittance control, or stiffness control (Hogan, 1985; Sciavicco & Siciliano, 1996). In contrast to the approach introduced in this contribution most works on the Cartesian impedance control problem consider a robot model which does not include joint flexibility.

In this work a decoupling based control approach for flexible joint robots is described. The considered control objective is the achievement of a desired compliant behavior between external generalized forces and the Cartesian end-effector motion of the robot. The design method will be based on some results from control theory for cascaded systems. The proposed controller will be designed in two steps. First, an inner feedback loop is used to bring the flexible joint robot model into cascaded form. Then, an additional outer control loop implements the desired compliant behaviour (Ott et al., 2003). The stability theory for cascaded control systems (Seibert & Suarez, 1990; Loria, 2001) can be applied to analyze the closed-loop system.

When dealing with a robot model with flexible joints, the maybe most obvious approach for the design of an impedance controller is the singular perturbation approach (Spong, 1987; De Luca, 1996; Ott et al., 2002). Therein the fast subsystem, which is in our case the torque dynamics, is considered as a perturbation of the rigid body model. One can then use any controller designed for the rigid body robot dynamics and apply it in combination with an inner torque control loop to the flexible joint robot model. The main disadvantage of this approach is that it does not allow for a formal stability proof without referring to the approximate singular perturbation consideration.

The controller structure proposed herein is somewhat related to the singular perturbation based controller. Also herein an inner torque control loop is combined with an outer impedance control loop. But these control loops will be designed in such a way that one can give a proof of asymptotic stability, based on the stability theory for cascaded systems. The proposed controller structure is also related to the controllers presented in (Lin & Goldenberg, 1995; 1996). But the following analysis focuses on the design of an impedance controller, while Lin and Goldenberg considered a position controller respectively a hybrid position/force-controller. The procedure for the stability analysis from these works cannot be applied to the impedance control problem considered herein in a straightforward way. The chapter is organized as follows. In Section 2 some relevant results of the stability theory for cascaded systems are reviewed. The considered dynamical model of a flexible joint robot is described in Section 3. In Section 4 the design

idea based on an inner torque control loop is presented. Section 5 presents the Cartesian impedance controller. Some simulation results are given in Section 6. Finally, Section 7 gives a short summary of the presented work.

## 2. Stability Theory for Cascaded Control Systems

Consider an autonomous cascaded system in the form

$$\dot{x}_1 = f_1(x_1, x_2), \quad (1)$$

$$\dot{x}_2 = f_2(x_2), \quad (2)$$

where  $x_1 \in \mathbb{R}^{n_1}$  and  $x_2 \in \mathbb{R}^{n_2}$  are the state variables. It is assumed that the functions  $f_1(x_1, x_2)$  and  $f_2(x_2)$  are locally Lipschitz and that all solutions exist for all times  $t > 0$ . Furthermore, it is assumed that the origin is an equilibrium point of (1)-(2), i.e.  $f_1(0,0) = 0$  and  $f_2(0) = 0$ . In the following the situation is analyzed when the uncoupled system (2) and

$$\dot{x}_1 = f_1(x_1, 0), \quad (3)$$

are globally asymptotically stable. Then the question arises, under which conditions also the coupled system (1)-(2) will be asymptotically stable. Locally this is always true, see the references in (Seibert & Suarez, 1990). In order to ensure that this holds also globally it was proven in (Seibert & Suarez, 1990) that it is sufficient to show that all solutions of the coupled system remain bounded. This is formulated in a more general form in the following theorem, taken from (Seibert & Suarez, 1990).

**Theorem 1.** *If the system (3) is globally asymptotically stable, and if (2) is asymptotically stable with region of attraction  $A \subseteq \mathbb{R}^{n_2}$ , and if every orbit of (1)-(2) with initial point in  $\mathbb{R}^{n_1} \times A$  is bounded for  $t > 0$ , then the system (1)-(2) is asymptotically stable with region of attraction  $\mathbb{R}^{n_1} \times A$ .*

Notice that Theorem 1 also handles the case of global asymptotic stability, if the region of attraction of (2) is the whole state space  $\mathbb{R}^{n_2}$ . Theorem 1 considers only the autonomous case. In case of the tracking control problem, on the other hand, a time-varying system must be considered. An extension of Theorem 1 to a special class of time-varying systems was presented in (Loria, 2001).

**Theorem 2.** *Consider the system*

$$\dot{x}_1 = f_1(x_1, t) + h(x_1, x_2, t)x_2, \quad (4)$$

$$\dot{x}_2 = f_2(x_2, t), \quad (5)$$

with state  $(x_1, x_2) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$ . The functions  $f_1(x_1, t)$ ,  $f_2(x_2, t)$ , and  $h(x_1, x_2, t)$  are continuous in their arguments, locally Lipschitz in  $(x_1, x_2)$ , uniformly in  $t$ , and  $f_1(x_1, t)$  is continuously differentiable in both arguments. This system is uniformly globally asymptotically stable if and only if the following holds:

- There exists a non-decreasing function  $H(\cdot)$  such that

$$\|h(x_1, x_2, t)\| \leq H(\|(x_1, x_2)\|). \quad (6)$$

- The systems (5) and  $\dot{x}_1 = f_1(x_1, t)$  are uniformly globally asymptotically stable.
- The solutions of (4)-(5) are uniformly globally bounded.

