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# A Robust State Feedback Adaptive Controller with Improved Transient Tracking Error Bounds for Plants with Unknown Varying Control Gain

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## 1. Introduction

The design of robust model reference adaptive control (MRAC) schemes for plants in controllable form, comprising unknown varying but bounded coefficients and varying control gain has attracted a great deal of research. Many nonlinear systems may be described by the controllable form; for instance, second order plants (see (Hong & Yao, 2007), (Hsu et al., 2006), (Yao & Tomizuka, 1994), (Jiang & Hill, 1999)) and systems whose nonlinear behavior or part of it, is represented by some function approximation technique (cf. Nakanishi et al. (2005), (Chen et al., 2008), (Tong et al., 2000), (Huang & Kuo, 2001), (Yousef & Wahba, 2009), (Hsu et al., 2006), (Koo, 2001), (Labiod & Guerra, 2007)).

The state adaptive backstepping (SAB) of (Kanellakopoulos et al., 1991) is a common framework for the design of adaptive controllers for plants in controllable form. As is well known, a major difficulty in introducing robustness techniques to SAB based schemes is that the states  $z_i$  and the stabilizing functions must be differentiable to certain extent (see (Yao & Tomizuka, 1997), (Yao, 1997), (Ge & Wang, 2003)).

The robust SAB scheme of (Zhou et al., 2004), (Su et al., 2009), (Feng, Hong, Chen & Su, 2008) has the advantage that the knowledge on the upper or lower bounds of the plant coefficients can be relaxed if the controller is properly designed and the control gain is constant or known. The approach is based on the truncation method of (Slotine & Li, 1991), pp. 309. The stabilizing functions are smoothed at each  $i$ -th step in order to render it differentiable enough. The following benefits are obtained: i) the scheme is robust with respect to unknown varying but bounded coefficients, ii) upper or lower bounds of the plant coefficients are not required to be known, and iii) the tracking error converges to a residual set whose size is user-defined.

The specific case of unknown varying control gain is an important issue, more difficult to handle than other unknown varying coefficients. The varying control gain is usually handled by means of robustness techniques (cf. (Wang et al., 2004), (Huang & Kuo, 2001), (Bechlioulis

& Rovithakis, 2009), (Li et al., 2004)) or the Nussbaum gain technique (cf. (Su et al., 2009), (Feng, Hong, Chen & Su, 2008), (Feng et al., 2006), (Ge & Wang, 2003)). The above methods are applicable to plants in parametric–pure feedback or controllable form, and with controllers that use the SAB or the MRAC as the control framework.

In (Wang et al., 2004), a system with dead zone in the actuator is considered, assuming that both dead zone slopes have the same value. The input term is rewritten as the sum of an input term with constant control gain plus a bounded disturbance-like term. The disturbance term is rejected by means of a robust technique, based on (Slotine & Li, 1991) pp. 309. Nevertheless, this strategy is not valid for different values of the slopes. Other robustness techniques comprise a control law with compensating terms and either a projection modification of the update law, as in cf. (Huang & Kuo, 2001), or a  $\sigma$  modification as in (Bechlioulis & Rovithakis, 2009), (Li et al., 2004). Nevertheless, some lower or upper bounds of the plant coefficients are required to be known.

The Nussbaum gain technique can relax this requirement, as can be noticed from (Su et al., 2009), (Feng, Hong, Chen & Su, 2008), (Feng, Su & Hong, 2008). The main drawbacks of the Nussbaum gain method are (see (Su et al., 2009), (Feng, Hong, Chen & Su, 2008), (Feng et al., 2006), (Ge & Wang, 2003), (Feng et al., 2007), (Feng, Su & Hong, 2008), (Ren et al., 2008), (Zhang & Ge, 2009), (Du et al., 2010)): i) the upper bound of the transient behavior of the tracking error is significantly modified in comparison with that of the disturbance-free case: the value of this bound depends on the time integral of terms that comprise Nussbaum terms, and ii) the controller involves an additional state, which is necessary to compute the Nussbaum function.

Other drawbacks are: i) the control gain is assumed to be the product of a unknown constant and a known function, as in (Tong et al., 2010), (Liu & Tong, 2010), ii) the control gain is assumed upper bounded by some unknown constant, as in (Zhang & Ge, 2009), (Du et al., 2010), (Su et al., 2009), (Feng, Hong, Chen & Su, 2008), (Feng et al., 2006), (Ge & Wang, 2003), (Feng et al., 2007), (Feng, Su & Hong, 2008), (Ren et al., 2008), iii) the control gain is assumed upper bounded by a known function, as in (Ge & Tee, 2007), (Psillakis, 2010), iv) upper or lower bounds of the plant coefficients are required to be known to achieve asymptotic convergence of the tracking error to a residual of user-defined size, as in (Ge & Tee, 2007), (Chen et al., 2009), (Feng et al., 2006), (Ge & Wang, 2003), (Feng et al., 2007), (Ren et al., 2008), (Ge & Tee, 2007), (Tong et al., 2010), (Liu & Tong, 2010), iv) the control or update law involves signum type signals, as in (Zhang & Ge, 2009), (Du et al., 2010), (Psillakis, 2010), (Su et al., 2009), (Feng, Hong, Chen & Su, 2008), (Feng, Su & Hong, 2008).

Recent adaptive control schemes based on the direct Lyapunov method achieve improved transient performance. For instance,  $L_1$  adaptive control, with the drawback that the control gain is assumed constant, as in (Cao & Hovakimyan, 2006), (Cao & Hovakimyan, 2008a), (Cao & Hovakimyan, 2008b), (Dobrokhodov et al., 2008), (Li & Hovakimyan, 2008).

Other works have the following drawbacks:

- i) The control gain is assumed constant, as in (Zhou et al., 2009), (Wen et al., 2009), (Bashash & Jalili, 2009).
- ii) The control gain is assumed upper bounded by some unknown constant, as in (Chen, 2009), (Ho et al., 2009) and (Park et al., 2009).

iii) The control gain is assumed upper bounded by some known function as in (Bechlioulis & Rovithakis, 2009).

iv) Upper or lower bounds of plant parameters are required to be known to achieve asymptotic convergence of the tracking error to a residual set of user-defined size, as in (Bashash & Jalili, 2009), (Chen, 2009), (Ho et al., 2009), (Park et al., 2009) and (Bechlioulis & Rovithakis, 2009).

In this chapter, we develop a controller that overcomes the above drawbacks, so that:

Bi) The upper bound of tracking error transient value does not depend on time integral terms.

Bii) Additional states are not used in the controller.

Biii) The control gain is not required to be upper bounded by a constant.

Biv) The control gain is not required to be bounded by a known function.

Bv) Upper or lower bounds of the plant parameters are not required to be known.

Bvi) The control and update laws do not involve signum type signals.

Bvii) The tracking error converges to a residual set whose size is user-defined.

We consider systems described by the controllable form model with arbitrary relative degree, unknown varying but bounded coefficients and varying control gain. We use the SAB of (Kanellakopoulos et al., 1991) as a basic framework for the control design, preserving a simple definition of the states resulting from the backstepping procedure. We use the Lyapunov-like function method to handle the unknown time varying behavior of the plant parameters. All closed loop signals remain bounded so that parameter drifting is prevented.

