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Finite Element Analysis of Multi-Stable Structures

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1. Introduction

1.1 Conception of multi-stable structures

A structure that has at least two stable geometric configurations is called multi-stable. The critical load level has to be reached before snap-through occurs, which means the stable states can carry load. The multi-stable structure is a system owning more than two stable configurations. The first important aspect is that large shape change can be accomplished with small energy input and without complicated actuators. Instead of the power being supplied to elastically deform over its entire range, power is only needed to snap the structure from one stable configuration to another. A reduction in weight of the overall structure is possible, since the whole structure can serve as both the base structure and the control surface.

It is receiving the attention of scientists in developing the multi-stable structure which enables a number of operational shapes (Hufenbach et al., 2002; Portela et al., 2008). It is potentially suitable for a wide variety of systems, such as morphing aircraft (Yokozeki et al., 2006; Diaconu et al., 2008; Schultz, 2008), deployable structures (Lei & Yao, 2010) and mechanical switches. The studies of multi-stable structures reported in open literatures are dominated by the bi-stable unsymmetric composite laminates (Jun & Hong, 1992; Dano &

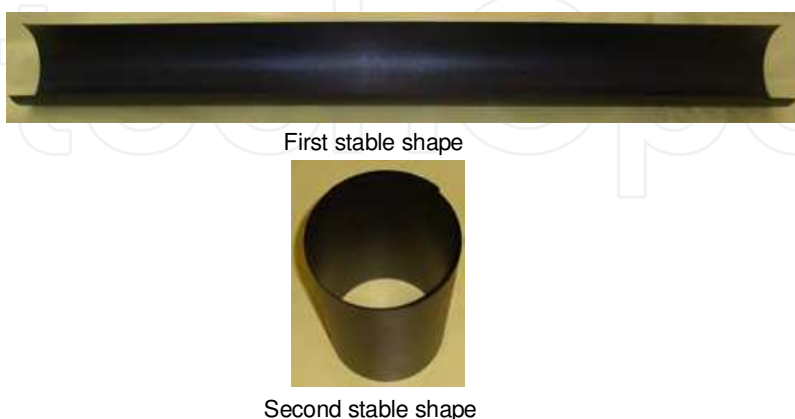


Fig. 1. The bi-stable unsymmetric composite laminate

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Hyer, 1998; Maenghyo et al., 1998; Hufenbach et al., 2002, 2006; Ren, & Parvizi-Majidi, 2003, 2006; Gigliotti et al., 2004; Dai et al. 2007; Etches et al., 2009). A typical bi-stable composite laminate is shown as in Fig.1, which means that applying an external force the room temperature cylindrical shape can be snapped into another cylinder with curvature of equal magnitude and opposite sign.

1.2 Design and analysis method

How to design and analyse the multi-stable behaviour is still a challenging problem. It is actually a bifurcation problem. First issue about the multi-stable structures is to predict their multiple shape configurations. Second important issue is to simulate the snap-through behaviors of multi-stable structures.

1.2.1 Analytical method

Hyer observed the large deformation of thin $[0/90]$ cross-ply unsymmetric composite laminates in curing experiments (Hyer, 1981). They found that the shapes of cured unsymmetric composite laminates were a part of cylinder, which were stable at room temperature. To take into account the large multi-stable deformations, which are often many times over the laminate thickness, the linear strain-displacement relations must be extended by non-linear terms. Since then, many researchers had employed the Rayleigh-Ritz approach proposed by Hyer to predict the shape of unsymmetric laminates (Hyer, 1982; Maenghyo & Hee, 2003). Jun (Jun & Hong, 1992) modified Hyer's theory by including more terms in the polynomials to take plane shear strain into account. Dano (Dano & Hyer, 1998) used a higher-order polynomial to calculate the displacement field. Dai and Daynes (Dai et al., 2009; Daynes & Weaver 2010) predicted the cured shape of the bi-stable hybrid composite laminate. There is a very good agreement between the experimental and analytical shapes predicted by Rayleigh-Ritz method. The cured shape and bifurcation point can be obtained. An example of bistable analytical solutions is shown as in Fig.2 (Dai et al., 2009). It can be seen that there only exist one group of solutions when the side length is less than some critical value (35 mm), as the branch curve AB and A'B' plotted in Fig.2. Here the curvature in x-direction is opposite to that in y-direction. There exist three groups of solutions when the side length is more than some critical value, as shown with the branch

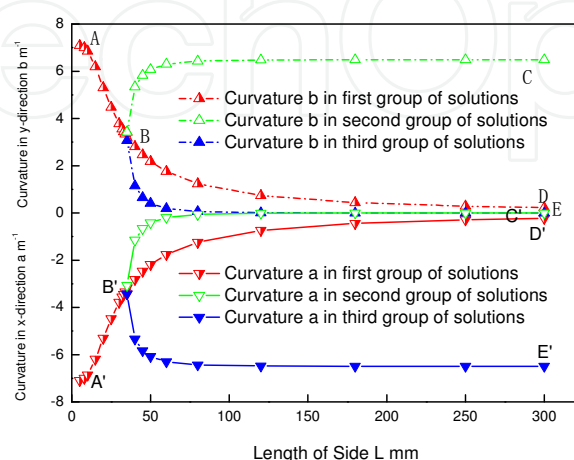


Fig. 2. An example of analytical solution curves for bi-stable composite laminate

curves of BD and B'D', BC and B'C', BE and B'E'. The first group of solutions as shown with the branch curve of BD and B'D' mean a saddle shape, which is not in reality. The second and third group of solutions mean that snap-through phenomenon occurs. The curvature of curve BC is 6.49 and the corresponding value of curve B'C' is approximately 0. It indicates the cured shape of unsymmetric hybrid composite laminates is similar to a half cylinder.

