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Tuning Three-Term Controllers for Integrating Processes with both Inverse Response and Dead Time

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1. Introduction

Most industrial processes respond to the actions of a feedback controller by moving the process variable in the same direction as the control effort. However, there are some interesting exceptions where the process variable first drops, then rises after an increase in the control effort. This peculiar behaviour that is well known as "inverse response", is due to the non-minimum phase zeros appearing in the process transfer function and representing part of the process dynamics. Second order dead-time inverse response process models (SODT-IR) are used to represent the dynamics of several chemical processes (such as level control loops in distillation columns and temperature control loops in chemical reactors), as well as the dynamics of PWM based DC-DC boost converters in industrial electronics. In the extant literature, there is a number of studies regarding the design and tuning of three-term controllers for SOPD-IR processes (Chen et al, 2005, 2006; Chien et al, 2003; Luyben, 2000; Padma Sree & Chidambaram, 2004; Scali & Rachid, 1998; Waller & Nygardas, 1975; Zhang et al, 2000). On the other hand, integrating models with both inverse response and dead-time (IPDT-IR models) was found to be suitable for a variety of engineering processes, encountered in the process industry. The common examples of such processes are chemical reactors, distillation columns and, especially, level control of the boiler steam drum. In recent years, identification and tuning of controllers for such process models has not received the appropriate attention as compared to other types of inverse response processes, although some very interesting results have been reported in the literature (Gu et al, 2006; Luyben, 2003; Shamsuzzoha & Lee, 2006; Srivastava & Verma, 2007). In particular, the method reported by (Luyben, 2003) determines the integral time of a series form PID controller as a fraction of the minimum PI integral time, while the controller proportional gain is obtained by satisfying the specification of +2 dB maximum closed-loop log modulus, and the derivative time is given as the one maximizing the controller gain. In the work proposed by (Gu et al, 2006), PID controller tuning for IPDT-IR processes is performed based on H∞ optimization and Internal Model Control (IMC) theory. In the work

proposed by (Shamsuzzoha & Lee, 2006), set-point weighted PID controllers are designed on the basis of the IMC theory. Finally, in the work proposed by (Srivastava & Verma, 2007), a method involving numerical integration has been proposed in order to identify process parameters of IPDT-IR process models.

From the preceding literature review, it becomes clear that results on controller tuning for IPDT-IR processes are limited, and that there is a need for new efficient tuning methods for such processes. The aim of this work is to present innovative methods of tuning three-term controllers for integrating processes incorporating both time-delay and a non-minimum phase zero. The three-term controller configuration applied in this work is the well known I-PD (or Pseudo-Derivative Feedback, PDF) controller configuration (Phelan, 1978) due its advantages over the conventional PID controller configuration (Paraskevopoulos et al, 2004; Arvanitis et al, 2005; Arvanitis et al, 2009a). A series of innovative controller tuning methods is presented in the present work. These methods can be classified in two main categories: (a) methods based on the analysis of the phase margin of the closed-loop system, and (b) methods based on a direct synthesis approach. According to the first class of proposed tuning methods, the controller parameters are selected in order to meet the desired specifications in the time domain, in terms of the damping ratio or by minimizing various integral criteria (Wilton, 1999). In addition, the proportional gain of the controller is chosen in such a way, that the resulting closed-loop system achieves the maximum phase margin for the given specification in the time domain, thus resulting in robust closed-loop performance. Controller parameters are involved in nonlinear equations that are hard to solve analytically. For that reason, iterative algorithms are proposed in order to obtain the optimal controller settings. However, in order to apply the proposed methods in the case of on-line tuning, simple approximations of the exact controller settings obtained by the aforementioned iterative algorithms are proposed, as functions of the process parameters. The second class of proposed tuning methods is based on the manipulation of the closed-loop transfer function through appropriate approximations and cancellations, in order to obtain a second order dead-time closed-loop system. On the basis of this method, the parameters of the I-PD controller is obtained in terms of an adjustable parameter that can be further appropriately selected in order either to achieve a desired damping ratio for the closed-loop system or to ensure the minimization of conventional integral criteria. Finally, In order to assess the effectiveness of the proposed control scheme and associated tuning methods and to provide a comparison with existing tuning methods for PID controllers, a simulation study on the problem of controlling a boiler steam drum modelled by an IPDT-IR process model is presented. Simulation results reveal that the proposed controller and tuning methods provide considerably smoother response than known design methods for standard PID controllers, in case of set point tracking, as well as lower maximum error in case of regulatory control. This is particular true for the proposed direct synthesis method, which outperforms existing tuning methods for PID controllers.

2. IPDT-IR process models and the I-PD controller configuration

This work elaborates on IPDT-IR process models of the form

$$G_P(s) = \frac{\bar{K}\left(-s\bar{\tau}_Z + 1\right)\exp(-\bar{d}s)}{s\left(s\bar{\tau}_p + 1\right)} \tag{1}$$

where $\bar{K}$, $\bar{d}$, $\bar{\tau}_P$ and $\bar{\tau}_Z$ are the process gain, the time delay, the time constant and the zero's time constant, respectively, controlled using the configuration of Fig. 1, i.e. the so-called I-PD or PDF controller. In this controller configuration, the three controller actions are separated. Integral action, which is dedicated to steady state error elimination, is located in the forward path of the loop, whereas proportional and derivative actions, which are mainly dedicated in assigning the desired closed-loop performance in terms of stability, responsiveness, disturbance attenuation, etc, are located in the feedback path. This separation leads to a better understanding of the role of each particular controller action. Moreover, the I-PD controller has some distinct advantages over the conventional PID controller, as reported in the works by (Paraskevopoulos et al, 2004; Arvanitis et al, 2005; Arvanitis et al, 2009a).

Observe now that applying the I-PD control strategy to the process model of the form (1), the following closed-loop transfer function is obtained

$$G_{CL}(s) = \frac{\bar{K}\bar{K}_I\left(-s\bar{\tau}_Z + 1\right)\exp(-\bar{d}s)}{s^2\left(\bar{\tau}_P s + 1\right) + \bar{K}\left(\bar{K}_D s^2 + \bar{K}_P s + \bar{K}_I\right)\left(-s\bar{\tau}_Z + 1\right)\exp(-\bar{d}s)} \tag{2}$$

It is not difficult to see that the action of the I-PD controller is equivalent to that of a PID controller in series form having the transfer function

$$G_{C,PID}(s) = \bar{K}_C\left(1 + \frac{1}{\bar{\tau}_I s}\right)\left(1 + \bar{\tau}_D s\right) \tag{3}$$

with a second order set-point pre-filter of the form $G_{C,SPF}(s) = 1/\left(\bar{\tau}_I s + 1\right)\left(1 + \bar{\tau}_D s\right)$, provided that the following relations hold
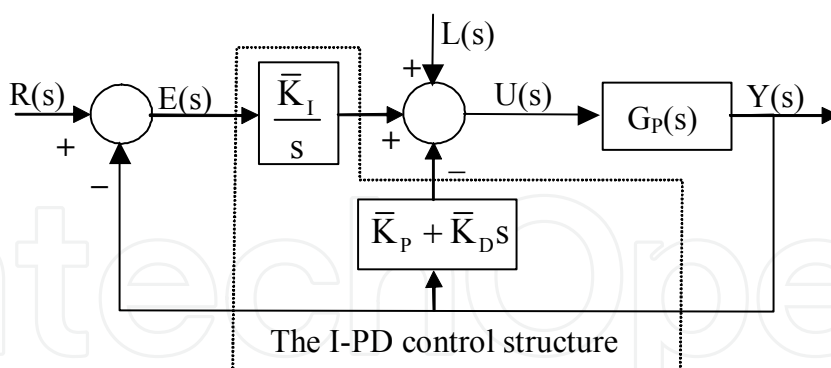

Fig. 1. The I-PD or PDF control strategy.

$$\bar{K}_P = \bar{K}_C\left(\bar{\tau}_D + \bar{\tau}_I\right)/\bar{\tau}_I \ , \ \bar{K}_I = \bar{K}_P/\left(\bar{\tau}_D + \bar{\tau}_I\right) = \bar{K}_C/\bar{\tau}_I \ , \ \bar{K}_D = \bar{K}_P\bar{\tau}_D\bar{\tau}_I/\left(\bar{\tau}_D + \bar{\tau}_I\right) = \bar{K}_C\bar{\tau}_D \tag{4}$$

Taking into account the above equivalence, the loop transfer function of the proposed feedback structure is given by

$$G_L(s) = \frac{\bar{K}\bar{K}_C\left(-s\bar{\tau}_Z + 1\right)\left(\bar{\tau}_I s + 1\right)\left(\bar{\tau}_D s + 1\right)\exp(-\bar{d}s)}{\bar{\tau}_I s^2\left(\bar{\tau}_P s + 1\right)} \tag{5}$$

Relations (2) and (5) are used in the next Sections for the derivation of the tuning methods proposed in this work.

3. Frequency domain analysis for closed-loop IPDT-IR processes

The equivalence between the PD-1F controller and the set-point pre-filtered PID controller provides us the possibility, to work with $\bar{K}_C$, $\bar{\tau}_I$ and $\bar{\tau}_D$ and not directly with $\bar{K}_P$, $\bar{K}_I$ and $\bar{K}_D$. Furthermore, in order to facilitate comparisons, let all system and controller parameters be normalized with respect to $\bar{\tau}_P$ and $\bar{K}$. Thus, the original process and controller parameters are replaced with the dimensionless parameters shown in Table 1. Then, relations (2) and (5) yield

$$G_{CL}(\hat{s}) = \frac{K_C\left(-\hat{s}\tau_Z + 1\right)\exp(-d\hat{s})}{\tau_I\hat{s}^2\left(\hat{s}+1\right) + K_C\left(\tau_I\hat{s}+1\right)\left(\tau_D\hat{s}+1\right)\left(-\hat{s}\tau_Z+1\right)\exp(-d\hat{s})} \tag{6}$$

$$G_L(\hat{s}) = \frac{K_C\left(-\hat{s}\tau_Z + 1\right)\left(\tau_I\hat{s}+1\right)\left(\tau_D\hat{s}+1\right)\exp(-d\hat{s})}{\tau_I\hat{s}^2\left(\hat{s}+1\right)} \tag{7}$$

From relation (7), the argument and the magnitude of the loop transfer function are given by

$$\varphi_L(w) = -\pi - dw - atan(w) - atan(\tau_Z w) + atan(\tau_I w) + atan(\tau_D w) \tag{8}$$

Original Parameters	Normalized Parameters	Original Parameters	Normalized Parameters
$\bar{\tau}_p$	$\tau_p = 1$	S	$\hat{s} = s\bar{\tau}_P$
$\bar{\tau}_Z$	$\tau_Z = \bar{\tau}_Z / \bar{\tau}_P$	$\bar{K}$	$K = 1$
$\bar{d}$	$d = \bar{d} / \bar{\tau}_P$	$\bar{K}_C$	$K_C = \bar{K}\bar{K}_C$
$\bar{\tau}_I$	$\tau_I = \bar{\tau}_I / \bar{\tau}_P$	$\bar{K}_D$	$K_D = \bar{K}\bar{K}_D$
$\bar{\tau}_D$	$\tau_D = \bar{\tau}_D / \bar{\tau}_P$	$\bar{K}_P$	$K_P = \bar{\tau}_p\bar{K}\bar{K}_P$
ω	$w = \omega\bar{\tau}_P$	$\bar{K}_I$	$K_I = \bar{\tau}_p^2\bar{K}\bar{K}_I$

Table 1. Normalized vs. original system parameters.

$$A_L\left(w\right) = \left|G_L\left(jw\right)\right| = K_C\frac{\sqrt{1+\left(\tau_I w\right)^2}\sqrt{1+\left(\tau_D w\right)^2}\sqrt{1+\left(\tau_Z w\right)^2}}{\tau_I w^2\sqrt{1+w^2}} \tag{9}$$

In Fig. 2, the Nyquist plots of $G_L(\hat{s})$ for typical IPDT-IR processes are depicted for several values of the parameter τ_I. From this figure, it becomes clear that, for specific d and τ_Z, and for τ_I greater than a critical value, say $\tau_{I,min}$, there exists a crossover point of the Nyquist plot with the negative real axis. In this case, the system can be stabilized, with an appropriate choice of K_C. Moreover, from these Nyquist plots, one can observe that the stability region is reduced when τ_I is decreased, starting from the maximum region of stability when $\tau_I \to \infty$, (which corresponds to a PD-controller). The Nyquist plot does not have any crossover point

with the negative real axis, in the case where $\tau_I=\tau_{I,min}$; that is, for $\tau_I \leq \tau_{I,min}$, the process cannot be stabilized. Moreover, in Fig. 3, the Nyquist plots of $G_L(\hat{s})$ are depicted for several values of the parameter τ_D. From these plots, it becomes clear that, for small values of τ_D, the stability region is increased with τ_D, whereas for larger values of τ_D the stability region decreases when τ_D increases.

