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The Weibull – Log-Weibull Transition of Interoccurrence Time of Earthquakes

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1. Introduction

Earthquakes are great complex phenomenon characterized by several empirical statistical laws (1). One of the most important statistical law is the Gutenberg - Richter law (2), where the cumulative number of $n(> m)$ of magnitude m satisfy the following relation:

$$\log n(> m) = a - bm, \quad (1)$$

where a and b are constants. b is so-called b -value and is similar to unity. Another important statistical law is a power law decay of the occurrence of aftershocks, called Omori law (3).

The time intervals between successive earthquakes can be classified into two types: interoccurrence times and recurrence times (4). Interoccurrence times are the interval times between earthquakes on all faults in a region, and recurrence times are the time intervals between earthquakes in a single fault or fault segment. For seismology, recurrence times mean the interval times of characteristic earthquakes that occur quasi-periodically in a single fault. Recently, a unified scaling law of interoccurrence times was found using the Southern California earthquake catalogue (5) and worldwide earthquake catalogues (6). In Corral's paper (6), the probability distribution of interoccurrence time, $P(\tau)$, can be written as

$$P(\tau) = Rf(R\tau), \quad (2)$$

$$f(R\tau) = C \frac{1}{(R\tau)^{1-\gamma}} \exp(-(R\tau)^\delta / B), \quad (3)$$

where R is the seismicity rate. He has found that $f(R\tau)$ follows the generalized gamma distribution. In equation (3), C is a normalized constant and is $C = 0.50 \pm 0.05$. γ , δ , and B are parameters estimated to be $\gamma = 0.67 \pm 0.05$, $\delta = 0.98 \pm 0.05$, and $B = 1.58 \pm 0.15$. It should

Catalog Name	Coverage	Term	Number of Earthquakes	m_{min}	m_c^0
JMA	25° –50° N and 125° –150° E	01/01/2001–31/1/2010	170,801	2.0	2.0
SCEDC	32° –37° N and 114° –122° W	01/01/2001–28/2/2010	116,089	0.0	1.4
TCWB	21° N–26° N and 119° –123° E	01/01/2001–28/2/2010	189,980	0.0	1.9

Table 1. Information on earthquake catalogues.

be noted that the interoccurrence times were analyzed for the events with the magnitude m above a certain threshold m_c under the following two assumptions: (a) earthquakes can be considered as a point process in space and time; (b) there is no distinction between foreshocks, mainshocks, and aftershocks. It has been shown that the distribution of the interoccurrence time is also obtained by analyzing aftershock data (7) and is derived approximately from a theoretical framework proposed by Saichev and Sornette (8; 9). Abe and Suzuki showed that the distribution of the interoccurrence time, $P(> \tau)$, can be described by q -exponential distribution with $q > 1$, corresponding to a power law distribution (10), namely,

$$P(> \tau') = \frac{1}{(1 + \epsilon \tau')^\gamma} = e_q(-\tau'/\tau_0) = [(1 + (1 - q)(-\tau'/\tau_0))^{\frac{1}{1-q}}]_+, \tag{4}$$

where q_t , τ_0 , γ , and ϵ are positive constants and $([a]_+ \equiv \max[0, a])$.

It has been reported that the sequence of aftershocks and successive independent earthquakes is a Poisson process (11; 12). However, recent works show that interoccurrence times are not independent random variables, but have “long-term memory” (13–16). Since an interoccurrence time depends on the past, it is difficult to determine the distribution of interoccurrence times theoretically. Therefore, the determination of the distribution of interoccurrence times is still an open problem. Moreover, an effect of a threshold of magnitude on the interoccurrence time statistics is unknown. We study the distribution of interoccurrence times by changing the threshold of magnitude. In this chapter, we review our previous studies (17; 18) and clarify the Weibull - log-Weibull transition and its implication by reanalyzing the latest earthquake catalogues, JMA catalogue (19), SCEDC catalogue (20), and TCWB catalogue (21). This study focuses on the interoccurrence time statistics for middle or big mainshocks.

2. Data and methodology

2.1 Earthquake catalogue

To study the interoccurrence time statistics, we analyzed three natural earthquake catalogues of the Japan Metrological Agency (JMA) (19), the Southern California Earthquake Data Center (SCEDC) (20) and the Taiwan Central Weather Bureau (TCWB) (21). Information on each catalogue is listed in Table 1, where m_{min} corresponds to the minimum magnitude in the catalogue and m_c^0 is the magnitude of completeness, that is the lowest magnitude at which the Gutenberg - Richter law holds. We basically consider events with magnitude greater than and equal to m_c^0 because events whose magnitudes are smaller than m_c^0 are supposedly incomplete for recording.

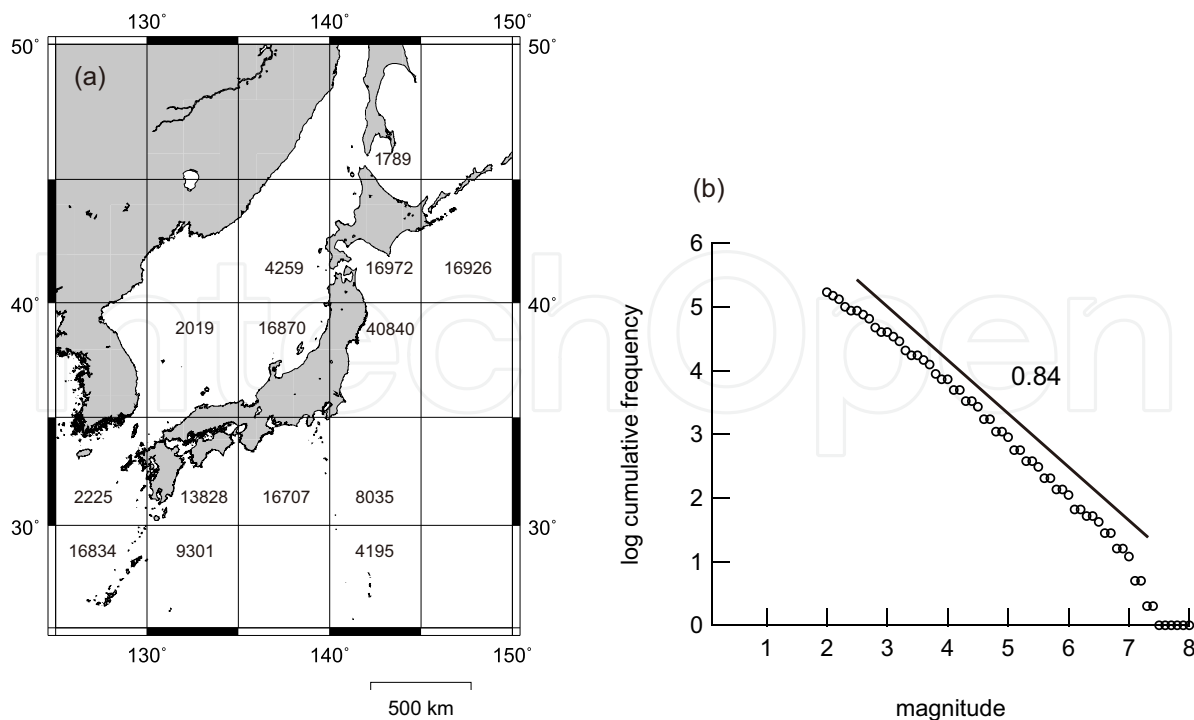


Fig. 1. Information on the Japan Meteorological Agency (JMA) earthquake catalogues. (a) covering region. The number of each cell means the number of earthquakes. (b) the magnitude distribution. $b = 0.84$ is calculated from the slope of the distribution.

2.1.1 Japan Meteorological Agency (JMA) earthquake catalogue

JMA catalogue is maintained by the Japan Meteorological Agency, covering from 25° to 50° N for latitude, and from 130° to 150° E for longitude [see Figure 1 (a)] during from 1923 to latest. This catalogue consists of an occurrence of times, a hypocenter, a depth, and a magnitude. In this chapter, we use the data from 1st January 2001 to 31st January 2010. As can be seen from Figure 1 (b), the distribution of magnitude obeys the Gutenberg - Richter law and m_c^0 is estimated to be 2.0.

2.1.2 Southern California Earthquake Data Center (SCEDC) earthquake catalogue

SCEDC catalogue is maintained by the Southern California Earthquake Data Center, covering from 32° to 37° N for latitude, and from 114° to 122° W for longitude [see Figure 2 (a)] during from 1932 to latest. The information of an earthquake, such as an occurrence of times, a hypocenter, a depth, and a magnitude, is listed. Here, we analyze the earthquake data from 1st January 2001 to 28th February 2010. In Figure 2 (b), we demonstrate the magnitude distribution, and we obtain $b = 0.97$.

