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Describing Function Recording with Simulink and MATLAB

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1. Introduction

Describing function is an equivalent gain of nonlinear element, defined by the harmonic linearization method of nonlinear static characteristic (Novogranov, 1986, Slotine and Li, 1991, Schwarz and Gran, 2001, Vukic et al. 2003 and many others). It is a known method of analysis and synthesis when nonlinear system can be decoupled into linear and nonlinear parts (Fig. 1). If the linear part of the system has the characteristics of low-pass filter and if we apply periodical signal to the system, output signal will have the same base frequency as input signal with damped higher frequencies.

Fig. 1. Nonlinear system represented with decoupled nonlinear $F(x, px)$ and linear parts $G_L(p)$, $p=d/dt$.

If the amplitudes of higher harmonics are relatively small when compared to the amplitude of the first harmonic, i.e. if filter hypothesis holds, output signal can be approximated by its first harmonic. Mathematically, first harmonic of the output signal, for the sinusoidal input signal $X_m \sin \omega t$, can be expressed by the Fourier expressions:

$$y_N(t) \approx Y_{P1} \sin \omega t + Y_{Q1} \cos \omega t$$

$$y_N(t) \approx \text{Im} \left\{ (Y_{P1} + jY_{Q1}) e^{j\omega t} \right\} \quad (1)$$

$$Y_{P1} = \frac{1}{\pi} \int_0^{2\pi} F(X_m \sin \omega t) \sin \omega t d(\omega t) \quad (2)$$

$$Y_{Q1} = \frac{1}{\pi} \int_0^{2\pi} F(X_m \sin \omega t) \cos \omega t d(\omega t) \quad (3)$$

where Y_{P1} and Y_{Q1} are first Fourier coefficients.

Describing function is the ratio between first harmonic of the output signal and input signal in complex form:

$$G_N(X_m) = P(X_m) + jQ(X_m) = \frac{Y_{P1}}{X_m} + j \frac{Y_{Q1}}{X_m} \tag{4}$$

where $P(X_m)$ and $Q(X_m)$ are coefficients of the harmonic linearization (Novogranov, 1986, Slotine and Li, 1991, Schwarz and Gran, 2001, Vukic et al., 2003 and many others).

Determination of describing function boils down to the determination of integral expressions for the known static characteristic of the nonlinear part of the system.

There are many conventional nonlinearities for which static characteristics and describing functions are theoretically derived and given in analytical form.

The problem arises when the static characteristic of nonlinear system cannot be analytically expressed or integral expressions can not be solved. In that case describing function can be determined by experiment or simulation (Kuljaca et al., Sijak et al.k 2004, 2005, 2007a, 2007b) or some method of numerical integration (Schwarz and Gran, 2001).

2. Nonlinear elements in Simulink

Nonlinear elements are given in Simulink library Discontinuities. Twelve discontinuities given there are shown in Fig. 2.

Fig. 2. Discontinuities (nonlinearities) given in Simulink.

We can see that there are three elements called "Dynamic" (Dead Zone Dynamic, Saturation Dynamic and Rate Limiter Dynamic). The term dynamic is considered at these elements with respect to change of nonlinearity limits (output values limits or rate change limits), not as dynamics in control systems sense (i.e. behavior in time domain). Never the less, we will not deal with such elements here. Describing function method analysis or synthesis is not suitable for systems with varying parameters.

One can see that Simulink gives only basic non-dynamic nonlinearities. More nonlinearities and their describing function can be found in work by Vukic et al., (2003). It is clear that one needs to build them out of basic Simulink models or write them as m or s functions. Since given functions don't have dynamics it is recommended to build them out of given Simulink blocks, or if that is not possible, then using type 2 s-functions. Even in that case some real time extension software for Simulink and Matlab might not be able to handle user designed s-functions.