Let $PM=\varphi(w_G)+\pi$ be the phase margin of the closed-loop system, where w_G is the frequency, at which the magnitude of the loop transfer function $G_L(\hat{s})$ equals unity. Taking into account (8), we obtain $PM = -dw_G – atan(w_G) - atan(\tau_Z w_G) + atan(\tau_I w_G) + atan(\tau_D w_G)$. From Fig. 2, it can be readily observed that for given d, τ_I, τ_D, τ_Z, there exists one point of the Nyquist plot corresponding to the maximum argument $\varphi_{L,max}(d, \tau_I, \tau_D, \tau_Z)$. The frequency w_P at which the argument (8) is maximized is given by the maximum real root of the equation $\left[d\varphi_L(w) / dw \right]\big|_{w=w_p} = 0$, which, after some easy algebraic manipulations, yields

$$-d - \frac{1}{1+w_P^2} - \frac{\tau_Z}{1+\tau_Z^2 w_P^2} + \frac{\tau_I}{1+\tau_I^2 w_P^2} + \frac{\tau_D}{1+\tau_D^2 w_P^2} = 0 \tag{10}$$

Hence, substituting w_P, as obtained by (10), in (8), the respective argument $\varphi_L(w_P)$ is computed. Consequently, the maximum phase margin PM_{max} for given d, τ_I, τ_D, τ_Z, can be obtained if we put $w_P=w_G$, i.e. choosing the controller proportional gain K_C according to

$$K_C = \frac{\tau_I w_P^2 \sqrt{1+w_P^2}}{\sqrt{1+(\tau_D w_P)^2}\sqrt{1+(\tau_I w_P)^2}\sqrt{1+(\tau_Z w_P)^2}} \tag{11}$$

With this choice for K_C, the phase margin is given by

$$PM(d,\tau_I,\tau_D,\tau_Z) = -dw_P - a\tan(w_P) - a\tan(\tau_Z w_P)$$
$$+a\tan(\tau_I w_P) + a\tan(\tau_D w_P) = PM_{max}(d,\tau_I,\tau_D,\tau_Z) \tag{12}$$

Obviously, in the case where $\tau_I=\tau_{I,min}$ and K_C is obtained by (11), then the closed-loop system is marginally stable, that is $PM_{max}=0$. Note that, from (12), $PM_{max}=0$, when $w_P=0$. Therefore, from (10), for $w_P=0$, we obtain $\tau_{I,min}=d+\tau_Z+1-\tau_D$. Since, for all values of τ_I, larger than $\tau_{I,min}=d+\tau_Z+1-\tau_D$, it holds $\left(d\left[PM(d,\tau_I,\tau_D,\tau_Z) \right] / dw \right)\big|_{w=0} = \tau_I+\tau_D-d-\tau_Z-1>0$, one can readily conclude that $PM_{max}>0$. Moreover, PM_{max} is an increasing function of both τ_I and τ_D. This is illustrated in Fig. 4, where PM_{max} is given as a function of τ_I and τ_D, for a typical IPDT-IR process.

As it was previously mentioned, the stability region of the closed-loop system increases with τ_D. This is due to the fact that the closed-loop system gain margin increases with τ_D, as one can verify from the variation of the crossover point of the Nyquist plot with the negative real axis as τ_D varies. Furthermore, there is one value of τ_D, say $\tau_{D,GM_{max}}(\tau_I, \tau_Z, d)$, for which the closed loop gain margin starts to decrease. In addition, from Fig. 2, it can be easily verified that the closed-loop gain margin increases arbitrarily as τ_I increases. These observations can also become evident from Fig. 5 that illustrates the maximum closed-loop gain margin GM_{max} as a function of the controller parameters τ_D and τ_I.
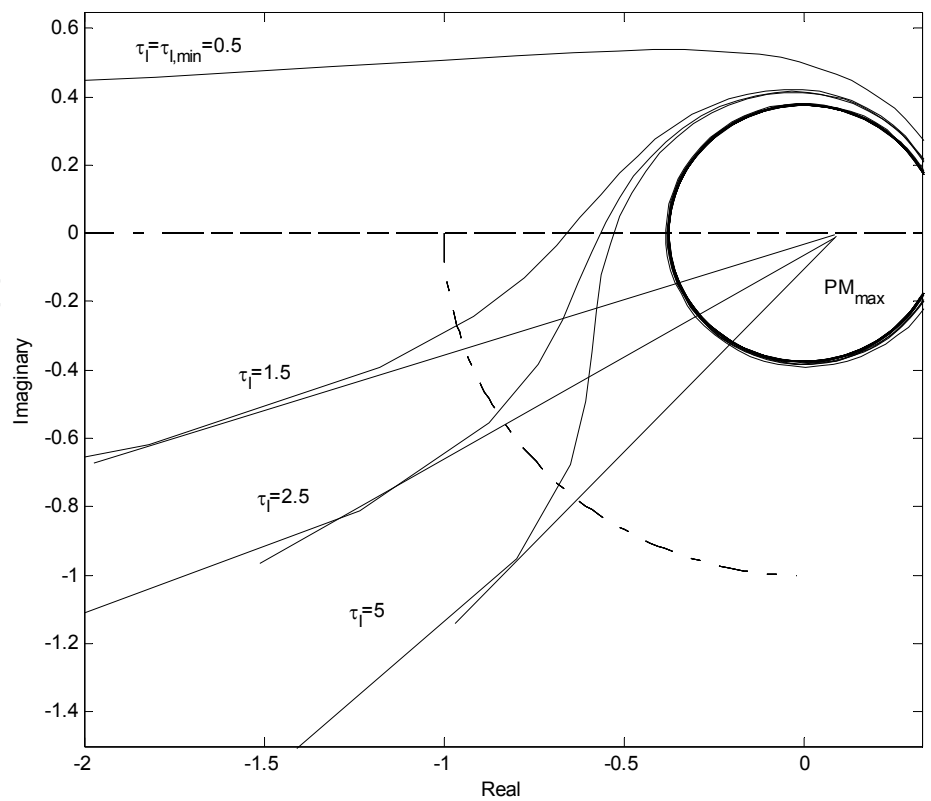
Fig. 2. Nyquist plots of a typical IPDT-IR process controlled by a PD1-F controller with K_C=0.5, d=0.5, τ_Z=0.5, τ_D=1.5, for various values of τ_I.
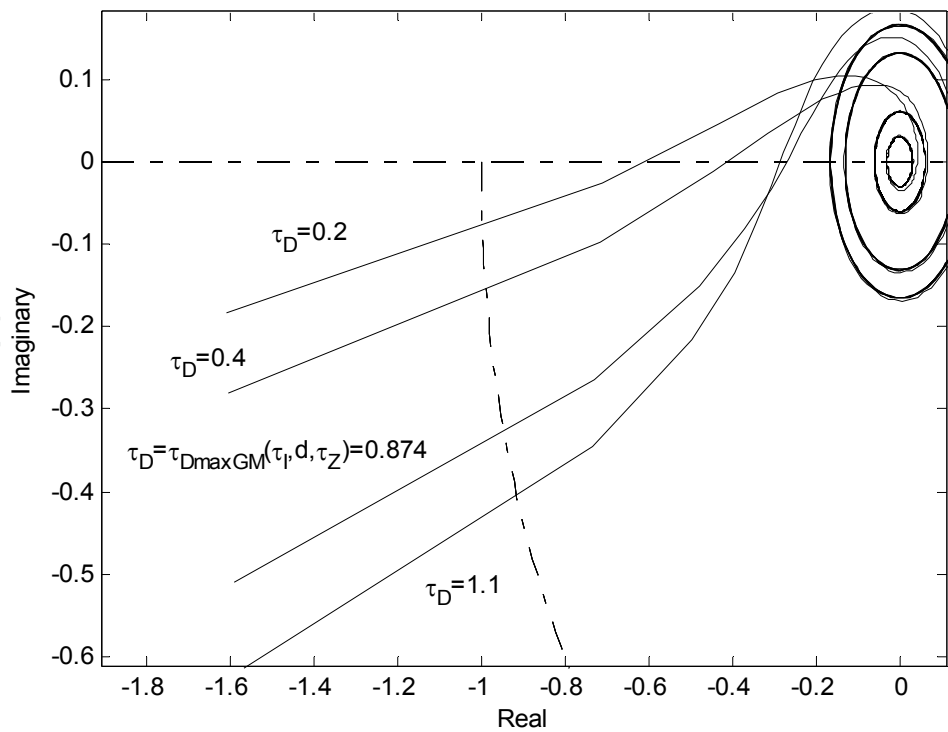

Fig. 3. Nyquist plots of a typical IPDT-IR process controlled by a PD-1F controller with K_C=0.3, d=0.5, τ_Z=0.5, τ_I=2.5, for various values of τ_D.

It can also be observed, from Fig. 5, that the value of $\tau_{D,GM_{max}}(\tau_I,\tau_Z,d)$ decreases as τ_I increases and it takes its minimum value, denoted by $\min\tau_{D,GM_{max}}(\tau_Z,d)$, when $\tau_I\to\infty$. This is also evident from Fig. 6, where $\tau_{D,GM_{max}}(\tau_I,\tau_Z,d)$ is depicted, as a function of τ_I, for several values of τ_Z and d. Applying optimization techniques using MATLAB®, it is plausible to provide accurate approximations of that limit as a function of the normalized parameters d and τ_Z. These approximations are summarized in Table 2. Note that, the maximum normalized error (M.N.E.), defined by

$$\max\left\{\frac{\left|\min\hat{\tau}_{D,GM_{max}}(\tau_Z,d)\text{-}\min\tau_{D,GM_{max}}(\tau_Z,d)\right|}{\min\tau_{D,GM_{max}}(\tau_Z,d)}\right\}, \text{where } \min\hat{\tau}_{D,GM_{max}}(\tau_Z,d)$$

denotes the approximate value, never exceeds 3%, for a wide range of d and τ_Z.

Finally, another interesting value relative to τ_D, is $\tau_{D,maxGM,I\text{-}P}(\tau_I,\tau_D,d)$ that denotes the maximum value of τ_D, for which the gain margin obtained by an I-PD controller is larger than the gain margin obtained by an I-P controller. It is worth noticing that the value of $\tau_{D,maxGM,I\text{-}P}(\tau_I,\tau_D,d)$ decreases as τ_I increases and it takes its minimum value, which is denoted by $min\tau_{D,maxGM,I\text{-}P}(\tau_I,\tau_D,d)$, when $\tau_I\to\infty$. This is evident from Fig. 7, which illustrates $\tau_{D,maxGM,I\text{-}P}(\tau_I,\tau_D,d)$ as a function of τ_I, for several pairs (τ_Z,d). Application of optimization techniques yields some useful approximations of this parameter, which are summarized in Table 3. These approximations are quite accurate, since the respective maximum normalized errors never exceed 5%, for a wide range of d and τ_Z.