2.1.3 Taiwan Central Weather Bureau (TCWB) earthquake catalogue

TCWB catalogue is maintained by the Central Weather Bureau, covering from 21° to 26° N for latitude, and from 119° to 123° E for longitude [see Figure 3 (a)]. This catalogue consists of an occurrence of times, a hypocenter, a depth, and a magnitude. We use the data from 1st January 2001 to 28th February 2010. As shown in Figure 3 (b), the Gutenberg - Richter law is valid in a

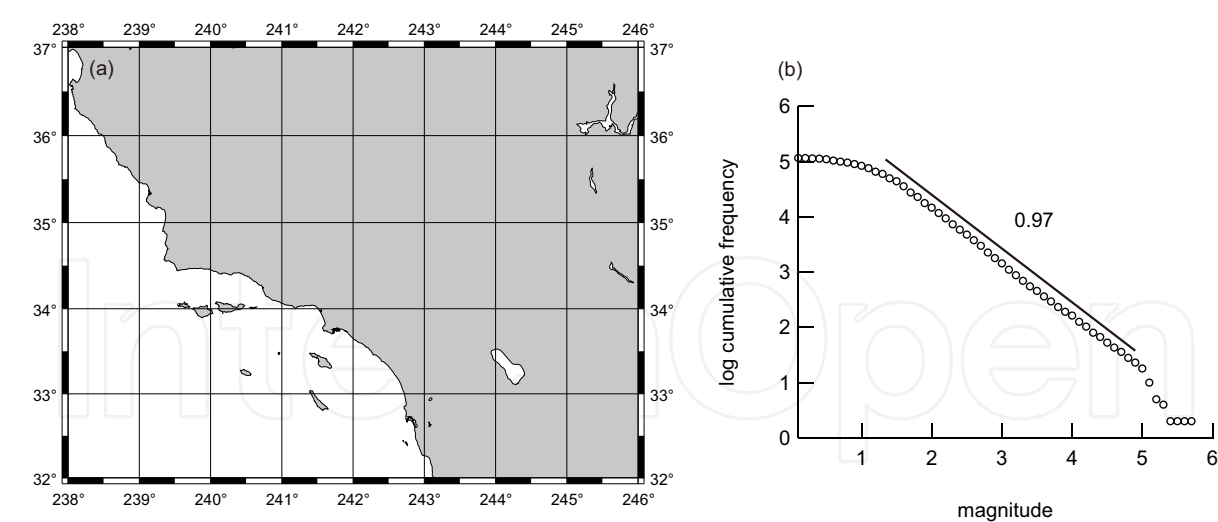


Fig. 2. Southern California Earthquake Data Center (SCEDC) earthquake catalogue information. (a) covering region. (b) the magnitude distribution. $b = 0.97$ is calculated from the slope of the distribution.

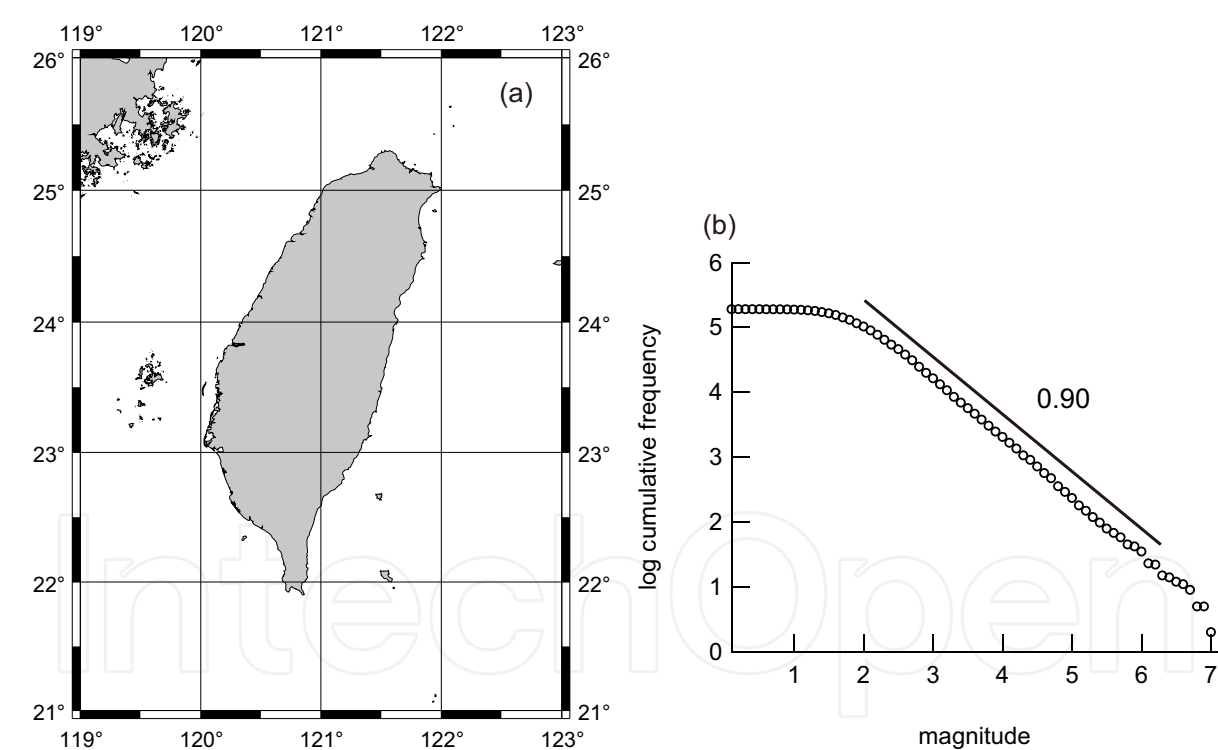


Fig. 3. Information on the Taiwan Central Weather Bureau (TCWB) earthquake catalogue. (a) covering region. (b) the magnitude distribution. $b = 0.90$ is calculated from the slope of the distribution.

magnitude range, $1.9 \leq m \leq 6.7$. b -value is calculated from the slope of the distribution, and is estimated to be 0.90.

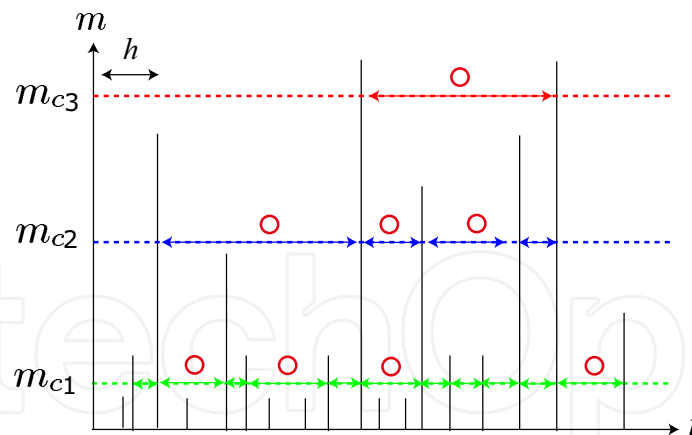


Fig. 4. Schematic diagram of the interoccurrence time of our analysis for different threshold of magnitude m_c . Circles (\circ) satisfy the condition. We analyze interoccurrence times greater than or equals to h .

2.2 Methodology (How to detect the appropriate distributions)

Our method is similar to the that of previous works (17; 18) (see Figure 4).

1. We divided the spatial areas into a window of L degrees in longitude and L degrees in latitude.
2. For each bin, earthquakes with magnitude m above a certain cutoff magnitude m_c were considered.
3. We analyzed interoccurrence times greater than and equals to h day.

For each bin, we analyzed interoccurrence times using at least 100 events to avoid statistical errors. h and L are taken to be 0.5 and 5, respectively. It is noted that for SCEDC and TCWB, we analyze earthquake covering the whole region. As shown in Figure 4, we investigated the interoccurrence time statistics for different 16 regions (14 regions in Japan, Southern California, and Taiwan). Aftershocks might be excluded from the study based on the information from previous studies (6; 12).

One of our main goals in this chapter is to determine the distribution function of the interoccurrence time. Here, we will focus our attention on the applicability of the Weibull distribution P_w , the log-Weibull distribution P_{lw} (22), the power law distribution P_{pow} (10), the gamma distribution P_{gam} (in the case of $\delta = 1$ in the paper (6)), and the log normal distribution P_{ln} (23), which are defined as

$$P_w(\tau) = \left(\frac{\tau}{\beta_1}\right)^{\alpha_1-1} \frac{\alpha_1}{\beta_1} \exp\left[-\left(\frac{\tau}{\beta_1}\right)^{\alpha_1}\right], \quad (5)$$

$$P_{lw}(\tau) = \frac{(\log(\tau/h))^{\alpha_2-1}}{(\log \beta_2)^{\alpha_2}} \frac{\alpha_2}{\tau} \exp\left[-\left(\frac{\log(\tau/h)}{\log \beta_2}\right)^{\alpha_2}\right], \quad (6)$$

$$P_{pow}(\tau) = \frac{1}{(1 + \beta_3 \tau)^{\alpha_3}}, \quad (7)$$

$$P_{gam}(\tau) = \tau^{\alpha_4-1} \frac{\exp(-\tau/\beta_4)}{\Gamma(\alpha_4)\beta_4^{\alpha_4}}, \quad (8)$$

$$P_{ln}(\tau) = \frac{1}{\tau\beta_5\sqrt{2\pi}} \exp\left[-\frac{(\ln(\tau) - \alpha_5)^2}{2\beta_5^2}\right], \quad (9)$$

where α_i , β_i , and h are constants and characterize the distribution. $\Gamma(x)$ is the gamma function. i stands for an index number; $i = 1, 2, 3, 4$, and 5 correspond to the Weibull distribution, the log-Weibull distribution, the power law distribution, the gamma distribution, and the log normal distribution, respectively.

The Weibull distribution is well known as a description of the distribution of failure-occurrence times (24). In seismology, the distribution of ultimate strain (25), the recurrence time distribution (26; 27), and the damage mechanics of rocks (28) show the Weibull distribution. In numerical studies, the recurrence time distribution in the 1D (4) and 2D (29) spring-block model, and in the "Virtual California model" (30) also exhibit the Weibull distribution. For $\alpha_1 = 1$ and $\alpha_1 < 1$, the tail of the Weibull distribution is equivalent to the exponential distribution and the stretched exponential distribution, respectively. The log-Weibull distribution is constructed by a logarithmic modification of the cumulative distribution of the Weibull distribution. In general, the tail of the log-Weibull distribution is much longer than that of the Weibull distribution. As for $\alpha_2 = 1$, the log-Weibull distribution is equal to a power law distribution. It has been shown that the log-Weibull distribution can be derived from the chain-reaction model proposed by Huillet and Raynaud (22).

