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Quantum Mechanical Three-Body Systems and Its Application in Muon Catalyzed Fusion

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1. Introduction

A negative muon is a lepton of the second generation with mass number about times heavier than that of electrons, and has a finite lifetime of $\tau_\mu = 2.197 \times 10^{-6}\,\text{sec}$. This lifetime is amply long for most experiments. Muon catalyzed fusion (μCF) is a physical phenomenon in which the negative muon is able to cause fusion at room temperature and thereby eliminating the need for high temperature plasmas or powerful lasers (Owski, 2007; Imo et al., 2006; Filchenkov et al., 2005; Filipowicz et al., 2008; Pahlavani & Motevalli, 2008, 2009; Marshal, 2001; Bystritsky et al., 2006, Nagamine et al., 1987; Nagamine, 2001; Ponomarev, 2001). In comparison with (μCF), hot fusion schemes are made difficulty by the electrostatic (Coulomb) repulsion between positively charged nuclei. In the two conventional approaches to control fusion namely, Magnetic Confinement Fusion (MCF) and Inertial Confinement Fusion (ICF), barrier is partially surmounted by energetic collisions. The particle densities, n and confinement times, τ in the plasma, ($T \geq 10^8 K$) are typically more than ten orders of magnitude difference for these two schemes but the product of these quantities required for d−t fusion is $n\tau \geq 10^{14}\,\text{sec}/cm^3$. For the μCF, effectively $n\tau \approx 10^{25}\,\text{sec}/cm^3$, but this criterion does not tell the whole truth because, in μCF the objective is to tunnel through the barrier without the benefit of kinetic energy. It is known that the $d-t$ fusion by the usual magnetic or inertia confinement suffering a lot of difficulties and problems causing from tritium handling, neutron damage to materials and neutron-induced radioactivity, etc.

Study of the muon catalyzed fusion reactions is of great interest and carried out in many laboratories of the world recently (Ishida et al., 1999; Petitjean et al., 1992, 1993; Bystritsky et al., 2005; Pahlavani & Motevalli, 2008, Bystritsky et al., 2000; Matsuzaki et al., 2001). Muons can be created by the decay of pion which is generated in the collision of intermediate-energy proton with target nuclei. In the muon catalyzed fusion process, after injection of muon in to deuterium and tritium mixture, either $d\mu$ or a $t\mu$ atom is formed, with a probability proportional to the relative concentrations of D and T in the mixture. These atoms are formed in exited states (Breunlich et al., 1989; Korenman, 1996) and then, due to cascade processes, de-excite to ground states. The following reactions illustrate direct formation of muonic dμ and tμ atoms

$$\mu^- + D \rightarrow d\mu + e^- \quad (\lambda_d) \tag{1}$$

$$\mu^- + T \rightarrow t\mu + e^- \quad (\lambda_t) \tag{2}$$

where e^- denotes an electron and λ_d and λ_t are the rate of reactions (1) and (2). The probability of formation of the $d\mu$ atom that will reach its *1s* ground state is quantified by the parameter q_{1s}, which is a function of target density, φ and tritium concentration, C_t. Also it is very sensitive to the $d\mu$ kinetic energy distribution (Menshikov & Ponomarev, 1984; Czaplinski et al., 1994). The difference between binding energies of $t\mu$ and $d\mu$ is about *48.1 eV* (Bom et al., 2005). Therefore, the transfer of a muon from $d\mu$ to a triton is favorable for all temperatures in the given processes