4. Controller tuning based on the maximum phase margin specification

The above analysis provides us the means to propose an efficient method for tuning I-PD controllers for IPDT-IR processes. The main characteristic of the proposed tuning method, which is designated as Method I in the sequel, is the selection of the controller gain K_C using (11). The remaining two parameters τ_I and τ_D can be selected such that a specific closed-loop performance is achieved. In particular, one can select the parameter τ_I in order to achieve a specific closed-loop response while selecting the parameter τ_D in order to improve this response in terms of the achievable gain margin or in terms of the minimization of several integral criteria, such as the well-known ISE criterion, the integral of squared error plus the normalized squared controller output deviation from its final value u_∞ (ISENSCOD criterion) of the form

$$J_{ISENSCOD} = \int_0^\infty \left\{ \left[y(t) - r(t) \right]^2 + K^2 \left[u(t) - u_\infty \right]^2 \right\} dt \tag{13}$$

or the integral of squared error plus the normalized squared derivative of the controller output (ISENSDCO criterion), having the form

$$J_{ISENSDCO} = \int_0^\infty \left\{ \left[y(t) - r(t) \right]^2 + K^2 \bar{\tau}_p^2 \dot{u}(t)^2 \right\} dt \tag{14}$$
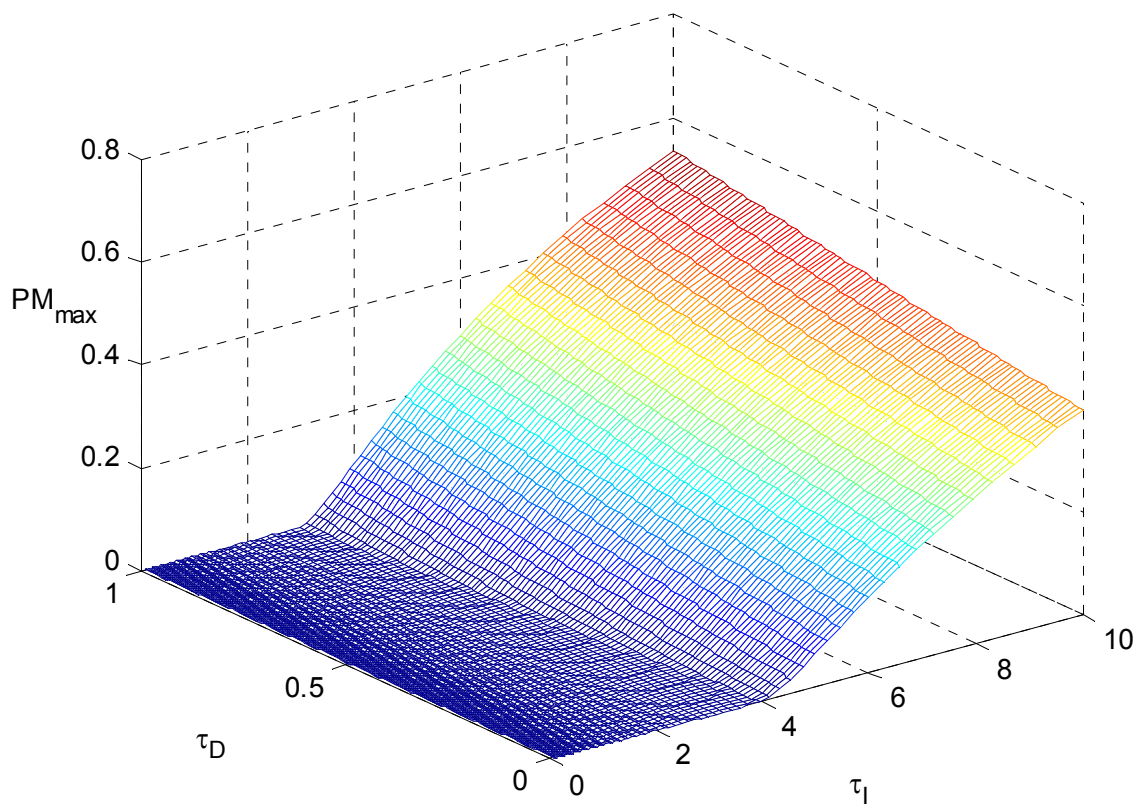
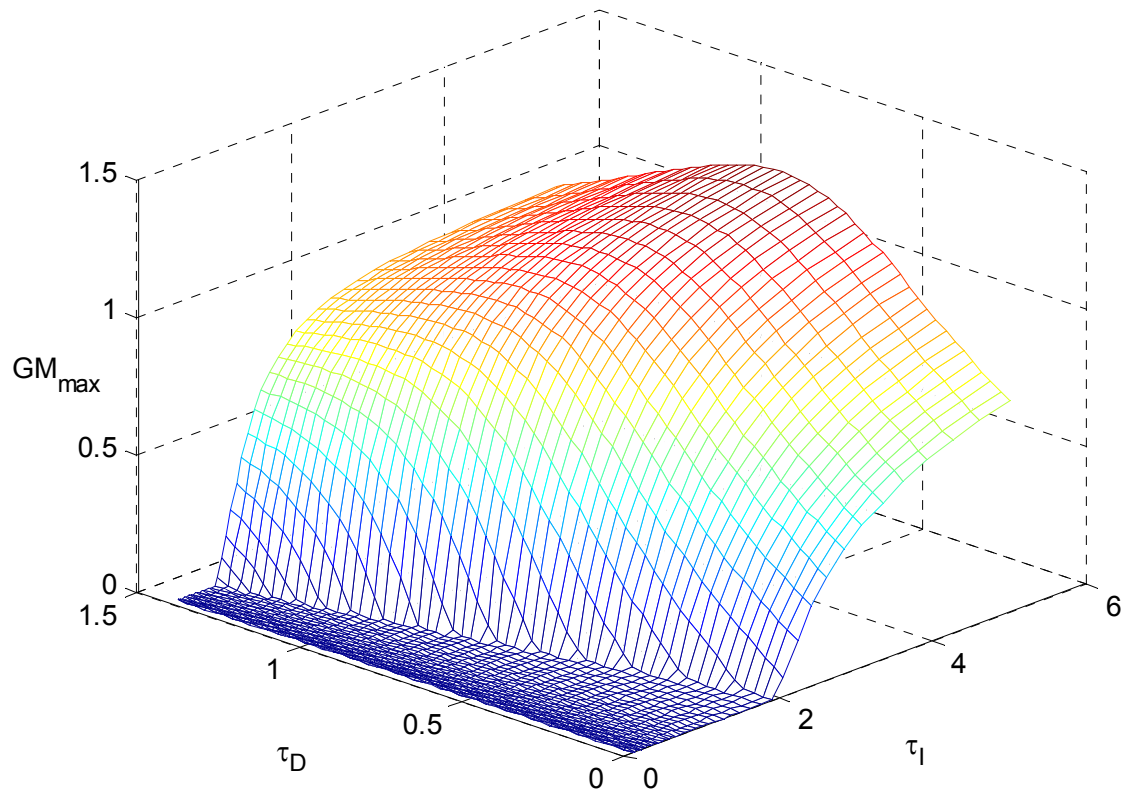Fig. 4. PM_{max} as a function of τ_I and τ_D for a typical IPDT-IR with $d=\tau_Z$=1.5.

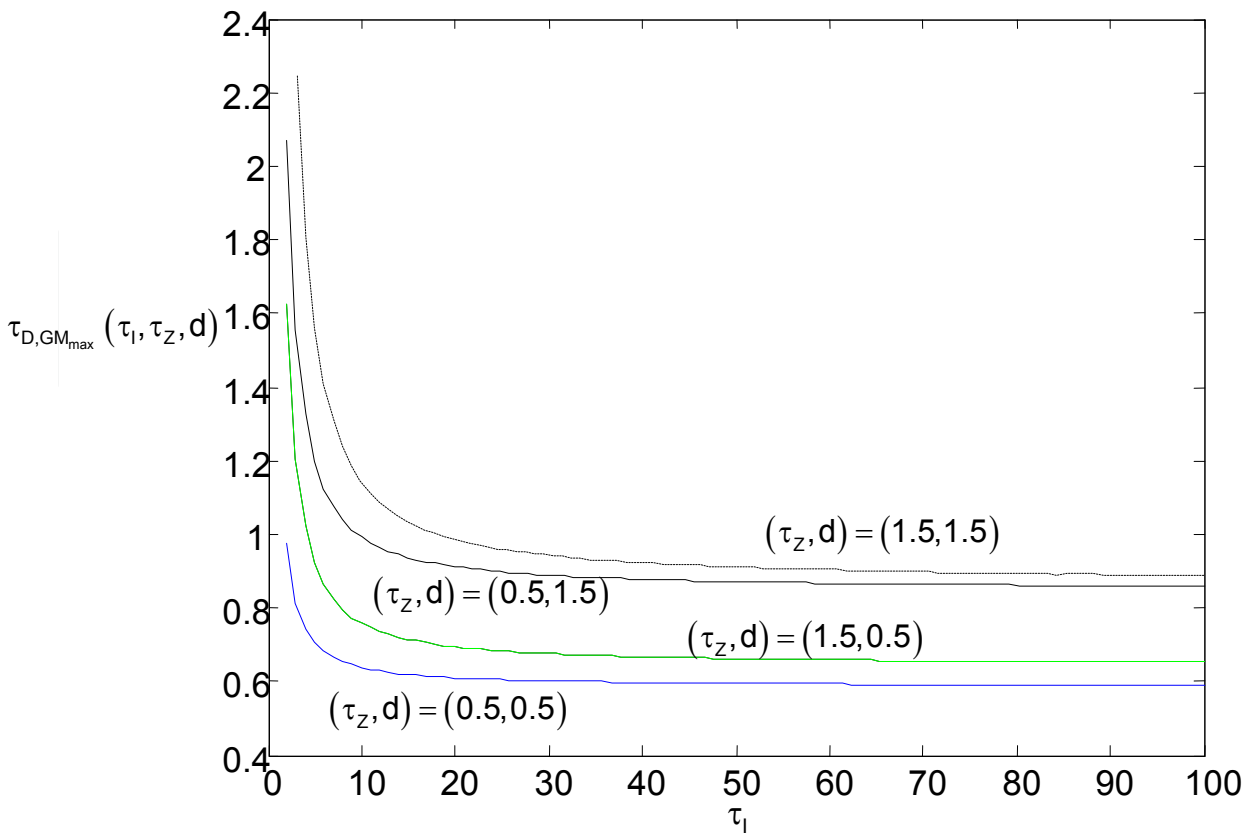Fig. 5. GM_{max} as a function of τ_I and τ_D for a typical IPDT-IR process with $d=\tau_Z$=0.5.

Fig. 6. Parameter $\tau_{D,GM_{max}}(\tau_I, \tau_Z, d)$ as a function of τ_I, for several values of τ_Z and d.

$\min\tau_{D,GMmax}(d,\tau_Z)$	M.N.E.
$-0.0561+0.555d-0.0629\tau_Z^2+0.1308\tau_Z-0.372d^2-0.1171(d+\tau_Z)^3+0.5144(d+\tau_Z)^{0.5}$ for $0<\tau_Z<0.5$ & $0<d<0.5$	2.8%
$0.4722+0.2213d-0.0045\tau_Z^2+0.0375\tau_Z+0.0026d^2+0.00004093(d+\tau_Z)^3$ for $0.5<\tau_Z<5$ & $0.5<d<5$	2.5%

Table 2. Proposed approximations for $\min\tau_{D,GMmax}(d,\tau_Z)$ for several values of τ_Z and d.

$\min\tau_{D,maxGM,I-P}(d,\tau_Z)$	M.N.E.
$1.3335-1.311d-0.755\tau_Z^2+1.7764\tau_Z+0.634d^2+1.484d^{0.5}-1.7531\tau_Z^{0.5}$ for $0<\tau_Z<0.5$ & $0<d<0.5$	2.8%
$1.1393+0.4528d-0.107\tau_Z+0.0114\tau_Z^2+0.0044d^2$ for $0.5<\tau_Z<5$ & $0.5<d<5$	2.5%

Table 3. Proposed approximations for $\min\tau_{D,maxGM,I-P}(d,\tau_Z)$ for several values of τ_Z and d.