Notice that, if the uncoupled systems  $\dot{x}_1 = f_1(x_1, t)$  and  $\dot{x}_2 = f_2(x_2, t)$  can be shown to be exponentially stable, then there exist also other results in the literature which can be applied to such a triangular system, see, e.g., (Vidyasagar, 1993). The impedance control problem from Section 5 though leads to a closed-loop system for which only (uniform

global) asymptotic stability, instead of exponential stability, can be shown for the respective subsystem  $\dot{x}_1 = f_1(x_1, t)$ . In the following it will be shown how Theorem 2 can be used for the design of a Cartesian impedance controller for the flexible joint robot model.

### 3. Considered Robot Model

In this work the so-called *reduced model* of a robot with  $n$  flexible joints is considered as proposed by (Spong, 1987):

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau + \tau_{ext}, \quad (7)$$

$$B\ddot{\theta} + \tau = \tau_m, \quad (8)$$

$$\tau = K(\theta - q). \quad (9)$$

Herein,  $q \in \mathbb{R}^n$  is the vector of link positions and  $\theta \in \mathbb{R}^n$  the vector of motor positions. The vector of transmission torques is denoted by  $\tau$ . The link side dynamics (7) contains the symmetric and positive definite inertia matrix  $M(q)$ , the vector of Coriolis and centripetal torques  $C(q, \dot{q})\dot{q}$  and the vector of gravitational torques  $g(q)$ . Furthermore,  $B$  and  $K$  are diagonal matrices containing the motor inertias and the stiffness values for the joints. The vector of motor torques  $\tau_m$  will serve as the control input and  $\tau_{ext}$  is the vector of external torques being exerted by the manipulator's environment.

Notice that herein friction effects have been neglected which may be justified by a sufficiently accurate friction compensation. It is further assumed that the external torques  $\tau_{ext}$  can be measured. If these torques are generated solely by external forces and torques at the end-effector, this can be realized by the use of a 6DOF force/torque-sensor mounted on the tip of the robot.

For the further analysis, the model (7)-(8) may be rewritten by choosing the state variables  $(q, \dot{q}, \tau, \dot{\tau})$  in the form

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau + \tau_{ext}, \quad (10)$$

$$BK^{-1}\ddot{\tau} + \tau = \tau_m - BM(q)^{-1}(\tau + \tau_{ext} - C(q, \dot{q})\dot{q} - g(q)). \quad (11)$$

Based on this model, the following controller design procedure is motivated by considering Theorem 2. An inner torque control loop is used to decouple the torque dynamics (11) exactly from the link dynamics (10). The (time-varying) set-point  $\tau_d$  for this inner control loop is generated by an impedance controller in the outer loop, cf. Section 5. In the next section the design of the inner loop torque controller is treated.

### 4. Controller Design Idea

In the following a torque controller is designed in such a way that the inner closed-loop system becomes uniformly globally asymptotically stable and is decoupled from the link dynamics. Obviously, some undesired terms in (11) can be eliminated by a feedback compensation of the form

$$\tau_m = u + BM(q)^{-1}(\tau + \tau_{ext} - C(q, \dot{q})\dot{q} - g(q)), \quad (12)$$

where  $u$  is an intermediate control input. This transforms (11) into

$$BK^{-1}\ddot{\tau} + \tau = u. \quad (13)$$

By introducing the torque set-point  $\tau_d$  and a torque error variable  $z = \tau - \tau_d$ , one obtains

$$BK^{-1}(\ddot{z} + \ddot{\tau}_d) + z + \tau_d = u. \quad (14)$$

A controller which makes this system globally asymptotically stable can easily be found. Consider for instance the control law

$$u = \tau_d + BK^{-1}(\ddot{\tau}_d - K_s \dot{z} - K_t z) , \quad (15)$$

with symmetric and positive definite gain matrices  $K_s$  and  $K_t$ . This leads to a closed-loop system of the form

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = z + \tau_d + \tau_{ext} , \quad (16)$$

$$\ddot{z} + K_s \dot{z} + (K_t + KB^{-1})z = 0. \quad (17)$$

Clearly, the decoupled torque error dynamics (17) is an exponentially stable linear system. Next, the torque set-point  $\tau_d$  is to be chosen such that a particular control goal is achieved. In order to get a uniformly globally asymptotically stable closed-loop system one must ensure that the conditions of Theorem 2 are fulfilled. Notice at this point also that the conditions of Theorem 2 are necessary and sufficient.

