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Avalanche Process in Semiconductor Photo Detectors in the Context of the Feedback Theory

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1. Introduction

The feedback theory is a quite advantageous tool in the process of description of complex systems' behaviour or the procedure of complex processes. We can characterize the complexity of the system from its structural properties when there is a large amount of bounds between many elements of system. We can also characterize the complexity of the system as its functional complexity when the current state of system is defined as a result of many self-consistent states originating during the evolution of the system. It can be said in most general words that the feedback theory (FBT) describes the behaviour of a system or a process when the current status is defined as a result of achieving the self-consistency of the main system parameters. The feedback theory is most fruitfully applied to the description of those complex systems for which, due to their statistical character it is impossible to construct the full physical-mathematical model with a simple solution, but for which it is easy to select statistical parameters of the system and define their connections with the system properties that are of our interest. This approach allows applying FBT to describe the evolution of complex systems in chemistry, physics, biology, sociology and economics [1].

In this paper, we discuss the avalanche process in semiconductor avalanche photo detectors (APD) proceeding from three different points of view on this process. Then, we sum up the conclusions made in relation to the avalanche process in connection with avalanche photo detectors. We present APD as a converter of photo radiation to electric current with further current amplification. As a result, we can interpret the APD operation from various points that will allow us, finally, both to make several interesting conclusions on their practical application and analyze a possibility to choose optimal parameters in APD manufacturing.

To describe efficiently the processes in the frames of the feedback theory, it is necessary to fetch out the main physical parameters that determine the process procedure and establish quantitative ratios between the character of the process procedure and the change of these parameters. Further, we discuss the main characteristic parameter – the multiplication factor of the avalanche photo detectors M as an applied biasing voltage function (of the material parameters and topology) and attempt to describe the conditions for the achievement of self-consistency with the choice of the feedback coefficient function. It should be mentioned that at present FBT is most widely used in the control theory [2], i.e. in the description of the

behaviour of the systems for which it is necessary to achieve the desired character of behaviour. It can be usually attained by the choice of the corresponding system feedback function if the system transfer function without feedback is known. From this point of view, the application of FBT should allow us to approach the designing of avalanche photo detectors from the taken characteristics. We will obtain an opportunity to build the necessary APD transfer function and define the necessary feedback function with it. We have to learn to connect the topology and technology with the parameters of the established Feedback Function (FBF) and then, proceeding from the given FBF, we will be able to obtain topological and technological parameters. The practical application of the described physical-technological algorithm is not discussed in this paper. Our task is to show how it is possible to come from different APD models to their generalized description using FBT.

2. Feedback theory basics

Let us regard the main notions of the feedback theory in radio technique. Hereinafter, we will have to describe an avalanche photodiode as an amplifying device with feedback and present it in the form of the corresponding equivalent scheme.

In radio technique the behaviour of a system with feedback is described by the function of a special type - the system function (in a complex form, as the transfer function $H(s)$). Let us discuss a simple scheme of the signal amplifier shown in Fig.2.1 to whose input a signal is given that depends on time $x(t)$, and the signal $y(t)$ is received in the output.

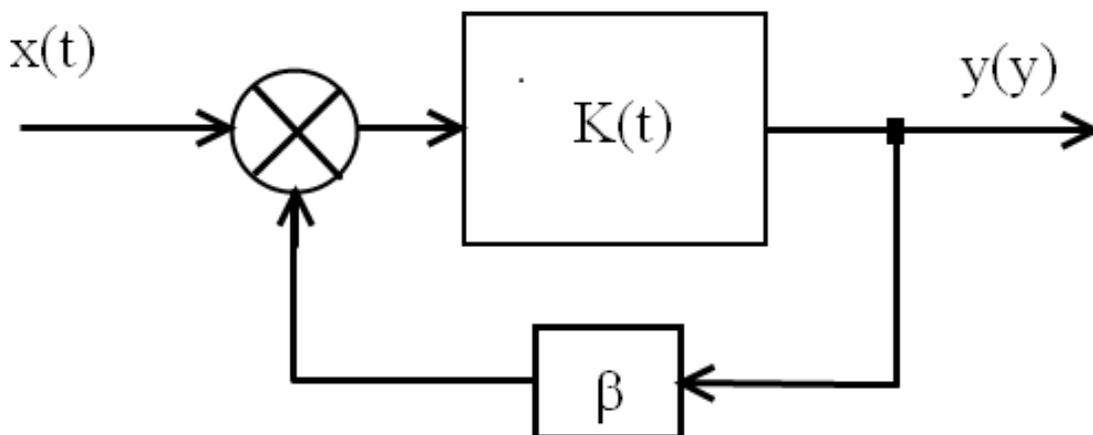


Fig. 2.1. Feedback in amplifier.

The system coefficient of gain function is defined as a ration $K(t)=y(t)/x(t)$. If a part of the signal from the system output is transmitted to the input it is supposed that the feedback is active in the system and the feedback coefficient is defined as $\beta(t)=((x'(t)-x(t))/y(t))$, where $x'(t)$ - is the signal in the system input in the action of the feedback in the system. At $\beta(t)>0$ the feedback is positive; at $\beta(t)<0$ the feedback is negative; at $\beta(t)=0$ there is no feedback in the system. The system gain is written in the system with acting feedback in the form (2.1)

$$K=K_0/(1-\beta*K_0) \quad (2.1)$$

where K_0 is gain of the system without the feedback. Negative feedback (NFB) is most often used in electronic systems as in this case the system stability increases. Besides, providing

$K_o \gg 1$ the gain is defined mainly with the feedback coefficient $K \approx 1/\beta$ that allows a desired coefficient of gain of the system be set by the choice of only one parameter β .

It should be stressed that the transfer function (1.1) in radio technology is given in frequency representation [3]. Speaking more accurately, it is regarded as a result of Laplace transformations on the complex plane. In this case the transfer function is equivalent to the frequency characteristic of the chain if active sources are absent.

$$H(s) = Y(s)/X(s) = L[y(t)]/L[x(t)] \tag{2.2}$$

The frequency representation for radio technical amplifying systems is more informative and suitable for their designing and study. We will discuss here only the time representation of the system coefficient of gain (amplifying) for avalanche detectors. The reason is the suitability and simplicity of this representation for the description of the avalanche photo detectors' operation and sufficiently simple procedure of measuring of the APD pulse characteristic. The time representation of the system gain is easily obtained if the APD pulse characteristic is known. In this stage, we do not consider the transfer function for APD, not to make the description and comprehension of the idea to describe the process of avalanche multiplication in the FBT frames too complicated.

3. Probability model of the avalanche multiplication

Let us consider an idealized model of avalanche multiplication of electrons in a homogeneous semiconductor with the following suppositions: electron multiplication takes place in a limited region with the length L_a with constant electron field voltage $E_a = \text{const}$ in this region. The probability of multiplication along the L_f electron free flight is $P < 1$. See Figure 3.1.

If there are N_0 electrons at the beginning of multiplication in the output of this region there will be $N \gg N_0$ electrons, as a result of multiplication. It can be expressed quantitatively in the form (3.1).

The multiplication factor is defined as $M = N/N_0$

$$N = N_0 * (1 + P + P^2 + P^3 + \dots + P^n) \text{ where } n = L_a / L_f \tag{3.1}$$

For $n \gg 1$ this expression can be written in the form

$$N = N_0 (1 - P)^{-1} \tag{3.2}$$

Formally, let us re-write (2.1) in the form

$$N_x = N_0 * (1 + (P - \beta) + (P - \beta)^2 + (P - \beta)^3 + \dots + (P - \beta)^n) = N_0 * (1 - (P - \beta))^{-1} \tag{3.3}$$

In this expression $\beta < 1$ and it describes a probability of electron multiplication suppression along its free flight. From expressions (3.1) and (3.3) we get

$$M = (N_x / N_0) = M_0 / (1 + \beta * M_0), \text{ where } M_0 = (1 - P)^{-1} \tag{3.4}$$

Therefore, we see that with this simple model the physical sense of the feedback coefficient is obvious. It describes a probability of electron multiplication suppression in its free flight.

It is to imagine a situation when to account for the multiplication probability of not only electrons but also holes, we upgrade the expression (3.3) so as $P=P_e+P_h$ and $\beta=\beta_e+\beta_h$, while P_e, P_h are multiplicity probabilities for electrons and holes in their free flight length. In this case $N_x=N_e+N_h$.

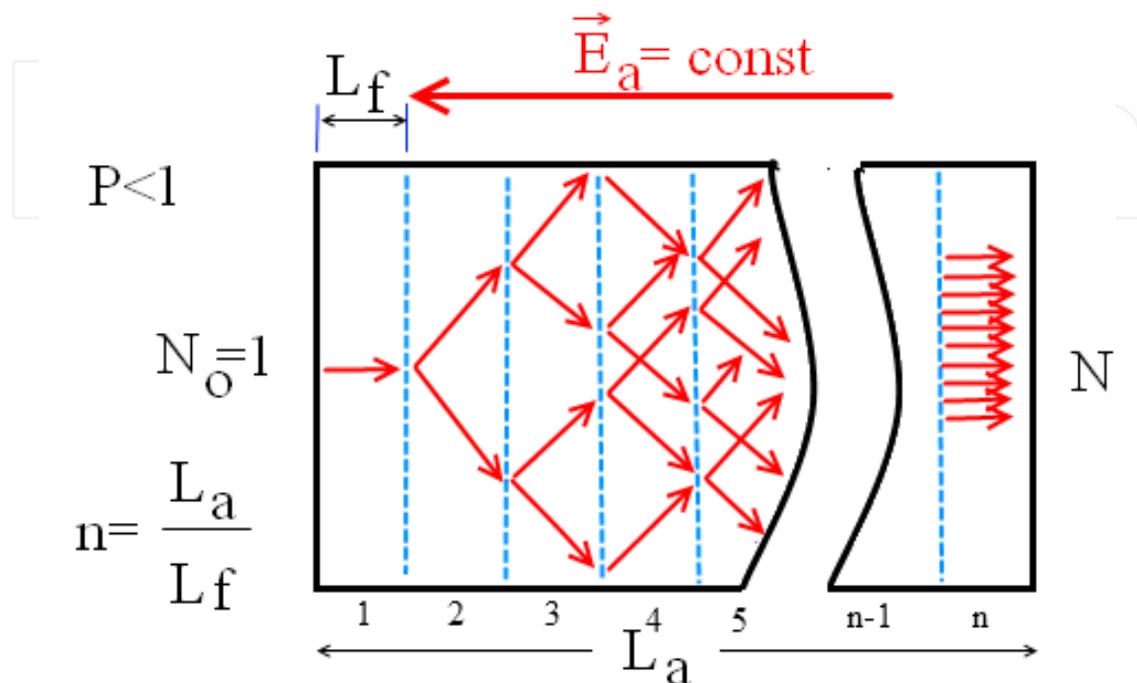


Fig. 3.1. Illustration of Probability model for avalanche process.