It is worth to notice, at this point, that tuning methods based on the minimization of ISE guarantee small error and very fast response, particularly useful in the case of regulatory control. However, the closed-loop step response is very oscillatory, and the tuning can lead to excessive controller output swings that cause process disturbances in other control loops. In contrast, minimization of criteria (13) or (14) leads to smoother closed-loop responses that are less demanding for the process actuators.
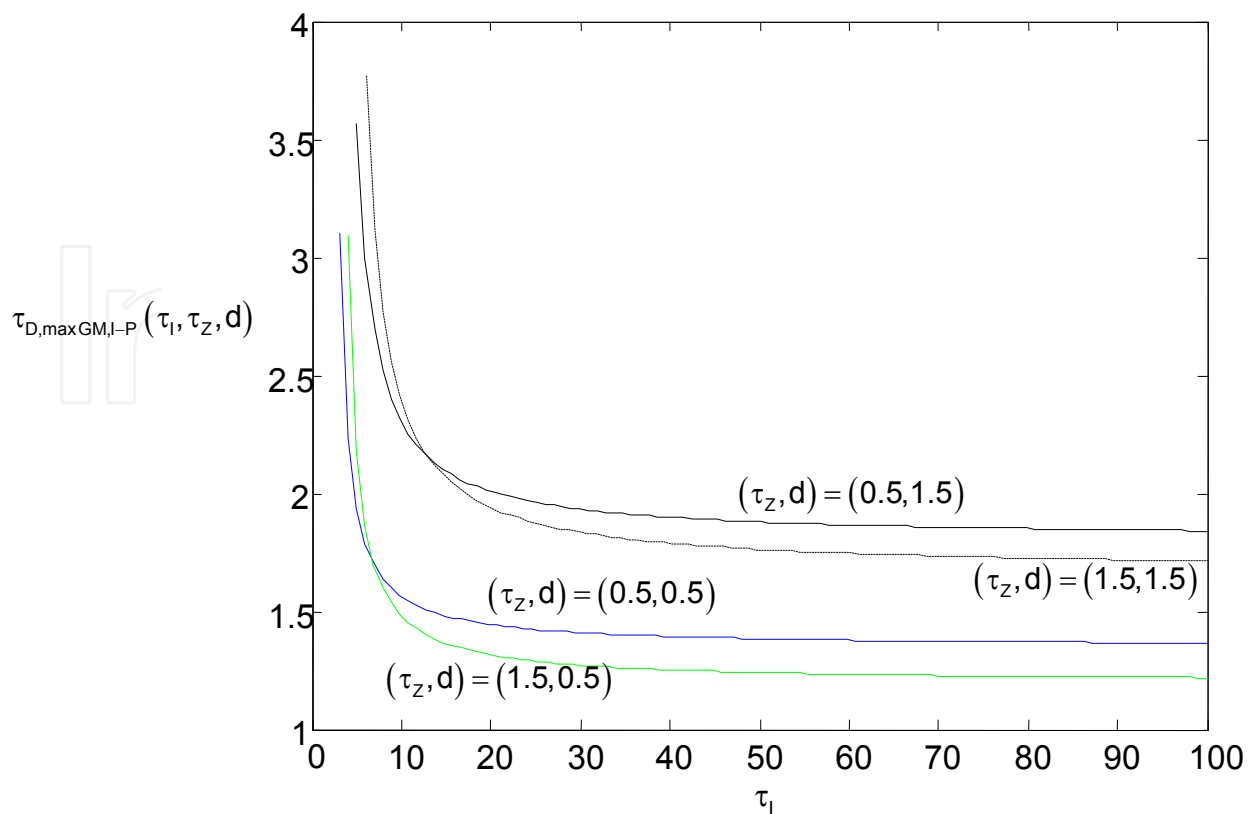
Fig. 7. Parameter $\tau_{D,maxGM,I\text{-}P}(\tau_I,\tau_D,d)$ as a function of τ_I, for several values of τ_Z and d.

In order to present systematically Method I, observe first that (6) can also be written as

$$G_{CL}(\hat{s}) = \frac{(-\hat{s}\tau_Z+1)(\hat{s}+1)^{-1}\exp(-d\hat{s})}{K_C^{-1}\tau_I\hat{s}^2+(-\hat{s}\tau_Z+1)(\tau_I\hat{s}+1)(\tau_D\hat{s}+1)(\hat{s}+1)^{-1}\exp(-d\hat{s})} \tag{15}$$

Setting $(-\hat{s}\tau_Z+1)(\hat{s}+1)^{-1} \approx 1-(1+\tau_Z)\hat{s}$ in the numerator of (15), we obtain

$$G_{CL}(\hat{s}) \simeq \frac{\left[1-(1+\tau_Z)\hat{s}\right]\exp(-d\hat{s})}{K_C^{-1}\tau_I\hat{s}^2+P(\hat{s})} \tag{16}$$

where $P(\hat{s})=(-\hat{s}\tau_Z+1)(\tau_I\hat{s}+1)(\tau_D\hat{s}+1)(\hat{s}+1)^{-1}\exp(-d\hat{s})$. Let us now perform a second order MacLaurin approximation of $P(\hat{s})$. That is $P(\hat{s}) \approx P_0+P_1\hat{s}+(P_2/2)\hat{s}^2$. Simple algebra yields $P_0=1$, $P_1=\tau_I+\tau_D-\tau_Z-d-1$, $P_2=2\tau_I(\tau_D-\tau_Z-d-1)+2(d+\tau_Z+1)(1-\tau_D)+d(2\tau_Z+d)$. Substituting the above relations in (16), we obtain

$$G_{CL}(\hat{s}) \simeq \frac{\left[1-(1+\tau_Z)\hat{s}\right]}{\tau_e^2\hat{s}^2+2\zeta\tau_e\hat{s}+1}\exp(-d\hat{s}) \tag{17}$$

where

$$\tau_e = \sqrt{\tau_I(K_C^{-1}+\tau_D-\tau_Z-d-1)+(d+\tau_Z+1)(1-\tau_D)+d(\tau_Z+0.5d)} \tag{18}$$

$$\zeta = \frac{(\tau_I + \tau_D - \tau_Z - d - 1)}{2\sqrt{\tau_I(K_C^{-1} + \tau_D - \tau_Z - d - 1) + (d + \tau_Z + 1)(1 - \tau_D) + d(\tau_Z + 0.5d)}} \qquad (19)$$

It is now clear that, the parameter τ_I can be selected in such a way that a desired damping ratio ζ_{des} is obtained for the second order approximation (17), of the closed-loop transfer function. With this design specification, the parameter τ_I must be selected as the maximum real root of the quadratic equation

$$\tau_I^2 + \tau_I \left[2(\tau_D - \tau_Z - d - 1) - 4\zeta_{des}^2(K_C^{-1} + \tau_D - \tau_Z - d - 1) \right]$$
$$+ (\tau_D - \tau_Z - d - 1)^2 - 4\zeta_{des}^2 \left[(d + \tau_Z + 1)(1 - \tau_D) + d(\tau_Z + 0.5d) \right] = 0 \qquad (20)$$

Relation (20) suggests that τ_I depends on τ_D and K_C. Since, here, it is proposed to select K_C according to (10) and (11), it remains to consider how to select τ_D. With regard to the parameter τ_D, there are several possible alternative ways to select it:

A first way, is to select $\tau_D = \min\tau_{D,maxGM,I-P}(\tau_Z, d)$. In this case, in order to calculate the parameters K_C and τ_I, it is necessary to solve equations (10), (11) and (20). To this end, the following iterative algorithm is applied:

4.1 Algorithm I

Step 1.   Set $\tau_D = \min\tau_{D,maxGM,I-P}(\tau_Z, d)$ and $\zeta = \zeta_{des}$.

Step 2.   Start with an initial values of τ_I. An appropriate choice is $\tau_{I,init} = 1.2\tau_{I,min} = 1.2(d + \tau_Z + 1 - \tau_D) = 1.2[d + \tau_Z + 1 - \min\tau_{D,maxGM,I-P}(\tau_Z, d)]$

Step 3.   For these values of τ_D and τ_I, calculate K_C from relation (11), using relation (10).

Step 4.   Calculate the integral term τ_I as the maximum real root of (20).

Step 5.   Repeat Steps 3 and 4 until the algorithm converges to a certain value for τ_I and K_C.

Note that the above algorithm always converges to values of K_C and τ_I that satisfy equations (10), (11) and (20). The parameters K_C and τ_I, obtained by the above algorithm, for several values of the desired damping ratio ζ_{des} can be approximated by the functions summarized in Table 4. These functions have been obtained by applying optimization techniques, which intent to minimize the respective M.N.E.s. These errors never exceed 5%. For intermediate values of ζ_{des}, a simple linear interpolation provides sufficiently accurate estimates of $\bar{K}_C$ and $\bar{\tau}_I$. The above approximations are very useful, since the need of iterations is avoided, when the proposed method is applied.

It is worth to notice, at this point, that the values of the parameter $\tau_D = \min\tau_{D,maxGM,I-P}(\tau_Z, d)$ are, in general, a bit small. Additionally, in the case of regulatory control, the I-PD controller settings obtained by Algorithm I, give somehow large maximum errors, although provide rather satisfactory settling times,. Simulation results show that larger values of τ_D give a better performance in terms of maximum error, whereas the settling time is larger. In this case, a good trade-off between settling time and maximum error, is obtained when $\tau_D = 1.25\min\tau_{D,maxGM,I-P}(\tau_Z, d)$. Table 5 summarizes the estimations of K_C and τ_I as functions of τ_Z

ζ_{des}	$K_C(\tau_Z,d)$ (for $0.2<\tau_Z<2$ and $0.1<d<2$)	M.N.E.
0.6	$0.1994+0.0832d-0.0143\tau_Z-0.0179d^2-0.1038\tau_Z^{0.5}+0.3591(d+\tau_Z+0.1)^{-1}$ $-0.197d(d+\tau_Z)^{-1}$	3.8%
0.707	$0.1711+0.0648d+0.0041\tau_Z-0.0157d^2-0.1135\tau_Z^{0.5}+0.3412(d+\tau_Z+0.1)^{-1}$ $-0.1353d(d+\tau_Z)^{-1}$	3.82%
0.85	$0.0885+0.0372d-0.0128\tau_Z-0.0094d^2-0.0449\tau_Z^{0.5}+0.3131(d+\tau_Z+0.1)^{-1}$ $-0.0484d(d+\tau_Z)^{-1}$	4%
1	$0.0124-0.0048d-0.008\tau_Z-0.0002d^2-0.005\tau_Z^{0.5}+0.2823(d+\tau_Z+0.1)^{-1}$ $-0.0509d(d+\tau_Z)^{-1}$	3.86%
1.2	$-0.0887-0.0339d+0.0072\tau_Z+0.0039d^2+0.0282\tau_Z^{0.5}+0.256(d+\tau_Z+0.1)^{-1}$ $+0.1593d(d+\tau_Z)^{-1}$	3.1%
ζ_{des}	$\tau_I(\tau_Z,d)$ (for $0.2<\tau_Z<2$ and $0.1<d<2$)	M.N.E.
0.6	$1.4954+3.1027d-3.8488\tau_Z-0.1729d^2-0.5138\tau_Z^{0.5}-1.4686(d+\tau_Z+1)^{-1}$	2.6%
0.707	$2.5628+4.329d+6.2521\tau_Z-0.4398d^2-2.5004\tau_Z^{0.5}-1.585(d+\tau_Z+1)^{-1}$	2.86%
0.85	$2.2014+7.5718d+10.1797\tau_Z-0.9624d^2-4.3353\tau_Z^{0.5}+0.3528(d+\tau_Z+1)^{-1}$	3.32%
1	$1.0227+4.2832d+9.2825\tau_Z+0.6779d^2-0.1484\tau_Z^{0.5}+2.0726\tau_Z^2+5.7469d^{0.5}$	3.14%
1.2	$3.0653+11.6507d+22.9691\tau_Z+0.9966d^2-3.453\tau_Z^{0.5}+3.0497\tau_Z^2+4.7546d^{0.5}$	3.15%

Table 4. Proposed approximations for $K_C(\tau_Z,d)$ and $\tau_I(\tau_Z,d)$ obtained by Algorithm I, for $0.2<\tau_Z<2$ and $0.1<d<2$, in the case where $\tau_D=\min\tau_{D,maxGM,I-P}(\tau_Z,d)$.

and d, in the case where, in Step 1 of Algorithm I, parameter τ_D is selected as $\tau_D=1.25\min\tau_{D,maxGM,I-P}(\tau_Z,d)$ instead of $\tau_D=\min\tau_{D,maxGM,I-P}(\tau_Z,d)$.