To determine the best fitting for the distribution of the interoccurrence time data, we used the root mean square (rms) and Kolomogorov-Smirnov (KS) tests as the measure of goodness-of-fit. The definition of the rms value is

$$\text{rms} = \sqrt{\frac{\sum_{i=1}^n (x_i - x'_i)^2}{n - k}}, \quad (10)$$

where x_i is actual data and x'_i is estimated data obtained from $P(\tau)$. n and k indicate the numbers of the data points and of the fitting parameters, respectively. In this study, the rms value is calculated using the cumulative distribution for decreasing the fluctuation of the data. The most appropriate distribution is, by definition, the smallest rms value. Also, in order to use the KS test, we define the maximum deviation of static DKS, which is so-called Kolomogorov-Smirnov statistic, as

$$\text{DKS} = \max_i |y_i - y'_i|, \quad (11)$$

where y_i and y'_i mean the actual data of the cumulative distribution and the data estimated from the fitting distribution, respectively. Then, the significance level of probability of the goodness-of-fit, Q , is defined as

$$Q = 2 \sum_{i=1}^{\infty} (-1)^{i-1} e^{2i^2\lambda^2}, \quad (12)$$

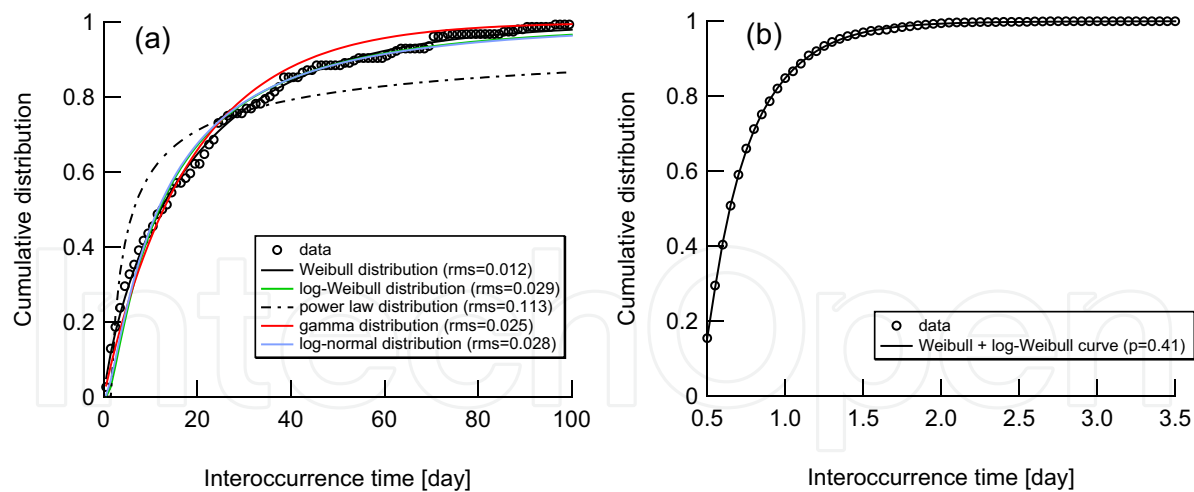


Fig. 5. Cumulative distribution of interoccurrence time at Okinawa region different m_c . The cumulative distribution is plotted by circles. (a) Several fitting curves are represented by lines ($m_c = 4.5$). (b) The superposition of the Weibull and the log-Weibull distribution is represented by line ($m_c = 2.0$).

where

$$\lambda = DKS \left(\sqrt{n'} + 0.12 + \frac{0.11}{\sqrt{n'}} \right), \tag{13}$$

where n' stands for the number of data points.

It is known that the preferred distribution shows the smallest value of DKS and the largest value of Q (31).

3. Results

3.1 Interoccurrence time statistics in Japan

First, we analyze the JMA data. Here, we consider the two region; Okinawa region (125°–130°E and 25°–30°N), and Chuetsu region (135°–140°E and 35°–40°N). The total number of earthquakes in Okinawa and Chuetsu are 16,834 and 16,870, respectively.

The cumulative distributions of the interoccurrence times for different m_c in Okinawa region and in Chuetsu region are displayed in Figures 5 and 6, respectively. We carried out two statistical tests, the rms and the KS test so as to determine the distribution function. The results for large magnitude ($m_c = 4.5$) in Okinawa and Chuetsu are shown in Table 2 and 3, respectively. For Okinawa, we found that the most suitable distribution is the Weibull distribution in all tests. In general, there is a possibility that the preferred distribution is not unique but depends on the test we use. However, the results obtained in Table 2 provide evidence that the Weibull distribution is the most appropriate distribution. As for Chuetsu, by two tests, the preferred distribution is suited to be the Weibull distribution as shown in Table 3, where the Weibull distribution is the most prominent distribution in the two tests. It follows that the Weibull distribution is preferred.

m_c		Distribution X		RMS test	KS test	
Region	distribution X	α_i	β_i [day]	rms [$\times 10^{-3}$]	DKS	Q
4.5 Okinawa	$P_w (i = 1)$	0.82 ± 0.007	19.0 ± 0.13	12	0.03	1
	$P_{lw} (i = 2)$	3.08 ± 0.06	35.3 ± 0.57	29	0.1	0.15
	$P_{pow} (i = 3)$	1.48 ± 0.02	1.04 ± 0.12	113	0.23	3.8×10^{-3}
	$P_{gam} (i = 4)$	0.96 ± 0.005	19.5 ± 0.24	24	0.07	0.55
	$P_{ln} (i = 5)$	2.45 ± 0.02	1.20 ± 0.02	28	0.09	0.33

Table 2. Results of rms value, DKS, and Q for different distribution functions for Okinawa area ($m_c = 4.5$). The error bars mean the 95% confidence level of fit.

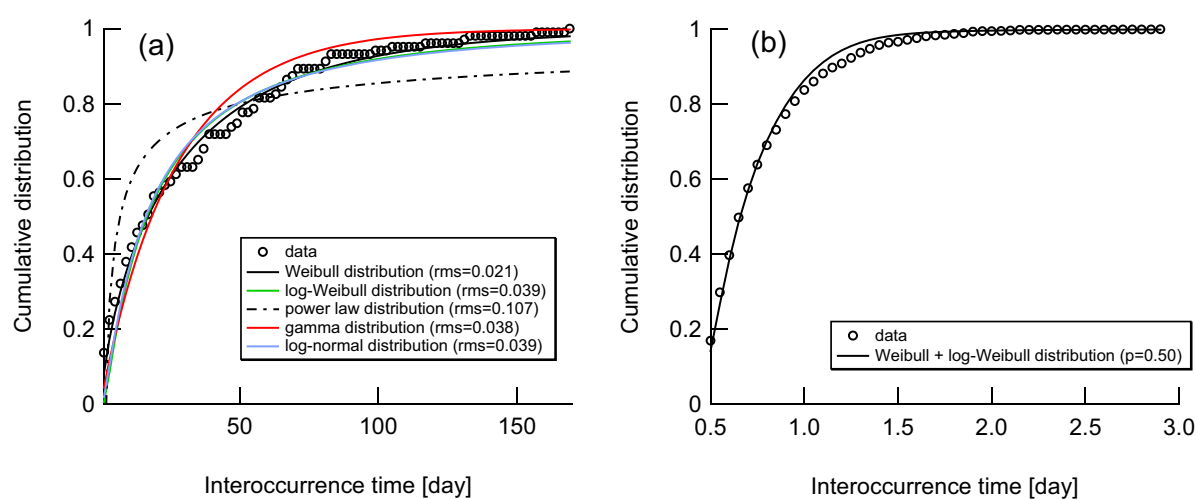


Fig. 6. Cumulative distribution of interoccurrence time for Chuetsu area different m_c . The cumulative distribution is plotted by circles. (a) Several fitting curves are represented by lines ($m_c = 4.5$). (b) The superposition of the Weibull and the log-Weibull distribution is represented by line ($m_c = 2.0$).

m_c		Distribution X		RMS test	KS test	
Region	distribution X	α_i	β_i [day]	rms [$\times 10^{-3}$]	DKS	Q
4.5 Chuetsu	$P_l (i = 1)$	0.75 ± 0.01	27.6 ± 0.40	21	0.06	0.94
	$P_{lw} (i = 2)$	3.12 ± 0.10	51.4 ± 1.39	39	0.14	0.06
	$P_{pow} (i = 3)$	1.47 ± 0.03	1.51 ± 0.21	107	0.19	4.8×10^{-3}
	$P_{gam} (i = 4)$	0.94 ± 0.009	28.9 ± 0.67	38	0.11	0.27
	$P_{ln} (i = 5)$	2.78 ± 0.03	1.33 ± 0.04	39	0.12	0.15

Table 3. Results of rms value, DKS, and Q for different distribution functions for Chuetsu area ($m_c = 4.5$). The error bars mean the 95% confidence level of fit.