This simple model presupposes independence of multiplication processes of electrons and holes. It is not easy to implement it in practice.

$$N_x=N_e+N_h=N_{0e}*(1-(P_e-\beta_e))^{-1} + N_{0h}*(1-(P_h-\beta_h))^{-1} \quad (3.5)$$

We would like to note that it is the first illustration of moving from a certain definite (in this case, simple probability) model for avalanche photo-detectors to further interpretation of this model in the frames of FBT.

We shall discuss in more detail some not very obvious statements of the suggested model. First, we suppose that all electron multiplication events are independent as a whole. The second supposition comes from the sum row notion in the form of the algebraic expression (3.2). This expression has uncertainty at $P=1$. It leads to a natural restriction $P<1$ and shows that the multiplication probability can be as close to the entity but cannot be equal to it. Both statements are not significant for the main conclusion - the result of the mathematical description of the avalanche generation process can be presented in the form of a formula for the coefficient of gain of the amplifier with feedback after some identical transformations, and, consequently, we can apply it for the analysis of the operation of FBT avalanche photo-detectors. To illustrate some advantages of the interpretation, we consider (3.4) in the case $M_0 \gg 1$. Then $M \sim 1/\beta$ and, consequently, the multiplication factor is definitely determined by the feedback coefficient β . In other words, the avalanche suppression processes in APD define the multiplication factor. As such requirements correspond to the operation principle by convention Geiger Mode Avalanche Photo Diode

(GAPD) [4], the probability interpretation of multiplication coefficient has quite a definite meaning. Namely, as GAPD operates in the region where the field intensity is critical and $P \sim 1$, the avalanche suppressing should be defined with a certain internal process whose probability should be practically constant to provide for low noise of the detector. The cell structure of GAPD is the simplest and most natural way to develop them, as the cell dimensions and the intensity of the electric field in it define the maximal amplifying. The smaller is the dimension of the cell the better are noise properties of GAPD due to electron fluctuations decrease in it. Further, we will continue the discussion of this subject in the phase of more accurate mathematical calculations.

4. Physical model of the avalanche multiplication

Let us discuss a classical one-dimensional system of continuity equations for P-N transition shown in Fig. 4.1 [5]

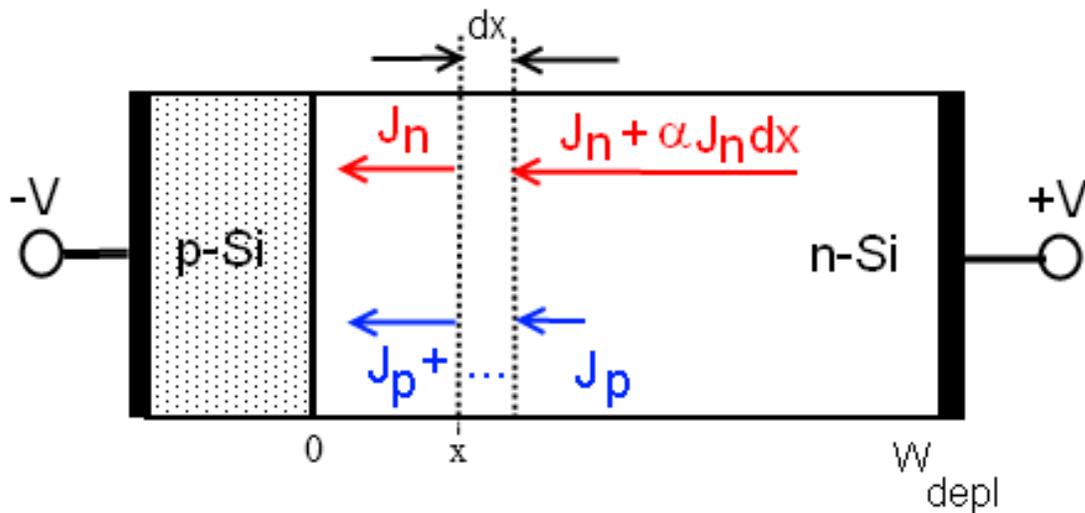


Fig. 4.1. Avalanche multiplications of electrons in high electrical field region of P-N junction.

$$\frac{dJ_n}{dx} = \frac{dJ_p}{dx} = \alpha_n \cdot J_n + \alpha_p \cdot J_p \tag{4.1}$$

at boundary conditions

$$\begin{aligned} J_n(0) &= J_{no} & J_p(0) &= J - J_{po} \\ J_n(W_{depl}) &= J - J_{po} & J_p(W_{depl}) &= J_{po} \\ J(x) &= J_n(x) + J_p(x) = \text{const} \end{aligned}$$

The solution of this system will be interesting for us in the form of multiplication factor $M_n = J_n / J_{no}$ for the case of purely electronic injection.

$$M_n = \frac{1}{1 - \int \left(\alpha_n \cdot \exp \left(- \int (\alpha_n - \alpha_p) dx' \right) \right) dx'} \tag{4.2}$$

if M_{no} is written in the form

$$M_{no} = \frac{1}{1 - \int (\alpha_n \cdot \exp(-\int (\alpha_n) dx)) dx'} \quad (4.3)$$

It is possible to represent (4.2) in the following form

$$M_n = \frac{M_{no}}{1 + \beta_n \cdot M_{no}} \quad (4.4)$$

In this expression the feedback coefficient is in the form

$$\beta_n = \int \alpha_n \cdot e^{-A(x)} \cdot (1 - e^{-B(x)}) dx \quad (4.5)$$

where

$$A(x) = \int \alpha_n dx \quad \text{and} \quad B(x) = \int \alpha_p dx$$

We can easily obtain the expression for the avalanche multiplication factor in the case of purely "holes" injection, simply changing n to p in formula (4.4) and vice versa. This formal transformation, as it may seem, of a well-known expression makes it possible for us to regard electron and hole multiplication from the FBT point of view.

Let us study some limit cases for formula (4.5).

Firstly, the trivial case $e^{B(x)} = 1$ means that there is no feedback or $B(x) = \int \alpha_p dx = 0$ - there is no multiplication of the hole component. On the other hand, the presence of the holes multiplication process is a factor that means the presence of positive feedback. This fact easily shows the case $e^{B(x)} \gg 1$ when we get from formula (4.5)

$$\beta_n = -\int \alpha_n \cdot e^{-(A(x) - B(x))} dx \quad (4.6)$$

In the obtained expression, the minus mark indicates the feedback type - namely positive feedback, and the ratio of two components of the exponent shows the FB depth.

Concerning the restrictions on the transfer from representation (4.2) to representation (4.4), they are connected with the necessity to follow the implementation of requirement

$$\int (\alpha_p \exp(-\int \alpha_n) dx) dx' \neq 1 \quad (4.7)$$

For FB coefficient the following condition should be implemented

$$\beta_n \cdot M_n > 1 \quad \text{or}$$

$$0 \leq \int \alpha_n \cdot e^{-(A(x) - B(x))} dx < \frac{1}{M_{no}}$$

Let us consider the character of possible behaviour of the feedback coefficient in the region of avalanche multiplication. We differentiate (4.6) in coordinate. As a result we get three variants of behaviour.

Case $A(x) > B(x)$ $d\beta_n/dx$ – the rate of hole multiplication in the direction of differentiation gradually decreases.

Case $A(x) = B(x)$ $d\beta_n/dx$ – the rate of hole multiplication in the direction of differentiation is constant.

Case $A(x) < B(x)$ $d\beta_n/dx$ – the rate of hole multiplication in the direction of differentiation gradually increases.

Thus, having done a formally identical transformation of the initial equation, we get an opportunity to mark three characteristic cases for positive FB. The method discussed above will be further applied and allow us to make a clear and generalizing classification of various APD types.

There two reasons that show that there is no need to analyze deeper the obtained expressions. Firstly, our main aim is an acquaintance with a possibility itself to interpret an avalanche process on various levels of abstraction and for various models. Secondly, further studies will have sense if we use a specific APD topology. There exists a simpler model of FBT for avalanche detectors to make general description of some common properties of APD. This model allows one to use a more illustrative and simpler approach, from the point of practical application of APD.

5. Miller's formula

It is well-known in APD applications that in practice the use of the physical model to describe a specific APD will be too complex and far from informative. That is why various empiric or semi-empiric models are used. Miller suggested the first model of this type in 1955 [6]. It turns out that the new approach based on the identical transformation of the main formula of this model allows one to obtain sufficiently interesting results.

