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Application of Hankel Transform for Solving a Fracture Problem of a Cracked Piezoelectric Strip Under Thermal Loading

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1. Introduction

In this chapter, an example of the application of Hankel transform for solving a fracture problem will be explained. In discussing axisymmetric problems, it is advantageous to use polar coordinates, and the Hankel transform method is powerful to solve the general equations in polar coordinates. A brief account of the Hankel transform will be given. Here f is a function of r, its transform is indicated by a capital F, J_ν is the ν th order Bessel function of the first kind, and the nature of the transformation either by a suffix or by a characteristic new variable s. It will be assumed without comment that the integrals in question exist, and that, if necessary, the functions and their derivatives tend to zero as the variable tends to infinity. The Hankel transform of order $\nu > -1/2$, $H_\nu[f(r)]$ or $F_\nu(s)$, of a function $f(r)$ is defined as

$$H_\nu[f(r)] \equiv F_\nu(s) = \int_0^\infty r J_\nu(sr) f(r) dr$$

and its inversion formula is

$$f(r) = \int_0^\infty s J_\nu(sr) F_\nu(s) ds$$

Also, integrating by parts twice gives

$$H_\nu\left[\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr} - \frac{\nu^2}{r^2}f\right] = -s^2 F_\nu(s)$$

provided that $rf(r)$ and $rdf(r)/dr$ tend to zero as $r \to 0$ and as $r \to \infty$.

The piezoelectric materials have attracted considerable attention recently. Owing to the coupling effect between the thermo-elastic and electric fields in piezoelectric materials, thermo-mechanical disturbances can be determined form measurement of the induced electric potential, and the ensuing response can be controlled through application of an appropriate electric field (Rao & Sunar, 1994). For successful and efficient utilization of

piezoelectric as sensors and actuators in intelligent systems, several researches on piezo-thermo-elastic behavior have been reported (Tauchert, 1992).

Moreover a better understanding of the mechanics of fracture in piezoelectric materials under thermal load conditions is needed for the requirements of reliability and lifetime of these systems. Using the Fourier transform, the present author studied the thermally induced fracture of a piezoelectric strip with a two-dimensional crack (Ueda, 2006a, 2006b).

Here the mixed-mode thermo-electro-mechanical fracture problem for a piezoelectric material strip with a penny-shaped crack is considered. It is assumed that the strip is under the thermal loading. The crack faces are supposed to be insulated thermally and electrically. By using the Hankel transform (Sneddon & Lowengrub, 1969), the thermal and electro-mechanical problems are reduced to a singular integral equation and a system of singular integral equations (Erdogan & Wu, 1996), respectively, which are solved numerically (Sih, 1972). Numerical calculations are carried out, and detailed results are presented to illustrate the influence of the crack size and the crack location on the stress and electric displacement intensity factors. The temperature, stress and electric displacement distributions are also presented.

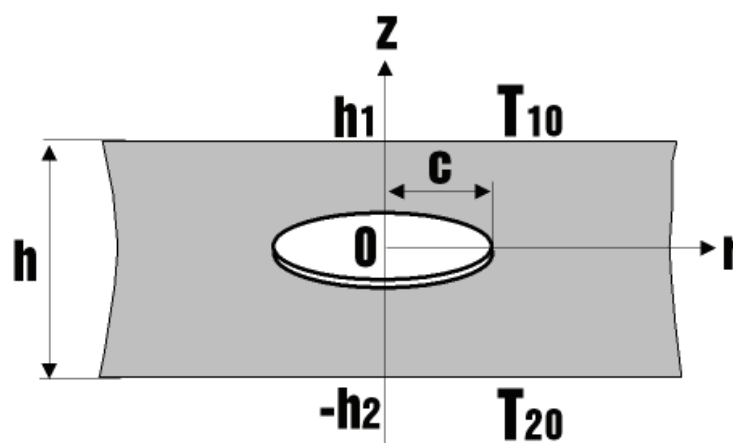## 2. Formulation of the problem

Fig. 1. Penny-shaped crack in a piezoelectric strip

A penny-shaped crack of radius c is embedded in an infinite long piezoelectric strip of thickness $h = h_1 + h_2$ as shown in Figure 1. The crack is located parallel to the boundaries and at an arbitrary position in the strip, and the crack faces are supposed to be insulated thermally and electrically. The cylindrical coordinate system is denoted by (r, θ, z) with its origin at the center of the crack face and the plane $r - \theta$ along the crack plane, where z is the poling axis. It is assumed that uniform temperatures T_{10} and T_{20} are maintained over the stress-free boundaries. In the following, the subscripts r, θ, z will be used to refer to the direction of coordinates. The material properties, such as the elastic stiffness constants, the piezoelectric constants, the dielectric constants, the stress-temperature coefficients, the coefficients of heat conduction, and the pyroelectric constant, are denoted by c_{kl}, e_{kl}, ε_{kk}, λ_{kk} $(k, l = 1, 2, 3)$, κ_r, κ_z, and p_z, respectively.

The constitutive equations for the elastic field are

$$\left.\begin{array}{l}
\sigma_{rri} = c_{11}\dfrac{\partial u_{ri}}{\partial r} + c_{12}\dfrac{u_{ri}}{r} + c_{13}\dfrac{\partial u_{zi}}{\partial z} + e_{31}\dfrac{\partial \phi_i}{\partial z} - \lambda_{11}T_i, \\[3mm]
\sigma_{\theta\theta i} = c_{12}\dfrac{\partial u_{ri}}{\partial r} + c_{11}\dfrac{u_{ri}}{r} + c_{13}\dfrac{\partial u_{zi}}{\partial z} + e_{31}\dfrac{\partial \phi_i}{\partial z} - \lambda_{11}T_i, \\[3mm]
\sigma_{zzi} = c_{13}\dfrac{\partial u_{ri}}{\partial r} + c_{13}\dfrac{u_{ri}}{r} + c_{33}\dfrac{\partial u_{zi}}{\partial z} + e_{33}\dfrac{\partial \phi_i}{\partial z} - \lambda_{33}T_i, \\[3mm]
\sigma_{zri} = c_{44}\left(\dfrac{\partial u_{zi}}{\partial r} + \dfrac{\partial u_{ri}}{\partial z}\right) + e_{15}\dfrac{\partial \phi_i}{\partial r}
\end{array}\right\} \quad (i=1,2) \qquad (1)$$

where $T_i(r,z)$ is the temperature, $\phi_i(r,z)$ is the electric potential, $u_{ri}(r,z)$, $u_{zi}(r,z)$ are the displacement components, $\sigma_{rri}(r,z)$, $\sigma_{\theta\theta i}(r,z)$, $\sigma_{zzi}(r,z)$, $\sigma_{zri}(r,z)$ $(i=1,2)$ are the stress components. The subscript $i=1,2$ denotes the thermo-electro-elastic fields in $0 \le z \le h_1$ and $-h_2 \le z \le 0$, respectively. For the electric field, the constitutive relations are

$$\left.\begin{array}{l}
D_{ri} = e_{15}\left(\dfrac{\partial u_{zi}}{\partial r} + \dfrac{\partial u_{ri}}{\partial z}\right) - \varepsilon_{11}\dfrac{\partial \phi_i}{\partial r}, \\[3mm]
D_{zi} = e_{31}\dfrac{\partial u_{ri}}{\partial r} + e_{31}\dfrac{u_{ri}}{r} + e_{33}\dfrac{\partial u_{zi}}{\partial z} - \varepsilon_{33}\dfrac{\partial \phi_i}{\partial z} + p_z T_i
\end{array}\right\} \quad (i=1,2) \qquad (2)$$

where $D_{ri}(r,z)$, $D_{zi}(r,z)$ $(i=1,2)$ are the electric displacement components.

