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Fuzzy Control Applied to Aluminum Smelting

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1. Introduction

Aluminum is a modern and new metal, since it has been produced for industry no earlier than 1886, when Hall and Héroult concurrently found out a method to produce free Aluminum through electrolysis (Beck, 2008). In 1900, the Aluminum production worldwide had reached a thousand tons. Nevertheless, at the beginning of the 21st century, global production reached 32 million tons encompassed by 24 million of primary Aluminum and 8 million of recycled material. This fact puts Aluminum at the second place in the list of the most used metals on earth. The world without Aluminum became inacceptable: the businessmen, the tourists, the delivery offices fly over the world in airplanes made of Aluminum, as well as many enterprises and industries are strongly dependent of this metal. Figure 1 shows in a widely perspective where Aluminum is most used.
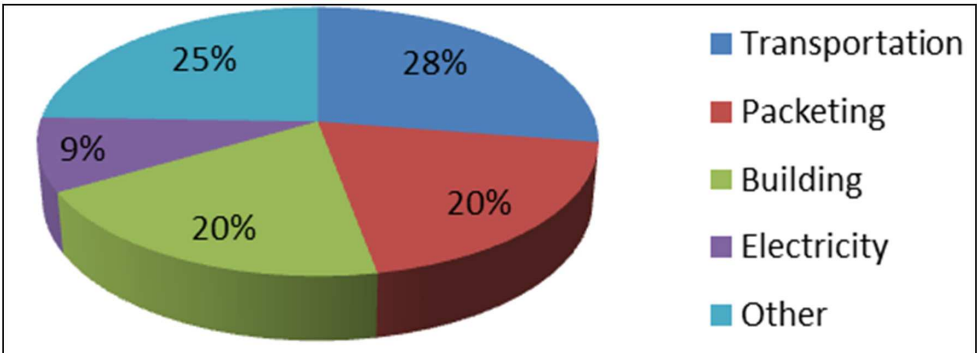

Fig. 1. Fields where Aluminum is most used (source: IAI, 2010)

This metal has contributed to low fuel consumption in cars and trucks, as well as allowing high speeds for trains and ships due to their weight reduction. Since it is a light metal, Aluminum eases the construction of buildings resistant to corrosion and low need for maintenance. Everywhere in the world, the electricity transmission lines for great distances are made of Aluminum, in part or whole. Food quality is preserved by Aluminum packages, reducing waste and giving comfort to users. This metal protects food, cosmetics and pharmaceutical products from ultraviolet rays, bad smells and bacteria. Food waste is avoided 30% when Aluminum packages are used.

Aluminum is a global commodity; its industry employs directly at least one million people, and indirectly more than four million. It is a slight compact industry, provided that around

20 smelters are responsible for 65% or world production. Most companies work only with Aluminum, but 20% of them work with Aluminum with other metals or mines. Half of Aluminum production is done by companies vertically integrated, from bauxite mining to metal recycling (IAI, 2010).

For all these reasons, the Aluminum can be considered a highly important metal, and therefore, its production is a target for many research activities. Researchers all around the world make efforts in making Aluminum production a less costly process, since it spends a lot of energy, and is very complex. In this chapter, we are going to present the whole context, and why and where fuzzy control is important to assist plant operators.

The impact and consequences of this work is the use of rules defined by process operators indirectly through the huge database which provides historic information including control decisions made by them. Since this strategy emulates process operator, it can be said that an expert system can provide this personnel more time to concentrate on other activities. Moreover, this technique will be continually improved by revising its rules and evaluation, provided that fuzzy decisions will also have an impact, and this should be analysed and adjusted.

1.1 Aluminum production process

Aluminum has been produced through the Hall-Héroult process, named after its inventors. So far, this is the only industrial way to produce this metal. Primary Aluminum is produced in a liquid form, through an electrolytic reduction of alumina (Al_2O_3) in a cryolite bath (Na_3AlF_6). This reaction takes place in electrolytic pots, as shown in Figure 2.
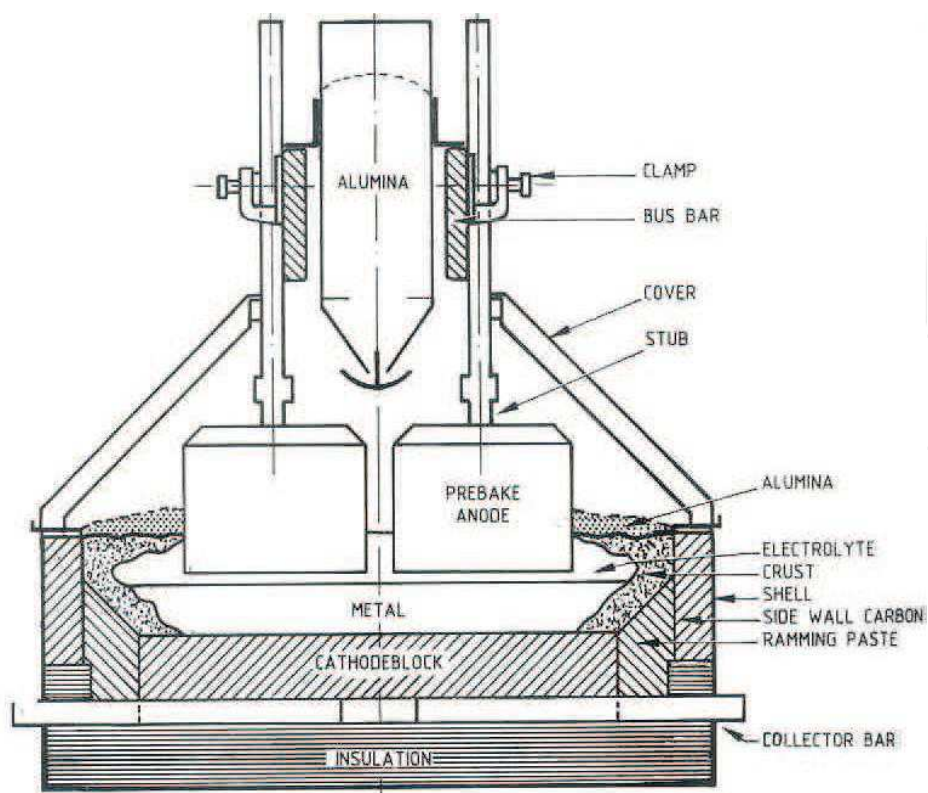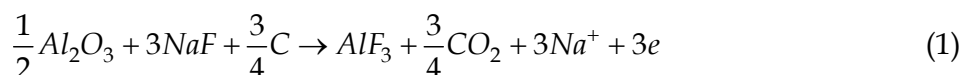

Fig. 2. Sketch of an Alumina reduction pot of prebake type (adapted from Kola & Store, 2009).

Inside these pots, also often called cells, Alumina is fed through silo and it is electrically consumed by the carbon anodes (Solheim, 2005), and as shown in the equation (1), the anode is also consumed during the electrolytic process.

$$\frac{1}{2}Al_2O_3 + 3NaF + \frac{3}{4}C \rightarrow AlF_3 + \frac{3}{4}CO_2 + 3Na^+ + 3e \qquad (1)$$

At the bottom part of the cell, there is a thermal isolated steel covering made of refractory material, named cathode block. The liquid Aluminum is formed above the cathode, and under the anode the electrolytic bath is formed. The cathode, in an electrochemical sense, is an interface between liquid Aluminum and the electrolytic bath, according to equation (2).

$$AlF_3 + 3Na^+ + 3e \rightarrow Al + 3NaF \qquad (2)$$
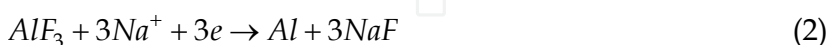
The full reaction inside the reduction pot is shown in equation (3).

$$\frac{1}{2}Al_2O_3 + \frac{3}{4}C \rightarrow Al + \frac{3}{4}CO_2 \qquad (3)$$

The pure electrolytic bath, i.e. cryolite, has a melting point at 1,011°C. In order to lower this point, called *liquidus* temperature, some additives are added into the bath, from which the main are Aluminum Fluoride (AlF_3) and Calcium Fluoride (CaF_2). The chemical composition of the bath in the reduction pot is 6-13% of AlF_3, 4-6% of CaF_2 and 2-4% of Al_2O_3. With a low liquidus temperature, pot operation is performed with low bath temperature, allowing reducing alumina solubility inside the bath. Therefore a good alumina concentration control system is required. Usually an aluminum reduction pot is operated under temperatures from 940°C to 970°C.

Bath is not consumed during the process, but part of it is lost, during vaporization, constituted of $NaAlF_4$. Moreover, part of bath is lost by drops dragging, by water present in fed alumina and the air aspired from inside the cell to form HF. In order to protect the environment, the gas is collected and cleared by a gas washing system. More than 98% of AlF_3 is retrieved in the gas washing system (Hyland et al, 2001), and recycled back to the pot. Moreover, Sodium Oxide (Na_2O) and Calcium Fluoride (CaF_2) at alumina feeding neutralizes AlF_3. The neutralized amount is also dependent on sodium penetration into the cathode which is, the pot age.