In the first variant of formula (5.1) the influence on avalanche amplification was not considered of the APD internal base and the contact resistance (the equivalent APD scheme is in Fig. 5.1a).

$$M_{no} = \frac{1}{1 - \left(\frac{V}{V_{br}}\right)^n} \quad (5.1)$$

Later, the formula was modified and presented in the form (5.2) for equivalent APD scheme shown in Fig. 5.1b.

$$M_n = \frac{1}{1 - \left(\frac{V - i \cdot R_{fb}}{V_{br}}\right)^n} \quad (5.2)$$

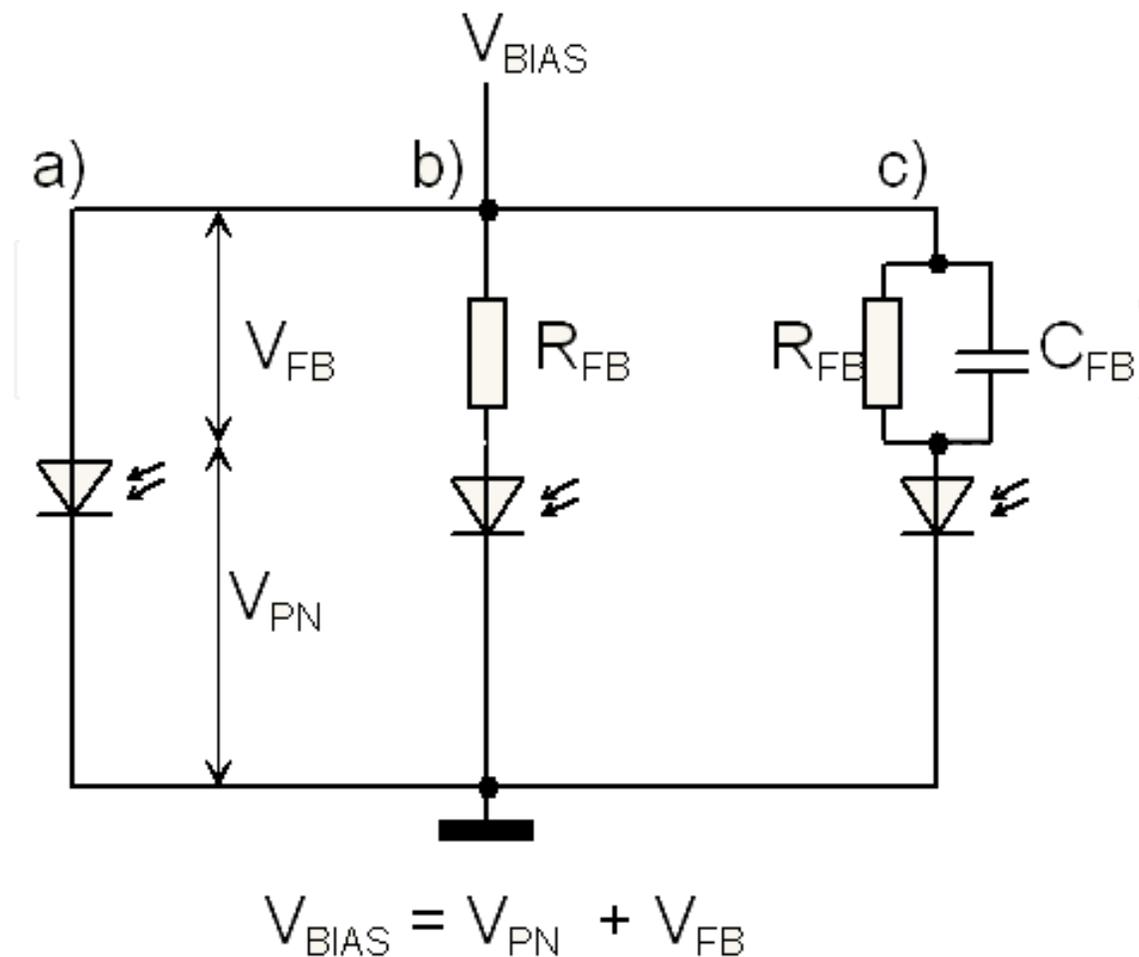


Fig. 5.1. The evolution of equivalent circuit in Miller's formula.

It turns out that the Miller formula can be transformed to the form of expression (2.0). Let us do transformation for formula (5.3) (the equivalent APD scheme is suggested in [7] and shown in Fig. 5.1c).

$$M_n = \frac{1}{1 - \left(\frac{V - V_{fb}}{V_{br}} \right)^n} \quad (5.3)$$

In the case under consideration the transformation has sense as the model exactly describes the APD operation with an internal FB. In (5.4) we show the feedback coefficient in the general form to indicate that the feedback value is defined by the change of the voltage V_{fb} . On the other hand, in this expression a significant FB difference is clearly seen if the FB resistance consists of purely active component and if it contains the reactive component [8].

$$M(V) = \frac{M_o(V)}{1 - \beta \cdot M_o(V)} \quad (5.4)$$

$$\text{here } \beta = \frac{V_{fb}}{V_{br}} = \frac{i \cdot R_{fb}}{V_{br}}$$

In the first case external chains of APD connection define the rate of the avalanche multiplication suppression; in the second case, the influence of the reactive component on the rate of the avalanche multiplication suppression is strong. Due to this, the suggested model allows one to describe accurately the operation of APD with internal local negative FB. The FB locality implies that the process of the avalanche suppressing is much quicker than the changes in the parameters of external chains of APD power supply. Thus, we can choose parameters C_{fb} and R_{fb} in such a way that we can accurately describe the APD behaviour with negative local FB. A step further is more complicated as it is necessary to connect the obtained distributed parameters with topological and technological parameters of the APD under study.

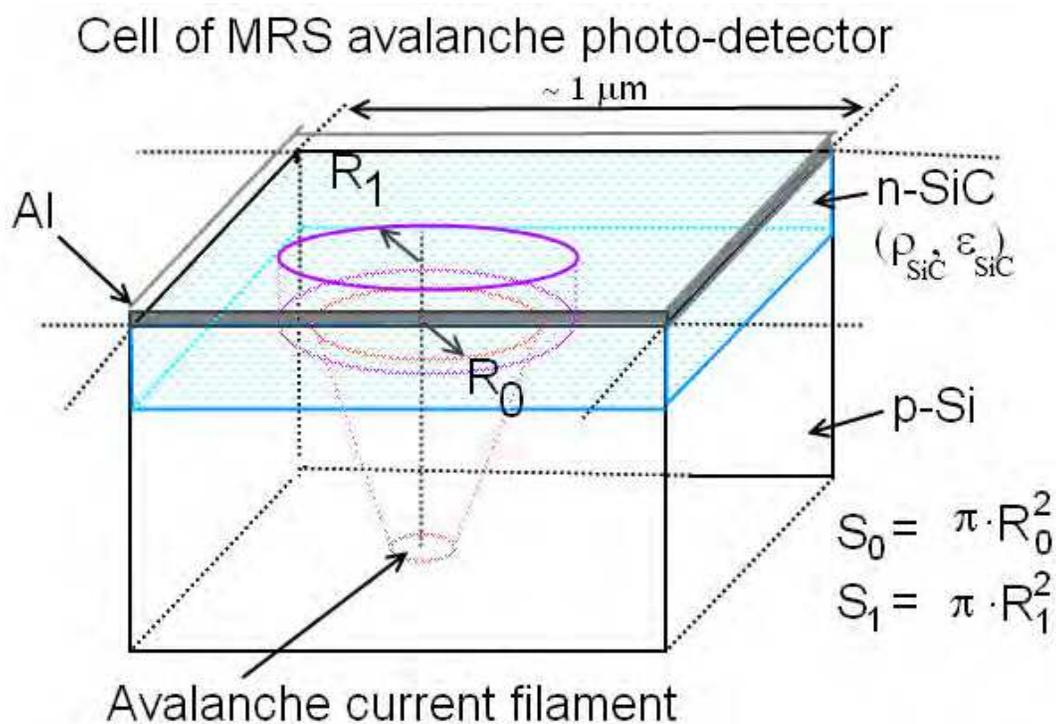


Fig. 5.2. Metal - Resistive Layer - Silicon (MRS APD) cell. The diameter of current filament must be change on boundary n-SiC - p-Si. ($\rho_{\text{SiC}} / \rho_{\text{p-Si}} \sim 10$)

In the discussed example (Figure 5.2) we can proceed from the fact that we have already known the parameters of the resistive layer (resistivity) ρ_{SiC} and area N^+ region S_N . The value C_{fb} is more significant for the description of the dynamic mode, while for the static one R_{fb} is more important, so we can expect that $R_{fb} = \rho_{\text{SiC}} \cdot \frac{d_{\text{SiC}}}{S_0}$ where S_0 determines the cross section area of the avalanche in stationary mode (avalanche current is constant).

To describe the dynamic mode, the value C_{fb} is more significant, while for the static mode R_{fb} is more significant. We can expect here that $R_{fb} = \rho_{\text{SiC}} \cdot \frac{d_{\text{SiC}}}{S_1}$ where S_1 determines the

dimension of the spreading area of the filament of the current on the boundary N_{SiC}-P_{Si}, proceeding from the term of achieving the current stationary when $M(t)=\text{const}$, and

$$C_{fb} = \frac{\varepsilon_0 \cdot \varepsilon_{SiC} \cdot S_1}{d_{SiC}}$$

where S_1 defines the maximum dimension of the avalanche current from which the avalanche suppression starts.

We can define a characteristic constant τ_{fb} (5.5) that we will call the time of feedback of the process of avalanche multiplication suppression. This value is determined by the APD technology and topology. It should be noted that the constant does not depend on the resistive layer thickness, but does depend on the spreading area cross-section and the thickness of the avalanche current area cross-section.

$$\tau_{fb} = \varepsilon_0 \cdot \varepsilon_{SiC} \cdot \rho_{SiC} \cdot \frac{S_1}{S_0} = \tau_{SiC} \cdot \frac{S_1}{S_0} \quad (5.5)$$

It seems obvious that for local negative FB it is necessary to implement the condition $\tau_{fb} \geq \tau_{av}$, namely, the time of FB suppression must be a little longer than the time of the avalanche generation τ_{av} . If we consider the model of the abrupt P-N junction and presuppose that the avalanche region $W_{av} \sim 0.1$ of the thickness of the depletion region W_{depl} and the velocity of the electron movement in the multiplication region V_{sat} we get an evaluation in the form (5.6).

$$\varepsilon_0 \cdot \varepsilon_{SiC} \cdot \rho_{SiC} \cdot \frac{S_1}{S_0} \geq \frac{W_{av}}{V_{sat}} \quad (5.6)$$

As V_{br} is depend on the silicon parameters [9], and $V_{sat}=\text{const}$, we get the evaluation (for the n-SiC/p-Si structure) of the minimal concentration of acceptors in p-Si in the form (5.7).

$$W_{av} \leq V_{sat} \cdot \tau_{SiC} \frac{S_1^2}{S_0^2} \quad (5.7)$$

$\tau_{SiC} = \varepsilon_0 \cdot \varepsilon_{SiC} \cdot \rho_{SiC}$ – Maxwell time for n-SiC.

This implies (for simplicity we suppose that $S_1=S_0$) that at $\rho_{SiC}=156$ [ohm/cm] the acceptors concentration in silicon must be lower $N_p < 2 \cdot 10^{15}$ 1/cm³. But at the same time the concentration cannot be too low. Otherwise FB will not have time to suppress locally the avalanche production process. If we take into account that an agreement of this kind is necessary for each new value ρ_{SiC} and, besides, that in the general case $S_1 \neq S_0$ depends on the impurity concentration and conditions on the border n-SiC/p-Si, it becomes clear why the suggested FB mechanism was implemented by only one research group [10].