The governing equations for the thermo-electro-elastic fields of the medium may be expressed as follows:

$$\kappa^2\left(\frac{\partial^2 T_i}{\partial r^2} + \frac{1}{r}\frac{\partial T_i}{\partial r}\right) + \frac{\partial^2 T_i}{\partial z^2} = 0 \quad (i=1,2) \qquad (3)$$

$$\left.\begin{array}{l}
c_{11}\left(\dfrac{\partial^2 u_{ri}}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u_{ri}}{\partial r} - \dfrac{u_{ri}}{r^2}\right) + c_{44}\dfrac{\partial^2 u_{ri}}{\partial z^2} + (c_{13}+c_{44})\dfrac{\partial^2 u_{zi}}{\partial r\partial z} + (e_{31}+e_{15})\dfrac{\partial^2 \phi_i}{\partial r\partial z} = \lambda_{11}\dfrac{\partial T_i}{\partial r}, \\[4mm]
c_{44}\left(\dfrac{\partial^2 u_{zi}}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u_{zi}}{\partial r}\right) + c_{33}\dfrac{\partial^2 u_{zi}}{\partial z^2} + (c_{13}+c_{44})\left(\dfrac{\partial^2 u_{ri}}{\partial r\partial z} + \dfrac{1}{r}\dfrac{\partial u_{ri}}{\partial z}\right) + e_{15}\left(\dfrac{\partial^2 \phi_i}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial \phi_i}{\partial r}\right) + e_{33}\dfrac{\partial^2 \phi_i}{\partial z^2} = \lambda_{33}\dfrac{\partial T_i}{\partial z}, \\[4mm]
e_{15}\left(\dfrac{\partial^2 u_{zi}}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u_{zi}}{\partial r}\right) + e_{33}\dfrac{\partial^2 u_{zi}}{\partial z^2} + (e_{15}+e_{31})\left(\dfrac{\partial^2 u_{ri}}{\partial r\partial z} + \dfrac{1}{r}\dfrac{\partial u_{ri}}{\partial z}\right) - \varepsilon_{11}\left(\dfrac{\partial^2 \phi_i}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial \phi_i}{\partial r}\right) - \varepsilon_{33}\dfrac{\partial^2 \phi_i}{\partial z^2} = -p_z\dfrac{\partial T_i}{\partial z}
\end{array}\right\} (i=1,2) \quad (4)$$

where $\kappa^2 = \kappa_r / \kappa_z$.

The boundary conditions can be written as

$$\left.\begin{array}{ll}
\dfrac{\partial}{\partial z}T_1(r,0) = 0 & (0 \le r < c) \\[3mm]
T_1(r,0) = T_2(r,0) & (c \le r < \infty)
\end{array}\right\} \qquad (5)$$

$$T_1(r,h_1) = T_{10},$$
$$\left. \frac{\partial}{\partial z} T_1(r,0) = \frac{\partial}{\partial z} T_2(r,0), \right\} \quad (0 \le r < \infty) \tag{6}$$
$$T_2(r,-h_2) = T_{20}$$

for thermal loading conditions and

$$\left. \begin{array}{ll} \sigma_{zz1}(r,0) = 0 & (0 \le r < c) \\ u_{z1}(r,0) = u_{z2}(r,0) & (c \le r < \infty) \end{array} \right\} \tag{7}$$

$$\left. \begin{array}{ll} \sigma_{zr1}(r,0) = 0 & (0 \le r < c) \\ u_{r1}(r,0) = u_{r2}(r,0) & (c \le r < \infty) \end{array} \right\} \tag{8}$$

$$\left. \begin{array}{ll} D_{z1}(r,0) = 0 & (0 \le r < c) \\ \phi_1(r,0) = \phi_2(r,0) & (c \le r < \infty) \end{array} \right\} \tag{9}$$

$$\left. \begin{array}{lll} \sigma_{zz1}(r,0) = \sigma_{zz2}(r,0), & \sigma_{zz1}(r,h_1) = 0, & \sigma_{zz2}(r,-h_2) = 0, \\ \sigma_{zr1}(r,0) = \sigma_{zr2}(r,0), & \sigma_{zr1}(r,h_1) = 0, & \sigma_{zr2}(r,-h_2) = 0, \\ D_{z1}(r,0) = D_{z2}(r,0), & D_{z1}(r,h_1) = 0, & D_{z2}(r,-h_2) = 0 \end{array} \right\} \quad (0 \le r < \infty) \tag{10}$$

for the electromechanical conditions.

3. Temperature field

For the problem considered here, it is convenient to represent the temperature as the sum of two functions.

$$T_i(r,z) = T^{(1)}(z) + T_i^{(2)}(r,z) \quad (i = 1,2) \tag{11}$$

where $T^{(1)}(z)$ satisfies the following equation and boundary conditions:

$$\frac{d^2 T^{(1)}}{dz^2} = 0 \tag{12}$$

$$\left. \begin{array}{l} T^{(1)}(h_1) = T_{10}, \\ T^{(1)}(-h_2) = T_{20} \end{array} \right\} \tag{13}$$

and $T_i^{(2)}(r,z) \, (i = 1,2)$ is subjected to the relations:

$$\kappa^2 \left(\frac{\partial^2 T_i^{(2)}}{\partial r^2} + \frac{1}{r} \frac{\partial T_i^{(2)}}{\partial r} \right) + \frac{\partial^2 T_i^{(2)}}{\partial z^2} = 0 \quad (i = 1,2) \tag{14}$$

$$\left. \begin{array}{ll} \dfrac{\partial}{\partial z} T_1^{(2)}(r,0) = -\dfrac{d}{dz} T^{(1)}(0) & (0 \le r < c) \\ T_1^{(2)}(r,0) = T_2^{(2)}(r,0) & (c \le r < \infty) \end{array} \right\} \tag{15}$$

$$\left.\begin{array}{l} T_1^{(2)}(r,h_1) = 0, \\[2mm] \dfrac{\partial}{\partial z}T_1^{(2)}(r,0) = \dfrac{\partial}{\partial z}T_2^{(2)}(r,0), \\[2mm] T_2^{(2)}(r,-h_2) = 0 \end{array}\right\} \quad (0 \le r < \infty) \tag{16}$$

It is easy to find from Eqs.(12) and (13) that

$$T^{(1)}(z) = \frac{1}{h_1 + h_2}\left\{(T_{10} - T_{20})z + T_{10}h_2 + T_{20}h_1\right\} \tag{17}$$

By applying the Hankel transform to Eq.(14) (Sneddon & Lowengrub, 1969), we have

$$T_i^{(2)}(r,z) = \sum_{j=1}^{2}\int_0^{\infty} D_{ij}(s)J_0(sr)\exp(s\tau_{ij}z)ds \quad (i = 1,2) \tag{18}$$

where $D_{ij}(s)\,(i,j = 1,2)$ are unknown functions to be solved and $\tau_{ij}\,(i,j = 1,2)$ are given by

$$\left.\begin{array}{l} \tau_{11} = \tau_{22} = -\kappa, \\[2mm] \tau_{12} = \tau_{21} = \kappa \end{array}\right\} \tag{19}$$