3. Describing function by numerical integration

An interesting approach to obtaining describing function using Simulink is given by Schwartz and Gran. The approach is very simple, yet effective. In essence, equations (2) and (3) are directly applied in Simulink on nonlinear element outputs. A simulation scheme adapted from Schwartz is given in Fig. 3. In this case dead zone nonlinearity is analyzed. Instead of dead zone, any other nonlinearity can be included in model.

Fig. 3. Simulink model 'csm' for generating describing function (Schwartz)

Model is called from Matlab script (adapted from Schwartz and Gran as well) as shown in Fig. 4. simulation parameters are passed to Simulink schematic from Matlab script, as given in Fig. 5.

```
E=[0.15:.01:0.25],[0.3:0.1:0.9],[1:.5:3];%amplitudes
omega=[.5:.5:5 5.5:.1:6.2 2*pi 6.2:.1:7.5 7:.5:10];
%omega are frequencies
Nf=length(omega);
array=zeros(1,1,length(omega),length(E));
s=zeros(length(omega),length(E));
for j=1:Nf
    omega1=omega(j);
    sim('csm');
    DF = (DFreal+sqrt(-1)*DFimag);
    s(j,:)=DF;
    array(1,1,j,:)=DF;
end
figure(1); surf(E,omega,abs(s))
shading interp
ylabel('Frequency (rad/sec)'); xlabel('Amplitude');
zlabel('Describing Function Gain')
title('Describing Function Amplitude vs. Frequency and
Amplitude of Input')
figure(2)
surf(E,omega,angle(s))
shading interp
ylabel('Frequency (rad/sec)'); xlabel('Amplitude');
zlabel('Describing Function Phase')
title('Describing Function Phase vs. Frequency and
Amplitude of Input')
```

Fig. 4. Matlab script for describing function generation

Fig. 5. Simulation configuration parameters – parameter omega1 is taken from Matlab script

Triggered subsystem is shown in Fig. 6. The whole file is built to stop simulation after one full period for a given frequency.

Fig. 6. Triggered subsystem

Let us now see how Simulink file with Matlab function will perform for a common nonlinearity, saturation with saturation limit 0.5. Saturation type nonlinearity is shown in Fig. 7.

Fig. 7. Saturation

Saturation has known describing function given by (5). It can be seen that imaginary part of the gain is zero. That is a case with all symmetrical nonlinearities.

$$G = G(Z_m) = \frac{2 \cdot \text{tg} \alpha}{\Pi} \left(\arcsin \frac{c}{A_m} + \frac{c}{A_m} \cdot \sqrt{1 - \frac{c^2}{A_m^2}} \right); A_m > c \tag{5}$$

where A_m is amplitude of signal entering saturation.

Let us now see the results of running our simulation script in order to obtain describing function for saturation. Simulation results are shown in Fig. 8 and Fig. 9.

Fig. 8. Describing function gain as function of amplitude and frequency

Fig. 9. Describing function phase as function of amplitude and frequency

It can be seen from Fig. 8 that describing function in this case is not dependent on frequency. Also, phase is zero since saturation is symmetrical nonlinearity. That can be seen from very small values of describing function phase as given in Fig. 9. These values are practically

zero. Obtained result from simulation is given complex matrix s . In case like this, where there is no frequency dependency and phase is zero, there is no need for plotting describing function in three dimensions. To make function plot in two dimensions we run the following short script:

```
gdf=s(1,:);
plot(E,real(gdf)),grid, ylabel('Describing function gain');
xlabel('Amplitude');
```

Two dimensional describing function gain is shown in

Fig. 10. Describing function gain – two dimensional representation

We will see in subsequent sections that this representation of symmetrical nonlinearities as gain dependent on amplitude of nonlinearity input signal is important for stability analysis.

This technique can be used to obtain describing functions of real nonlinearities by using Simulink model from Fig. 3 with appropriate real time additions with Real Time Window Target, xPc Target or other real time systems for use with Simulink. Of course, in that case amplitudes will have to be given one by one, not as here as one vector.