The key elements to handle the varying behavior of the control gain are: i) introduce the control gain in the term involving the adjusted parameter vector, by means of the inequality that relates the control gain and its lower bound, and ii) apply the Young's inequality.

In current works that deal with plants in controllable form and time varying parameters and use the state transformation based on the backstepping procedure, they modify the defined states at each step of the state transformation in order to tackle the unknown time varying behavior of the plant parameters. Instead of altering the state transformation, we formulate a Lyapunov-like function, such that its magnitude and time derivative vanish when the states resulting from the state transformation reach a target region.

The control design and proof of boundedness and convergence properties are simpler in comparison to current works that use the Nussbaum gain method. The controller is also simpler as it does not introduce additional states that would be necessary to handle the unknown time varying control gain.

The chapter is organized as follows. In section 2 we detail the plant model. In section 3 we present the goal of the control design. In section 4 we carry out a state transformation, based on the state backstepping procedure. In section 5 we derive the control and update laws. In section 6 we prove the boundedness of the closed loop signals. In section 7 we prove the convergence of the tracking error  $e$ , finally, in section 8 we present an example.

## 2. Problem statement

In this section we detail the plant and the reference model. Consider the following plant in controllable form:

$$y^{(n)} = \gamma_n^\top a + bu + d \quad (1)$$

where  $y(t) \in \mathbb{R}$  is the system output,  $u(t) \in \mathbb{R}$  the input,  $a$  a vector of varying entries,  $\gamma_n$  a known vector,  $b$  the control gain, and  $d$  a disturbance-like term. We make the following assumptions:

Ai) The vector  $a$  involves unknown, time varying, bounded entries  $a_1, \dots, a_j$ , which satisfy:  $|a_1| \leq \bar{\mu}_1, \dots, |a_j| \leq \bar{\mu}_j$ , where  $\bar{\mu}_1, \dots, \bar{\mu}_j$  are unknown, positive constants.

Aii) The entries of the vector  $\gamma_n$  are known linear or nonlinear functions of  $y, \dots, y^{(n-1)}$ .

Aiii) The terms  $y, \dot{y}, \dots, y^{(n-1)}$  are available for measurement.

Aiv) The term  $d$  represents either external disturbances or unknown model terms that satisfy:

$$|d| \leq \mu_d f_d, \quad (2)$$

where  $\mu_d$  is an unknown positive constant, and  $f_d$  is a known function that depends on  $y, \dots, y^{(n-1)}$ . In the case that  $d$  is bounded, we have  $f_d = 1$ . The term  $d$  may come from the product of a known function  $g_d$  with an unknown varying but bounded coefficient  $c_g$ :  $d = c_g g_d$ ,  $|c_g| \leq \mu_d$ , so that  $f_d = |g_d|$ , where  $\mu_d$  is an unknown positive constant whereas  $g_d$  is a known function.

Av) The control gain  $b$  satisfies:

$$|b| \geq b_m > 0, \quad b \neq 0 \quad \forall t \geq t_0 \quad (3)$$

where  $b_m$  is an unknown lower bound, and the value of the signum of  $b$  is constant and known.

*Remark.* We recall that  $\mu_d, b_m, \bar{\mu}_1, \dots, \bar{\mu}_j$  are unknown constants. In contrast, the values of  $y, \dots, y^{(n-1)}, \gamma_n, f_d, \text{sgn}(b)$  are required to be known. Notice in assumption Av that we do not require the control gain  $b$  to be upper bounded by any constant. That is a major contribution with respect to current works that use the Nussbaum gain method, e.g (Su et al., 2009), (Feng, Hong, Chen & Su, 2008), (Feng, Su & Hong, 2008), (Feng et al., 2007), (Ge & Wang, 2003). The requirement about the value of the signum of  $b$  is a common and acceptable requirement.

## 3. Control goal

Let

$$e(t) = y(t) - y_d(t) \quad (4)$$

$$y_d^{(n)} + a_{m,n-1}y_d^{(n-1)} + \dots + a_{m,0}y_d = a_{m,0}r \quad (5)$$

$$\Omega_e = \{e : |e| \leq C_{be}\} \quad (6)$$

where  $e(t)$  is the tracking error,  $y_d(t)$  is the desired output,  $\Omega_e$  is a residual set,  $r$  is the reference signal. Moreover,  $a_{m,n-1}, \dots, a_{m,0}$  are constant coefficients defined by the user, such that the polynomial  $K(p)$  is Hurwitz, being  $K(p)$  defined as  $K(p) = p^n + a_{m,n-1}p^{n-1} + \dots + a_{m,0}$ . The reference signal  $r(t)$  is bounded and user-defined. The constant  $C_{be}$  is positive and user-defined.

The objective of the MRAC design is to formulate a controller, provided by the plant model (1) subject to assumptions Ai to Av, such that:

- i) The tracking error  $e$  converges asymptotically to the residual set  $\Omega_e$ .
- ii) The control signals are bounded and do not involve discontinuous signals.

#### 4. State transformation based on the state backstepping

In this section we carry out a state transformation by following the steps  $0, \dots, n$  of the backstepping procedure. The plant model (1) can be rewritten as follows:

$$\dot{x}_i = x_{i+1}, \quad 1 \leq i \leq n-1 \quad (7)$$

$$\dot{x}_n = a^\top \gamma_n(x_1, \dots, x_n) + bu + d \quad (8)$$

$$x_1 = y, \quad x_2 = \dot{y}, \quad \dots, \quad x_n = y^{(n-1)}$$

The model (7, 8) can be obtained by making  $\gamma_1 = \dots = \gamma_{n-1} = 0$  in the parametric - pure feedback form of (Kanellakopoulos et al., 1991). We use the SAB of (Kanellakopoulos et al., 1991) as the basic framework for the formulation of the control and update laws.

We develop the SAB for the plant model (7, 8), and introduce a new robustness technique. Since the order of the plant is  $n$ , the procedure comprises the steps  $0, \dots, n$ , to be carried out in a sequential manner.

*Step 0.* We begin by defining the state  $z_1$  as the tracking error:

$$z_1 = e = y - y_d = x_1 - y_d \quad (9)$$

*Step i* ( $1 \leq i \leq n-1$ ). At each  $i$ -th step, we obtain the dynamics of the state  $z_i$  by deriving it with respect to time, and using the definitions of  $\dot{x}_{i+1}$  provided by (7). For the sake of clarity, we develop the step 1 and then we state a generalization for ( $1 \leq i \leq n-1$ ).