Taking the second boundary condition (15) into consideration, the problem may be reduced to a singular integral equation by defining the following new unknown function $G_0(r)$ (Erdogan & Wu, 1996):

$$G_0(r) = \left\{\begin{array}{ll} \dfrac{\partial}{\partial r}\left\{T_1^{(2)}(r,0) - T_2^{(2)}(r,0)\right\} & (0 \le r < c) \\[4mm] 0 & (c \le r < \infty) \end{array}\right\} \tag{20}$$

Making use of the first boundary condition (15) with Eqs.(16), we have the following singular integral equation for the determination of the unknown function $G_0(t)$:

$$\int_0^c t\left\{M_0^{(1)}(t,r) + M_0^{(2)}(t,r)\right\}G_0(t)dt = -\frac{2}{\kappa}\frac{T_{10} - T_{20}}{h_1 + h_2} \quad (0 \le r < c) \tag{21}$$

In Eq.(21), the kernel functions $M_0^{(1)}(t,r)$ and $M_0^{(2)}(t,r)$ are given by

$$M_0^{(1)}(t,r) = \left\{\begin{array}{ll} \dfrac{2}{\pi}\dfrac{1}{t^2 - r^2}E\left(\dfrac{r}{t}\right) & (r < t), \\[4mm] \dfrac{2}{\pi}\left\{\dfrac{r}{t(t^2 - r^2)}E\left(\dfrac{t}{r}\right) + \dfrac{1}{rt}K\left(\dfrac{t}{r}\right)\right\} & (r > t) \end{array}\right\} \tag{22}$$

$$M_0^{(2)}(t,r) = -\int_0^{\infty} s\left\{\frac{2\rho_1(s)\rho_2(s)}{\kappa\rho_0(s)} + 1\right\}J_0(sr)J_1(st)ds \tag{23}$$

where K and E are complete elliptic integrals of the first and second kind, and $\rho_k(s)\,(k = 0,1,2)$ are given by

$$\left.\begin{array}{l} \rho_0(s) = \rho_2(s)\{1 - \exp(-2s\kappa h_1)\} - \rho_1(s)\{1 - \exp(-2s\kappa h_2)\}, \\[12pt] \rho_i(s) = \tau_{i1} - \tau_{i2}\exp(-2s\kappa h_i) \quad (i = 1, 2) \end{array}\right\} \tag{24}$$

Once $G_0(t)$ is obtained from Eq.(21), the temperature field can be easily calculated as follows:

$$T_i^{(2)}(r,z) = \sum_{j=1}^{2} T_{ij}^{(2)}(r,z) \quad (i = 1, 2) \tag{25}$$

where

$$T_{ij}^{(2)}(r,z) = \int_0^\infty R_{ij}(s)R_0(s)J_0(sr)\exp(s\tau_{ij}z)ds \quad (i,j = 1, 2) \tag{26}$$

with

$$\left.\begin{array}{ll} R_0(s) = \int_0^c tG_0(t)J_1(st)dt, & \\[10pt] R_{11}(s) = -\dfrac{\rho_2(s)}{\rho_0(s)}, & R_{12}(s) = \dfrac{\rho_2(s)}{\rho_0(s)}\exp(-2s\kappa h_1), \\[12pt] R_{21}(s) = -\dfrac{\rho_1(s)}{\rho_0(s)}, & R_{22}(s) = \dfrac{\rho_1(s)}{\rho_0(s)}\exp(-2s\kappa h_2) \end{array}\right\} \tag{27}$$

On the plane $z = 0$, the temperatures $T_i^{(2)}(r,0)$ $(i = 1, 2)$ are reduced to

$$T_i^{(2)}(r,0) = \frac{(-1)^i}{2}\int_r^c G_0(t)dt + \int_0^\infty \left\{\sum_{j=1}^{2} R_{ij}(s) - \frac{(-1)^i}{2}\right\} R_0(s)J_0(sr)ds \quad (i = 1, 2) \tag{28}$$

4. Thermally induced elastic and electric fields

The non-disturbed temperature filed $T^{(1)}(z)$ given by Eq.(17) does not induce the stress and electric displacement components, which affect the singular field. Thus, we consider the elastic and electric fields due to the disturbed temperature distribution $T_i^{(2)}(r,z)$ $(i = 1, 2)$ only. It is convenient to represent the solutions $u_{zi}(r,z)$, $u_{ri}(r,z)$ and $\phi_i(r,z)$ $(i = 1, 2)$ as the sum of two functions, respectively.

$$\left.\begin{array}{l} u_{zi}(r,z) = u_{zi}^{(1)}(r,z) + u_{zi}^{(2)}(r,z), \\[12pt] u_{ri}(r,z) = u_{ri}^{(1)}(r,z) + u_{ri}^{(2)}(r,z), \\[12pt] \phi_i(r,z) = \phi_i^{(1)}(r,z) + \phi_i^{(2)}(r,z) \end{array}\right\} \quad (i = 1, 2) \tag{29}$$

where $u_{zi}^{(1)}(r,z)$, $u_{ri}^{(1)}(r,z)$, $\phi_i^{(1)}(r,z)$ $(i = 1, 2)$ are the particular solutions of Eqs.(4) replaced T_i by $T_i^{(2)}$, and $u_{zi}^{(2)}(r,z)$, $u_{ri}^{(2)}(r,z)$, $\phi_i^{(2)}(r,z)$ $(i = 1, 2)$ are the general solutions of

homogeneous equations obtained by setting $T_i = 0$ $(i = 1, 2)$ in Eqs.(4). In the following, the superscripts (1) and (2) indicate the particular and general solutions of Eqs.(4). Substituting Eqs.(29) into Eqs.(1) and (2), one obtains stress $\sigma_{rri}(r,z)$, $\sigma_{\theta\theta i}(r,z)$, $\sigma_{zzi}(r,z)$, $\sigma_{zri}(r,z)$ and electric displacement $D_{ri}(r,z)$, $D_{zi}(r,z)$ $(i = 1, 2)$ expressions.

Using the displacement potential function method (Ueda, 2006a), the particular solutions can be obtained as follows:

$$
\left.
\begin{aligned}
\sigma_{zzi}^{(1)}(r,z) &= \sum_{j=1}^{2} \int_0^\infty p_{1ij}^{(1)} R_{ij}(s) R_0(s) J_0(sr) \exp(s\tau_{ij}z) ds, \\
\sigma_{zri}^{(1)}(r,z) &= \sum_{j=1}^{2} \int_0^\infty p_{2ij}^{(1)} R_{ij}(s) R_0(s) J_1(sr) \exp(s\tau_{ij}z) ds, \\
D_{zi}^{(1)}(r,z) &= \sum_{j=1}^{2} \int_0^\infty p_{3ij}^{(1)} R_{ij}(s) R_0(s) J_0(sr) \exp(s\tau_{ij}z) ds, \\
u_{zi}^{(1)}(r,z) &= \sum_{j=1}^{2} \int_0^\infty \frac{1}{s} p_{4ij}^{(1)} R_{ij}(s) R_0(s) J_0(sr) \exp(s\tau_{ij}z) ds, \\
u_{ri}^{(1)}(r,z) &= \sum_{j=1}^{2} \int_0^\infty \frac{1}{s} p_{5ij}^{(1)} R_{ij}(s) R_0(s) J_1(sr) \exp(s\tau_{ij}z) ds, \\
\phi_i^{(1)}(r,z) &= -\sum_{j=1}^{2} \int_0^\infty \frac{1}{s} p_{6ij}^{(1)} R_{ij}(s) R_0(s) J_0(sr) \exp(s\tau_{ij}z) ds
\end{aligned}
\right\} (i = 1, 2) \qquad (30)
$$