At the cathode sidewall there is a cool layer called ledge, which protects the sidewall from erosion. The ledge is composed of Na_3AlF_6 and CaF_2 (Thonstad & Rolseth, 1983). The ledge thickness is dependent on the heat flux through the cell sides, which is dependent on the bath temperature and liquidus temperature (that difference is called superheat). Once it is established that ledge composition is basically Na_3AlF_6, that means the total cryolite mass varies, while AlF3 and Al2O3 mass do not vary with the ledge thickness. In addition, once the additive concentration is the additive mass divided by the total bath mass, the ledge thickness variation triggers variation in the additives' concentrations. Then, changes in concentrations triggers changes in liquidus temperature, which in turn triggers changes in superheat, affecting ledge thickness. Thus, the challenge is to guarantee a stable pot operation which means a stable protection ledge minimizing energy input and maximizing production.

1.2 Current control systems

Regarding pot control, there are three main variables to be controlled: bath temperature, AlF_3 concentration and Al_2O_3 concentration. For that, there are three control inputs: anode beam moves (controlling energy input, by means of anode-to-cathode distance (ACD), AlF_3 addition and Al_2O_3 addition). The AlF_3 mass reduction dynamic process is slow, the AlF_3 concentration control system should deal with long response times (long delays) to control inputs which in this case are the changes to AlF_3 concentration. On the other hand, the Al_2O_3 mass reduction process is faster, and the Al_2O_3 concentration control system should deal with fast responses to the control inputs which in this case are the Al_2O_3 concentration changes. Usually, the Al_2O_3 concentration control system is considered an isolate problem, decoupled from the other control systems.

Bath temperature is measured manually, once a day or at least once a week. The AlF3 concentration (acidity) is typically measured manually once or twice a week, while Al_2O_3 concentration is not normally measured, except in special situations when process engineers need exceptionally. The only real time measurement is the bath pseudo-resistance (R_b), defined by equation (4),

$$R_b = \frac{U_f - 1,7}{I} \quad (\mu\,\Omega) \tag{4}$$

where U_f is cell voltage in Volts, and I is potline current in KA, these variables are also measured continually. The R_b measurement is used as input for anode-to-cathode distance adjustment, and acts as a control variable along with energy input into the cell.

Due to the fact that there is a strong relation between energy balance and mass balance through the ledge (see, e.g., Drenstig, 1997, Chapter 5), the reduction cell control must be considered as a multivariable non-linear control. A raise in the bath temperature causes acidity decrease and increases bath conductivity (Hives et al, 1993). According to Drenstig (1997, Chapter 5), acidity variation is ruled by bath temperature variation. Likewise, the control system logic should be bath temperature control through the additives (with negative or positive effects), around a setpoint, and Aluminum fluoride (AlF_3) constant addition. While this seems to be obvious and reasonable, there is a long way to go to transform this idea into a viable application in an alumina reduction cell.

1.3 Usage of fuzzy logic in aluminum industry

One easy and cheap method to perform a non-linear control system in an alumina reduction cell is to use fuzzy systems. With a qualitative approach, fuzzy systems offer a methodology to simulate a human expert operational behaviour and allow using available data from these experts' knowledge. Fuzzy expert systems have been largely used in control systems (Benyakhlef&Radouane, 2008; Chiu &Lian, 2009; Yu et al., 2010; Feng, 2010; Wang et al., 2011), since when Mamdani and Assilian developed a fuzzy controller for a boiler (Mamdani&Assilan, 1975).

In Aluminum industry, control strategies involve alumina addition neural control by cell states estimation (Meghlaoui et al., 1997), bath Aluminum fluoride control by mass balance differential equations and algebraic equations that deal with mass balance and thermal

balance (Drenstig et al, 1998), the use of LQR (Linear Quadratic Gaussian) to perform cell multivariable control by identifying dynamic models (McFadden et al, 2006); the use of regression models for bath temperature along with IF-THEN rules to add Aluminum fluoride into the cell (Yongbo et al, 2008), and PID (Proportional, Integral and Derivative) control along with a feed-forward loop for Aluminum fluoride addition and a PI (Proportional and Integral) control for bath temperature (Kola & Store, 2009). The use of fuzzy controllers in the cell is also often used (Meghlaoui&Aljabri, 2003; Yan &Taishan, 2006; Shuiping&Jinhong, 2008; Shuiping et al. 2010; Xiaodong et al, 2010; Dan Yang et al., 2011). However these works have not exploited any operational experience stored in process database, and the existing data mining works in the Aluminum Industry (Zhuo et al., 2008) are not addressed to the fluoride addition problem.

1.4 The novelty proposed in this work

In this chapter we propose a data-oriented fuzzy-based strategy applied to one of the Aluminum smelting sub-processes. Aluminum industries usually maintain huge databases which provide historic information regarding the process, including control decisions made by process operators. It can be said that these information contain the system's dynamics and the process team's knowledge. This knowledge can be exploited to develop an expert system, provided that most of process decision makers control the plant based on their own experience in a fuzzy approach. This work shows the whole design of the fuzzy system, their rules formation and fuzzy sets selection, and its results. This work was performed in a Brazilian company whose aim was to develop a fuzzy controller based on an expert system whose rules were generated from the company's process database and interviews with process operators. This work is also fully based on the literature of Gomes et al, 2010. The control system is aimed at adding Aluminum fluoride into alumina reduction cells. The results show more stability on bath temperature and AlF$_3$ concentration.

2. Fuzzy controllers and systems: An overview

The inaccuracy and uncertainty are two aspects that may be part of the information. There are two theories used to deal with inaccuracy and uncertainty: classic sets (crisp) theory and probabilities theory, respectively. However, these theories do not always capture the information content provided by humans in natural language. The classic sets theory cannot deal with the fuzzy aspect of information while the probabilities theory is more suited to handle frequency information than those provided by humans.

The fuzzy sets theory, developed by LoftiZadeh in 1965 (Zadeh, 1965), aimed at dealing with the fuzzy aspect of information, while, in 1978, Zadeh also developed the probabilities theory that deals with information uncertainty (Zadeh, 1978). These theories have been used in systems that use human-provided information. These theories are closely linked with each other. When the fuzzy sets theory is used in a logic context, as knowledge-based systems, it is known as fuzzy logic (term used in this chapter). The fuzzy logic is currently one of the most successful technologies for the development of process control systems, due to low implementation cost, easy maintenance and the fact that complex requirements may be implemented in simple controllers.

In the broad sense, a fuzzy controller is a rule-based fuzzy system, composed of a set of inference rules of the type If <Condition> Then <Action>, that define the control actions according to several ranges the controlled variables in the problem may assume. These ranges (usually poor defined) are modeled by fuzzy sets and named as linguistic terms. In this section, we present all the theoretic aspects for the development of the fuzzy controller.

2.1 Theoretic aspects

2.1.1 Fuzzy sets

Crisp sets have hard defined membership functions (either 0 or 1), while fuzzy set have soft defined membership functions. Given a set A in a universe U, the elements of this universe just belong or not to that set. That is, the element x is true $\left(f_A(x)=1\right)$, or false $\left(f_A(x)=0\right)$. This can be expressed as

$$f_A(x) = \begin{cases} 1 & if \quad x \in A; \\ 0 & if \quad x \notin A \end{cases}$$

(5)

Zadeh(Zadeh, 1965) proposed a more general approach, so the characteristic function could yield float point values in the interval [0,1]. A fuzzy set A in a universe U is defined by a membership function $\mu_A(x) \rightarrow [0,1]$, that amounts the element x for the fuzzy set. Fuzzy sets can be defined in continuous or discrete universes. If the universe U is discrete and finite, the fuzzy set A is usually denoted by expression:

$$A = \sum_{i=1}^{m} \frac{\mu_A(x_i)}{x_i}$$

$$A = \frac{\mu_A(x_i)}{x_i} + \cdots + \frac{\mu_A(x_m)}{x_m}$$

(6)

If U is a continuous universe, the fuzzy set A is denoted by expression:

$$A = \int \frac{\mu_A(x)}{x}$$

(7)

Where $\mu_A(x_i)$ is known as membership function which may show how much x belongs to the set A, and U is known as the universe of discourse. In other words, the element x may belong to more than one fuzzy set, but with different membership values.

2.1.2 Linguistic variables

A linguistic variable has its value expressed qualitatively by a linguistic term and quantitatively by a membership function. A linguistic function is characterized by {n,T,X,m(n)} where n is the variable's name, T is the set of linguistic terms of n (Cold, Normal, Hot, Very Hot), X is the domain (Universe of Discourse) of n values which the linguistic term meaning is determined on (the temperature may be between 970º and 975ºC)

and m(t) is a semantic function that assigns each linguistic term $t \in T$ its meaning, what is a fuzzy set in X (that is, m: T→(X) where (X) is the fuzzy sets space).

2.1.3 Fuzzy sets operation

Given fuzzy sets A and B contained in a universe of μ_A and μ_B, respectively, their operation are defined as sets theoretic operation (union, intersection and complement) as follows:

Equality: If for every $x \in U$, $\mu_A(x) = \mu_B(x)$, then the set A is equal to set B.