## 5. A Cartesian Impedance Controller

### 5.1 Task Description

The considered task for the controller design is an impedance controller in Cartesian coordinates  $x \in \mathbb{R}^m$  describing the end-effector pose. These Cartesian coordinates are given by a mapping  $x = f(q)$  from the configuration space to the task space. The Jacobian of this mapping is denoted by  $J(q) = \partial f(q) / \partial q$ . The Cartesian velocity and acceleration are then given by

$$\dot{x} = J(q)\dot{q}, \quad (18)$$

$$\ddot{x} = J(q)\ddot{q} + \dot{J}(q)\dot{q}. \quad (19)$$

In this work the non-redundant and non-singular case is treated. It is thus considered that the number  $n$  of configuration coordinates and the number  $m$  of task coordinates are equal and that the Jacobian matrix  $J(q)$  is non-singular in the considered region of the workspace. Furthermore, the mapping  $f(q)$  is assumed to be invertible such that the Cartesian coordinates can be used as a set of generalized coordinates for the further analysis. Formally, the Jacobian matrix  $J(q)$  is then expressed in terms of Cartesian coordinates as  $\bar{J}(x) = J(f^{-1}(x)) = J(q)$ .

These assumptions are quite common in designing controllers with respect to Cartesian coordinates. In practice they clearly can only be fulfilled in a limited region of the workspace which depends on the particular choice of Cartesian coordinates  $x = f(q)$ . However, in the following the control objective is to achieve a uniformly globally asymptotically stable closed-loop system. Thereby, the validity of the control law is analyzed for a globally valid set of coordinates while disregarding the problem of finding an appropriate set of Cartesian coordinates suitable for a given task).

According to the coordinates  $x$ , the external torques  $\tau_{ext}$  can be written in terms of a generalized Cartesian force vector  $F_{ext}$  defined by the relationship  $\tau_{ext} = \bar{J}(x)^T F_{ext}$ .

Equation (16) can then be rewritten in terms of Cartesian coordinates as

$$\Lambda(x)\ddot{x} + \mu(x, \dot{x})\dot{x} + p(x) = \bar{J}(x)^{-T}(z + \tau_d) + F_{ext} , \quad (20)$$

where the Cartesian inertia matrix  $\Lambda(x)$  and the matrix  $\mu(x, \dot{x})$  are given by



$$\Lambda(x) = \left( J(q)^{-T} M(q) J(q)^{-1} \right)_{q=f^{-1}(x)}, \quad (21)$$

$$\mu(x, \dot{x}) = \left( J(q)^{-T} C(q, \dot{q}) J(q)^{-1} - \Lambda(q) \dot{J}(q) J(q)^{-1} \right)_{q=f^{-1}(x)}, \quad (22)$$

and  $p(x) = \bar{J}(x)^{-T} g(f^{-1}(x)) = J(q)^{-T} g(q)$  is the Cartesian gravity vector.

Based on the Cartesian model (20), the desired closed-loop behavior is formulated next. The considered impedance control problem is specified by means of symmetric and positive definite stiffness and damping matrices  $K_d$  and  $D_d$ , and a (possibly time-varying) desired trajectory  $x_d(t)$ . The desired trajectory is assumed to be four times continuously differentiable. For many applications the shaping of the desired stiffness and damping behavior is sufficient, and no shaping of the inertia matrix is required. Therefore, the Cartesian robot inertia matrix  $\Lambda(x)$  as well as the matrix  $\mu(x, \dot{x})$  are preserved in the desired impedance behavior. With the Cartesian error vector  $e_x = x - x_d$  the considered desired impedance can be written as

$$\Lambda(x)\ddot{e}_x + (\mu(x, \dot{x}) + D_d)\dot{e}_x + K_d e_x = F_{ext}. \quad (23)$$

For this system the following two important properties hold.

**Property 1.** For  $F_{ext} = 0$ , the system (23) with the symmetric and positive definite stiffness and damping matrices  $K_d$  and  $D_d$  is uniformly globally asymptotically stable.

**Property 2.** For  $\dot{x}_d(t) = 0$ , the system (23) with the symmetric and positive definite stiffness and damping matrices  $K_d$  and  $D_d$  gets time-invariant and represents a passive mapping from the external force  $F_{ext}$  to the velocity error  $\dot{e}_x$ .

Property 1 was proven in (Paden & Panja, 1988; Santibanez & Kelly, 1997) for a system like (23) but in configuration coordinates. This result obviously also holds for the considered Cartesian coordinates. Property 2 corresponds to the well known passivity property of mechanical systems and is based on the fact that the matrix  $\dot{\Lambda}(x) - 2\mu(x, \dot{x})$  is skew symmetric (Sciavicco & Siciliano, 1996).

These two properties verify that the desired impedance behavior is chosen properly. Property 2 is of particular importance for an impedance controller, since it implies that the feedback interconnection with any passive environment, considered as a mapping  $\dot{e}_x \rightarrow -F_{ext}$ , results in a passive closed-loop system.