where the constants $p_{kij}^{(1)}$ $(i, j = 1, 2, k = 1, 2, ..., 6)$ are given in Appendix A. The general solutions are obtained by using the Hankel transform technique (Sneddon & Lowengrub, 1969):

$$
\left.
\begin{aligned}
\sigma_{zzi}^{(2)}(r,z) &= \sum_{j=1}^{6} \int_0^\infty s p_{1ij}^{(2)} A_{ij}(s) J_0(sr) \exp(s\gamma_{ij}z) ds, \\
\sigma_{zri}^{(2)}(r,z) &= \sum_{j=1}^{6} \int_0^\infty s p_{2ij}^{(2)} A_{ij}(s) J_1(sr) \exp(s\gamma_{ij}z) ds, \\
D_{zi}^{(2)}(r,z) &= \sum_{j=1}^{6} \int_0^\infty s p_{3ij}^{(2)} A_{ij}(s) J_0(sr) \exp(s\gamma_{ij}z) ds, \\
u_{zi}^{(2)}(r,z) &= \sum_{j=1}^{6} \int_0^\infty p_{4ij}^{(2)} A_{ij}(s) J_0(sr) \exp(s\gamma_{ij}z) ds, \\
u_{ri}^{(2)}(r,z) &= \sum_{j=1}^{6} \int_0^\infty p_{5ij}^{(2)} A_{ij}(s) J_1(sr) \exp(s\gamma_{ij}z) ds, \\
\phi_i^{(2)}(r,z) &= -\sum_{j=1}^{6} \int_0^\infty p_{6ij}^{(2)} A_{ij}(s) J_0(sr) \exp(s\gamma_{ij}z) ds
\end{aligned}
\right\} (i = 1, 2) \qquad (31)
$$

where $A_{ij}(s)$ $(i = 1, 2, j = 1, 2, ..., 6)$ are the unknown functions to be solved, and the constants γ_{ij} and $p_{kij}^{(2)}$ $(i = 1, 2, j, k = 1, 2, ..., 6)$ are given in Appendix B.

Similar to the temperature analysis, the problem may be reduced to a system of singular integral equations by taking the second boundary conditions (7)-(9) into consideration and by defining the following new unknown functions $G_l(r)$ $(l = 1, 2, 3)$:

$$G_1(r) = \begin{cases} \dfrac{\partial}{\partial r}\left\{ u_{z1}^{(2)}(r,0) - u_{z2}^{(2)}(r,0) \right\} & (0 \leq r < c) \\ 0 & (c \leq r < \infty) \end{cases} \tag{32}$$

$$G_2(r) = \begin{cases} r\dfrac{\partial}{\partial r}\left[\dfrac{1}{r}\left\{ u_{r1}^{(2)}(r,0) - u_{r2}^{(2)}(r,0) \right\} \right] & (0 \leq r < c) \\ 0 & (c \leq r < \infty) \end{cases} \tag{33}$$

$$G_3(r) = \begin{cases} -\dfrac{\partial}{\partial r}\left\{ \phi_1^{(2)}(r,0) - \phi_2^{(2)}(r,0) \right\} & (0 \leq r < c) \\ 0 & (c \leq r < \infty) \end{cases} \tag{34}$$

Making use of the first boundary conditions (7)-(9) with Eqs.(10), we have the following system of singular integral equations for the determination of the unknown functions $G_l(t)$ $(l = 1, 2, 3)$:

$$\int_0^c t[\{ Z_{11}^\infty M_0^{(1)}(t,r) + M_{11}(t,r) \} G_1(t) + M_{12}(t,r)G_2(t) + \\ + \{ Z_{13}^\infty M_0^{(1)}(t,r) + M_{13}(t,r) \} G_3(t)]dt = \sigma_{zz0}(r) \quad (0 \leq r < c) \tag{35}$$

$$\int_0^c t[M_{21}(t,r)G_1(t) + \{ Z_{22}^\infty M_1^{(1)}(t,r) + M_{22}(t,r) \} G_2(t) + \\ + M_{23}(t,r)G_3(t)]dt = \sigma_{zr0}(r) \quad (0 \leq r < c) \tag{36}$$

$$\int_0^c t[\{ Z_{31}^\infty M_0^{(1)}(t,r) + M_{31}(t,r) \} G_1(t) + M_{32}(t,r)G_2(t) + \\ + \{ Z_{33}^\infty M_0^{(1)}(t,r) + M_{33}(t,r) \} G_3(t)]dt = D_{z0}(r) \quad (0 \leq r < c) \tag{37}$$

where the kernel functions $M_1^{(1)}(t,r)$, $M_{kl}(t,r)$ and the constants Z_{kl}^∞ $(k, l = 1, 2, 3)$ are given by

$$M_1^{(1)}(t,r) = \begin{cases} \dfrac{4}{\pi r t}\left\{ K\left(\dfrac{r}{t}\right) - E\left(\dfrac{r}{t}\right) \right\} + \dfrac{2}{\pi r t}\left\{ \dfrac{t^2}{t^2 - r^2}E\left(\dfrac{r}{t}\right) - K\left(\dfrac{r}{t}\right) \right\} & (r < t), \\ \dfrac{4}{\pi t^2}\left\{ K\left(\dfrac{t}{r}\right) - E\left(\dfrac{t}{r}\right) \right\} + \dfrac{2}{\pi}\dfrac{1}{t^2 - r^2}E\left(\dfrac{t}{r}\right) & (r > t) \end{cases} \tag{38}$$

$$M_{kl}(t,r) = \begin{cases} \displaystyle\int_0^\infty s\left\{ Z_{kl}(s) - Z_{kl}^\infty \right\} J_0(sr)J_1(st)ds & (k = 1,3, l = 1,3), \\ \displaystyle\int_0^\infty s\left\{ Z_{kl}(s) - Z_{kl}^\infty \right\} J_0(sr)J_2(st)ds & (k = 1,3, l = 2), \\ \displaystyle\int_0^\infty s\left\{ Z_{kl}(s) - Z_{kl}^\infty \right\} J_1(sr)J_1(st)ds & (k = 2, l = 1,3), \\ \displaystyle\int_0^\infty s\left\{ Z_{kl}(s) - Z_{kl}^\infty \right\} J_1(sr)J_2(st)ds & (k = 2, l = 2) \end{cases} \tag{39}$$

$$Z_{kl}(s) = \sum_{j=1}^{6} p_{k1j}^{(2)} d_{1jl}(s), \quad Z_{kl}^{\infty} = \lim_{s \to \infty} Z_{kl}(s) \quad (k,l=1,2,3) \tag{40}$$

In Eq.(40), the functions $d_{1jl}(s)$ $(j=1,2,...,6, l=1,2,3)$ are given in Appendix C.

