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Correspondences of Scale Relativity Theory with Quantum Mechanics

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1. Introduction

We perform a critical analysis of some quantum mechanical models such as the hydrodynamic model (Madelung's model), de Broglie's theory of double solution etc., specifying both mathematical and physical inconsistencies that occur in their construction.
These inconsistencies are eliminated by means of the fractal approximation of motion (physical objects moving on continuous and non-differentiable curves, i.e. fractal curves) developed in the framework of Scale Relativity (SR) (Nottalle, L., 1993; Chaline, J. et al, 2009; Chaline, J. et al, 2000; Nottale, L., 2004; Nottale, L. & Schneider J., 1984; Nottale, L., 1989; Nottale, L., 1996). The following original results are obtained: i) separation of the physical motion of objects in wave and particle components depending on the scale of resolution (differentiable as waves and non-differentiable as particles) - see paragraphs 5-7; ii) solidar motion of the wave and particle (wave-particle duality) - see paragraph 8, the mechanisms of duality (in phase wave-particle coherence, paragraphs 9 and 10 and wave-particle incoherence, see paragraph 11); iii) the particle as a clock, its incorporation into the wave and the implications of such a process - see paragraphs 12 and 13; iv) Lorentz-type mechanisms of wave-particle duality - see paragraph 14.
The original results of this work are published in references (Harabagiu A. et al , 2010; Agop, M. et al, 2008; Harabagiu, A. & Agop, M., 2005;Harabagiu, A. et al, 2009; Agop, M. et al, 2008). Explicitely, Eulerian's approximation of motions on fractal curves is presented in (Agop, M. et al, 2008), the hydrodynamic model in a second order approximation of motion in (Harabagiu, A. & Agop, M., 2005), wave-particle duality for „coherent" fractal fluids with the explanation of the potential gap in (Harabagiu, A. et al, 2009), the physical self-consistence of wave-particle duality in various approximations of motion and for various fractal curves in (Agop, M. et al, 2008). A unitary treatment of both the problems listed above and their various mathematical and physical extensions are developed in (Harabagiu A. et al , 2010).

2. Hydrodynamic model of quantum mechanics (Madelung's model)

Quantum mechanics is substantiated by the Schrődinger wave equation (Țițeica, S., 1984; Felsager, B., 1981; Peres, A., 1993; Sakurai J.J. & San Fu Taun, 1994)

$$i\hbar\frac{\partial\Psi}{\partial t}=U\Psi-\frac{\hbar^2}{2m_0}\Delta\Psi \tag{1}$$

where $\hbar$ is the reduced Planck's constant, m_0 the rest mass of the test particle, U the external scalar field and Ψ the wave-function associated to the physical system. This differential equation is linear and complex.
Starting from this equation, Madelung (Halbwacs, F., 1960; Madelung R., 1927) constructed the following model. One separates real and imaginary parts by choosing Ψ of the form:

$$\Psi(\mathbf{r},t)=R(\mathbf{r},t)e^{iS(\mathbf{r},t)} \tag{2}$$

which induces the velocity field:

$$\mathbf{v}=\frac{\hbar}{m_0}\nabla S \tag{3}$$

and the density of the probability field:

$$\rho(\mathbf{r},t)=R^2(\mathbf{r},t) \tag{4}$$

Using these fields one gets the hydrodynamic version of quantum mechanics (Madelung's model)

$$\frac{\partial}{\partial t}(m_0\rho\mathbf{v})+\nabla(m_0\rho\mathbf{v}\mathbf{v})=-\rho\nabla(U+Q) \tag{5}$$

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{v})=0 \tag{6}$$

where

$$Q=-\frac{\hbar^2}{2m_0}\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}} \tag{7}$$

is called the quantum potential. Equation (5) corresponds to the momentum conservation law and equation (6) to the conservation law of the probability's density field (quantum hydrodynamics equations).
We have the following: i) any micro-particle is in constant interaction with an environment called „subquantic medium" through the quantum potential Q, ii) the „subquantic medium" is identified with a nonrelativistic quantum fluid described by the equations of quantum hydrodynamics. In other words, the propagation of the Ψ field from wave mechanics is replaced by a fictitious fluid flow having the density ρ and the speed $\mathbf{v}$, the fluid being in a field of forces $\nabla(U+Q)$. Moreover, the following model of particle states (Bohm D. & Hiley B.J., 1993; Dörr D. et al,1992; Holland P.R., 1993; Albert D.Z., 1994; Berndl K. et al, 1993; Berndl K. et al, 1994; Bell J.S., 1987; Dörr D. et al, 1993): Madelung type fluid in „interaction" with its own „shell" (there is no space limitation of the fluid, though of the particle).

3. DeBroglie's theory of double solution. The need for introducing the model of Bohm and Vigier

One of the key observations that de Broglie left in the development of quantum mechanics, is the difference between the relativistic transformation of the frequency of a wave and that of a clock's frequency (de Broglie L., 1956; de Broglie L., 1957; de Broglie L., 1959; de Broglie L., 1963; de Broglie L., 1964; de Broglie L., 1980). It is well known that, if υ_0 is the frequency of a clock in its own framework, the frequency confered by an observer who sees it passing with the speed $v = \beta c$ is

$$\nu_c = \nu_0 \sqrt{1-\beta^2} \ .$$

This is what is called the phenomenon of "slowing down of horologes". This phenomenon takes place due to the relative motion of horologes. On the contrary, if a wave within a certain reference system is a stationary one, with frequency υ_0 and is noticed in a reference system animated with speed $v = \beta c$, as compared with the first one, it will appear as a progressive wave that propagates in the sense of the relative motion, with frequency

$$\nu = \frac{\nu_0}{\sqrt{1-\beta^2}}$$

and with the phase speed

$$V = \frac{c}{\beta} = \frac{c^2}{v} .$$

If the corpuscle, according to relation $W = h\nu$, is given an internal frequency

$$\nu_0 = \frac{m_o c^2}{h}$$

and if we admit that within the appropriate system of the corpuscle the associated wave is a stationary one, with frequency υ_0, all the fundamental relations of undulatory mechanics and in particular $\lambda = \frac{h}{p}$, in which p is the impulse of the corpuscle, are immediately obtained from the previous relations.

Since de Broglie considers that the corpuscle is constantly located in the wave, he notices the following consequence: the motion of the corpuscle has such a nature that it ensures the permanent concordance between the phase of the surrounding wave and the internal phase of the corpuscle considered as a small horologe. This relation can be immediately verified in the simple case of a corpuscle in uniform motion, accompanied by a monochromatic plain wave. Thus, when the wave has the general form

$$\Psi = A(x,y,z,t)e^{\frac{2\pi i}{h}\Phi(x,y,z,t)}$$

in which A and Φ are real, the phase concordance between the corpuscle and its wave requires that the speed of the corpuscle in each point of its trajectory be given by the relation

$$\mathbf{v} = -\frac{1}{m_0}\nabla\Phi$$

Nevertheless it was not enough to superpose the corpuscle with the wave, imposing it to be guided by the propagation of the wave: the corpuscle had to be represented as being incorporated in the wave, i.e. as being a part of the structure of the wave. De Broglie was thus directed to what he himself called the theory of "double solution". This theory admits that the real wave is not a homogeneous one, that it has a very small area of high concentration of the field that represents the corpuscle and that, besides this very small area, the wave appreciably coincides with the homogeneous wave as formulated by the usual undulatory mechanics.
The phenomenon of guiding the particle by the surrounding undulatory field results from the fact that the equations of the field are not linear ones and that this lack of linearity, that almost exclusively shows itself in the corpuscular area, solidarizes the motion of the particle with the propagation of the surrounding wave (de Broglie L., 1963; de Broglie L., 1964; de Broglie L., 1980).
Nevertheless there is a consequence of "guidance" upon which we should insist. Even if a particle is not submitted to any external field, if the wave that surrounds it is not an appreciably plain and monochromatic one (therefore if this wave has to be represented through a superposition of monochromatic plain waves) the motion that the guidance formula imposes is not rectilinear and uniform. The corpuscle is subjected by the surrounding wave, to a force that curves its trajectory: this "quantum force" equals the gradient with the changed sign of the quantum potential Q given by (7). Therefore, the uniform motion of the wave has to be superposed with a "Brownian" motion having random character that is specific to the corpuscle.
Under the influence of Q, the corpuscle, instead of uniformly following one of the trajectories that are defined by the guidance law, constantly jumps from one of these trajectories to another, thus passing in a very short period of time, a considerably big number of sections within these trajectories and, while the wave remains isolated in a finite area of the space, this zigzag trajectory hurries to explore completely all this region. In this manner, one can justify that the probability of the particle to be present in a volume element $d\tau$ of the physical space is equal to $|\Psi|^2 d\tau$. This is what Bohm and Vigier did in their statement: therefore they showed that the probability of repartition in $|\Psi|^2$ must take place very quickly. The success of this demonstration must be correlated with the characteristics if "Markov's chains."(Bohm, D., 1952; Bohm D. & Hiley B.J., 1993; Bohm D., 1952; Bohm D., 1953).

4. Comments

In his attempt to built the theory of the double solution, de Broglie admits certain assertions (de Broglie L., 1956; de Broglie L., 1957; de Broglie L., 1959; de Broglie L., 1963; de Broglie L., 1964; de Broglie L., 1980):

i. the frequency of the corpuscle that is assimilated to a small horologe must be identified with the frequency of the associated progressive wave;
ii. the coherence of the inner phase of the corpuscle-horologe with the phase of the associated wave;
iii. the corpuscle must be "incorporated" into the progressive associated wave through the "singularity" state. Thus, the motion of the corpuscle "solidarizes" with the propagation of the associated progressive wave. Nevertheless, once we admit these statements, de Broglie's theory does not answer a series of problems, such as, for example:

1. What are the mechanisms through which either the undulatory feature or the corpuscular one impose, either both of them in the stationary case as well as in the non stationary one?;
2. The limits in the wave-corpuscle system of the corpuscular component as well as the undulatory one and their correspondence;
3. How is the "solidarity" between the motion of the corpuscle and the one of the associated progressive wave naturally induced?

iv. What are the consequences of this "solidarity"? And we could continue … . Moreover, Madelung's theory (Halbwacs, F., 1960; Madelung R., 1927) brings new problems. How can we built a pattern of a corpuscle (framework + Madelung liquid) endlessly extended in space?

Here are some of the "drawbacks" of the patterns in paragraphs 2 and 3 which we shall analyze and remove by means of introducing the fractal approximation of the motion.

