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Relativistic Metrology:
From Earth to Astrophysics

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1. Introduction

Till twenty years ago the basic concepts of special and general relativity were introduced by speaking of *clocks* and *rods* with an unspecified structure. Then the advances in atomic physics and in space navigation led to a revolution in metrology with the elimination of these old idealized notions and their replacements with realistic standards. For instance one can compare the 1965 point of view of Ref. (Basri S.A., 1965) with the 1997 one of Ref. (Guinot B., 1997).

However also today many scientists still think in terms of the *absolute notions of time and space* present in the Galilei space-time used in Newtonian physics, due to the fact that on Earth non-relativistic quantum mechanics is able to treat consistently problems ranging from molecular physics till quantum information without taking into account gravity (when needed Newtonian gravity is used). Only the description of light in atomic physics requires relativity (the trajectories of photons do not exist in Galilei space-time). Therefore most of the problems are formulated in inertial frames centered on inertial observers (having a constant velocity) in Galilei space-time (they are connected by the group of Galilei transformations containing space and time translations, spatial rotations and boosts) and, if needed, extended to non-relativistic accelerated frames taking into account the associated inertial apparent forces. The rotation of the Earth and its motion around the Sun are negligible effects for this type of physics.

Instead particle physics must face high speed objects and needs the Minkowski space-time of special relativity. Now the notions of space and time are no longer absolute: only the *global space-time is an absolute notion*. The Large Hadron Collider LHC particle accelerator at CERN is described with the coordinate time and the coordinate position of an inertial frame of Minkowski space-time centered on an inertial observer. To get the description with respect to another inertial observer one needs the group of Poincaré transformations (space and time translations plus Lorentz transformations Λ, i.e. spatial rotations and boosts; $x^{'\mu} = a^\mu + \Lambda^\mu{}_\nu x^\nu$). In the new inertial frame the new coordinate time (and also the coordinate position) depends on both the old coordinate time and positions. This has generated an endless (and still going on) philosophical discussion on the meaning of time. Since the Lorentz-scalar line element joining two nearby points in an inertial frame of Minkowski space-time is $ds^2 = (dx^o)^2 - \sum_r (dx^r)^2$ (with the particle physics conventions; $ds^2 = -(dx^o)^2 + \sum_r (dx^r)^2$ with the general relativity ones; x^μ are inertial time, $x^o = ct$, and space, x^r, coordinates), space and time increments have different sign (Lorentz signature). The

only intrinsic structure of Minkowski space-time is that in each point A there is a light-cone (or null cone) defined by $ds^2 = 0$, which is the locus of the ray of light (traveling with the velocity of light) arriving in that point from the past or emanating from that point towards the future. The points inside the light-cone in A have a time-like distance from A and can be reached (if in the future) by traveling with a velocity less than the velocity of light. The points outside the light-cone of A have space-like distance from A and could be reached only with super-luminal velocity. If there is an atomic clock in A moving along a time-like curve towards the future its Lorentz-scalar *proper time* is defined as $d\tau^2 = ds^2$ ($d\tau^2 = -ds^2$ with the other convention) and coincides with the coordinate time x^o only in the inertial rest frame of the clock. However, since time is not absolute, there is no intrinsic notion of 3-space and of synchronization of clocks: both of them have to be defined with some convention. As a consequence the *1-way velocity of light* from one observer A to an observer B has a meaning only after a choice of a convention for synchronizing the clock in A with the one in B. Therefore the crucial quantity in special relativity is the *2-way (or round trip) velocity of light c* involving only one clock: the observer A emits a ray of light which is reflected somewhere and then reabsorbed by A so that only the clock of A is implied in measuring the time of flight. It is this velocity which is isotropic and constant in special relativity.

In Minkowski space-time the Euclidean 3-spaces of the inertial frames centered on an inertial observer A are identified by means of Einstein convention for the synchronization of clocks: the inertial observer A sends a ray of light at x_i^o towards the (in general accelerated) observer B; the ray is reflected towards A at a point P of B world-line and then reabsorbed by A at x_f^o; by convention P is synchronous with the mid-point between emission and absorption on A's world-line, i.e. $x_P^o = x_i^o + \frac{1}{2}\left(x_f^o - x_i^o\right) = \frac{1}{2}\left(x_i^o + x_f^o\right)$. This convention selects the Euclidean instantaneous 3-spaces $x^o = ct = const.$ of the inertial frames centered on A. Only in this case the one-way velocity of light between A and B coincides with the two-way one, *c*. However, if the observer A is accelerated, the convention breaks down and we need a theory of non-inertial frames in Minkowski space-time as the one developed in Ref.(Alba D. et al, 2010, 2007). In this theory the transition from an inertial to a non-inertial frame (with its relativistic inertial forces and its non-Euclidean 3-spaces) can be described as a gauge transformation connecting two different generalized conventions for clock synchronization: therefore physics does not change, only the appearances of phenomena change.

However the International Space Station ISS near the Earth and space navigation in the Solar System require general relativity (at least its Post-Newtonian approximation) to take into account the effects of the gravitational field which is missing in special relativity. Now also space-time is no longer an absolute notion but is dynamically determined by Einstein's equations. Einstein's space-times have Lorentz signature but the structure of the light-cones changes from a point to another one. However rays of light, moving along null geodesics, are assumed to have the same 2-way velocity of light *c* as in special relativity, being an eikonal approximation to Maxwell equations. The equivalence principle implies that global inertial frames cannot exist: only locally near a particle in free fall we can have a local inertial frame and a local special relativistic approximation. Again there is the problem of clock synchronization for the definition of the non-Euclidean 3-spaces: even if the space-time is dynamically determined by Einstein's equations, each solution can be presented in arbitrary systems of 4-coordinates, since this is the gauge freedom of general relativity (form invariance of Einstein's equations under general coordinate transformations).

Therefore the presentation (gauge choice) of a solution of Einstein's equations is nothing else that a *metrology choice of a standard of space-time*, i.e. a choice of the time and space 4-coordinates.

Moreover this choice is fundamental for the description of macroscopic matter (its energy-momentum tensor is the source term in Einstein's equations) at the experimental level: physicists, space engineers and astronomers use an *intrinsically coordinate-dependent* (i.e. dependent on the chosen conventions) description of the trajectory of every macroscopic body (from spacecrafts to satellites, planets, stars,....).

Around the Earth *GPS (Global Positioning System) is a space-time standard* (Ashby N., 2003), relying on the time and length standards on the Earth surface. There is an array of 24 satellites around the Earth each one with an atomic clock with an accuracy which for modern commercial devices is today less than 30 nanoseconds. The satellites are at an altitude of 20,000 Km and have a mean velocity of 14,000 km/hr. Special relativity implies that these clocks tick more slowly (about 7 microseconds per day) than clocks on Earth. But general relativity implies that they tick faster (about 48 microseconds per day), so that a satellite clock advances faster than a clock on ground by about 38 microseconds per day. If we forget general relativity the precision of GPS localization (less that 15 meters) is lost within two minutes.