Subset: If for every $x \in U$, $\mu_A(x) \leq \mu_B(x)$, then the set B contains set A.

Union: This operation is similar to the union between two classic sets $A \cup B$. The union between fuzzy sets may be written with membership functions of sets A and B, as follows:

$$\mu_{A \cup B}(x) = \max\left[\mu_A(x), \mu_B(x)\right] \tag{8}$$

Intersection: This operation is similar to the intersection between two classic sets $A \cap B$. The intersection between fuzzy sets may be written with membership functions of sets A and B, as follows:

$$\mu_{A \cap B}(x) = \min\left[\mu_A(x), \mu_B(x)\right] \tag{9}$$

Complement: The complement set of A, named as $\bar{A}$, is defined by the membership function:

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \tag{10}$$

s-Norms: These are combinations of membership functions of two fuzzy sets A and B, resulting in the union $A \cup B$ of set membership functions:

$$s\left[\mu_A(x), \mu_B(x)\right] = \mu_{A \cup B}(x) \tag{11}$$

The combination s should match these properties:

1. s[1,1]=1,s[a,0]=a
2. s[a,b] = s[b,a]
3. s[a,b] ≤ s[a',b'], if a < a' and b < b'
4. s[s[a,b],c]=s[a,s[b,c]]

t-Norms: These are combinations of membership functions of two fuzzy sets A and B, resulting in the intersection $A \cap B$ of two set membership functions:

$$t\left[\mu_A(x), \mu_B(x)\right] = \mu_{A \cap B}(x) \tag{12}$$

The combination t should match these properties:

1. t[1,1]=1,t[a,0]=a
2. t[a,b] = t[b,a]

3.   t[a,b] ≤ t[a′,b′], if a < a′ and b < b′
4.   t[t[a,b],c]=t[a,t[b,c]]

2.1.4 Fuzzy relations and compositions

A fuzzy relation describes the presence or absence of an association (or interaction) between two or more sets. Likewise, given two universes U and V, the relation R defined in U x V is a subset of the Cartesian product of the two universes, so that R: U x V →{0,1}. That is, if any $x \in U$ and $y \in V$ are related, R(x,y)=1; otherwise R(x,y)=0. This relation (U,V) can be defined by the following characteristic function.

$$f_A(x) = \begin{cases} 1 \text{ if } \text{ and } \text{ only } \text{ if}(x,y) \in R(U,Y); \\ 0 \quad otherwise \end{cases} \tag{13}$$

Fuzzy relations represent the association degree between two or more fuzzy sets. The fuzzy operations (union, intersection and complement) are similarly defined. Given two fuzzy relations R(x,y) and S(x,y) defined in one space U x Y, the resulting membership functions are:

$$\mu_{R \cap S}(x,y) = \mu_R(x,y) * \mu_S(x,y)$$

$$\mu_{R \cup S}(x,y) = \mu_R(x,y) \oplus \mu_S(x,y) \tag{14}$$

where * is any t-norm and ⊕ is any t-co-norm.

Given U, V, and W as three universes of discourses, R as a relation on U x V, and S another relation on V x W, in order to obtain the composition R o S, that relates U and W, it is initially extended R and S to U x V x W. Since the relations R and S have now the same domain, then we can determine the relation support between the universes U x W by the following expression:

$$\mu_{R0S}(x,z) = \sup \left[\min \left\{ \mu_R^{ext}(x,y,z), \mu_S^{ext}(x,y,z) \right\} \right] \tag{15}$$

Where

$$\mu_R^{ext}(x,y,z) = \mu_R(x,y)$$

$$\mu_S^{ext}(x,y,z) = \mu_S(x,y) \tag{16}$$

The main difference between the fuzzy relation and the classic relation is that the latter $\mu_R(x,y)$ assumes values 0 or 1, while fuzzy relation may assume infinite values between 0 and 1.

2.1.5 Fuzzy implications

Fuzzy rules are conditional structures that use heuristic methods through linguistic expressions in rule forms, composed by a condition (IF) and a consequence (THEN), forming the following structure

$$IF\{condition\}THEN\{consequence\} \tag{17}$$

where conditions and consequences are fuzzy propositions built by linguistic expressions:

1. x is Low
2. y is NOT Tall
3. x is Low AND y is Tall
4. x is Low OR y is Tall

The rules 1 and 2 define "immediate" propositions, the rules 3 and 4 define combined propositions. These propositions use fuzzy operators NOT, OR and AND, respectively in 2, 3, and 4.

Mamdani (Mamdani & Assilan, 1975) defined the use of fuzzy relations R_{MM} and R_{PM} in U x V as an interpretation for the rule IF <pert₁> THEN <pert₂>, where R_{MM} and R_{PM} are defined as

$$\mu_{QMM}(x,y) = \min\left[\mu_{pert1}(x), \mu_{pert2}(y)\right] \tag{18}$$

$$\mu_{QPM}(x,y) = \min\left[\mu_{pert1}(x), \mu_{pert2}(y)\right] \tag{19}$$

where $x \in U$ and $y \in V$.

2.2 Fuzzy system structure

Figure 3 shows the structure of a basic model of fuzzy system applied in industrial process. The fuzzy system structure consists of four subsystems: Input Fuzzification, Rule Database, Inference Machine and Defuzzification.
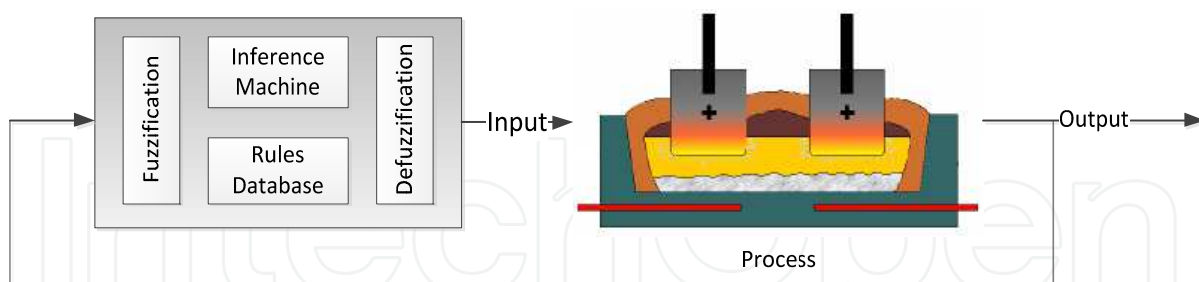

Fig. 3. Fuzzy System Structure

2.2.1 Input fuzzification

In this stage, the input variables (crisp variables) are converted into fuzzy values through a real numbers mapping $x^* \in U \subset R^n$ for a fuzzy set $A' \subset R^n$. The steps for fuzzification are presented:

1. acquire numeric values of input variables (crisp values);
2. map these variables in a universe of discourse U;
3. determine membership functions and linguistic variables.

The variables mapping (crisp) is characterized by membership function $\mu_A(x) \rightarrow [0,1]$. Such functions may be classified in: Triangle-shaped, Trapezoidal, and Gaussian. These functions are shown in Figure 4.
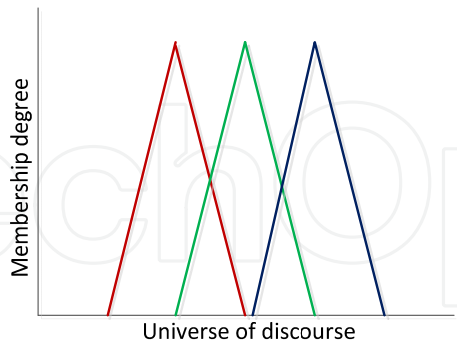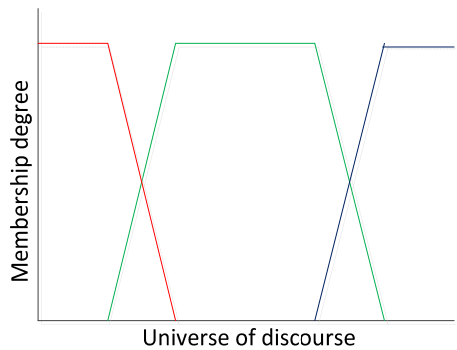

Fig. 4a. Triangle-shaped function
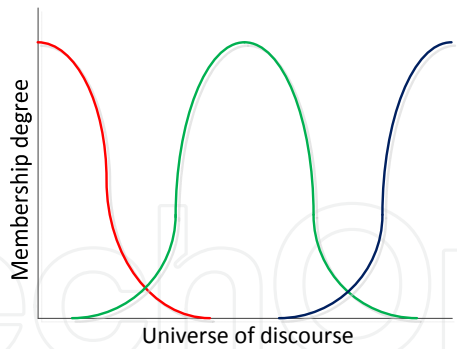

Fig. 4b. Trapezoidal Function

Fig. 4c. Gaussian Function

The Triangle-shaped and Trapezoidal functions use the triangle fuzzificator:

$$\mu_{A'}(x) = \begin{cases} \left(1 - \dfrac{\left|x_1 - x_1^*\right|}{b_1}\right) \cdots \left(1 - \dfrac{\left|x_n - x_n^*\right|}{b_n}\right) & if \quad x = \left|x_i - x_i^*\right| \leq b_i; \\ 0 & if \quad x = \left|x_i - x_i^*\right| > b_i \end{cases} \tag{20}$$

The Gaussian function uses the Gaussian fuzzificator:

$$\mu_{A'}(x) = \exp^{\left(\frac{|x_1 - x_1^*|}{a_1}\right)^2} \cdots \exp^{\left(\frac{|x_n - x_n^*|}{a_n}\right)^2} \qquad (21)$$

2.2.2 Fuzzy rule database

A fuzzy rule database is a collection of IF-THEN rules that can be expressed as:

$$R^{(l)} : IF \quad x_1 \quad is \quad A_1^l \quad AND \cdots AND \quad x_x \quad is \quad A_n^l \quad THEN \quad y \quad is \quad B^l \qquad (22)$$

Where $l = 1, 2, \ldots, M$, A_1^l and B^l are fuzzy sets in $U_i \subset R$ and $U \subset R$ respectively, $x = col(u_1, \ldots, u_n) \in U_1 \times \ldots \times U_n$, and $y \in V$. x and y are linguistic variables. The knowledge of an expert is stored in this rule database, since all decisions taken by an expert can be written as rules. In essence, the rules model the fuzzy system behaviour.