1.2.2 Finite element method

In addition to analytical method, Finite Element Analysis (FEA) is found to be a robust way to study the bi-stable laminates. Snap through analysis of multi-stable structures requires the solution of a nonlinear system of equations. The iterative techniques are used to solve these equations, where small incremental changes in displacement are found by imposing small incremental changes in load on the structure. The arc length methods were commonly used to overcome the problem of tracing the equilibrium path in the neighborhood of limit points (Riks, 1979; Wempner, 1971). Schlecht employed the FEA to predict the cured shape of unsymmetric laminates with the commercial finite element code Marc (Schlecht et al., 1995, 1999). Giddings combined a finite element model of a Macro-Fiber Composite (MFC) with a bi-stable asymmetric laminate model. Both predicted shape and snap-through voltage of a piezo-actuated [0/90] laminate agreed well with experimental results (Giddings et al. 2011). In recent years, another commercial finite element code ABAQUS is widely employed, which is also accurate enough in shape prediction of bi-stable laminates (Dai, 2011; Mattioni, 2008, 2009; Portela et al., 2008). ABAQUS offers two procedures, namely RIKS and STABILIZE, both are capable of solving a nonlinear system of equations and hence suitable procedures for studying unstable post-buckling problems. The formulation behind each procedure is rather different and hence, requires different sensitivity of their respective input parameters. Tawfik systematically discussed the two methods (Tawfik et al., 2007), namely "RIKS" and "Stabilize", both were capable of studying the snap-through of bi-stable laminates. Mattioni pointed out that the "Stabilize" algorithm is more suitable (Mattioni, 2008, 2009). Portela studied the snap-through of laminate with actuator (MFC) by FEA (Portela et al., 2008).

2. Finite Element Analysis of bi-stable laminates

There are two Finite Element Methods to solve the bi-stable bifurcation problem: one-step method and two-step method. In one-step method, the geometrical model without internal thermal stress is directly built, which is actually a stable structure. Then the loads are applied on the structure and the critical load can be solved. In two-step method, the cool-down process is firstly simulated and the room temperature bi-stable geometrical models are built. Then, the snap-through simulation of the stable configurations can be conducted. The following section gives the detail examples for the both models.

2.1 Experiments of bi-stable laminates

The experimental set up is given as follows before performing finite element analysis. The testing specimen is [0/90] cross-ply epoxy matrix carbon fiber-reinforced composite laminate with the size of 140mm×140mm. The materials in the experiments are T300/Epoxy prepreg with longitudinal elastic modulus E_1 of 137.47 GPa, transverse elastic modulus E_2 of

10.07 GPa, Possion’s ratio ν_{12} of 0.23, the longitudinal linear expansion α_1 of $0.37\times10^{-6}/^{\circ}\text{C}$ and transverse linear expansion α_2 of $24.91\times10^{-6}/^{\circ}\text{C}$.

As shown in Fig.3, the composite laminate is put on a pair of slippery orbits to ensure that the laminate is supported only at four corner points. A concentrated force is applied on the geometrical center of the laminate, by a special loading apparatus which can increase the load gradually. The concentrated force is gradually increased to the critical load of snap-through and the height of the center point at every step is measured.



Fig. 3. Schematics of concentrated force load experiments

The load-displacement curve is shown in Fig.4. The displacement denotes the deflection at the middle of laminate’s span. It can be observed that the curve is almost straight up at the beginning, and the load reaches its maximum at the end of curve, which is the critical load of snap-through. The critical load is 1.32N and the corresponding displacement at the center of the laminate is 21mm, which denotes the critical deformation of snap-through. Because of the limitation of loading apparatus, the unloading part of the curve is not obtained.

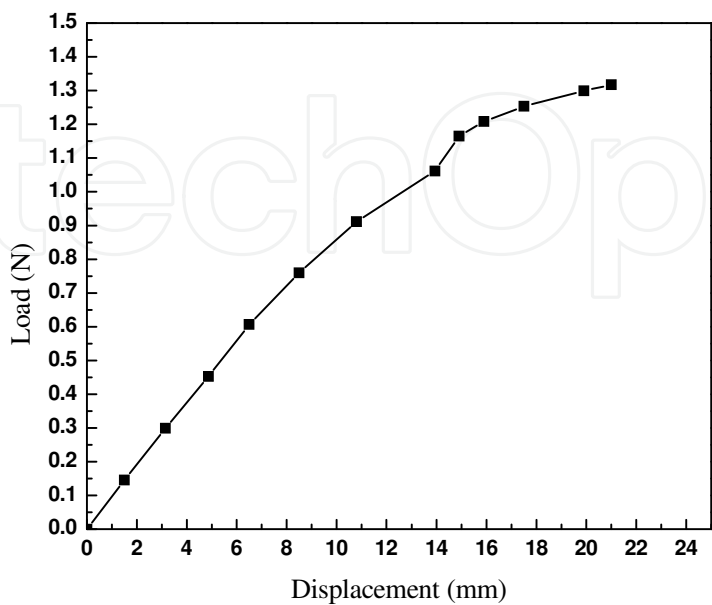


Fig. 4. Load-displacement curve of bi-stable laminates

2.2 Two-step FEA method

In this section, the commercial finite element code ABAQUS is employed to simulate the snap-through of bi-stable laminate, and the predicted results such as critical load and deformation of the composite laminate are presented.

The first step is to simulate the cool-down process of composite laminate with internal thermal stress. Although the practical composite laminate is a square plate with the side length of 140mm, a geometrical imperfection in the finite element model should be introduced. Otherwise the simulation result will be a saddle-shape rather than the expected cylindrical shape. The cool-down process in practice is time-consuming (a few hours depending on the curing cycle that the material requires). The first analysis step is “Static”. To perform the geometrical non-linear analysis, the option “NLGEOM” is employed. The element type is S4R and the center node of the FEA model is fully constrained. During the analysis, the model is cooled down from 160°C to 20 °C, and the deformation is merely generated by the temperature change, then first stable shape can be obtained. To get another stable shape, the “Static, Stabilized” step with an automatic stabilization which is based on the addition of viscous forces to the global equilibrium equation is used and the solution scheme is able to converge to another stable shape.

The simulation results and the measured results are listed in Table 1. It can be seen that the numerical results agree well with the measured results.