## 5.2 Controller Design

The remaining part of the controller design aims at choosing a control input  $\tau_d$  such that the system

$$\Lambda(x)\ddot{x} + \mu(x, \dot{x})\dot{x} + p(x) = \bar{J}(x)^{-T} (z + \tau_d) + F_{ext}, \quad (24)$$

$$\ddot{z} + K_s \dot{z} + (K_t + KB^{-1})z = 0, \quad (25)$$

which already contains the torque feedback action, resembles the desired dynamics (23) as closely as possible. If the systems (24) and (25) were completely uncoupled (i.e. for  $z = 0$  in (24)), the control objective could be exactly fulfilled by a feedback of the form

$$\begin{aligned} \tau_d &= \bar{J}(x)^T p(x) + \bar{J}(x)^T (\Lambda(x)\ddot{x}_d + \mu(x, \dot{x})\dot{x}_d - D_d\dot{e}_x - K_d e_x) \\ &= g(q) + J(q)^T (\Lambda(x)\ddot{x}_d + \mu(x, \dot{x})\dot{x}_d - D_d\dot{e}_x - K_d e_x). \end{aligned} \quad (26)$$

In the following it will be shown that the feedback law (26) leads to a uniformly globally asymptotically stable closed-loop system also for the flexible joint robot model (24)-(25). Thereby, the cascaded structure of (24)-(25) will be utilized by applying Theorem 2. Moreover, it will be shown that a passivity property analogous to Property 2 holds. The closed-loop system containing the torque feedback action, cf. (12) and (15), and the impedance control law (26) is given by

$$\Lambda(x)\ddot{e}_x + \mu(x, \dot{x})\dot{e}_x + D_d\dot{e}_x + K_d e_x = F_{ext} + \bar{J}(x)^{-T} z, \quad (27)$$

$$\ddot{z} + K_s \dot{z} + (K_t + KB^{-1})z = 0, \quad (28)$$

Notice that this system is time-varying due to the dependence on both  $x$  and  $e_x$ .

The main results of the following analysis are formulated in form of two propositions:

**Proposition 1.** For  $F_{ext} = 0$ , the system (27)-(28) with the symmetric and positive definite matrices  $K_s$ ,  $K_t$ ,  $K_d$ , and  $D_d$  is uniformly globally asymptotically stable.

**Proposition 2.** For  $\dot{x}_d(t) = 0$ , the system (27)-(28) with the symmetric and positive definite matrices  $K_s$ ,  $K_t$ ,  $K_d$ , and  $D_d$  gets time-invariant and represents a passive mapping from the external force  $F_{ext}$  to the velocity error  $\dot{e}_x$ .

### 5.3 Proof of Proposition 1

Before the actual proof is started, two well known technical lemmata are quoted for further reference (Horn & Johnson, 1990; Vidyasagar, 1993).

**Lemma 1.** Suppose that a symmetric matrix  $A$  is partitioned as

$$A = \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{1,2}^T & A_{2,2} \end{bmatrix}$$

where  $A_{1,1}$  and  $A_{2,2}$  are square. Then the matrix  $A$  is positive definite if and only if  $A_{1,1}$  is positive definite and  $A_{2,2} - A_{1,2}A_{1,1}^{-1}A_{1,2}^T > 0$  (positive definite).

**Lemma 2.** Given an arbitrary positive definite matrix  $Q$ , one can find a unique positive definite solution  $P$  of the Lyapunov equation  $A^T P + PA = -Q$  if and only if the matrix  $A$  is Hurwitz.

For the stability analysis of the system (27)-(28) it is convenient to rewrite it in the state variables  $(e_x, \dot{e}_x, z, \dot{z})$ . Therefore, the following substitutions are made:

$J(e_x, t) = J(f^{-1}(x)) = J(q)$ ,  $\Lambda(e_x, t) = \Lambda(x)$ , and  $\mu(e_x, \dot{e}_x, t) = \mu(x, \dot{x})$ . With  $w = (z, \dot{z})$  and

$$A = \begin{bmatrix} 0 & I \\ -K_s & -(K_t + KB^{-1}) \end{bmatrix}$$

(28) can be written as  $\dot{w} = Aw$ . Thus, for  $F_{ext} = 0$  we have

$$\Lambda(e_x, t)\ddot{e}_x + \mu(e_x, \dot{e}_x, t)\dot{e}_x + D_d\dot{e}_x + K_d e_x = J(e_x, t)^{-T} z, \quad (29)$$

$$\dot{w} = Aw. \quad (30)$$

In this form Theorem 2 can be applied. The first condition in Theorem 2 is the existence of a function  $H(\cdot)$ , for which (6) holds. This is ensured here by the assumption that the Jacobian matrix is non-singular for all times  $t$ . Hence there exists a  $\delta \in \mathbb{R}$ ,  $0 < \delta < \infty$  such that

$$\|J(e_x, t)^{-T}\| \leq \sup_{t \in [0, \infty)} \sqrt{\lambda_{\max}(J(e_x, t)^{-1} J(e_x, t)^{-T})} \leq \delta,$$

with  $\lambda_{\max}(A(t))$  the maximum eigenvalue of the matrix  $A(t)$  at time  $t$ .

Uniform global asymptotic stability of each of the two uncoupled subsystems is ensured by Property 1 and the fact that the linear system  $\dot{w} = Aw$  is even globally exponentially stable for positive definite matrices  $K_s$  and  $K_t$ .