The presented derivations are one more example of FBT application for the description of APD operation. In the Chapter 6 that considers examples of application of the models based on FBT ideas we discuss the model of the n-SiC/p-Si structure in more detail.

Let us regard the general analysis of formula (5.4)[11]. We mark that in the first approximation we thought that $n=1$. If we take the general case $n>1$, the feedback coefficient must be presented in the form (5.8)

$$\beta = \frac{V^n - (V - V_{br})^n}{V_{br}^n} \tag{5.8}$$

We study the character of behaviour of the FB coefficient depending on the voltage changes V .

If we define the avalanche gain efficiency coefficient in the form

$$G_{ef} = \frac{dM_0}{dV} - M_0^2 \cdot \frac{d\beta}{dV} \text{ the derivative on } V \text{ from (4.4) will be in the form (5.9)}$$

$$\frac{dM}{dV} = \frac{G_{ef}}{(1 - \beta \cdot M)^2} \tag{5.9}$$

If we define the FB efficiency parameter in the form (5.10) it is possible to outline three characteristic cases of FB demonstration in APD in Fig. 5.3.

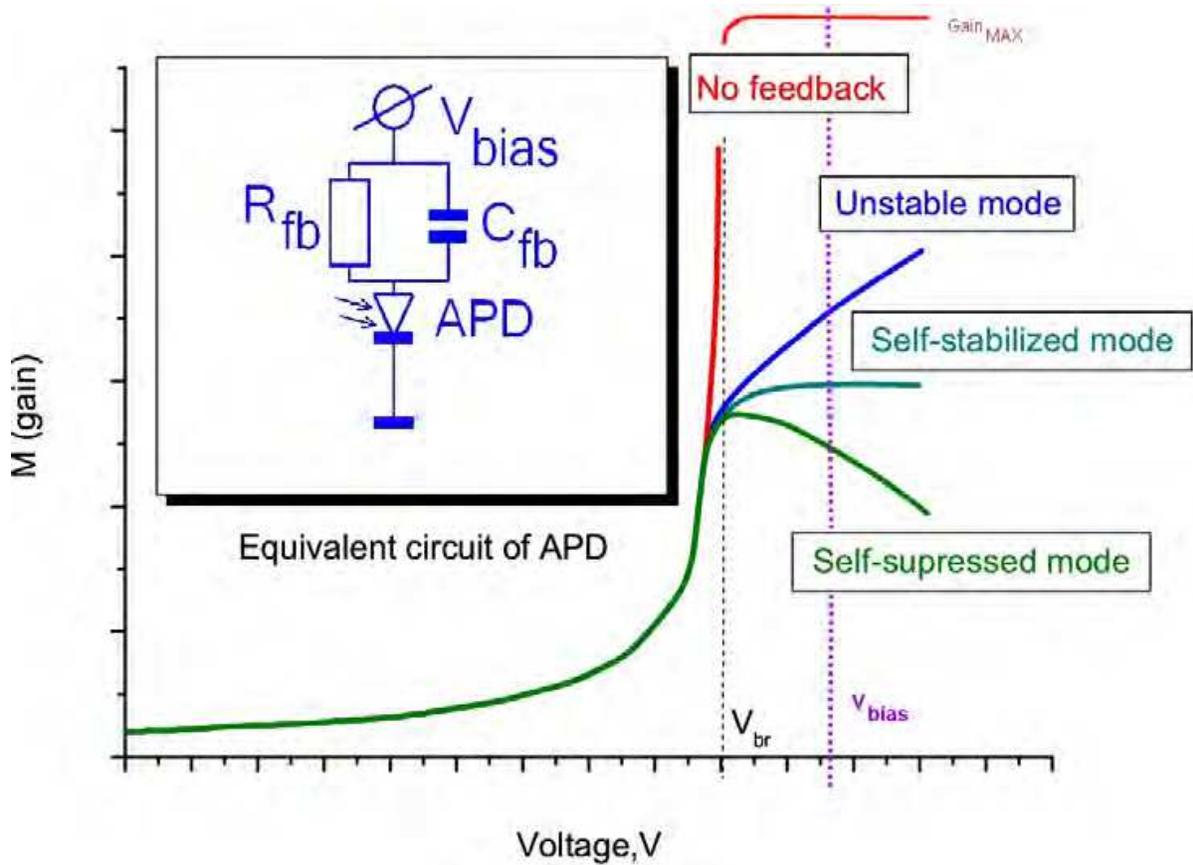


Fig. 5.3. Classification of APD using model of FB.

$$K_{ef} = \frac{\frac{d\beta}{dV}}{1 - \frac{dM_0}{dV} \cdot M_0} \tag{5.10}$$

$K_{ef} < 1$ - with the growth of voltage the feedback increases slower than the avalanche gain that corresponds to the APD operation in the unstable mode;

$K_{ef} = 1$ - with the growth of voltage the feedback increases as quickly as the avalanche gain that corresponds to the APD operation in the self stabilization mode;

$K_{ef} > 1$ - with the growth of voltage the feedback increases quicker than the avalanche gain that corresponds to APD operation in the self-suppressing mode.

A classification of this type allows us to define the character of the FB behavior. For some particular cases $\frac{d\beta}{dV} \approx \frac{1}{M_0} \cdot \frac{dM_0}{dV}$ for example we can state:

$\frac{d\beta}{dV} > 0$ FB coefficient monotonously grows with the voltage increase; thus, theoretically, there must exist voltage when, if higher than its value, relative changes of the avalanche gain $\frac{1}{M_0} \cdot \frac{dM_0}{dV}$ are lower than the FB introduced suppressing. Initially, the condition

$\frac{d\beta}{dV} < \frac{1}{M_0} \cdot \frac{dM_0}{dV}$ is implemented.

$\frac{d\beta}{dV} = 0$ The FB coefficient is constant with the voltage growth; therefore, theoretically there

must be voltage for which relative changes in avalanche gain $\frac{1}{M_0} \cdot \frac{dM_0}{dV}$ correspond to FB

introduced suppressing. Initially, the condition $\frac{d\beta}{dV} < \frac{1}{M_0} \cdot \frac{dM_0}{dV}$ is implemented.

$\frac{d\beta}{dV} < 0$ The FB coefficient monotonously decreases with the voltage growth; thus, theoretically there must exist voltage when, if high than its value, relative changes of the avalanche gain $\frac{1}{M_0} \cdot \frac{dM_0}{dV}$ become higher than the FB introduced suppressing. Initially, the

condition $\frac{d\beta}{dV} > \frac{1}{M_0} \cdot \frac{dM_0}{dV}$ is implemented.

All the above stated, despite the fact that only cases of quite particular character were considered, can further assist in sorting out correctly the main mechanism of FB for each specific case. And this will be the first and basic step how to connect experimental data with the APD physics, topology and technology.

6. Examples of FBT application for APD

6.1 Logistic model of front of avalanche in GAPD

In this chapter we consider model that describes the process of avalanche generating in GAPD. The peculiarity of the model is in the fact that we will not solve the Fundamental Equation System for semiconductors and will not actually use the Poisson equation. The dynamics of an avalanche process in GAPD is determined by two important characteristics of this detector. The first is the difference of the bias voltage and breakdown voltage and the

second is the dimension and structure of the GAPD cell. Let us discuss a GAPD cell as it is shown in Fig. 6.1.

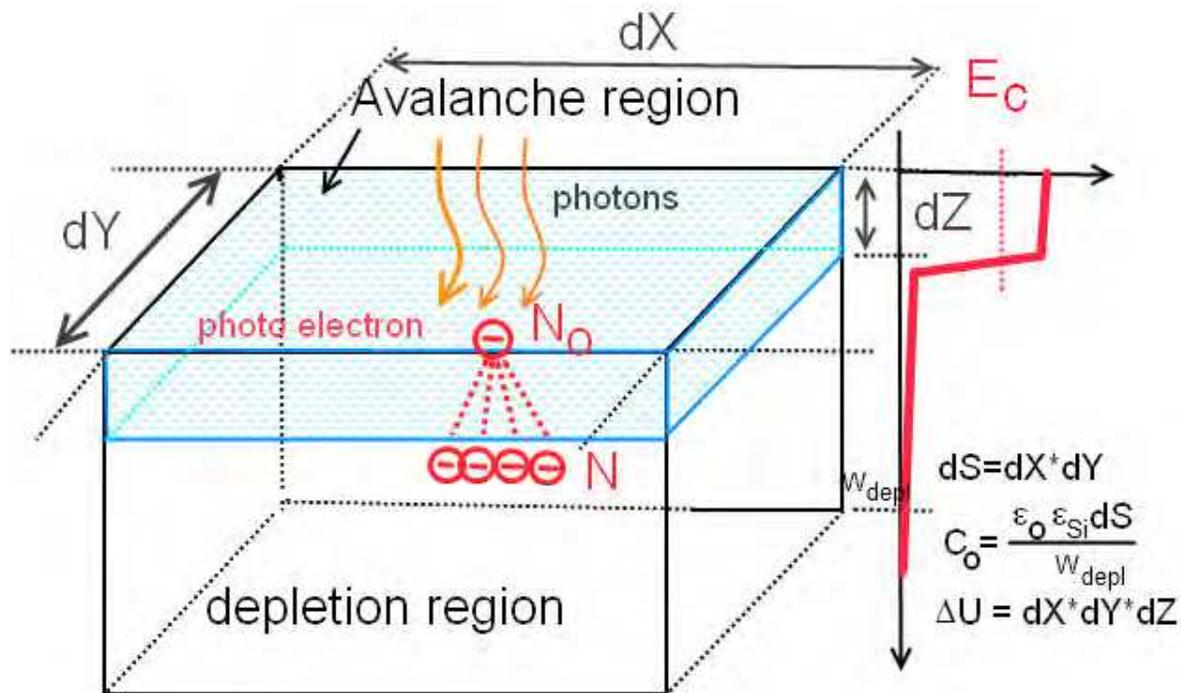


Fig. 6.1.1. Structure of GAPD for model.

