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# Polyoptimal Multiperiodic Control of Complex Systems with Inventory Couplings Via the Ideal Point Evolutionary Algorithm

Marek Skowron and Krystyn Styczeń  
Wrocław University of Technology  
Poland

## 1. Introduction

The paper is devoted to the polyoptimal control of a complex system with inventory couplings, which transfer the by-products of some subsystems to other subsystems as their input components or energy carriers. The cooperation of the subsystems on the recycling may enhance desired ecological features of complex production processes reducing the waste stream endangering the natural environment. The consideration of such systems is connected with the tendency of the rearrangement of complex industrial production systems from an open loop form with many waste products to a closed loop form guaranteeing their beneficial utilization (Ignatenko et al., 2007; Salmiaton & Garforth, 2007; Tan et al., 2008; Tatara et al., 2007; Yi & Luyben, 1996). The networks of interconnected chemical or biochemical reactors can be mentioned as the examples of systems discussed (Diaconescu et al., 2002; Russo et al., 2006; Smith & Waltman, 1995). The recycling problem is analyzed for various operation modes of the networks. Because of the flexible couplings the subsystems have high autonomy degree and can be operated in their own mode. In particular the following three nested operation kinds of the flexibly coupled network can be distinguished (Skowron & Styczeń, 2009): the steady state process (the low intensity production process), the periodic process (the increased intensity production process with the same operation period for all subsystems), and the multiperiodic process (the high intensity production process with different operation periods for the subsystems adjusted to their particular dynamic properties).

Since each of the subsystems has its own objective function composed of the product value, the recycled loop cost, and the waste neutralization cost, the polyoptimal (multiobjective) formulation of the control problem for the complex dynamic network comes to mind. The importance of the search of a compromise solution for a set of conflicting objectives has been widely emphasized in the literature (Huang & Yang, 2001; Sanchis et al., 2008; Sawaragi et al., 1985; Zitzler & Thiele, 1999). The extension of the admissible control processes may yield an essential improvement of the optimal objective function. To this end the periodic dynamic processes of the subsystems (the cycles) are represented by the finite-dimensional vectors encompassing their periods, their initial states, their local controls, and their inventory interactions. The three nested control problems are considered, namely the polyoptimal steady-state control problem, the polyoptimal periodic control problem, and the polyoptimal multiperiodic control problem. The evolutionary optimization algorithm is proposed, which finds the approximations of the polyoptimal ideal point for steady-state, periodic, and

multiperiodic processes. The proper dominance for such three polyoptimal points in the objective function space is analyzed. The algorithm is generalized to the case of the improper dominance of the approximated nested ideal points. It uses increased aspiration levels of the objective functions for suitably chosen subsystems.

The application of the evolutionary algorithm to the nested polyoptimization has the advantage of the searching for a globally optimal solutions on each nested stage of the system operation. The easiness of the incorporation of various side constraints including the stability conditions for the optimized control process should also be emphasized (Skowron & Styczeń, 2006). On the other hand the algorithm proposed is time consuming. It deals with dynamic interconnected processes, it evaluates the objective function of the complex recycled systems implementing the globalized Gauss-Newton method for the finding of periodic control processes of the subsystems, and it reconstructs both the averaged control constraints as well as the interaction constraints. This complicates its application for advanced polyoptimal approaches requiring a broad scanning of the Pareto set, the niching technique or the nondominated sorting technique (Audet et al., 2008; Sarkar & Modak, 2005; Tarafder et al., 2005; 2007; Zhang & Li, 2007). In this context the ideal point method shows the advantage of the moderate extent of the computations necessary for the determination of a nested polyoptimal solution.

The theoretical and algorithmic developments are illustrated by the illustrative example of the nested polyoptimization of production processes performed in systems of cross-recycled chemical reactors.

## 2. Polyoptimal multiperiodic control problem for recycled systems

Consider the following polyoptimal multiperiodic control (POMC) problem for systems composed of  $N$  subsystems with the inventory couplings (IC): minimize the vector objective function

$$G(z) \doteq (G_1(z), G_2(z), \dots, G_N(z)) \quad (1)$$

composed of the  $\tau_i$ -averaged objective functions of the particular subsystems

$$G_i(z_i) = \frac{1}{\tau_i} \int_0^{\tau_i} g_i(x_i(t), u_i(t), v_i(t)) dt \quad (i = 1, 2, \dots, N), \quad (2)$$

and subject for  $i = 1, 2, \dots, N$  to the  $\tau_i$ -periodic state equations of the subsystems

$$\dot{x}_i(t) = f_i(x_i(t), u_i(t), v_i(t)), \quad t \in [0, \tau_i], \quad x_i(\tau_i) = x_i(0), \quad (3)$$

to the resource constraints

$$\frac{1}{\tau_i} \int_0^{\tau_i} u_i(t) dt = b_i, \quad (4)$$

to the stability constraints

$$|s_i(\Phi_i(z_i))|_\infty \leq \alpha_i, \quad (5)$$

to the box constraints

$$\tau_i \in \mathcal{T}_i, \quad x_i(t) \in X_i, \quad u_i(t) \in U_i, \quad v_i(t) \in V_i, \quad t \in [0, \tau_i], \quad (6)$$

and to the averaged inventory interaction constraints

$$\frac{1}{\tau_i} \int_0^{\tau_i} v_i(t) dt \leq \sum_{j=1}^N \frac{1}{\tau_j} \int_0^{\tau_j} h_{ij}(x_j(t), u_j(t), v_j(t)) dt, \quad (7)$$

where  $\tau_i \in R_+$  is the operation period of the  $i$ -th subsystem,  $x_i \in W_{\infty}^{1, m_i}(0, \tau_i)$  is its state trajectory,  $u_i(t) \in L_{\infty}^{m_i}(0, \tau_i)$  is its control,  $v_i(t) \in L_{\infty}^{r_i}(0, \tau_i)$  is its inventory interaction,  $z_i \doteq (\tau_i, x_i, u_i, v_i)$  is its control process,  $b_i \in R^{m_i}$  is its averaged level of the resource availability, and  $\mathcal{T}_i \doteq [\tau_i^-, \tau_i^+]$ ,  $X_i \doteq [x_i^-, x_i^+]$ ,  $U_i \doteq [u_i^-, u_i^+]$  and  $V_i \doteq [v_i^-, v_i^+]$  are the box sets with the bounds  $\tau_i^{\pm} \in R_+$ ,  $x_i^{\pm} \in R^{n_i}$ ,  $u_i^{\pm} \in R^{m_i}$  and  $v_i^{\pm} \in R^{r_i}$ , and  $s_i \doteq (s_{ij})_{j=1}^{n_i}$  is the vector of the Floquet's multipliers of the state equation (3) i.e. the vector of the eigenvalues of its monodromy matrix  $\Phi_i(z_i)$  endowed with the norm  $|s_i|_{\infty} \doteq \max_{j=1,2,\dots,n_i} |s_{ij}|$ , and  $\alpha_i \in R_+$  is the local stability F-level of the  $i$ -th subsystem, and