For the case  $i = 1$ , we differentiate  $z_1$  defined in (9) and use the definition of  $\dot{x}_1$  provided by (7) with  $i = 1$ :

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_d = x_2 - \dot{y}_d = x_2 + \varphi_1 \quad (10)$$

$$\varphi_1 = \varphi_1(\dot{y}_d) = -\dot{y}_d$$

where  $\varphi_1$  is a known function of  $\dot{y}_d$ . Equation (10) can be rewritten as:

$$\dot{z}_1 = -c_1 z_1 + z_2 \quad (11)$$

$$z_2 = x_2 + c_1 z_1 - \dot{y}_d \quad (12)$$

where  $c_1 \geq 2$  is a positive constant of the user choice. The dynamic equation of  $z_2$  is obtained by differentiating it with respect to time. The same procedure must be followed until the step



$i = n - 1$ . To state a generalization, we express  $\dot{z}_i$  as:

$$\dot{z}_i = x_{i+1} + \varphi_i, \quad (13)$$

$$\varphi_i = \varphi_i(z_1, \dots, z_i, y_d, \dot{y}_d, \dots, y_d^{(i)}), \quad (14)$$

where  $\varphi_i$  is a known scalar term, that is function of  $z_1, \dots, z_i, y_d, \dot{y}_d, \dots, y_d^{(i)}$ . Equation (13) can be rewritten as:

$$\dot{z}_i = -c_i z_i + z_{i+1}, \quad (15)$$

$$z_{i+1} = x_{i+1} + \varphi_i + c_i z_i \quad (16)$$

where  $c_i \geq 2$  is a positive constant of the user choice. At the step  $i = n - 1$  we obtain the dynamic equation for  $z_{n-1}$  and the expression for  $z_n$  as indicated by (15, 16).

**Remark.** Notice that the definition of the states  $z_i$  is similar to that of the disturbance free case, so that a simple design is preserved. This is due to the following facts:

- i) Disturbance like terms are absent in the dynamics  $\dot{x}_1, \dots, \dot{x}_{n-1}$  given by (7), so that they are also absent in the dynamics  $\dot{z}_1, \dots, \dot{z}_{n-1}$ , as can be noticed in (13).
- ii) Dead zone functions of the states  $z_i$  are not used.

*Step n.* We obtain the dynamics of  $z_n$  by differentiating it with respect to time and using the expression of  $\dot{x}_n$  provided by (8):

$$\dot{z}_n = bu + a^\top \gamma_n + \varphi_n + d \quad (17)$$

$$\varphi_n = \varphi_n(z_1, \dots, z_n, y_d, \dots, y_d^{(n)}) \quad (18)$$

where  $\varphi_n$  is a known scalar that is function of  $z_1, \dots, z_n, y_d, \dots, y_d^{(n)}$ . Notice that the disturbance like term  $d$  and the control input  $u$  appear explicitly at the dynamics of  $z_n$ , at the step  $n$  of the procedure. Thus, we have completed the state transformation, which allows us to develop the controller.

## 5. Control and update laws

In this section we develop the control and update laws, taking into account the assumptions stated in section 2 and the goals of section 3. The key elements of the procedure are:

- i) Incorporate the assumptions  $A_i$  and  $A_{iv}$ , concerning the unknown time varying parameter  $a_1, \dots, a_j$  and the disturbance like term  $d$ .
- ii) Carry out a linear parameterization.
- iii) Express the parameterization in terms of adjustment error and adjusted parameter vector.
- iv) Introduce the control gain  $b$  within the adjusted parameter vector.
- v) Formulate the control law.
- vi) Formulate a Lyapunov-like function and find its time derivative.

vii) Formulate the update law.

We begin by rewriting (17) as follows:

$$\dot{z}_n = -c_n z_n + bu + a^\top \gamma_n + \varphi_n + c_n z_n + d, \quad (19)$$

where  $c_n \geq 2$  is a positive constant of the user choice. Multiplying (19) by  $z_n$ , we obtain:

$$z_n \dot{z}_n = -c_n z_n^2 + bz_n u + z_n a^\top \gamma_n + z_n(\varphi_n + c_n z_n) + z_n d \quad (20)$$

The term  $z_n a^\top \gamma_n + z_n(\varphi_n + c_n z_n) + z_n d$  can be rewritten as follows:

$$\begin{aligned} z_n a^\top \gamma_n + z_n(\varphi_n + c_n z_n) + z_n d &= z_n(a^\top \gamma_n + d + \varphi_n + c_n z_n) \\ &= z_n(a_{[1]} \gamma_{n[1]} + \cdots + a_{[j]} \gamma_{n[j]} + d + \varphi_n + c_n z_n) \\ &\leq |z_n| \left( |a_{[1]} \gamma_{n[1]}| + \cdots + |a_{[j]} \gamma_{n[j]}| + |d| + |\varphi_n + c_n z_n| \right) \end{aligned} \quad (21)$$

using assumptions Ai and Aiv of section 2 and parameterizing, we obtain:

$$\begin{aligned} z_n a^\top \gamma_n + z_n(\varphi_n + c_n z_n) + z_n d &\leq |z_n| \left( \bar{\mu}_1 |\gamma_{n[1]}| + \cdots + \bar{\mu}_j |\gamma_{n[j]}| + \mu_d f_d + |\varphi_n + c_n z_n| \right) \\ &= \sqrt{b_{mn}} |z_n| \bar{\varphi}^\top \theta \end{aligned} \quad (22)$$

where

$$\bar{\varphi} = \left[ |\gamma_{n[1]}|, \cdots, |\gamma_{n[j]}|, f_d, |\varphi_n + c_n z_n| \right]^\top \quad (23)$$

$$\theta = (1/\sqrt{b_{mn}}) \left[ \bar{\mu}_1, \cdots, \bar{\mu}_j, \mu_d, 1 \right]^\top \quad (24)$$

Notice that the entries of the vector  $\theta$  are unknown, positive, constant, because bounds of the time varying parameters  $a_i$  and  $d$  have been introduced, according to the properties in assumptions Ai and Aiv of section 2. Now, we express (22) in terms of adjustment error and adjusted parameter vector:

$$z_n a^\top \gamma_n + z_n(\varphi_n + c_n z_n) + z_n d \leq -\sqrt{b_{mn}} |z_n| \bar{\varphi}^\top \tilde{\theta} + \sqrt{b_{mn}} |z_n| \bar{\varphi}^\top \hat{\theta} \quad (25)$$

where

$$\begin{aligned} \tilde{\theta} &= \hat{\theta} - \theta \\ &= \hat{\theta} - \frac{1}{\sqrt{b_{mn}}} \left[ \bar{\mu}_1, \cdots, \bar{\mu}_j, \mu_d, 1 \right]^\top \end{aligned} \quad (26)$$

being  $\hat{\theta}$  an adjusted parameter vector and  $\tilde{\theta}$  an adjustment error. Using the property (3) in the term  $\sqrt{b_{mn}} |z_n| \bar{\varphi}^\top \hat{\theta}$  of (25), we obtain:

$$\sqrt{b_{mn}} |z_n| |\bar{\varphi}^\top \hat{\theta}| \leq \sqrt{\frac{3C_{bvz}}{2}} \sqrt{\frac{2}{3C_{bvz}}} \sqrt{|b|} |z_n| |\bar{\varphi}^\top \hat{\theta}| \quad (27)$$

$$\text{where } C_{bvz} = (1/2)C_{be}^2 \quad (28)$$



using the Young's inequality (cf. (Royden, 1988) pp. 123), we obtain:

$$\sqrt{b_{mn}}|z_n||\bar{\varphi}^\top \hat{\theta}| \leq \frac{3}{4}C_{bvz} + \frac{1}{3C_{bvz}}|b|z_n^2(\bar{\varphi}^\top \hat{\theta})^2 \quad (29)$$