The electric field in the cell is close or more to the critical one; therefore, the probability of the avalanche generation of a secondary carrier $P \sim 1$. Moreover, there exists a certain threshold value of photo-generated electrons N_0 necessary for the start of the avalanche process. Thus, to suppress the avalanche in cell from thermal noises for the thermo-generated carriers N_T the $N_T < N_0$ inequation should be implemented. We can evaluate N_T supposing that the noise generates only by the detector dark current. This supposition will allow us evaluate theoretically the dependence of the size of cell on the concentration of the doping applied for the APD material. It is obvious that the higher is the doping concentration the lower is the maximal cell volume $\Delta \bar{U}$ at the same cell area. At backward bias voltage V_{bias} , the dark current J_d and equals to N_T will determine the concentration of minor carriers generated per time unit in the depletion region. For APD with a long base, presupposing that the dark current is defined by the volume component from [12] it follows

$J_d \approx \frac{e \cdot N_i \cdot \Delta U}{2 \cdot \tau_{bulk}}$ Let us consider to be definite a cell of the S area in the form of a abrupt P-N junction on silicon of the N type with the donor concentration N_d and depletion region size $W_{depl}(N_d, V_{br})$ at break-down voltage V_{br} .

$$N_T \approx \frac{N_i \cdot \Delta U}{2 \cdot \tau_{bulk} \cdot U_{sat}} \tag{6.1.1}$$

Consequently $N_0 > \frac{N_i \cdot \Delta U}{2 \cdot \tau_{bulk} \cdot U_{sat}}$ or $\Delta U = S \cdot W_{depl}(N_d, V_{br}) < \frac{2 \cdot \tau_{bulk} \cdot U_{sat}}{N_i}$

It should be noted that, if the threshold value N_0 is large, the avalanche probability is small and the front time increases. Big lifetime of the minor charge carriers in SCR (Space-Charge Region) makes it possible to produce GAPD cells of a larger area. After we have evaluated the maximal cell area let us consider the following model.

Let the maximal number of electrons in the cell available for multiplication be equal to N . At the moment of time t after the start of the avalanche multiplication process there are already x electrons; therefore, the number of electrons potentially available for multiplication will be $(N-x)$. If the length of the avalanche gain region is L_{av} and the velocity of electron in the electric field $U_{sat} = \text{const}$, (the time of front of avalanche in the order $T_{av} \sim L_{av}/U_{sat}$). The differential equation for the described situation is given in the form (6.1.2).

$$\frac{dx}{dt} = k \cdot x \cdot (N - x) \quad (6.1.2)$$

with the initial requirement $x(t=0) = N_0$

The solution of this equation is well known as a logistic equation [13].

$$x(t) = \frac{N}{(1 - a \cdot \exp(-k \cdot N \cdot t))} \quad (6.1.3)$$

where $a = \frac{N - N_0}{N_0}$

The constant $1/k$ must have time dimensionality, so we input a certain constant of the time of the avalanche front and define it on the basis of the dimension analysis.

$\tau_0 = \frac{1}{k} = T_{av} \cdot \frac{N_0}{N} = \frac{L_{av}}{V_{sat}} \cdot \frac{N_0}{N}$ (at $N_0=1$ $\tau_0 = T_{av}/N$ has a simple physical interpretation - it is the time of one multiplication act, and if the threshold number N_0 grows the time of one multiplication act becomes longer). Now we write (5.1.3) in the form

$$x(t) = \frac{N}{(1 - a \cdot \exp(-\frac{t}{\tau_0}))} \quad (6.1.4)$$

To transfer to the interpretation of the solution from the FBT point of view, we introduce new variables. $M(t) = x(t)/N_0$ - the avalanche gain coefficient at the moment of time t , $M_0 = N/N_0$ - maximal avalanche gain in GAPD.

We write (6.1.4) in the form

$$M(t) = \frac{M_0}{(1 + \beta(t) \cdot M_0)} \quad (6.1.5)$$

where $\beta(t) = (1 - \frac{1}{M_0}) \cdot \exp(-\frac{t}{\tau_0})$

The model given above describes the exponential change of the FB factor from the maximal value $\beta_{\max} = (1 - \frac{1}{M_0})$ to zero that corresponds to the maximal current through GAPD.

In Fig. 6.1.2 we show the results of calculations of the avalanche front line for different values of N_0 . The lower are the threshold values the faster augment the avalanche front. It corresponds to the idea that higher probability of the charge carrier ionization in APD corresponds to the fast front of the avalanche process.

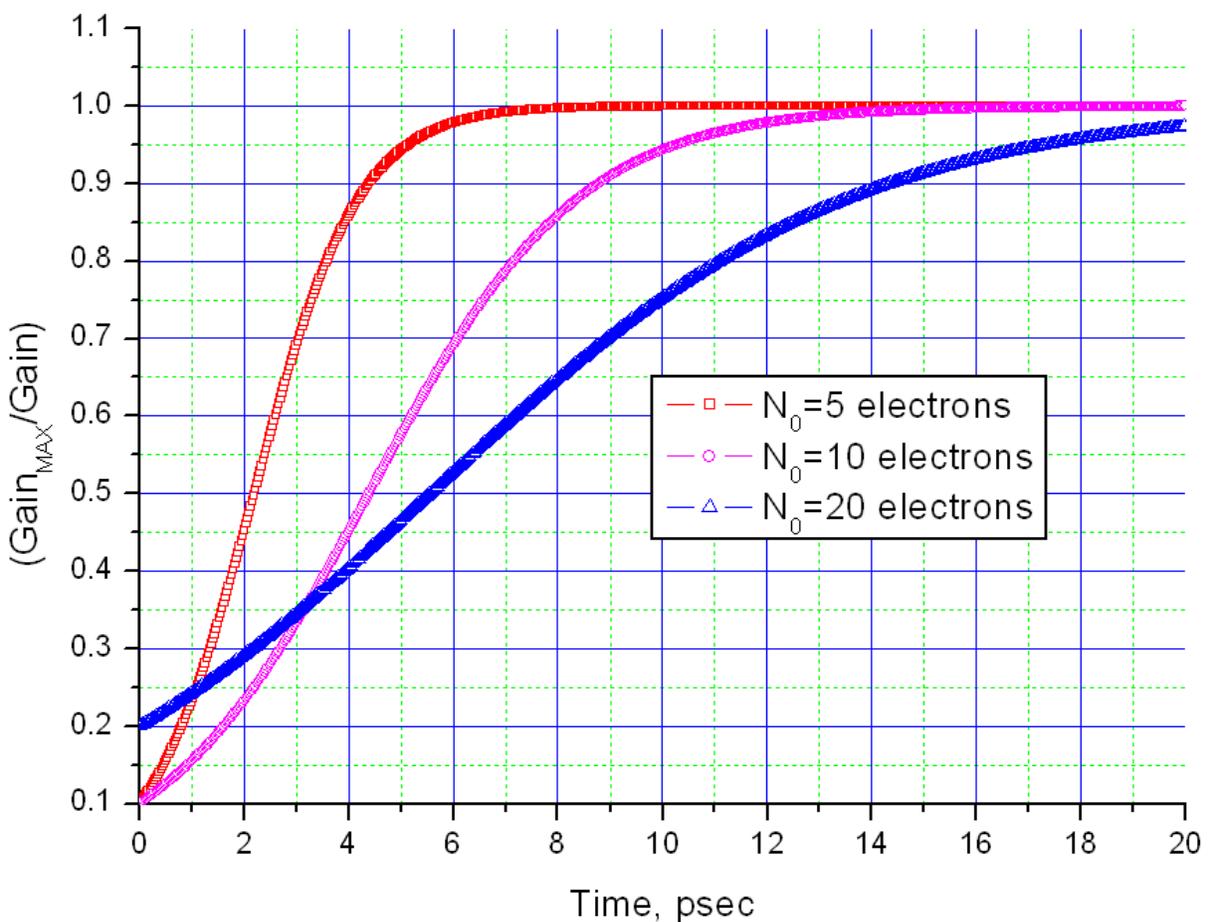


Fig. 6.1.2. Simulation of raising current in avalanche versus time by using Logistic model

6.2 GAPD Analysis. FBT interpretation

Paper [11] suggests a model where the feedback factor was given in the form $\beta = \frac{V_{fb}}{V_{br}}$.

Proceeding from the supposition that in GAPD the voltage should change by the value $V - V_{br}$ with the avalanche suppressing due to the voltage drop on the amplifying cell to suppress the avalanche, we think it is the very similar FB voltage

$V - V_{br} = V_{fb}$. Let the FB factor be equal to $\beta = V_{fb} / V_{br}$. We consider the charge gain in the GAPD cell whose capacity is C_0 . The cell discharge charge is $Q_0 = C_0 * V_{fb}$. If the threshold number of electrons N_0 , the avalanche gain coefficient of the charge $M_x \sim Q_0 / e * N_0 = C_0 * V_{fb} / e * N_0$. On

the other hand, we can define the voltage of a separate gain at $V=V_{br}$ when $\beta \sim 1$, so the choice $\beta = \frac{V_{fb}}{V_{br}}$ will be justified if M_0 is chosen in the corresponding way. From the expression (6.2.1) we obtain the value M_0 .

$$M = \frac{C_0 \cdot V_{fb}}{e \cdot N} = \frac{M_0}{1 + \frac{V_{fb}}{V_{br}} \cdot M_0} \quad (6.2.1)$$

The result of the solution is given by way of the charge gain coefficient.

$$M_0 = \frac{C_0 \cdot V_{fb}}{e \cdot N_0 - C_0 \cdot V_{fb} \cdot \frac{V_{fb}}{V_{br}}} = \frac{Q_0}{Q_i + \frac{V_{fb}}{V_{br}} \cdot Q_0} \quad (6.2.2)$$

In this expression Q_0 is an intensified charge from the cell and Q_i is the initial charge that switches the avalanche in GAPD. Thus, we have managed to present the FB factor in the desired form and, at the same time, keep the general kind of the expression for the avalanche gain in the form $M = \frac{M_0}{1 + \beta \cdot M_0}$.

Let us discuss now the feedback model when the avalanche restrictions happen due to limitation of the charge available for the avalanche multiplication in the cell. As the main parameter is the type and concentration of GAPD material doping, we will try to connect our abstract model with silicon parameters. The maximal charge available for multiplication in the cell is defined by the cell and SCR dimensions and equals to $Q_{max} = C_0 \cdot V_{fb} \sim N_a^{1/2}$ (we suppose that the basic material is silicon of the N type with donor concentration N_d). The possible charge of the avalanche gain Q_{av} is defined by the electric field intensity in the gain area, $Q_{av} = e \cdot N_0 \cdot M \sim N_a^m$ where $m > 1.2$ than the length of the avalanche region.