$$g_i : R^{n_i} \times R^{m_i} \times R^{r_i} \rightarrow R, \quad f_i : R^{n_i} \times R^{m_i} \times R^{r_i} \rightarrow R^{n_i},$$

$$h_{ij} : R^{n_j} \times R^{m_j} \times R^{r_j} \rightarrow R^{r_i}$$

are continuous functions on the sets  $X_i \times U_i \times V_i$  and  $X_j \times U_j \times V_j$ , respectively, while  $z \doteq (z_i)_{i=1}^N$  is the control process of the IC system.

The objective functions  $G_i(z_i)$  for the particular systems are combined from such quantities as, for example, the averaged yield of the major products and the by-products, the averaged selectivity of the production process, and the averaged energy consumption or its dissipation.

The dynamics of the subsystems is governed by the  $\tau_i$ -periodic state equations (3), the periods of which can be chosen independently for each of the subsystems according to their dynamic properties. This is guaranteed by the flexible inventory couplings between the subsystems, which enable to stock up on some output products of the subsystems to recycle them in a complex production system. The inequalities (7) restrict the averaged outflows of the inventory couplings by their averaged inflows.

The constraints (4) mirror the averaged availability of the resources used in the process operation. The relationships (5) are responsible for the local asymptotic stability of periodic control processes for particular subsystems. The constraints for the local stability F-levels are important to ensure practical applicability of optimized processes.

To depict the ideal point evolutionary algorithm implementable for the POMC problem we apply the time scaling  $t := \tau_i t$  independently to each subsystem. We reduce this way the IC system to the computationally convenient unit time interval  $[0, 1]$ . We convert the continuous-time control of the  $i$ -th subsystem  $u_i(t)$  and its inventory interaction  $v_i(t)$  to the discrete-time form  $\tilde{u}_i^k$  and  $\tilde{v}_i^k$  for  $t \in [k/K, (k+1)/K)$ , where  $u_i^k \in R^{m_i}$  and  $v_i^k \in R^{r_i}$  ( $k = 0, 1, \dots, K-1$ ). We set  $u_i^K \doteq (u_i^0, u_i^1, \dots, u_i^{K-1})$  and  $v_i^K \doteq (v_i^0, v_i^1, \dots, v_i^{K-1})$ .

We assume that the normalized nonlinear state equation of each subsystem

$$\dot{x}_i(t) = \tau_i f_i(x_i(t), u_i(t), v_i(t)), \quad t \in [0, 1] \quad (8)$$

has the uniquely determined solution  $x_i(t, \tau_i, x_i(0), u_i, v_i)$  for every optimization argument  $(\tau_i, x_i(0), u_i, v_i)$  satisfying the constraints (6). Thus we can treat the states of the subsystems as

the resolvable variables  $x_i(t, z_i)$  found by high accuracy integration procedures for nonlinear differential equations with the given initial state and the input functions.

We convert this way the POMC problem to the following normalized and discretized form: minimize the vector objective function

$$G(z) \doteq (G_1(z), G_2(z), \dots, G_N(z)) \quad (9)$$

composed of the normalized objective functions of the subsystems

$$G_i(z_i) \doteq \int_0^1 g_i(x_i(t, z_i), \tilde{u}_i(t), \tilde{v}_i(t)) dt \quad (i = 1, 2, \dots, N) \quad (10)$$

and subject for  $i = 1, 2, \dots, N$  to the normalized process periodicity constraints

$$x_i(0) - x_i(1, z_i) = 0, \quad (11)$$

to the normalized and discretized resource constraints

$$\frac{1}{K} \sum_{k=0}^{K-1} u_i^k = b_i, \quad (12)$$

to the normalized stability constraints

$$|s_i(\Phi_i(1, z_i))|_\infty \leq \alpha_i, \quad (13)$$

to the normalized box constraints

$$\begin{aligned} \tau_i \in \mathcal{T}_i, \quad x_i(0) \in X_i, \quad u_i^k \in U_i, \quad v_i^k \in V_i \quad (k = 0, 1, \dots, K-1), \\ x_i(t_l) \in X_i \quad (t_l \doteq l/L, \quad l = 1, 2, \dots, L), \end{aligned} \quad (14)$$

and to the normalized and discretized inventory interaction constraints

$$\frac{1}{K} \sum_{k=0}^{K-1} v_i^k \leq \sum_{j=1}^N \int_0^1 h_{ij}(x_j(t, z_j), \tilde{u}_j(t), \tilde{v}_j(t)) dt, \quad (15)$$

where an additional dense time grid  $\{t_l\}_{l=1}^L$  is used in (14) to approximate sufficiently exactly the state constraints within the normalized control horizon, and

$$z_i \doteq (\tau_i, x_i(0), u_i^K, v_i^K) \in R^{M_i} \quad (M_i \doteq 1 + n_i + (m_i + r_i)K)$$

is the discrete representation of a controlled cycle of the  $i$ -th subsystem encompassing its period, its initial state, its discretized control, and its discretized inventory interaction, while

$$z \doteq (z_i)_{i=1}^N \in \prod_{i=1}^N R^{M_i}$$

is the normalized discretized control process of the IC system.