m_c		Weibull distribution		Distribution X		p	RMS test	KS test	
Region	distribution X	α_1	β_1 [day]	α_i	β_i [day]		rms [$\times 10^{-3}$]	DKS	Q
4.5 Okinawa	$P_{lw} (i = 2)$	0.82 ± 0.007	19.0 ± 0.13	—	—	1	12	0.03	1
	$P_{pow} (i = 3)$	0.82 ± 0.007	19.0 ± 0.13	—	—	1	12	0.03	1
	$P_{gam} (i = 4)$	0.82 ± 0.007	19.0 ± 0.13	—	—	1	12	0.03	1
	$P_{ln} (i = 5)$	0.82 ± 0.007	19.0 ± 0.13	—	—	1	12	0.03	1
4.0 Okinawa	$P_{lw} (i = 2)$	0.93 ± 0.009	8.47 ± 0.06	—	—	1	13	0.03	1
	$P_{pow} (i = 3)$	0.93 ± 0.009	8.47 ± 0.06	—	—	1	13	0.03	1
	$P_{gam} (i = 4)$	0.93 ± 0.009	8.47 ± 0.06	—	—	1	13	0.03	1
	$P_{ln} (i = 5)$	0.93 ± 0.009	8.47 ± 0.06	—	—	1	13	0.03	1
3.5 Okinawa	$P_{lw} (i = 2)$	1.07 ± 0.008	3.45 ± 0.02	2.07 ± 0.04	6.25 ± 0.09	0.77 ± 0.02	5.3	0.02	1
	$P_{pow} (i = 3)$	1.07 ± 0.008	3.45 ± 0.02	1.81 ± 0.04	0.64 ± 0.04	0.94 ± 0.009	7.3	0.03	1
	$P_{gam} (i = 4)$	1.07 ± 0.008	3.45 ± 0.02	—	—	1	8.8	0.04	1
	$P_{ln} (i = 5)$	1.07 ± 0.008	3.45 ± 0.02	0.85 ± 0.009	0.94 ± 0.01	0.65 ± 0.03	5.6	0.02	1
3.0 Okinawa	$P_{lw} (i = 2)$	1.44 ± 0.02	1.77 ± 0.02	1.40 ± 0.03	2.57 ± 0.04	0.58 ± 0.01	3.8	0.01	1
	$P_{pow} (i = 3)$	1.41 ± 0.02	1.63 ± 0.01	2.22 ± 0.04	0.55 ± 0.01	0.79 ± 0.01	8.8	0.07	0.73
	$P_{gam} (i = 4)$	1.41 ± 0.02	1.63 ± 0.01	—	—	1	17	0.1	0.34
	$P_{ln} (i = 5)$	1.41 ± 0.02	1.63 ± 0.01	0.20 ± 0.003	0.72 ± 0.003	0.04 ± 0.03	5.4	0.04	1
2.5 Okinawa	$P_{lw} (i = 2)$	1.72 ± 0.02	1.14 ± 0.008	1.25 ± 0.01	1.68 ± 0.01	0.47 ± 0.006	2.1	0.007	1
	$P_{pow} (i = 3)$	1.90 ± 0.05	1.01 ± 0.008	2.84 ± 0.04	0.51 ± 0.005	0.58 ± 0.02	9.2	0.04	1
	$P_{gam} (i = 4)$	1.90 ± 0.05	1.01 ± 0.008	1.07 ± 0.02	0.94 ± 0.02	0.99 ± 0.04	22	0.14	0.04
	$P_{ln} (i = 5)$	—	—	-0.20 ± 0.004	0.52 ± 0.005	0	12	0.07	0.61
2.0 Okinawa	$P_{lw} (i = 2)$	1.78 ± 0.03	0.76 ± 0.009	1.16 ± 0.01	1.44 ± 0.008	0.41 ± 0.008	2.0	0.007	1
	$P_{pow} (i = 3)$	2.57 ± 0.10	0.77 ± 0.007	3.61 ± 0.05	0.48 ± 0.003	0.40 ± 0.02	7.0	0.03	1
	$P_{gam} (i = 4)$	2.57 ± 0.10	0.77 ± 0.007	1.09 ± 0.03	0.68 ± 0.03	0.96 ± 0.05	25	0.14	0.18
	$P_{ln} (i = 5)$	—	—	-0.41 ± 0.005	0.39 ± 0.008	0	15	0.08	0.78

Table 4. Interoccurrence time statistics of earthquakes in Okinawa region. The error bars mean the 95% confidence level of fit.

However, the fitting accuracy of the Weibull distribution becomes worse with a gradual decrease in m_c . We now propose a possible explanation which states that “the interoccurrence time distribution can be described by the superposition of the Weibull distribution and another distribution, hereafter referred to as the distribution $P_X(\tau)$,

$$\begin{aligned} P(\tau) &= p \times \text{Weibull distribution} + (1 - p) \times \text{distribution X} \\ &= p \times P_w(\tau) + (1 - p) \times P_X(\tau) \end{aligned}$$

(14)

where p is a parameter in the range, $0 \leq p \leq 1$ and stands for the ratio of P_w divided by $P(\tau)$. The interoccurrence time distribution obeys the Weibull distribution for $p = 1$. On the other hand, it follows the distribution $P_X(\tau)$ for $p = 0$. Here, the log-Weibull distribution, the power law distribution, the gamma distribution, and the log normal distribution are candidates for the distribution $P_X(\tau)$.

Next we shall explain the parameter estimation procedures;

(A); the optimal parameters are estimated so as to minimize the differences between the data and the test function by varying five parameters, $\alpha_1, \beta_1, \alpha_i, \beta_i$ and p .

m_c		Weibull distribution		Distribution X		p	RMS test	KS test	
Region	distribution X	α_1	β_1 [day]	α_i	β_i [day]		rms [$\times 10^{-3}$]	DKS	Q
4.5 Chuetsu	$P_{lw} (i = 2)$	0.75 ± 0.01	27.6 ± 0.40	—	—	1	21	0.06	0.94
	$P_{pow} (i = 3)$	0.75 ± 0.01	27.6 ± 0.40	—	—	1	21	0.06	0.94
	$P_{gam} (i = 4)$	0.75 ± 0.01	27.6 ± 0.40	—	—	1	21	0.06	0.94
	$P_{ln} (i = 5)$	0.75 ± 0.01	27.6 ± 0.40	—	—	1	21	0.06	0.94
4.0 Chuetsu	$P_{lw} (i = 2)$	0.81 ± 0.01	10.6 ± 0.13	—	—	1	17	0.03	1
	$P_{pow} (i = 3)$	0.81 ± 0.01	10.6 ± 0.13	—	—	1	17	0.03	1
	$P_{gam} (i = 4)$	0.81 ± 0.01	10.6 ± 0.13	—	—	1	17	0.03	1
	$P_{ln} (i = 5)$	0.81 ± 0.01	10.6 ± 0.13	—	—	1	17	0.03	1
3.5 Chuetsu	$P_{lw} (i = 2)$	0.89 ± 0.006	4.84 ± 0.02	2.08 ± 0.06	8.70 ± 0.19	0.93 ± 0.02	6.1	0.03	1
	$P_{pow} (i = 3)$	0.89 ± 0.006	4.84 ± 0.02	1.66 ± 0.04	0.62 ± 0.05	0.98 ± 0.009	6.3	0.03	1
	$P_{gam} (i = 4)$	0.89 ± 0.006	4.84 ± 0.02	—	—	1	6.9	0.04	1
	$P_{ln} (i = 5)$	0.89 ± 0.006	4.84 ± 0.02	1.12 ± 0.02	1.11 ± 0.02	0.90 ± 0.04	6.2	0.03	1
3.0 Chuetsu	$P_{lw} (i = 2)$	1.06 ± 0.02	2.00 ± 0.06	1.90 ± 0.16	5.13 ± 0.39	0.82 ± 0.03	3.9	0.012	1
	$P_{pow} (i = 3)$	1.09 ± 0.008	2.14 ± 0.01	1.98 ± 0.04	0.52 ± 0.02	0.92 ± 0.009	5.0	0.02	1
	$P_{gam} (i = 4)$	1.09 ± 0.008	2.14 ± 0.01	—	—	1	6.5	0.03	1
	$P_{ln} (i = 5)$	1.09 ± 0.008	2.14 ± 0.01	0.39 ± 0.008	0.91 ± 0.02	0.63 ± 0.03	3.7	0.014	1
2.5 Chuetsu	$P_{lw} (i = 2)$	1.48 ± 0.02	1.21 ± 0.009	1.19 ± 0.02	1.91 ± 0.02	0.62 ± 0.008	2.5	0.01	1
	$P_{pow} (i = 3)$	1.56 ± 0.03	1.16 ± 0.009	2.53 ± 0.04	0.49 ± 0.008	0.71 ± 0.01	6.4	0.03	1
	$P_{gam} (i = 4)$	1.56 ± 0.03	1.16 ± 0.009	1.03 ± 0.009	1.09 ± 0.03	0.99 ± 0.04	15	0.01	0.38
	$P_{ln} (i = 5)$	—	—	-0.11 ± 0.003	0.64 ± 0.004	0	5.1	0.04	1
2.0 Chuetsu	$P_{lw} (i = 2)$	1.85 ± 0.03	0.80 ± 0.009	1.18 ± 0.02	1.43 ± 0.01	0.50 ± 0.01	2.2	0.007	1
	$P_{pow} (i = 3)$	2.46 ± 0.09	0.78 ± 0.007	3.51 ± 0.06	0.48 ± 0.004	0.47 ± 0.03	8.2	0.02	1
	$P_{gam} (i = 4)$	2.46 ± 0.09	0.78 ± 0.007	1.19 ± 0.05	0.69 ± 0.02	0.97 ± 0.05	2.6	0.12	0.46
	$P_{ln} (i = 5)$	—	—	-0.40 ± 0.005	0.40 ± 0.008	0	14	0.06	0.99

Table 5. Interoccurrence time statistics of earthquakes in Chuetsu area. The error bars mean the 95% confidence level of fit.

If there is a parameter, where C_v , the ratio of the standard deviation divided by the mean for a parameter exceeds 0.1, another estimation procedure, (B), is performed.

(B); the Weibull parameters, α_1 and β_1 , and the parameters of $P_X(\tau)$, α_i and β_i , are optimized dependently and then p is estimated.

According to those procedures (A) and (B), we obtain the fitting of results of $P(\tau)$. The results for Okinawa and Chuetsu region are listed in Table 4 and 5. We assume that the Weibull distribution is a fundamental distribution, because p becomes unity for large m_c , which means that the effect of the distribution $P_X(\tau)$ is negligible. As observed in Table 4 and 5, the log-Weibull distribution is the most suitable distribution for the distribution $P_X(\tau)$ according to the two goodness-of-fit tests. Thus, we find that the interoccurrence times distribution can be described by the superposition of the Weibull distribution and the log-Weibull distribution, namely,

$$\begin{aligned} P(\tau) &= p \times \text{Weibull distribution} + (1 - p) \times \text{log-Weibull distribution}, \\ &= p \times P_w + (1 - p) \times P_{lw}, \end{aligned}$$

(15)

$P(\tau)$ is controlled by five parameters, $\alpha_1, \alpha_2, \beta_1, \beta_2$, and p .