	Experimental	Numerical	Error (%)
Deflection (mm)	25.90	27.268	--
Span (mm)	126.3	126.06	--
K (m ⁻¹)	11.19	11.49	2.64

Table 1. Experimental results and FEA results of cured bi-stable laminates

After the cool-down simulation, the temperature is kept at 20°C and the following boundary condition and displacement load are applied on the model.

1. The center node of the laminate is fully constrained.
2. Four same displacement loads are applied on the four corner nodes, parallel to the vertical axis, pointing to the opposite direction of the corner nodes’ current position.

The “Static, Stabilize” step is employed here, under the boundary conditions above, the reaction force of the center node is the corresponding center load in experiment. The simulation process is shown in Fig-5.

The curve of the reaction force of center node versus its relative displacement is presented in Fig-6. It is observed that the FEA results are greater than the experimental and the curve is more complex. The error of critical load is about 28% since finite element model is an ideal model while the specimen contains some defects which are generated in the manufacturing process. In FEA, the thickness of the model is 0.25mm, which is equal to the sum of two plies. However, the thickness of the specimen is not equal to 0.25mm exactly. The actual thickness is about 0.23±0.02mm. Therefore the modified thickness 0.23mm is used to

calculate again. The results are shown in Fig.6. The results of modified model are much improved and agree well with experiment. The calculated critical load is 1.33N, and the error of critical load decreases to 0.7%.

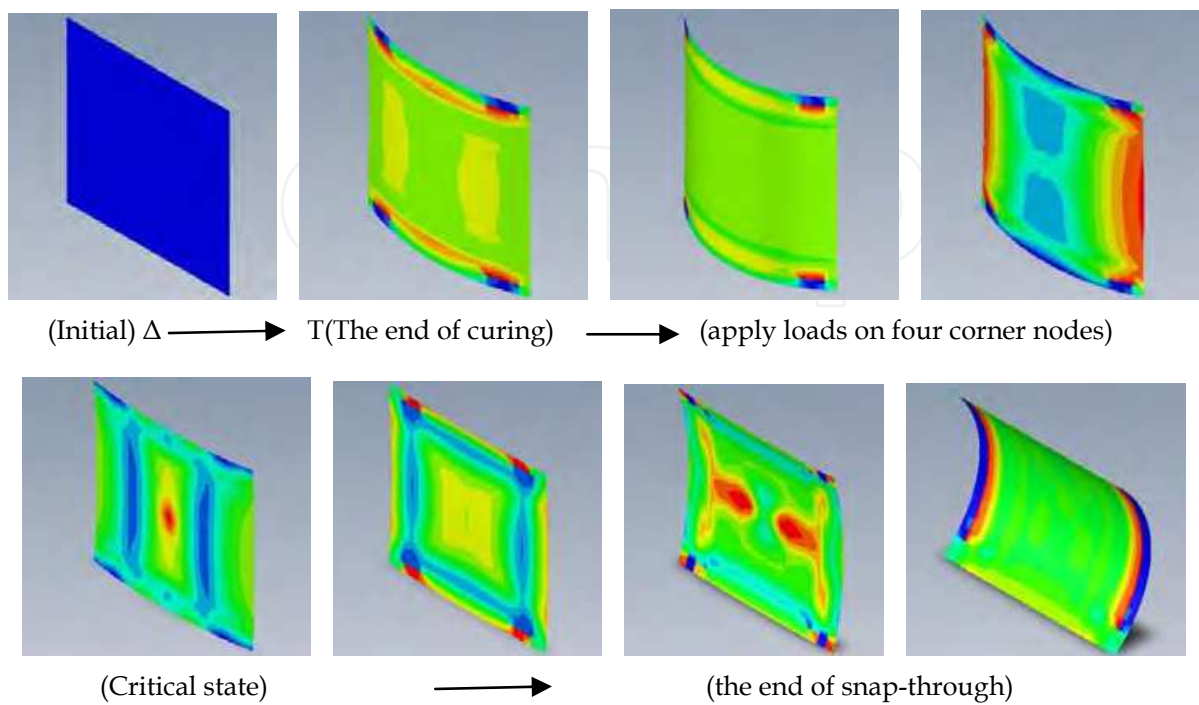


Fig. 5. Different stages of snap-through simulation in two-step model

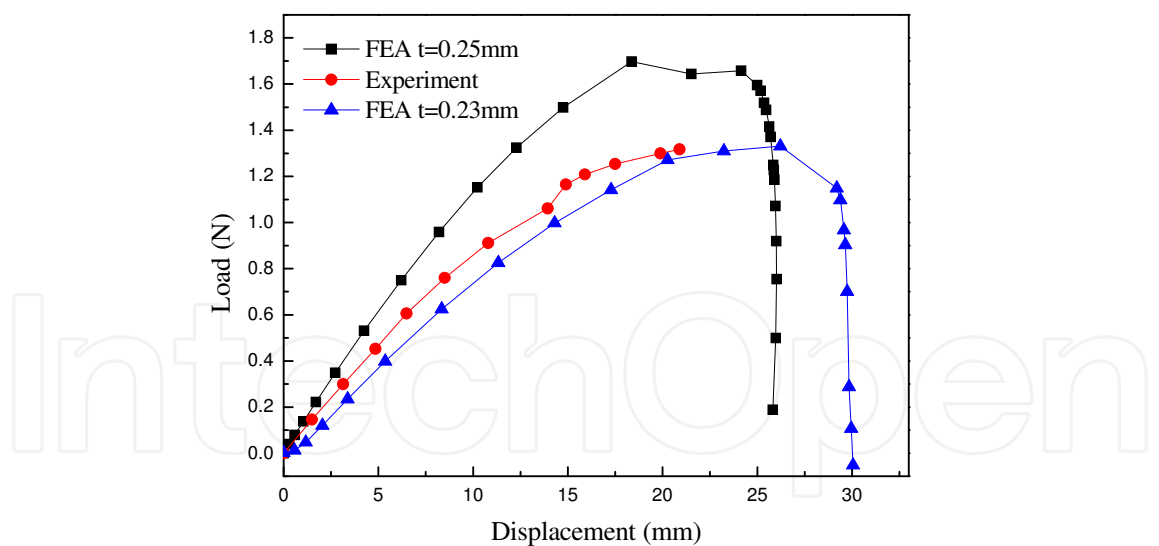


Fig. 6. Load-displacement curves of numerical simulation and experiment

2.3 One-step FEA method

Here, a part of cylindrical model with no internal thermal stress is directly built. The initial state of the model has the same geometrical configuration as the specimen in experiment, such as dimensions and curvature. The thickness is chosen to be 0.23mm. The snap-through process is shown in Fig.7.