Let  $\tilde{Z}$  be the set of all the admissible solutions of the POMC problem, i.e. the set of all the multiperiodic cycles  $\tilde{z}$  satisfying the constraints (11)-(15). We determine the ideal point  $\tilde{G}^* \doteq (\tilde{G}_1^*, \tilde{G}_2^*, \dots, \tilde{G}_N^*)$  in the objective space of the POMC problem by the computation of  $N$  optimal

values of the objective functions of the particular subsystems for the multiperiodic operation of the IC system:

$$\tilde{G}_i^* = \min_{\tilde{z} \in \tilde{Z}} G_i(z) \quad (i = 1, 2, \dots, N).$$

We define the compromise multiperiodic solution  $\tilde{z}^*$  for the POMC problem as the solution minimizing the distance to the ideal point

$$\tilde{z}^* = \arg \min_{\tilde{z} \in \tilde{Z}} |G(z) - \tilde{G}^*|_\infty,$$

where the distance is defined with the help of the uniform norm  $|G(z) - \tilde{G}^*|_\infty \doteq \max_{i=1,2,\dots,N} |G_i(z) - \tilde{G}_i^*|$ .

The multiperiodic control process of the IC system may ensure high productivity of particular subsystems and it may be characterized as the high intensity production process with different operation periods for the subsystems adjusted to their particular dynamic properties. On the other hand its implementation is connected with increased requirements for inventory capacities and their maintenance. It may also have low stability margins for the periodic state trajectories, which involve the need of the design of high quality stabilizing loops for the subsystems.

For such reasons we consider the nested control processes "sitting inside" the multiperiodic control process i.e. the periodic control process and the static control process.

The periodic control process may be interpreted as the synchronized operation mode of the IC system. It requires moderate inventory capacities and facilitates the balancing of the inventory interactions.

The POMC problem is converted to the polyoptimal periodic control (POPC) problem by the setting  $\tau_i = \tau$  ( $i = 1, \dots, N$ ). Such a choice of the operation periods may be convenient for the balancing of the inventory interactions. It reduces, however, the set of admissible solutions to the set  $\tilde{Z}$  of all the periodic cycles  $\tilde{z}$  satisfying the constraints (11)-(15) with the equal periods  $\tau_i = \tau$ . We determine the ideal point  $\tilde{G}^* \doteq (\tilde{G}_1^*, \tilde{G}_2^*, \dots, \tilde{G}_N^*)$  in the objective space of the POPC problem by the computation of  $N$  optimal values of the objectives functions connected with the  $\tau$ -periodic operation of particular subsystems:

$$\tilde{G}_i^* = \min_{\tilde{z} \in \tilde{Z}} G_i(z) \quad (i = 1, 2, \dots, N).$$

We define the compromise periodic solution  $\tilde{z}^*$  for the POPC problem as the solution minimizing the uniform distance to the ideal point

$$\tilde{z}^* = \arg \min_{\tilde{z} \in \tilde{Z}} |G(z) - \tilde{G}^*|_\infty.$$

Fixing in time all the process variables leads to the simplified system with direct interconnections and without inventories. The steady-state control processes may be implemented with the help of simple stabilization loops, for example, of relay type. However, such processes ignore the optimization potential underlying in the process dynamics.

The POPC problem is converted to the polyoptimal steady-state control (POSS) problem by the fixing in time all the process variables, which is equivalent to the minimization of the vector steady-state objective function

$$G(\bar{z}) \doteq (G_1(\bar{z}), G_2(\bar{z}), \dots, G_N(\bar{z})) \quad (16)$$

having the components

$$G_i(\bar{z}) \doteq g_i(\bar{x}_i, \bar{u}_i, \bar{v}_i) \quad (17)$$

and subject for  $i = 1, \dots, N$  to the steady-state constraints

$$f_i(\bar{x}_i, \bar{u}_i, \bar{v}_i) = 0, \quad (18)$$

$$B_i \bar{u}_i = b_i, \quad (19)$$

$$|s_i(e^{f_{i,x_i}(\bar{x}_i, \bar{u}_i, \bar{v}_i)})|_\infty \leq \alpha_i, \quad (20)$$

$$\bar{x}_i \in X_i, \quad \bar{u}_i \in U_i, \quad \bar{v}_i \in V_i, \quad (21)$$

$$\bar{v}_i \leq \sum_{j=1}^N g_j(\bar{x}_j, \bar{u}_j, \bar{v}_j), \quad (22)$$

where

$$\bar{z}_i \doteq (\bar{x}_i, \bar{u}_i, \bar{v}_i)_{i=1}^N \in R^{n_i} \times R^{m_i} \times R^{r_i}$$

is the steady-state control process of the  $i$ -th subsystem, and

$$\bar{z} \doteq (\bar{z}_i)_{i=1}^N \in \prod_{i=1}^N R^{n_i} \times R^{m_i} \times R^{r_i}$$

is the steady-state control process for the IC system.

Let  $\bar{Z}$  be the set of all the admissible solutions of the POSS problem, i.e. the set of all the steady-state processes  $\bar{z}$  satisfying the constraints (18)-(22). We determine the ideal point  $\bar{G}^* \doteq (\bar{G}_1^*, \bar{G}_2^*, \dots, \bar{G}_N^*)$  in the objective space of the POSS problem by the computation of  $N$  optimal values of the objectives functions connected with the steady-state operation of particular subsystems:

$$\bar{G}_i^* = \min_{\bar{z} \in \bar{Z}} \bar{G}_i(\bar{z}) \quad (i = 1, 2, \dots, N).$$

We define the compromise steady-state solution  $\bar{z}^*$  for the POSS problem as the solution minimizing the distance to the ideal point

$$\bar{z}^* = \arg \min_{\bar{z} \in \bar{Z}} |G(\bar{z}) - \bar{G}^*|_\infty.$$

**Definition 1:** The triple of compromise nested control processes  $\mathbf{z}^* \doteq (\tilde{z}^*, \bar{z}^*, \bar{z}^*)$  is said to be

- strongly proper if it satisfies the relationships

$$G(\tilde{z}^*) < G(\bar{z}^*) < G(\bar{z}^*),$$

- partially strongly proper if it satisfies the relationships

$$G(\tilde{z}^*) \leq G(\bar{z}^*) < G(\bar{z}^*) \quad \text{or} \quad G(\tilde{z}^*) < G(\bar{z}^*) \leq G(\bar{z}^*),$$

- proper if it satisfies the relationships

$$G(\tilde{z}^*) \leq G(\bar{z}^*) \leq G(\bar{z}^*),$$

- weakly proper if it satisfies the relationships

$$G(\tilde{z}^*) \leq G(\bar{z}^*) \leq G(\bar{z}^*) \quad \text{or} \quad G(\tilde{z}^*) \leq G(\bar{z}^*) \leq G(\bar{z}^*),$$



- improper if it satisfies the relationship

$$G(\bar{z}^*) \leq G(\tilde{z}^*),$$

where the vector inequality  $\leq$  means that the inequality  $\leq$  holds for all the components with the strict inequality for some of them, and  $\leq$  means that higher level compromise solutions may improve objective functions for some subsystems at polyoptimal static solution, but deteriorate for other subsystems at this solution.