As the change of the field intensity per length unit in low-ohm material is higher than in high-ohm material, the rate of the avalanche gain change in the low-ohm material in the SCR avalanche part is higher and the avalanche generates in smaller space. It leads to its greater instability and imposes strict requirements to the time of FB establishment, as it was indicated above (formula (5.7)). In this case the FB mechanism due to the limitation of the charge available for gain in the cell is more preferable.

There should exist conditions when FB due to charge quantity limitation in the cell is possible and this condition will be $Q_{av} \geq Q_{max}$. In carrying out this inequation, the maximal charge is defined by the cell geometry and not by electric field intensity in it. Meeting this requirement will allow one to develop GAPD with high field intensity without fear of destroying the cell with high currents due to local instability of the avalanche process.

Let us study a simple model shown in Fig. 6.1 the abrupt P-N junction on the N-type silicon. If $N_d \gg N_a$ the dimension W_{depl} of SCR at the bias voltage equal to breakdown voltage is known and the maximal charge accumulated in the cell at the excess of breakdown voltage by $\Delta V = V - V_{br}$ is defined from equation (6.2.3). Let us suppose that the region of the

avalanche multiplication is $W_{av} \sim 0.1 \cdot W_{depl}(V_{br})$, and R_o - is the dimension of the region of avalanche current filament. The maximal cell charge is:

$$Q_{max} = C_0(N_d, V_{br}) \cdot \Delta V \approx \frac{\epsilon_0 \cdot \epsilon_{Si} \cdot S}{W_{depl}(V_{br})} \cdot \Delta V = \frac{\epsilon_0 \cdot \epsilon_{Si} \cdot \pi \cdot R_o^2}{\sqrt{\frac{2 \cdot \epsilon_0 \cdot \epsilon_{Si} \cdot V_{br}}{e \cdot N_d}}} \cdot \Delta V \tag{6.2.3}$$

To calculate the maximally possible charge of the avalanche gain let us suppose that the electric field in the region of the avalanche gain W_{av} is constant $E = \text{const}$ and the ionization coefficient $\alpha(E)$ is known. We think that an avalanche appears as a micro-plasma current filament of a constant and independent of material thickness and $S_0/S_1 = 1$ (see chapter 6). Then, the maximally achievable charge of the avalanche multiplication can be calculated from equation (6.2.4).

$$Q_{av} \approx e \cdot N_0 \cdot \alpha(E) \cdot S_1 \cdot W_{av} = e \cdot N_0 \cdot \alpha \cdot \left(-\frac{E_c}{E}\right) \cdot S_1 \cdot W_{av} \tag{6.2.4}$$

In the expression α_o , E_c the model parameters and e - is the electron charge. Let us suppose that the field intensity E is equal to the maximal for the given PN junction at voltage $V = V_{br}$.

$$E = E_{max} = \frac{e \cdot N_d \cdot W_d(N_d, V_{br})}{\epsilon_0 \cdot \epsilon_{Si}}$$

As a result, we obtain expression (6.2.5)

$$Q_{av} = e \cdot N_0 \cdot \alpha \cdot \left(-\frac{\epsilon_0 \cdot \epsilon_{Si} \cdot E_c}{e \cdot N_d \cdot W_d(N_d, V_{br})}\right) \cdot S_1 \cdot W_{av}(N_d, V_{br}) \tag{6.2.5}$$

where $W_{av}(N_d, V_{br}) \approx 0.1 \cdot \sqrt{\frac{2 \cdot \epsilon_0 \cdot \epsilon_{Si} \cdot V_{br}}{e \cdot N_d}}$

In Fig. (6.2.1) and (6.2.2) calculation results are presented in the given formulas (6.2.3) and (6.2.5) for $N_o = 1$, $R_o = 2, 3, 5, 7, 10, 15$ μm , $S_1 = 1 \text{ mkm}^2$ and $\Delta V = 0.1, 0.25, 0.5, 1.0, 2.0, 3.0, 5.0$ V. Although the dimension of the current filament is not taken into account in calculations of the dependence that shows the maximal charge change of the avalanche gain $Q_{av}(N_d, W_{av}(N_d, V_{br}))$ achievable in multiplying, while it is easy to make the corresponding correction, the main thing is to connect it rightly with real GAPD topology and technology. The observed decrease of the charge at high N_d concentrations is explained by the effect of decrease $W_{av}(N_d, V_{br})$.

In Fig. (6.2.1) the red line shows how the maximal charge $Q_{av}(N_d, W_{av}(N_d, V_{br}))$ achievable in multiplying changes if the donor concentration in the base silicon changes in the limits from 10^{12} cm^{-3} to 10^{16} cm^{-3} , and the straight lines show how the available charge in the cell changes $Q_{max}(N_d, W_{av}(N_d, V_{br}, R_o))$ at various values of the avalanche region radius R_o and $\Delta V = 1$. The region where the feedback mechanism operates to restrict the avalanche at the expense of the limitation of the charge available for gain in the cell is defined by the inequation $Q_{av} \geq Q_{max}$.

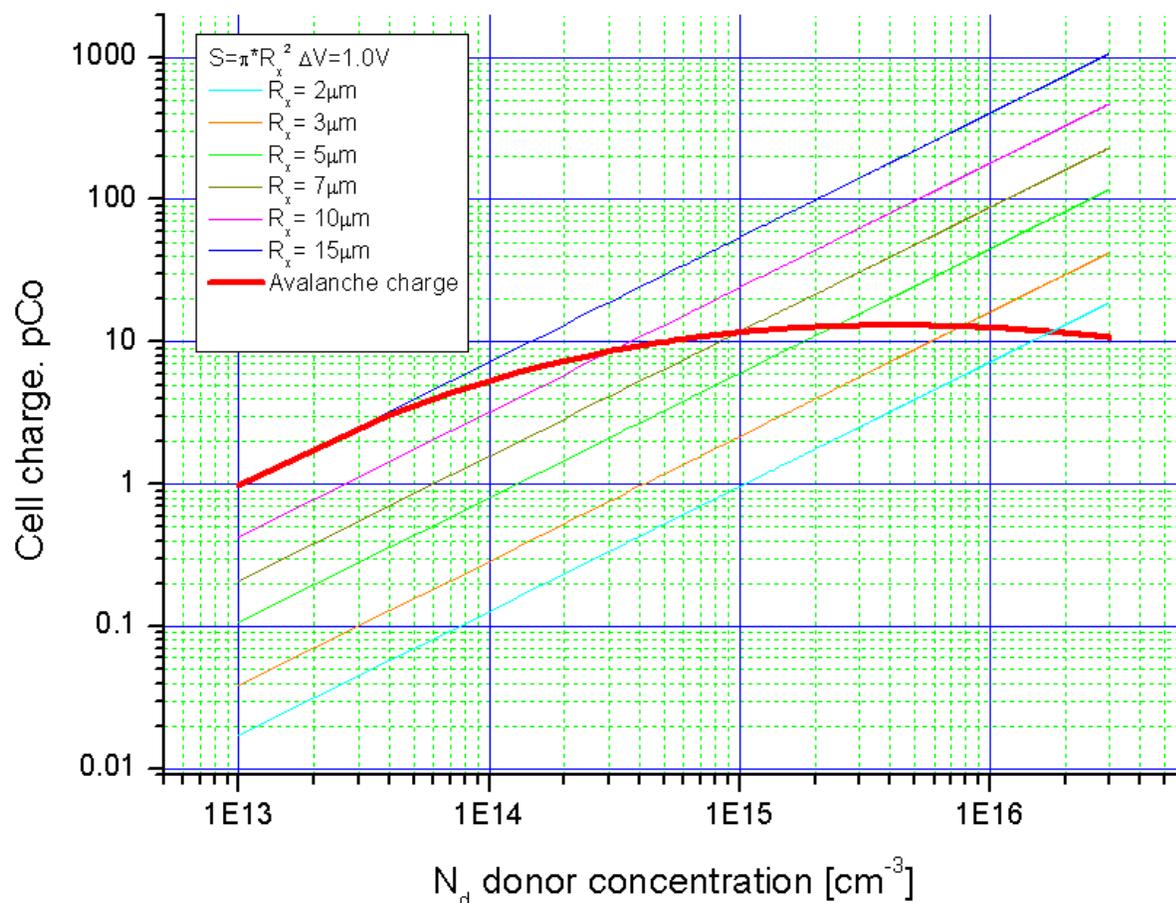


Fig. 6.2.1. The selection optimal size of cell for GAPD. The concentration of donors in cell must be selected near cross point of red line of maximum avalanche charge and the lines of different radius of cell.

In Fig. (6.2.2) the red line shows how the maximal charge $Q_{av}(N_d, W_{av}(N_d, V_{br}))$ changes if it is attained in multiplying and the donor concentration in the base silicon changes in the limits from 10^{12} cm^{-3} to 10^{16} cm^{-3} , while the straight lines show how the charge available in the cell changes $Q_{max}(N_d, W_{av}(N_d, V_{br}, R_o))$ at various voltages ΔV if $R_o=5$. The region of application of the feedback mechanism to restrict the avalanche at the expense of the limitation of the charge available for gain in the cell is defined by the inequation $Q_{av} \geq Q_{max}$.

Let us discuss an example of application of the obtained dependences. If we choose silicon with $N_d=10^{16} \text{ cm}^{-3}$ at the expected $\Delta V=1$ the size R_o of the charge region available for multiplication should be less than $3 \mu\text{m}$ (for example, $2 \mu\text{m}$) and the mode is implemented when the maximal avalanche gain is restricted by the cell geometry and not by the intensity of the cell electric field. At $\Delta V=2$ the size R_o of the charge region available for multiplication should be less than $2 \mu\text{m}$. In another Figure we see that if we choose silicon with $N_d=10^{16} \text{ cm}^{-3}$ at the chosen $R_o=5$ the excess voltage ΔV for the charge multiplication should be less than 0.5 (for example, 0.25); if $R_o=2$ we can choose $\Delta V=2$ but not larger. The discussed examples show that the implementation of the restriction mode of the charge in the cell is quite complicated due to the complex interconnections of many parameters. It is necessary

to calculate the GAPD construction from the point that the emerged avalanche will discharge the cell to the full extent and switch off due to the carriers' deficit, with no unstable unlimited by anything avalanche process. In practice, the discussed above mechanisms of avalanche quenching are implemented simultaneously, both in the form of decreasing the voltage in the cell and achieving depletion of free multiplication carriers in the avalanche region. To answer this question it is necessary to conduct more elaborate studies.