2.2.3 Fuzzy inference machine

The fuzzy inference machine acts on a set of rules, denoted in (22), maps inputs (conditions) into outputs (consequences). In this stage, called inference, the fuzzy operations are performed on these variables. The conditions will trigger some rules then the variables of the triggered rules are combined, performing the implication and summing up the result of all rules. The fuzzy rule database with m rules does:

- Determine the membership value $\mu_{A_1^l \times \ldots \times A_n^l}(x_1, \ldots, x_n)$ for the fuzzy sets triggered for the m rules.

- Perform the fuzzy inference of $A' \subset U$ for $B' \subset V$ based on each rule that compose the fuzzy rule database:

$$\mu_{B^l}^l(y) = \sup_{x \in U} t\left[\mu_{A'}(x), \mu_R^l(x, y)\right] \qquad (23)$$

The inference machine combines the m fired fuzzy sets, as expressed in:

$$\mu_{B'}(y) = \mu_{B_1}(y) \oplus \ldots \otimes \mu_{B_n}(y) \qquad (24)$$

where $\oplus$ denotes the t-norm operator.

There are two main types of inference machine: Product and Minimum.

In the product Inference Machine, we use:

a. inference of rule database individually
b. Mamdani implication (19)
c. Algebraic product for all t-norm operators and maximum for all s-norm operators. This inference machine can be represented as follows:

$$\mu_{B'}(y) = \max_{l=1}^{m}\left[\sup_{x \in u}\left[\mu_{A'}(x)\prod_{i=1}^{n}\mu_{A'}^l(x)\mu_{B^l}(y)\right]\right] \qquad (25)$$

In the Minimum Inference machine, we use:

a.   inference of rule database individually
b.   Mamdani implication (19)
c.   Algebric product for all t-norm operators and minimum for all s-norm operators. This inference machine can be represented as follows:

$$\mu_{B'}(y) = \max_{l=1}^{m}\left[\sup_{x \in u}\min\left(\mu_{A'}(x), \mu_{A_1^l}, \ldots, \mu_{A_n^l}(x_n), \mu_{B^L}(y)\right)\right] \tag{26}$$

2.2.4 Defuzzification

In this stage, fuzzy output values are converted back in real values. This conversion is done through mapping, $B' \subset V$ for a point $y^* \in V$. There are many methods for defuzzification, namely Centre of Gravity (or Centre of Area), Centre of Maxima, Average of Maxima, to name a few.

The method Centre of Gravity evaluates the center of area corresponding to the union of fuzzy sets that contributed to the result. It is mathematically represented by the formula:

$$\bar{y} = \frac{\sum_{i=1}^{N} y_i \mu_B(y_i)}{\sum_{i=1}^{N} \mu_B(y_i)} \tag{27}$$

where $\bar{y}$ is the resulting center of gravity, y_i is the center of the individual membership function and $\mu_B(y_i)$ is the area of a membership function modified by the fuzzy inference result (not null values).

The Centre of Maxima method uses the higher values of membership functions. The not null values are considered weights and the result is obtained as a support point among them. It is evaluated by the following equation:

$$\bar{y} = \frac{\sum_{i=1}^{N} y_i \sum_{i=1}^{N} \mu_M(y_i)}{\sum_{i=1}^{N}\sum_{i=1}^{N} \mu_M(y_i)} \tag{28}$$

where $\mu_M(y_i)$ are the membership functions maximum (height) points.

The Average of Maxima method uses the maximum point of each membership function and takes the mean value as the defuzzified value. It is represented by the following formula:

$$\bar{y} = \sum_{i=1}^{M} \frac{y_i}{M} \tag{29}$$

where y_i is the i-th element corresponding to the membership functions maximum and M is the total of elements.

3. Bath chemistry control in aluminum reduction cells

During the Aluminum production process, several chemical additives are used in reduction industries to contro bath chemical and physical composition. These additives' aim to lower the liquidus temperature (Haupin&Kvande, 1993), i.e., to decline the melting point of cryolite (Na_3AlF_6), allowing the solubilisation of alumina (Al_2O_3) and therefore better energy use. There are two strategies for bath chemistry control: Heat Balance and Mass Balance. Any change in the cell's heat balancet results in changes in the bath chemical composition, as well as any change in the bath chemical composition causes changes in the heat balance. It is noted that there is a relationship between cell's heat balance and its current chemical composition, influencing the cells' productivity (Dias, 2002). The current model used for control strategy is based on correlation between bath temperature and fluoride excess (%AlF_3) in the bath. Besides these correlations, there are other variables having some influence in the bath chemistry, which are also used in the control strategy.

The electrolyte used in Aluminium reduction pots is basically composed of melted cryolite (Na_3AlF_6), Aluminum fluoride (AlF_3), calcium fluoride (CaF_2) and alumina (Al_2O_3), and its major concentration is formed by cryolite. The bath components' percentages are directly related to stability. The fluoride percentage has the property of lowering the cryolite melting point from about 1100^oC down to. Likewise, the bath is composed of a solid part (non-melted cryolite) and liquid part (melted cryolite) which may vary according to the percentage of fluoride present in the bath. The greater the percentage of fluoride is, the lower the bath melting point is, therefore emphasizing the presence of liquid part in comparison with the solid part (mass balance), leading to a cooling of the cell (heat balance). Similarly, low quantities of fluoride emphasize the solid part regarding the liquid part, causing a heat of the cell (heat balance).

There are many factors contributing to the Aluminum fluoride consumption, which is added in the pot during the Aluminum reduction process. In other to stabilize such situations during the process, a theoretical calculation is defined, considering the following factors:

- Addition due to the absorption by pot lining (Hyland et al, 2001).
- Addition due to the sodium and calcium oxide present in alumina (Al_2O_3), according to the equations 30 and 31:

$$3Na_2O + 2AlF_3 = 6NAF + Al_2O \tag{30}$$

$$3CaO + 2AlF_3 = 3CaF_2 + Al_2O_3 \tag{31}$$
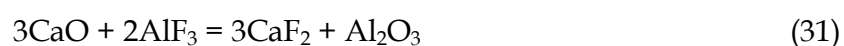
Based on these information, the theoretical consumption is determined by the following expression:

$$AlF3[kg] = A*\%Na_2O + B*\%CaO + C*\%AlF_3 \tag{32}$$

where A, B and C are constants and $\%Na_2O$, $\%CaO$ and $\%AlF_3$ represent respectively the percentages of sodium oxide, calcium oxide and Aluminum fluoride. The electrolyte composition control represents a challenge in Aluminum reduction industries, due to the intrinsic relation between heat and mass balance.