What remains is to show that all solutions of (29)-(30) remain uniformly globally bounded. Consider therefore the following positive definite function

$$V(e_x, \dot{e}_x, w, t) = \frac{1}{2} \dot{e}_x^T \Lambda(e_x, t) \dot{e}_x + \frac{1}{2} e_x^T K_d e_x + w^T P w, \quad (31)$$

with a positive definite matrix  $P$ . Considering the well known skew symmetry property of  $\dot{\Lambda}(x) - 2\mu(x, \dot{x})$ , one can derive the time derivative of (31) along the solutions of (29)-(30) as

$$\dot{V}(e_x, \dot{e}_x, w, t) = -\dot{e}_x^T D_d \dot{e}_x - w^T Q w + \dot{e}_x^T J(e_x, t)^{-T} z,$$

where  $Q = -(A^T P + PA)$  can be an arbitrary positive definite matrix, since  $A$  is Hurwitz (see Lemma 2). Then,  $\dot{V}(e_x, \dot{e}_x, w, t)$  can be written as

$$\dot{V}(e_x, \dot{e}_x, w, t) = - \begin{bmatrix} \dot{e}_x \\ z \\ \dot{z} \end{bmatrix}^T N(e_x, t) \begin{bmatrix} \dot{e}_x \\ z \\ \dot{z} \end{bmatrix},$$

with

$$N(e_x, t) = \begin{bmatrix} D_d & \begin{bmatrix} -\frac{1}{2} J(e_x, t)^{-T} & 0 \end{bmatrix} \\ \begin{bmatrix} -\frac{1}{2} J(e_x, t)^{-1} \\ 0 \end{bmatrix} & Q \end{bmatrix}.$$

From Lemma 1 it follows that  $N(e_x, t)$  is positive definite if and only if

$$Q - \frac{1}{4} J(e_x, t)^{-1} D_d^{-1} J(e_x, t)^{-T} > 0. \quad (32)$$

Condition (32) can be fulfilled for every positive definite matrix  $D_d$ , because the Jacobian does not get singular and the matrix  $Q$  is a positive definite matrix which may be chosen arbitrarily. Hence, one can conclude

$$\dot{V}(e_x, \dot{e}_x, w, t) \leq 0. \quad (33)$$

At this point it is worth mentioning that  $V(e_x, \dot{e}_x, w, t)$  is bounded from above and below by some time-invariant functions  $W_1(e_x, \dot{e}_x, w)$  and  $W_2(e_x, \dot{e}_x, w)$ , i.e.

$$\begin{aligned} W_1(e_x, \dot{e}_x, w) &\leq V(e_x, \dot{e}_x, w, t) \leq W_2(e_x, \dot{e}_x, w), \\ W_1(e_x, \dot{e}_x, w) &= \frac{1}{2} \lambda_1 \|\dot{e}_x\|_2^2 + \frac{1}{2} e_x^T K_d e_x + w^T P w, \\ W_2(e_x, \dot{e}_x, w) &= \frac{1}{2} \lambda_2 \|\dot{e}_x\|_2^2 + \frac{1}{2} e_x^T K_d e_x + w^T P w, \end{aligned}$$

where

$$0 < \lambda_1 < \inf_{t \in [0, \infty)} \lambda_{\min}(\Lambda(e_x, t)) < \sup_{t \in [0, \infty)} \lambda_{\max}(\Lambda(e_x, t)) < \lambda_2 < \infty,$$



with  $\lambda_{\min}(A(t))$  and  $\lambda_{\max}(A(t))$  as the minimum and maximum eigenvalue of the matrix  $A(t)$  at time  $t$ .

Based on these properties of  $V(e_x, \dot{e}_x, w, t)$ , Lemma A.1 from the Appendix, and  $\dot{V}(e_x, \dot{e}_x, w, t) \leq 0$  one can show that all the solutions of (29)-(30) are uniformly globally bounded. Proposition 1 follows then from Theorem 2.

It is important to mention that the need for referring to Theorem 2 in this stability analysis results from the fact that, on the one hand, the considered system is time-varying and, on the other hand, the time derivative of the chosen function  $V(e_x, \dot{e}_x, w, t)$  is only negative semi-definite. This situation, and the exploitation of the fact that the matrix  $Q$  can be chosen arbitrarily, are the most important differences to the stability proofs in (Lin & Goldenberg, 1995; 1996).

## 5.4 Proof of Proposition 2

Proposition 2 can be shown by considering  $V(e_x, \dot{e}_x, w, t)$  of (31) as a candidate storage function. In case of  $F_{ext} \neq 0$  the time derivative of  $V(e_x, \dot{e}_x, w, t)$  along the solutions of (27)-(28) gets

$$\dot{V}(e_x, \dot{e}_x, w, t) = - \begin{bmatrix} \dot{e}_x \\ z \\ \dot{z} \end{bmatrix}^T N(e_x, t) \begin{bmatrix} \dot{e}_x \\ z \\ \dot{z} \end{bmatrix} + \dot{e}_x^T F_{ext} .$$

The matrix  $N(e_x, t)$  has already been shown to be positive definite. From this one can immediately conclude the passivity property from Proposition 2.