Substituting (29) into (25), we obtain:

$$\begin{aligned} & z_n a^\top \gamma_n + z_n(\varphi_n + c_n z_n) + z_n d \\ & \leq -\sqrt{b_{mn}}|z_n|\bar{\varphi}^\top \tilde{\theta} + \frac{3}{4}C_{bvz} + \frac{1}{3C_{bvz}}|b|z_n^2(\bar{\varphi}^\top \hat{\theta})^2 \end{aligned} \quad (30)$$

**Remark.** We have proposed a new method to handle the unknown varying behavior of the control gain  $b$ , alternative to the current Nussbaum gain method. We parameterized the model term  $z_n a^\top \gamma_n + z_n(\varphi_n + c_n z_n) + z_n d$  in terms of adjustment error  $\tilde{\theta}$  and adjusted parameter vector  $\hat{\theta}$ , and developed the following steps:

- i) Introduce the constant  $\sqrt{b_{mn}}$  in the parameterization, see (22).
- ii) Introduce the inequality  $\sqrt{b_{mn}} \leq \sqrt{|b|}$ , see (27).
- iii) Apply the Young's inequality to obtain  $b$ , see (29).

Recall that the value of  $b_{mn}$  is not required to be known.

Substituting (30) into (20), we obtain:

$$\begin{aligned} z_n \dot{z}_n & \leq -c_n z_n^2 + (3/4)C_{bvz} + b z_n \left( u + \frac{1}{3C_{bvz}} \text{sgn}(b) z_n (\bar{\varphi}^\top \hat{\theta})^2 \right) \\ & \quad - \sqrt{b_{mn}}|z_n|\bar{\varphi}^\top \tilde{\theta} \end{aligned} \quad (31)$$

we choose the following control law:

$$u = -\frac{1}{3C_{bvz}} \text{sgn}(b) z_n (\bar{\varphi}^\top \hat{\theta})^2 \quad (32)$$

where  $\bar{\varphi}$ ,  $z_n$  are defined in (23), (16), respectively. Substituting (32) into (31), we obtain:

$$z_n \dot{z}_n \leq -c_n z_n^2 + (3/4)C_{bvz} - \sqrt{b_{mn}}|z_n|\bar{\varphi}^\top \tilde{\theta} \quad (33)$$

To handle the effect of the constant  $(3/4)C_{bvz}$ , we formulate the following Lyapunov-like function:

$$\bar{V}_z = \begin{cases} (1/2)(\sqrt{V_z} - \sqrt{C_{bvz}})^2 & \text{if } V_z \geq C_{bvz} \\ 0 & \text{otherwise} \end{cases} \quad (34)$$

$$V_z = (1/2)(z_1^2 + \cdots + z_n^2) \quad (35)$$

where  $C_{bvz}$  is defined in (28). We need the following properties:

**Proposition 5.1.** *The function  $\bar{V}_z$  defined in (34) has the following properties:*

$$i) \bar{V}_z \geq 0 \quad (36)$$

$$ii) V_z \leq 3C_{bvz} + 3\bar{V}_z \quad (37)$$

$$iii) \bar{V}_z, (\partial \bar{V}_z / \partial V_z) \text{ are continuous with respect to } V_z \quad (38)$$

*Proof.* From (34) it follows that  $\bar{V}_z \geq 0 \forall t \geq t_0$ , the property i of proposition 5.1. In addition, from (34) it follows that

$$V_z \leq (\sqrt{2\bar{V}_z} + \sqrt{C_{bvz}})^2 \quad (39)$$

Applying the Young's inequality (cf. (Royden, 1988) pp. 123), we obtain:

$$V_z = C_{bvz} + 2\sqrt{C_{bvz}}\sqrt{2\bar{V}_z} + 2\bar{V}_z \leq 3C_{bvz} + 3\bar{V}_z \quad (40)$$

This completes the proof of property ii. From (42) it follows that  $\partial \bar{V}_z / \partial V_z = 0$  if  $V_z = C_{bvz}$  and that  $\partial \bar{V}_z / \partial V_z$  is continuous. From (34) it follows that  $\bar{V}_z$  is continuous. This completes the proof of property iii of proposition 5.1.

□

Differentiating (34) with respect to time, we obtain:

$$\frac{d\bar{V}_z}{dt} = \frac{\partial \bar{V}_z}{\partial V_z} \dot{V}_z \quad (41)$$

$$\frac{\partial \bar{V}_z}{\partial V_z} = \begin{cases} (1/2)(1/\sqrt{V_z})(\sqrt{V_z} - \sqrt{C_{bvz}}) & \text{if } V_z \geq C_{bvz} \\ 0 & \text{otherwise} \end{cases} \quad (42)$$

To compute  $\dot{V}_z$ , we differentiate (35) with respect to time:  $\dot{V}_z = z_1\dot{z}_1 + \dots + z_n\dot{z}_n$ . Introducing (11) and (15), we obtain

$$\dot{V}_z = -c_1z_1^2 + z_1z_2 - c_2z_2^2 + z_2z_3 + \dots + z_n\dot{z}_n \quad (43)$$

substituting (33), we obtain:

$$\begin{aligned} \dot{V}_z \leq & -c_1z_1^2 + z_1z_2 - c_2z_2^2 + z_2z_3 + \dots - c_nz_n^2 \\ & + (3/4)C_{bvz} - \sqrt{b_{mn}}|z_n|\bar{\varphi}^\top \tilde{\theta} \end{aligned} \quad (44)$$

Provided that  $c_1 \geq 2, c_2 \geq 2, \dots, c_n \geq 2$  and completing the squares yields:

$$\begin{aligned} & -c_1z_1^2 + z_1z_2 - c_2z_2^2 + z_2z_3 \dots - c_nz_n^2 \\ & \leq -z_1^2 - (3/4)z_2^2 + \dots - (3/4)z_n^2 \leq -(3/2)V_z \end{aligned}$$

substituting into (44), we obtain:

$$\dot{V}_z \leq -(3/2)V_z + (3/4)C_{bvz} - \sqrt{b_{mn}}|z_n|\bar{\varphi}^\top \tilde{\theta} \quad (45)$$

Since  $\partial \bar{V}_z / \partial V_z$  is non-negative, we can multiply it into (45) without changing the direction of the inequality:

$$\frac{\partial \bar{V}_z}{\partial V_z} \dot{V}_z \leq -(3/2)V_z \frac{\partial \bar{V}_z}{\partial V_z} + (3/4)C_{bvz} \frac{\partial \bar{V}_z}{\partial V_z} - \sqrt{b_{mn}}|z_n| \bar{\varphi}^\top \bar{\theta} \frac{\partial \bar{V}_z}{\partial V_z} \quad (46)$$

Substituting into (41), we obtain:

$$\frac{d\bar{V}_z}{dt} \leq -(3/2)V_z \frac{\partial \bar{V}_z}{\partial V_z} + (3/4)C_{bvz} \frac{\partial \bar{V}_z}{\partial V_z} - \sqrt{b_{mn}}|z_n| \bar{\varphi}^\top \bar{\theta} \frac{\partial \bar{V}_z}{\partial V_z} \quad (47)$$

we choose the update law so as to reject the effect of the term involving the adjustment error  $\bar{\theta}$ :

$$\dot{\hat{\theta}} = \Gamma \bar{\varphi} |z_n| \frac{\partial \bar{V}_z}{\partial V_z} \quad (48)$$

where  $\Gamma$  is a diagonal matrix whose diagonal elements are positive constants defined by the user, whereas  $\bar{\varphi}$ ,  $z_n$ ,  $\partial \bar{V}_z / \partial V_z$  are defined in (23), (16), (42), respectively.