Usually the bath chemistry control is performed daily or weekly, collecting all the information about thermal and mass balance (Bath Temperature, Liquidus Temperature, Super Heat, Fluoride, Bath Composition and so on). With this information, the process team should take decisions on how much should be added into the bath in order to keep temperature and fluoride under control near a setpoint. Figure 5 shows a scheme of this process.
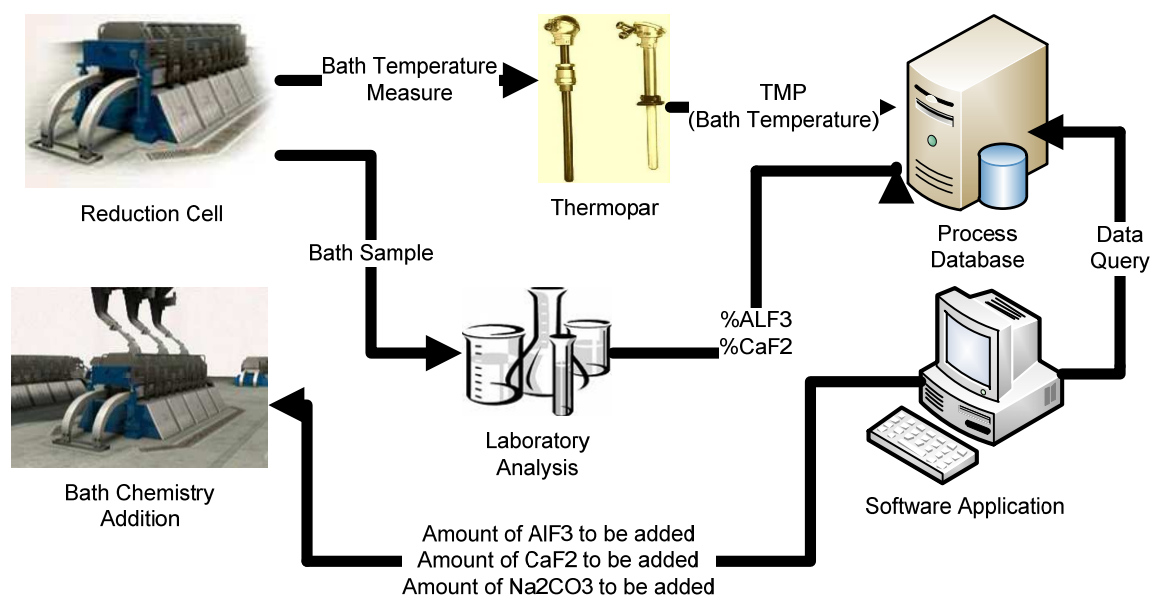

Fig. 5. Bath Chemistry Process Schematic Diagram

Variable	Description
TMP	BathTemperature
%ALF3	Percentage of AluminumFluoride in the bath
%CaF2	Percentage of Calcium Fluoride in the bath
AlF3A	Amount of Aluminum Fluoride to be added
CaF2A	Amount of Calcium Fluoride to be added
Na2CO3A	Amount of Sodium Carbonate to be added
LIFE	Time elasped (in days) since cell startup

Table 1. Variables used in the Bath Chemistry Control Process

3.1 Challenges on this control

The strongest impact of this process in Aluminum smelting is the direct influence on Current Efficiency and on ledge. Because of that, a careful control is required in order to keep both bath temperature and Aluminum fluoride stable. The Current Efficiency means literally how much is produced from the maximum allowed, according to equation 32.

$$Kg_{Al} = \left(\frac{I*86400}{96485} \right) *0,009 \tag{33}$$

where I is the current in Amperes. A hypothetically Current Efficiency of 100% means that production is equal to the theoretical maximum. However, part of the Aluminum formed in the bath is recombined again with carbon gas, as showed in the equation (34).

$$2Al^+ + 3CO_2 \rightarrow Al_2O_3 + 3CO \tag{34}$$

The optimum point is reached when the variables are stabilized around a setpoint. Each variable is assigned a setpoint, but the cells are subjected to many disturbances that have effect on every controlled variable. This makes the process even harder to control and more complex to model (Prasad, 2000; McFadden et al., 2001; Welch, 2002). Process experts take actions, sometimes predefined, to control the process based on their experience in the process. This means their decisions are usually taken without any model of the system. For that reason, an AI technique approach is useful since it does not need to model analytically the whole process but it can represent it with some accuracy and yield good results. To address the fluoride addition problem, we can build a fuzzy system in which all the process knowledge can be included as rules, and provided that process operators usually refer to variables using linguistic terms, Fuzzy sets can be used to represent these linguistic terms.

4. Fuzzy control applied for fluoride addition in aluminum reduction cells

Fuzzy Controllers have been applied in industrial plants, since many solutions are sold with this technology as part of it(Cao et al, 2010). In Aluminum industry, the Aluminum fluoride addition control is usually performed by parameterized equations, confidentially protected. These are made by data collection and numeric approximation. This model has a poor performance since the plant is very nonlinear and complex and its modeling is very difficult. Very often the process operators must take manual actions to control the process. This decision making process for fluoride addition in reduction cells is a routine for adjusting the bath composition and hence its performance.

In order to maintain performance and stability of electrolytic cells, some action on thermal balance and mass balance is required (Welch, 2000), acting on process variables. These variables are used to determine how much Aluminum fluoride should be added into the bath. Bath chemistry control stands as a great challenge for Aluminum smelters, since it is intrinsic to the thermal balance of electrolytic cells.

4.1 Design procedure

Since human intervention in this process is often required, a fuzzy controller must follow the actions operators usually take when analyzing recent data from the cells. In this sense, a linguistic processing is needed to represent the process data under a fuzzy view. Also, a survey with process engineers responsible for the bath chemistry control is performed in order to find out which data the process operators usually look at before performing a fluoride addition. TThese data can also represent the process dynamic behaviour. In this

work they are: Bath Temperature (TMP), Percentage of aluminum fluoride in the bath(ALF), Cell operation time (also known as Pot life) (LIFE). Moreover, the Temperature and Fluoride trend information (TTMP and TALF, respectively)are also viewed by process operators and should be taken into account for fuzzy processing, provided that Bath temperature and Aluminum Fluoride are negatively correlated, which means as one is rising the other is falling. The past fluoride additions are also considered in a separate variable called Accumulated Aluminum Fluoride (ALF3AC), so the information of how much fluoride has been added into the bath in the last three cycles is considered for fuzzy processing. And finally, the output variable for the fuzzy system is Fluoride addition (ALF3A), which is the control variable. It is important to note that the variables TMP, ALF and LIFE are measured, but TTMP, TALF and ALF3AC are calculated from TMP and ALF, as shown in equations.

$$TTMP(t) = TMP(t) - TMP(t-1) \tag{35}$$

$$TALF(t) = ALF(t) - ALF(t-1) \tag{36}$$

$$ALF3AC(t) = \sum_{i}^{3} ALF3A(t-i) \tag{37}$$

After these variables have been chosen, each one is assigned linguistic terms like process operators usually call, as shown in table 2.

Input Variables		Linguistic terms
TMP	Bath Temperature	Very Cold, Cold, Normal, Hot, Very Hot
ALF	Aluminum Fluoride	Very Low, Low, Normal, High, Very High
LIFE	Cell Life	Young, Average, Old
ALF3AC	Accumulated Aluminum Fluoride	Very Low, Low, Normal, High, Very High, Ultra High
TTMP	Bath Temperature Trend	Rise, Fall
TALF	Aluminum Fluoride Trend	Rise, Fall
Output Variable		Linguistic terms
ALF3A	Aluminum Fluoride to be added	No Add, Very Low, Low, Mid-Low, Normal, Mid-High, High, Very High, Super High, Ultra High

Table 2. Fuzzy Variables used in this system and their linguistic terms

4.1.1 Fuzzy sets

The linguistic terms for each process variable are used to form the fuzzy sets, which are characterized by membership functions, as described in 2.1. The membership functions related to each fuzzy set were determined by the dynamic behaviour of each variable as the process evolves. All sets are represented by trapezoidal functions whose limits are based on a qualitative knowledge on the plant. Figures 6a-6g show the fuzzy sets plots for each input variable and for the output variable.
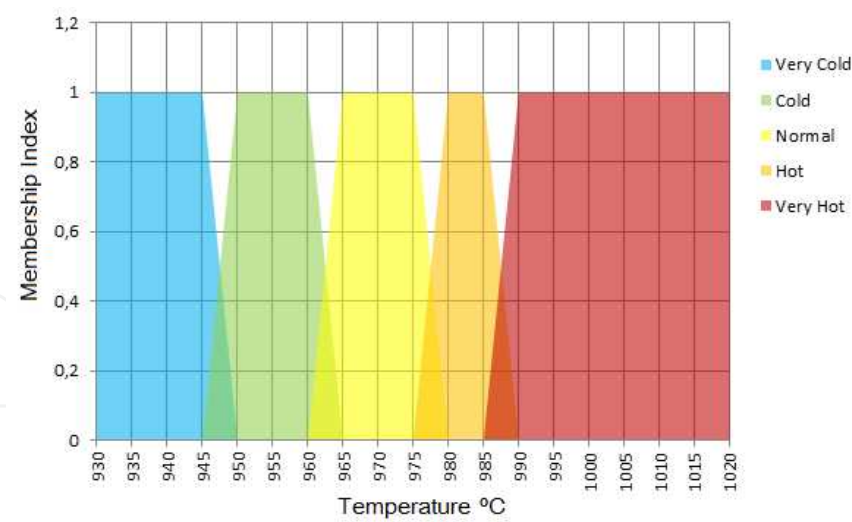
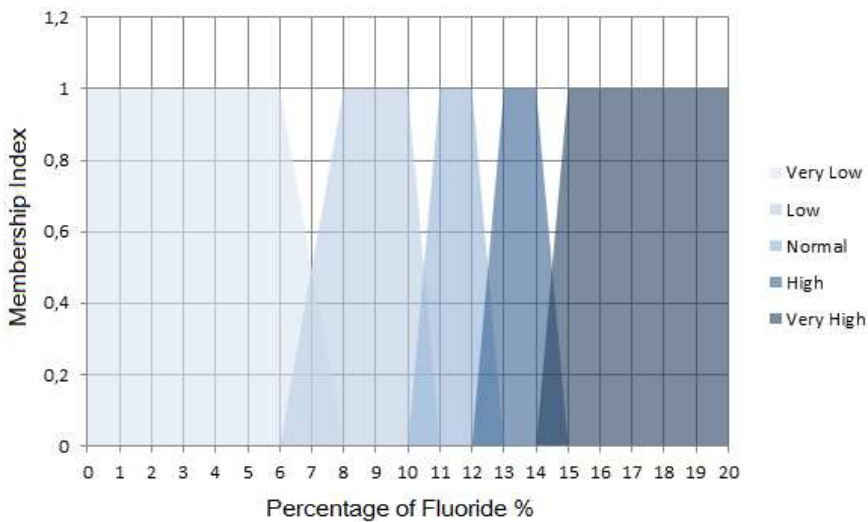Fig. 6a. Fuzzy sets for the bath temperature