## 6. Simulation Results

In this section a simulation study of the proposed impedance control law is presented. The controller is evaluated for a flexible joint robot model of the seven-axis DLR-Lightweight-Robot-III (Hirzinger et al., 2002) (see Fig. 1). This robot is equipped with joint torque sensors in addition to the common motor position sensors and thus is ideally suited for the verification of the presented controller. In the present simulation only the first six joints of the robot are considered.

It is assumed that the complete state of the system (10)-(11) is available and that also the (generalized) external forces can be measured by a force-torque-sensor mounted on the tip of the robot.

For the desired impedance behavior (23) diagonal stiffness and damping matrices  $K_d$  and  $D_d$  are chosen. The stiffness values for the translational components of  $K_d$  were set to a value of 2000 N/m, while the rotational components were set to 100 Nm/rad. The damping values were chosen as 150 Ns/m and 50 Nms/rad for the translational and the rotational components, respectively. The Cartesian coordinates are composed of three translational and three rotational coordinates, in which the modified Euler-angles, see, e.g., (Natale, 2003) according to the common roll-pitch-yaw representation were used.

In the following two different gains for the torque control loop are evaluated and compared to the response of the desired impedance behavior (23). In both cases the torque control gain matrices  $K_s$  and  $K_t$  were chosen as diagonal. In the first case a rather small proportional gain of  $K_t = I$  was chosen, while in the second case a higher gain of

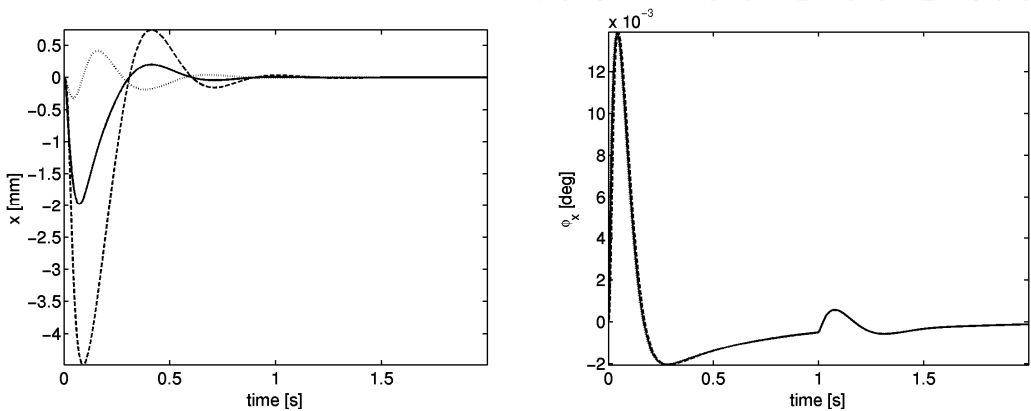
$K_t = 10 \cdot I$  was used. In both cases the torque damping matrix  $K_s$  was chosen according to an overall damping factor of 0.7 for the linear system (28).



Figure 1. DLR-Lightweight-Robot-III. The picture on the right hand side shows the initial configuration of the robot in the simulation study

In the simulation two step responses are presented, starting from a joint configuration as shown at the right hand side of Fig. 1. First, a step for the virtual equilibrium position of 5 cm in z-direction is commanded at time instant  $t = 0$ . Apart from this step the virtual equilibrium position is constant. Then, after a delay of one second, a step-wise excitation by an external force of 1 N in y-direction is simulated. Notice that this force step results in an excitation of the link side dynamics (27) only, while the torque error dynamics (28) keeps unaffected.

Fig. 2 shows the simulation results for the two decoupling based controllers (with low and high gains) in comparison with the desired impedance behavior. The translational end-effector coordinates are denoted by  $x$ ,  $y$ , and  $z$ , while the orientation coordinates are denoted by  $\phi_x$ ,  $\phi_y$ , and  $\phi_z$ . Due to the coupling via the (fully occupied) inertia matrix, all the coordinates deviate from their initial values in the step responses. As expected, the closed-loop behavior resembles the desired behavior better for higher torque control gains. The step of the external force, instead, does not affect the torque control loop, and