We are aimed at the comparison of the ideal point compromise solutions of the POMC problem for the steady-state processes, for the periodic processes, and for the multiperiodic processes. The finding of the strongly proper nested triple  $z^*$  means the uniform improvement of the ideal point compromise solutions between all the levels of nested optimization problem. It may be the basis for the application of the compromise multiperiodic control process. The other types of the nested triple  $z^*$  determine weaker possibilities of the polyoptimal nested optimization of the IC system. The practitioner choosing a definitive process for the implementation takes into account the degree of the improvement of the objective functions for the subsystems between the nested compromise solutions.

### 3. Ideal point evolutionary polyoptimal multiperiodic optimization

The general scheme of the ideal point evolutionary algorithm for the nested polyoptimal multiperiodic optimization can be stated as follows :

**Algorithm 1:** Finding of the ideal point compromise solution for the POMC problem.

*Step 1:* Choose randomly an initial steady-state control process population  $\bar{z}^0 \doteq (\bar{x}_i^0, \bar{u}_i^0, \bar{v}_i^0)_{i=1}^N$  and apply the evolutionary global optimization (EGO) algorithm of (Skowron & Styczeń, 2009) to solve  $N$  single objective static optimization problems

$$\min_{\bar{z} \in \bar{Z}} \bar{G}_i(\bar{z}) \quad (i = 1, 2, \dots, N)$$

to obtain the ideal point  $\bar{G}^* \doteq (\bar{G}_1^*, \bar{G}_2^*, \dots, \bar{G}_N^*)$  in the objective functions space of the POSS problem. Apply the EGO algorithm to find the ideal point compromise solution for the POSS problem

$$\bar{z}^* = \arg \min_{\bar{z} \in \bar{Z}} |G(\bar{z}) - \bar{G}^*|_\infty.$$

*Step 2:* Using the  $\pi$ -test ((Bernstein & Gilbert, 1980; Sterman & Ydstie, 1991)) evaluate the intervals  $[\tau_{i-}, \tau_{i+}]$  of period values guaranteeing the local improvement of the subsystems objective functions by the periodic operation, and choose  $\tau^0 \in [\tau_{i-}, \tau_{i+}]$  ( $i = 1, 2, \dots, N$ ).

*Step 3:* Choose randomly an initial periodic control process population  $\tilde{z}^0 = (\tau^0, \tilde{x}_i^0, \tilde{u}_i^0, \tilde{v}_i^0)_{i=1}^N$  and apply the evolutionary global optimization (EGO) algorithm of (Skowron & Styczeń, 2009) to solve  $N$  single objective periodic optimization problems

$$\min_{\tilde{z} \in \tilde{Z}} \tilde{G}_i(\tilde{z}) \quad (i = 1, 2, \dots, N)$$

to obtain the ideal point  $\tilde{G}^* \doteq (\tilde{G}_1^*, \tilde{G}_2^*, \dots, \tilde{G}_N^*)$  in the objective functions space of the POPC problem. Apply the EGO algorithm to find the ideal point compromise solution  $\tilde{z}^* = (\tau^*, \tilde{x}_i^*, \tilde{u}_i^*, \tilde{v}_i^*)_{i=1}^N$  for the POPC problem as

$$\tilde{z}^* = \arg \min_{\tilde{z} \in \tilde{Z}} |G(\tilde{z}) - \tilde{G}^*|_\infty.$$



*Step 4:* Choose randomly an initial multiperiodic control process population  $\tilde{z}^0 = (\tau^0, \tilde{x}_i^0, \tilde{u}_i^0, \tilde{v}_i^0)_{i=1}^N$  and apply the evolutionary global optimization (EGO) algorithm to solve  $N$  single objective multiperiodic optimization problems

$$\min_{\tilde{z} \in \tilde{Z}} \tilde{G}_i(\tilde{z}) \quad (i = 1, 2, \dots, N)$$

to obtain the ideal point  $\tilde{G}^* \doteq (\tilde{G}_1^*, \tilde{G}_2^*, \dots, \tilde{G}_N^*)$  in the objective functions space of the POMC problem. Apply the EGO algorithm to find the ideal point compromise solution for the POMC problem

$$\tilde{z}^* = \arg \min_{\tilde{z} \in \tilde{Z}} |G(\tilde{z}) - \tilde{G}^*|_\infty.$$

*Step 5:* Determine the properness of the determined nested triple  $z^*$  on the basis of the Definition 1.

If the nested triple  $z^*$  turns out to be improper the following regularization may improve its properness.

**Algorithm 2:** Combined the ideal point compromise solution and aspiration levels approach for the POMC problem.

*Step 1:* Modify the set of admissible solutions for the POPC problem as follows:

$$\tilde{\underline{Z}} \doteq \{\tilde{z} \in \tilde{Z} : G_i(\tilde{z}) \leq G_i(\tilde{z}^*) - \tilde{\Delta}_i \quad (i \in \tilde{\mathcal{N}}),$$

where  $\tilde{\mathcal{N}} \subset \{1, 2, \dots, N\}$  is the set of indices of the subsystems, the objective functions of which are deteriorated by the ideal point compromise solution of the POPC problem at the point  $\tilde{z}^*$ , and the corrections  $\tilde{\Delta}_i > 0$  determine the aspiration levels for the subsystems with deteriorated objective functions on the periodic optimization level.