5. The motion equation of the physical object in the fractal approximation of motion. The Eulerian separation of motion on resolution scales

The fractal approximation of motion refers to the movement of physical objects (wave + corpuscle) on continuous and non differentiable curves (fractal curves). This approximation is based on the scale Relativity theory (RS) (Nottalle, L., 1993; Chaline, J. et al, 2009; Chaline, J. et al, 2000; Nottale, L., 2004, Nottale, L. & Schneider J., 1984; Nottale, L., 1989; Nottale, L., 1996). Thus, the fractal differential operator can be introduced

$$\frac{\hat{d}}{dt}=\frac{\partial}{\partial t}+\hat{\mathbf{V}}\cdot\nabla-i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1} \tag{8}$$

where $\hat{\mathbf{V}}$ is the complex speed field

$$\hat{\mathrm{V}}=\mathrm{V}-\mathrm{iU} \tag{9}$$

λ is the scale length, dt is the temporary resolution scale, τ is the specific time to fractal-non fractal transition, and D_F is the arbitrary and constant fractal dimension. Regarding the fractal dimension, we can use any of Hausdorff-Bezicovici, Minkowski-Bouligand or Kolmogoroff dimensions, etc. (Budei, L., 2000; Barnsley, M., 1988; Le Mehante A., 1990; Heck, A. & Perdang, J.M., 1991; Feder, J. & Aharony, A., 1990; Berge, P. et al, 1984; Gouyet J.F., 1992; El Naschie, M.S. et al, 1995; Weibel, P. et al, 2005; Nelson, E., 1985; Nottalle, L., 1993; Chaline, J. et al, 2009; Chaline, J. et al, 2000; Nottale, L., 2004; Agop, M. et al, 2009). The only restriction refers to the maintaining of the same type of fractal dimension during the dynamic analysis. The real part of the speed field V is differentiable and independent as compared with the resolution scale, while the imaginary scale U is non differentiable (fractal) and depends on the resolution scale.

Now we can apply the principle of scale covariance by substituting the standard time derivate (d/dt) with the complex operator $\hat{\mathrm{d}}/\mathrm{dt}$. Accordingly, the equation of fractal space-time geodesics (the motion equation in second order approximation, where second order derivates are used) in a covariant form:

$$\frac{d\hat{\mathbf{V}}}{dt}=\frac{\partial\hat{\mathbf{V}}}{\partial t}+\hat{\mathbf{V}}\cdot\nabla\hat{\mathbf{V}}-i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\nabla^2\hat{\mathbf{V}}\equiv 0 \tag{10}$$

This means that the sum of the local acceleration $\partial\hat{\mathbf{V}}/\partial t$, convection $\hat{\mathbf{V}}\cdot\nabla\hat{\mathbf{V}}$ and "dissipation" $\nabla^2\hat{\mathbf{V}}$ reciprocally compensate in any point of the arbitrarily fractal chosen trajectory of a physical object.
Formally, (10) is a Navier-Stokes type equation, with an imaginary viscosity coefficient,

$$\eta = i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1} \tag{11}$$

This coefficient depends on two temporary scales, as well as on a length scale. The existence of a pure imaginary structured coefficient specifies the fact that "the environment" has rheological features (viscoelastic and hysteretic ones (Chioroiu, V. et al, 2005; Ferry, D. K. & Goodnick, S. M., 2001; Imry, Y., 2002)).
For

$$\frac{\lambda^2}{2\tau}\cdot\left(\frac{dt}{\tau}\right)^{(2/D_F)-1} \to 0 \tag{12}$$

equation (10) reduces to Euclidian form (Harabagiu A. et al , 2010; Agop, M. et al, 2008):

$$\frac{\partial\hat{\mathbf{V}}}{\partial\mathbf{t}} + \hat{\mathbf{V}}\cdot\nabla\hat{\mathbf{V}} \equiv 0 \tag{13}$$

and, hence, separating the real part from the imaginary one

$$\begin{aligned} &\frac{\partial\mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V} - \mathbf{U}\cdot\nabla\mathbf{U} = 0 \\ &\frac{\partial\mathbf{U}}{\partial t} + \mathbf{U}\cdot\nabla\mathbf{V} + \mathbf{V}\cdot\nabla\mathbf{U} = 0 \end{aligned} \tag{14a,b}$$

Equation (14a) corresponds to the law of the impulse conservation at differentiable scale (the undulatory component), while (14b) corresponds to the same law, but at a non differentiable scale (corpuscular component). As we will later show, in the case of irotational movements (14) it will be assimilated to the law of mass conservation.

6. Rotational motions and flow regimes of a fractal fluid

For rotational motions, $\nabla\times\hat{\mathbf{V}} \neq 0$ relation (10) with (9) through separating the real part from the imaginary one, i.e. through separating the motions at a differential scale (undulatory characteristic) and non differential one (corpuscular characteristic), results (Harabagiu A. et al , 2010)

$$\begin{aligned} &\frac{\partial\mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V} - \mathbf{U}\cdot\nabla\mathbf{U} - \frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\Delta\mathbf{U} = 0 \\ &\frac{\partial\mathbf{U}}{\partial t} + \mathbf{U}\cdot\nabla\mathbf{V} + \mathbf{V}\cdot\nabla\mathbf{U} + \frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\Delta\mathbf{V} = 0 \end{aligned} \tag{15a,b}$$

According to the operator relations

$$\mathbf{V}\cdot\nabla\mathbf{V}=\nabla\left(\frac{\mathbf{V}^2}{2}\right)-\mathbf{V}\times(\nabla\times\mathbf{V})$$
$$\mathbf{U}\cdot\nabla\mathbf{U}=\nabla\left(\frac{\mathbf{U}^2}{2}\right)-\mathbf{U}\times(\nabla\times\mathbf{U}) \tag{16a-c}$$
$$\mathbf{U}\cdot\nabla\mathbf{V}+\mathbf{V}\cdot\nabla\mathbf{U}=\nabla(\mathbf{U}\cdot\mathbf{V})-\mathbf{V}\times(\nabla\times\mathbf{U})-\mathbf{U}\times(\nabla\times\mathbf{V})$$

equations (15) take equivalent forms

$$\frac{\partial\mathbf{V}}{\partial t}+\nabla\left(\frac{\mathbf{V}^2}{2}-\frac{\mathbf{U}^2}{2}\right)-\mathbf{V}\times(\nabla\times\mathbf{V})-\mathbf{U}\times(\nabla\times\mathbf{U})-\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\Delta\mathbf{U}=0$$
$$\frac{\partial\mathbf{U}}{\partial t}+\nabla(\mathbf{V}\cdot\mathbf{U})-\mathbf{V}\times(\nabla\times\mathbf{U})-\mathbf{U}(\nabla\times\mathbf{V})+\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\Delta\mathbf{V}=0 \tag{17a,b}$$

We can now characterize the flow regimes of the fractal fluid at different scales, using some classes of Reynolds numbers. At a differential scale we have

$$R(differential-nondifferential)=R(D-N)=\frac{|V\cdot\nabla V|}{D|\Delta U|}\approx\frac{V^2l^2}{DUL} \tag{18}$$

$$R(nondifferential-nondifferential)=R(N-N)=\frac{|U\cdot\nabla U|}{D|\Delta U|}\approx\frac{Ul}{D} \tag{19}$$

with

$$D=\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1} \tag{20}$$

and at nondifferential scale

$$\text{R (differential-non differential-differential transition)} = \text{R(TDN-D)}=\frac{|U\cdot\nabla V|}{D|\Delta V|}\approx\frac{UL}{D} \tag{21}$$

$$\text{R (non differential-differential-differential transition)} = \text{R(TND-D)}=\frac{|V\cdot\nabla U|}{D|\Delta V|}\approx\frac{UL^2}{Dl} \tag{22}$$

In previous relations V, L, D, are the specific parameters, while U, l, D are the parameters of the non differential scale. The parameters V, U are specific speeds, L, l specific lengths and D is a viscosity coefficient. Moreover, the common "element" for R(D-N), R(N-N), R(TDN-D) and R(TND-D) is the "viscosity" which, through (20) is imposed by the resolution scale.
Equations (15) are simplified in the case of the stationary motion for small Reynolds numbers. Thus, equation (15) for small R (D-N) becomes

$$-\mathbf{U}\cdot\nabla\mathbf{U}-\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\Delta\mathbf{U}=0 \tag{23}$$

and for small R(N-N)

$$-\mathbf{V}\cdot\nabla\mathbf{V}-\frac{\lambda^2}{2\tau}\left(\frac{\mathrm{dt}}{\tau}\right)^{(2/D_F)-1}\Delta\mathbf{U}=0 \tag{24}$$

Equation (15b) for small R(TDN-D) takes the form

$$\mathbf{V}\cdot\nabla\mathbf{U}+\frac{\lambda^2}{2\tau}\left(\frac{\mathrm{dt}}{\tau}\right)^{(2/D_F)-1}\Delta\mathbf{V}=0 \tag{25}$$

and for small R(TND-D)

$$\mathbf{U}\cdot\nabla\mathbf{V}+\frac{\lambda^2}{2\tau}\left(\frac{\mathrm{dt}}{\tau}\right)^{(2/D_F)-1}\Delta\mathbf{V}=0 \tag{26}$$

7. Irotational motions of a fractal fluid. The incorporation of the associate wave corpuscle through the solidarity of movements and generation of Schrodinger equation

For irotational motions

$$\nabla\times\hat{\mathbf{V}}=0 \tag{27}$$

which implies

$$\nabla\times\mathbf{V}=0,\nabla\times\mathbf{U}=0 \tag{28 a,b}$$

equation (10) (condition of solidarity of movements) becomes (Harabagiu A. et al , 2010)

$$\frac{\partial\mathbf{V}}{\partial t}+\nabla\left(\frac{\mathbf{V}^2}{2}\right)-i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\Delta\mathbf{V}=0 \tag{29}$$

Since through (27) the complex speed field is expressed by means of a scalar function gradient Φ,

$$\hat{\mathbf{V}}=\nabla\Phi \tag{30}$$

equation (29) taking into account the operator identities

$$\frac{\partial}{\partial t}\nabla=\nabla\frac{\partial}{\partial t},\quad \nabla\Delta=\Delta\nabla \tag{31}$$

takes the form

$$\nabla\left[\frac{\partial\Phi}{\partial t}+\frac{1}{2}(\nabla\Phi)^2-\mathrm{i}\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\Delta\Phi\right]=0 \tag{32}$$

or furthermore, through integration

$$\frac{\partial\Phi}{\partial t}+\frac{1}{2}\left(\nabla\Phi\right)^2-i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\Delta\Phi=F\left(t\right) \tag{33}$$

where F(t) is an arbitrary function depending only on time.
In particular, for Φ having the form

$$\Phi=-2i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\ln\Psi \tag{34}$$

where Ψ is a new complex scalar function, equation (46), with the operator identity

$$\frac{\Delta\Psi}{\Psi}=\Delta\ln\Psi+\left(\nabla\ln\Psi\right)^2 \tag{35}$$

takes the form :

$$\frac{\lambda^4}{4\tau^2}\left(\frac{dt}{\tau}\right)^{(4/D_F)-2}\Delta\Psi+i\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\frac{\partial\Psi}{\partial t}+\frac{F\left(t\right)}{2}\Psi=0 \tag{36}$$

The Schrodinger "geodesics" can be obtained as a particular case of equation (36), based on the following hypothesis (conditions of solidarity of the motion, incorporating the associated wave corpuscle):

i. the motions of the micro-particles take place on fractal curves with the fractal dimension D_F=2, i.e. the Peano curves (Nottalle, L., 1993; Nottale, L., 2004);
ii. $d_{\pm}\xi^i$ are the Markov-Wiener type stochastic variables (Nottalle, L., 1993; Nottale, L., 2004) that satisfy the rule

$$\left\langle d_{\pm}\xi^i d_{\pm}\xi^l\right\rangle=\pm\delta^{il}\frac{\lambda^2}{\tau}dt \tag{37}$$

iii. space scale λ and temporary one τ are specific for the Compton scale

$$\lambda=\frac{\hbar}{m_0c},\quad \tau=\frac{\hbar}{m_0c^2} \tag{38}$$

with m_0 the rest mass of the microparticle, c the speed of light in vacuum and $\hbar$ the reduced Planck constant. The parameters (38) should not be understood as "structures" of the standard space-time, but as standards of scale space-time; iv) function F(t) from (36) is null. Under these circumstances, (36) is reduced to the standard form of Schrodinger's equation (Ţiţeica, S., 1984; Peres, A., 1993)

$$\frac{\hbar^2}{2m_0}\Delta\Psi+i\hbar\frac{\partial\Psi}{\partial t}=0 \tag{39}$$

In such a context, the scale potential of the complex speeds plays the role of the wave function.