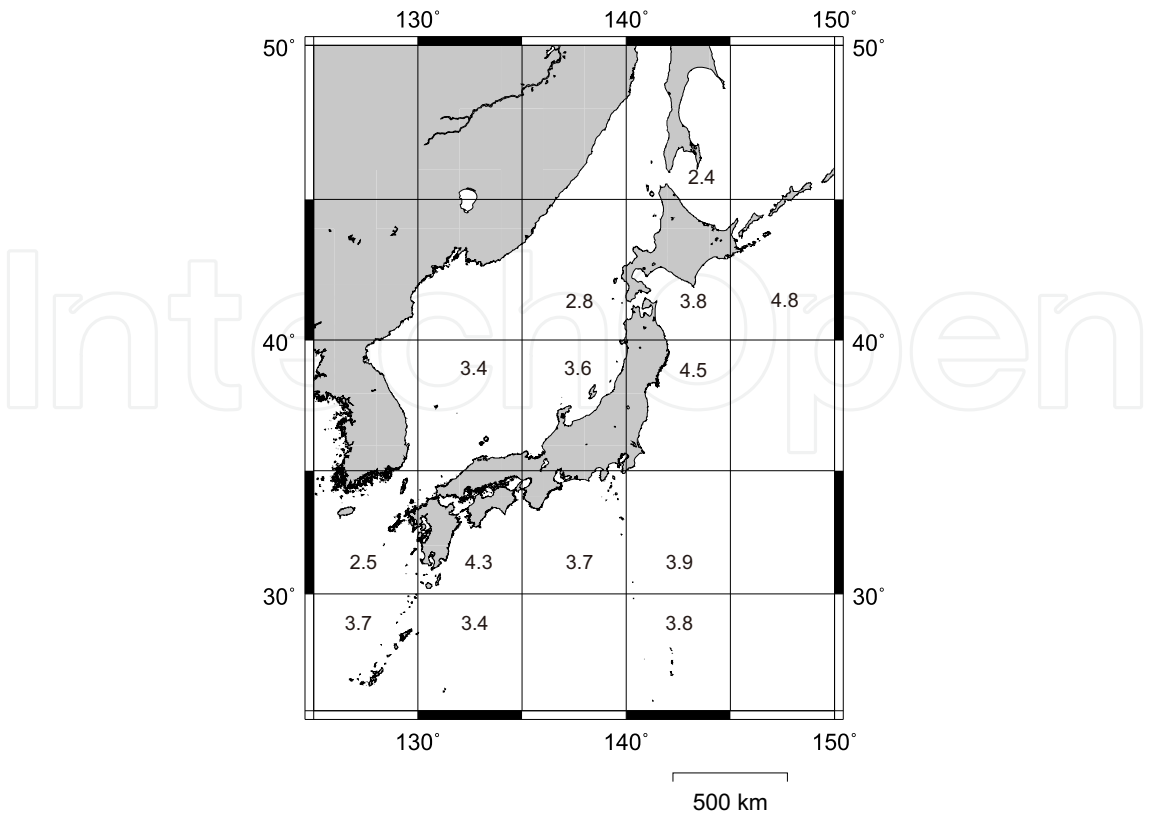


Fig. 7. The crossover magnitude from the superposition regime to the (pure) Weibull regime, denoted m_c^{**} , map around Japan.

It has been shown that the interoccurrence time distribution of earthquakes with large m_c obeys the Weibull distribution with the exponent $\alpha_1 < 1$. As shown in Tables 4 and 5, we stress the point that the distribution function of the interoccurrence time changes by varying m_c . This indicates that the interoccurrence time statistics basically contains both Weibull and log-Weibull statistics, and a dominant distribution function is changed according to the ratio p . In this case, the dominant distribution of the interoccurrence time changes from the log-Weibull distribution to the Weibull distribution when m_c is increased. Thus, the interoccurrence time statistics exhibit transition from the Weibull regime to the log-Weibull regime. The crossover magnitude from the superposition regime to the Weibull regime, denoted by m_c^{**} , depends on the spatial area. We demonstrate the values of m_c^{**} for each regime in Figure 7. As can be seen from the figure, m_c^{**} depends on the region and ranges from 2.4 (140°–145°E and 45°–50°N) to 4.8 (145°–150°E and 40°–45°N). Comparing Figure 1 (a) and Figure 7, we have found that the Weibull - log-Weibull transition occurs in all region, where we analyzed.

3.2 Interoccurrence time statistics in Southern California

Second, we analyze the interoccurrence time statistics using the SCEDC data. The cumulative distributions of interoccurrence time for $m_c = 4.0$ and $m_c = 2.0$ are shown in Figure 8 (a) and (b), respectively. By the rms test and KS test, we confirmed that the Weibull distribution is preferred for large m_c ($m_c = 4.0$) [see Table 6], which is the same result as that from JMA

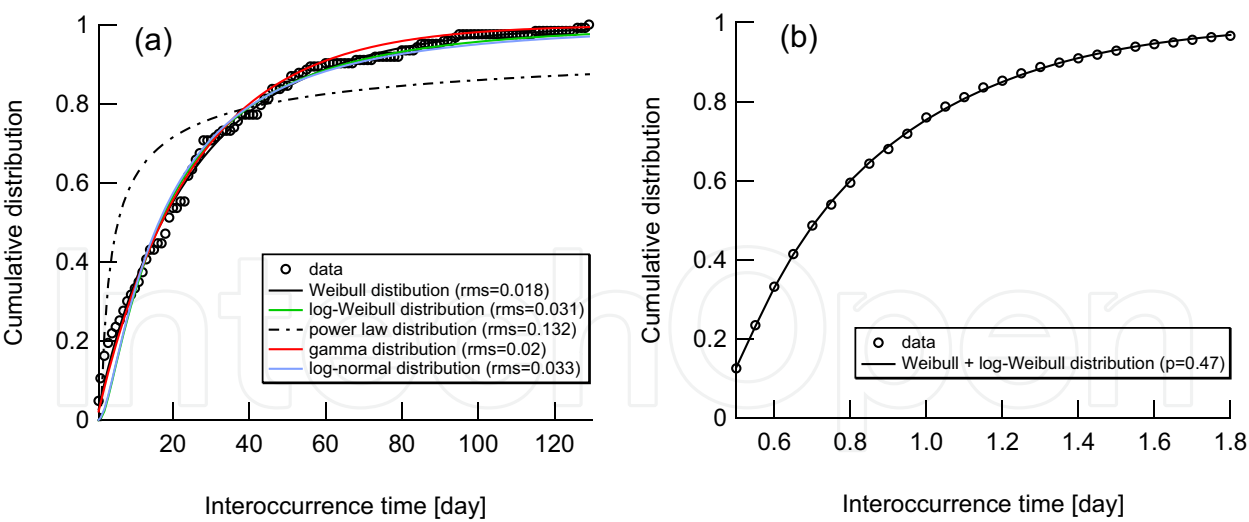


Fig. 8. Cumulative distribution of interoccurrence time in Southern California region different m_c and distribution functions. (a) $m_c = 4.0$ and (b) $m_c = 2.0$.

m_c		Distribution X		RMS test	KS test	
Region	distribution X	α_i	β_i [day]	rms [$\times 10^{-3}$]	DKS	Q
SCEDC	$P_l (i = 1)$	0.91 ± 0.01	25.4 ± 0.21	18	0.07	0.56
	$P_{lw} (i = 2)$	3.66 ± 0.07	48.2 ± 0.68	31	0.14	0.11
	$P_{pow} (i = 3)$	1.44 ± 0.02	1.15 ± 0.15	133	0.49	1.6×10^{-9}
	$P_{gam} (i = 4)$	0.97 ± 0.004	25.6 ± 0.22	20	0.08	0.36
	$P_{ln} (i = 5)$	2.79 ± 0.02	1.10 ± 0.02	33	0.13	0.02

Table 6. Results of rms value, DKS, and Q for different distribution functions in the case of $m_c = 4.0$ for Southern California earthquakes. The error bars mean the 95% confidence level of fit.

data. Unfortunately, the fitting accuracy of the Weibull distribution gets worse by decreasing a threshold m_c . We propose the following hypothesis, "the interoccurrence time distribution can be described by the superposition of the Weibull distribution and the distribution $P_X(\tau)$ " which is the same in 3.1. As shown in Table 7, the log-Weibull distribution is the most suitable for the distribution $P_X(\tau)$, because the smallest rms-value, the smallest DKS value and the largest Q value can be obtained. Therefore, we find that the Weibull - log-Weibull transition shows in Southern California earthquakes. The crossover magnitude m_c^{**} is estimated to be 3.3.