*Step 2:* Apply the EGO algorithm to find the combined ideal point compromise and aspiration levels solution  $\tilde{\underline{z}}^* = (\underline{\tau}^*, \tilde{x}_i^*, \tilde{u}_i^*, \tilde{v}_i^*)_{i=1}^N$  for the POPC problem as

$$\tilde{\underline{z}}^* = \arg \min_{\tilde{\underline{z}} \in \tilde{\underline{Z}}} |G(\tilde{\underline{z}}) - \tilde{\underline{G}}^*|_\infty.$$

*Step 3:* Modify the set of admissible solutions for the POMC problem as follows:

$$\tilde{\underline{\underline{Z}}} \doteq \{\tilde{\underline{z}} \in \tilde{\underline{Z}} : G_i(\tilde{\underline{z}}) \leq G_i(\tilde{\underline{z}}^*) - \tilde{\Delta}_i \quad (i \in \tilde{\mathcal{N}}),$$

where  $\tilde{\mathcal{N}} \subset \{1, 2, \dots, N\}$  is the set of indices of the subsystems, the objective functions of which are deteriorated by the ideal point compromise solution of the POMC problem at the point  $\tilde{\underline{z}}^*$ , and the corrections  $\tilde{\Delta}_i > 0$  determine the aspiration levels for the subsystems with deteriorated objective functions on the multiperiodic optimization level.

*Step 4:* Apply the EGO algorithm to find the combined ideal point compromise and aspiration levels solution  $\tilde{\underline{\underline{z}}}^* = (\underline{\tau}_i^*, \tilde{x}_i^*, \tilde{u}_i^*, \tilde{v}_i^*)_{i=1}^N$  for the POMC problem as

$$\tilde{\underline{\underline{z}}}^* = \arg \min_{\tilde{\underline{\underline{z}}} \in \tilde{\underline{\underline{Z}}}} |G(\tilde{\underline{\underline{z}}}) - \tilde{\underline{\underline{G}}}^*|_\infty.$$

Of course it may suffice to solve one of the corrected problems POPC or POMC.

From the evolutionary algorithm perspective, the most important thing is the way of coding an individual. We propose to represent an individual of the problem (9)-(15) by the vector (Skowron & Styczeń, 2009)  $\tilde{z} \doteq (\tilde{z}_\mu)_{\mu=1}^{\tilde{N}}$ , where  $\tilde{N} = \sum_{i=1}^N M_i$ ,  $\tilde{z}_i \doteq \tau_i$  ( $i = 1, 2, \dots, N$ ) are operation periods of all subsystems,  $\tilde{z}_{j+N+\sum_{l=1}^{i-1} n_l} \doteq x_{ij}(0)$  ( $j = 1, 2, \dots, n_i$ ;  $i = 1, 2, \dots, N$ ) are coordinates of initial states of all subsystems,  $\tilde{z}_{k+1+(j-1)K+K\sum_{l=1}^{i-1} m_l+N+\sum_{l=1}^N n_l} \doteq u_{ij}^k$  ( $k = 0, 1, \dots, K-1$ ;  $j = 1, 2, \dots, m_i$ ;  $i = 1, 2, \dots, N$ ) are discrete-time control coordinates of all subsystems,  $\tilde{z}_{k+1+(j-1)K+K\sum_{l=1}^{i-1} r_l+K\sum_{l=1}^N m_l+N+\sum_{l=1}^N n_l} \doteq v_{ij}^k$  ( $k = 0, 1, \dots, K-1$ ;  $j = 1, 2, \dots, r_i$ ;  $i = 1, 2, \dots, N$ ) are discrete-time inventory interactions of all subsystems. Values of the individual's genes are bounded by the set  $Z \doteq [\tilde{z}^-, \tilde{z}^+]$  ( $\tilde{z}^\pm \in R^{\tilde{N}}$ ), which results from a set of inclusion constraints  $\mathcal{T} \times X \times U \times V$ .

The form of the individual  $\tilde{z}$  allows to use known crossing and mutation operators (Michalewicz, 1996). However, basing on several experiments we performed, we propose to use a uniform crossing operator and a non-uniform mutation operator. Unfortunately the available operators deliver an individual which violate the constraints (11)-(15). In case of the periodic constraint (11) we propose to use the Newton method as the reconstruction algorithm. The averaged constraints (12) and (15) can be preserved with the help of the reconstruction algorithm described by Skowron and Styczeń (Skowron & Styczeń, 2009). This reconstruction algorithm is not sufficient for the inventory interaction (15) constraint and that's why it shall be used together with the penalty term. The penalty term shall be also applied for stability (13) and box state (14) constraints (Skowron & Styczeń, 2006).

#### 4. Illustrative example

Let two continuous stirred tank reactors cooperate with the help of the inventory interactions of the mixed catalytic-resource type. The series reaction  $A_1 \rightleftharpoons B_1 \rightarrow C_1$  takes place in the first reactor, and the parallel reactions  $A_2 \rightleftharpoons B_2$  and  $A_2 \rightarrow C_2$  take place in the second reactor, where the main reactions are reversible, and  $A_i$  is the raw material of the  $i$ th reactor,  $B_i$  is its desired product, and  $C_i$  is its by-product. Assume that the  $i$ -th reactor is  $\tau_i$ -periodically operated, and denote by  $x_{i1}(t), x_{i2}(t), x_{i3}(t)$  its concentrations of  $A_i, B_i, C_i$ , respectively, by  $u_{i1}(t)$  its input concentration of  $A_i$ , by  $u_{i2}(t)$  its flow rate, by  $v_i(t)$  its inventory interaction. The interaction of the first reactor uses the by-product of the second reactor as the catalyst of the own reactions, while the interaction of the second reactor uses the by-product of the first reactor as the supplement of the own raw material. Consider the following POMC problem for the discussed system: minimize the vector objective function

$$G(z) \doteq (G_1(z), G_2(z))$$

having the components

$$G_i(z) \doteq -\frac{1}{\tau_i} \int_0^{\tau_i} u_{i2}(t)x_{i2}(t)dt,$$

and subject to the  $\tau_1$ -periodic state equations of the first subsystem

$$\begin{aligned}\dot{x}_{11}(t) &= u_{12}(t)(u_{11}(t) - x_{11}(t)) - \kappa_{11}v_1(t)^2x_{11}^{p_{11}}(t) + \kappa_{12}v_1(t)x_{12}^{p_{12}}(t), \\ \dot{x}_{12}(t) &= -u_{12}(t)x_{12}(t) + \kappa_{11}v_1(t)^2x_{11}^{p_{11}}(t) - \kappa_{12}v_1(t)x_{12}^{p_{12}}(t) - \kappa_{13}v_1(t)x_{12}^{p_{13}}(t), \\ \dot{x}_{13}(t) &= -u_{12}(t)x_{13}(t) + \kappa_{12}v_1(t)x_{12}^{p_{13}}(t),\end{aligned}$$