8. Extended hydrodynamic model of scale relativity and incorporation of associated wave corpuscle through fractal potential. The correspondence with Madelung model

Substituting the complex speed (9) with the restriction (27) and separating the real part with the imaginary one, we obtain the set of differential equations (Harabagiu A. et al , 2010)

$$m_0\frac{\partial \mathbf{V}}{\partial t}+m_0\nabla\left(\frac{\mathbf{V}^2}{2}\right)=-\nabla(\mathrm{Q})$$
$$\frac{\partial \mathbf{U}}{\partial t}+\nabla(\mathbf{V}\cdot\mathbf{U})+\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\Delta\mathbf{V}=0 \tag{40a,b}$$

where Q is the fractal potential, expressed as follows

$$Q=-\frac{m_0\mathbf{U}^2}{2}-\frac{m_0}{2}\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\nabla\cdot\mathbf{U} \tag{41}$$

For

$$\Psi=\sqrt{\rho}e^{iS} \tag{42}$$

with $\sqrt{\rho}$ an amplitude and S a phase, then (34) under the form

$$\Phi=-i\frac{\lambda^2}{\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\ln\left(\sqrt{\rho}e^{iS}\right)$$

implies the complex speed fields of components

$$\mathbf{V}=\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\nabla S,\quad \mathbf{U}=\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\nabla\ln\rho \tag{43a,b}$$

From the perspective of equations (43), the equation (40) keeps its form, and the fractal potential is given by the simple expression

$$Q=-m_0\frac{\lambda^2}{\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}} \tag{44}$$

Again through equations (43), equation (40b) takes the form:

$$\nabla\left(\frac{\partial\ln\rho}{\partial t}+\mathbf{V}\cdot\nabla\ln\rho+\nabla\cdot\mathbf{V}\right)=0$$

or, still, through integration with $\rho\neq 0$

$$\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\mathbf{V})=T(t) \tag{45}$$

with T(t), an exclusively time dependent function
Equation (40) corresponds to the impulse conservation law at differential scale (the classical one), while the impulse conservation law at non differential scale is expressed through (45) with $T(t) \equiv 0$, as a probability density conservation law
Therefore, equations

$$\begin{aligned} m_0\left(\frac{\partial \mathbf{V}}{\partial t}+\mathbf{V}\cdot\nabla\left(\frac{\mathbf{V}^2}{2}\right)\right) &= -\nabla(Q) \\ \frac{\partial \rho}{\partial t}+\nabla\cdot(\rho\mathbf{V}) &= 0 \end{aligned} \tag{46a,b}$$

with Q given by (41) or (44) forms the set of equations of scale relativity extended hydrodynamics in fractal dimension D_F. We mention that in references (Nottalle, L., 1993; Chaline, J. et al, 2009; Chaline, J. et al, 2000; Nottale, L., 2004) the model has been extended only for D_F=2. The fractal potential (41) or (44) is induced by the non differentiability of space-time.
In an external scalar field U, the system of equations (46) modifies as follows

$$\begin{aligned} m_0\left[\frac{\partial \mathbf{V}}{\partial t}+\nabla\left(\frac{\mathbf{V}^2}{2}\right)\right] &= -\nabla(Q+U) \\ \frac{\partial \rho}{\partial t}+\nabla(\rho\mathbf{V}) &= 0 \end{aligned} \tag{47a,b}$$

Now the quantum mechanics in hydrodynamic formula (Madelung's model (Halbwacs, F., 1960)) is obtained as a particular case of relations (47), using the following hypothesis: i) the motion of the micro-particles takes place on Peano curves with D_F=2; ii) $d_{\pm}\xi^i$ are the Markov-Wiener variables (Nottalle, L., 1993; Chaline, J. et al, 2009; Chaline, J. et al, 2000; Nottale, L., 2004); iii) the time space scale is a Compton one. Then, (38) have the expressions

$$\mathbf{V}=\frac{\hbar}{m_0}\nabla S, \quad \mathbf{U}=\frac{\hbar}{2m_0}\nabla \ln\rho \tag{48}$$

and (41),

$$Q=-\frac{m_0\mathbf{U}^2}{2}-\frac{\hbar}{2}\nabla\cdot\mathbf{U} \tag{49}$$

9. "Mechanisms" of duality through coherence in corpuscle-wave phase

In the stationary case, the system of equations (46) becomes (Harabagiu A. et al , 2010)

$$\begin{aligned} \nabla\left(\frac{\mathbf{V}^2}{2}+Q\right) &= 0 \\ \nabla(\rho\mathbf{V}) &= 0 \end{aligned} \tag{50a,b}$$

or, still, through integration

$$\frac{\mathbf{V}^2}{2}+Q=E=const.$$
$$\rho\mathbf{V}=const. \tag{51a,b}$$

Let us choose the null power density in (51b). Then there is no impulse transport at differential scale between corpuscle and wave. Moreover, for $\rho \neq 0$

$$\mathbf{V}=0 \tag{52}$$

which implies through relation (43)

$$S=const. \tag{53}$$

In other words, the fluid becomes coherent (the fluid particles have the same phase). Such a state is specific for quantum fluids (Ciuti C. & Camsotto I., 2005; Benoit Deveand, 2007), such as superconductors, superfluids, etc. (Felsager, B., 1981; Poole, C. P. et al, 1995). Under such circumstances, the phase of the corpuscle considered as a small horologe equals the phase of the associated wave (coherence in corpuscle-wave phase).
At non-differential scale, equation (51) , with restriction (52) takes the form

$$Q=-\frac{2m_0\mathrm{D}^2\Delta\sqrt{\rho}}{\sqrt{\rho}}=-\frac{m_0\mathbf{U}^2}{2}-m_0D\nabla\mathbf{U}=E=const$$
$$D=\frac{\lambda^2}{\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1} \tag{54 a,b}$$

or, still, by applying the gradient operator

$$\mathbf{A}=\nabla(\sqrt{\rho}) \tag{55}$$

$$\Delta\mathbf{A}+\frac{E}{2m_0D^2}\mathbf{A}=0 \tag{56}$$

We distinguish the following situations

i. For E>0 and with substitution

$$\frac{1}{\Lambda^2}=\frac{E}{2m_0D^2} \tag{57}$$

equation (56) becomes

$$\Delta\mathbf{A}+\frac{1}{\Lambda^2}\mathbf{A}=0 \tag{58}$$

Therefore:

1. the space oscillations of field **A** and, therefore the space associated with the motion of coherent fluid particles is endowed with regular non homogeneities (of lattice type). In other words, the field **A** crystallizes ("periodicizes") the space. The one dimensional space "crystal" has the constant of the network

$$\Lambda = \frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\left(\frac{2m_0}{E}\right)^{1/2} \tag{59}$$

that depends both on the "viscosity" – $i\eta$ given by (11) and on the energy of the particle;

2. the one dimensional geodesics of the "crystallized" space given by the expression

$$\rho(x) = A^2 \sin^2(kx + \delta) \tag{60}$$

implies both fractal speed

$$U_x = D\frac{d\ln\rho}{dx} = 2Dkctg(kx + \delta) \tag{61}$$

and fractal potential

$$\begin{aligned} Q_x &= -\frac{m_0 U_x^2}{2} - m_0 D\frac{dU_x}{dx} = -2m_0 D^2 k^2 ctg^2(kx + \delta) \\ &+2m_0 D^2 k^2 \frac{1}{\sin^2(kx + \delta)} = 2m_0 D^2 k^2 \end{aligned} \tag{62}$$

with A and δ and the integration constants

$$k = \frac{1}{\Lambda} \tag{63}$$

3. for the movements of microparticles on Peano curves (D_F=2) at Compton scale

$$\mathrm{D} = 2m_0\hbar \ ,$$

therefore, through (62) under the form

$$Q_x = 2m_0 D^2 k^2 = \frac{p_x^2}{2m_0}, \quad p_x = 2m_0 Dk \tag{64a,b}$$

de Broglie "quantum" impulse is found

$$p_x = \frac{\hbar}{\Lambda} \tag{65}$$

4. the dominant of the undulatory characteristic is achieved by the "self diffraction" mechanism of the fractal field, ρ, on the one dimensional space "crystal" of constant Λ induced by the same field. Indeed, relation (61) with notations

$$\Phi = kx + \delta \quad , \quad k = \frac{1}{\Lambda} \tag{66a,b}$$

in approximation Φ«1 , i.e. for $tg\Phi \approx \sin\Phi$ and using Nottale's relation (Nottalle, L., 1993; Chaline, J. et al, 2009; Chaline, J. et al, 2000; Nottale, L., 2004; Nottale, L. &

Schneider J., 1984; Nottale, L., 1989; Nottale, L., 1996) $2D/U_x \approx n\lambda$ it takes the common form (Bragg's relation)

$$\Lambda \sin\Phi \approx n\lambda; \tag{67}$$

This result is in concordance with the recently expressed opinion in (Mandelis A. et al, 2001; Grössing G., 2008; Mandelis A., 2000);

5. there is impulse transfer on the fractal field between the corpuscle and the wave;
6. according to Taylor's criterion (Popescu, S., 2004) self-organization (crystallization and self diffraction of the space) appears when the energy of the system is minimal. This can be immediately verified using relation (51a);

ii. For E=0, equations (51a) and (56) have the same form

$$\Delta\sqrt{\rho}=0 \quad \Delta\mathbf{A}=0 \tag{68}$$

It follows that:

1. the geodesics are expressed through harmonic functions and the particle finds itself in a critical state, i.e. the one that corresponds to the wave-corpuscle transition;
2. in the one –dimensional case, the geodesics have the form

$$\rho(x)=kx+\delta \tag{69}$$

which induces the fractal speed field

$$U_x=\frac{D}{kx+\delta} \tag{70}$$

namely the null value of the fractal potential

$$Q_x=-\frac{m_0}{2}\frac{D^2}{(kx+\delta)^2}+\frac{m_0}{2}\frac{D^2}{(kx+\delta)^2}=0 \tag{71}$$

3. although the energy is null, there is impulse transfer between corpuscle and wave on the fractal component of the speed field

iii. For E<0 and with notations

$$\frac{1}{\bar{\Lambda}^2}=\frac{\bar{E}}{2m_0D^2}, \quad E=-\bar{E} \tag{72}$$

equation (56) takes the form

$$\Delta\mathbf{A}-\frac{1}{\bar{\Lambda}^2}\mathbf{A}=0 \tag{73}$$

The following aspects result:

1. field A is expelled from the structure, its penetration depth being

$$\bar{\Lambda}=\frac{\lambda^2}{2\tau}\left(\frac{dt}{\tau}\right)^{(2/D_F)-1}\left(\frac{2m_0}{\bar{E}}\right)^{1/2} \tag{74}$$

2. the one-dimensional geodesics of the space are described through function

$$\rho(x) = \bar{A}^2 sh^2(\bar{k}x + \bar{\delta}) \tag{75}$$

and lead to the fractal speed

$$U_x = 2D\bar{k}cth(\bar{k}x + \bar{\delta}) \tag{76}$$

the fractal potential respectively

$$Q_x = -2m_0D^2\bar{k}^2cth^2(\bar{k}x + \bar{\delta}) + 2m_0D^2\frac{\bar{k}^2}{sh^2(\bar{k}x + \bar{\delta})} = -2m_0D^2\bar{k}^2 \tag{77}$$

where $\bar{A}$, $\bar{\delta}$ are two integration constants and

$$\bar{k} = \frac{1}{\bar{\Lambda}} \tag{78}$$

3. the dominant of the corpuscular characteristic is accomplished by means of "self-expulsion" mechanism of the fractal field from its own structure that it generates (that is the corpuscle), the penetration depth being $\bar{\Lambda}$. The identification

$$Q_x = -2m_0D^2\bar{k}^2 = \frac{\bar{p}^2}{2m_0} \tag{79}$$

implies the purely imaginary impulse

$$\bar{p} = -2im_0D\bar{k} \tag{80}$$

that suggests ultra rapid virtual states (ultra rapid motions in the wave field, resulting in the "singularity"of the field, i.e. the corpuscle). As a matter of fact, if we consider de Broglie's original theory (motions on Peano curves with D_F=2, at Compton's scale), singularity (the corpuscle) moves "suddenly" and chaotically in the wave field, the wave-corpuscle coupling being accomplished through the fractal potential. The corpuscle "tunnels" the potential barrier imposed by the field of the associate progressive wave, generating particle-antiparticle type pairs (ghost type fields (Bittner E.R., 2000)). Nevertheless this model cannot specify the type of the physical process by means of which we reach such a situation: it is only the second quantification that can do this (Ciuti C. & Camsotto I., 2005; Benoit Deveand Ed., 2007; Mandelis A. et al, 2001; Grössing G., 2008; Mandelis A., 2000; Bittner E.R., 2000);

4. there is an impulse transfer between the corpuscle and the wave on the fractal component of the speed field, so that all the attributes of the differential speed could be transferred on the fractal speed.

All the above results indicate that wave-particle duality is an intrinsic property of space and not of the particle.

10. Wave-corpuscle duality through flowing stationary regimes of a coherent fractal fluid in phase. The potential well

According to the previous paragraph, let us study the particle in a potential well with infinite width and walls. Then the speed complex field has the form (Harabagiu A. et al, 2010; Agop, M. et al, 2008; Harabagiu, A. & Agop, M., 2005; Harabagiu, A. et al, 2009)

$$\hat{V}_x = V_x - iU_x = 0 - 2iD\left(\frac{n\pi}{a}\right)ctg\left(\frac{n\pi}{a}\right)x \tag{81}$$

and generates the fractal potential (the energy of the structure) under the form of the noticeable

$$Q_n = 2m_0D^2\left(\frac{n\pi}{a}\right)^2 = E_n \tag{82}$$

The last relation (82) allows the implementation of Reynold's criterion

$$R(n) = \frac{V_c a}{D} = 2n\pi, \quad V_c = \left(\frac{2E_n}{m_0}\right)^{\frac{1}{2}} \tag{83a,b}$$

For movements on Peano curves (D_F=2) at Compton scale $(2m_0D = \hbar)$ (83) with substitutions

$$m_0V_c = \Delta P_x, \quad a = \Delta x \tag{84a,b}$$

and n=1 reduces to Heinsenberg's relation of uncertainty under equal form

$$\Delta p_x \Delta x = \frac{h}{2} \tag{85}$$

while for $n < +\infty$ it implies a Ruelle-Takens' type criterion of evolution towards chaos (Ruelle D. & Takens, F., 1971; Ruelle, D., 1975). Therefore, the wave-corpuscle duality is accomplished through the flowing regimes of a fractal fluid that is coherent in phase. Thus, the laminar flow (small n) induces a dominant ondulatory characteristic, while the turbulent flow (big n) induces a dominant corpuscular characteristic.

11. Wave-corpuscle duality through non-stationary regimes of an incoherent fractal fluid

In the one dimensional case the equations of hydrodynamics (46) take the form

$$m_0\left(\frac{\partial V}{\partial t} + V\cdot\frac{\partial V}{\partial x}\right) = -\frac{\partial}{\partial x}\left[-2\mathrm{m}_0D^2\frac{1}{\rho^{1/2}}\frac{\partial^2}{\partial x^2}\left(\rho^{1/2}\right)\right]; \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho V) = 0 \tag{86a,b}$$

Imposing the initial conditions

$$\begin{aligned} V(x, t=0) &= c = const \\ \rho(x, t=0) &= \frac{1}{\pi^{1/2}\alpha}e^{-\left(\frac{x}{\alpha}\right)^2} = \rho_0 \end{aligned} \tag{87a,b}$$

and on the frontier

$$V\left(x=ct,t\right)=c$$
$$\rho(x=-\infty,t)=\rho(x=+\infty,t)=0 \tag{88a,b}$$

the solutions of the system (86), using the method in (Munceleanu, C.V. et al, 2010), have the expressions

$$\rho\left(x,t\right)=\frac{1}{\pi^{1/2}\left[\alpha^2+\left(\frac{2D}{\alpha}t\right)^2\right]^{1/2}}\exp\left[\frac{\left(x-ct\right)^2}{\alpha^2+\left(\frac{2D}{\alpha}t\right)^2}\right]$$
$$V=\frac{c\alpha^2+\left(\frac{2D}{\alpha}\right)^2 tx}{\alpha^2+\left(\frac{2D}{\alpha}t\right)^2} \tag{89a,b}$$

The complex speed field is obtained

$$\hat{V}=V-iU=\frac{c\alpha^2+\left(\frac{2D}{\alpha}\right)^2 tx}{\alpha^2+\left(\frac{2D}{\alpha}t\right)^2}+2iD\frac{x-ct}{\alpha^2+\left(\frac{2D}{\alpha}t\right)^2} \tag{90}$$

and the field of fractal forces

$$F=4m_0D^2\frac{\left(x-ct\right)}{\left[\alpha^2+\left(\frac{2D}{\alpha}t\right)^2\right]^2} \tag{91}$$

Therefore:

i. both differential scale speed V and non-differential one U are not homogeneous in x and t. Under the action of fractal force F, the corpuscle is assimilated to the wave, is a part of its structure, so that it joins the movement of the corpuscle with the propagation of the associated progressive wave;
ii. the timing of the movements at the two scales, V=U implies the space-time homographic dependence

$$x=\frac{c\alpha^2}{2D}\frac{1+\frac{2D}{\alpha^2}t}{1-\frac{2D}{\alpha^2}t} \tag{92}$$

in the field of forces

$$F = \frac{2m_0 Dc}{\left(1-\frac{2D}{\alpha^2}t\right)\left[\alpha^2+\left(\frac{2D}{\alpha}t\right)^2\right]} \tag{93}$$

Considering that the type (92) changes are implied in gravitational interaction (Ernst, F.J., 1968; Ernst, F.J., 1971), it follows that the solidarity of the corpuscle movement with the movement of the associated progressive wave is accomplished by means of the appropriate gravitational field of the physical object;

iii. the uniform movement V=c is obtained for null fractal force F=0 and fractal speed U=0, using condition x=ct. The fractal forces in the semi space. $-\infty \le x \le \bar{x}$ and $\bar{x} \le x \le +\infty$ are reciprocally compensated.

$$F\Big|_{-\infty}^{\bar{x}} = F\Big|_{\bar{x}}^{+\infty}$$

This means that the corpuscle in "free" motion simultaneously polarizes the "environment" of the wave behind $x \le ct$ and in front of $x \ge ct$, in such a manner that the resulting force has a symmetrical distribution as compared with the plane that contains the position of the noticeable object $\bar{x} = ct$ at any time moment t. Under such circumstances, the physical object uniformly moves (the corpuscle is located in the field of the associated wave).

12. The corpuscle as a horologe and its incorporation in the associated wave. Consequences

According to de Broglie's theory, the corpuscle must be associated to a horologe having the frequency equal to that of the associated progressive wave. Mathematically we can describe such an oscillator through the differential equation

$$\ddot{q} + \omega^2 q = 0 \tag{94}$$

where ω defines the natural frequency of the oscillator as it is dictated by the environment (the wave), and the point above the symbol referes to the differential as compared with time. The most general solution of equation (94) generally depends not on two arbitrary constants, as it is usually considered, but on three: the initial relevant coordinate, the initial speed and the phase of the harmonic oscillatory within the ensemble that structurally represents the environment (the isolated oscillator is an abstraction !). Such a solution gives the relevant co-ordinate

$$q(t) = he^{i(\omega t+\Phi)} + \bar{h}e^{-i(\omega t+\Phi)} \tag{95}$$

where $\bar{h}$ refers to the complex conjugate of h and Φ is an initial phase specific to the individual movement of the oscillator. Such a notation allows us to solve a problem that we could name "the oscillators with the same frequency", such as Planck's resonators' ensemble-the basis of the quantum theory arguments in their old shape. That is, given an ensemble of oscillators having the same frequency in a space region, which is the relation between them?