A second obvious way to select τ_D is through the relation $\tau_D=\min\tau_{D,GM_{max}}(\tau_Z,d)$. However, in this case, the obtained values of τ_D, are quite small. It is worth to remember that the value $\min\tau_{D,GM_{max}}(\tau_Z,d)$ is obtained when $\tau_I\to\infty$. So, in order to obtain a larger and more efficient value for the parameter τ_D, we propose here to select $\tau_D=2\tau_{D,GM_{max}}(\tau_I,\tau_Z,d)$. This value of τ_D, which is twice the one that maximizes the gain margin for a given τ_I, is obtained without the assumption $\tau_I\to\infty$, and of course it is always greater than the value $\min\tau_{D,GM_{max}}(\tau_Z,d)$. However, that choice causes additional difficulties on our attempts to obtain the parameters K_C, τ_I and τ_D, since τ_D, now, depends implicitly on τ_I, whereas the choice of $\tau_D=\min\tau_{D,GM_{max}}(\tau_Z,d)$ or $\tau_D=\min\tau_{D,maxGM,I-P}(\tau_Z,d)$ are independent of τ_I. Therefore, additional iterations are necessary, in the case where the selection $\tau_D=2\tau_{D,GM_{max}}(\tau_I,\tau_Z,d)$ is made. More precisely, in this case, the following algorithm is proposed to obtain admissible controller settings:

ζ_{des}	$K_C(\tau_Z,d)$	M.N.E.
0.6	$0.2574+0.0349d-0.0356\tau_Z-0.0085d^2-0.0559\tau_Z^{0.5}+0.2538(d+\tau_Z+0.1)^{-1}$ $-0.1702d(d+\tau_Z)^{-1}$ for $0.3<\tau_Z<2$ & $0.3<d<2$	3%
0.6	$0.0439+0.1017d+0.0268\tau_Z-0.0155d^2-0.1347\tau_Z^{0.5}+0.9583(d+\tau_Z+1)^{-1}$ $-0.2309d(d+\tau_Z)^{-1}$ for $0.4<\tau_Z<2$ & $0.2<d<2$	1.3%
0.6	$0.0225+0.0943d-0.0025\tau_Z-0.0152d^2-0.0707\tau_Z^{0.5}+0.9161(d+\tau_Z+1)^{-1}$ $-0.2117d(d+\tau_Z)^{-1}$ for $0.2<\tau_Z<2$ & $0.4<d<2$	1.1%
0.707	$0.2814+0.0362d-0.046\tau_Z-0.0087d^2-0.0552\tau_Z^{0.5}+0.2225(d+\tau_Z+0.1)^{-1}$ $-0.1841d(d+\tau_Z)^{-1}$ for $0.2<\tau_Z<2$ & $0.06<d<2$	4.8%
0.85	$0.1698+0.0457d-0.0349\tau_Z-0.0099d^2-0.0413\tau_Z^{0.5}+0.3256(d+\tau_Z+0.3)^{-1}$ $-0.1381d(d+\tau_Z)^{-1}$ for $0.2<\tau_Z<2$ & $0.1<d<2$	4%
1	$0.0939+0.0305d-0.0366\tau_Z-0.0065d^2-0.0076\tau_Z^{0.5}+0.3198(d+\tau_Z+0.3)^{-1}$ $-0.0709d(d+\tau_Z)^{-1}$ for $0.2<\tau_Z<2$ & $0.1<d<2$	4.72%
1.2	$0.0195+0.0205d-0.0153\tau_Z-0.0047d^2-0.0159\tau_Z^{0.5}+0.3219(d+\tau_Z+0.3)^{-1}$ $+0.0024d(d+\tau_Z)^{-1}$ for $0.2<\tau_Z<2$ & $0.12<d<2$	4.96%
ζ_{des}	$\tau_I(\tau_Z,d)$	M.N.E.
0.6	$-0.4713+1.8218d+1.8329\tau_Z+0.0067d^2+1.4335\tau_Z^{0.5}+0.3006\tau_Z^2+1.5613d^{0.5}$ for $0.3<\tau_Z<2$ & $0.3<d<2$	1.13%
0.707	$0.2867+0.2494d-3.2074\tau_Z+0.4602d^2+6.44\tau_Z^{0.5}+1.5289\tau_Z^2+3.6189d^{0.5}$ $-2.2731(d+\tau_Z+1)^{-1}$ for $0.2<\tau_Z<2$ & $0.06<d<2$	4.17%
0.85	$1.9692+1.1533d+5.4948\tau_Z+0.4359d^2-2.9027\tau_Z^{0.5}+0.998\tau_Z^2+5.6043d^{0.5}$ $+0.8839(d+\tau_Z+1)^{-1}$ for $0.2<\tau_Z<2$ & $0.1<d<2$	1.7%
1	$2.0741+3.515d+3.3445\tau_Z+0.3835d^2+0.2227\tau_Z^{0.5}+3.2956\tau_Z^2+6.4788d^{0.5}$ $+2.326(d+\tau_Z+1)^{-1}$ for $0.2<\tau_Z<2$ & $0.1<d<2$	2.4%
1.2	$-1.3108+8.9972d+19.7612\tau_Z+0.2125d^2-10.1307\tau_Z^{0.5}+4.4869\tau_Z^2$ $+9.6879d^{0.5}+12.725(d+\tau_Z+1)^{-1}$ for $0.2<\tau_Z<2$ & $0.12<d<2$	2.5%

Table 5. Proposed approximations for $K_C(\tau_Z,d)$ and $\tau_I(\tau_Z,d)$ obtained by Algorithm I, in the case where $\tau_D=1.25\min\tau_{D,maxGM,I-P}(\tau_Z,d)$.

4.2 Algorithm II

Step 1. Set $\zeta=\zeta_{des}$ and start with some initial values of τ_D and τ_I. Appropriate choices are

$$\tau_{D,init} = \min\tau_{D,GM_{max}}\left(\tau_Z,d\right) \tag{21}$$

$$\tau_{I,init} = 1.2\tau_{I,\min} = 1.2\left[d+\tau_Z+1-\min\tau_{D,GM_{max}}\left(\tau_Z,d\right)\right] \tag{22}$$

Step 2. For these values of τ_D and τ_I, calculate K_C from relation (11) using relation (10).

Step 3. Calculate the integral term τ_I as the maximum real root of (20).

Step 4. Repeat Steps 2 and 3 until the algorithm converges to a certain value for τ_I and K_C.

Step 5. For the obtained value of τ_I, select $\tau_D = 2\tau_{D,GM_{max}}\left(\tau_I, \tau_Z, d\right)$.

Step 6. Repeat Steps 2 to 5 until convergence.

Similarly to Algorithm I, Algorithm II always converges to values of K_C, τ_I and τ_D that satisfy equations (10), (11) and (20). The parameters K_C, τ_I and τ_D, obtained by the above algorithm, for several values of the desired damping ratio ζ_{des} can be approximated by the functions summarized in Table 6, using optimization techniques. Once again, the maximum normalized errors never exceed 5%. For intermediate values of ζ_{des}, a simple linear interpolation provides sufficiently accurate estimates of K_C, τ_I and τ_D.

Finally, a third way to choose τ_D, is in order to minimize some integral criteria, such as the criteria (13) or (14). In this case, the following iterative algorithm can be applied to achieve admissible controller settings:

4.3 Algorithm III

Step 1. Set $\zeta=\zeta_{des}$ and start with some initial values of τ_D and τ_I. Appropriate initial values are given by relations (21) and (22), respectively. Alternatively, on can initialize the algorithm by using $\tau_{D,init}=\min\tau_{D,maxGM,I-P}(\tau_Z,d)$ and $\tau_{I,init}=1.2\tau_{I,min}=1.2[d+\tau_Z+1-\min\tau_{D,maxGM,I-P}(\tau_Z,d)]$.

Step 2. For these values of τ_D and τ_I, calculate K_C from relation (11) using relation (10).

Step 3. Calculate the integral term τ_I as the maximum real root of (20).

Step 4. Repeat Steps 2 and 3 until the algorithm converges to a certain value for τ_I and K_C.

Step 5. For the obtained value of τ_I, select τ_D in order to minimize (13) or (14).

Step 6. Repeat Steps 2 to 5 until convergence.

Note that, Step 5 of the above iterative algorithm is an iterative algorithm by itself. This is due to the fact that optimization algorithms as well as extensive simulations (since there are no close form solution for such integrals in the case of time-delay systems), are used to obtain the optimal values of τ_D that minimize them. Iterative algorithms that minimize the aforementioned integral criteria are usually based on the golden section method.

Algorithm III always converges to values of K_C, τ_I, τ_D that satisfy equations (10), (11) and (20). Note also that convergence of the algorithm is independent of the initial pair ($\tau_{D,init}$, $\tau_{I,init}$). The parameters K_C, τ_I and τ_D, obtained by the above algorithm, for several values of the desired damping ratio ζ_{des} can be approximated by the functions summarized in Table 7, using optimization techniques. Once again, the maximum normalized errors never exceed 5%. For intermediate values of ζ_{des}, a simple linear interpolation provides sufficiently accurate estimates of K_C, τ_I and τ_D.

Finally, once K_C, τ_I and τ_D are obtained according to the Algorithms I-III, the original I-PD controller parameters $\bar{K}_P$, $\bar{K}_I$ and $\bar{K}_D$, can be easily obtained by using the relations summarized in Table 1, as well as relations (4), interrelating controller gains for the I-PD controller and the series PID controller with set-point pre-filter.

ζ_{des}	$K_C(\tau_Z,d)$ (for $0.2<\tau_Z<2$ & $0.1<d<2$)	M.N.E.
0.6	$0.1432+0.0072d-0.0363\tau_Z-0.0035d^2-0.0207\tau_Z^{0.5}+0.2477(d+\tau_Z-0.1)^{-1}$ $-0.064d(d+\tau_Z)^{-1}$	4.57%
0.707	$0.2163+0.036d-0.0025\tau_Z-0.008d^2-0.0914\tau_Z^{0.5}+0.2353(d+\tau_Z-0.1)^{-1}$ $-0.1212d(d+\tau_Z)^{-1}$	4.3%
0.85	$0.1454-0.0477d-0.0313\tau_Z+0.0082d^2+0.0092\tau_Z^{0.5}+0.1569(d+\tau_Z-0.15)^{-1}$ $+0.0439d(d+\tau_Z)^{-1}$	4.7%
1	$-0.1131-0.0476d-0.0211\tau_Z+0.0113d^2+0.089\tau_Z^{0.5}+0.3852(d+\tau_Z+0.3)^{-1}$ $+0.1452d(d+\tau_Z)^{-1}$	2.6%
1.2	$-0.1858-0.0375d+0.0029\tau_Z+0.0091d^2+0.0607\tau_Z^{0.5}+0.5999(d+\tau_Z+1)^{-1}$ $+0.1459d(d+\tau_Z)^{-1}$	4.65%
ζ_{des}	$\tau_I(\tau_Z,d)$	M.N.E.
0.6	$-0.5878+1.7259d+1.5127\tau_Z+0.0187d^2+2.0613\tau_Z^{0.5}+0.11\tau_Z^2+1.6319d^{0.5}$ $+0.0603(d+\tau_Z+1)^{-1}$ for $0.2<\tau_Z<2$ & $0.1<d<2$	1.8%
0.707	$-4.8923+3.223d+2.7225\tau_Z-0.4963d^2-0.1359\tau_Z^2+4.8271(d+\tau_Z)^{0.5}$ $+3.418(d+\tau_Z+1)^{-0.5}$ for $0.2<\tau_Z<2$ & $0.02<d<2$	5%
0.85	$-0.7157+1.6486d+4.1163\tau_Z+0.3192d^2+0.8445\tau_Z^{0.5}+2.5935d^{0.5}$ $+4.3918(d+\tau_Z)^{0.5}-1.5426(d+\tau_Z+1)^{-0.5}$ for $0.2<\tau_Z<1.9$ & $0.14<d<2$	3.6%
1	$-0.3735+5.9535d+11.0894\tau_Z+0.3347d^2+0.9352\tau_Z^{0.5}+3.7375d^{0.5}$ $+0.1432(d+\tau_Z)^{0.5}+1.438(d+\tau_Z+1)^{-0.5}$ for $0.2<\tau_Z<1.9$ & $0.14<d<2$	2.38%
1.2	$-10.2543+26.9196d+21.2015\tau_Z-2.3514d^2+7.6005\tau_Z^{0.5}-6.1009d^{0.5}$ $+9.7159(d+\tau_Z)^{-0.5}$ for $0.2<\tau_Z<2$ & $0.02<d<2$	4.37%
ζ_{des}	$\tau_D(\tau_Z,d)$ (for $0.2<\tau_Z<2$ & $0.1<d<2$)	M.N.E.
0.6	$1.664-0.1961d+0.0583\tau_Z+0.1392d^2+0.0145\tau_Z^{0.5}+0.7743d^{0.5}$ $-1.3746(d+\tau_Z+1)^{-1}$	2.9%
0.707	$1.9179+0.524d+1.8374\tau_Z-0.0061d^2-0.2274\tau_Z^2-2.1731\tau_Z^{0.5}+0.045d^{0.5}$ $-0.178(d+\tau_Z+1)^{-1}$	4.37%
0.85	$2.2066+0.2973d-0.0407\tau_Z+0.0079d^2+0.0105\tau_Z^2-0.1954\tau_Z^{0.5}$ $+0.0203d^{0.5}-1.8737(d+\tau_Z+1)^{-1}$	4.4%
1	$2.2783-0.2417d-0.1019\tau_Z+0.1367d^2-0.1256\tau_Z^{0.5}+0.362d^{0.5}$ $-2.1512(d+\tau_Z+1)^{-1}$	4.9%
1.2	$1.8375+0.3488d+0.2776\tau_Z+0.022d^2-0.5294\tau_Z^{0.5}-0.0578d^{0.5}$ $-0.5849(d+\tau_Z)^{-0.5}$	4.8%