While in non-inertial frames in Galilei and Minkowski space-times there is a good understanding of the apparent inertial forces, in general relativity the gravitational field is described by the 4-metric tensor $^4g_{\mu\nu}(x)$ in an arbitrary 4-coordinate system centered on an arbitrary observer (the line element is now $ds^2 = {}^4g_{\mu\nu}(x)\,dx^\mu\,dx^\nu$) and it is not clear how to introduce a distinction between gravitational and inertial effects. However this is possible at the Hamiltonian level for the globally hyperbolic, asymptotically Minkowskian space-times, where it is possible to define global 3+1 splittings of the space-time, namely a foliation with 3-spaces evolving in time. The study of the 4-metric tensor in this framework allows one to disentangle the two physical degrees of freedom (or *tidal* variables) of the gravitational field (the two polarizations of gravitational waves in the linearized theory) and the gauge (or *inertial* variables) degrees of freedom describing the arbitrariness in the choice of the 4-coordinates. As shown in Ref. (Lusanna L., 2011) among the inertial variables there is the so-called York time (the trace of the extrinsic curvature of the 3-space as a 3-sub-manifold of space-time): it describes the remnant of the special relativistic gauge freedom in clock synchronization in this class of general relativistic space-times.

For the physics in the Solar System one assumes that the relevant Einstein space-times are globally hyperbolic (namely admitting a global definition of time) and asymptotically flat (namely tending to Minkowsky space-time at spatial infinity) space-times containing *N* bodies (the Sun and the planets) treated as point-like objects carrying multipoles (spin, moment of inertia,... of the extended body). A Post-Newtonian approximation is used in solving Einstein's equations in harmonic gauges and the gravitational waves inside the Solar System are shown to be negligible.

We have spoken only about Einstein general relativity. See Ref. (Will C.M., 2006, 1993) for the status of alternative theories of gravity inside the Solar System and Refs. (Soffel M.H., 1989; Brumberg V.A., 1991; Damour T. et al, 1991, 1992, 1993, 1994) for a treatment going from geodesy to celestial mechanics.

In what follows there will be a sketch, with update bibliography, of relativistic metrology inside the Solar System. It includes

A) Space and time standards.

B) The conventions needed for the description of satellites around the Earth: it is done by means of NASA (USA National Aeronautics and Space Administration) coordinates (Moyer T.D., 2003) firstly in the International Terrestrial Reference System ITRS (with an associated frame ITRF fixed on the Earth surface; see Ref. (IERS, 2004) for the IERS2003 conventions of the International Earth Rotation and Reference System Service IERS) and then in the Geocentric Celestial Reference System GCRS (with an associated non-rotating frame GCRF centered on the Earth center; see Ref. (Soffel M.H. et al, 2003) for the International Astronomic Units IAU2000).

C) The conventions needed for the description of planets and other objects in the Solar System uses the Barycentric Celestial Reference System BCRS (with an associated quasi-inertial Minkowski frame BCRF, if perturbations from the Milky Way are ignored, centered in the barycenter of the Solar System and with the axes specified by means of fixed stars (quasars) in the Hypparcos catalog (Kovalesky J. et al, 1989; Sovers O.J. et al, 1998; Ma C. et al, 1998; Johnstone K.J. et al, 1999; Fey A. et al, 2009)) and ephemerides (see IAU2000 in Ref. (Soffel M.H. et al, 2003)).

Ref. (Kaplan G.H., 2005) contains all the relevant aspects of these conventions.

While ITRF is essentially realized as a non-relativistic non-inertial frame in Galilei space-time, BCRF is defined as a *quasi-inertial frame*, *non-rotating* with respect to some selected fixed stars, in Minkowski space-time with nearly-Euclidean Newton 3-spaces. The qualification *quasi-inertial* is introduced to take into account general relativity, where inertial frames exist only locally. It can also be considered as a Post-Minkowskian space-time with 3-spaces having a very small extrinsic curvature. GCRF is obtained from BCRF by taking into account Earth's rotation around the Sun with a suitable Lorentz boost with corrections from Post-Newtonian gravity. By taking into account the extension of the geoid and Earth revolution around its axis one goes from the quasi-Minkowskian GCRF to the quasi-Galilean ITRF.

New problems emerge by going outside the Solar System. In astronomy the positions of stars and galaxies are determined from the data (luminosity, light spectrum, angles) on the sky, i.e. on a 2-dimensional spherical surface around the Earth with the relations between it and the observatory on the Earth done with GPS.

Then one needs a description of stars and galaxies as living in a 4-dimensional nearly-Galilei space-time with the International Celestial Reference System ICRS (see Refs. (Kovalesky J. et al, 1989; Sovers O.J. et al, 1998; Ma C. et al, 1998; Johnstone K.J. et al, 1999; Fey A. et al, 2009)), whose materialization ICRF is considered as a "quasi-inertial frame" in a "quasi-Galilei space-time", in accord with the assumed validity of the cosmological and Copernican principles. Namely one assumes a homogeneous and isotropic cosmological Friedmann-Robertson - Walker solution of Einstein equations (the standard ΛCDM cosmological model). In it the constant intrinsic 3-curvature of instantaneous 3-spaces is nearly zero as implied by the CMB data (Bartelmann M., 2010; Bean R., 2009), so that Euclidean 3-spaces (and Newtonian gravity) can be used (all galactic dynamics is Newtonian gravity). See Ref. (Lindegren L. et al, 2003) for the IAU conventions for defining the astrometric radial

velocity of stars taking into account astrometric positions, spectroscopy of star light and light propagation in gravitational fields.

However, to reconcile all the data with this 4-dimensional reconstruction one must postulate the existence of dark matter and dark energy as the dominant components of the classical universe (Durrer R., 2011; Bonvin C. et al, 2011; Garret K. et al, 2011; Ross M., 2010) after the recombination 3-surface (a kind of Heisenberg cut between quantum cosmology and classical astrophysics)!

2. Standards of length and time

In this Section we discuss the existing standards for length and time.

2.1 Standard of length

In 1975 in the 15th Meeting CGPM of the General Conference on Weights and Measures (Meeting 15, 1975) the conventional value of the 2-way velocity of light was fixed to be $c = 299792458\, m\, s^{-1}$.

In 1983 the 17th Meeting CGPM of the General Conference on Weights and Measures (Meeting 17, 1983) adopted the following standard of length

The meter is the length of the path traveled by light in vacuum during a time interval of $1/c$ of a second.

To measure the 3-distance between two objects in an inertial frame we send a ray of light from the first object, to which is associated an atomic clock, to the second one, where it is reflected and then reabsorbed by the first object. The measure of the flight time and the 2-way velocity of light determine the 3-distance between the objects.

This convention is compatible with the Euclidean 3-space of inertial frames in Minkowski space-time. When the technology will allow one to measure the deviations from Euclidean 3-space implied by Post-Newtonian gravity we will need a modified convention taking into account a general relativistic notion of length.

In astronomy the unit of length, defined in the IAU (1976) System of Astronomical Constants, is the *astronomical unit AU*, approximately equal to the mean Earth-Sun distance (Resolution 10, 1976; IBWM, 2006). It is the radius of an unperturbed circular Newtonian orbit about the Sun of a particle having infinitesimal mass, moving with an angular frequency of 0.017 202 098 95 radians per day. Measurements of the relative positions of planets in the Solar System are done by radar (or by telemetry from space probes): one measures the time taken for light to be reflected from an object using the conventional value of the velocity of light c.

Both for objects inside the Solar System and for the nearest stars one measure the distance with the *trigonometric parallax* by using the propagation of light and its velocity c in inertial frames. One measures the difference (the inclination angle) in the apparent position of an object viewed along two different lines of sight at two different times and then uses Euclidean geometry to evaluate the distance. The used unit in astronomy is the parsec, which is 3.26 light-years or $3.26\, 10^{16}$ meters.