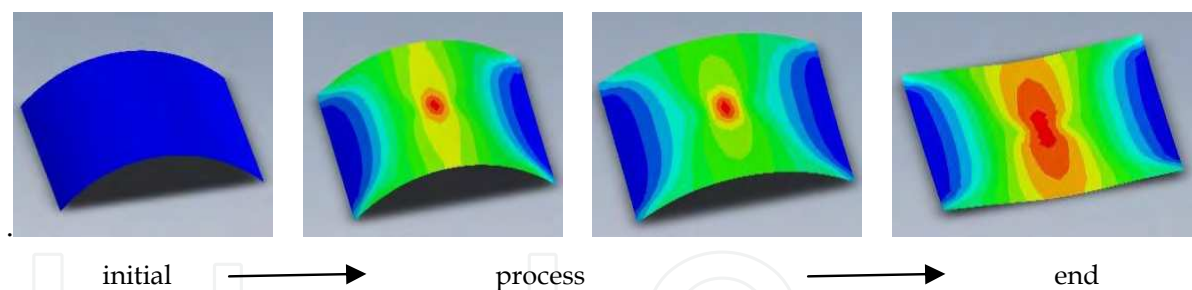


Fig. 7. Different stages of snap-through simulation in one-step model

The results of one-step model are compared with two-step model and experiments. In Fig.8, it can be observed that the curve of one-step model is almost linear and has no critical point. Interestingly, the load-displacement curves of one-step model and two-step model almost agree with each other when the deformation is small, however, as the displacement increases, the difference between the two kinds of models increases rapidly. Thus, to study an unsymmetric composite laminate, when the deformation is small, it is more convenient to directly build a model with the same curve, and the results will be accurate enough.

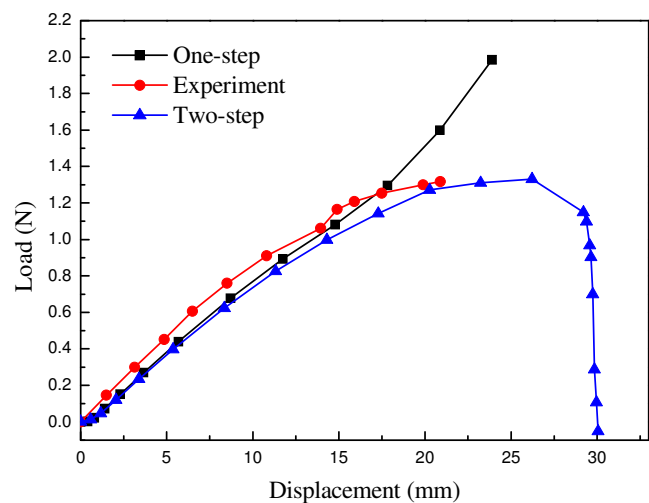


Fig. 8. Load-displacement curves in one-step model, two-step model and experiment

3. Snap-through FEA of multi-stable lattice structures

The studies of multi-stable structures are dominated by the bi-stable unsymmetric composite laminates. In some applications there may be a need to fabricate multi-stable structures with more than 2 stable configurations. One of important issue is how to fabricate the multi-stable structures. The additional constraint and the change of stresses distribution may cause the loss of multi-stable capability when the bi-stable laminates are bonded to other structural components. Here, the tri-stable lattice structures based on bi-stable laminates are presented and some simulation studies are given.

3.1 Design and fabrication of tri-stable lattice structures

The lattice structure is made up by four same rectangular composite laminates with the size of 140mmx35mmx0.25mm. The stacking sequence is [0/90]. Due to the fact that the size of laminates is too small, it is not convenient to manufacture the laminates. In experiment, we

first manufactured a square laminate with the size of 140mm×140mm×0.25mm. Then the square laminate is divided into four same rectangular laminates. The rectangular laminates have two stable configurations, namely configuration A and B in Fig.9. The lattice element is composed of four narrow rectangular bi-stable laminates. The measured average curvature of rectangular laminates is 8.85m⁻¹.

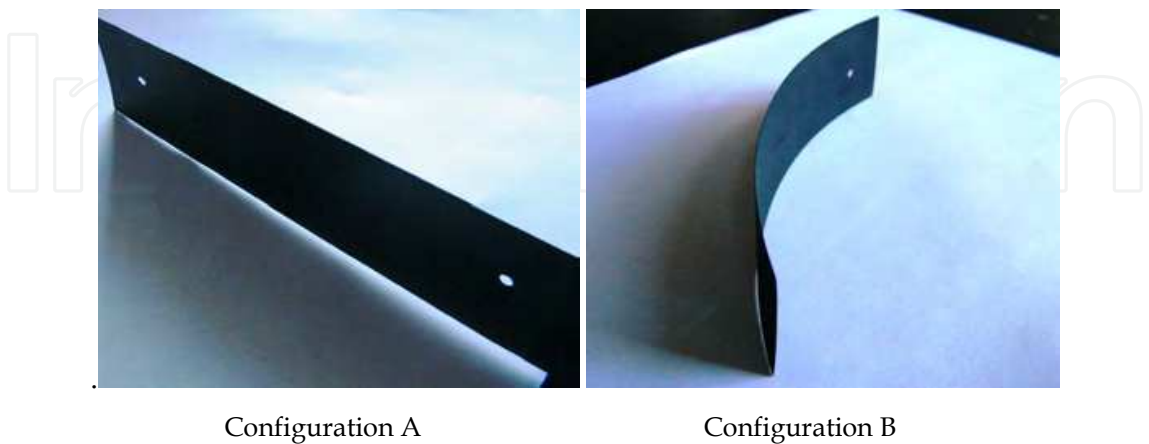


Fig. 9. Two stable configurations of rectangular cross-ply composite laminate

The four rectangular laminates are connected by the bolts. The narrow rectangular laminate has the same curvature in both the longitudinal and transvers directions. Thus it can offer the smooth and close contact between two laminates when the narrow rectangular laminate is bent along the longitudinal direction. The lattice structure, as presented in Fig.10, has three stable configurations without external constraint. The first stable shape is like a plane rectangular lattice. The second stable shape is like a concave lattice. The third stable shape is like a convex lattice.