Table 6. Proposed approximations for $K_C(\tau_Z,d)$, $\tau_I(\tau_Z,d)$ and $\tau_D(\tau_Z,d)$ obtained by Algorithm II

ζ_{des}	$K_C(\tau_Z, d)$ $\tau_I(\tau_Z, d)$	M.N.E.
0.6	$0.1786-0.0462d+0.0034\tau_Z+0.0085d^2-0.0403\tau_Z^{0.5}+0.2234(d+\tau_Z)^{-1}$ $+0.0271(d+\tau_Z)^{-2}$ $0.0717+1.5545d+2.5829\tau_Z+0.0439d^2-0.0783\tau_Z^2$ $+0.2783\tau_Z^{0.5}+1.6007d^{0.5}+0.1138(d+\tau_Z+1)^{-1}$ for $0.2<\tau_Z<2$ & $0.2<d<2$	2.35% 3.3%
0.707	$0.1471-0.0324d+0.0012\tau_Z+0.0059d^2-0.0352\tau_Z^{0.5}+0.2616(d+\tau_Z)^{-1}$ $-0.0005(d+\tau_Z)^{-2}$ $-0.457+2.5379d+2.6464\tau_Z+0.0252d^2-0.4502\tau_Z^2$ $+1.7881d^{0.5}+2.5791\tau_Z^{0.5}$ for $0.3<\tau_Z<1.65$ & $0.3<d<1.65$	0.5% 2.12%
0.85	$0.1269-0.0244d-0.0017\tau_Z+0.0037d^2-0.0261\tau_Z^{0.5}+0.2567(d+\tau_Z)^{-1}$ $-0.0136(d+\tau_Z)^{-2}$ $-0.3892+3.6961d+3.4709\tau_Z-0.0198d^2-0.2082\tau_Z^2$ $+3.1789d^{0.5}+3.5233\tau_Z^{0.5}$ for $0.3<\tau_Z<1.69$ & $0.3<d<1.69$	0.5% 1.36%
1	$0.1101-0.0187d-0.0015\tau_Z+0.0023d^2-0.0224\tau_Z^{0.5}+0.2471(d+\tau_Z)^{-1}$ $-0.0255(d+\tau_Z)^{-2}$ $0.2357+5.5238d+5.3907\tau_Z+0.0616d^2-0.2819\tau_Z^2$ $+4.5751d^{0.5}+4.998\tau_Z^{0.5}$ for $0.3<\tau_Z<1.69$ & $0.3<d<1.69$	0.5% 1.29%
1.2	$0.1042-0.0162d-0.0068\tau_Z+0.0014d^2-0.0112\tau_Z^{0.5}+0.2081(d+\tau_Z)^{-1}$ $-0.0212(d+\tau_Z)^{-2}$ $0.3879+14.3387d+1.3617\tau_Z-1.0061d^2+0.9472\tau_Z^2$ $+1.9009d^{0.5}+16.4489\tau_Z^{0.5}$ for $0.2<\tau_Z<2$ & $0.2<d<2$	1.0% 2.5%
ζ_{des}	$\tau_D(\tau_Z, d)$	M.N.E.
0.6	$0.7388+1.1169d+1.5105\tau_Z-0.0632d^2-0.3234\tau_Z^2-0.4559\tau_Z^{0.5}$ for $0.2<\tau_Z<1.7$ & $0.2<d<2$	2.3%
0.707	$0.3239+0.758d+0.5251\tau_Z-0.0065d^2-0.0819\tau_Z^2+0.393d^{0.5}+0.5963\tau_Z^{0.5}$ for $0.3<\tau_Z<1.65$ & $0.3<d<1.65$	1.42%
0.85	$0.333+0.7693d+0.4648\tau_Z+0.007d^2-0.043\tau_Z^2+0.3902d^{0.5}+0.7295\tau_Z^{0.5}$ for $0.3<\tau_Z<1.69$ & $0.3<d<1.69$	1.35%
1	$0.3745+0.7965d+0.4051\tau_Z+0.0164d^2+0.0029\tau_Z^2+0.4242d^{0.5}+0.0029\tau_Z^{0.5}$ for $0.3<\tau_Z<1.69$ & $0.3<d<1.69$	1.2%
1.2	$0.6193+0.9377d+1.0697\tau_Z+0.006d^2-0.1332\tau_Z^2+0.4388d^{0.5}-0.1332\tau_Z^{0.5}$ for $0.2<\tau_Z<2$ & $0.2<d<2$	2.1%

Table 7. Proposed approximations for $K_C(\tau_Z,d)$, $\tau_I(\tau_Z,d)$, $\tau_D(\tau_Z,d)$ obtained by Algorithm III.

5. A direct synthesis tuning method

The tuning methods presented in the previous Section are somehow complicated, since they are based on iterative algorithms. In what follows, our aim is to present a rather simpler

tuning method, which is called here Method II, and it is based on the direct synthesis approach. To this end, observe that after parameter normalization according to Table 1 relation (2) may further be written as

$$G_{CL}(\hat{s}) = \frac{(-\hat{s}\tau_Z + 1)(\hat{s}+1)^{-1}\exp(-d\hat{s})}{K_I^{-1}\hat{s}^2 + \left(K_D K_I^{-1}\hat{s}^2 + K_P K_I^{-1}\hat{s} + 1\right)(-\hat{s}\tau_Z + 1)(\hat{s}+1)^{-1}\exp(-d\hat{s})} \tag{23}$$

Relation (23) may be approximated as

$$G_{CL}(\hat{s}) = \frac{\left[1-(\tau_Z+1)\hat{s}\right]\exp(-d\hat{s})}{K_I^{-1}\hat{s}^2 + \left(K_D K_I^{-1}\hat{s}^2 + K_P K_I^{-1}\hat{s} + 1\right)(-\hat{s}\tau_Z + 1)(\hat{s}+1)^{-1}(1+d\hat{s})^{-1}} \tag{24}$$

where the approximations $(-\hat{s}\tau_Z+1)(\hat{s}+1)^{-1}\approx 1-(\tau_Z+1)\hat{s}$ and $\exp(-d\hat{s}) \approx 1/(d\hat{s}+1)$ are used to obtain relation (24). In what follows, define $d_{max}=\max\{1,d\}$ and $d_{min}=\min\{1,d\}$. Then, relation (24) yields

$$G_{CL}(\hat{s}) = \frac{\left[1-(\tau_Z+1)\hat{s}\right]\exp(-d\hat{s})}{K_I^{-1}\hat{s}^2 + \left(K_D K_I^{-1}\hat{s}^2 + K_P K_I^{-1}\hat{s} + 1\right)\left[1-(\tau_Z+d_{min})\hat{s}\right](\hat{s}d_{max}+1)^{-1}} \tag{25}$$

where, the approximation $(-\hat{s}\tau_Z+1)(\hat{s}d_{min}+1)^{-1} \approx 1-(\tau_Z+d_{min})\hat{s}$ is used to produce (25). By performing appropriate division, relation (25) becomes

$$G_{CL}(\hat{s}) = \frac{\left[1-(\tau_Z+1)\hat{s}\right]\exp(-d\hat{s})}{K_I^{-1}\hat{s}^2 + \left[d_{max}^{-1}K_D K_I^{-1}\hat{s} + \left(d_{max}^{-1}K_P - d_{max}^{-2}K_D K_I^{-1}\right) + Q(\hat{s})\right]\left[1-(\tau_Z+d_{min})\right]\hat{s}} \tag{26}$$

where $Q(\hat{s}) = \left(1 - d_{max}^{-1}K_P K_I^{-1} + d_{max}^{-2}K_D K_I^{-1}\right)(d_{max}\hat{s}+1)^{-1}$. Now, selecting

$$K_D = d_{max}K_P - d_{max}^2 K_I \tag{27}$$

we obtain $Q(\hat{s})=0$ and $G_{CL}(\hat{s}) = \dfrac{\left[1-(\tau_Z+1)\hat{s}\right]\exp(-d\hat{s})}{K_I^{-1}\hat{s}^2 + \left[\left(K_P K_I^{-1} - d_{max}\right)\hat{s} + 1\right]\left[1-(\tau_Z+d_{min})\hat{s}\right]}$, which yields

$$G_{CL}(\hat{s}) = \frac{\left[1-(\tau_Z+1)\hat{s}\right]\exp(-d\hat{s})}{\lambda^2\hat{s}^2 + 2\xi\lambda\hat{s} + 1} \tag{28}$$

where, $\lambda = \sqrt{K_I^{-1} - \left(K_P K_I^{-1} - d_{max}\right)(\tau_Z+d_{min})}$ and $\xi = \left(K_P K_I^{-1} - d - \tau_Z - 1\right)/(2\lambda)$, since $d_{max}+d_{min}=d+1$.

The Routh stability criterion about relation (28) yields

$$K_P>(d+\tau_Z+1)K_I=K_{P,min} \text{ and } K_P<d_{max}K_I+(\tau_Z+d_{min})^{-1}=K_{P,max} \tag{29}$$

Hence, as for K_P, one can select the middle value of the range given by inequalities (29), i.e. $K_P=(K_{P,min}+K_{P,max})/2=[(d+\tau_Z+d_{max}+1)K_I +(\tau_Z+d_{min})^{-1}]/2$. This relation, further yields

$$\theta=K_P/K_I=[(d+\tau_Z+d_{max}+1)K_I +(\tau_Z+d_{min})^{-1}K_I^{-1}]/2 \tag{30}$$

Solving (30) with respect to K_I and taking into account the definition of θ and (27), we obtain

$$K_I=[(\tau_Z+d_{min})(2\theta-d-\tau_Z-d_{max}-1)]^{-1}$$

$$K_P=\theta[(\tau_Z+d_{min})(2\theta-d-\tau_Z-d_{max}-1)]^{-1} \tag{31}$$

$$K_D=(\theta d_{max}-d^2_{max})[(\tau_Z+d_{min})(2\theta-d-\tau_Z-d_{max}-1)]^{-1}$$

provided that $\theta>(d+\tau_Z+d_{max}+1)/2=\theta_{min}$.