However this convention cannot be used for more distant either galactic or extra-galactic objects (UCLA, 2007; Carrol B.W. et al, 2007). New notions like standard candles, dynamical parallax, spectroscopic parallax, luminosity distance,..... are needed. These notions involve

both aspects of light propagation in curved space-times and cosmological assumptions like the Hubble law (velocity-redshift linear relation). Therefore they belong to another type of metrology.

2.2 Atomic clocks and ACES

The time scales like the SI (International System of Units) *atomic second* are based on frequency standards for *microwave* atomic clocks based on isotopes like cesium (^{133}Cs) and rubidium (^{87}Rb) and with frequencies of the order of GigaHertz ($10^9 Hz$). While the national standard agencies (National Institute of Standards and Technology NIST in USA, National Physical Laboratory NPL in United Kingdom, Paris Observatory in France, Physikalisch-Technische Bundesanstalt PTB in Germany, Istituto Nazionale di Ricerca Metrologica INRIM in Italy) maintain an accuracy of 1 nanosecond per day ($1ns = 10^{-9}s$), many primary cesium atomic clocks using laser cooled atomic fountains have an inaccuracy less than 100 picoseconds per day ($1ps = 10^{-12}s$) with the best ones approaching 10 ps per day (Bize S. et al, 2005; Parker T.E., 2010).

If atomic clocks operating on different quantum transitions are considered as ideal clocks in general relativity, then they measure the same proper time (and not a coordinate time) along their trajectory (Guinot B., 1997). See Ref. (Reynaud S. et al, 2009) and its bibliography for the experiments on the *universality of clock rates* (relative frequency ratios between different clocks are constant at a level of the order of 10^{-16} per year). See also Ref. (Perlick V., 1987, 1994) for another general relativistic effect, the *second clock effect*, according to which two clocks synchronized at the same point, then separated and finally rejoined remain synchronized in Riemannian space-times like Einstein's ones but not in Weyl space-times.

A new family of *optical* atomic clocks in the region of $10^{15} Hz$ is developing quickly with the help of optical frequency-combs for direct optical frequency measurements. They allow one to reach a fractional frequency inaccuracy of better than 10^{-17} (corresponding to better than 1 ps per day) (Gill P., 2005; Rosenbad T. et al, 2008; Ludlow A.D. et al, 2008; Chou C.W. et al 2010a) and will become relevant for metrology in the near future. Moreover optical clocks allow to verify the "time dilation effect" for relative speeds of less than 10 m/s or for a change in height near the Earth's surface of less than 1 meter (Chou C.W., 2010b).

In Ref. (Arias E.F., 2005) there is a review of time metrology with a comparison of various time scales, the use of GPS receiver for time transfer (see also Ref. (Petit G. et al, 2005)) and on the dissemination and access to the international time scales. See also Refs. (Lemonde P. et al, 2001; SIGRAV 2006) for the status of atomic clocks in space near the Earth or on spacecrafts inside the Solar System.

The Atomic Clock Ensemble in Space (ACES) mission of the European Space Agency ESA (ACES 2010; Cacciapuoti L. et al, 2007, 2008; Blanchet L. et al, 2000), to be launched in 2015, aims to put a new microwave atomic clock (PHARAO, Projet d'Horloge Atomique par Refroidissement d'Atome en Orbite) together with an active hydrogen maser (SHM, Space active Hydrogen Maser) on the International Space Station (ISS; height 400 Km, rotation period 90 min, inclination angle 51.6^o). The two clocks will generate an on-board timescale with an expected frequency instability and inaccuracy at the 10^{-16} level. There will be a frequency comparison between the space clocks and ground clocks using microwave links: in particular ACES will give the first precision measurement of the gravitational redshift of the geoid, namely of the $1/c^2$ deformation of Minkowski light-cone caused by the geo-potential.

2.3 Time scales

The fundamental conceptual time scale is the *SI atomic second* whose definition is (Resolution 1, 1956)

The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom. This definition refers to a cesium atom at rest at a temperature of 0° K.

It gives a precise and constant rate of time measurement for *observers local to the apparatus on the surface of the Earth, i.e. on the rotating geoid* (for them it is a unit of *proper time* (Guinot B., 1997)), in which such seconds are counted.

However from 1971 the conventional practical high-precision atomic time standard is a coordinate time (Guinot B., 1997), the *International Atomic Time TAI*. TAI is defined as a suitable weighted average of the SI time kept by over 200 atomic clocks (mainly cesium clocks) in about 70 national laboratories worldwide (Circular T263, 2009). The comparison of the clocks is done using GPS signals and two-way satellite time and frequency transfer.

The next step is to connect TAI to the time scales based the *Earth Rotation*, which were used in astronomical applications as telescope pointing, depended on the geographical location of the observer and were based on observing celestial bodies crossing the meridian every day. Two such scales are:

Greenwich sidereal time is the hour angle of the equinox measured with respect to the Greenwich meridian.

Local sidereal time is the local hour angle of the equinox or the Greenwich sidereal time plus the longitude (east positive) of the observer, expressed in time units. Sidereal time appears in two forms, *mean* (GAST Greenwich Apparent mean Sidereal Time or LMST Local Mean Sidereal Time) and *apparent* (LAST, Local Apparent Sidereal Time), depending on whether the mean or true equinox is the reference point. The position of the mean equinox is affected only by *precession* while the true equinox is affected by both *precession and nutation*. Let us remember that the equinox is a direction in space along the nodal line defined by the intersection of the ecliptic (the plane of the Earth's orbit) and equatorial planes. The difference between true and mean sidereal time is the *equation of the equinoxes*, which is a complex periodic function with a maximum amplitude of about 1sec. Of the two forms, apparent sidereal time is more relevant to actual observations, since it includes the effect of nutation. Greenwich (or local) apparent sidereal time can be operationally obtained from the right ascensions of celestial objects transiting the Greenwich (or local) meridian.

Nowadays *Universal Time UT* is the generic timescale based on Earth's rotation. It is determined by Very Long Baseline Interferometry (VLBI) observations of distant quasars with an accuracy of microseconds. There are various variants of UT. The most used is UT1, based on VLBI observations of quasars, on Lunar Laser Ranging (LLR), on determination of GPS satellite orbits. UT1 is the same everywhere on Earth and is proportional to the rotation angle of the Earth with respect to distant quasars.

An approximate version of UT1 is the *Coordinate Universal Time UTC*. It is an atomic time scale and the international standard for *civil time*. It is a hybrid time scale (ITUR, 2007), which uses SI atomic seconds on the geoid (it usually has 86 400 SI seconds per day), but subject to occasional 1 second adjustments (the so-called *leap second*) to keep it within 0.9 seconds from

UT1 ($UT1 \approx UTC + DUT1$ with $DUT1 \approx \pm 0.1 sec$) and to have $TAI = UTC + \triangle AT$ ($\triangle AT$ is an integer number of leap seconds).

Other civil times given in Ref. (Moyer T.D., 2003) are

GPS Master Time: it is an atomic time for GPS receiving station on Earth and for GPS satellites - $TAI = GPS + const.$.

ST Station Time: it is an atomic time at a Deep Space Network (DSN) tracking station on Earth. It is assumed $UTC \; or \; GPS = ST + a + b \left(t - t_o\right) + c \left(t - t_o\right)^2$.

As UT is slightly irregular in its rate, astronomers introduced Ephemeris Time and then replaced it with Terrestrial Time TT.