Fig. 10. Three stable configurations of lattice structures

3.2 Experiments of tri-stable lattice structures

The concave configuration is unable to directly transform to convex configuration. It has to transform to the plane configuration first, then to the convex configuration. So we can just investigate the snap-through between the plane configuration and the two curving configurations. We name the plane as configuration A, and name the convex configuration as configuration B, as seen in Fig.11 and 12. For loading scheme of configuration A, A concentrated force is applied on the center of rectangular laminate, and the two supports are set symmetrically. For the lattice structure, two same concentrated loads are applied

simultaneously on the two centers of rectangular laminates and the four supports are set symmetrically as well. The critical loads to snap through the rectangular laminate and the lattice structure are measured. The relationship between the critical loads and the position of supports is also investigated by changing the distance between the load and supports. For loading scheme of configuration B, the specimens are put on a slippery glass, and a concentrated load is also applied on the centers of rectangular laminates.



Fig. 11. The loading scheme for configuration A

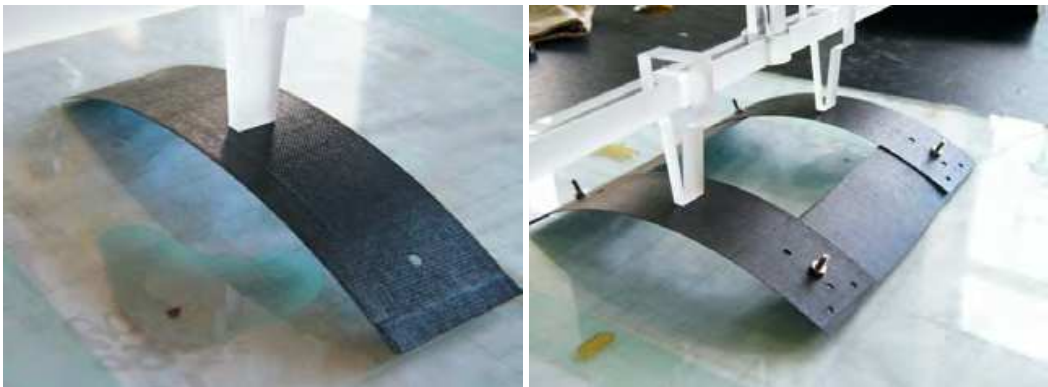


Fig. 12. The loading scheme for configuration B

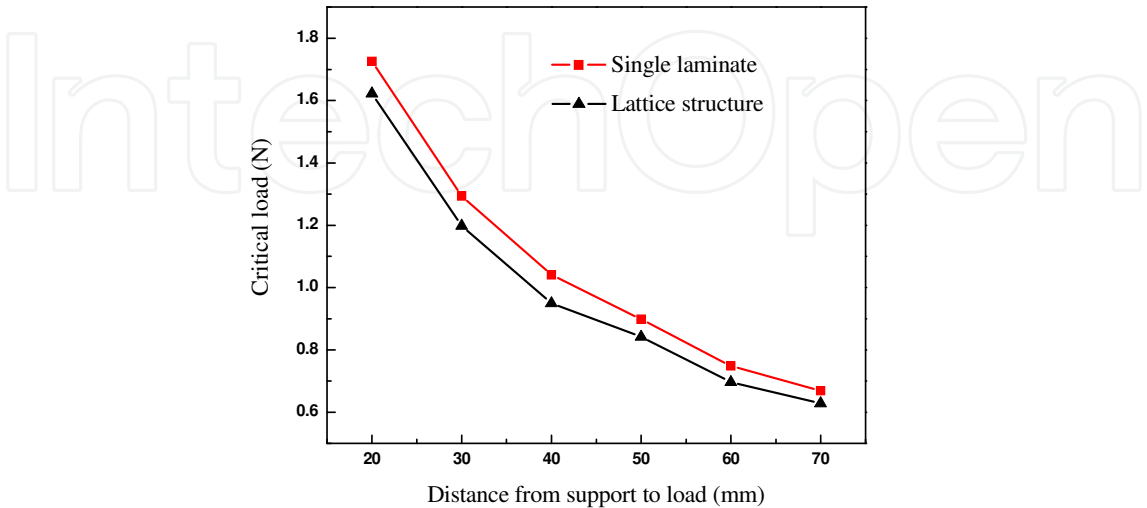


Fig. 13. Measured average critical loads of configuration A of both separate laminates and lattice structure

For configuration A, when load slowly increases to the critical load, the rectangular laminates quickly snap from configuration A to B. Because of some unpredictable defects introduced in manufacture process, the mechanical properties of four rectangular laminates vary from each other, which give rise to difference of their critical loads. The average measured critical loads for configuration A are presented in Fig.13. In order to compare with the separate rectangular laminates, half of the critical loads of lattice structure are given. The experimental results reveal that the critical load increases as the distance from support to load decreases and the relationship is not linear.

For configuration B, the experimental phenomenon is much different from configuration A. The deformations of separate laminates and lattice structures before snap-through are much greater than those of configuration A while the critical loads are much smaller. Under the loading scheme of configuration B, the measured critical loads of four separate laminates are presented in Table 1, and the measured critical loads of lattice structure are presented in Table 2. It shows that the average critical load of lattice structure, divided by 2, 0.177N, is 6.8% smaller than the average critical loads of separate laminates, 0.190N.