Now, it only remains to specify θ. Note that, θ can be arbitrarily selected as an adjustable parameter in the interval $[\theta_{min}, \infty)$, thus permitting on-line tuning. However, it would be useful for the designer to follow certain rules, based on some criteria relative to the closed-loop system performance, in order to select the adjustable parameter θ.

A first criterion for the selection of θ is related to the responsiveness of the closed-loop system. In particular, parameter θ can be selected in such a way that a desired damping ratio ξ_{des} is obtained for the second order approximation (28), of the closed-loop transfer function. In this case, using the definitions of θ, λ and ξ and the first of (31), after some trivial algebraic manipulations, on can conclude that parameter θ must be selected as the maximum positive real root of the quadratic equation

$$\theta^2 - 2\left[2\xi_{des}^2\left(\tau_Z + d_{\min}\right)+d+\tau_Z+1\right]\theta + 4\xi_{des}^2\left(d+\tau_Z+1\right)=0 \tag{32}$$

An alternative tuning can be obtained from the minimization of the integral criteria (13) and (14). Let w_u be the ultimate frequency of the normalized open-loop system, with transfer function $G_p(\hat{s})=(-\hat{s}\tau_Z+1)\exp(-d\hat{s})/[\hat{s}(\hat{s}+1)]$, i.e. the frequency at which $\arg\left(G_P(\hat{s})\big|_{\hat{s}=jw_u}\right)=-\pi$, and set $\theta=2\pi/(w_u\beta)$, where β is a parameter to be specified. Note that w_u is the solution of

$$dw_u + a\tan(\tau_Z w_u) + a\tan(w_u)= \pi/2 \tag{33}$$

By defining $\tau_{min}=\min\{\tau_Z,1\}$, $\tau_{max}=\max\{\tau_Z,1\}$, and by appropriately using the approximations $atan(\tau_{min}w_u)\approx\tau_{min}w_u$, $atan(x) \approx x+x^2[\pi(0.5\pi-x)]^{-1}$ and the fact that $\tau_{min}+\tau_{max}=\tau_Z+1$, we finally obtain (see Arvanitis et al, 2009b, for details)

$$w_u \approx \frac{(0.5\pi-1)}{2\left[d+\tau_z+1-\left(d+\tau_{\min}\right)\pi^{-1}\right]}\left[1+\sqrt{1+\frac{\pi\left[d+\tau_z+1-\left(d+\tau_{\min}\right)\pi^{-1}\right]}{\left(d+\tau_{\min}\right)\left(0.5\pi-1\right)^2}}\right] \tag{34}$$

Having obtained a sufficiently accurate estimation of w_u as a function of the normalized process parameters, we now focus our attention on the determination of β that minimizes (13) or (14). Since there is no closed form solution for the minimization of the above integrals in the case of time-delay systems, simulation must be used instead. Here, optimization

algorithms are used to obtain the parameter β that minimizes (13) or (14), as a function of d and τ_Z. Table 8 summarizes the estimated parameters β minimizing criteria (13) and (14) in the case of regulatory control (ISENSCOD-L and ISENDCO-L criteria), together with the respective maximum normalized errors.

Criterion	$\beta(\tau_Z,d)$	M.N.E.
ISENSCOD-L (Eq. (13))	$-0.1131-1.4126d-2.0935\tau_Z-0.2022d^2+0.0486\tau_Z^{0.5}$ $-0.5246d^{0.5}+4.2676(d+\tau_Z)^{0.5}+0.0233(d+\tau_Z)^3$ for $0<\tau_Z<2$ & $0<d<1$	4.26%
ISENSCOD-L (Eq. (13))	$2.1108+1.4284d+0.5002\tau_Z-0.6778d^2+0.9347\tau_Z^{0.5}$ $+1.5618d^{0.5}-2.651(d+\tau_Z)^{0.5}+0.0209(d+\tau_Z)^3-0.269\tau_Z^2$ for $0<\tau_Z<2$ & $1<d<2$	4.96%
ISENSDCO-L (Eq. (14))	$-0.0409+4.5635d+4.1371\tau_Z-0.4161d^2+0.8091\tau_Z^{0.5}+2.8912d^{0.5}$ $-3.8217(d+\tau_Z)^{0.5}-1.5426(d+\tau_Z)^2+0.1682(d+\tau_Z)^3+0.304\tau_Z^2$ for $0<\tau_Z<2$ & $0.13<d<1$	4.5%
ISENSDCO-L (Eq. (14))	$0.886-0.6479d-1.0903\tau_Z-0.5933d^2+1.2604\tau_Z^{0.5}$ $+2.6081d^{0.5}-0.759(d+\tau_Z)^{0.5}+0.2357(d+\tau_Z)^3-0.2545\tau_Z^2$ for $0<\tau_Z<2$ & $1<d<2$	4.7%

Table 8. Proposed approximations of the parameter $\beta(\tau_Z,d)$ for Method II.

6. Simulation application to a boiler steam drum

The most typical example of an IPDT-IR process is a boiler steam drum. The level (output) is controlled by manipulating the boiler feed water (BFW) to the drum. The drum is located near the top of the boiler and is connected to it by a large number of tubes. Liquid and vapour water circulate between the drum and the boiler as a result of the density difference between the liquid in the down-comer pipes leading from the bottom of the drum to the base of the boiler and the vapour/liquid mixture in the riser pipes going up through the boiler and back into the steam drum. In has been suggested by Luyben (2003) that the transfer function model of the process takes the form (1), with $\bar{K}$ =0.547, $\bar{\tau}_P$ =1.06, $\bar{\tau}_Z$ =0.418, $\bar{d}$ =0.1.

We next apply the tuning methods presented in Sections 4 and 5 in order to tune I-PD controllers for the above IPDT-IR model of boiler steam drum, as well to provide a comparison with existing tuning methods for PID controllers. Note that, for the above process model, the method proposed in the work (Luyben, 2003), for an integral time constant equal to 25% of the minimum PI integral time, yields the conventional PID controller settings K_C=1.61, τ_I=11.5 and τ_D=1.15. The method reported in the work (Shamsuzzoha & Lee, 2006), for λ=0.798, ξ=1, ψ=25, yields the two-degrees-of-freedom PID controller settings, K_C=2.3892, τ_I=3.5778, τ_D=0.7249, in the case where the set-point weighting parameter is selected as b=0.3. Finally, the method proposed in the work (Gu et al, 2006), yields the following IMC based PID controller settings K_C=2.0883, τ_I=3.8664, τ_D=0.6879, with the filter time constant having the value λ=0.8.

Applying Algorithm I of Method I, with τ_D=min$\tau_{D,maxGM,I-P}(\tau_Z,d)$=1.1535 (as calculated by Table 3), $\tau_{I,init}$=0.4022 and with the desired damping ratio of (17) having the value ζ_{des}=0.6, we obtain the I-PD controller settings $\bar{K}_P$ =2.104, $\bar{K}_I$ =0.6391 and $\bar{K}_D$ =1.5876. Moreover, applying the same Algorithm with τ_D=1.25min$\tau_{D,maxGM,I-P}(\tau_Z,d)$=1.4419 (as calculated by Table

3), $\tau_{I,init}$=0.0561 and with the same closed-loop system specification, we obtain the I-PD controller settings $\bar{K}_P$=2.0026, $\bar{K}_I$=0.5695 and $\bar{K}_D$=1.7289. Fig. 8 illustrates the set-point tracking responses as well as the regulatory control responses of the closed-loop system, in the case of a step load disturbance L=1 at time t=30. Obviously, Method I based on Algorithm I with τ_D=1.25$min\tau_{D,maxGM,I-P}(\tau_Z,d)$ provides a better performance, as compared to Method I relying on Algorithm I with τ_D=$min\tau_{D,maxGM,I-P}(\tau_Z,d)$. Both methods give a better performance, in terms of overshoot, maximum error and settling time, as compared to the method reported in the work by (Luyben, 2003). They also provide less overshoot in terms of set-point tracking as compared to the method reported in the work by (Gu et al, 2006). The method reported in the work by (Shamsuzzoha & Lee, 2006) gives the better overall performance, in the present case. However, the proposed method shows a better performance, as compared to known tuning methods, in terms of the initial jump in the closed-loop response.

We next apply Algorithm II of Method I in order to obtain admissible I-PD controller settings in the case where ζ_{des}=0.6. Algorithm II has been initialized with $\tau_{D,init}$=0.3807 and $\tau_{I,init}$=1.3296 and yields the controller parameters $\bar{K}_P$=2.1071, $\bar{K}_I$=0.6521 and $\bar{K}_D$=1.504. The set-point tracking responses as well as the regulatory control responses of the closed-loop system are illustrated in Fig. 9. The proposed method outperforms the method reported by (Luyben, 2003), and it is comparable to that reported by (Gu et al, 2006). Our method shows a better performance in terms of initial jump. The method by (Shamsuzzoha & Lee, 2006) shows, once again, a better performance in terms of overshoot and settling time in case of set-point tracking, while it performs better in terms of maximum error in the case of regulatory control.

The above conclusions hold also in the case where Algorithm III of Method I is applied in order to obtain ζ_{des}=0.6 and, simultaneously, to minimize the integral criterion of the form (13) in case of disturbance rejection. Algorithm III is initialized here $\tau_{D,init}$=0.3807, and $\tau_{I,init}$=1.3296, and provides the controller parameters $\bar{K}_P$=2.1069, $\bar{K}_I$=0.6426 and $\bar{K}_D$=1.5732. Fig. 10 illustrates the closed-loop set-point tracking and regulator control responses for the various methods applied to control the process.