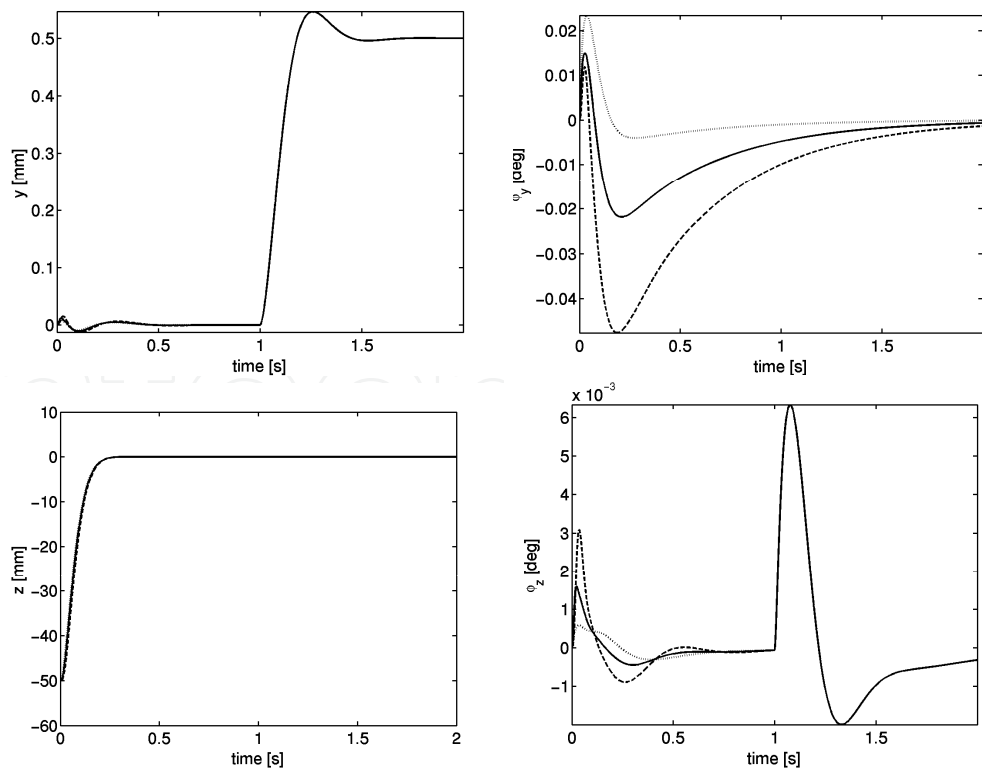
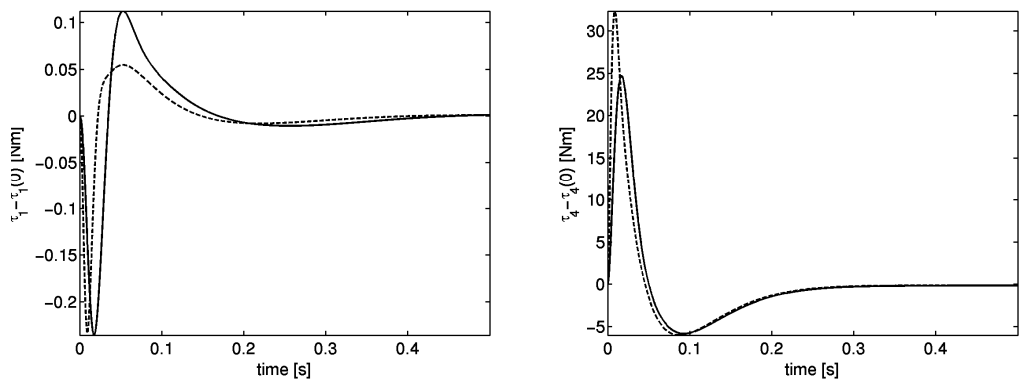


Figure 2. Simulation results for the two step responses (in the virtual equilibrium position and in the external force). The dotted lines show the simulation results of the desired impedance (23). The dashed and solid lines show the results for the decoupling based impedance controller with low and high gains, respectively

thus the closed-loop behaviors of the two controllers correspond exactly to the desired behavior for this excitation, see Fig. 2. The difference between the two controllers is also shown in Fig. 3, where a comparison of the resulting joint torques is given.

In the presented simulation study it was assumed that all the state variables, as well as the external forces, are available for the controller. In case of a typical industrial robot usually only the motor position can be measured, and the motor velocity can be computed from this via numerical differentiation. All the other state variables, including the link acceleration and the jerk, must be estimated. Some advanced modern robot arms, like for instance the DLR lightweight robots, are instead also equipped with joint torque sensors. Apart from the implementation of the inner torque feedback loop, these sensors can also be used for a more reliable estimation of the link acceleration and the jerk. Even more promising would be the use of acceleration sensors, either of a six-dof Cartesian sensor or of individual joint acceleration sensors.