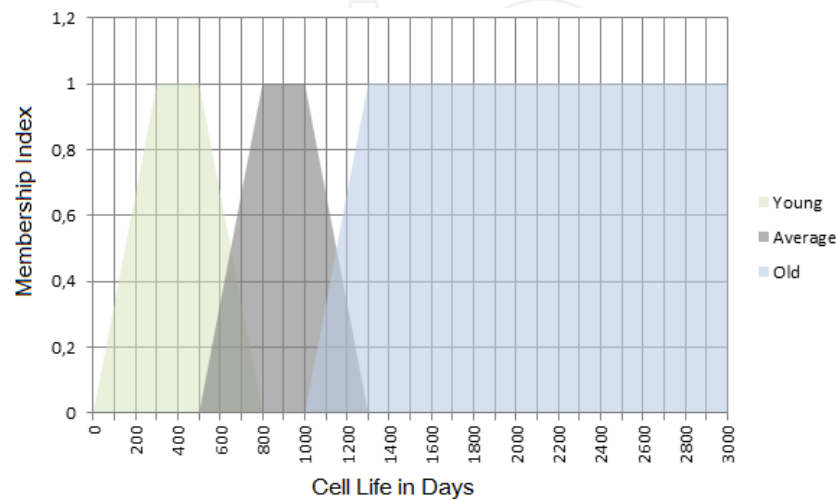Fig. 6b. Fuzzy sets for Percentage of Aluminum fluoride in the bath
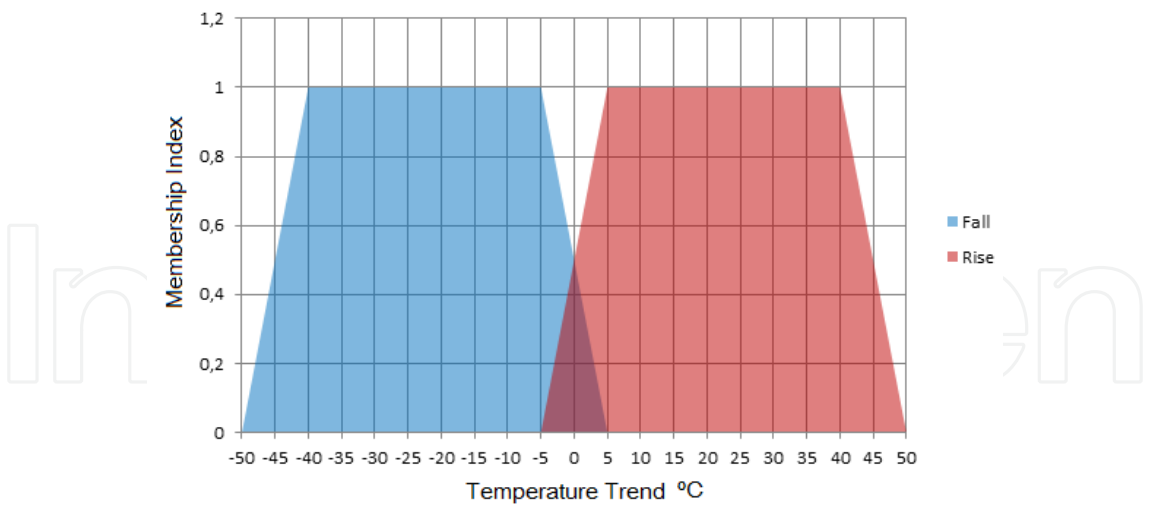

Fig. 6c. Fuzzy sets for Life
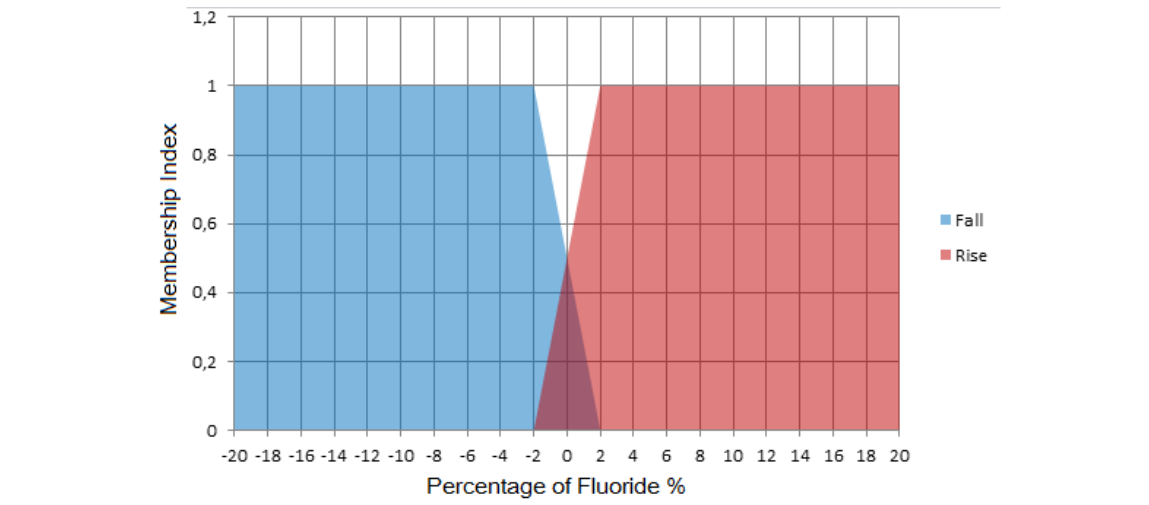
Fig. 6d. Fuzzy sets for Temperature Trend

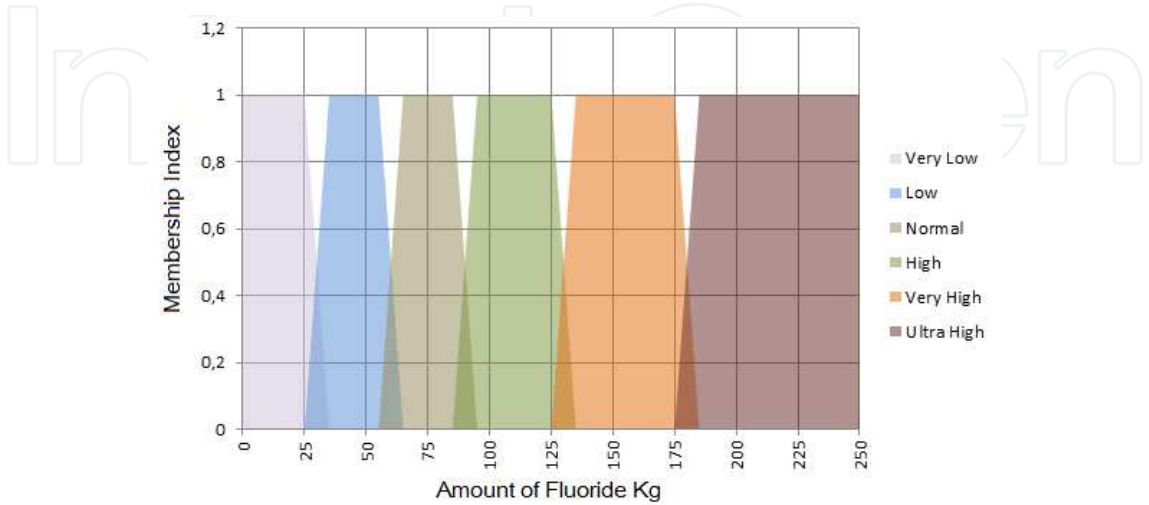Fig. 6e. Fuzzy sets for Fluoride Trend

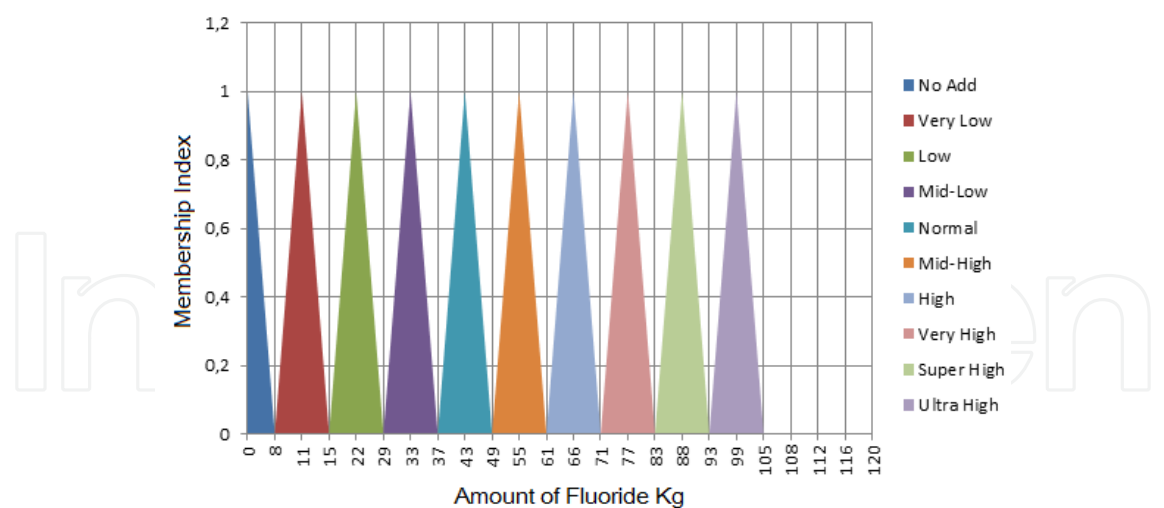
Fig. 6f. Fuzzy sets for Accumulated Aluminum fluoride