Let us now apply Method II reported in Section 5 to the above IPDT-IR process model of the boiler steam drum and perform a comparison with known methods of tuning PID controllers. To this end, suppose first that the design specification is given in terms of the damping ratio of the closed-loop system, which is selected here as ξ_{des}=$\sqrt{2}/2$=0.707. In this case, application of Method II gives the I-PD controller parameters $\bar{K}_P$=3.5624, $\bar{K}_I$=1.363 and $\bar{K}_D$=2.2447. An alternative design is obtained by applying Method II with the specification of minimizing the integral criterion (13) in case of disturbance rejection (ISENCOD-L) criterion. In this case, the obtained controller parameters are $\bar{K}_P$=3.352, $\bar{K}_I$=1.2034 and $\bar{K}_D$=2.2009. Figs. 11 and 12 illustrate the closed-loop set-point tracking as well as the regulatory control responses obtained from the application of above I-PD controllers to the boiler steam drum process, together with those obtained from the application of known tuning methods for conventional and appropriately modified PID controllers. From these figures, it becomes obvious that the proposed Method II gives a very smooth response and outperforms most existing tuning methods in terms of overshoot,
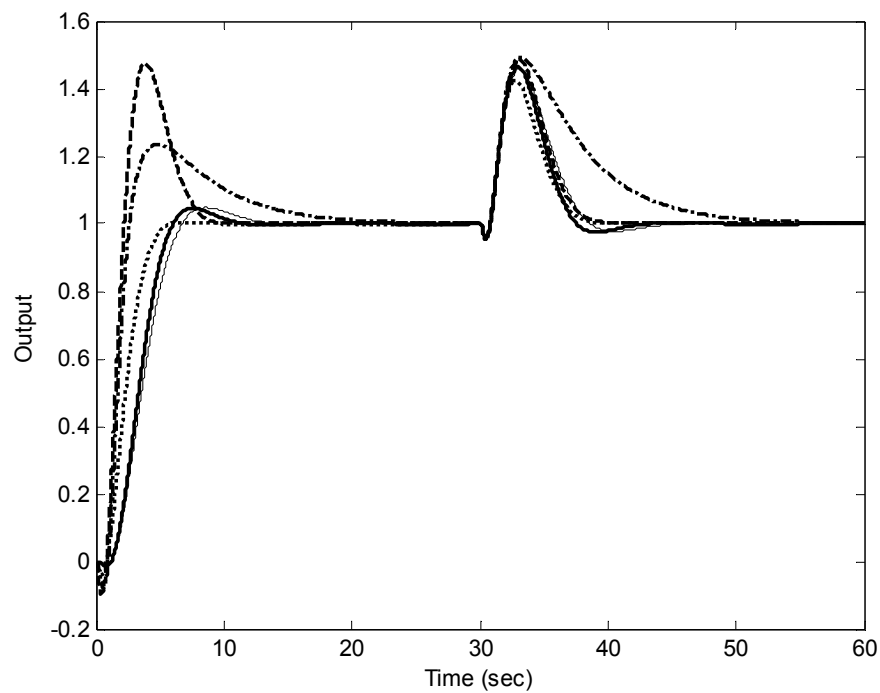
Fig. 8. Set-point tracking and regulatory control closed-loop responses of the boiler steam drum in the case where Algorithm I of Method I is applied to tune the I-PD controller. Solid-thick line: Algorithm I of Method I, with $\tau_D=\min\tau_{D,maxGM,I\text{-}P}(\tau_Z,d)$ and $\zeta_{des}=0.6$. Solid-thin line: Algorithm I of Method I, with $\tau_D=1.25\min\tau_{D,maxGM,I\text{-}P}(\tau_Z,d)$ and $\zeta_{des}=0.6$. Dash line: Method of Gu et al (2006). Dash-dot line: Method of Luyben (2003). Dot line: Method of Shamsuzzoha & Lee (2006).
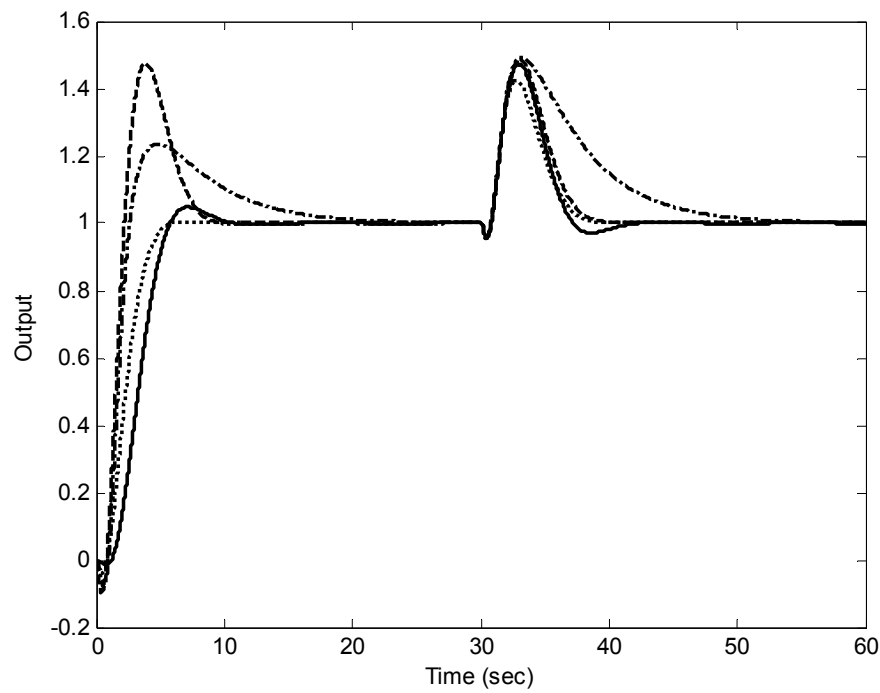
Fig. 9. Set-point tracking and regulatory control responses of the closed-loop system, when Algorithm II of Method I is applied for $\zeta_{des}=0.6$. Solid line: Proposed method. Other legend as in Fig. 8.

settling time, maximum error (in case of regulatory control) as well as initial jump. Finally, it provides the same set-point tracking capabilities as the method reported in the work (Shamsuzzoha & Lee, 2006), through the design of a simpler three-term controller structure.
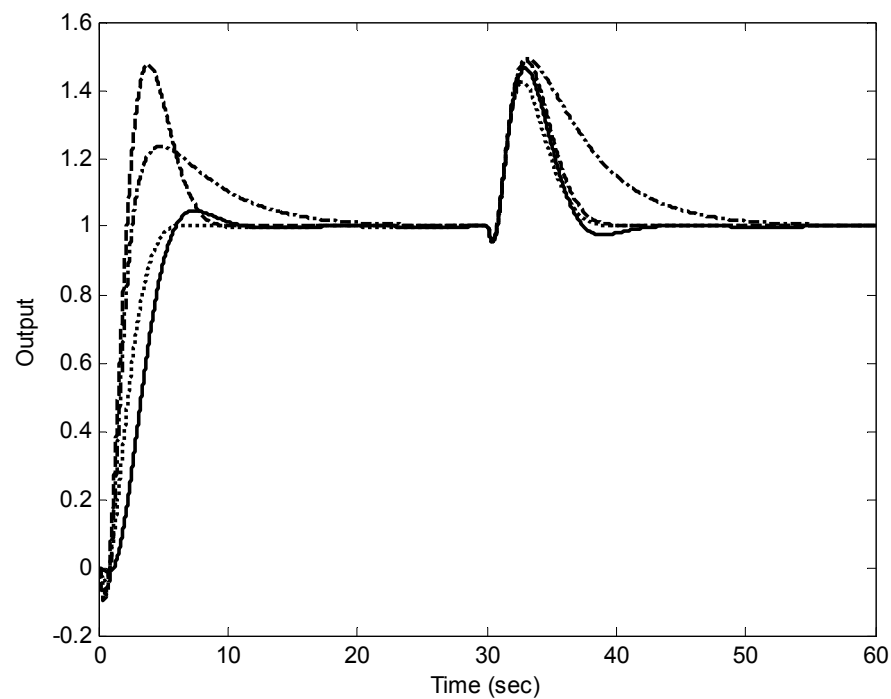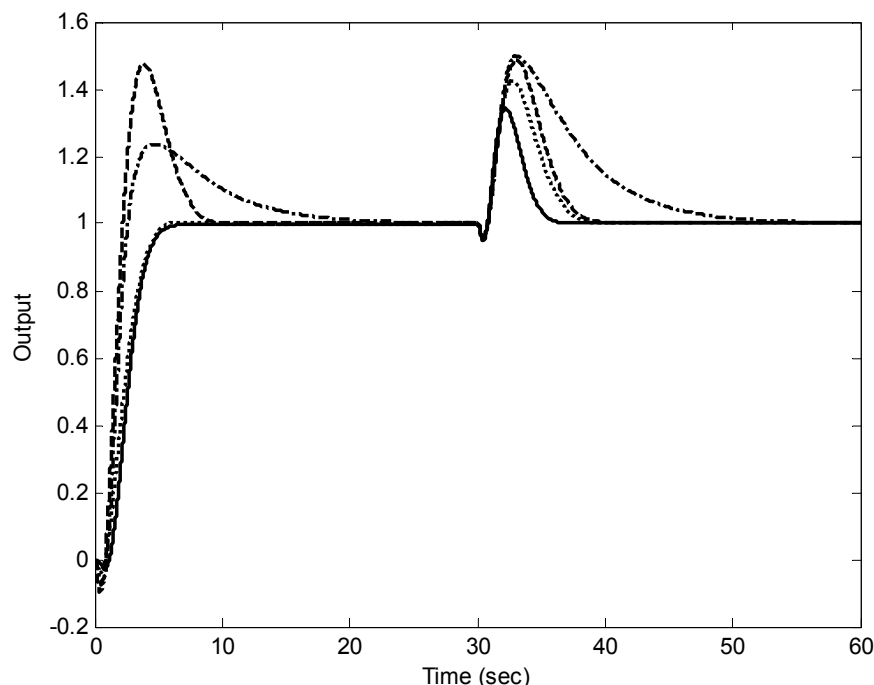

Fig. 10. Closed-loop set-point tracking and regulatory control responses, when Algorithm III is applied to minimize (13). Solid line: Proposed method. Other legend as in Fig. 8.
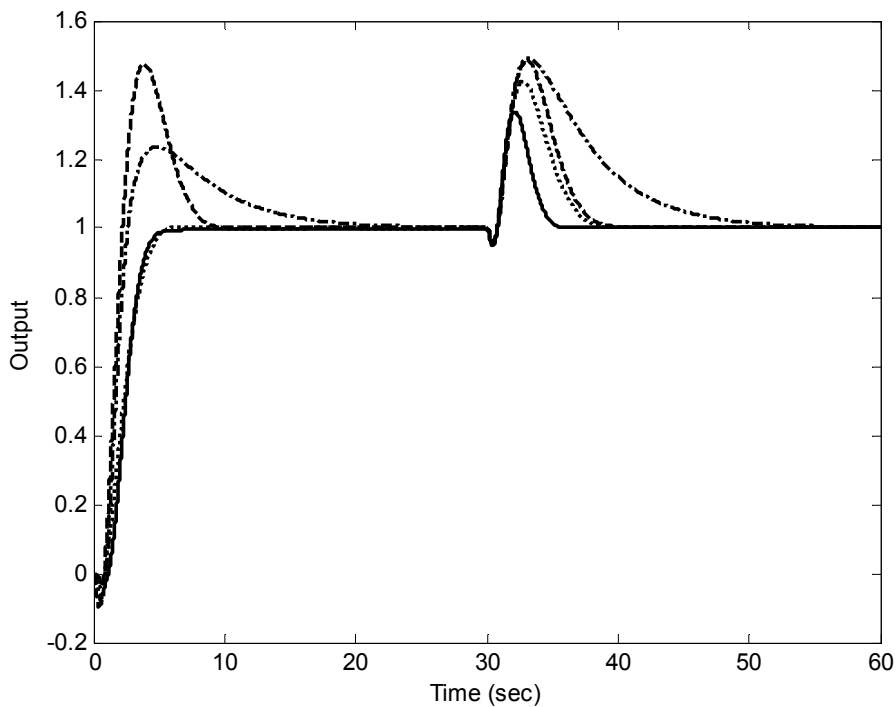
Fig. 11. Closed-loop set-point tracking and regulatory control responses, when Method II is applied with ξ_{des}=0.707. Solid line: Proposed method. Other legend as in Fig. 8.

Fig. 12. Closed-loop set-point tracking and regulatory control responses, when Method II is applied in order to minimize (13). Solid line: Proposed method. Other legend as in Fig. 8.

7. Conclusions

In this work, a variety of new methods of tuning three-term controllers for integrating dead-time processes with inverse response have been presented. These methods can be classified in two main categories: (a) methods that guarantee the maximum phase margin specification in the frequency domain together with some desired specification in the time domain, and (b) methods based on direct synthesis. With regard to the first class of tuning methods, in order to obtain the optimal controller settings, iterative algorithms are used. In addition, several accurate approximations of these settings, useful for on-line tuning, are derived as functions of the process parameters. On the other hand, the proposed method based on direct synthesis provides explicit relations of the three-term controller settings in terms of the process parameters and of adjustable parameters that can be appropriately assigned to obtain the desired closed-loop performance. The performance of the above tuning methods is tested through their simulation application on a typical IPDT-IR process model of a boiler steam drum. Several successful comparisons of the proposed methods with existing tuning formulas for conventional, IMC-based, and two-degrees-of freedom PID controllers are also reported. From these comparisons, on can readily conclude that the proposed direct synthesis method outperforms existing tuning methods, while the methods based on the analysis of the phase margin of the closed-loop system provide a performance comparable to that of most PID controller tuning methods reported in the extant literature.

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PID Controller Design Approaches - Theory, Tuning and Application to Frontier Areas
Edited by Dr. Marialena Vagia

ISBN 978-953-51-0405-6
Hard cover, 286 pages
Publisher InTech
Published online 28, March, 2012
Published in print edition March, 2012

First placed on the market in 1939, the design of PID controllers remains a challenging area that requires new approaches to solving PID tuning problems while capturing the effects of noise and process variations. The augmented complexity of modern applications concerning areas like automotive applications, microsystems technology, pneumatic mechanisms, dc motors, industry processes, require controllers that incorporate into their design important characteristics of the systems. These characteristics include but are not limited to: model uncertainties, system's nonlinearities, time delays, disturbance rejection requirements and performance criteria. The scope of this book is to propose different PID controllers designs for numerous modern technology applications in order to cover the needs of an audience including researchers, scholars and professionals who are interested in advances in PID controllers and related topics.

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