**Remark.** So far, we have developed the controller, which involves the control law (32) and the update law (48). Other parameters necessary for its implementation are:  $V_z$  defined in (35);  $z_1, z_2, \dots, z_n$  defined in (9), (12),  $\dots$ , (16), respectively;  $C_{bvz}$  defined in (28). Recall that  $c_1 \geq 2, \dots, c_n \geq 2$  are user-defined positive constants.

**Remark.** The control and update laws stated in (32) and (48) have the following features:

- i) The control law uses the adjusted parameter vector  $\hat{\theta}$ , so that it does not rely on upper or lower bounds of the plant coefficients, i.e.  $\bar{\mu}_1, \dots, \bar{\mu}_j, \mu_d, b_{mn}$ , and excessive control effort is also avoided.
- ii) Additional states are not required to handle the unknown varying behavior of the control gain, what is an important benefit with respect to closely related schemes that use the Nussbaum gain method.
- iii) The control and update laws do not involve discontinuous signals. In fact, the vectorial field of the closed loop system is Lipschitz continuous, so that trajectory unicity is preserved.

## 6. Boundedness analysis

In this section we prove that the closed loop signals  $z_1, \dots, z_n, \hat{\theta}, u$  are bounded if the developed controller is applied.

**Theorem 6.1. Boundedness of the closed loop signals.** Consider the plant (1) subject to assumptions A<sub>i</sub> to A<sub>v</sub>; the signals  $z_1, \dots, z_n$  defined in (9), (12) and (16); the signals  $\bar{\varphi}$ ,  $V_z$ ,  $\partial \bar{V}_z / \partial V_z$ ,  $C_{bvz}$  defined in (23), (35), (42) and (28), respectively. If the controller (32), (48) is applied, then the signals  $z_1, \dots, z_n, \hat{\theta}$ , and  $u$  remain bounded and  $|e|$  is upper bounded as follows:

$$|e| \leq \sqrt{2} \left( \sqrt{C_{bvz}} + \sqrt{2V(\bar{x}(t_0))} \right)^2 \quad (49)$$

*Proof.* We choose the following Lyapunov-like function:

$$V(\bar{x}(t)) = \bar{V}_z + V_\theta \quad (50)$$

$$V_\theta = (1/2)\sqrt{b_{mn}}\tilde{\theta}^\top \Gamma^{-1}\tilde{\theta} \quad (51)$$

$$\bar{x}(t) = [z_1, \dots, z_n, \tilde{\theta}^\top]^\top \quad (52)$$

where  $\bar{V}_z$  is defined in (34) and  $\tilde{\theta}$  in (26). The time derivative of  $V_\theta$  is:

$$\dot{V}_\theta = (1/2)\sqrt{b_{mn}}(\dot{\tilde{\theta}}^\top \Gamma^{-1}\tilde{\theta} + \tilde{\theta}^\top \Gamma^{-1}\dot{\tilde{\theta}}) \quad (53)$$

Since  $\Gamma$  is diagonal, then  $\Gamma^{-1}$  is diagonal,  $(\Gamma^{-1})^\top = \Gamma^{-1}$ ,  $\dot{\tilde{\theta}}^\top \Gamma^{-1}\tilde{\theta} = \tilde{\theta}^\top \Gamma^{-1}\dot{\tilde{\theta}}$ . In view of this and the update law (48), we have:

$$\dot{V}_\theta = \sqrt{b_{mn}}\tilde{\theta}^\top \Gamma^{-1}\dot{\tilde{\theta}} = \sqrt{b_{mn}}\tilde{\theta}^\top \bar{\varphi}|z_n| \frac{\partial \bar{V}_z}{\partial V_z} \quad (54)$$

Differentiating (50) with respect to time, we obtain:  $\dot{V} = \dot{\bar{V}}_z + \dot{V}_\theta$ . Substituting equations (47) and (54), we obtain:

$$\begin{aligned} \dot{V} &\leq -(3/2)V_z \frac{\partial \bar{V}_z}{\partial V_z} + (3/4)C_{bvz} \frac{\partial \bar{V}_z}{\partial V_z} \\ &= -\frac{3}{2} \frac{\partial \bar{V}_z}{\partial V_z} \left( \frac{V_z}{2} + \frac{V_z}{2} - \frac{C_{bvz}}{2} \right) \end{aligned} \quad (55)$$

From (42) it follows that

$$\partial \bar{V}_z / \partial V_z = 0 \text{ for } V_z \leq C_{bvz} \quad (56)$$

$$\partial \bar{V}_z / \partial V_z > 0 \text{ for } V_z > C_{bvz}. \quad (57)$$

In view of this and (55), we obtain:

$$\begin{aligned} \dot{V} &\leq -\frac{3}{2} \frac{\partial \bar{V}_z}{\partial V_z} \left( \frac{V_z}{2} \right) \text{ if } V_z \geq C_{bvz} \\ \dot{V} &\leq 0 = -\frac{3}{2} \frac{\partial \bar{V}_z}{\partial V_z} \left( \frac{V_z}{2} \right) \text{ if } V_z < C_{bvz} \\ \Rightarrow \dot{V} &\leq -\frac{3}{4} \frac{\partial \bar{V}_z}{\partial V_z} V_z \leq 0 \end{aligned} \quad (58)$$

Thus,  $\dot{V} + 0V \leq 0$ . Using the Lemma in (Slotine & Li, 1991) pp. 91, we obtain:

$$V(\bar{x}(t)) \leq V(\bar{x}(t_0)) \exp(-0t) = V(\bar{x}(t_0)) \quad (59)$$

where

$$V(\bar{x}(t_0)) = \bar{V}_{z0} + V_{\theta0} \quad (60)$$

$$\bar{V}_{z0} = \begin{cases} (1/2)(\sqrt{V_{z0}} - \sqrt{C_{bvz}})^2 & \text{if } V_{z0} \geq C_{bvz} \\ 0 & \text{otherwise} \end{cases} \quad (61)$$

$$V_{z0} = (1/2)(z_1(t_0)^2 + \dots + z_n(t_0)^2) \quad (62)$$

$$V_{\theta0} = (1/2)\sqrt{b_{mn}}(\hat{\theta}(t_0) - \theta)^\top \Gamma^{-1}(\hat{\theta}(t_0) - \theta) \quad (63)$$