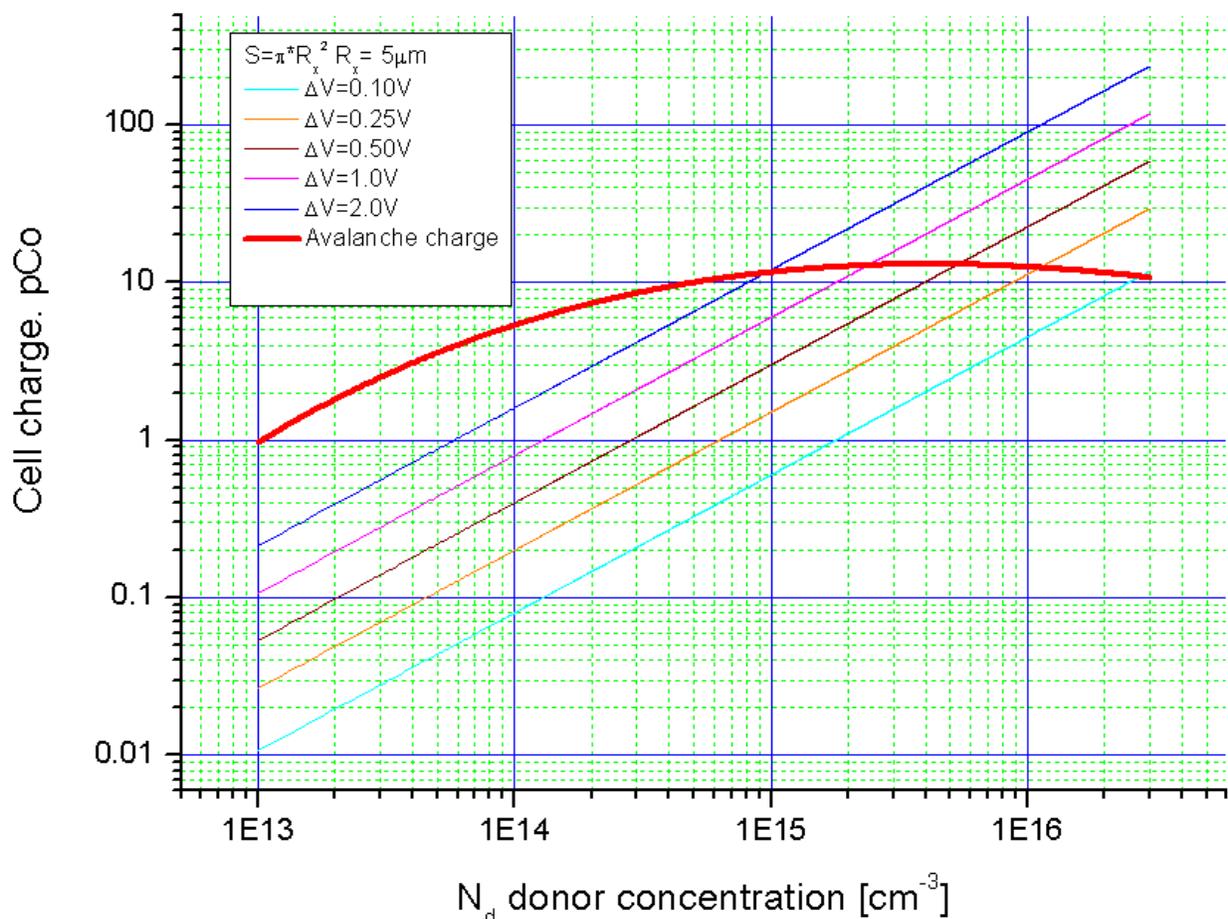


Fig. 6.2.2. The selection optimal over voltage $\Delta V = (V_{\text{bias}} - V_{\text{br}})$ for GAPD. The red line is the maximal avalanche charge can be the limit for over voltage for given GAPD with fixed size and concentration donor in cell.

6.3 APD equivalent scheme – SPICE model

A model to shape an avalanche front in APD on the basis of n-SiC/p-Si P-N junction is suggested for application in paper [11]. The form of the FB factor (its time dependence during the front shaping) was obtained on the basis of experimental data, with an initial supposition that the n-SiC layer properties determine the characteristic time of FB establishment.

The full equivalent schemes of MRS APD and Micro Channel Avalanche Photo Diode (MCAPD) are shown in Fig. 6.3.1. We are interested in one of its elements – a nonlinear

source of current $I(t)$ (APD) controlled by current (R_{fb} , D_{fb} , C_{fb}). The source amplifies the photocurrent $I_0(t)$ M times and by a certain time dependence of the FB factor $\beta(t)$ it correctly describes the shaping of the APD front.

Let us consider the method of definition of time dependence of the FB factor to be applied in the further suggested model. The method can be applied to linear APD, but not to GAPD.

Let the APD under study be under the effect of a rectangular light pulse that generates current $I_0(t)$ in APD. An intensified current pulse $I(t)$ can be given in the form

$$I(t) = I_0(t) \cdot \frac{M_0}{1 + \beta(t) \cdot M_0} \quad (6.3.1)$$

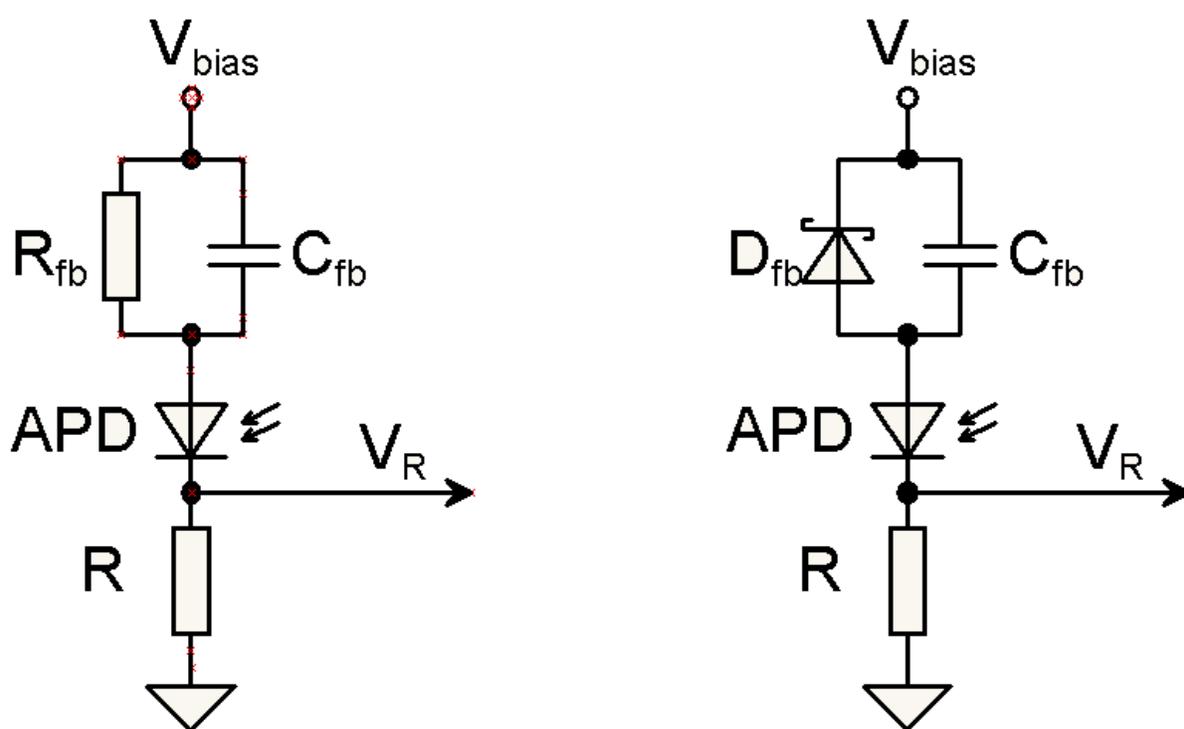


Fig. 6.3.1. Two different model were investigated for analysis by SPICE. APD - is source of current that controlled by voltage. R_{fb} - linear element of FB can be used for MRS APD [14], the D_{fb} is not linear element of FB and can be used for MC APD [15].

We suppose that $M_0 = \text{const}$ does not depend on time $M_0 \neq M_0(t)$ and only $\beta = \beta(t)$.

Then it is easy to obtain the dependence for the FB factor from (5.3.1)

$$\frac{I_0(t)}{I(t)} = \beta(t) + \frac{1}{M_0} \quad (6.3.2)$$

As we suppose that $M_0^{-1} = \text{const}$ any time dependence of the FB factor is obviously reflected by equation (6.3.2).

For test of method were defined of FB factors for APD of two types described in papers [14] and [15].

The dependences that correspond to the definition are presented in Table 6.1.1 as fragments of the SPICE code and the simulation result and its comparison with experimental data is given in Fig. 6.3.2 and 6.3.3.