Fig. 6g. Fuzzy sets for Amount of Aluminum fluoride to be added

4.1.2 Fuzzy rules definition

In order to define the fuzzy rules, a database T was built by taking the process variables records from the chosen inputs and outputs. This database encompasses three years of operation and has over 800,000 records. This huge number of records allows querying each combination of variables' fuzzy sets against the database in order to find which output value was chosen in the most of times. This means that the rules definition cannot be performed by interviews as fuzzy system designers usually do, however some adjusts on the rules may be made by process experts. Table 3 shows the number of fuzzy sets for each variable and the number of combinations:

Variable (VAR)	Number of Fuzzy Sets (NVAR)
TMP	5
ALF	5
LIFE	3
TTMP	2
TALF	2
ALF3AC	6
Total of combinations (NTMP x NALF x NLIFE x NTTMP x NTALF x NALF3AC)	1800

Table 3. Combinations of Fuzzy Sets

Through these combinations, one can perform a statistical research in the process database and find which output fuzzy set has more occurrences for every single combination. Table 4 shows how the fuzzy rule database look like, taking into account these combinations. It is assumed for interpretation the connector AND for all rules. Table 5 shows a case for defining an output for a given rule.

The statistical research may fall into three cases:

- **Case 1** - there is only one most frequent set for a condition of a given rule R^l, which is going to be the rule's output.
- **Case 2** – there are two or more frequent set for a condition of a given rule R^l, whose output should be chosen later by a process expert.
- **Case 3** – there are no records matching the condition of a given rule, which means the rule output should be chosen later a process expert, however it is likely that this situation won't happen, implying no need for adjustment.

#	Conditional Variables						Consequence
	Bath Temp.	Aluminum Fluoride	Cell Life	Accum. Fluoride	Fluoride Trend	Temp. Trend	Fluoride to be added
1	Very Cold	Very Low	Young	Very Low	Fall	Fall	Det. by queries
2	Very Cold	Very Low	Young	Very Low	Fall	Rise	Det. by queries
3	Very Cold	Very Low	Young	Very Low	Rise	Fall	Det. by queries
4	Very Cold	Very Low	Young	Very Low	Rise	Rise	Det. by queries
5	Very Cold	Very Low	Young	Low	Fall	Fall	Det. by queries
…	…	…	…	…	…	…	…
1800	Very Hot	Very High	Old	Ultra High	Rise	Rise	Det. by queries

Table 4. Fuzzy Rule Database Structure

"if **TMP** is *Normal* and **ALF** is *Very Low* and **LIFE** is *Normal* and **ALF3AC** is *Normal* and **TTMP** is *Fall* and **TALF** is *Rise*"	
A query against a database is performed, and the following result is found:	
ALF3A is *Normal*	Twice
ALF3A is *High*	3 Times
ALF3A is *Very High*	6 Times
Thus, as for this rule, the output is chosen as *Very High*, since it is the decision more often.	

Table 5. Fuzzy rule definition upon database research

4.2 Fuzzy operations

Real world (crisp) values are fuzzified by the membership functions defined in figures 6a-6f, which may yield fuzzy values in one or two sets. We used the minimum operator to apply the fuzzy values. Table 6 shows an example of fuzzification and table 7 show an example of the fuzzy minimum operator:

Variable	Crisp Value	Fuzzy Values (with membership indexes)
Bath Temperature (TMP)	962ºC	Cold (0.6) and Normal (0.4)
Aluminum Fluoride (ALF)	9.82 %	Low (1)
Cell Life (LIFE)	596 days	Young (0.68) and Normal (0.32)
Accumulated Aluminum Fluoride (ALF3AC)	70 Kg	Normal (1)
Temperature Trend (TTMP)	-13ºC	Fall (1)
Fluoride Trend (TALF)	- 3%	Fall (1)

Table 6. Fuzzy Values for a case

Rule	Rule output (Least membership index)
"if **TMP** is *Cold(0.6)* and **ALF** is *Low(1)* and **LIFE** is *Young(0.68)* and **ALF3AC** is *Normal(1)* and **TTMP** is *Fall(1)* and **TALF** is *Fall(1)*"	Mid-High (0.6)
"if **TMP** is *Normal(0.4)* and **ALF** is *Low(1)* and **LIFE** is *Young(0.68)* and **ALF3AC** is *Normal(1)* and **TTMP** is *Fall(1)* and **TALF** is *Fall(1)*"	Very Low (0.4)
"if **TMP** is *Cold(0.6)* and **ALF** is *Low(1)* and **LIFE** is *Normal(0.32)* and **ALF3AC** is *Normal(1)* and **TTMP** is *Fall(1)* and **TALF** is *Fall(1)*"	Mid-High (0.32)
"if TMP is *Normal(0.4)* and ALF is *Low(1)* and LIFE is *Normal(0.32)* and ALF3AC is *Normal(1)* and TTMP is *Fall(1)* and TALF is *Fall(1)*"	High (0.32)

Table 7. Rules triggered for the fuzzy values in the case of table 6

The implication operation chosen in this work is the product method, meaning that every output set is multiplied by the rule's least membership value. And for the aggregation operation, the output sets build the geometric shape by the maximum. The defuzzification method is the centre of area. Figure 7 shows the geometric shape made by the output sets with their least membership index in table 7.
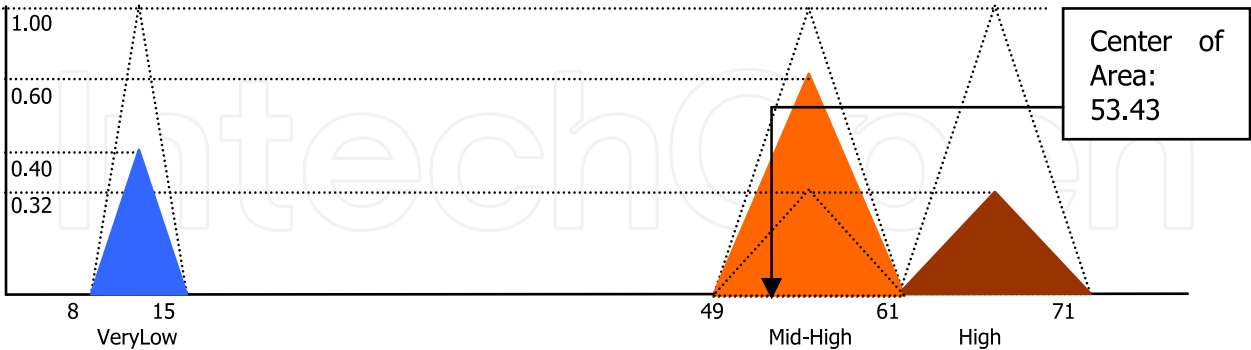