to the  $\tau_2$ -periodic state equations of the second subsystem

$$\begin{aligned}\dot{x}_{21}(t) &= u_{22}(t)(u_{21}(t) + v_2(t) - x_{21}(t)) - \kappa_{21}x_{21}^{p_{21}}(t) + \kappa_{22}x_{22}^{p_{22}}(t) - \kappa_{23}x_{21}/(1 + x_{21}), \\ \dot{x}_{22}(t) &= -u_{22}(t)x_{22}(t) + \kappa_{21}x_{21}^{p_{21}}(t) - \kappa_{22}x_{22}^{p_{22}}(t), \\ \dot{x}_{23}(t) &= -u_{22}(t)x_{23}(t) + \kappa_{23}x_{21}/(1 + x_{21}),\end{aligned}$$

to the resource constraints

$$\frac{1}{\tau_i} \int_0^{\tau_i} u_{ij}(t)dt = 1 \quad (i, j = 1, 2),$$

to the box constraints

$$\begin{aligned}\tau_i &\in [0.1, 20], \quad 0 \leq x_{ik}(t) \quad (i = 1, 2; k = 1, 2, 3), \\ 0 \leq u_{ij}(t) &\leq 2, \quad 0 \leq v_i(t) \leq 2 \quad (j = 1, 2), \quad t \in [0, \tau_i],\end{aligned}$$

to the stability constraints

$$|s_i(\Phi_i(\tau_i, x_i, u_i, v_i))|_\infty \leq 0.8 \quad (i = 1, 2),$$

and to the inventory interaction constraints

$$\begin{aligned}\frac{1}{\tau_1} \int_0^{\tau_1} v_1(t)dt &\leq \frac{1}{\tau_2} \int_0^{\tau_2} x_{23}(t)dt, \\ \frac{1}{\tau_2} \int_0^{\tau_2} v_2(t)dt &\leq \frac{1}{\tau_1} \int_0^{\tau_1} x_{13}(t)dt,\end{aligned}$$

where the reactions obey the power law with the exponents  $p_{ij}$ . The optimization goal is equivalent to the maximization of the averaged yield of the useful product for each of the reactors. We compare the nested polyoptimal steady-state, periodic, and multiperiodic control processes for such cross-recycled reactors.

The evaluation of the initial advantageous duration of the operation periods for the subsystems can be found with the help of the  $\pi$ -test. Assuming the unit mean value of the inventory interactions and the subsystem parameters  $n_i = 3, m_i = 2, p_{i1} = 2, p_{i2} = 1, p_{i3} = 1.5, \kappa_{11} = 40, \kappa_{12} = 12, \kappa_{13} = 10, \kappa_{21} = 30, \kappa_{22} = 15, \kappa_{23} = 2$  we obtain the  $\pi$ -curves for the subsystems (Fig. 1) with the suboptimal operation periods  $\tau_1 = 2.5, \tau_2 = 3.1$ . The form of the  $\pi$ -curves shows that optimal operation periods for the case considered should be searched within the intermediate frequencies.

	Ideal point			Compromise solution		
	POSS	POPC	POMC	POSS	POPC	POMC
$G_1^*$	-0.3399	-0.7352	-0.7442	-0.2846	-0.5143	-0.6461
$G_2^*$	-0.4693	-2.1912	-2.1914	-0.4109	-2.0758	-2.0854

Table 1. The values of the objective functions obtained with the help of evolutionary algorithm

Table 1 shows the results which were achieved with the aid of the evolutionary algorithm described Skowron and Styczeń (Skowron & Styczeń, 2009). It is easy to notice that applying the periodic control for the considered system of two continuous stirred tank reactors cooperated with the help of the inventory interactions significantly improves the productivity

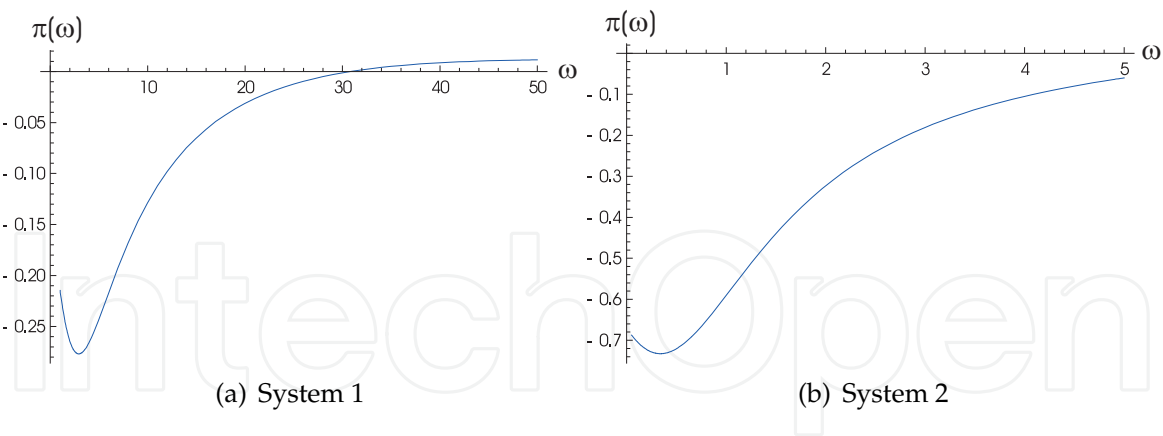


Fig. 1. The results of  $\pi$ -test

of the system in comparison to the steady-state approach. Comparing ideal points for POSS and POPC problems we can see that productivity of the first system is improved about 116%. Much greater improvement is observed for the second system. Applying the periodic control improves that the productivity of the second about 366%.

The results confirm also that efficiency of the system can be increased by applying multiperiodic control. The ideal point of the first system is improved about 1.2% and for the second system we see improvement equal 0.009%. The improvement after applying the multiperiodic control is not such spectacular like for the case when the steady-state control is replaced by the periodic control. But for some systems improvement of the efficiency of the process about 1-2% can give very huge economical profits.