The *Ephemeris Time T_{eph} = ET*, replacing an old barycentric dynamical time TDB, is a relativistic coordinate time based on high-precision ephemerides, which are lists of instantaneous positions of the centers of mass of Sun, Moon and planets with respect to (equatorial rectangular 3-coordinates of) BCRS for any date and time between 1600 and 2001, developed at the Jet Propulsion Laboratory (JPL) and denoted DE405/LE405 (Kaplan G.H., 2005; Standish E.M., 1998). Lunar rotation angles are also provided. The DE405 coordinate system has been aligned to the ICRS. The JPL ephemerides are computed by an N-body numerical integration carried out in BCRS.

Terrestrial Time TT, which is an astronomical time scale used for geocentric and topo-centric ephemerides. The "standard epoch" for modern astrometric reference data, designated J2000.0 is expressed as a TT instant: J2000.0 means 2000 January 1, 12^h TT at geo-center (Julian date JD 24515450 TT; J2000.0 is shorthand for the celestial reference system defined by the mean dynamical equator and equinox of J2000.0) (Kaplan G.H., 2005). TT is an idealized form of TAI ($TT = TAI + 32^s.184$). TT runs at the same rate as a time scale based on the SI second on the surface of the Earth.

As shown in Eq.(2.6) of Ref. (Kaplan G.H., 2005) we have $T_{eph} \approx TDB \approx TT + F(T)$, where $F(T)$ is a given function of the number T of Julian centuries of TT from J2000.0 ($T = \left(JD(TT) - 2451545.0\right)/36525$).

See also Ref.(Moyer T.D., 2003), p.18, where the following chains of transformations are defined

$T_{eph} \rightarrow TAI (\rightarrow UT1) \rightarrow UTC, GPS \rightarrow ST$ (ST = time scale of a tracking station on the Earth),

$T_{eph} \rightarrow TAI \rightarrow GPS \rightarrow ST$ (ST = time scale at an Earth satellite)

Let us remark that the astronomical universal time UT1 is defined by using the new earth precession-nutation theory denoted IAU2000A (relating the International Celestial Reference Frame ICRF to the International Terrestrial Reference Frame ITRF from 2003), which has been replaced in 2009 with a more dynamically consistent precession model denoted IAU2006 (Coppola V., 2009; IAU, 2006).

According to IAU2006 UT1 is *linear* in the Earth rotation angle θ, a geocentric angle (such that $\dot{\theta} = \omega_{earth}$ is the average angular velocity of rotation of the Earth) with a *Non-Rotating Origin NLO* in the equatorial plane orthogonal to the *Celestial Intermediate Pole CIP* from the axes centered in the *Celestial Intermediate Origin CIO* with no instantaneous rotation around

the Earth axis to the axes centered in the *Terrestrial Intermediate Origin TIO* rotating with the Earth (in IAU2000A CIO and TIO were called CEO, Celestial Ephemeris Origin, and TEO, Terrestrial Ephemeris Origin, respectively).

The lengths of the sidereal (θ) and UT1 seconds, and the value of $\dot{\theta}$, are not precisely constant when expressed in a uniform time scale such as TT. The accumulated difference in time measured by a clock keeping SI seconds on the geoid from that measured by the rotation of the Earth is $\triangle T = TT - UT1$. The long-term trend is for $\triangle T$ to increase gradually because of the tidal deceleration of the Earth's rotation, which causes UT1 to lag increasingly behind TT. In predicting the precise times of topo-centric phenomena, like solar eclipse contacts, both TT and UT1 come into play, and this requires assumptions about the value of $\triangle T$ at the time of the phenomenon. Alternatively, the circumstances of such phenomena can be expressed in terms of an *imaginary system of geographic meridians* that rotate uniformly about the Earth's axis ($\triangle T$ is assumed zero, so that UT1 = TT) rather than with the real Earth; the real value of $\triangle T$ then does not need to be known when the predictions are made. The zero-longitude meridian of the uniformly rotating system is called the *ephemeris meridian*.

Finally the astronomical conventions IUA2000 (Soffel M.H. et al, 2003) for the description of the Solar System (BCRS) and of the space near the Earth (GCRS) introduced the following two theoretical time scales not taken by any real clock but connected with Post-Newtonian solutions of Einstein's equations in special harmonic gauges with Sun, Earth, Moon, planets as matter.

Barycentric Coordinate Time - $t_B = TCB$ - it advances at a rate $1.55 \ 10^{-8}$ faster with respect to SI seconds on the surface of the Earth and is the time coordinate in BCRS.

Geocentric Coordinate Time - $t_G = TCG$ - it advances at a rate $6.97 \ 10^{-10}$ faster with respect to SI seconds on the surface of the Earth and is the time coordinate in GCRS. The connection to the terrestrial time is assumed to be $TT = TCG - L_G (TCG - t_o)$ with a constant rate $\frac{d\,TT}{dt_G} = 1 - L_G$ with $L_G = 6.969290134 \ 10^{-10}$, while the transformation connecting TCB and TCG is given in the next Section.

Let us notice that the discussion whether it is better to use primary conventions based on atomic clocks or to revert to astronomical conventions is still open (Finkleman D. et al, 2011) and will be discussed again in 2012. For a recent update on the problem of time see Ref. (McCarthy D.D., 2009). At this stage it is difficult to say which point of view will become more relevant in the near future: how to compare astronomic precisions connected to VLBI and LLR with theoretical problems of atomic clocks like whether an atomic fountain clock can be approximated with a mass-point with a well defined proper time?

3. The space-time in the Solar System and near the Earth

In this Section we will describe the conventions used to describe physics on the Earth's surface and space physics near the Earth and in the Solar System. Instead of starting from the Earth, where Newtonian gravity is dominating, we shall begin with the general relativistic description of the Solar System.

The IAU conventions (Soffel M.H. et al, 2003) for the Solar System identify a system of harmonic coordinates (a BCRF frame) centered on the solar system barycenter and a Post-Newtonian solution of Einstein's equations in a special harmonic gauge at the $O(1/c^3)$

order, which can be interpreted as an asymptotically-Minkowskian Post-Newtonian Einstein space-time. With a suitable coordinate transformation this solution is transformed in a description of the same space-time with new harmonic coordinates centered on the center of the Earth (a GCRF frame). However the presentation of this Einstein's space-time is strongly special relativistic (just $O(1/c^2)$ for the NASA coordinates of spacecrafts (Moyer T.D., 2003)) and becomes Galilean when one makes the transition from the coordinates with origin in the center of the Earth to coordinates fixed on the crust of the Earth (a ITRF frame in the IERS conventions) (IERS, 2003).

3.1 BCRS - Barycentric Celestial Reference System

The resolution B1.3 of IAU2000 (Soffel M.H. et al, 2003) states that the Barycentric Celestial Reference System BCRS is a global reference system of barycentric space-time coordinates for the Solar System within the framework of general relativity. It is centered in the barycenter of the Solar System, which can be considered as a quasi-inertial Minkowski observer with a constant 4-velocity (the time axis of the barycentric time $t_B = TCB$), because the effects of the Milky Way are negligible. Its spatial axes (in the instantaneous 3-spaces Σ_{t_B} with $t_B = const.$ with rectangular 3-coordinates x_B^i) are restricted to be *kinematically non-rotating*, namely they have no systematic rotation with respect to distant objects in the universe. For all practical applications the spatial axes are assumed to be oriented like the spatial axes of ICRS (see next Section). Therefore to each ICRF frame giving a materialization of ICRS is associated a BCRF frame.