4. Fuzzy control systems and their describing functions in Simulink

Use of fuzzy control systems is today well known in control systems design. Unfortunately, while there are well developed methods for design or training of fuzzy systems, stability analysis remains problematic. If the fuzzy system is of Mamdani type, than its mathematical description can be very complicated and stability analysis is complicated as well. However, if we can obtain describing function of fuzzy system, then we can use known stability analysis methods developed for linear systems. One type of experimental method of obtaining describing function for fuzzy elements is given in (Kuljaca et al.) and (Sijak et al., 2005, 2007b). Here, we will use numerical integration in Simulink as given by Schwartz. Simulation file will have to be reworked since we cannot use vectors in this simulation due to fuzzy element.

Of course, there are different fuzzy controllers and elements, but describing function for all of them can be achieved using this method. One should note that here we are not talking about adaptive fuzzy controllers, but about fuzzy controllers with fixed membership functions and rule base.

Let us first look on one example of fuzzy controller, as shown in Fig. 11 (Kuljaca et al., Sijak et al. 2007a), where k_p and k_d are gains.

Fig. 11. The block diagram of fuzzy controller

First of all, it can be seen that fuzzy element here has two inputs. Fuzzy elements in general can have more inputs and outputs than here, but, this structure is most often in control systems (either as given here with derivative of the error or with integral of the error). When more inputs are used it becomes extremely complicated to tune the controller.

Numerical integration Simulink model (Schwartz) with fuzzy controller will look like in Fig. 12, without vector inputs for amplitudes since fuzzy block in Simulink cannot handle such input type. This is also much closer to real measurement, since in real measurement we would not be able to use vector inputs. Meaning of this is that we need to use adjusted script to run the Simulink model. Adjusted script is given in Fig. 13.

Fuzzy Logic controller Simulink block is regular block from Simulink Fuzzy Logic toolbox library.

Fuzzy element itself is developed using FIS editor from Matlab Fuzzy Logic Toolbox.

This is an illustrative example; however, it is quite useful in giving an insight in use of describing function for fuzzy controllers. The given method can give a graphic representation of any fuzzy controller based on error and derivatives of error signal. In most cases fuzzy controllers are also symmetrical, thus subsequent stability analysis will be simpler. Amplitudes and frequencies are to be chosen to satisfy expected operational environment of controller.

Fig. 12. Numerical integration model 'csm_fuzzy'– describing function recording

```
E=[0.15:.01:0.25],[0.3:0.1:0.9],[1:.5:3];
omega=[.5:.5:5 5.5:.1:6.2 2*pi 6.2:.1:7.5 7:.5:10];
for ke=1:max(size(E))
    A=E(ke);
    for kw=1:max(size(omega))
        omegal=omega(kw)
        sim('csm_fuzzy')
        s(ke,kw) = (DFreal+sqrt(-1)*DFimag);
    end
end
figure(1); surf(omega,E,abs(s))
shading interp
xlabel('Frequency (rad/sec)'); ylabel('Amplitude');
zlabel('Describing Function Gain')
title('Describing Function Amplitude vs. Frequency and Amplitude of Input')
figure(2); surf(omega,E,angle(s))
shading interp
xlabel('Frequency (rad/sec)'); ylabel('Amplitude');
zlabel('Describing Function Phase')
title('Describing Function Phase vs. Frequency and Amplitude of Input')
```

Fig. 13. Matlab script for running 'csm_fuzzy' model

In example given here gains k_p and k_d are set to 1.2 and 0.001 respectively, min-max and centroid defuzzification Mamdani type fuzzy controller membership functions are given in Fig. 14 and rulebase is given in Fig. 15.

Fig. 14. Membership functions of the fuzzy element: a) proportional input, b) derivative input and c) output

$x_d \backslash x_p$	NV	NS	NM	ZE	PM	PS	PV
NV	NV	NV	NV	NV	NS	NM	ZE
NS	NV	NV	NV	NS	NM	ZE	PM
NM	NV	NV	NS	NM	ZE	PM	PS
ZE	NV	NS	NM	ZE	PM	PS	PV
PM	NS	NM	ZE	PM	PS	PV	PV
PS	NM	ZE	PM	PS	PV	PV	PV
PV	ZE	PM	PS	PV	PV	PV	PV

Fig. 15. Rulebase of the fuzzy element

Describing function for given fuzzy controller is given in Fig. 16 and Fig. 17. It can be seen from Fig. 16 that describing function gain is function of amplitude and it really does not depend on frequency in this case.