The mathematical answer to this problem can be obtained if we note that what we want here is to find a mean to pass from a triplet of numbers –the initial conditions- of an oscillator

towards the same triplet of another oscillator with the same frequency. This process (passing) implies a simple transitive continuous group with three parameters that can be built using a certain definition of the frequency. We start from the idea that the ratio of two fundamental solutions of equation (94) is a solution of Schwarts' non linear equation (Agop, M. & Mazilu, N., 1989; Agop, M. & Mazilu, N., 2010; Mihăileanu, N., 1972)

$$\frac{d}{dt}\left(\frac{\ddot{\tau}_0}{\dot{\tau}_0}\right)-\frac{1}{2}\left(\frac{\ddot{\tau}_0}{\dot{\tau}_0}\right)^2=2\omega^2,\quad \tau_0(t)\equiv e^{-2i\omega t} \tag{96}$$

This equation proves to be a veritable definition of frequency as a general characteristic of an ensemble of oscillators that can be scanned through a continuous group of three parameters. Indeed equation (96) is invariant to the change of the dependent variable

$$\tau(t)=\frac{a\tau_0(t)+b}{c\tau_0(t)+d} \tag{97}$$

which can be verified through direct calculation. Thus, $\tau(t)$ characterizes another oscillator with the same frequency which allows us to say that, starting from a standard oscillator we can scan the whole ensemble of oscillators of the same frequency when we let loose the three ratios a: b: c: d in equation (97). We can make a more precise correspondence between a homographic change and an oscillator, by means of associating to each oscillator a personal $\tau(t)$ through equation

$$\tau_1(t)=\frac{h+\bar{h}k\tau_0(t)}{1+k\tau_0(t)} \qquad k\equiv e^{-2i\Phi} \tag{98}$$

Let us notice that τ_0,τ_1 can be freely used one instead the other, which leads to the next group of changes for the initial conditions

$$h'\to\frac{ah+b}{ch+d}\quad \bar{h}'\to\frac{a\bar{h}+b}{c\bar{h}+d}\quad k'\to k\cdot\frac{c\bar{h}+d}{ch+d}\qquad a,b,c,d\in R \tag{99a-d}$$

This is a simple transitive group: one and only one change of the group (the Barbilian group (Agop, M. & Mazilu, N., 1989; Agop, M. & Mazilu, N., 2010; Barbilian, D., 1935; Barbilian, D., 1935; Barbilian, D., 1938; Barbilian, D., 1971)) corresponds to a given set of values (a/c, b/c, d/c).

This group admits the 1-differential forms, absolutely invariant through the group (Agop, M. & Mazilu, N., 1989)

$$\omega_0=i\left(\frac{dk}{k}-\frac{dh+d\bar{h}}{h-\bar{h}}\right),\quad \omega_1=\overline{\omega_2}=\frac{dh}{k(h-\bar{h})} \tag{100}$$

and the 2- differential form

$$\frac{ds^2}{\alpha^2}=\omega_0{}^2-4\omega_1\omega_2=-\left(\frac{dk}{k}-\frac{dh-d\bar{h}}{h-\bar{h}}\right)^2+4\frac{dhd\bar{h}}{\left(h-\bar{h}\right)^2},\quad \alpha=const. \tag{101}$$

respectively.
If we restrict the definition of a parallelism of directions in Levi-Civita manner (Agop, M. & Mazilu, N., 1989)

$$d\varphi = -\frac{du}{v} \tag{102}$$

with

$$h=u+iv\ ,\ \bar{h}=u-iv,\ k=\mathrm{e}^{-i\varphi} \tag{103}$$

Barbilian's group invariates the metrics of Lobacevski's plane (Agop, M. & Mazilu, N., 1989),

$$\frac{ds^2}{\alpha^2} = -\frac{du^2+dv^2}{v^2} \tag{104}$$

Metrics (104) coincides with the differential invariant that is built with the complex scalar field of the speed,

$$\frac{ds^2}{\alpha^2} \equiv d\phi d\bar{\phi} = (2Dds - iDd\ln\rho)(2Dds + iDd\ln\rho) = 4D^2(ds)^2 + D^2\left(\frac{d\rho}{\rho}\right)^2 \tag{105}$$

which admits the identities

$$\alpha = D,\ 2ds \equiv d\Phi = -\frac{du}{v},\ d\ln\rho \equiv d\ln v \tag{106a-c}$$

Now, through a Matzner-Misner type principle one can obtain Ernst's principle of generating the symmetrical axial metrics (Ernst, F.J., 1968; Ernst, F.J., 1971)

$$\delta\int\frac{\nabla h\nabla\bar{h}}{(h-\bar{h})^2}\gamma^{1/2}d^3x = 0 \tag{106d}$$

where $\gamma = \det\gamma_{\alpha\beta}$ with $\gamma_{\alpha\beta}$ the metrics of the "environment".
Therefore, the incorporation of the corpuscle in the wave, considering that it functions as a horologe with the same frequency as that of the associated progressive wave, implies gravitation through Einstein's vacuum equations (equivalent to Ernst's principle (106d)). On the contrary, when the frequencies do not coincide, there is an induction of Stoler's group from the theory of coherent states (the parameter of the change is the very ratio of frequencies when creation and annihilation operators refer to a harmonic oscillator (Agop, M. & Mazilu, N., 1989)).
Let us note that the homographic changes (99) generalize the result (92). Moreover, if $a,b,c,d\ \epsilon\mathbb{Z}$ then the Ernst type equations describe supergravitation N=1 (Green, M.B. et al, 1998).

13. Informational energy through the fractal potential of complex scalar speed field. The generation of forces

The informational energy of a distribution is defined through the known relation (Mazilu N. & Agop M., 1994),

$$E = -\int \rho \ln \rho dx \tag{107}$$

where $\rho(\mathrm{x})$ is the density of distributions, and we note by x, on the whole, the random variables of the problem, dx being the elementary measure of their field.
This functional represents a measure of the uncertainty degree, when defining the probabilities, i.e. it is positive, it increases when uncertainty also incresases taken in the sense of expanding distribution and it is additive for sources that are independent as compared to uncertainity. If we admit the maximum of informational energy in the inference against probabilities, having at our disposal only a partial piece of information this is equivalent to frankly admitting the fact that we cannot know more. Through this, the distributions that we obtain must be at least displaced, as compared to the real ones, because there is no restrictive hypothesis regarding the lacking information. In other words, such a distribution can be accomplished in the highest number of possible modalities. The partial piece of information we have at our disposal, is given, in most cases, in the form of a f(x) function or of more functions.

$$\overline{f} = \int \rho(x) f(x) dx \tag{108}$$

Relation (108), together with the standard relation of distribution density

$$\int \rho(x) dx = 1 \tag{109}$$

are now constraints the variation of the functional (107) has to subject to, in order to offer the distribution density corresponding to the maximum of informational energy. In this concrete case, Lagrange's non determined multipliers method directly leads to the well known exponential distribution

$$\rho(x) = \exp(-x - \mu f(x)) \tag{110}$$

Let us notice that through the fractal component of the complex scalar of speed field

$$\Phi = D \ln \rho \tag{111}$$

expression (107), ignoring the scale factor D, is identical with the average mean of (111)

$$E = -\frac{\overline{\Phi}}{D} = -\int \rho \ln \rho dx \tag{112}$$

In the particular case of a radial symmetry, imposing the constraints

$$\overline{r} = \int \rho(r) r dr \tag{113}$$

$$\int \rho(r) dr \equiv 1 \tag{114}$$

the distribution density $\rho(r)$ through the maximum of informational energy implies the expression

$$\rho(r) = \exp(-\lambda - \mu r), \quad \lambda, \mu = const. \tag{115}$$

or in notations

$$\exp(-\lambda) \equiv \rho_0, \quad \mu = 2 / a \tag{116}$$

$$\rho(r) = \rho_0 e^{-\frac{2r}{a}} \tag{117}$$

Then the fractal speed

$$u = D\frac{d}{dr}(\ln \rho) = -\frac{2D}{a} = const \tag{118}$$

through the fractal potential

$$Q = -\frac{m_0 u^2}{2} - m_0 D^2 \left[\frac{d^2}{dr^2}(\ln \rho) + \frac{2}{r}\frac{d}{dr}(\ln \rho) \right] = -\frac{2m_0 D^2}{a}\left(\frac{1}{a} - \frac{2}{r} \right) \tag{119}$$

implies the fractal field of central forces

$$F(r) = -\frac{dQ}{dr} = -\frac{4m_0 D^2}{ar^2} \tag{120}$$

Consequently, the fractal "medium" by maximization of the informational energy becomes a source of central forces (gravitational or electric type).

14. Lorenz type mechanism of wave-corpuscle duality in non stationary systems

Impulse conservation law

Let us rewrite the system of equations (15) for an external scalar field U under the form

$$\begin{aligned} &\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} - \mathbf{U} \cdot \nabla \mathbf{U} - D\Delta \mathbf{U} = -\nabla U \\ &\frac{\partial \mathbf{U}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{U} + \mathbf{U} \cdot \nabla \mathbf{V} + D\Delta \mathbf{V} = 0 \end{aligned} \tag{121a,b}$$

with D given by relation (54). Hence, through their decrease and using substitution

$$\overline{V} = V - \mathbf{U} \tag{122}$$

we find

$$\frac{\partial \overline{\mathbf{V}}}{\partial t} + \overline{\mathbf{V}} \cdot \nabla \overline{\mathbf{V}} = 2\mathbf{U} \cdot \nabla \mathbf{U} + 2D\Delta \mathbf{U} + D\Delta \overline{\mathbf{V}} - \nabla U \tag{123}$$

Taking into account that the fractal term, $2\mathbf{U} \cdot \nabla \mathbf{U} + 2D\Delta \mathbf{U}$ intervenes as a pressure (for details see the kinetic significance of fractal potential Q (Bohm, D., 1952)) then we can admit the relation

$$2\mathbf{U} \cdot \nabla \mathbf{U} + 2D\Delta \mathbf{U} = -2\left(-\frac{\mathbf{U}^2}{2} - D\nabla \cdot \mathbf{U} \right) = -2\nabla\left(\frac{Q}{m_0} \right) = -\frac{\nabla p}{\rho} \tag{124}$$

then equation (123) takes the usual form

$$\frac{\partial \mathbf{V}}{\partial t} + \overline{\mathbf{V}} \cdot \nabla \overline{\mathbf{V}} = -\frac{\nabla p}{\rho} - \nabla U + D\Delta \overline{\mathbf{V}} \tag{125}$$

In particular, if $\nabla U = \mathbf{g}$ is a gravitational accelaration (125) becomes

$$\frac{\partial \overline{\mathbf{V}}}{\partial t} + \overline{\mathbf{V}}.\nabla \overline{\mathbf{V}} = -\frac{\nabla p}{\rho} - \mathbf{g} + D\Delta \overline{\mathbf{V}} \tag{126}$$

Energy conservation law

Energy conservation law, ε in the case of movements on fractal curves of fractal dimension D_F is written under the form

$$\frac{\hat{d}\varepsilon}{dt} = \frac{d\varepsilon}{dt} + \hat{\mathbf{V}} \cdot \nabla \varepsilon - iD\Delta\varepsilon = 0 \tag{127}$$

or, still, by separating the real part from the imaginary one

$$\frac{\partial \varepsilon}{\partial t} + \mathbf{V} \cdot \nabla \varepsilon = 0, \quad -\mathbf{U} \cdot \nabla \varepsilon = D\Delta\varepsilon \tag{128}$$