The harmonic 4-coordinates and the retarded Post-Newtonian solution of Einstein's equations for the 4-metric $g_{B\mu\nu}(x_B)$ given in the IAU2000 conventions are

$$x_B^\mu = \left(x_B^o = c\, t_B; x_B^i\right),$$

$$g_{Boo}(x_B) = \epsilon \left[1 - \frac{2\, w_B(x_B)}{c^2} - \frac{2\, w_B^2(x_B)}{c^4} + O(c^{-5})\right],$$

$$g_{Boi}(x_B) = \epsilon \left[\frac{4\, w_{Bi}(x_B)}{c^3} + O(c^{-5})\right],$$

$$g_{Bij}(x_B) = -\epsilon\,^3 g_{Bij}(x_B) = -\epsilon \left[(1 + \frac{2\, w_B(x_B)}{c^2})\, \delta_{ij} + O(c^{-4})\right].$$

The signature of the 4-metric is the same as for Minkowski metric $\eta^{\mu\nu} = \epsilon\,(+---)$ ($\epsilon = +$ is the particle physics convention, $\epsilon = -$ is the general relativity one). The 3-metric $^3g_{Bij}(x_B)$ on the 3-spaces Σ_{t_B} is positive-definite.

See Appendix A of Ref. (Soffel M.H. et al, 2003) for the Post-Newtonian gravitational potentials $w_B(x_B)$ and $w_B^i(x_B)$ generated by the Sun and the planets. These extended bodies are usually approximated with their center of mass (mass monopole) carrying, when needed like for Saturn, a spin dipole.

The barycenter of the Solar System has coordinates $x_{B(B)}^\mu = \left(x_B^o; 0^i\right)$ and its world-line is a straight-line (the time axis) approximating a time-like geodesic of the 4-metric if we neglect galactic and extra-galactic influences. In each point of the barycentric world-line there is an *orthonormal tetrad* with the time-like 4-vector given by the barycenter 4-velocity $u_{B(B)}^\mu = \left(1; \vec{0}\right)$

and with the 3 mutually orthogonal spatial axes $\epsilon^{\mu}_{B(B)r} = \left(0; \vec{\epsilon}_{B(B)r}\right)$ whose orientation is determined by ICRS. The instantaneous 3-spaces Σ_{t_B} are considered as nearly Euclidean inertial 3-spaces. However their extrinsic 3-curvature as 3-sub-manifolds of the space-time is not zero but of order $O(c^{-2})$, so that strictly speaking they do not correspond to Einstein's clock synchronization convention.

The world-line $x^{\mu}_{B(G)}(x^o_B) = \left(x^o_B; \vec{x}_{B(G)}(x^o_B)\right)$ of the Earth's geo-center (a time-like geodesics of the Post-Newtonian 4-metric $g_{B\mu\nu}(x_B)$)) is determined by the *JPL ephemerides as solution of the equations of motion of the solar system bodies.* The geo-center has the 4-velocity $u^{\mu}_{B(G)}(x^o_B) = \frac{d\,x^{\mu}_{B(G)}(x^o_B)}{dx^o_B} = \left(1; \vec{v}_{B(G)}(x^o_B)\right)$ and carries spatial 3-axes $\epsilon^{\mu}_{B(G)r}(x^o_B) = \left(0; \vec{\epsilon}_{B(G)r}\right)$ assumed parallel to the axes $\epsilon^{\mu}_{B(B)r}$ of the barycenter. For an arbitrary point in the solar system with coordinates $\left(x^o_B; \vec{x}_B\right)$ we have $\vec{x}_B = \vec{x}_{B(G)}(x^o_B) + \vec{r}_{B(G)}(x^o_B)$.

The global reference system BCRS is the reference system in which the positions and motions of bodies outside the immediate environment of the Earth have to be expressed. It is appropriate for the solution of the equations of motion of solar system bodies (the development of the solar system ephemerides). Within it the positions and motions of galactic and extra-galactic objects are most simply expressed. It is the system to be used for most positional-astronomy reference data, e.g. star catalogues.

3.2 GCRS - Geocentric Celestial Reference System

The resolution B1.3 of IAU2000 (Soffel M.H. et al, 2003) states that the Geocentric Celestial Reference System BCRS is a global reference system of space-time coordinates for Earth based measurements and the solution of the equations of motion of bodies in the near-Earth environment (artificial satellites) within the framework of general relativity. The GCRS is defined such that the transformation between BCRS and GCRS spatial coordinates *contains no rotation component*, so that GCRS is kinematically non-rotating with respect to BCRS. The equations of motion of an Earth satellite with respect to GCRS will contain relativistic Coriolis forces that come mainly from geodesic precession. The spatial orientation of the GCRS is derived from that of BCRS, that is by the orientation of the ICRS. Its origin is the world-line of the geo-center (fictitious observer at the center of the Earth): it is the time axis of the geocentric time $t_G = TCG$ and the instantaneous 3-spaces Σ_{t_G} with $t_G = const.$ are inertial hyper-planes (Einstein's convention for clock synchronization) only at the lowest order in $1/c$.

GCRS has the following 4-coordinates and retarded Post-Newtonian solution of Einstein's equations for the 4-metric $g_{G\mu\nu}(x_G)$

$$x^{\mu}_G = \left(x^o_G = c\,t_G; x^a_G\right),$$

$$g_{Goo}(x_G) = \epsilon\left[1 - \frac{2\,w_G(x_G)}{c^2} - \frac{2\,w^2_G(x_G)}{c^4} + O(c^{-5})\right],$$

$$g_{Goa}(x_G) = \epsilon\left[\frac{4\,w^a_G(x_G)}{c^3} + O(c^{-5})\right],$$

$$g_{Gab}(x_G) = -\epsilon^3 g_{Gab}(x_G) = -\epsilon\left[(1 + \frac{2\,w_G(x_G)}{c^2})\,\delta_{ab} + O(c^{-4})\right].$$

See Appendix A of Ref. (Soffel M.H. et al, 2003) for the GCRS Post-Newtonian gravitational potentials $w_G(x_G)$ and $w_G^a(x_G)$. While $w_G(x_G)$ generalizes the Newton potential, the components $g_{Goa}(x_G)$ (i.e. $w_G^a(x_G)$) are responsible for the *gravito-magnetic* effects near the Earth like the Lense-Thirring or frame-dragging effect (Ciufolini I. et al, 1995; Will C.M., 2011). The Post-Newtonian solution $g_{G\mu\nu}(x_G)$ describes the *exterior* gravitational field outside the Earth surface, not inside.

The geo-center has coordinates $x_{G(G)}^\mu = \left(x_G^o; 0^a\right)$ with tangent time-like vector (the unit 4-velocity) $u_{G(G)}^\mu = (1; 0^a)$, while the spatial axes have the 3 orthogonal tangent space-like unit vectors $\epsilon_{G(G)r}^\mu$. It is a time-like geodesic of the Post-Newtonian 4-metric $g_{G\mu\nu}(x_G)$, if the Earth is approximated as a *mass monopole*. Otherwise the Earth mass and spin multipoles will create a deviation of the geo-center world-line from a time-like geodesic.