3.3 Interoccurrence time statistics in Taiwan

Finally, the TCWB data was analyzed to investigate the interoccurrence time statistics in Taiwan. Figure 9 shows the cumulative distribution of interoccurrence time for $m_c = 4.5$ and $m_c = 3.0$, respectively. For large m_c , the Weibull distribution is preferred on the basis of the rms and KS test [see Table 8]. As the threshold of magnitude m_c decreases, the fitting accuracy of the Weibull distribution is getting worse, as is common in JMA and SCEDC. According to a hypothesis that the interoccurrence time distribution can be described by the superposition of

m_c		Weibull distribution		Distribution X		p	RMS test	KS test	
Region	distribution X	α_1	β_1 [day]	α_i	β_i [day]		rms [$\times 10^{-3}$]	DKS	Q
4.0 SCEDC	$P_{lw} (i = 2)$	0.91 ± 0.01	25.4 ± 0.21	—	—	1	18	0.07	0.56
	$P_{pow} (i = 3)$	0.91 ± 0.01	25.4 ± 0.21	—	—	1	18	0.07	0.56
	$P_{gam} (i = 4)$	0.91 ± 0.01	25.4 ± 0.21	—	—	1	18	0.07	0.56
	$P_{ln} (i = 5)$	0.91 ± 0.01	25.4 ± 0.21	—	—	1	18	0.07	0.56
3.5 SCEDC	$P_{lw} (i = 2)$	0.83 ± 0.006	9.29 ± 0.05	—	—	1	11	0.03	1
	$P_{pow} (i = 3)$	0.83 ± 0.006	9.29 ± 0.05	—	—	1	11	0.03	1
	$P_{gam} (i = 4)$	0.83 ± 0.006	9.29 ± 0.05	—	—	1	11	0.03	1
	$P_{ln} (i = 5)$	0.83 ± 0.006	9.29 ± 0.05	—	—	1	11	0.03	1
3.0 SCEDC	$P_{lw} (i = 2)$	1.01 ± 0.01	3.08 ± 0.04	1.37 ± 0.07	3.49 ± 0.19	0.80 ± 0.01	4.1	0.01	1
	$P_{pow} (i = 3)$	0.98 ± 0.008	2.85 ± 0.02	1.85 ± 0.03	0.58 ± 0.03	0.91 ± 0.009	6.1	0.03	1
	$P_{gam} (i = 4)$	—	—	0.99 ± 0.002	2.84 ± 0.01	1	8.2	0.05	1
	$P_{ln} (i = 5)$	0.98 ± 0.008	2.85 ± 0.02	0.63 ± 0.009	1.00 ± 0.01	0.60 ± 0.03	5.0	0.03	1
2.5 SCEDC	$P_{lw} (i = 2)$	1.32 ± 0.01	1.72 ± 0.01	1.35 ± 0.01	2.40 ± 0.02	0.57 ± 0.005	2.0	0.01	1
	$P_{pow} (i = 3)$	1.33 ± 0.02	1.55 ± 0.009	2.27 ± 0.03	0.54 ± 0.008	0.76 ± 0.01	6.4	0.04	0.96
	$P_{gam} (i = 4)$	1.33 ± 0.02	1.17 ± 0.009	—	—	1	12	0.10	0.04
	$P_{ln} (i = 5)$	—	—	0.13 ± 0.002	0.75 ± 0.003	0	4.4	0.04	0.92
2.0 SCEDC	$P_{lw} (i = 2)$	1.88 ± 0.03	0.92 ± 0.008	1.15 ± 0.02	1.56 ± 0.01	0.47 ± 0.008	2.7	0.007	1.1
	$P_{pow} (i = 3)$	2.18 ± 0.07	0.88 ± 0.008	3.14 ± 0.05	0.49 ± 0.005	0.52 ± 0.02	8.3	0.11	0.88
	$P_{gam} (i = 4)$	2.18 ± 0.07	0.88 ± 0.008	1.15 ± 0.04	0.80 ± 0.02	0.98 ± 0.05	26	0.13	0.72
	$P_{ln} (i = 5)$	—	—	-0.31 ± 0.005	0.46 ± 0.007	0	7.5	0.007	1

Table 7. Interoccurrence time statistics of earthquakes in Southern California area. The error bars mean the 95% confidence level of fit.

the Weibull distribution and another distribution, we investigate the distribution $P_X(\tau)$. As shown in Table 9, the log-Weibull distribution is preferred as the distribution on the basis of the two goodness-of-fit tests. We estimated the crossover magnitude m_c^{**} to be 4.9.

3.4 Brief summary of the interoccurrence time statistics for earthquakes

Taken all together, we clarified that distribution of interoccurrence time is well fitted by the superposition of the Weibull distribution and log-Weibull distribution. For large m_c , $P(\tau)$ obeys the Weibull distribution with $\alpha_1 < 1$, indicating that the occurrence of earthquakes is not a Poisson process. When the threshold of magnitude m_c decreases, the ratio of the Weibull distribution of $P(\tau)$ gradually increases. We suggest that the Weibull statistics and log-Weibull statistics coexist in interoccurrence time statistics, where the change of the distribution means the change of a dominant distribution. In this case, the dominant distribution changes from the log-Weibull distribution to the Weibull distribution by increasing the m_c . It follows that the Weibull - log-Weibull transition exists in Japan, Southern California, and Taiwan.

4. Discussion

4.1 Size dependency

To investigate the region-size, L dependency of the Weibull - log-Weibull transition, we change the window size L is varied from 3° to 25° (17). In (17), we use JMA data we used is from

m_c		Distribution X		RMS test	KS test	
Region	distribution X	α_i	β_i [day]	rms [$\times 10^{-3}$]	DKS	Q
5.0 TCWB	$P_l (i = 1)$	0.86 ± 0.006	15.1 ± 0.08	8.4	0.02	1
	$P_{lw} (i = 2)$	3.01 ± 0.06	27.9 ± 0.42	24	0.10	0.24
	$P_{pow} (i = 3)$	1.51 ± 0.03	0.93 ± 0.11	111	0.25	3.6×10^{-6}
	$P_{gam} (i = 4)$	0.97 ± 0.003	15.3 ± 0.14	16	0.05	0.95
	$P_{ln} (i = 5)$	2.23 ± 0.02	1.15 ± 0.02	24	0.08	0.53

Table 8. Results of rms value, DKS, and Q for different distribution functions in the case of $m_c = 5.0$ for Taiwan earthquakes. The error bars mean the 95% confidence level of fit.

m_c		Weibull distribution		Distribution X			RMS test	KS test	
Region	distribution X	α_1	β_1 [day]	α_i	β_i [day]	p	rms [$\times 10^{-3}$]	DKS	Q
5.0 TCWB	$P_{lw} (i = 2)$	0.86 ± 0.006	15.0 ± 0.08	—	—	1	8.4	0.02	1
	$P_{pow} (i = 3)$	0.86 ± 0.006	15.0 ± 0.08	—	—	1	8.4	0.02	1
	$P_{gam} (i = 4)$	0.86 ± 0.006	15.0 ± 0.08	—	—	1	8.4	0.02	1
	$P_{ln} (i = 5)$	0.86 ± 0.006	15.0 ± 0.08	—	—	1	8.4	0.02	1
4.5 TCWB	$P_{lw} (i = 2)$	0.88 ± 0.004	5.34 ± 0.02	—	—	1	4.3	0.01	1
	$P_{pow} (i = 3)$	0.88 ± 0.004	5.34 ± 0.02	—	—	1	4.3	0.01	1
	$P_{gam} (i = 4)$	0.88 ± 0.004	5.34 ± 0.02	—	—	1	4.3	0.01	1
	$P_{ln} (i = 5)$	0.88 ± 0.004	5.34 ± 0.02	—	—	1	4.3	0.01	1
4.0 TCWB	$P_{lw} (i = 2)$	1.00 ± 0.01	2.30 ± 0.04	1.82 ± 0.05	4.06 ± 0.13	0.68 ± 0.01	2.8	0.009	1
	$P_{pow} (i = 3)$	1.08 ± 0.01	2.30 ± 0.02	1.95 ± 0.04	0.55 ± 0.02	0.89 ± 0.01	6.9	0.03	1
	$P_{gam} (i = 4)$	1.08 ± 0.01	2.30 ± 0.02	—	—	1	10	0.05	0.99
	$P_{ln} (i = 5)$	1.08 ± 0.01	2.30 ± 0.02	0.46 ± 0.005	0.92 ± 0.006	0.39 ± 0.02	2.8	0.007	1
3.5 TCWB	$P_{lw} (i = 2)$	1.44 ± 0.03	1.29 ± 0.02	1.32 ± 0.05	2.15 ± 0.05	0.61 ± 0.02	4.5	0.01	1
	$P_{pow} (i = 3)$	1.52 ± 0.03	1.25 ± 0.01	2.41 ± 0.05	0.50 ± 0.01	0.74 ± 0.02	8.6	0.07	0.94
	$P_{gam} (i = 4)$	1.52 ± 0.03	1.25 ± 0.01	—	—	1	18	0.09	0.73
	$P_{ln} (i = 5)$	—	—	-0.04 ± 0.003	0.66 ± 0.004	0	5.4	0.03	1
3.0 TCWB	$P_{lw} (i = 2)$	2.07 ± 0.06	0.75 ± 0.009	1.18 ± 0.03	1.47 ± 0.01	0.51 ± 0.01	3.2	0.01	1
	$P_{pow} (i = 3)$	2.62 ± 0.09	0.78 ± 0.006	3.58 ± 0.08	0.48 ± 0.005	0.53 ± 0.02	7.5	0.02	1
	$P_{gam} (i = 4)$	2.62 ± 0.09	0.78 ± 0.006	1.26 ± 0.08	0.66 ± 0.03	0.97 ± 0.05	23	0.10	0.76
	$P_{ln} (i = 5)$	—	—	-0.41 ± 0.004	0.38 ± 0.006	0	12	0.05	1
2.5 TCWB	$P_{lw} (i = 2)$	5.35 ± 0.41	0.60 ± 0.005	0.86 ± 0.05	1.16 ± 0.007	0.43 ± 0.04	16	0.037	1
	$P_{pow} (i = 3)$	5.35 ± 0.41	0.60 ± 0.005	6.54 ± 0.09	0.49 ± 0.001	0.10 ± 0.04	10	0.041	1
	$P_{gam} (i = 4)$	5.35 ± 0.41	0.60 ± 0.005	1.69 ± 2.04	0.33 ± 0.61	0.94 ± 0.07	46	0.17	0.41
	$P_{ln} (i = 5)$	—	—	-0.58 ± 0.006	0.19 ± 0.01	0	35	0.11	0.87

Table 9. Interoccurrence time statistics of earthquakes in Taiwan area. The error bars mean the 95% confidence level of fit.