Hence, through addition and taking into account relation (122), we obtain the expression

$$\frac{\partial \varepsilon}{\partial t} + \overline{\mathbf{V}} \cdot \nabla \varepsilon = D\Delta\varepsilon \tag{129}$$

In particular, for $\varepsilon = 2m_0 D\Omega$ with Ω the wave pulsation (for movements on Peano curves with D_F=2 at Compton scale $\varepsilon = \hbar\Omega$) the previous relation becomes

$$\frac{\partial \Omega}{\partial t} + \overline{\mathbf{V}} \cdot \nabla \Omega = D\Delta\Omega \tag{130}$$

Lorenz type "mechanism"

For an incompressible fractal fluid, the balance equations of the "impulse" -see (126), of the energy -see (129) and "mass" – see (46) with $\rho = const.$ and $\nabla \cdot \mathbf{U} = 0$ become

$$\begin{aligned} &\frac{\partial \overline{\mathbf{V}}}{\partial t} + \overline{\mathbf{V}} \cdot \nabla \overline{\mathbf{V}} = -\frac{\nabla p}{\rho} - \mathbf{g} + D\Delta \overline{\mathbf{V}} \\ &\frac{\partial \varepsilon}{\partial t} + \overline{\mathbf{V}} \cdot \nabla \varepsilon = D\Delta\varepsilon \\ &\nabla \overline{\mathbf{V}} = 0 \end{aligned} \tag{131a-c}$$

Let us take into account the following simplyfing hypothesis:

i. constant density, $\rho = \rho_0 = const.$ excepting the balance equation of the impulse where density is disturbed according to relation

$$\rho = \rho_0 + \delta\rho \tag{132}$$

ii. the energy "expansion" is a linear one

$$\rho = \rho_0 \left[1 - \alpha \left(\varepsilon - \varepsilon_0 \right) \right] \tag{133}$$

with α the energy "dilatation" constant.
Under such circumstances, system (131) becomes

$$\begin{aligned} &\rho_0 \left(\frac{\partial \overline{\mathbf{V}}}{\partial \mathrm{t}} + \overline{\mathbf{V}} \cdot \nabla \overline{\mathbf{V}} \right) + \nabla \mathrm{p} = \left(\rho_0 + \delta\rho \right) \mathbf{g} + \rho_0 D \Delta \overline{\mathbf{V}} \\ &\frac{\partial \varepsilon}{\partial \mathrm{t}} + \overline{\mathbf{V}} \cdot \nabla \varepsilon = D \Delta \varepsilon \\ &\nabla \overline{\mathbf{V}} = 0 \end{aligned} \tag{134a-c}$$

In order to study the dynamics of system (134), our description closely follows the approach in (Bârzu, A. et al, 2003).
The convection in the fractal fluid takes place when the ascending force that results from energy "dilatation" overcomes the viscous forces. Then we can define the Rayleigh number

$$R = \frac{\left| F_{asc} \right|}{\left| F_{visc} \right|} \approx \frac{\left| \frac{\delta\rho g}{\rho_0} \right|}{\left| D \Delta \overline{\mathbf{V}} \right|} \tag{135}$$

The variation of the density satisfies through (133) the relation

$$\frac{\delta\rho}{\rho_0} \approx \alpha \Delta\varepsilon \tag{136}$$

and the "energy" balance equation (134c) implies

$$\overline{V} \approx \frac{D}{d} \tag{137}$$

where d is the thickness of the fractal fluid level. Substituting (136) and (137) in (135) we obtain Rayleigh's number under the form

$$R = \frac{\alpha \beta g d^4}{D^2} \tag{138}$$

where $\beta = \Delta\varepsilon / \mathrm{d} < 0$ is the energy gradient between the superior and inferior frontiers of fluid layer. In the case of convection, Rayleigh's number plays the role of control parameter and takes place for

$$R > R_{critic}$$

In general, R is controlled through the gradient β of the energy.
As reference state, let us choose the stationary rest state $(\overline{\mathbf{V}} = 0)$, for which equations (134a-c) take the form

$$\begin{cases} \nabla p_S = -\rho_S g\hat{z} = -\rho_0[1-\alpha(\varepsilon_S-\varepsilon_0)]g\hat{z} \\ \Delta\varepsilon_S = 0 \end{cases} \tag{139a,b}$$

where $\hat{z}$ represents the versor of vertical direction. We take into account that pressure and ε vary only in vertical direction due to the considered symmetry. For ε the conditions on the frontier are

$$\varepsilon(x,y,0)=\varepsilon_0, \quad \varepsilon(x,y,d)=\varepsilon_1 \tag{140a,b}$$

Integrating equation (139b) with these conditions on the frontier, it will follow that in the reference rest state, the profile of ε on vertical direction is linear.

$$\varepsilon_S = \varepsilon_0 - \beta z \tag{141}$$

Substituting (141) in (139) and integrating, we obtain

$$p_S(z) = p_0 - \rho_0 g\left(1+\frac{\alpha\beta z}{2}\right)z \tag{142}$$

The features of the system in this state do not depend on coefficient D that appears in balance equations.

We study now the stability of the reference state using the method of small perturbations (Bârzu, A. et al, 2003). The perturbed state is characterized by

$$\begin{cases} \varepsilon = \varepsilon_S(z)+\theta(r,t) \\ \rho = \rho_S(z)+\delta\rho(r,t) \\ p = p_S(z)+\partial p(r,t) \\ \overline{\mathbf{V}} = \delta\overline{\mathbf{V}}(r,t) = (u,v,w) \end{cases} \tag{143a-d}$$

One can notice that the perturbations are time and position functions. Substituting (143) in equations (134) and taking into account (141) and (142) the following equations for perturbations (in linear approximation) are obtained:

$$\begin{aligned} &\nabla\cdot\delta\overline{\mathbf{V}} = 0 \\ &\frac{\partial\theta}{\partial t} = \beta w + D\nabla^2\theta \\ &\frac{\partial\delta\overline{\mathbf{V}}}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p + D\nabla^2\delta\overline{\mathbf{V}} + g\alpha\theta\hat{z} \end{aligned} \tag{144a-d}$$

We introduce adimensional variables $\overline{\mathbf{r}}', t', \theta', \delta\overline{\mathbf{V}}', \delta p'$ through the changes

$$\mathbf{r}' = \frac{\mathbf{r}}{d};\ \ t' = \frac{t}{d^2/D};\ \ \theta' = \frac{\theta}{\left(\frac{D^2}{g\alpha d^3}\right)};\ \ \delta\overline{\mathbf{V}}' = \frac{\delta\overline{\mathbf{V}}}{D/d};\ \ \delta p' = \frac{\delta p}{\left(\rho_0\frac{D^2}{d^2}\right)}$$

Replacing these changes and renouncing, for simplicity, at the prime symbol, the adimensional perturbations satisfy the equations

$$\begin{aligned}
&\frac{\partial \overline{\mathbf{V}}}{\partial \mathrm{t}} + \overline{\mathbf{V}} \cdot \nabla \overline{\mathbf{V}} = -\nabla p + \theta \hat{z} + \nabla^2 \overline{\mathbf{V}} \\
&\frac{\partial \theta}{\partial \mathrm{t}} + (\overline{\mathbf{V}} \cdot \nabla)\theta = Rw + \nabla^2 \theta \\
&\nabla \cdot \overline{\mathbf{V}} = 0
\end{aligned} \tag{145}$$

where R is Rayleigh's number.
For $R>R_C$, the reference state becomes unstable, and the convection "patterns" appear. We consider them as being parallel therefore the speed vector will be always perpendicular to their axis. We assume the patterns parallel to the y axis, i.e., the speed component along this direction is zero.
The incompressibility condition becomes

$$u_x + w_z = 0 \tag{146}$$

Equation (146) is satisfied if and only if

$$u = -\psi_z;\ w = \psi_x \tag{147}$$

where $\psi(x,y,z)$ defines Lagrange's current function. The speed field must satisfy the conditions on frontiers (the inferior and superior surfaces)

$$w|_{z=\pm 1/2} = 0 \tag{148}$$

If the frontiers are considered free (the superficial tension forces are neglected), the "shear" component of the pressure tensor is annulated

$$\frac{\partial u}{\partial z}|_{z=\pm 1/2} = 0 \tag{149}$$

Using Lagrange's function, $\psi(x,y,z)$ the limit conditions (148) and (149) become

$$\begin{aligned}
&\Psi_x|_{z=\pm 1/2} = 0 \\
&\Psi_{zz}|_{z=\pm 1/2} = 0
\end{aligned}$$

Let us choose ψ with the form

$$\psi(x,z,t) = \psi_1(t)\cos(\pi z)\sin(qx)$$

According to (147), the components of the speed field are

$$\begin{cases}
u = \pi \Psi_1(t)\sin(\pi z)\sin(qx) \\
w = q\Psi_1(t)\cos(\pi z)\cos(qx)
\end{cases}$$

The impulse conservation equation (for equation (145)) for directions x and z becomes

$$\begin{aligned}
&(u_t + uu_x + wu_z) = -p_x + \Delta u \\
&(w_t + uw_x + ww_z) = -p_z + \Delta w + \theta
\end{aligned} \tag{150a,b}$$

We derive (150 a) according to z and (150) according to x. One finds

$$\left[u_{tz}+\frac{\partial}{\partial z}\left(uu_x+wu_z\right)\right]=-p_{xz}+\frac{\partial}{\partial z}\left(\Delta u\right)$$
$$\left[w_{tz}+\frac{\partial}{\partial x}\left(uw_x+ww_z\right)\right]=-p_{zx}+\frac{\partial}{\partial x}\left(\Delta w\right)+\frac{\partial\theta}{\partial x}$$

Through the sum we obtain

$$\left[-\left(\Delta\Psi\right)_t+\frac{\partial}{\partial z}\left(uu_x+wu_z\right)-\frac{\partial}{\partial x}\left(uw_x+ww_z\right)\right]=-\Delta^2\Psi-\theta_x \tag{151}$$

The value ε being fixed on the two frontiers, we shall have

$$\theta\left|_{z=\pm1/2}\right.=0$$

We consider θ having the form

$$\theta\left(x,z,t\right)=\theta_1\left(t\right)\cos\left(\pi z\right)\cos\left(qx\right)+\theta_2\left(t\right)\sin\left(2\pi z\right) \tag{152}$$

If we consider in (151) the expressions for u, w, θ and ψ it follows that

$$\dot{\psi}_1=\frac{q\theta_1}{\pi^2+q^2}-(\pi^2+q^2)\psi_1 \tag{153}$$

The balance equation for the energy becomes

$$\begin{aligned}\dot{\theta}_1&=-\pi q\psi_1\theta_2+qR\psi_1-(\pi^2+q^2)\theta_1\\ \dot{\theta}_2&=\frac{1}{2}\pi q\psi_1\theta_1-4\pi^2\theta_2\end{aligned} \tag{154}$$

In (153) and (154) we change the variables

$$t'=(\pi^2+q^2)t;\quad X=\frac{\pi q}{\sqrt{2}(\pi^2+q^2)}\psi_1$$
$$Y=\frac{\pi q^2}{\sqrt{2}(\pi^2+q^2)^3}\theta_1;\quad Z=\frac{\pi q^2}{(\pi^2+q^2)^3}\theta_2$$

We obtain the Lorenz type system

$$\begin{aligned}\dot{X}&=\left(Y-X\right)\\ \dot{Y}&=-XZ+rX-Y\\ \dot{Z}&=XY-bZ\end{aligned} \tag{155}$$

where

$$r=\frac{q^2}{(\pi^2+q^2)^3}R,\quad b=\frac{4\pi^2}{\pi^2+q^2}$$

The Lorenz system

$$\dot{X} = \sigma(Y - X)$$
$$\dot{Y} = -XZ + rX - Y$$
$$\dot{Z} = XY - bZ$$

reduces to (155) for $\sigma \equiv 1$.