The tetrad carried by the geo-center is obtained from the BCRS tetrad with the tensorial transformation law of 4-vectors, i.e. with the matrix $\frac{\partial x_G^\mu}{\partial x_B^\nu}|_{geocenter}$. To evaluate it one needs the transformation between BCRS and GRCS coordinates. If the barycentric 3-velocity and 3-acceleration of the geo-center are $\vec{v}_{B(G)} = \frac{d\,\vec{x}_{B(G)}}{dt_B}$ and $\vec{a}_{B(G)} = \frac{d\,\vec{v}_{B(G)}}{dt_B}$ respectively and if we introduce the relative 3-vector $\vec{r}_{B(G)}(x_B^o) = \vec{x}_B - \vec{x}_{B(G)}(x_B^o)$, the BCRS-GCRS coordinate transformation is (Soffel M.H. et al, 2003)

$$
\begin{aligned}
t_G &= t_B - \frac{1}{c^2}\left[A(t_B) + v_{B(G)}^i r_{B(G)}^i\right] + \\
&\quad + \frac{1}{c^4}\left[B(t_B) + B^i(t_B)\, r_{B(G)}^i + B^{ij}(t_B)\, r_{B(G)}^i\, r_{B(G)}^j + C(t_b, \vec{x}_B)\right] + O(c^{-5}) = \\
&= t_B - \frac{1}{c^2}\left[\int_{t_{Bo}}^{t_B} dt\left(\frac{\vec{v}_{B(G)}^2}{2} + w_{Bext}(\vec{x}_{B(G)})\right) + v_{B(G)}^i r_{B(G)}^i\right] + O(c^{-4}),
\end{aligned}
$$

$$
\begin{aligned}
x_G^a &= \delta_{ai}\Big[r_{B(G)}^i + \frac{1}{c^2}\Big(\frac{1}{2} v_{B(G)}^i v_{B(G)}^j r_{B(G)}^j + w_{Bext}(\vec{x}_{B(G)})\, r_{B(G)}^i + \\
&\quad + r_{B(G)}^i a_{B(G)}^j r_{B(G)}^j - \frac{1}{2} a_{B(G)}^i r_{B(G)}^2\Big)\Big] + O(c^{-4}),
\end{aligned}
$$

The functions $A(t_B)$, $B(t_B)$, $B^i(t_B)$, $B^{ij}(t_B)$ and $C(t_B, \vec{x}_{B(G)})$, depending on the BCRS gravitational potentials $w_{Bext}(\vec{x}_{B(G)})$ (the BCRS Newtonian potential of all solar system bodies apart from the Earth acting on the geo-center) and $w_{Bext}^i(\vec{x}_{B(G)})$ (the BCRS gravito-magnetic potential) are given in Ref. (Soffel M.H. et al, 2003).

As shown in Ref. (Soffel M.H. et al, 2003), this transformation reduces to a pure Lorentz boost without rotation modulo terms of order $O(c^{-4})$ in the limit of no acceleration due to the gravitational field (i.e. with $\vec{x}_{B(G)}(t_B) = \vec{v}_{B(G)}\, t_B$, $\vec{v}_{B(G)} = const.$, $v_{B(G)} = |\vec{v}_{B(G)}|$, $\beta_{B(G)} = v_{B(G)}/C$, $\gamma_{B(G)} = (1 - \beta_{B(G)}^2)^{-1/2}$)

$$
t_G = \gamma_{B(G)}\left(t_B - \frac{\vec{v}_{B(G)} \cdot \vec{x}_B}{c^2}\right) + O(c^{-4}),
$$

$$
\vec{x}_G = \vec{x}_B - \gamma_{B(G)}\,\vec{v}_{B(G)}\, t_B + \frac{\gamma_{B(G)} - 1}{v_{B(G)}^2}\,\vec{v}_{B(G)} \cdot \vec{x}_B\,\vec{v}_{B(G)} + O(c^{-4}).
$$

Without the kinematically non-rotating constraint, GCRS would have a *slow rotation* (≈ 1.9 arcsec/century) with respect to the BCRS, the largest component of which is the *geodetic (DeSitter-Fokker) precession*, i.e. it would be *dynamically non-rotating* (and Coriolis terms should be added to the equations of motion of bodies in GCRS). Instead in the *kinematically non-rotating* version the *motion of the celestial pole* is defined in GCRS and the *geodetic precession* appears in the *precession-nutation theory* rather than in the transformation between GCRS and BCRS.

In the GCRS of IERS2003 (IERS, 2003) there are small velocities allowing one to use Galilean calculations plus relativistic corrections. However the *ecliptic plane* was redefined only in the IAU2006 resolutions (IAU, 2006) and a nearly relativistic dynamical theory of Earth rotation appeared only in 2009 with IAU2006 (Coppola V., 2009).

Therefore there are still open problems in the relativistic formulation of angular variables (Kaplan G.H., 2005):

1) the algorithms for *space motion, parallax, light-time and gravitational deflection* (for the observer at the geo-center the gravity field of the Earth is neglected in evaluating the deflection(star catalogs and ephemerides use 3-vectors in BCRS));

2) the series of rotations for *precession, nutation, Earth rotation and polar motion* (in this order) use 3-vectors in GCRS;

3) the *aberration* calculation connects the two systems because it contains the transformation between them: its input are two 3-vectors in BCRS and its output is a 3-vector in GCRS;

4) in the VLBI case *aberration* does not appear explicitly, but the conventional algorithm for the *delay* observable incorporates 3-vectors expressed in both systems.

3.3 ITRS - International Terrestrial Reference System

The International Terrestrial Reference System ITRS is the Earth-fixed geodetic system which matches the *reference ellipsoid* WGS-84 (basis of the terrestrial coordinates latitude, longitude, height, obtainable from GPS; it has equatorial radius $6,378.137$ m and polar flattening $1/298.257223563$) to several centimeters and is defined on the instantaneous 3-spaces Σ_{t_G} of constant geocentric time $t_G = TCG = const.$. It uses geocentric rectangular 3-coordinates $\vec{x}_T = \vec{x}_{ITRS}$ on Σ_{t_G} connected to the geocentric ones $\vec{x}_G$ by time-dependent rotations. It is centered on the geo-center like GCRS with the center of mass defined for the whole Earth including oceans and atmosphere. The coordinates of $ITRS \approx WGS84 \approx GPS$ are $\left(c\,t_G; \vec{x}_{ITRS}\right)$. GCRS is obtained from ITRS with a series of *time-dependent rotations* fixed by the conventions in IERS2003 (IERS, 2003) for the precession-nutation theory of Earth rotation. In chapter 5 of Ref. (Kaplan G.H., 2005) there is the old precession-nutation theory, while in chapter 6 there is the new theory of Earth rotation (updated with IAU2006). In Ref. (Coppola V., 2009) there is a more dynamical version IAU2006. Therefore the quasi-inertial relativistic 3-spaces Σ_{t_G} of GCRS are replaced with quasi-Euclidean non-relativistic 3-spaces (still denoted Σ_{t_G}) only by means of rotations.

The World Geodetic System WGS84 (WGS, 1984) is the latest revision (dated 1984 and revised in 2004) of a standard for use in cartography, geodesy and navigation. It comprises a standard coordinate frame for the Earth, a standard spheroidal reference surface (the reference ellipsoid) for raw altitude data and a gravitational equipotential surface (the *geoid*) that defines

the minimal sea level. The measurement of the form and dimensions of the Earth, the location of objects on its surface and the Earth gravity field are done by means of artificial satellites like the GPS ones (Seeber G., 2003). Let us remark that the gravitational field inside the Earth is evaluated in geodesy with Newtonian gravity, while the external GCRS gravitational potential is evaluated with Post-Newtonian general relativity and the junction of the two approaches has still to be done.