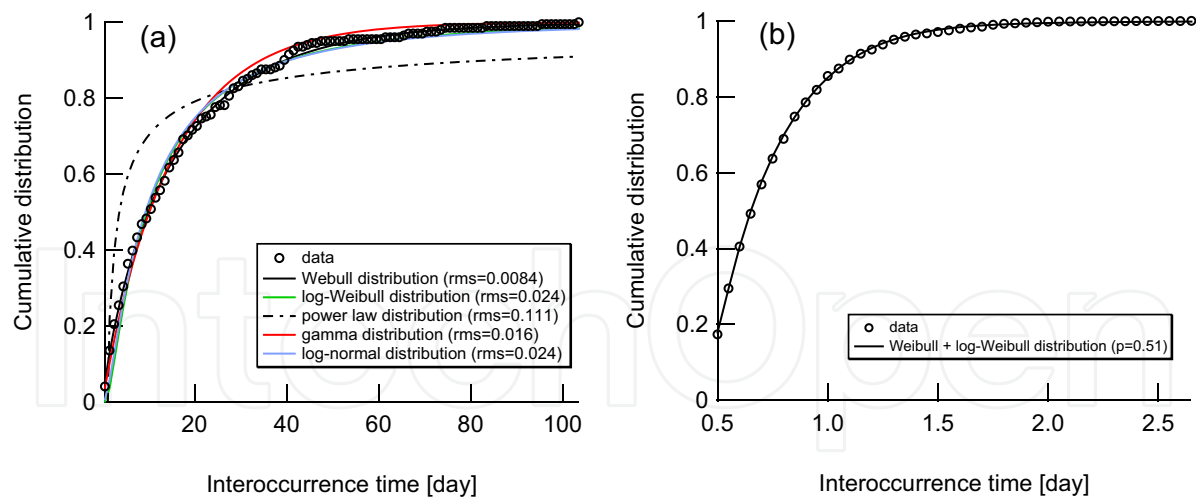


Fig. 9. Cumulative distribution of interoccurrence time for Taiwan for different m_c and distribution function. (a) and (b) represent interoccurrence time when $m_c = 4.5$ and $m_c = 3.0$, respectively.

L	Region	m_c	α_1	β_1 [day]	rms	m_c^{**}
$L = 3$	140°-143° E and 35°-38° N	3.9	0.88 ± 0.01	19.4 ± 0.18	0.011	4.6
$L = 5$	140°-145° E and 35°-40° N	4.0	0.75 ± 0.02	10 ± 0.19	0.014	4.7
$L = 10$	140°-150° E and 35°-45° N	4.2	0.94 ± 0.005	8.36 ± 0.04	0.0077	4.9
$L = 25$	125°-150° E and 25°-50° N	5.0	0.93 ± 0.01	17.8 ± 0.10	0.041	5.7

Table 10. Interoccurrence time statistics for different system size L by analyzing the JMA data from 1st January 2001 to 31st October 2007 (17).

1st January 2001 to 31st October 2007. We use the data covering the region 140°-143° E and 35°-38° N for $L = 3$, 140°-145° E and 35°-40° N for $L = 5$, 140°-150° E and 35°-45° N for $L = 10$, and 125°-150° E and 25°-50° N for $L = 25$. As for $L = 25$, the data covers the whole region of the JMA catalogue. The result of fitting parameters of $P(\tau)$, the crossover magnitude m_c^{**} , and the rms value are listed in Table 10. It is demonstrated that in all the cases, the Weibull exponent α_1 is less than unity and the Weibull - log-Weibull transition appears. m_c^{**} depends on L , namely $m_c^{**} = 3.9$ for $L = 3$, $m_c^{**} = 4.0$ for $L = 5$, $m_c^{**} = 4.2$ for $L = 10$, and $m_c^{**} = 5.0$ for $L = 25$. Therefore we can conclude that the interoccurrence time statistics, namely the Weibull - log-Weibull transition, presented here hold from $L = 3$ to $L = 25$.

4.2 Relation between the m_c^{**} and m_{max}

To study the feature of the Weibull - log-Weibull transition, we summarize our results obtained from 16 different regions (14 regions in Japan, Southern California, and Taiwan.) Interestingly, m_c^{**} is proportional to the maximum magnitude of an earthquake in a region, where we analyzed, denoted here m_{max} [see Figure 10]. We then obtain a region-independent relation between m_c^{**} and m_{max} ,

$$m_c^{**} / m_{max} = 0.56 \pm 0.08$$

(16)

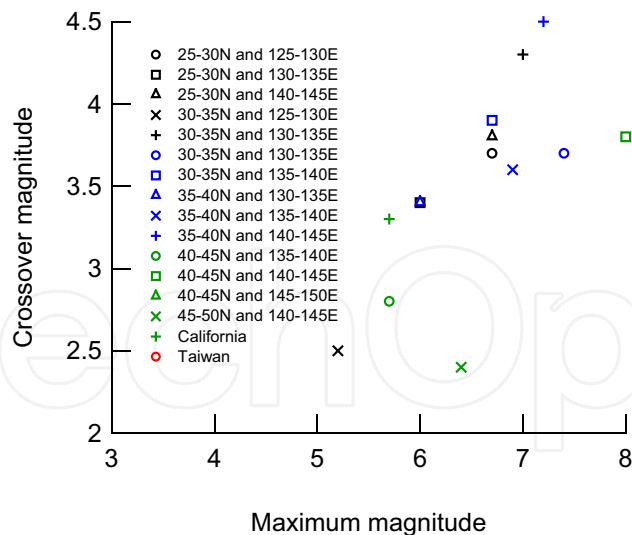


Fig. 10. The crossover magnitude m_c^{**} v.s the maximum magnitude m_{max} in 16 regions (14 region in Japan, Southern California, and Taiwan).

Region	relative plate motion	velocity [mm/yr]	m_c^{**}
Taiwan	PH-EU	71	4.90
East Japan	PA-PH	49	4.07 ¹
West Japan	PH-EU	47	3.74 ²
California	PA-NA	47	3.30

Table 11. List of the crossover magnitude, m_c^{**} and the plate velocity (32; 33). The notation of PH, EU, PA, and NA represent PHilippine Sea plate, EUrasian plate, PAcific plate, and North American plate, respectively.

1 We take an average using three regions; 25°–30°N, 140°–145°E ($m_c^{**} = 3.8$), 30°–35°N, 140°–145°E ($m_c^{**} = 3.9$), 35°–40°N, 140°–145°E ($m_c^{**} = 4.5$).

2 We take an average using five regions; 25°–30°N, 125°–130°E ($m_c^{**} = 3.7$), 25°–30°N, 130°–135°E ($m_c^{**} = 3.4$), 30°–35°N, 130°–135°E ($m_c^{**} = 4.3$), 30°–35°N, 135°–140°E ($m_c^{**} = 3.7$), 35°–40°N, 135°–140°E ($m_c^{**} = 3.6$).

This relation can be useful to interpret the Weibull - log-Weibull transition of geophysical meaning.

4.3 Interpretation of the Weibull - log-Weibull transition

Although the scaled crossover magnitudes m_c^{**}/m_{max} is region-independent, the crossover magnitude m_c^{**} from the superposition regime to the pure Weibull regime probably depends on the tectonic region (Figure 7). To investigate the Weibull - log-Weibull transition further, we consider the plate velocity with m_c^{**} , which can shed light on the geophysical implication of the region-dependent m_c^{**} . As shown in Table 11, m_c^{**} is on the average proportional to the plate velocity. That means that the maximum magnitude m_{max} for a tectonic region is more or less proportional to the plate velocity since $m_c^{**}/0.56 = m_{max}$. Such an interesting

consequence is reminiscent of the early study by Ruff and Kanamori (1980) (34). They showed a relation that the magnitude of characteristic earthquake occurred in the subduction-zone, M_w is directly proportional to the plate-velocity, V , and is directly inversely proportional to plate-age T , namely,

$$M_w = -0.000953T + 1.43V + 8.01. \quad (17)$$

The relation $m_c^{**}/0.56 = m_{max}$ can thus be explained on the basis of their early observation about the velocity-dependence of the characteristic earthquake magnitude. The physical interpretation of the Weibull - log-Weibull transition remains open. However, it might suggest that the occurrence mechanism of earthquake could probably depend on its magnitude then, inevitably, the distribution of the interoccurrence time statistics changes as the threshold of magnitude m_c is varied. It is well known that the Weibull distribution for life-time of materials can be derived in the framework of damage mechanics (4; 24; 35–37). Our present results thus suggest that larger earthquakes might be caused by the damage mechanism driven by the plate motion, whereas the effect of the plate-driven damaging process might become minor for smaller earthquakes. Hence, the transition from the Weibull regime to the log-Weibull regime could be interpreted from the geophysical sense as the decrement of the plate-driven damaging mechanics.

4.4 A universal relation and intrinsic meanings of the Gutenberg-Richter parameter

Here we consider the interrelation between the Gutenberg-Richter law, denoted in this subsection $P(m) \propto e^{-bm}$ and the Weibull distribution for the interoccurrence time ($P(\tau) \propto t^{-\alpha-1} \cdot e^{-(\tau/\beta)^\alpha}$). We assume that these two statistics are correct over wide ranges, and the parameters (α, β) are depending on the magnitude, i.e., $\alpha(m)$ and $\beta(m)$, then the following relation is easily obtained from the calculation of the mean interoccurrence time between two earthquakes whose magnitude is larger than m ,

$$\beta(m_1)e^{-bm_1}\Gamma\left(1 + \frac{1}{\alpha_1}\right) = \beta(m_2)e^{-bm_2}\Gamma\left(1 + \frac{1}{\alpha_2}\right), \quad (18)$$

where m_1 and m_2 are arbitrary values of m . This implies that the quantity defined by $\beta(m_1)e^{-bm_1}\Gamma\left(1 + \frac{1}{\alpha_1}\right)$ is a universal constant when we consider the local earthquakes in a relatively small area.