Lattice structure	Test 1	Test 2	Test 3	Test 4	Average
Critical load(N)	0.361	0.348	0.356	0.351	0.354

Table 2. Measured critical load of lattice structure for configuration B

3.3 Snap through FEA of tri-stable structures

The commercial finite element code ABAQUS is used to simulate the snap-through of rectangular laminates and lattice structure. The parameters used in the simulation are same as the above. A two-step method is proposed. First, it is required to simulate the cool-down process to build the initial finite element models of laminates and lattice structure. It often takes hours for the laminates to cool down from the curing temperature to room temperature. So the cool-down process is thought to be a quasi-static process. In the simulation, the laminate is modeled using four-node-square shell elements (S4R). The four bi-stable laminates are numerically generated during cool-down process by applying an initial curing temperature and a final room temperature. For lattice structures, the four components are bonded together before the cool-down simulation by using four “tie” constraints in ABAQUS, which constrain the relative displacements of the bonding points. Calculated curvature and measured curvatures of separate laminates are listed in Table 3.

Specimen	1	2	3	4	average
Measured curvature m ⁻¹	8.44	8.70	9.21	9.06	8.85
Simulated curvature m ⁻¹			9.18		
Error (%)	8.77	5.52	0.33	1.32	3.73

Table 3. Simulated and measured curvatures

In second step, after the cool-down simulation, the snap-through simulation of the stable configurations can be conducted. The snap-through problem of unsymmetric laminates is a buckling problem in essence, which can be unstable. However, if the instability is localized,

there will be a local transfer of strain energy from one region of the model to neighboring region, and global solution methods may not work. This problem has to be solved either dynamically or with the aid of artificial damping. Abaqus/Standard (Simulia 2009) provides an automatic mechanism for stabilizing unstable quasi-static problems through the automatic addition of volume-proportional damping to the model. To ensure an accurate solution is obtained with automatic stabilization, the ratio between the viscous damping energy and the total strain energy should not exceed the defined dissipated energy fraction or any reasonable amount. Fig.14 presents finite element models and the loading illustration for configuration A, which is used to simulate the snap-through and to compare with the experiments. According to loads and boundary conditions in experiments, a concentrated force is applied on the center point of finite element model, and vertical movements at supports are constrained. For lattice structures, two same concentrated forces are applied simultaneous on center points of two parts, vertical movements at supports are also constrained.

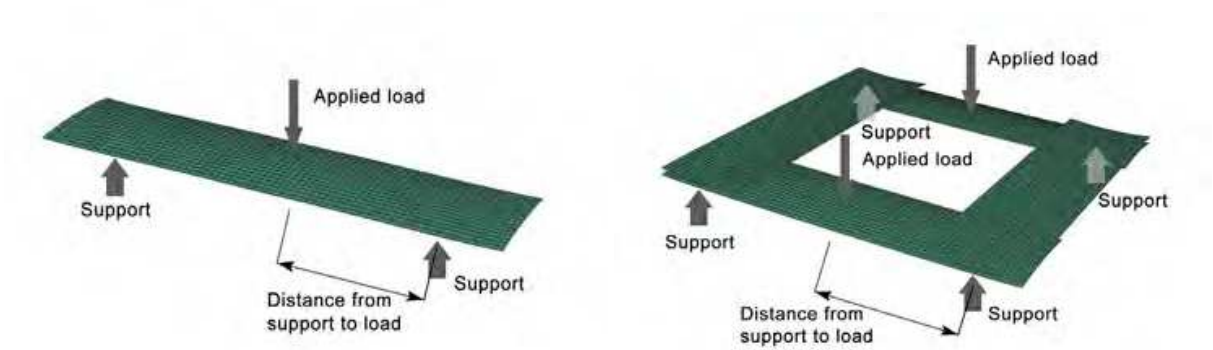


Fig. 14. Illustration of finite element model and loading scheme for configuration A

For configuration B, the snap-through simulation is a little more complicated, and it is required to build an analytical plane to simulate the glass plane in experiments. The surface to surface contact between the model of laminate and the analytical plane should be established to simulate the interaction between the specimens and the glass. Since the critical load of configuration B is very small, the gravity is considered here. The loading scheme is illustrated in Fig.15. In the models of both single laminate and lattice structure, concentrated forces are applied on the center points, and vertical movements at supports are constrained.

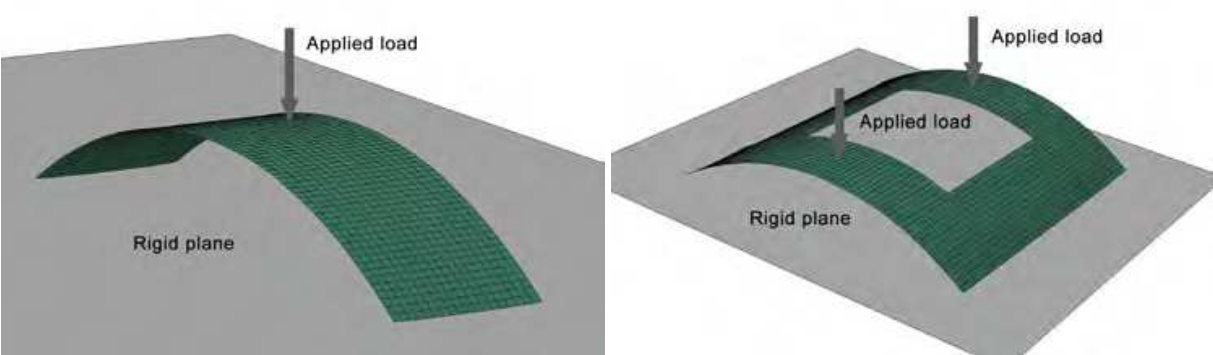


Fig. 15. Illustration of finite model and loading scheme for configuration B

In experiments, the specimens are snapped too fast to observe the process clearly, but through the FEA we can observe every step of snap-through under the presented loading scheme. As shown in Fig.16, for configuration A, when loads increase to certain value, local instability is observed firstly in the center of model and snap-through starts from there. Regions between snapped and unsnapped regions turn to be unstable, then unstable regions gradually move towards short edges and snapped region spread. During the process of snap-through, unstable regions move smoothly toward short edge. When they get to short edges, the plane configuration is snapped to be a convex configuration.