The functions $\sigma_{zz0}(r)$, $\sigma_{zr0}(r)$ and $D_{z0}(r)$, which correspond to the stress and electric displacement components induced by the disturbed temperature field $T_i^{(2)}(r,z)$ $(i=1,2)$ on the r-axis in the plate without crack, are obtained as follows:

$$\left.\begin{aligned}
\sigma_{zz0}(r) &= \int_0^{\infty} R_0(s) \left\{ \sum_{j=1}^{6} p_{11j}^{(2)} d_{1j}^T(s) + \sum_{j=1}^{2} p_{11j}^{(1)} R_{1j}(s) \right\} J_0(sr) ds, \\
\sigma_{zr0}(r) &= \int_0^{\infty} R_0(s) \left\{ \sum_{j=1}^{6} p_{21j}^{(2)} d_{1j}^T(s) + \sum_{j=1}^{2} p_{21j}^{(1)} R_{1j}(s) \right\} J_1(sr) ds, \\
D_{z0}(r) &= \int_0^{\infty} R_0(s) \left\{ \sum_{j=1}^{6} p_{31j}^{(2)} d_{1j}^T(s) - \sum_{j=1}^{2} p_{31j}^{(1)} R_{1j}(s) \right\} J_0(sr) ds
\end{aligned}\right\} \tag{41}$$

where the functions $d_{1j}^T(s)$ $(j=1,2,...,6)$ are also given in Appendix C. These components are superficial quantities and have no physical meaning in this analysis. However, they are equivalent to the crack face tractions in solving the crack problem by a proper superposition.

To solve the singular integral equations (21) and (35)- (37) by using the Gauss-Jacobi integration formula (Sih, 1972), we introduce the following functions $\Phi_l(t)$ $(l=0,1,2,3)$:

$$G_l(t) = \left(\frac{c+t}{c-t} \right)^{1/2} \Phi_l(t) \quad (l=0,1,2,3) \tag{42}$$

Then the stress intensity factors K_{I}, K_{II} and the electric displacement intensity factor K_{D} may be defined and evaluated as:

$$\left.\begin{aligned}
K_{\mathrm{I}} &= \lim_{r \to c^+} \{2\pi(c-r)\}^{1/2} \sigma_{zz1}(r,0) = (\pi c)^{1/2} \left\{ Z_{11}^{\infty} \Phi_1(c) + Z_{13}^{\infty} \Phi_3(c) \right\}, \\
K_{\mathrm{II}} &= \lim_{r \to c^+} \{2\pi(c-r)\}^{1/2} \sigma_{zr1}(r,0) = (\pi c)^{1/2} Z_{22}^{\infty} \Phi_2(c), \\
K_{\mathrm{D}} &= \lim_{r \to c^+} \{2\pi(c-r)\}^{1/2} D_{z1}(r,0) = (\pi c)^{1/2} \left\{ Z_{31}^{\infty} \Phi_1(c) + Z_{33}^{\infty} \Phi_3(c) \right\}
\end{aligned}\right\} \tag{43}$$

5. Numerical results and discussion

For the numerical calculations, the thermo-electro-elastic properties of the plate are assumed to be ones of cadmium selenide with the following properties (Ashida & Tauchert, 1998).

The values of the coefficients of heat conduction for cadmium selenide could not be found in the literature. Since the values of them for orthotropic Alumina (Al$_2$O$_3$) are $\kappa_r = 21.25 [\mathrm{W/mK}]$ and $\kappa_z = 29.82 [\mathrm{W/mK}]$ (Dag, 2006), the value $\kappa^2 = \kappa_r / \kappa_z = 1/1.5$ is assumed. To examine the effects of the normalized crack size c/h and the normalized crack

location h_1/h on the stress and electric displacement intensity factors, the solutions of the system of the singular integral equations have been computed numerically.

$$
\left.
\begin{aligned}
&c_{11} = 74.1 \times 10^9 [\mathrm{N/m^2}], \qquad c_{12} = 45.2 \times 10^9 [\mathrm{N/m^2}], \\
&c_{13} = 39.3 \times 10^9 [\mathrm{N/m^2}], \qquad c_{33} = 83.6 \times 10^9 [\mathrm{N/m^2}], \\
&c_{44} = 13.2 \times 10^9 [\mathrm{N/m^2}], \\
&\quad e_{31} = -0.16 [\mathrm{C/m^2}], \qquad\quad e_{33} = 0.347 [\mathrm{C/m^2}], \\
&\quad e_{15} = -0.138 [\mathrm{C/m^2}], \\
&\varepsilon_{11} = 82.6 \times 10^{-12} [\mathrm{C/Vm}], \quad \varepsilon_{33} = 90.3 \times 10^{-12} [\mathrm{C/Vm}], \\
&\lambda_{11} = 0.621 \times 10^6 [\mathrm{N/Km^2}], \quad \lambda_{33} = 0.551 \times 10^6 [\mathrm{N/Km^2}], \\
&p_z = -2.94 \times 10^{-6} [\mathrm{CK^{-1}m^{-2}}].
\end{aligned}
\right\}
\tag{44}
$$

In the first set of calculations, we consider the temperature field and the electro-elastic fields without crack. Figure 2 shows the normalized temperature $(T_i(x) - T_{20})/T_0$ $(i = 1,2)$ on the crack faces $(0 \le r < c, z \to 0^{\pm})$ and the crack extended line $(c \le r \le 2c, z = 0)$ for $h_1/h = 0.25$ and $c/h = 0.5$, where $T_0 = T_{10} - T_{20}$. The maximum local temperature difference across the crack occurs at the center of the crack.

Figure 3 exhibits the normalized stress components $(\sigma_{zz0}(r), \sigma_{zr0}(r))/\lambda_{33}T_0$ and the electric displacement component $D_{z0}(r)/p_z T_0$ on the r-axis in the strip without crack due to the temperature shown in Figure 2. The maximum absolute values of $\sigma_{zz0}(r)$ and $D_{z0}(r)$ occur at the center of the crack $(r/c = 0.0)$, whereas the maximum value of $\sigma_{zr0}(r)$ occurs at the crack tip $(r/c = 1.0)$.
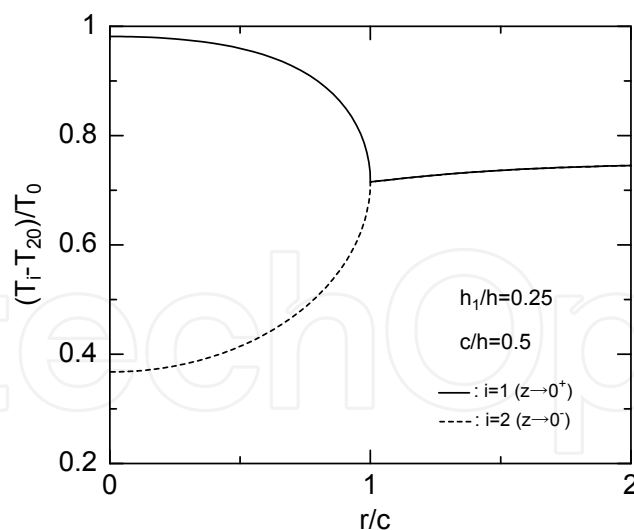
Fig. 2. The temperature on the crack faces and the crack extended line for $c/h = 0.5$ and $h_1/h = 0.25$