Encouraged by observed improvement after applying multiperiodic control we calculated also the compromise solution (Table 1, Fig. 2-5). For the first system the improvement is about 25% and for the second system is about 0.46%. We see that received compromise solution for POMC problem is strongly proper according Definition 1. Thus these results confirm that for the considered system of two continuous stirred tank reactors cooperated with the help of the inventory interactions it is wise to apply multiperiodic approach.

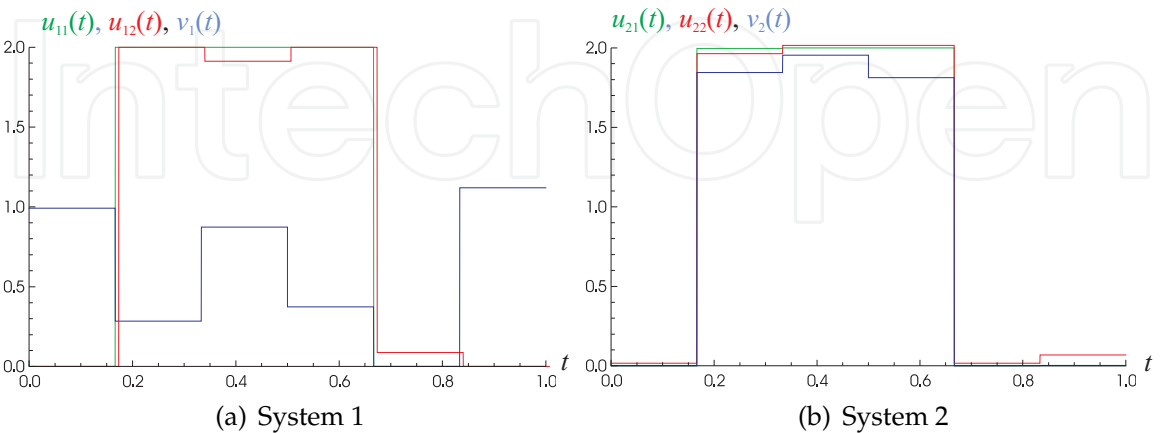


Fig. 2. The optimal control  $\tilde{u}_{ij}^*(t)$  and the inventory interaction  $\tilde{v}_i^*(t)$  ( $i, j = 1, 2$ ) for POPC problem

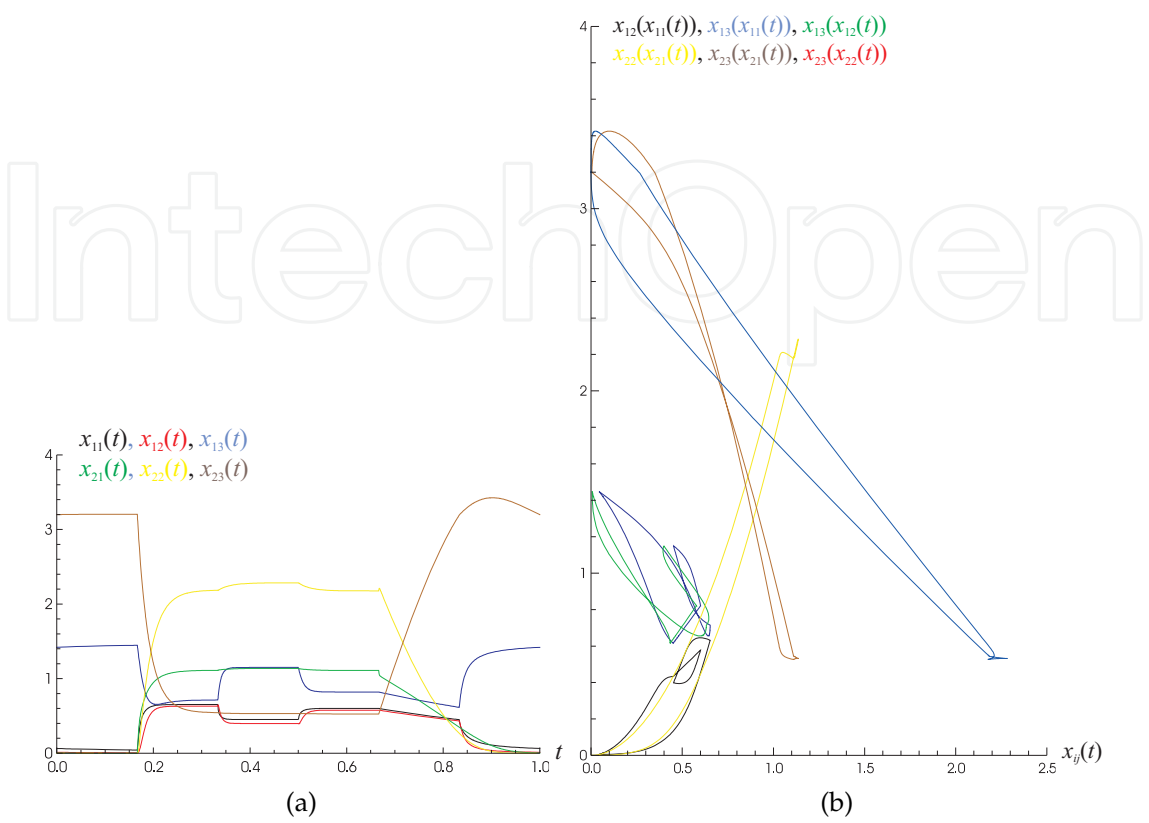


Fig. 3. The optimal state  $\tilde{x}_{ij}^*(t)$  ( $i = 1, 2; j = 1, 2, 3$ ) for POPC problem