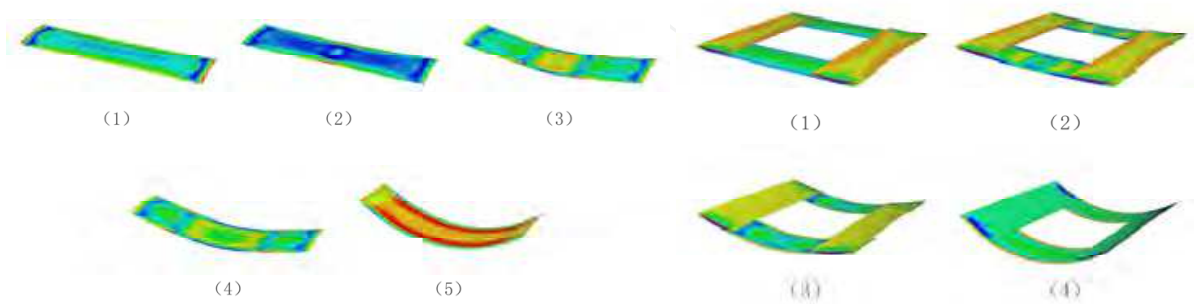


Fig. 16. Simulated snap-through process for configuration A

Because the finite element model is ideal, the loads applied on the two components of lattice structures are the same. The applied load-displacement curve of configuration A according to the different positions of supports is shown in Fig.17. The peak points of the curvatures are the critical loads of snap-through for loading scheme of configuration A. When the load increases to critical load, central regions lose stability

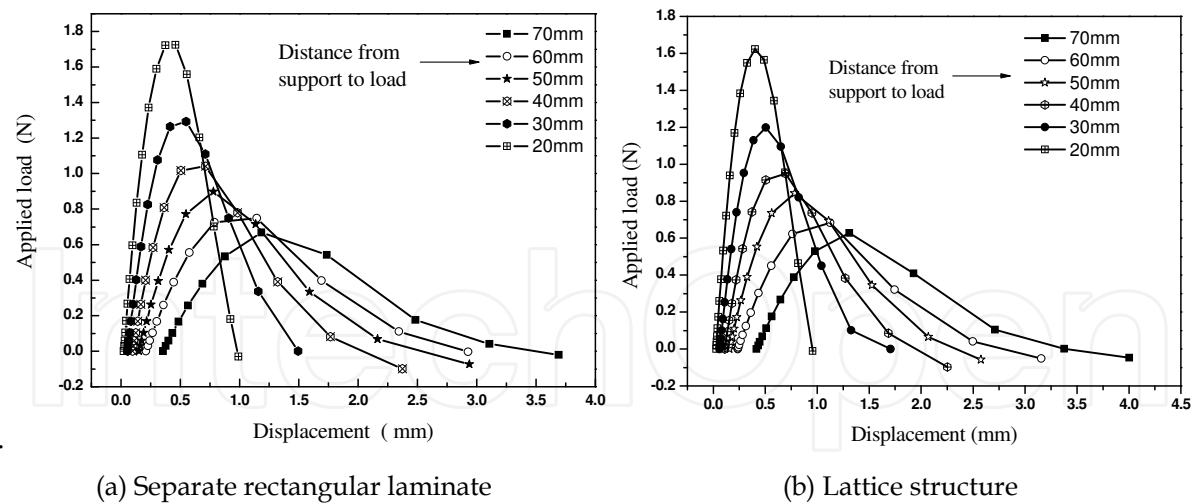


Fig. 17. Calculated applied load vs. displacement curves for configuration A

Under the loading scheme illustrated in Fig.15, the snap-through process of configuration B given by FEA is presented in Fig.18. The curvatures decrease with increasing loads. When the curvatures decrease to the critical value, and then center region starts to lose stability and snap-through also starts from the center region. Snap-through spreads gradually form the center region towards toward two short edges. Finally, the lattice structure is snapped to be a plane configuration when the unstable region spread to short edges.

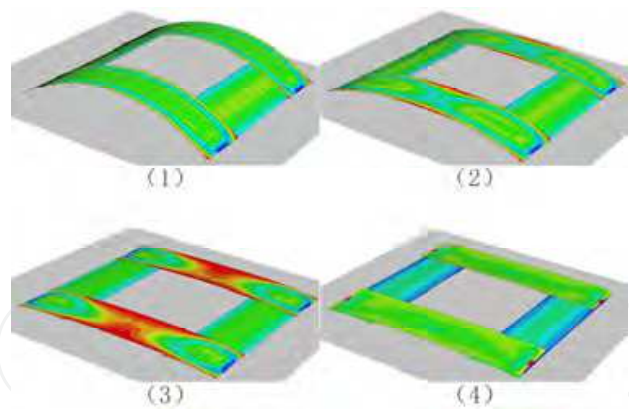


Fig. 18. Simulated snap-through process for configuration B

The calculated load-displacement curves of configuration B are given in Fig.19. The peak points of the two curves in Fig.13 are critical points of snap-through. As same as experiments, the calculated critical load of lattice structure is a little smaller than separate laminates, 0.1836N and 0.1942N respectively.