In the second set of calculations, we study the influence of the crack size on the stress and electric displacement intensity factors. Figures 4(a)-(c) show the plots of the normalized stress and electric displacement intensity factors $(K_I, K_{II})/\lambda_{33}T_0(\pi c)^{1/2}$, $K_D/p_z T_0(\pi c)^{1/2}$ versus c/h for $h_1/h = 0.25$, 0.5 and 0.75. Because of symmetry, the values of K_I and K_D
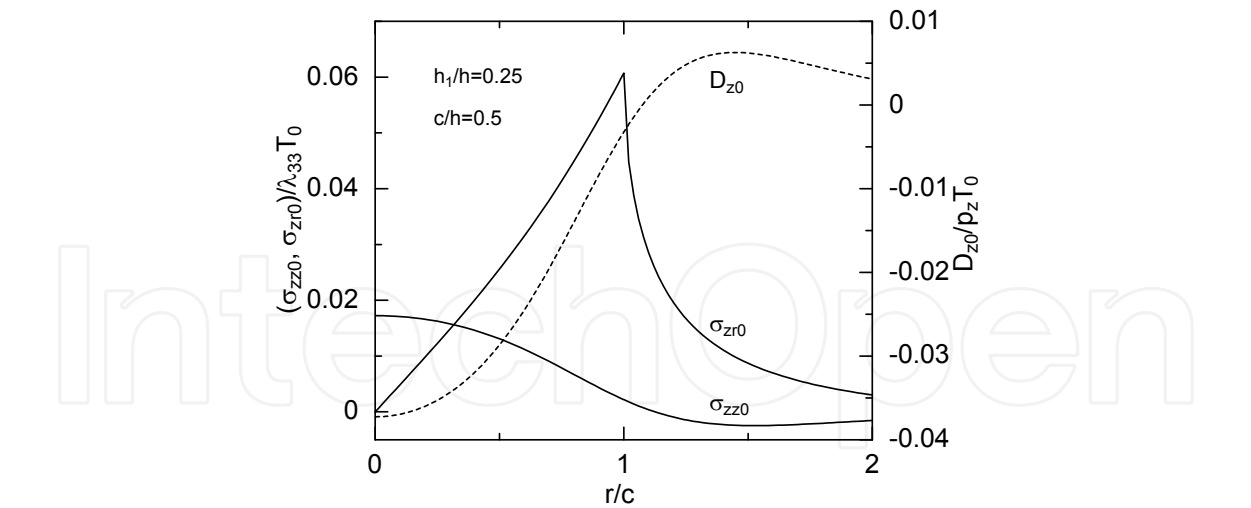
Fig. 3. The stress components σ_{zz0}, σ_{zr0} and the electric displacement component D_{z0} on the r-axis without crack due to the temperature shown in Fig. 2
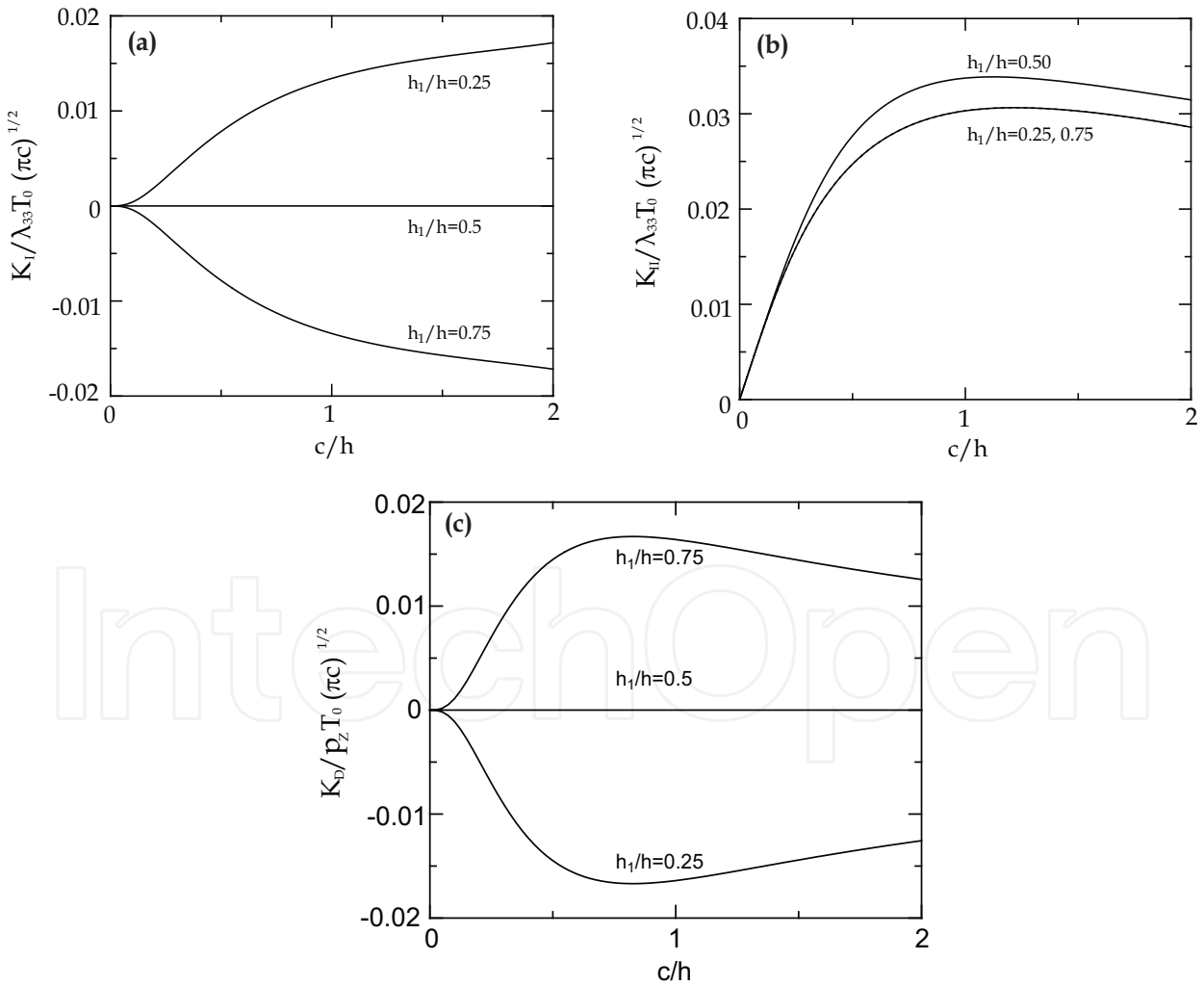
Fig. 4. (a) The effect of the crack size on the stress intensity factor K_{I}. (b) The effect of the crack size on the stress intensity factor K_{II}. (c) The effect of the crack size on the electric displacement intensity factor K_{D}

for $h_1 / h = 0.5$ are zero, and $[K_I]_{h_1/h=0.25} = -[K_I]_{h_1/h=0.75}$, $[K_{II}]_{h_1/h=0.25} = [K_{II}]_{h_1/h=0.75}$ $[K_D]_{h_1/h=0.25} = -[K_D]_{h_1/h=0.75}$. The absolute value of $K_I / \lambda_{33}T_0(\pi c)^{1/2}$ for $h_1 / h = 0.25$ and 0.75 monotonically increases with increasing c / h, but the value of $K_{II} / \lambda_{33}T_0(\pi c)^{1/2}$ and the absolute value of $K_D / p_z T_0(\pi c)^{1/2}$ increase at first, reach maximum values and then decrease with increasing c / h. The value of K_I for $h_1 / h = 0.75$ becomes negative so that the contact of the crack faces would occur. The results presented here without considering this effect may not be exact but would be more conservative. Since the contact of the crack faces will increase the friction between the faces and make thermo-electrical transfer across the crack faces easier, the stress and electric displacement intensity factors would be lowered by these two factors.