Describing function phase as function of amplitude and frequency shown in Fig. 17 shows very small values and for all practical purposes can be considered to be zero. Since our fuzzy controller design is symmetrical one that is in compliance with theoretical expectations.

Fig. 16. Describing function gain as function of amplitude and frequency

Fig. 17. Describing function phase as function of amplitude and frequency

5. Stability analysis with describing functions in Simulink – Case study

Once describing function is obtained, it could be used for stability analysis. We will use a simplified secondary frequency control model of the isolated thermo power system with generation rate constraint (Sijak et al.,2001). Control model is shown in Fig. 18.

Fig. 18. Power system secondary load-frequency control model

where:

- $G_G = \frac{1}{1 + sT_G}$ - the transfer function of the turbine governor
- T_{CH} - the steam turbine time constant
- ΔP_m - the change of the mechanical power of the turbine
- ΔP_L - the change of the power system load
- ΔP_r - the power system active power reference change
- Δf - the power system frequency change
- $G_s = \frac{K_s}{1 + sT_s}$ - the power system transfer function
- R - the static speed droop of the uncontrolled system
- $F(z)$ - the power system generation rate constraint, static characteristic of saturation nonlinearity.

The system given in Fig. 18 consists of nonlinear parts divided by linear parts for which the filter hypothesis is satisfied. Such system can be represented as in Fig. 19.

Fig. 19. The structure of the nonlinear system and fuzzy controller

where:

- $F_i(x_i, px_i), p = \frac{d}{dt}$, - nonlinear parts of the system
- $G_{Li}(s)$ - transfer functions of the linear parts of the system

Assuming that the filter hypothesis is valid for $G_{Li}(p)$, the nonlinearities can be harmonically linearized and their describing functions used instead (Vukic et al., Netushil et al.).

$$F_i(x_i, px_i) \approx F_i(X_{im}, \omega), x_i = X_{im} \sin(\omega t) \quad (6)$$

Harmonically linearized system is shown in Fig. 20.

Fig. 20. The structure of the harmonically linearised nonlinear system and fuzzy controller

The self-oscillations (limit cycles) of the system in Fig. 20 are described by solution of:

$$1 + F_1(X_{1m}, \omega) G_{L1}(j\omega) F_2(X_{2m}, \omega) G_{L2}(j\omega) = 0 \quad (7)$$

Assuming that $F_1(X_{1m}, \omega)$ and $F_2(X_{2m}, \omega)$ are describing functions of the fuzzy controller and nonlinear part of the plant respectively, then the stability analysis of the system can be conducted using the solution of complex equation (7).

Let us now deal with nonlinearities. Generation rate constraint is saturation type nonlinearity shown in Fig. 21. Describing function for such nonlinearities is known and it is given in (8).

Fig. 21. Generation rate constraint

$$G_N(Z_m) = \frac{4}{\pi} \left(\frac{\alpha}{2} - \frac{\sin 2\alpha}{4} + \frac{\cos \alpha}{Z_m/\delta} \right), \alpha = \arcsin \frac{\delta}{Z_m} \quad (8)$$

The parameters of the system are: $T_G = 0.08\text{ s}$, $T_{CH} = 0.3\text{ s}$, $\delta = 0.0017$, $K_s = 120\text{ Hz/p.u.}$, $T_s = 20\text{ s}$, $R = 2.4\text{ Hz/p.u.}$, $b_1 = 1$. With given parameters, describing function of generation rate constraint is shown in Fig. 22.

Fuzzy controller represents a bit more complex problem. We do not have its describing function in analytical form and we need to obtain it by experimental method. The method is described in Section 5 and describing function gain is plotted in Fig. 16 in three dimensional representation. Phase changes are zero, thus there is no complex component of describing function.