One of the most important results derived from equation (18) is that the GR parameter b is determined by two parameters, in other words, the parameters (α, β) depend on the magnitude m as well as on the GR parameter b ,

$$\begin{aligned} \alpha &= f_\alpha(m, b) \\ \beta &= f_\beta(m, b), \end{aligned} \quad (19)$$

where the functional forms of f_α and f_β characterize the time series of earthquakes under consideration.

It is difficult to determine those forms completely from any seismological relations known so far, but it is possible for us to obtain the universal aspects of f_α and f_β by a perturbational

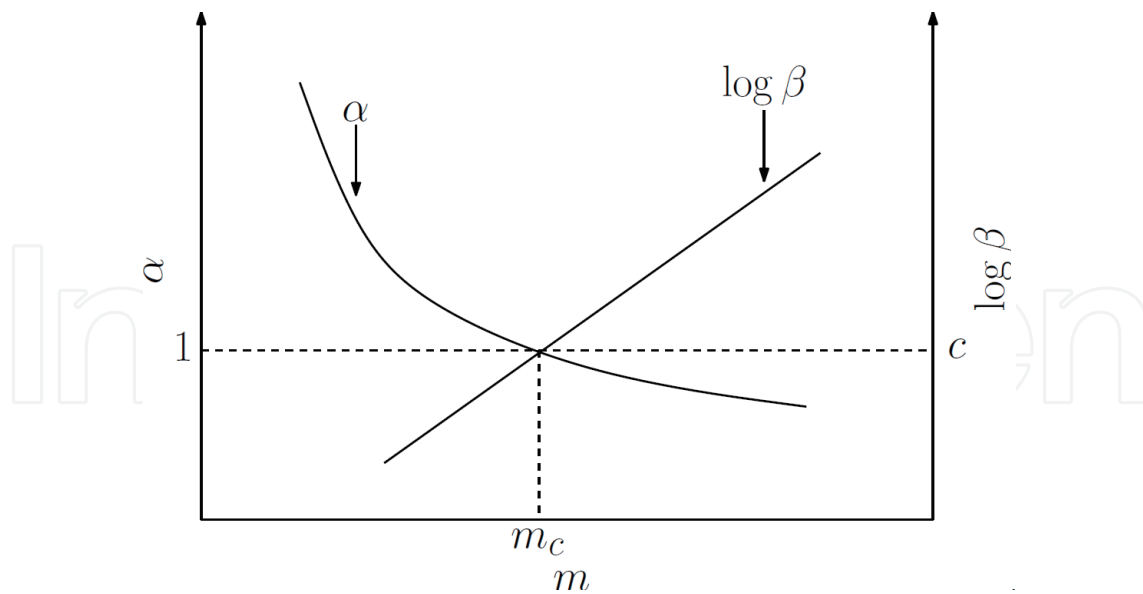


Fig. 11. Schematic picture of universal behavior of $f_\alpha(m, b)$ and $f_\beta(m, b)$ near $m \simeq m_c$.

approach. Here we consider a particular solution of equation (19) which satisfies the following conditions; $f_\beta(m, b) = \exp[b(m - m_c) + c]$ and $f_\alpha(m_c, b)$, namely, the characteristic time β is an exponentially increasing function of m , and the interoccurrence time distribution is an exponential one ($\alpha = 1$) at $m = m_c$, where b' and c are constant parameters. By use of this simplification, equation (18) is rewritten by putting $m_1 = m$ and $m_2 = m$,

$$\begin{aligned} (b' - b)(m - m_c) &= -\log \Gamma \left(1 + \frac{1}{\alpha(m)} \right) \\ &\cong \frac{1}{2}\Delta - \frac{3}{4}(\Delta)^2 + \dots, \quad (\Delta = \alpha(m) - 1). \end{aligned} \quad (20)$$

Here we used the Taylor expansion near $m \cong m_c$ (i.e., $\alpha(m) \cong \alpha(m_c)$). Figure 11 shows the schematic result of equation (20). One can see that the universal relation is recognized in many cases treated in this chapter (section 4.3.3.), though the exponential growth of β , $\log \beta(m) \simeq b'(m - m_c) + c$ is a little bit accelerated.

We have to remind that the solution mentioned above is not unique, but many other solutions for equation (19) are possible under the universal relation of equation (18). Further details will be studied in our forthcoming paper (38).

4.5 Comparison with previous works

Finally, we compared our results with those of previous studies. The unified scaling law shows a generalized gamma distribution [see in equations (2), and (3)] which is approximately the gamma distribution, because δ in Corral's paper (6) is close to unity ($\delta = 0.98 \pm 0.05$). For a long time domain, this distribution decays exponentially, supporting the view that an earthquake is a Poisson process. However, we have demonstrated that the Weibull distribution is more appropriate than the gamma distribution on the basis of two goodness-fit-tests. In addition, for large m_c , the distribution in a long time domain is similar

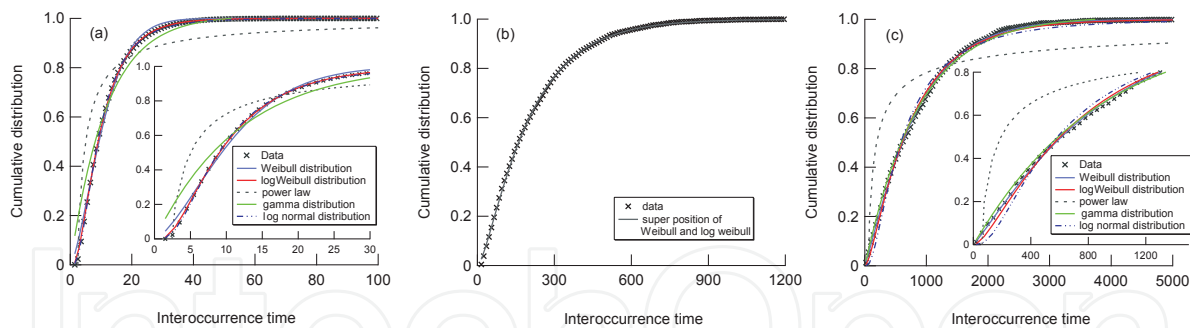


Fig. 12. Interoccurrence time statistics for different magnitude m_c by analyzing catalogue produced by the two-dimensional spring-block model (29).

to the stretched exponential distribution because α_1 is less than unity, suggesting that an occurrence of earthquake is not a Poisson process but has a memory. We provide the first evidence that the distribution changes from the Weibull to log-Weibull distribution by varying m_c , i.e., the Weibull - log-Weibull transition. Recently, Abaimov *et al.* showed that the recurrence time distribution is also well-fitted by the Weibull distribution (4) rather than the Brownian passage time (BPT) distribution (23) and the log normal distribution. Taken together, we infer that both the recurrence time statistics and the interoccurrence time statistics show the Weibull distribution.

In this chapter, we propose a new insight into the interoccurrence time statistics, stating that the interoccurrence statistics exhibit the Weibull - log-Weibull transition by analyzing the different tectonic settings, JMA, SCEDC, and TCWB. This stresses that the distribution function can be described by the superposition of the Weibull distribution and the log-Weibull distribution, and that the predominant distribution function changes from the log-Weibull distribution to the Weibull distribution as m_c is increased. Note that there is a possibility that a more suitable distribution might be found instead of the log-Weibull distribution. Furthermore, the Weibull - log-Weibull transition can be extracted more clearly by analyzing synthetic catalogs produced by the spring-block model [see Figure 12] (29).

5. Conclusion

In conclusion, we have proposed a new feature of interoccurrence time statistics by analyzing the Japan (JMA), Southern California, (SCEDC), and Taiwan (TCWB) for different tectonic conditions. We found that the distribution of the interoccurrence time can be described clearly by the superposition of the Weibull distribution and the log-Weibull distribution. Especially for large earthquakes, the interoccurrence time distribution obeys the Weibull distribution with the exponent $\alpha_1 < 1$, indicating that a large earthquake is not a Poisson process but a phenomenon exhibiting a long-tail distribution. As the threshold of magnitude m_c increases, the ratio of the Weibull distribution in the interoccurrence time distribution p gradually increases. Our findings support the view that the Weibull statistics and log-Weibull statistics coexist in the interoccurrence time statistics. We interpret the change of distribution function as the change of the predominant distribution function; the predominant distribution changes from the log-Weibull distribution to the Weibull distribution when m_c is increased. Therefore, we concluded that the interoccurrence time statistics exhibit a Weibull - log-Weibull

transition. We also find the region-independent relation, namely, $m_c^{**}/m_{max} = 0.56 \pm 0.08$. In addition, the crossover magnitude m_c^{**} is proportional to the plate velocity, which is consistent with an earlier observation about the velocity-dependence of the characteristic earthquake magnitude (34). Although the origins of both the log-Weibull distribution and the Weibull - log-Weibull transition remain open questions, we suggest the change in the distribution from the log-Weibull distribution to the Weibull distribution can be considered as the enhancement in the plate-driven damaging mechanics. We believe that this work is a first step toward a theoretical and geophysical understanding of this transition.

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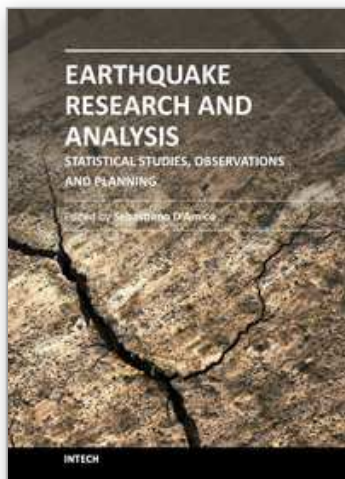
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