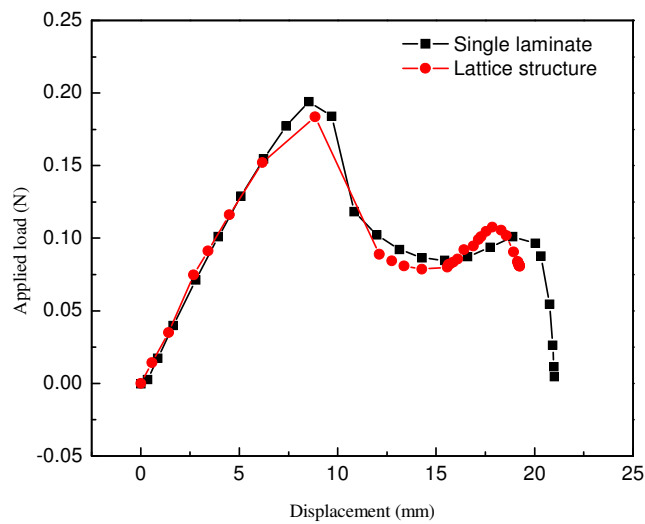


Fig. 19. Calculated applied load vs. displacement curves for configuration B

Compared Fig.17 with Fig.19, it can be found that the deformation of configuration A before snap-through is smaller than that of configuration B, however the critical load of configuration A is much greater than that of configuration B. The lattice structure is made up by the four rectangular composite laminates, so its mechanical behaviours are mainly dominated by the mechanical behaviours of rectangular laminates. Since the critical loads of four components of lattice structure are different, the two components may not snap at the same time even under symmetric loads. The part with relative smaller critical load will snap first. Apparently, the slower the load increases, the greater the time lag is, this will weaken the continuity of snap-through process of lattice structure. It is needed to find a better way to snap the lattice structure more continuous as an integral structure instead of tied parts.

The two configurations of rectangular bi-stable composite laminates curl along long edge and short edge respectively, and respond differently under centre load. In Fig.16 and Fig.18, it is found that both configurations snap locally in center first. In Fig.17, it shows that before

local snap the deformation is very small. In Fig.19, it shows the critical load is much smaller, and the deformation is much greater, and after the critical point the applied load does not decrease to zero as configuration A. The laminate has a displacement jump on a certain load then the load decreases to zero as snapped region spreads to the entire laminate. FEA results denote that the bending stiffness of configuration A is greater. The error between calculated curvature and measured curvature is 3.73%. It indicates that the initial finite element models are reasonable. The comparisons of critical loads of configuration A between FEA and experiments are given in Fig.14. The calculated critical loads under the same loading scheme are about 11-18% smaller than measured values. For configuration B, The calculated results are greater than the measured values. The measured critical load of separate rectangular laminate is 0.190N, while the calculated is 0.194N. The measured critical load of lattice structures is 0.177N, while the calculated is 0.183N. The error is about 3%~4%.

The main reason giving rise to errors of calculated critical loads is that the finite element models are ideal while the specimens inevitably contain some defects, in addition, complex boundary constraints may be not fully characterized in numerical models. How to consider the defects and the constrain effects in the snap-through simulation is needed to be studied further.

4. Conclusions

The finite element analysis based on general purpose software of ABAQUS was successfully used to simulate the snap-through behaviors of multi-stable structures. The two-step model and the one-step model were introduced to predict the critical load. The results from two-step model show a good agreement with experiment, the error of critical load decreases from 28% (ideal model) to 0.7% (modified model). The results of one-step model show that the model without internal thermal stress is not bi-stable. However, the comparison between the two kinds of models indicates that when the deformation is relative small (about half of the critical deformation), the results of the two kinds of models agreed well. Therefore, to study an unsymmetric bi-stable composite laminate, when the deformation is small, it is more convenient to directly build a model with the same curvature, and the results will be accurate enough. Otherwise, the model with internal thermal stress must be considered.

It is undoubtedly very important and requisite in engineering how to fabricate multi-stable (more than 2) structures and to get different stable configurations. A method for assembling the bi-stable laminates to fabricate a multi-stable structure has been presented. The snap-through of rectangular cross-ply composite laminates and a lattice structure was numerically and experimentally studied. The experiments to measure the snapping force levels of rectangular laminates and lattice structures were conducted. A two-step method based on commercial software ABAQUS to simulate the snap-through behaviors of lattice structures were presented. Snap through of lattice structures were successfully described with the use of this method. The same phenomenon between the FEA and experiments were observed. The predicted curvatures show a good agreement with experimental results while there are some errors between predicted and measured critical loads. The errors of critical loads are caused by unpredictable defects and complex boundary constrains. How to consider the defects and the constrain effects in the snap-through simulation is needed to be studied further. The proposed tri-stable structures and two-step method are instructive to

construct the multi-stable structures and to understand their snap-through behaviors. Finally, It should be noted that deep understanding of their behaviours will need to await more detailed studies, for example, the reliability and the fatigue properties, et al.

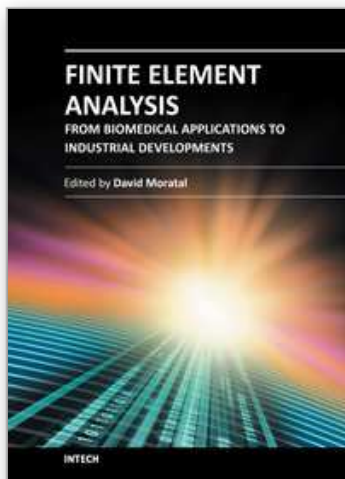
5. Acknowledgments

Thanks to support by National Natural Science Foundation of China (Grant No.10872058) and the Major State Basic Research Development Program of China (973 Program) under grant No. 2010CB631100

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Finite Element Analysis - From Biomedical Applications to Industrial Developments

Edited by Dr. David Moratal

ISBN 978-953-51-0474-2

Hard cover, 496 pages

Publisher InTech

Published online 30, March, 2012

Published in print edition March, 2012

Finite Element Analysis represents a numerical technique for finding approximate solutions to partial differential equations as well as integral equations, permitting the numerical analysis of complex structures based on their material properties. This book presents 20 different chapters in the application of Finite Elements, ranging from Biomedical Engineering to Manufacturing Industry and Industrial Developments. It has been written at a level suitable for use in a graduate course on applications of finite element modelling and analysis (mechanical, civil and biomedical engineering studies, for instance), without excluding its use by researchers or professional engineers interested in the field, seeking to gain a deeper understanding concerning Finite Element Analysis.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Fuhong Dai and Hao Li (2012). Finite Element Analysis of Multi-Stable Structures, Finite Element Analysis - From Biomedical Applications to Industrial Developments, Dr. David Moratal (Ed.), ISBN: 978-953-51-0474-2, InTech, Available from: <http://www.intechopen.com/books/finite-element-analysis-from-biomedical-applications-to-industrial-developments/finite-element-analysis-of-multi-stable-structures>

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