Fig. 7. Implication, Aggregation and Defuzzification operations

4.3 Result and validation

The fuzzy algorithm was directly implemented in an industrial plant of aluminum reduction. Initially 10 pots were chosen from one potline, to which the operators were instructed to intervene only when there is an extreme need. However, it is worth

mentioning, that the validation of new fluoride addition logic must be tested for at least seven months. By the time this paper was written, the pots used in these tests were operating for nearly five months with the new logic. The figure 8 show the real result obtained for one pot during the test period.
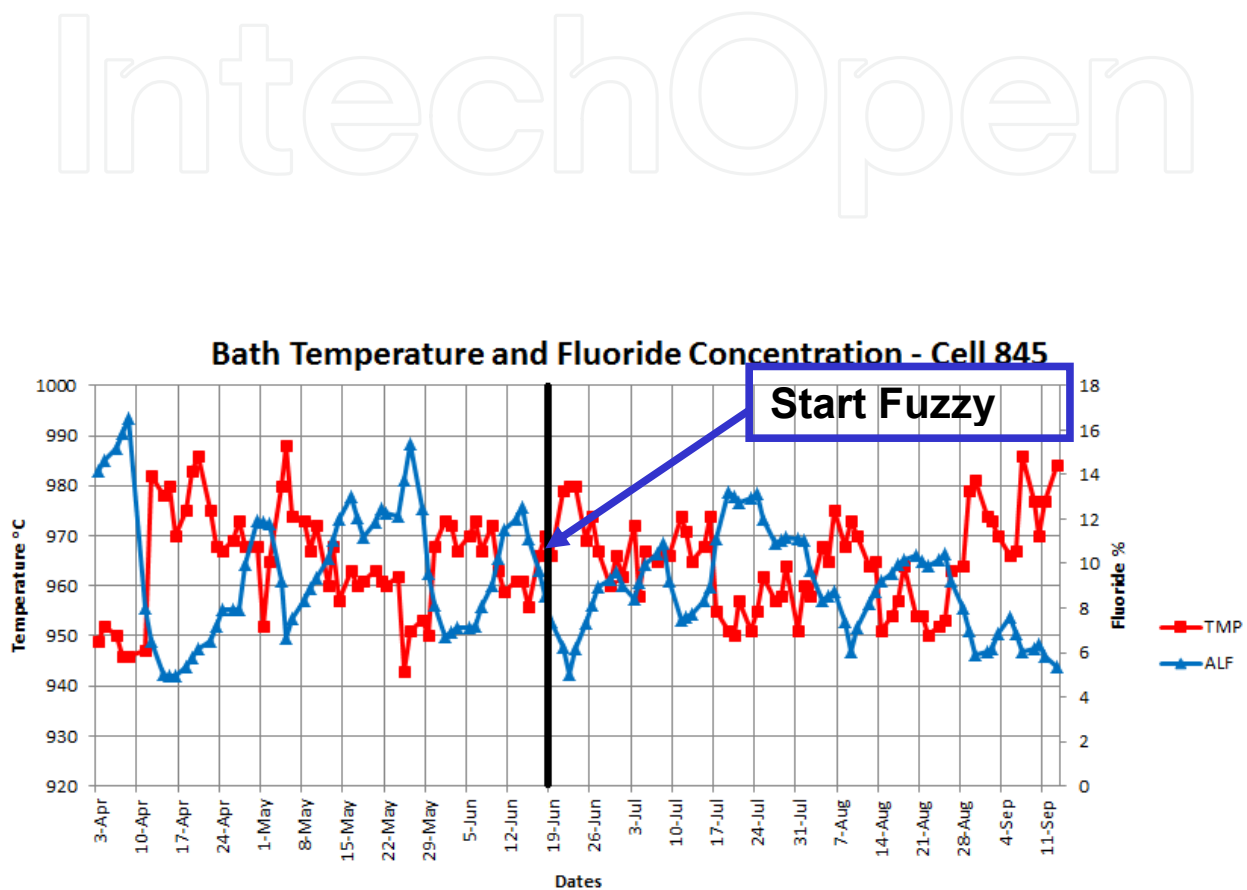

Fig. 8. Real result obtained during the tests

The figure 8 is divided in two regions defined by the date when the fuzzy system started. It is notable that the right (or later) region has less oscillation of the temperature variable (red line) and the percentage fluoride has been decreased. This has been one of the main expected results with the fuzzy logic, being interpreted by the process engineering as a safer operational condition.

Another desired goal was to reduce the human interventions in the process. In the previous control strategy, there was a high oscillation degree, which often required human intervention, by changing the proposed value to a quantity, sometimes, higher than needed, thus destabilizing the process. With the new strategy, the need of an analysis tool arose in order to show the membership values of each set, the activated rules and the corresponding defuzzified output. This tool allows monitoring the decisions made by the fuzzy system, and the historical analysis of past decisions. A screen of this tool is shown in figure 9.
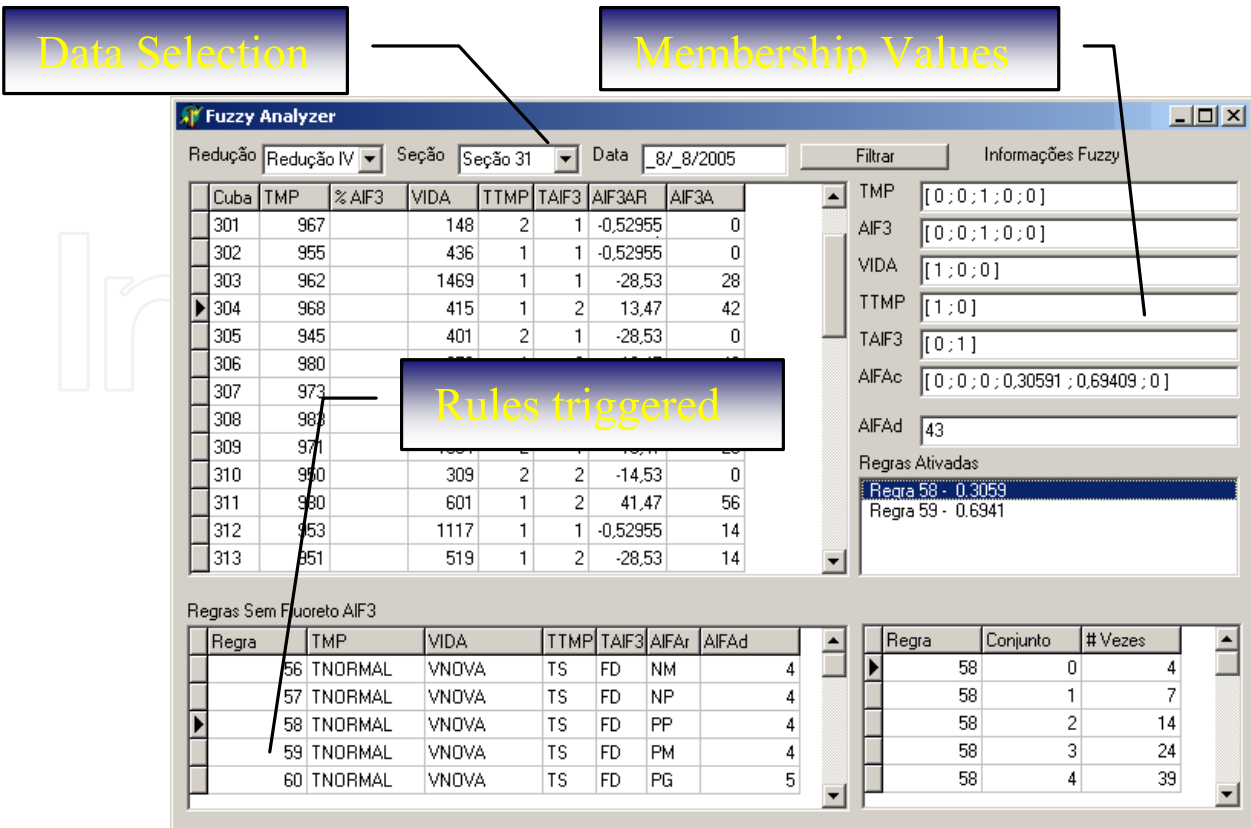
Fig. 9. Fuzzy Analyzer tool

5. Conclusion

There was a need in the Aluminum smelting process for fluoride addition and control using the process experts' knowledge, since the current methodologies does not address well this problem and there are always many human interventions on this process. The results presented by the fuzzy strategy show that it can match the process requirements once it aggregated the interventions or changes made by process technicians to the control variable. This positive result will give technicians more time for other activities, such as process improvements instead of always worried in analyzing, criticizing and change the suggested results by the current system.

The fuzzy strategy not only aggregated human knowledge to the system, but it has also improved the system stability as shown in results and validation, the temperature and fluoride variations declined. However, it is only possible to achieve a trustworthy degree of a new strategy after a period of at least 7 months. Meanwhile, the system is still in the observation state.

The impact of this work can be scaled to a higher level by considering the continual improvement of the rules and the fuzzy system as well, since it will be continually evaluated and adjusted. Thus there will be an efficient control on fluoride addition.

For future works, this methodology can be extended to other decision making process whose decision is taken based on human interpretation or consolidated data. Also we

suggest the use of other fuzzy settings such as inference machines, membership functions, and implication and aggregation methods for comparison.

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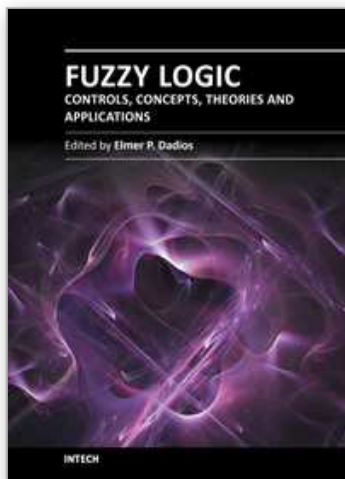
This book introduces new concepts and theories of Fuzzy Logic Control for the application and development of robotics and intelligent machines. The book consists of nineteen chapters categorized into 1) Robotics and Electrical Machines 2) Intelligent Control Systems with various applications, and 3) New Fuzzy Logic Concepts and Theories. The intended readers of this book are engineers, researchers, and graduate students interested in fuzzy logic control systems.

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