In the description of Earth rotation *precession* and *nutation* are really two aspects of a single phenomenon, the overall response of the spinning oblate, elastic Earth to external gravitational torques from the Moon, Sun and planets. As a result of these torques, the orientation of the Earth's rotation axis is constantly changing with respect to a space-fixed (locally inertial) reference system. The motion of the celestial pole among the stars is conventionally described as consisting of a smooth *long term motion called precession* upon which is superimposed a series of *small periodic components called nutation*.

In the old theory precession and nutation are described by 3×3 rotation matrices operating on column 3-vectors in a traditional equatorial celestial coordinate system. The 3-vectors have the form $\vec{x} = \begin{pmatrix} x_x \\ x_y \\ x_z \end{pmatrix} = \begin{pmatrix} d\cos\delta\cos\alpha \\ d\cos\delta\sin\alpha \\ d\sin\delta \end{pmatrix}$, where $\alpha = tg^{-1}\frac{x_y}{x_x}$ is the *right ascension*, $\delta = tg^{-1}\frac{x_z}{\sqrt{x_x^2 + x_y^2}}$ is the *declination* and d is the *distance* from the specific origin of the system. For stars and objects *at infinity* (beyond the solar system), d is often simply put to 1.

In these traditional systems the adjective *mean* is applied to quantities (pole, equator, equinox, coordinates) affected only by precession, while *true* describes quantities affected by both precession and nutation. Thus it is the *true* quantities that are directly relevant to observations; *mean* quantities now usually represent an intermediate step in the computation.

Let us now describe the rotations in the 3-spaces Σ_{t_G} connecting a GCRS 3-vector $\vec{x}_G$ to a ITRS 3-vector $\vec{x}_T$ according to the conventions of the new theory of Ref. (IERS, 2003). The new definitions were forced by the fact the the errors in the determination of the old quantities were too big.

A matrix **B**, called *frame bias matrix*, is required to convert ICRS data to the *dynamical mean equator and equinox J2000.0*: $\vec{x}_{mean(2000)} = \mathbf{B}\,\vec{x}_{ICRS}$. The same matrix is used in geocentric transformations to adjust 3-vectors in the GCRS so that they can be operated on by the conventional precession and nutation matrices. The matrix **B** corresponds to a fixed set of very small rotations: $\mathbf{B} = R_1(-\eta_o)\,R_2(\xi_o)\,R_3(d\,\alpha_o)$ with $d\alpha_o = -14.6mas$, $\xi_o = -16.6170mas$, $\eta_o = -6.8192$, all converted to radians (divide by 206264806.247).

If **B** is the frame bias matrix , $P(t_G)$ the GCRS matrix for precession and $N(t_G)$ the GCRS matrix for nutation, for a 3-vector $\vec{x}_G$ in GCRS we have

$$\vec{x}_G \xrightarrow{\mathbf{B}} MEAN\ EQUATOR\ and\ EQUINOX\ of\ J2000.0,$$

$$\xrightarrow{P(t_G)} MEAN\ EQUATOR\ and\ EQUINOX\ at\ t_G,$$

$$\xrightarrow{N(t_G)} \vec{E}_y = \left(TRUE\ EQUATOR\ and\ EQUINOX\ at\ t_G \right).$$

If the 3-vector $\vec{E}_y$ is decomposed on the basis $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, then by definition the

true equinox at t_G in GCRS is the unit 3-vector

$$\vec{y}_G = B^T P^T(t_G) N^T(t_G) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

By definition the *true celestial pole at date t_G* - the *Celestial Intermediate Pole CIP* - in GCRS is the unit 3-vector

$$\vec{n}_G = B^T P^T(t_G) N^T(t_G) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \sin d \cos E \\ \sin d \sin E \\ \cos d \end{pmatrix},$$

$$\vec{n}_G \cdot \vec{y}_G = 0.$$

By definition the *Earth's axis* is the line through the geo-center in direction of the CIP. The angle of rotation about this axis (θ, linear in UT1 and *independent from the precession-nutation model for the Earth*) must be measured with respect to some agreed-upon direction in space (CIO, see later on).

The reference point on the equator (origin of θ) must be defined in such a way that the rate of change of the Earth's rotation angle, measured with respect to this point, is the angular velocity of the Earth about the CIP. As the CIP moves, the point must move to remain in the equatorial plane (instantaneously orthogonal to the CIP axis); but the point motion must be such that the measured rotation angle is not contaminated by some component of the motion of the CIP itself. This leads to the concept of *Non-Rotating Origin* (NLO) on the equator: as the equator moves the point's instantaneous motion must always be orthogonal to the equator (whereas the equinox has a motion along the equator: the precession in right ascension). That is, the point motion at some time t_G must be directly toward or away from the position of the pole of rotation at t_G. The point is not unique.

The new conventions use the *Celestial Intermediate Reference System CIRS E_σ*, which has the NLO azimuthal origin at the *Celestial Intermediate Origin CIO or σ*, a well defined point on the equator of CIP with GCRS coordinates

$$\vec{\sigma}_G = \vec{y}_G \cos \mathcal{E}_o - (\vec{n}_G \times \vec{y}_G) \sin \mathcal{E}_o, \qquad \vec{n}_G \cdot \vec{\sigma}_G = 0,$$

where $\mathcal{E}_o$ is an angle, named *equation of the origins* (the arc on the instantaneous true equator of date t_G from the CIO at equinox; it is the right ascension of the true equinox relative to the CIO; it is also the difference $\theta - GAST$, where GAST is the angular equivalent of Greenwich apparent sidereal time), given at p.60 of Ref.(Kaplan G.H., 2005). Now we have an orthonormal triad: $\vec{n}_G$, $\vec{\sigma}_G$ and $\vec{y}_G = \vec{n}_G \times \vec{\sigma}_G$.

The coordinates in E_σ are

$$GCRS \xrightarrow{C} E_\sigma, \qquad \vec{x}_\sigma = C \vec{x}_G,$$

$$C^T = \left(\vec{\sigma}_G, \vec{y}_G, \vec{n}_G \right) = \begin{pmatrix} \sigma_1 & y_1 & x_{CIP} \\ \sigma_2 & y_2 & y_{CIP} \\ \sigma_3 & y_3 & z_{CIP} \end{pmatrix} = R_3(-E) R_2(-d) R_3(E) R_3(s),$$

with the angles d and E appearing in $\vec{n}_G$. The angle s, the CIO *locator*, given at p.62 of Ref. (Kaplan G.H., 2005), represents the difference between the length of the arc from the point N westward to the CIO (on the instantaneous equator) and the length of the arc from N westward to the GCRS origin of right ascension (on the GCRS equator).

On the celestial sphere, the Earth's instantaneous (moving) equator intersects the GCRS equator at two nodes. Let N be the ascending node of the instantaneous equator on the GCRS equator.

The matrix C is the CIO-based rotation taking into account nutation, precession and frame bias.