Since  $V(\bar{x}(t)) \geq 0$ , we have:  $0 \leq V(\bar{x}(t)) \leq V(\bar{x}(t_0))$ . Introducing the definition (50), we obtain

$$\bar{V}_z + V_\theta \leq V(\bar{x}(t_0)) \quad (64)$$

$$\Rightarrow \bar{V}_z \leq V(\bar{x}(t_0)), \quad V_\theta \leq V(\bar{x}(t_0)) \quad (65)$$

Thus, it follows from (51) that  $\tilde{\theta} \in L_\infty$ , and consequently  $\hat{\theta} \in L_\infty$ . The inequality  $\bar{V}_z \leq V(\bar{x}(t_0))$  implies that the tracking error  $e$  is bounded, as we show hereafter. We begin by solving (34) for  $V_z$ :

$$V_z = \left( \sqrt{C_{bvz}} + \sqrt{2\bar{V}_z} \right)^2 \text{ if } \bar{V}_z > 0 \quad (66)$$

$$V_z \leq C_{bvz} \text{ if } \bar{V}_z = 0 \quad (67)$$

Using the inequality  $\bar{V}_z \leq V(\bar{x}(t_0))$ , we obtain:

$$V_z \leq \left( \sqrt{C_{bvz}} + \sqrt{2V(\bar{x}(t_0))} \right)^2 \text{ if } \bar{V}_z > 0 \quad (68)$$

$$V_z \leq C_{bvz} \leq \left( \sqrt{C_{bvz}} + \sqrt{2V(\bar{x}(t_0))} \right)^2 \text{ if } \bar{V}_z = 0 \quad (69)$$

combining both inequalities, we obtain:

$$V_z \leq \left( \sqrt{C_{bvz}} + \sqrt{2V(\bar{x}(t_0))} \right)^2 \quad (70)$$

Introducing the definition (35), we obtain:

$$\sqrt{z_1^2 + \cdots + z_n^2} \leq \sqrt{2} \left( \sqrt{C_{bvz}} + \sqrt{2V(\bar{x}(t_0))} \right)^2 \quad (71)$$

so that  $z_1 \in L_\infty, \dots, z_n \in L_\infty$ . Since  $e^2 = z_1^2 \leq z_1^2 + \cdots + z_n^2$ , we obtain:

$$|e| \leq \sqrt{2} \left( \sqrt{C_{bvz}} + \sqrt{2V(\bar{x}(t_0))} \right)^2 \quad (72)$$

which indicates the upper bound for the tracking error  $e$ .

**Remark.** Notice that this upper bound does not involve integral terms, what is an important advantage with respect to the Nussbaum Gain method, see (Su et al., 2009), (Feng, Hong, Chen & Su, 2008), (Feng et al., 2006), (Ge & Wang, 2003), (Feng et al., 2007), (Feng, Su & Hong, 2008), (Ren et al., 2008).

Now, we proceed to show the boundedness of  $u$ . From (9), (12), (16) it follows that  $x_1 \in L_\infty$ ,  $x_2 \in L_\infty, \dots, x_n \in L_\infty$ . Therefore,  $\gamma_n \in L_\infty$ . It follows from (23) that  $\bar{\varphi} \in L_\infty$ . From (32) it follows that  $u \in L_\infty$ . This completes the proof.

□

## 7. Convergence analysis

In this section we prove that if the developed controller is applied, then the signal  $z_1$  converges asymptotically to  $\Omega_z$ , where  $\Omega_z = \{z_1 : |z_1| \leq C_{be}\}$ .

**Theorem 7.1. Convergence of the tracking error.** Consider the plant (1) subject to assumptions  $A_i$  to  $A_v$ ; the signals  $z_1, \dots, z_n$  defined in (9), (12) and (16); the signals  $\bar{\varphi}$ ,  $V_z$ ,  $\partial \bar{V}_z / \partial V_z$ ,  $C_{bvz}$  defined in (23), (35), (42) and (28), respectively. If the controller (32), (48) is applied, then the signal  $z_1$  converges asymptotically to  $\Omega_z$ , where  $\Omega_z = \{z_1 : |z_1| \leq C_{be}\}$ .

*Proof.* In view of (42), equation (58) can be rewritten as:

$$\dot{V} \leq -f_d \leq 0 \quad (73)$$

$$f_d = \begin{cases} (3/8)(\sqrt{V_z})(\sqrt{V_z} - \sqrt{C_{bvz}}) & \text{if } V_z \geq C_{bvz} \\ 0 & \text{otherwise} \end{cases} \quad (74)$$

The derivative  $\partial f_d / \partial V_z$  is not continuous, as it involves an abrupt change at  $V = C_{bvz}$ . Thus, the Barbalat's Lemma can not be applied on  $f_d$ . To remedy that, we shall express (73) in terms of a function with continuous derivative:

$$\dot{V} \leq -f_d \leq -f_g \leq 0 \quad (75)$$

$$f_g = \begin{cases} (3/8)(\sqrt{V_z} - \sqrt{C_{bvz}})^2 & \text{if } V_z \geq C_{bvz} \\ 0 & \text{otherwise} \end{cases} \quad (76)$$

Arranging and integrating (75), we obtain:

$$\begin{aligned} f_g &\leq -\dot{V} \\ \int_{t_0}^t f_g d\tau &\leq V(\bar{x}(t_0)) - V(\bar{x}(t)) \\ V(\bar{x}(t)) + \int_{t_0}^t f_g d\tau &\leq V(\bar{x}(t_0)) \end{aligned} \quad (77)$$

Thus,  $f_g \in L_1$ . We have to prove that  $f_g \in L_\infty$ ,  $\dot{f}_g \in L_\infty$  to apply the Barbalat's Lemma. Since  $V_z \in L_\infty$ , it follows from (76) that  $f_g \in L_\infty$ . Differentiating (76) with respect to time, we obtain:

$$\dot{f}_g = \frac{\partial f_g}{\partial V_z} \dot{V}_z \quad (78)$$

$$\frac{\partial f_g}{\partial V_z} = \begin{cases} (3/8)(1/\sqrt{V_z})(\sqrt{V_z} - \sqrt{C_{bvz}}) & \text{if } V_z \geq C_{bvz} \\ 0 & \text{otherwise} \end{cases} \quad (79)$$

Notice that  $\partial f_g / \partial V_z$  is continuous. Since  $V_z \in L_\infty$ , then  $\partial f_g / \partial V_z \in L_\infty$ . Since  $z_1 \in L_\infty, \dots, z_n \in L_\infty$ , it follows from (11), (15) that  $\dot{z}_1 \in L_\infty, \dots, \dot{z}_{n-1} \in L_\infty$ . Since  $u \in L_\infty$ , it follows from (17) that  $\dot{z}_n \in L_\infty$ . Therefore, from (43) it follows that  $\dot{V}_z \in L_\infty$ .