In the final set of calculations, we investigate the influence of the crack location on the intensity factors. Figure 5 indicates the effect of the crack location on K_I, K_{II} and K_D for $c / h = 0.5$. As h_1 / h increases, the values of K_I and K_D tend to decrease or increase monotonically. The value of $K_{II} / \lambda_{33}T_0(\pi c)^{1/2}$ decreases if the crack approaches the free boundaries $(h_1 / h \to 0.0 \text{ or } 1.0)$, and the peak value of $K_{II} / \lambda_{33}T_0(\pi c)^{1/2} = 0.0277$ occurs at $h_1 / h = 0.5$.
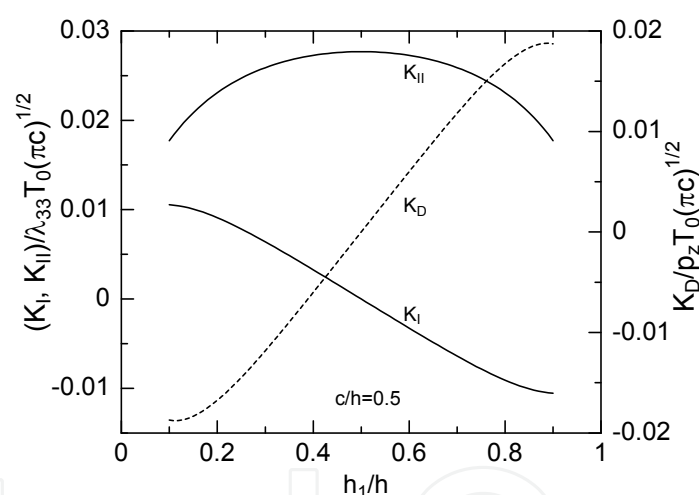

Fig. 5 The effect of the crack location on the stress intensity factors K_I, K_{II} and the electric displacement intensity factor K_D

6. Conclusion

An example of the application of Hankel transform for solving a mixed-mode thermo-electro-elastic fracture problem of a piezoelectric material strip with a parallel penny-shaped crack is explained. The effects of the crack size (c / h) and the crack location (h_1 / h) on the fracture behavior are analyzed. The following facts can be found from the numerical results.

1. The large shear stress occurs in the strip without crack due to the disturbed temperature field.
2. The normalized intensity factors are under the great influence of the geometric parameters h_1 / h and c / h.

3. For the case of $h_1/h > 0.5$, mode I stress intensity factor becomes negative so that the contact of the crack faces would occur.

4. The intensity factors of crack near the free surfaces due to the thermal load are not so large.

Appendix A

The constants $p_{kij}^{(1)}$ $(i,j=1,2,k=1,2,...,6)$ are

$$
\left.\begin{aligned}
p_{1ij}^{(1)} &= \left(c_{13} - c_{33}k_{ij}\tau_{ij}^{\ 2}\right)C_{ij} - e_{33}\tau_{ij}N_{ij} - \lambda_{33}, \\
p_{2ij}^{(1)} &= c_{44}\left(1+k_{ij}\right)\tau_{ij}C_{ij} + e_{15}N_{ij}, \\
p_{3ij}^{(1)} &= \left(e_{31} - e_{33}k_{ij}\tau_{ij}^{\ 2}\right)C_{ij} + \varepsilon_{33}\tau_{ij}N_{ij} + p_z, \\
p_{4ij}^{(1)} &= -k_{ij}\tau_{ij}C_{ij},\ p_{5ij}^{(1)} = C_{ij},\ p_{6ij}^{(1)} = N_{ij}
\end{aligned}\right\} \quad (i,j=1,2) \tag{A.1}
$$

where

$$
\left.\begin{aligned}
C_{ij} &= \frac{b'_{21} - b'_{22}\tau_{ij}^2}{m_{ij}^{(1)} + k_{ij}m_{ij}^{(2)}}, \\
N_{ij} &= \frac{n_{ij}^{(11)} + k_{ij}n_{ij}^{(12)}}{m_{ij}^{(1)} + k_{ij}m_{ij}^{(2)}}, \\
k_{ij} &= -\frac{\tau_{ij}^2 n_{ij}^{(11)} + n_{ij}^{(21)}}{\tau_{ij}^2 n_{ij}^{(12)} + n_{ij}^{(22)}}
\end{aligned}\right\} \quad (i,j=1,2) \tag{A.2}
$$

with

$$
\left.\begin{aligned}
m_{ij}^{(1)} &= a'_{41} - a'_{42}\tau_{ij}^2, \\
m_{ij}^{(2)} &= \left(a'_{44}\tau_{ij}^2 - a'_{43}\right)\tau_{ij}^2, \\
n_{ij}^{(11)} &= \left\{(H_{1r} + H_{3r})\left(b'_{21} - b'_{22}\tau_{ij}^2\right) + H_{4r}m_{ij}^{(1)}\right\}\tau_{ij}, \\
n_{ij}^{(21)} &= \left\{(H_{1z} + H_{3z})\left(b'_{21} - b'_{22}\tau_{ij}^2\right) + H_{4z}m_{ij}^{(1)}\right\}\tau_{ij}, \\
n_{ij}^{(12)} &= \left\{\left(H_{1r} - H_{2r}\tau_{ij}^2\right)\left(b'_{21} - b'_{22}\tau_{ij}^2\right) + H_{4r}m_{ij}^{(2)}\right\}\tau_{ij}, \\
n_{ij}^{(22)} &= \left\{\left(H_{1z} - H_{2z}\tau_{ij}^2\right)\left(b'_{21} - b'_{22}\tau_{ij}^2\right) + H_{4z}m_{ij}^{(2)}\right\}\tau_{ij}
\end{aligned}\right\} \quad (i,j=1,2) \tag{A.3}
$$

$$
\left.\begin{aligned}
H_{1r} &= \frac{c_{44}\varepsilon_{33} + e_{15}e_{33}}{e_{33}\varepsilon_{11} - e_{15}\varepsilon_{33}}, \quad H_{1z} = \frac{c_{44}\varepsilon_{11} + e_{15}^2}{e_{15}\varepsilon_{33} - e_{33}\varepsilon_{11}}, \quad H_{2r} = \frac{c_{33}\varepsilon_{33} + e_{33}^2}{e_{33}\varepsilon_{11} - e_{15}\varepsilon_{33}}, \quad H_{2z} = \frac{c_{33}\varepsilon_{11} + e_{15}e_{33}}{e_{15}\varepsilon_{33} - e_{33}\varepsilon_{11}}, \\
H_{3r} &= \frac{c_{13}\varepsilon_{33} + e_{31}e_{33}}{e_{33}\varepsilon_{11} - e_{15}\varepsilon_{33}}, \quad H_{3z} = \frac{c_{13}\varepsilon_{11} + e_{15}e_{31}}{e_{15}\varepsilon_{33} - e_{33}\varepsilon_{11}}, \quad H_{4r} = \frac{p_z e_{33} - \lambda_{33}\varepsilon_{33}}{e_{33}\varepsilon_{11} - e_{15}\varepsilon_{33}}, \quad H_{4z} = \frac{p_z e_{15} - \lambda_{33}\varepsilon_{11}}{e_{15}\varepsilon_{33} - e_{33}\varepsilon_{11}}
\end{aligned}\right\} \tag{A.4}
$$

$$a'_{41} = c_{11}, \qquad\qquad\qquad a'_{42} = c_{44} + (e_{15} + e_{31})(H_{1r} + H_{3r}),$$
$$a'_{43} = c_{13} + c_{44} + (e_{15} + e_{31})H_{1r}, \quad a'_{44} = (e_{15} + e_{31})H_{2r}, \qquad\qquad (A.5)$$
$$b'_{21} = \lambda_{11}, \qquad\qquad\qquad b'_{22} = -(e_{15} + e_{31})H_{4r}$$