Characteristics of Lorenz type system. Transitions towards chaos.

We consider the evolution equations of Lorenz type system (155) with the notation

$$\begin{aligned} \dot{x} &= (y - x) \\ \dot{y} &= rx - y - xz \\ \dot{z} &= xy - bz \end{aligned} \tag{156}$$

The system is a dissipative one, since the divergence (for details see (Bărzu, A. et al, 2003))

$$\nabla \cdot F = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = -2 - b < 0$$

since b>0.

Therefore, the phase volume exponentially diminishes in time, as the system tends towards the atractor. For any value of the control parameter r, the system (156) admits as a fixed point the origin

$$x_0 = y_0 = z_0 = 0 \tag{157}$$

The characteristic equation is

$$\begin{vmatrix} -1-\omega & 1 & 0 \\ r - z_0 & -1-\omega & -x_0 \\ y_0 & x_0 & -b-\omega \end{vmatrix} = 0 \tag{158}$$

For the fixed point (157), it takes the form

$$\begin{vmatrix} -1-\omega & 1 & 0 \\ r & -1-\omega & 0 \\ 0 & 0 & -b-\omega \end{vmatrix} = 0$$

from where we find

$$(b + \omega)\left[\omega^2 + 2\omega - (r - 1)\right] = 0 \tag{159}$$

Since parameters b and r are positive ones, it follows that the first eigenvalue $\omega_1 = -b$ is negative for any values of the parameters. The other two eigenvalues ω_2 and ω_3 satisfy the relations

$$\begin{cases} \omega_2 + \omega_3 = -2 < 0 \\ \omega_2 \omega_3 = -(r - 1) \end{cases} \tag{160}$$

According to (160), if $0 < r < 1$ the sum of the two eigenvalues is negative and the product is positive. Therefore, all the eigenvalues are negative and the origin is a stable node. For r > 1, according to (160), the origin becomes unstable and two new fixed points appear in a fork bifurcation. These points are noted with C^+ and C^- which corresponds to patterns

$$(C^+)\begin{cases} x_0 = y_0 = \sqrt{b(r-1)} \\ z_0 = r-1 \end{cases}, \quad (C^-)\begin{cases} x_0 = y_0 = -\sqrt{b(r-1)} \\ z_0 = r-1 \end{cases} \tag{161}$$

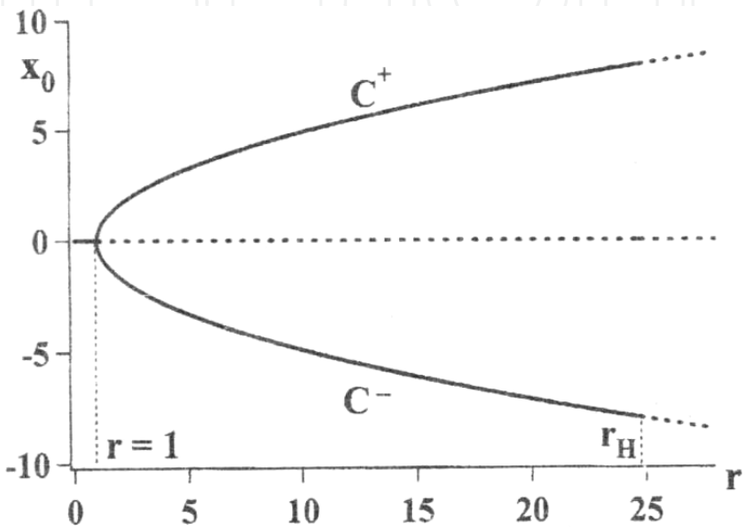

Fig. 1. (according to (Bărzu, A. et al, 2003))

Let us study their stability. Replacing the values that correspond to the branch (C^+) in (158), the characteristic equation becomes

$$\begin{vmatrix} -1-\omega & 1 & 0 \\ 1 & -1-\omega & -\sqrt{b(r-1)} \\ \sqrt{b(r-1)} & \sqrt{b(r-1)} & -b-\omega \end{vmatrix} = 0$$

from where it follows that

$$\omega^3 + \omega^2(b+2) + \omega b(1+r) + 2b(r-1) = 0 \tag{162}$$

If the fixed points (161) will bear a Hopf bifurcation, for a value of control parameter $r_H > 1$, there will be two complex conjugated purely imaginary eigenvalues. Replacing $\omega = i\beta$ in (162) we obtain

$$-i\beta^3 - \beta^2(b+2) + i\beta b(1+r) + 2b(r-1) = 0 \tag{163}$$

Separating the real part from the imaginary one in (163) we obtain the system

$$\begin{aligned} -\beta^3 + \beta b(1+r) &= 0 \\ -\beta^2(b+2) + 2b(r-1) &= 0 \end{aligned} \tag{164a,b}$$

From equation (164a) it follows that $\beta^2 = b(1+r)$. Replacing this value in equation (164), Hopf bifurcation takes place in

$$r_H = -\frac{b+4}{b} \tag{165}$$

Considering that $r_H > 1$ the condition for b results

$$b \leq 4 \tag{166}$$

For this value of the control parameter, the two fixed points C^+ and C^- lose their stability in a subcritical Hopf bifurcation. Beyond the bifurcation point all the periodical orbits are unstable and the system has a chaotic behavior. Figures 2a-c to 8a-c show the trajectories, the time evolutions, the phase portraits and the Fourier transform for the different values of the parameters. It follows that when the value of the parameter r increases, there is a complicated succession of chaotic regimes with certain periodicity windows. The limit cycle appears through a reverse subarmonic cascade and loses stability through intermittent transition towards a new chaotic window.