So far we have proved that  $\partial f_g / \partial V_z \in L_\infty$  and  $\dot{V}_z \in L_\infty$ , so that it follows from (78) that  $\dot{f}_g \in L_\infty$ . In view of  $f_g \in L_\infty$ ,  $\dot{f}_g \in L_\infty$ , application of Barbalat's Lemma (cf. (Ioannou & Sun, 1996) pp. 76), then indicates that  $f_g$  converges asymptotically to zero. Hence, from (76) it follows that  $V_z$  converges to  $\Omega_{vz}$ , where  $\Omega_{vz} = \{V_z : V_z \leq C_{bvz}\}$ . From the definition (35),



it follows that  $z_1$  converges asymptotically to  $\Omega_z$ , where  $\Omega_z = \{z_1 : |z_1| \leq \sqrt{2C_{bvz}}\}$ . Since  $C_{bvz} = (1/2)C_{be}^2$ , it follows that  $\Omega_z = \{z_1 : |z_1| \leq C_{be}\}$ . This completes the proof.  $\square$

## 8. Simulation example

Consider the following case of the plant (1):

$$\ddot{y} = \gamma_2^\top a + bu + d \quad (80)$$

$$\gamma_2 = [\dot{y}, y]^\top, \quad a = [a_1, a_2]^\top \quad (81)$$

$$\begin{aligned} a_1 &= -2(1 + 0.1\sin(2(\pi/8)t)), \quad a_2 = -1(1 + 0.1\sin(2(\pi/5)t)) \\ b &= 2(1 + 0.1\sin((2\pi/11)t)) + 0.6|y| \\ d &= -0.2(1 + 0.1\sin((2\pi/7)t))y \end{aligned} \quad (82)$$

The aim is that  $y$  converges towards  $y_d$ , with a threshold of 0.1. In figure 1 we present a simulation block diagram for the example.

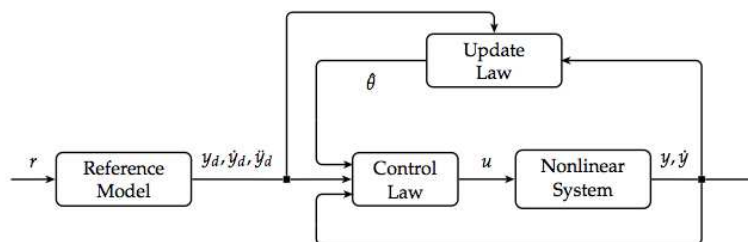


Fig. 1. Simulation block diagram.

The properties Ai, Aiv, Av of section 2 are analyzed at the following. From (82) it follows that  $a_1, d, b$  are bounded as:

$$|a_1| \leq 2(1.1) = 2.2, \quad |a_2| \leq 1(1.1) = 1.1 \quad (83)$$

$$|d| \leq 0.2(1.1)|y| = 0.22|y| \quad (84)$$

$$|b| \geq 2(0.9) = 1.8 > 0 \quad (85)$$

Hence, the upper bounds of  $a_1, a_2, d$ , and the lower bound of  $b$ , are:

$$|a_1| \leq \bar{\mu}_1 = 2.2, \quad |a_2| \leq \bar{\mu}_2 = 1.1, \quad |b| \geq b_{mn} = 1.8 \quad (86)$$

$$|d| \leq \mu_d f_d, \quad \mu_d = 0.22, \quad f_d = |y| \quad (87)$$

where  $f_d$  is not constant and known, whereas  $b_{mn}, \bar{\mu}_1, \bar{\mu}_2, \mu_d, b_{mn}$  are positive, constant and unknown to the controller. From (86), (87) it follows that assumptions Ai, Aiv, Av of section 2 are satisfied.

The procedure of section 4 is followed in order to establish the terms involved in the control and update laws, mentioned in remark 5. Eq. (80) can be rewritten as:

$$\dot{x}_1 = x_2 \quad (88)$$

$$\dot{x}_2 = \gamma_2^\top a + bu + d \quad (89)$$

$$x_1 = y, \quad x_2 = \dot{y}, \quad n = 2 \quad (90)$$

since  $n = 2$ , the state transformation based on the backstepping procedure involves the steps 0, 1, 2.

**Step 0.** Let

$$z_1 = e = y - y_d = x_1 - y_d \quad (91)$$

as in (9).

**Step 1.** Differentiating (91) with respect to time and arranging, yields:

$$\dot{z}_1 = -c_1 z_1 + z_2 \quad (92)$$

$$z_2 = x_2 + c_1 z_1 - \dot{y}_d \quad (93)$$

as in (11), (12).

**Step 2.** Since  $n = 2$ , the second step is the last one. Differentiating (93) with respect to time, using (89) and arranging, yields:

$$\begin{aligned} \dot{z}_2 &= \dot{x}_2 + c_1 \dot{z}_1 - \ddot{y}_d \\ &= \gamma_2^\top a + bu + d + c_1 \dot{z}_1 - \ddot{y}_d \\ &= \gamma_2^\top a + bu + d + c_1(x_2 + \varphi_1) - \ddot{y}_d \end{aligned} \quad (94)$$

using the definitions (91), (93), yields:

$$\dot{z}_2 = \gamma_2^\top a + bu + d + \varphi_2 \quad (95)$$

$$\varphi_2 = c_1(z_2 - c_1 z_1) - \ddot{y}_d \quad (96)$$

notice that the form of (95), (96) is that of (17), (18), respectively. This completes the state transformation based on the backstepping procedure.

The parameters defined above can be summarized as:

$$z_1 = y - y_d \quad (97)$$

$$z_2 = x_2 + c_1 z_1 - \dot{y}_d \quad (98)$$

$$x_1 = y, \quad x_2 = \dot{y}_d \quad (99)$$

$$\varphi_1 = -\dot{y}_d \quad (100)$$

$$\varphi_2 = c_1(z_2 - c_1 z_1) - \ddot{y}_d \quad (101)$$

According to remark 5, it remains to define  $\bar{\varphi}$ ,  $V_z$ . From (81), definition (23) and  $n = 2$ , it follows that

$$\bar{\varphi} = \left[ |\gamma_{2[1]}|, |\gamma_{2[2]}|, f_d, |\varphi_2 + c_2 z_2| \right]^\top = \left[ |\dot{y}|, |y|, f_d, |\varphi_2 + c_2 z_2| \right]^\top \quad (102)$$

From (35) and  $n = 2$  it follows that

$$V_z = (1/2)(z_1^2 + z_2^2) \quad (103)$$

Expressions (97) to (103) allow to define the control and update law. From (32), (48), (82) and  $n = 2$  it follows that

$$\text{sgn}(b) = +1 \quad (104)$$

$$u = -\frac{1}{3C_{bvz}} z_2 (\bar{\varphi}^\top \hat{\theta})^2 \quad (105)$$

$$\dot{\hat{\theta}} = \Gamma \bar{\varphi} |z_2| \frac{\partial \bar{V}_z}{\partial V_z} \quad (106)$$

the main parameters needed to compute  $u$  and  $\hat{\theta}$  are:  $\bar{\varphi}$  (102),  $\varphi_2$  (101),  $C_{bvz}$  (28),  $z_2$  (98),  $z_1$  (97),  $\partial \bar{V}_z / \partial V_z$  (42),  $V_z$  (103). In addition,  $\Gamma$  is a diagonal matrix whose diagonal elements are positive constants defined by the user.