The Earth rotation angle θ with the origin CIO rotates the CIO equatorial axis to an instantaneous axis (the *Terrestrial Intermediate Origin TIO or $\tilde{\omega}$*), which is a NLO azimuthal origin for the *Terrestrial Intermediate Reference System TIRS $E_{\tilde{\omega}}$*

$$E_\sigma \overset{R_3(\theta)}{\to} E_{\tilde{\omega}}, \qquad \vec{x}_{\tilde{\omega}} = R_3(\theta)\, \vec{x}_\sigma.$$

Then to arrive to ITRS from $E_{\tilde{\omega}}$ we must take into account the *polar motion*

$$E_{\tilde{\omega}} \overset{W^T(t_G)}{\to} ITRS, \qquad \vec{x}_T = W^T(t_G)\, \vec{x}_{\tilde{\omega}},$$

$$W(t_G) = R_3(-s')\, R_2(x_p)\, R_1(y_p),$$

where $W^T(t_G)$ is the *polar motion* (wobble) matrix and x_p and y_p are the coordinates of CIP in ITRS. This rotation reorients the pole from the ITRS z-axis to the CIP and moves the origin of longitude very slightly from the ITRS x-axis to TIO (the angle $s' \approx -47 microarcsec$). See p.63 of Ref. (Kaplan G.H., 2005).

In conclusion we have the following sequence of rotations connecting GCRS to ITRS

$$GCRS \to E_\sigma \to E_{\tilde{\omega}} \to ITRS,$$

$$\vec{x}_G \to \vec{x}_\sigma = C\, \vec{x}_G \to \vec{x}_{\tilde{\omega}} = R_3^T(-\theta)\, \vec{x}_\sigma \to \vec{x}_T = W^T\, \vec{x}_{\tilde{\omega}},$$

$$\vec{x}_T = W^T\, R_3^T(-\theta)\, C\, \vec{x}_G.$$

Therefore ITRS is defined by taking the instantaneous 3-spaces Σ_{t_G} of GCRS and by rotating the 3-coordinates in each 3-space to take into account the rotation of the Earth. However in this way all the clocks on the Earth surface have the same geocentric time t_G. A more relativistic formulation should replace the final rotation matrix R with a Lorentz transformation $\Lambda = B\,R$, where the Lorentz boost B would imply the transformation of the global GCRS time t_G into the different local coordinate times associated with the proper times (SI atomic seconds) of the atomic clocks in each point of the Earth surface.

4. The space-time outside the Solar System

Reference data for positional astronomy, such as the the data in astrometric star catalogs or barycentric planetary ephemerides, are specified in the International Celestial Reference

System ICRS (Kaplan G.H., 2005; Kovalevski J. et al, 1989; Sovers O.J. et al, 1998; Johnstone K.J. et al, 1999; Ma C. et al, 1998; Fey A. et al, 2009) with origin in the solar system barycenter and with spatial axes fixed with respect to space. A materialization as a ICRF is obtained by supposing that the origin is a quasi-inertial observer and that we have a quasi-inertial (essentially non-relativistic) reference frame with rectangular 3-coordinates (or equatorial geographical coordinates) in a nearly Galilei space-time whose 3-spaces are Euclidean. The directions of the spatial axes are effectively defined by the adopted coordinates (i.e. using the tabulated *right ascensions* and *declinations* and, in the case of a star catalogue, the *proper motions (ephemerides)*) of 212 extragalactic radio sources observed by VLBI . These radio sources (quasars and AGN, active galactic nuclei) are assumed to have no observable intrinsic angular momentum. At low accuracy one uses a star catalogue system such as the FK5 (Fey A. et al, 2009). At a more accurate level taking into account optical wavelengths, one has the Hipparcos Celestial Reference Frame HCRF, composed of the positions and proper motions of the astrometrically well-behaved stars in the Hipparcos catalog.

Thus, the ICRS is a *space-fixed* system, more precisely a *kinematically non-rotating* system, without an associated epoch. ICRS provides the orientation of BCRS and closely matches the conventional dynamical system defined by the Earth's mean equator and equinox of J2000.0: the alignment difference is at the 0.02 arcsecond level, negligible for many applications.

However if we take into account the description of the universe given by cosmology, the actual cosmological space-time cannot be a nearly Galilei space-time but it must be a cosmological space-time with some theoretical *cosmic time*. In the standard cosmological model (Bartelmann M., 2010; Bean R., 2009) it is a homogeneous and isotropic Friedmann-Robertson-Walker space-time whose instantaneous 3-spaces have nearly vanishing internal 3-curvature, so that may locally be replaced with Euclidean 3-spaces as it is done in galactic dynamics. However they have a time-dependent conformal factor (it is one in Galilei space-time) responsible for the Hubble constant regulating the expansion of the universe. Moreover the Hubble constant is also the negative of the trace of the external 3-curvature of the 3-space as 3-sub-manifold of the space-time. As a consequence the transition from the astronomical ICRS to an astrophysical description taking into account cosmology is far from being understood.

5. Concluding remarks

As we have seen relativistic metrology is a field in rapid evolution and subject to continuous refinements.

The existing standard of length will survive till when the technology will allow us to detect the deviations from Euclidean instantaneous 3-spaces implied by Post-Newtonian general relativity. For instance in Ref. (Turyschev S.G. et al, 2006) there is a proposal of a space mission LATOR (Laser Astrometric Test Of Relativity), in which two spacecrafts behind the Sum will form a triangle with the International Space Station ISS. This would allow us to measure the three angles of the triangle to see whether their sum is 2π as required by an instantaneous Euclidean 3-space.

The development of optical atomic clocks will allow us to develop a new generation of gravimeters for the local study of the gravitational field of the Earth (now also investigated with the satellites GOCE (Gravity field and steady-state Ocean Circulation Explorer, ESA), CHAMP (CHAllenging Mini-Satellite Payload, GeoForschungsZentrum GFZ), GRACE (Gravity Recovery and Climate Experiment, Center for Space Research, Austin Texas)). One

open problem to get a reliable theory of heights over the reference geoid is the comparison of the measurements of gravimeters on the two sides of an ocean. But a byproduct of the ACES mission will be the possibility of such a comparison, by synchronizing the optical atomic clocks of the gravimeters with the ACES clocks on the International Space Station ISS (Svelha D. et al, 2008). As a consequence standard non-relativistic geodesy will be replaced by relativistic geodesy.

Also the transformation from the non-relativistic ITRS on the Earth surface to the relativistic GCRS around the Earth will be accomplished. This will put full control on possible semi-relativistic precessional effects near the Earth surface.

Space navigation inside the Solar System will require refinements of BCRS. In particular to test deviations from Einstein theory of general relativity (the one used in BCRS). See for instance the recent interest in the Pioneer anomaly (Turyshev S.G. et al, 2010) and the endless number of proposals for its explanation.

Regarding ICRS we need a general relativistic relativistic version of it taking into account the non-Euclidean nature of the 3-space as 3-sub-manifolds of space-time. The unsolved problems of dark energy and dark matter, required by the standard ΛCDM cosmological model starting from the hypothesis of homogeneity and isotropy of space-time, are pushing towards inhomogeneous cosmological space-times in which the 3-spaces have small internal 3-curvature but a non zero external 3-curvature. The first step will be to face these problems inside the Milky Way finding a *relativistic* galactic celestial reference frame extending the existing BCRF. To this end the GAIA (Global Astrometric Interferometer for Astrophysics) mission of ESA (Jordi C., 2011; Jordan S., 2008; Klioner S.A. et al, 2005), to be launched in 2012, for the 3-dimensional cartography of our galaxy (position, proper velocity, radial velocity and spectroscopic data for about one billion stars) will be a first relevant step.

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