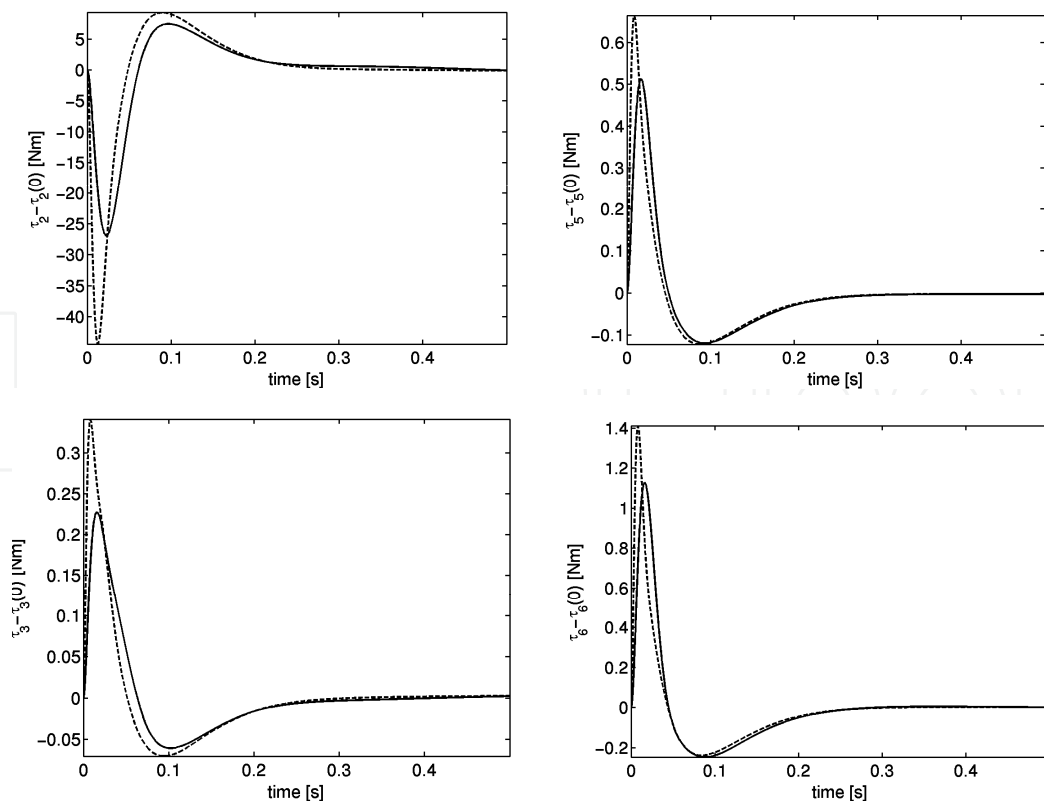


Figure 3. Deviation of the joint torques from their initial values for the decoupling based controllers. The dashed and solid lines show the results for the controller with high and low gains, respectively

## 7. Summary

In this work a Cartesian impedance controller for flexible joint robots was proposed. The control approach was based on the stability theory for cascaded systems. In the proposed controller an inner torque feedback loop decouples the torque dynamics from the rigid body dynamics. For the implementation of the impedance behaviour a control law well known from rigid body robotics is used in combination with the torque controller.

It should also be mentioned that, apart from the considered impedance control problem, different rigid body controllers can be applied to the flexible joint model analogously to the procedure described herein. For the proof of the asymptotic stability of the closed-loop system one can take advantage of Theorem 2.

## Appendix

In this appendix, a short lemma about uniform global boundedness of the solutions of time-varying differential equations is presented, which is used in Section 5.3 for the proof of uniform global asymptotic stability of the Cartesian impedance controller.

Consider a time-varying system of the form

$$\dot{x} = f(x, t) , \quad (A.1)$$

with state  $x \in \mathbb{R}^n$ . In order to show that the solutions of (A.1) are uniformly globally bounded (according to Definition A.1, which is taken from (Loria, 2001)), the following Lemma A.1 is usefull. This lemma can be proven easily based on the theorems presented in (Khalil, 2002).

**Definition A.1.** The solution  $x(t, t_0, x_0)$  of (A.1), with initial state  $x_0$  and initial time  $t_0$ , is said to be uniformly globally bounded if there exists a class  $K_\infty$  function  $\alpha$  and a number  $c > 0$  such that  $\|x(t, t_0, x_0)\| \leq \alpha(\|x_0\|) + c$  holds  $\forall t \geq t_0$ .

**Lemma A.1.** If there exists a continuously differentiable, positive definite, radially unbounded, and decrescent function  $V(x, t)$ , for which the time derivative along the solutions of (A.1) satisfies  $\dot{V}(x, t) = \partial V(x, t) / \partial x \cdot f(x) + \partial V(x, t) / \partial t \leq 0$ , then the solutions of (A.1) are uniformly globally bounded.

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## **Cutting Edge Robotics**

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This book is the result of inspirations and contributions from many researchers worldwide. It presents a collection of wide range research results of robotics scientific community. Various aspects of current research in robotics area are explored and discussed. The book begins with researches in robot modelling & design, in which different approaches in kinematical, dynamical and other design issues of mobile robots are discussed. Second chapter deals with various sensor systems, but the major part of the chapter is devoted to robotic vision systems. Chapter III is devoted to robot navigation and presents different navigation architectures. The chapter IV is devoted to research on adaptive and learning systems in mobile robots area. The chapter V speaks about different application areas of multi-robot systems. Other emerging field is discussed in chapter VI - the human- robot interaction. Chapter VII gives a great tutorial on legged robot systems and one research overview on design of a humanoid robot. The different examples of service robots are showed in chapter VIII. Chapter IX is oriented to industrial robots, i.e. robot manipulators. Different mechatronic systems oriented on robotics are explored in the last chapter of the book.

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