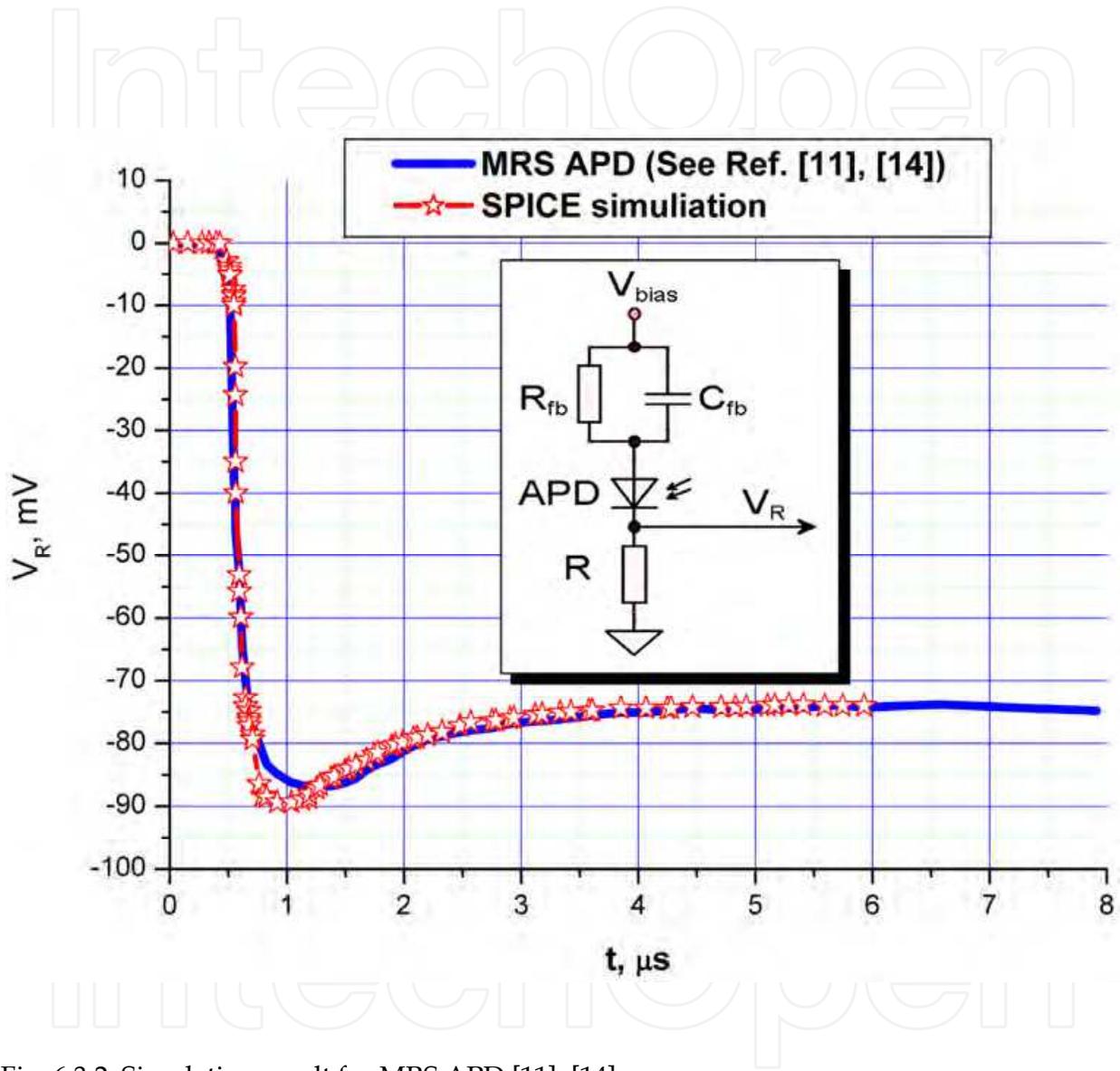


Fig. 6.3.2. Simulation result for MRS APD [11], [14].

Model MRS	$R_{fb}=1K$	$C_{fb}=1200pF$	$R=1K$	
Model MP APD	$C_{fb}=5pF$	$D_{fb}(CJO)=10pF$	$D_{fb}(BV)=100mV$	$D_{fb}(IBV)=20nA$

Table 6.1. (Parameters for simulation)

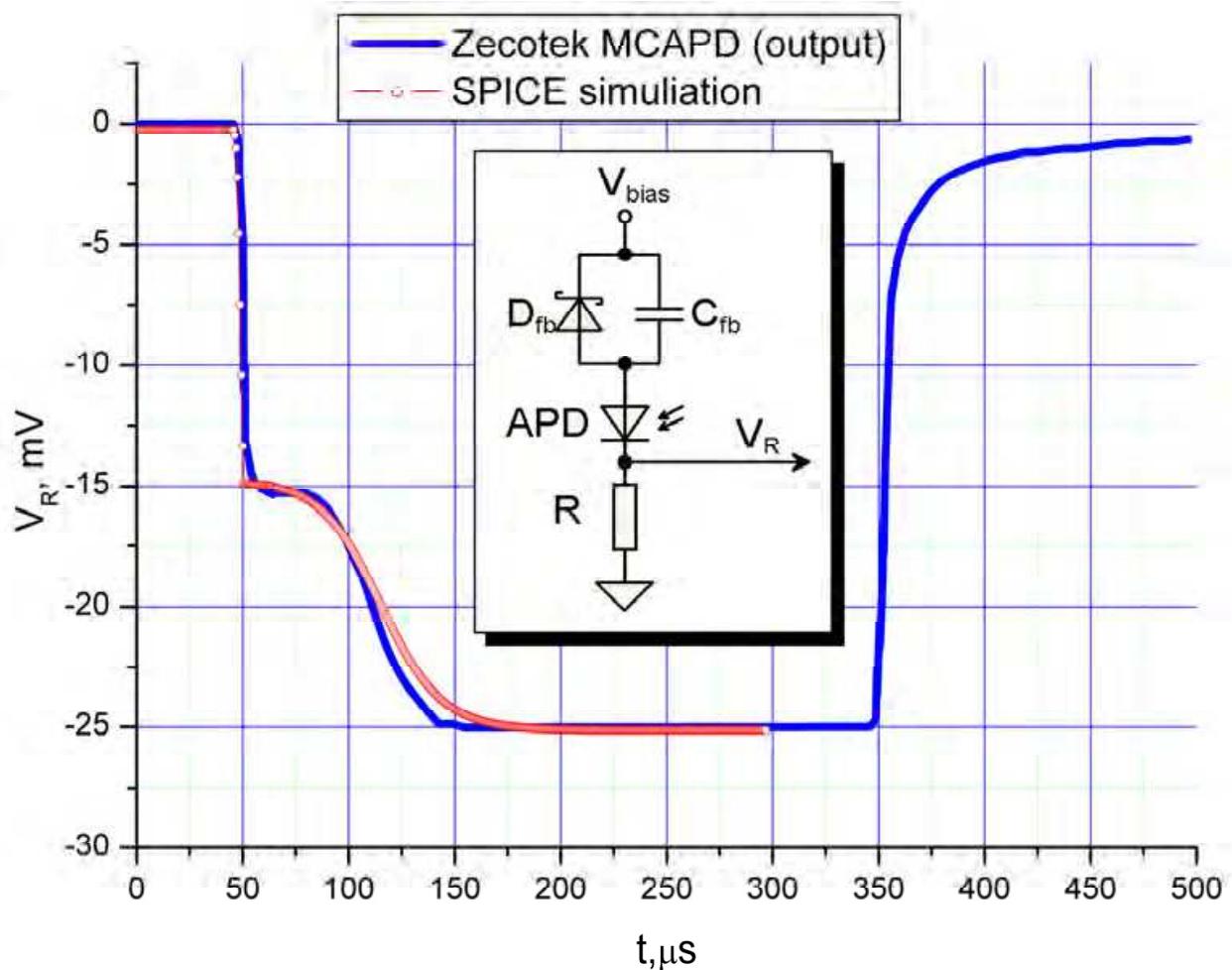


Fig. 6.3.3. Simulation result for MCAPD [15].

APD SPICE model:

```
.FUNC DPWR(D) {I(D)*V(D)}
.FUNC BPWR(Q) {IC(Q)*VCE(Q)+IB(Q)*VBE(Q)}
.FUNC FPWR(M) {ID(M)*VDS(M)}
.FUNC HOTD(D,MAX) {IF((V(D)*I(D)>MAX),1,0)}
.FUNC HOTB(Q,MAX) {IF((VCE(Q)*IC(Q)+IB(Q)*VBE(Q)>MAX),1,0)}
.FUNC HOTF(M,MAX) {IF((VDS(M)*ID(M)>MAX),1,0)}
.PARAM LOW3MIN={IMPORT(LOW3MIN.OUT,LOW3THRES)}
.PARAM HIGH3MAX={IMPORT(HIGH3MAX.OUT,HIGH3THRES)}
.PARAM LOWLVDS={IMPORT(LOWLVDS.OUT,LOWLIMIT)}
.PARAM HILVDS={IMPORT(HILVDS.OUT,HILIMIT)}
.PARAM LIMTLVDS={IMPORT(LIMTLVDS.OUT,LVDSLIMITS)}
.FUNC SKINAC(DCRES,RESISTIVITY,RELPERM,RADIUS) {((PI*RADIUS*RADIUS)/
((PI*RADIUS*RADIUS)-PI*(RADIUS-
SKINDEPTHAC(RESISTIVITY,RELPERM)**2))*DCRES}
.FUNC SKINDEPTHAC(RESISTIVITY,RELPERM)
{503.3*(SQRT(RESISTIVITY/(RELPERM*F)))}
.FUNC SKINTR(DCRES,RESISTIVITY,RELPERM,RADIUS,FREQ) {((PI*RADIUS*RADIUS)/
```

```

((PI*RADIUS*RADIUS)-PI*(RADIUS-
SKINDEPTHTR(RESISTIVITY,RELPERM,FREQ)**2))*DCRES}
.FUNC SKINDEPTHTR(RESISTIVITY,RELPERM,FREQ)
{503.3*(SQRT(RESISTIVITY/(RELPERM*FREQ)))}
BG1 0 PINA I = {V(IN)*POWER((40/(40-(V(0)-V(PINA))))),1)

```

7. Conclusion

The FBT approach allows to hide information about real physics processes in different types of APD. We can exchange a physical model describing a complex behavior of the system by a simple universal model with predictable results. The basic steps are in definition of main statistical parameters and their connections with external conditions of APD. Practically in all cases it is possible to implement a formal transfer from the physical model of an avalanche generation to the description in FBT. The formal separation of processes into those of avalanche multiplication and avalanche suppression or restriction allows a simpler understanding of the physical origin of the phenomena. Based on FBT notions, it is possible to construct simple enough model to describe processes of avalanche gain. This model can be connected with important parameters of APD under study and allows to determine the link of some technological parameters with APD properties.

Application of the FBT approach to construct the SPICE model allows one to facilitate the procedure of definition of the equivalent APD circuit main parameters and correctly describe the peculiarities of the front signal shaping.

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In this book some recent advances in development of photodetectors and photodetection systems for specific applications are included. In the first section of the book nine different types of photodetectors and their characteristics are presented. Next, some theoretical aspects and simulations are discussed. The last eight chapters are devoted to the development of photodetection systems for imaging, particle size analysis, transfers of time, measurement of vibrations, magnetic field, polarization of light, and particle energy. The book is addressed to students, engineers, and researchers working in the field of photonics and advanced technologies.

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