Appendix B

The constants γ_{ij} $(i = 1, 2, j = 1, 2, \ldots, 6)$ are the roots of the following characteristic equations:

$$(f_4 g'_2 + g_4 f'_2)\gamma_{ij}^6 + (f_4 g'_0 + f_2 g'_2 + g_4 f'_0 + g_2 f'_2)\gamma_{ij}^4 +$$
$$+ (f_2 g'_0 + f_0 g'_2 + g_2 f'_0 + g_0 f'_2)\gamma_j^2 + (f_0 g'_0 + g_0 f'_0) = 0 \qquad (i = 1, 2, \ j = 1, 2, \ldots, 6) \qquad (B.1)$$

where $\Re[\gamma_{1j}] < \Re[\gamma_{1j+1}]$, $\Re[\gamma_{2j}] > \Re[\gamma_{2j+1}]$ $(j = 1, 2, \ldots, 5)$ and

$$\begin{cases} f_4 = c_{44} e_{33}, \\ f_2 = (c_{13} + c_{44})(e_{15} + e_{31}) - c_{11} e_{33} - c_{44} e_{15}, \\ f_0 = c_{11} e_{15}, \\ f'_2 = c_{33}(e_{15} + e_{31}) - e_{33}(c_{13} + c_{44}), \\ f'_0 = -c_{44}(e_{15} + e_{31}) + e_{15}(c_{13} + c_{44}) \end{cases} \qquad (B.2)$$

$$\begin{cases} g_4 = c_{44}\varepsilon_{33}, \\ g_2 = -(e_{15} + e_{31})^2 - c_{11}\varepsilon_{33} - c_{44}\varepsilon_{11}, \\ g_0 = c_{11}\varepsilon_{11}, \\ g'_2 = e_{33}(e_{15} + e_{31}) + \varepsilon_{33}(c_{13} + c_{44}), \\ g'_0 = -e_{15}(e_{15} + e_{31}) - \varepsilon_{11}(c_{13} + c_{44}) \end{cases} \qquad (B.3)$$

The functions $p_{kij}^{(2)}(s)$ $(i = 1, 2, j, k = 1, 2, \ldots, 6)$ are

$$\begin{cases} p_{1ij}^{(2)} = c_{13} a_{ij} + \gamma_{ij}\left(c_{33} - e_{33} b_{ij}\right), \\ p_{2ij}^{(2)} = c_{44}\left(\gamma_{ij} a_{ij} - 1\right) + e_{15} b_{ij}, \\ p_{3ij}^{(2)} = e_{31} a_{ij} + \gamma_{ij}\left(e_{310} + \varepsilon_{330} b_{ij}\right), \\ p_{4ij}^{(2)} = 1, \\ p_{5ij}^{(2)} = a_{ij}, \\ p_{6ij}^{(2)} = b_{ij} \end{cases} \qquad (i = 1, 2, j = 1, 2, \ldots, 6) \qquad (B.4)$$

where a_{ij} and b_{ij} $(i = 1, 2, j = 1, 2, \ldots, 6)$ are given by

$$\begin{cases} a_{ij} = \dfrac{g'_2 \gamma_{ij}^2 + g'_0}{g_4 \gamma_{ij}^4 + g_2 \gamma_{ij}^2 + g_0}, \\ b_{ij} = -\dfrac{(c_{44}\gamma_{ij}^2 - c_{11})a_{ij} - c_{13} - c_{44}}{e_{15} + e_{31}} \end{cases} \qquad (i = 1, 2, j = 1, 2, \ldots, 6) \qquad (B.5)$$

Appendix C

The functions $d_{ijk}(s)$ $(i = 1, 2, j = 1, 2, ..., 6, k = 1, 2, 3)$ are given by

$$\left.\begin{array}{l} d_{1jk}(s) = q_{j,k+9}(s), \\ \\ d_{2jk}(s) = q_{j+6,k+9}(s) \end{array}\right\} \quad (j = 1, 2, ..., 6, k = 1, 2, 3) \qquad (C.1)$$

where the functions $q_{j,k}(s)$ $(j, k = 1, 2, ..., 12)$ are the elements of a square matrix $Q = \Delta^{-1}$ of order 12. The elements $\delta_{j,k}(s)$ $(j, k = 1, 2, ..., 12)$ of the square matrix Δ are given by

$$\left.\begin{array}{ll} \delta_{j,k}(s) = p_{j1k}^{(2)} \exp(s\gamma_{1k}h_1) & (j = 1, 2, 3), \\ \delta_{j+3,k+6}(s) = p_{j2k}^{(2)} \exp(-s\gamma_{2k}h_2) & (j = 1, 2, 3), \\ \delta_{j+6,k}(s) = p_{j1k}^{(2)} & (j = 1, 2, ..., 6), \\ \delta_{j+6,k+6}(s) = -p_{j2k}^{(2)} & (j = 1, 2, ..., 6) \end{array}\right\} (k = 1, 2, ..., 6) \qquad (C.2)$$

The functions $d_{ij}^T(s)$ $(i = 1, 2, j = 1, 2, ..., 6)$ are

$$\left.\begin{array}{l} d_{1j}^T(s) = \displaystyle\sum_{k=1}^{12} q_{j,k}(s)u_k(s), \\ \\ d_{2j}^T(s) = \displaystyle\sum_{k=1}^{12} q_{j+6,k}(s)u_k(s) \end{array}\right\} (j = 1, 2, ..., 6) \qquad (C.3)$$

where

$$\left.\begin{array}{ll} u_k(s) = -\dfrac{R_0(s)}{s} \displaystyle\sum_{j=1}^{2} p_{k1j}^{(1)} R_{1j}(s)\exp(s\tau_{1j}h_1) & (k = 1, 2, 3), \\ \\ u_{k+3}(s) = -\dfrac{R_0(s)}{s} \displaystyle\sum_{j=1}^{2} p_{k2j}^{(1)} R_{2j}(s)\exp(-s\tau_{2j}h_2) & (k = 1, 2, 3), \\ \\ u_{k+6}(s) = -\dfrac{R_0(s)}{s} \displaystyle\sum_{j=1}^{2} \left\{ p_{k1j}^{(1)} R_{1j}(s) - p_{k2j}^{(1)} R_{2j}(s) \right\} & (k = 1, 2, ..., 6) \end{array}\right\} \qquad (C.4)$$

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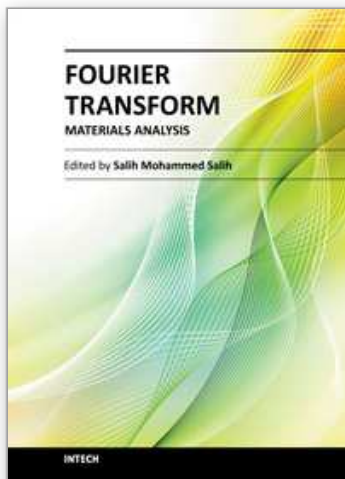
Fourier Transform - Materials Analysis

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The field of material analysis has seen explosive growth during the past decades. Almost all the textbooks on materials analysis have a section devoted to the Fourier transform theory. For this reason, the book focuses on the material analysis based on Fourier transform theory. The book chapters are related to FTIR and the other methods used for analyzing different types of materials. It is hoped that this book will provide the background, reference and incentive to encourage further research and results in this area as well as provide tools for practical applications. It provides an applications-oriented approach to materials analysis written primarily for physicist, Chemists, Agriculturalists, Electrical Engineers, Mechanical Engineers, Signal Processing Engineers, and the Academic Researchers and for the Graduate Students who will also find it useful as a reference for their research activities.

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