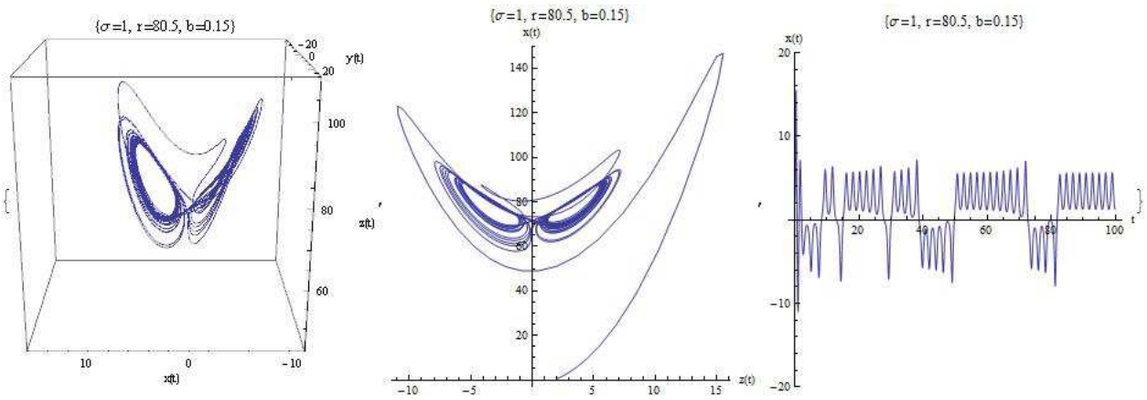

Fig. 2. a) Trajectory b) time evolution c) phase pattern for r=80, b=0.15

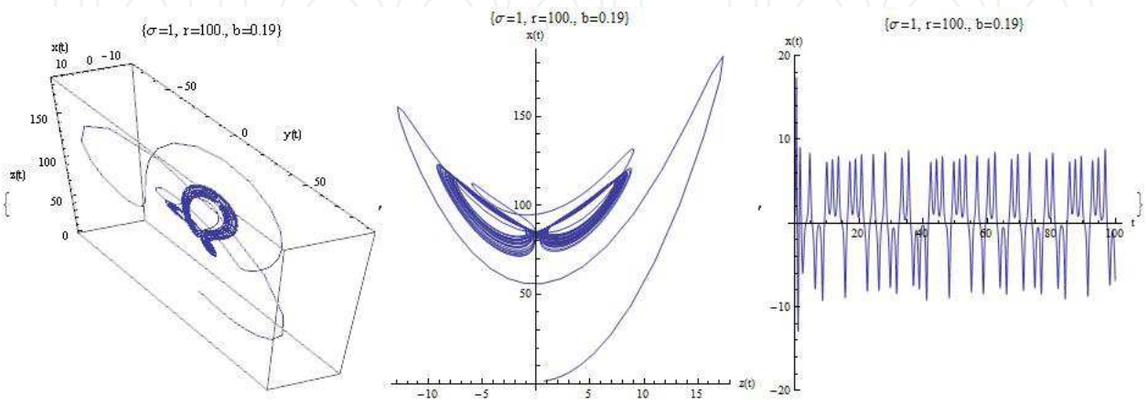

Fig. 3. a) Trajectory b) time evolution c) phase pattern for r=100, b=0.19

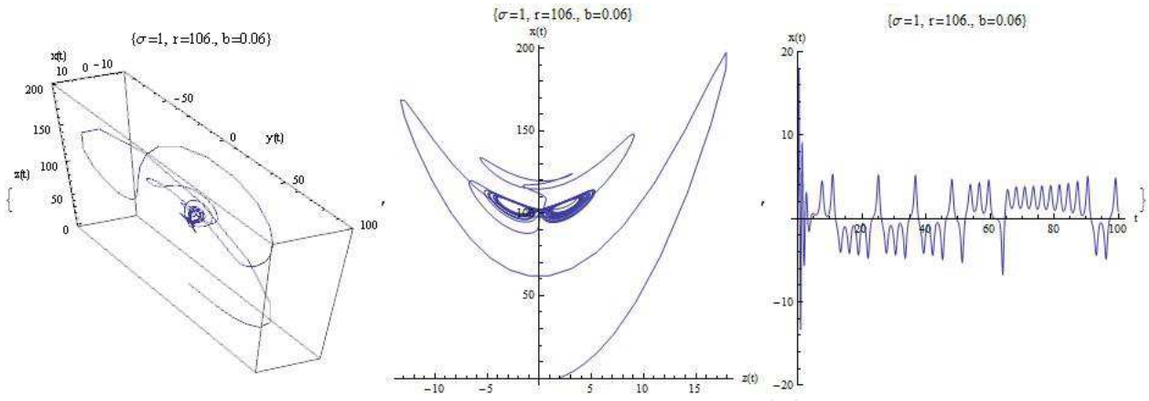

Fig. 4. a) Trajectory b) time evolution c) phase pattern for r=100, b=0.06

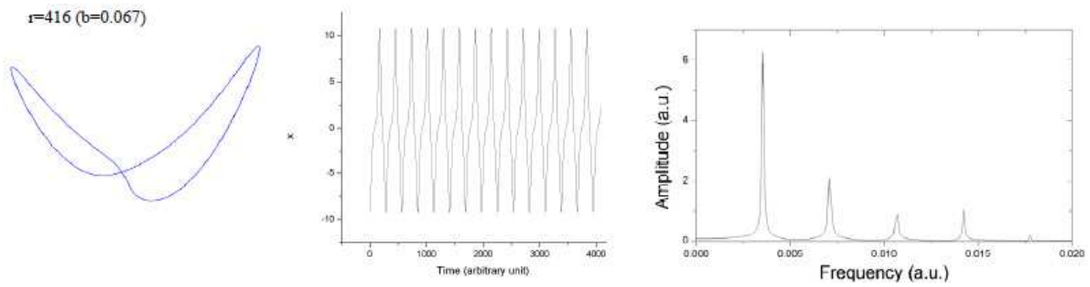

Fig. 5. a) Time evolution b) phase portrait c) the Fourier transform for r=416, b=0.067

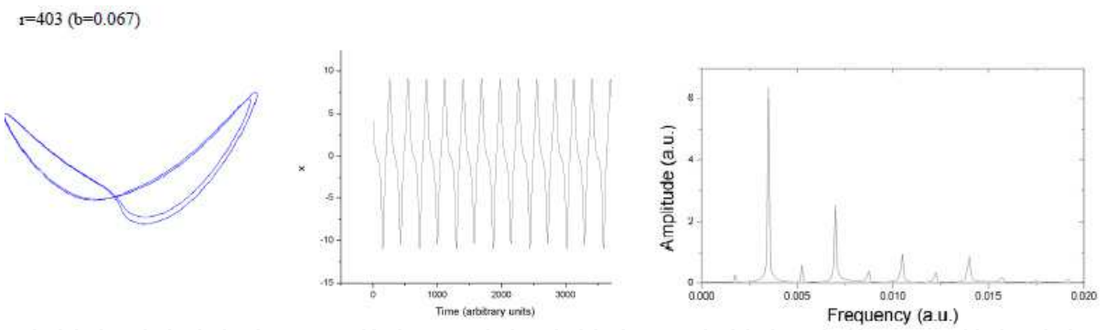

Fig. 6. a) Time evolution b) phase portrait c) the Fourier transform for r=403, b=0.067

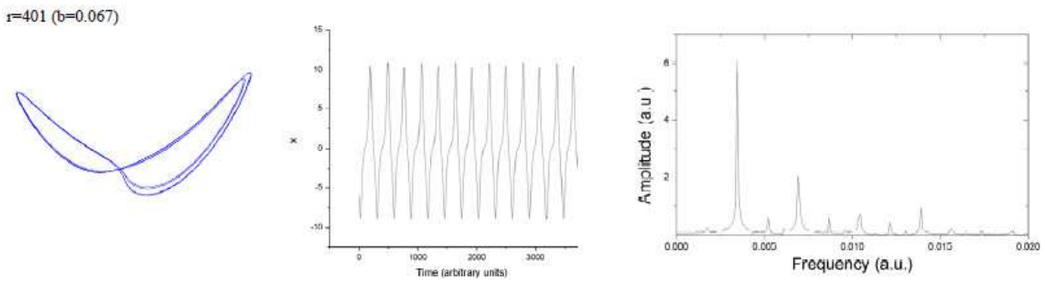

Fig. 7. a) Time evolution b) phase portrait c) the Fourier transform for r=401, b=0.067

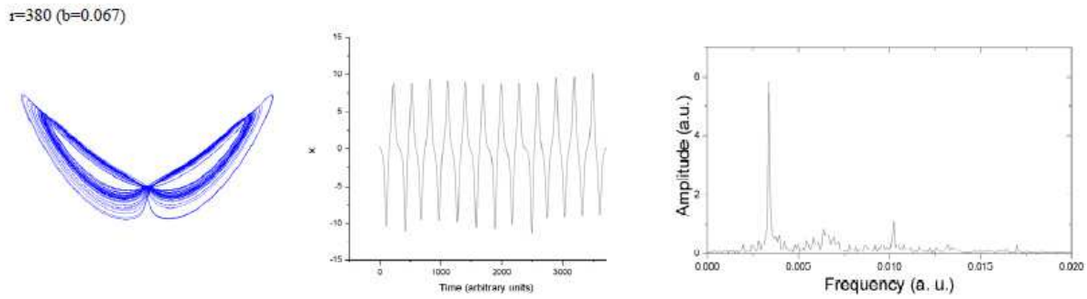

Fig. 8. a) Time evolution b) phase portrait c) the Fourier transform for r=380, b=0.067

In Fig.9 we present the map of the Lyapunov exponent with the value $\sigma = 1$ (the co-ordinates of the light points represent the pairs of values $(x, y) = (b, r)$ for which the probability of entering in a chaotic regime is very high.

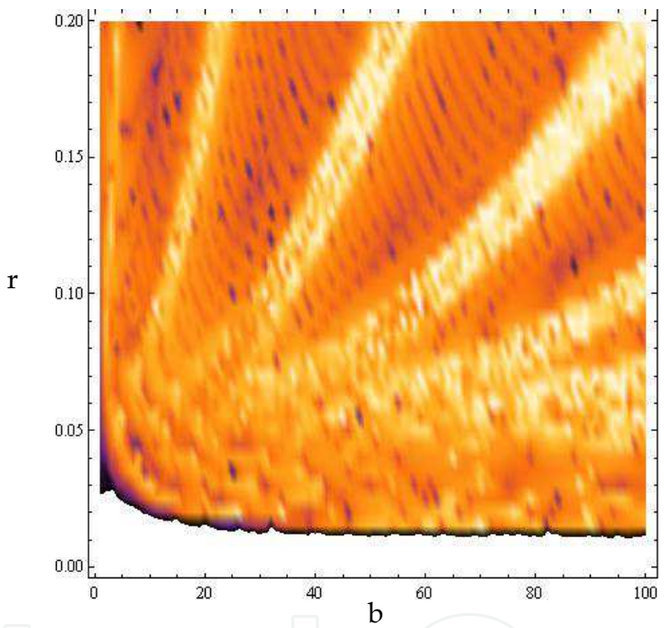

Fig. 9. The Lyapunov exponent map for value $\sigma = 1$ of the Lorenz system

Correspondences with quantum mechanics

The previous analysis states the following:

i. a model of a physical object can be imagined. This model is built from a Madelung type fluid limited by two carcases that are submitted to an energy "gradient", from the inferior carcase towards the superior one;
ii. for small energy gradients, i.e. $R<R_C$ the reference state is a stable one. The ascending force resulting from energy "dilatation" is much smaller than the dissipative one.
iii. for energy gradients that impose restriction $R>R_C$ the reference state becomes unstable through the generation of convective type "rolls". The ascensional force is bigger than the dissipative one;
iv. the increase of energy gradient destroys the convective type "patterns" and induces turbulence;

v. this behavior of fractal fluid can correspond to a Lorenz type "mechanism": limit cycles the convective type "rolls", intermitences ("jumps" between limit cycles) with the "destroy" of the convective type "rolls", chaos with "turbulence" of the convective type state etc.;
vi. the stability of solutions corresponds to the dominant undulatory feature, the wave-corpuscle duality can be correlated with the Lorenz type mechanism: self-organization of the structure through the generation of convective type "rolls" implies the wave-corpuscle transition, while the "jumps" among limit cycles, i.e. the intermittences induce a critical state that corresponds to chaos transition, thus ensuring the dominance of corpuscular effect.

15. Conclusions

Finally we can display the conclusions of this chapter as follows:
- a critical analisys of the hydrodinamic model of Madelung and of the double solution theory of de Broglie's theory of double solution was performed – departing from here, we built a fractal approximation of motion;
- we got the equation of motion of the physical object in the fractal approximation and the Eulerian case was studied;
- the flowing regimes of a rotational fractal fluid were studied;
- we studied the irotational regime of a fractal fluid and the incorporation of the particle into the associated wave by generating a Schrödinger equation;
- the extended hydrodinamic model of scale relativity was built and the role of the fractal potential in the process of incorporation of the particle into the wave, specified;
- we indicated the mechanisms of wave–particle duality by their in phase coherences;
- we studied the wave-particle duality by stationary flow regimes of a fractal fluid which is coherent in phase, and by non-stationary flow regimes of an incoherent fractal fluid by means of a „polarization" type mechanism;
- considering the particle as a singularity in the wave, we showed that its incorporation into the associated wave resulted in Einstein's equations in vacuum - contrary, its non-incorporation led to the second quantification;
- we established a relation between the informational energy and the fractal potential of the complex speed field - it resulted that the generation of forces implies the maximum of the information energy principle;
- we showed that a particle model in a fractal approximation of motion induced a Lorenz type mechanism.

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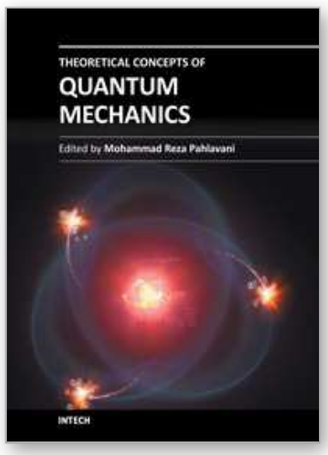

Theoretical Concepts of Quantum Mechanics
Edited by Prof. Mohammad Reza Pahlavani

ISBN 978-953-51-0088-1
Hard cover, 598 pages
Publisher InTech
Published online 24, February, 2012
Published in print edition February, 2012

Quantum theory as a scientific revolution profoundly influenced human thought about the universe and governed forces of nature. Perhaps the historical development of quantum mechanics mimics the history of human scientific struggles from their beginning. This book, which brought together an international community of invited authors, represents a rich account of foundation, scientific history of quantum mechanics, relativistic quantum mechanics and field theory, and different methods to solve the Schrodinger equation. We wish for this collected volume to become an important reference for students and researchers.

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