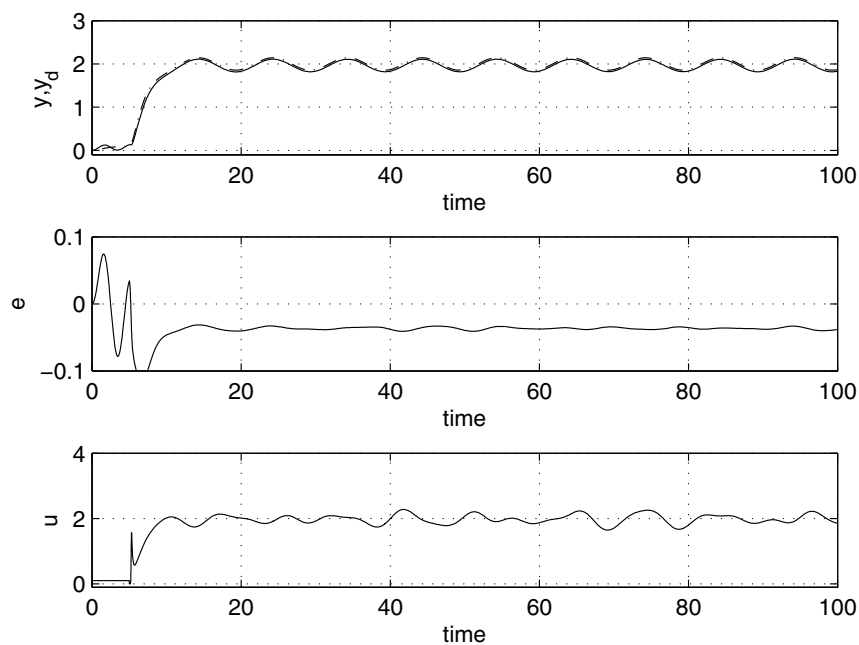


Fig. 2. Example 1, upper: output  $y$  (continuous line), desired output  $y_d$  (dash-dot line); middle: tracking error  $e$ ; lower: control input  $u$ .

Since the aim is that  $y$  converges towards  $y_d$ , with a threshold of 0.1, we set  $C_{be} = 0.1$ . We use the reference model (5) with  $y_d(t_0) = y(t_0)$ ,  $\dot{y}_d(t_0) = 0$ ,  $a_{m,1} = 1$ ,  $a_{m,0} = 1$ . We use the following parameter values for the control and update laws:  $c_1 = 2$ ,  $c_2 = 2$ ,  $\Gamma = \text{diag}\{1, 1, 1, 1\}$ .

The results are shown in figures 2 and 3. We have chosen  $y_d(t_0) \approx y(t_0)$  in order to obtain a rapid convergence of  $y$  towards  $y_d$ . Figure 2 shows that. *i)* the tracking error  $e$  converges asymptotically towards  $\Omega_e = \{e : |e| \leq 0.1\}$ . *ii)* The output  $y$  converges towards  $y_d$  with threshold 0.1 without large transient differences. Figure 3 shows that  $\hat{\theta}_1, \dots, \hat{\theta}_4$  are not decreasing with respect to time. This occurs because  $\dot{\hat{\theta}}$  is non-negative. The procedure for the sample plant (80) is simpler in comparison with adaptive controllers that use the Nussbaum gain method.

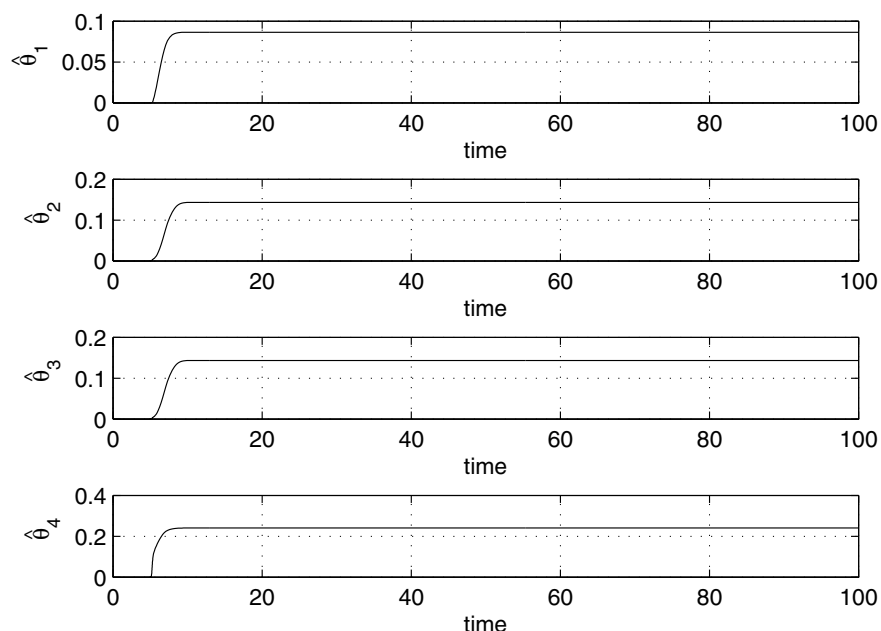


Fig. 3. Example 1, entries of the updated parameter vector  $\hat{\theta}$ , from upper to lower:  $\hat{\theta}_1$ ;  $\hat{\theta}_2$ ,  $\hat{\theta}_3$ ,  $\hat{\theta}_4$ .

## 9. Acknowledgements

A. Rincon acknowledges financial support provided by “Programa de becas para estudiantes sobresalientes de posgrado”, Universidad Nacional de Colombia - vicerrectoría de Investigación. This work was partially supported by Universidad Nacional de Colombia - Manizales, project 12475, Vicerrectoría de Investigación, DIMA.

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## **Applications of Nonlinear Control**

Edited by Dr. Meral Altınay

ISBN 978-953-51-0656-2

Hard cover, 202 pages

**Publisher** InTech

**Published online** 13, June, 2012

**Published in print edition** June, 2012

A trend of investigation of Nonlinear Control Systems has been present over the last few decades. As a result the methods for its analysis and design have improved rapidly. This book includes nonlinear design topics such as Feedback Linearization, Lyapunov Based Control, Adaptive Control, Optimal Control and Robust Control. All chapters discuss different applications that are basically independent of each other. The book will provide the reader with information on modern control techniques and results which cover a very wide application area. Each chapter attempts to demonstrate how one would apply these techniques to real-world systems through both simulations and experimental settings.

### **How to reference**

In order to correctly reference this scholarly work, feel free to copy and paste the following:

A. Rincon, F. Angulo and G. Osorio (2012). A Robust State Feedback Adaptive Controller with Improved Transient Tracking Error Bounds for Plants with Unknown Varying Control Gain, Applications of Nonlinear Control, Dr. Meral Altınay (Ed.), ISBN: 978-953-51-0656-2, InTech, Available from:  
<http://www.intechopen.com/books/applications-of-nonlinear-control/a-robust-state-feedback-adaptive-controller-with-improved-transient-tracking-error-bounds-for-pl>

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