Representation in three dimension in this case is not required since there is no dependency of describing function gain on frequency. After running simulation script given in Section 5, one should run the following Matlab code in order to obtain two-dimensional representation of fuzzy controller describing function:

```
gdf=s(:,1);  
plot(E,real(gdf)),grid, ylabel('Describing function gain');  
xlabel('Amplitude');
```

Describing function in two dimensional representation is given in Fig. 23.

Fig. 22. Generation rate constraint describing function

Fig. 23. Describing function of fuzzy controller in two dimensional representation

The static characteristics of the fuzzy regulator, i.e. its describing functions, shows that harmonically linearized fuzzy regulator can be regarded as input dependent variable gain element. Consequently, the stability analysis can be conducted in the plane $G_{NF} = f(G_{NZ})$, where

- $G_N(X_m) = G_{NF} = G_N(\Delta F_m)$ - describing function of the fuzzy regulator ,
- $G_{NZ} = G_N(Z_m)$ - describing function of the power system generation rate constraint.

The stability of the equilibrium point for the system given in Fig. 18 can be determined from the characteristic polynomial of the closed loop harmonically linearised equation (Kuljaca et al., Sijak et al., 2007):

$$T_{CH} + (K_0 + b_l G_{NF}) \cdot \frac{K_s}{1 + s \cdot T_s} \cdot \frac{1}{1 + s \cdot T_G} \cdot \frac{G_{NZ}}{s} + \frac{G_{NZ}}{s} = 0 \tag{9}$$

where: $K_0 = \frac{1}{R}$

From (9) the closed loop characteristic equation of the controlled system can be obtained:

$$\begin{aligned} &T_{CH}T_sT_Gs^3 + [T_GT_sG_{NZ} + T_{CH}(T_G + T_s)]s^2 + \\ &[T_{CH} + (T_G + T_s)G_{NZ}]s \\ &+ G_{NZ} + (K_0 + b_l G_{NF})K_sG_{NZ} = 0 \end{aligned} \tag{10}$$

By applying Hurwitz stability criterion the following inequality is obtained:

$$\begin{aligned} & \left[T_G T_s G_{NZ} + T_{CH} (T_G + T_s)\right] \left[T_{CH} + (T_G + T_s) G_{NZ}\right] - \\ & T_{CH} T_s T_G \left[G_{NZ} + (K_0 + b_l G_{NF}) K_s G_{NZ}\right] > 0 \end{aligned} \tag{11}$$

The system is stable as long as (11) is positive. Thus, the stability boundary can be derived as:

$$\begin{aligned} G_{NF} = & \frac{(T_G + T_s) G_{NZ}}{b_l T_{CH} K_s} + \frac{T_{CH} (T_G + T_s)^2 - T_G T_s T_{CH} K_0 K_s}{b_l T_{CH} T_s T_G K_s} \\ & + \frac{T_{CH} (T_G + T_s)}{b_l T_s T_G K_s G_{NZ}} \end{aligned} \tag{12}$$

With the parameters used for simulation, the stability boundary function $G_{NF} = f(G_{NZ})$ can be evaluated. The function $G_{NF} = f(G_{NZ})$ is shown in Fig. 24.

Fig. 24. Function $G_{NF} = f(G_{NZ})$

With given values system is stable in operating region of interest.

6. Conclusion

Chapter deals with use of Simulink for obtaining describing function for different elements. Described method of numerical simulation performed in Simulink is suitable

for obtaining describing function of any single input single output non-dynamic nonlinearity. Method used for conventional static nonlinearities is then adjusted to work with fuzzy systems (and other systems that cannot handle vector based batch simulation in Simulink). Matlab scripts required to run given Simulink model are also given in Chapter. This method can be also used to obtain describing function for single input single output static neural networks. Finally, an example from use describing function for stability analysis is given.

Described method can be used with practical controllers with Matlab real time tools.

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