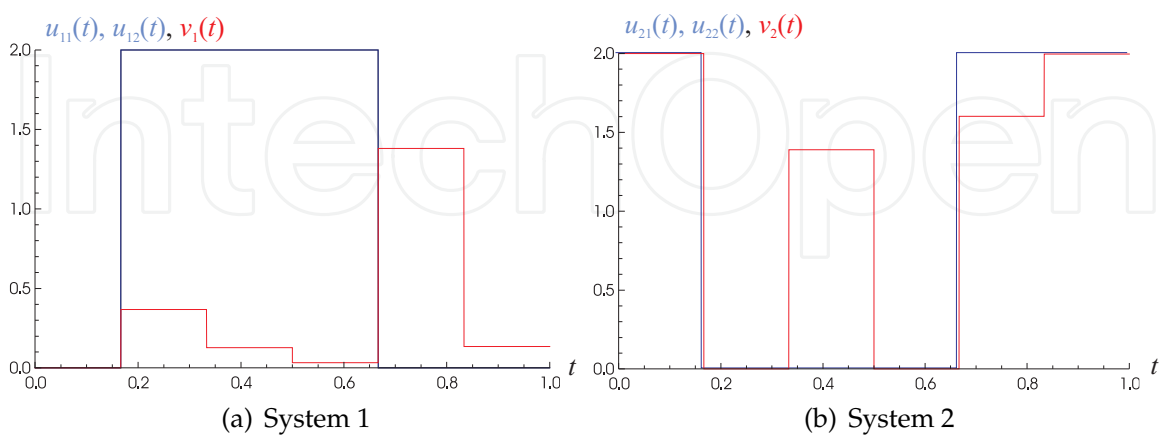


Fig. 4. The optimal control  $\tilde{u}_{ij}^*(t)$  and the inventory interaction  $\tilde{v}_i^*(t)$  ( $i, j = 1, 2$ ) for POMC problem

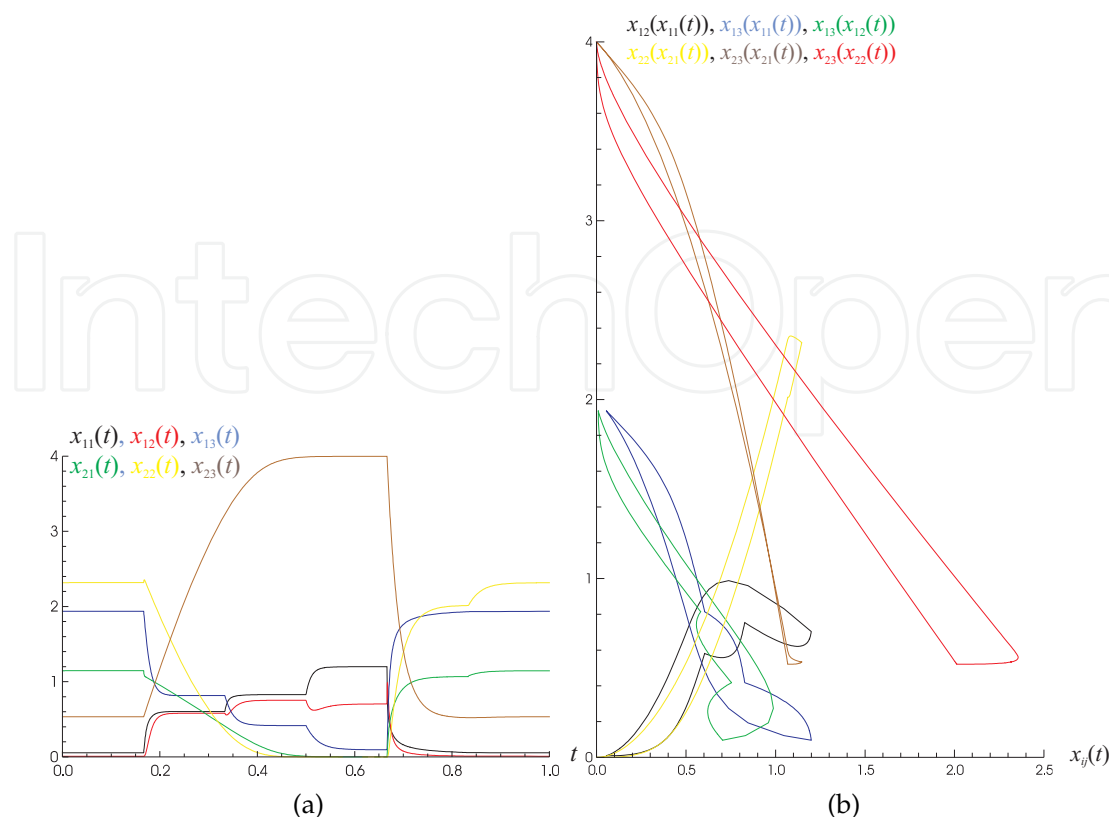


Fig. 5. The optimal state  $\tilde{x}_{ij}^*(t)$  ( $i = 1, 2; j = 1, 2, 30$  for POMC problem

## 5. Conclusion

The polyoptimal multiperiodic control problem for complex systems with the inventory couplings was analysed. The ideal point evolutionary algorithm was proposed for the solving of this problem. It has been shown that the multiperiodic operation of the complex cross-recycled chemical production systems may ensure the uniform improvement of the vector objective function as compared with the steady-state operation, and with the periodic operation. Such polyoptimal solution may be preferred by practitioners. The method applied shows the advantage of the moderate extent of the computational effort necessary for the finding of a best compromise solution. The solution obtained this way may be further exploited as the starting point for the implementation of some improved nested multiobjective optimization based, for example, on the verification of the attainability of the given aspiration levels for particular objective functions.

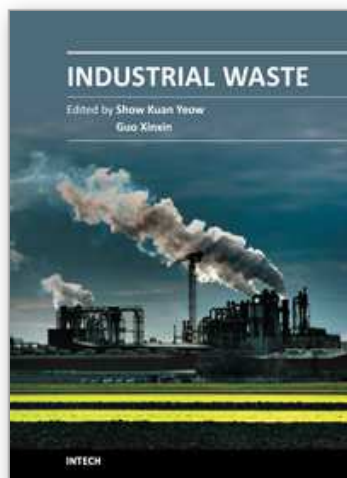
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## **Industrial Waste**

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This book is intended to fulfil the need for state-of-the-art development on the industrial wastes from different types of industries. Most of the chapters are based upon the ongoing research, how the different types of wastes are most efficiently treated and minimized, technologies of wastes control and abatement, and how they are released to the environment and their associated impact. A few chapters provide updated review summarizing the status and prospects of industrial waste problems from different perspectives. The book is comprehensive and not limited to a partial discussion of industrial waste, so the readers are acquainted with the latest information and development in the area, where different aspects are considered. The user can find both introductory material and more specific material based on interests and problems. For additional questions or comments, the users are encouraged to contact the authors.

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Phone: +86-21-62489820  
Fax: +86-21-62489821

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