$$d\mu + t \rightarrow t\mu + d + 48.1eV \quad (\lambda_{dt}) \tag{3}$$

with a rate of $\lambda_{dt} = 2.8 \times 10^8 \phi$ (Caffery et al., 1987; Jones et al., 1987; Bystritsky et al., 1980; Breunlich et al., 1987). The muon mass is about *206.77* times larger than the mass of electron. Consequently, the size of a muonic hydrogen atom is smaller than the one of the electronic hydrogen by the same rate approximately. These small muonic atoms can approach other hydrogen nuclei experiencing reduced Coulomb barrier and then induce *d–t* fusions. The process in which a muonic molecule is formed is the most important step in the μCF. The formation of muonic molecules of hydrogen isotopes and their nuclear reactions have been the subject of many experimental and theoretical studies (Caffery et al., 1987; Jones et al., 1987; Bystritsky et al., 1980). In collisions of $t\mu$ muonic atoms with D_2 and DT molecules, the muonic molecules $dt\mu$ are formed during a time interval $\tau_{dt\mu} \leq 10^{-8}$ sec (Jones et al., 1983; Eliezer & Henis, 1994) according to the following resonance reactions

$$t\mu + D_2 \rightarrow \left[(dt\mu)_{J\nu}\, d2e \right] \quad (\lambda_{dt\mu-d}) \tag{4}$$

$$t\mu + DT \rightarrow \left[(dt\mu)_{J\nu}\, t2e \right] \quad (\lambda_{dt\mu-t}) \tag{5}$$

$$\lambda_{dt\mu} = \lambda_{dt\mu-d} C_d + \lambda_{dt\mu-t} C_t \tag{6}$$

in the excited rotational–vibrational ($J\nu$) state with quantum number $J=\nu=1$, where C_d and C_t are concentrations of deuterium and tritium nuclei, respectively. A strong resonance effect appears due to degeneracy in the excited state of the $dt\mu$ and the electron molecule complex. The rate of formation of the $dt\mu$ molecules has been found to depend strongly on temperature, density and on whether collision of the $t\mu$ atom occurs with a D_2 or a DT molecule (Bom et al., 2005; Faifman et al., 1996; Ackerbauer et al., 1999).

In fact, the radius of a muonic hydrogen ion ($dt\mu$) is much smaller (about ≈ 200 times) than a usual electron molecule, therefore the nuclei may tunnel the coulomb barrier with a high probability and fuse with a rate of ≈ 10^{12} sec^{-1} (Bogdanova et al., 1982). Resonant formation of the $dd\mu$ molecule at very low temperatures was observed in solid and liquid D_2 targets (Bogdanova et al., 1982).

Developed methods in this field are based on detailed three-body equations which provide a correct description of the quantum mechanical three-body systems (Takahashi & Takatsuka, 2006; Kilic, Karr & Hilico, 2004; Nielsen et al., 2001; Pahlavani, 2010). Theoretical study of muonic three-body system comprises different theoretical methods, e.g. variational

methods (Viviani et al. 1998; Frolov, 1993), Born-Oppenheimer approximation (Beckel et al., 1970; Kilic et al., 2004) and adiabatic expansion (Fano, 1981; Lin, 1995).

The Born-Oppenheimer approach assumes the nuclei to be infinitely heavy with respect to the negatively charged particle. It should be kept in mind that the following Born-Oppenheimer approach is the simplest solution to the three-body coulomb system. This approach is expected to be a poor approximation for calculations of muonic molecule eigenvalues. In this work, we calculate binding energies of the bound states of the $dd\mu$ muonic three-body system molecule using the adiabatic expansion method.

2. Adiabatic expansion approximation for the three-body system

The exact Hamiltonian that describes muonic three-body system can be shown by following relation:

$$H = -\frac{1}{2m_1}\nabla^2_{R_1} - \frac{1}{2m_2}\nabla^2_{R_2} - \frac{1}{2m_\mu}\nabla_{r_\mu} - \frac{z_1 z_\mu}{\left|\vec{r}_\mu - \vec{R}_1\right|} - \frac{z_2 z_\mu}{\left|\vec{r}_\mu - \vec{R}_2\right|} + \frac{z_1 z_2}{\left|\vec{R}_1 - \vec{R}_2\right|} \tag{7}$$

where *1* and *2* denote the two nuclei, their position is given by R_1 and R_2, and the muon coordinate is r_μ. The center of mass coordinate R_{CM} is given by

$$\vec{R}_{CM} = \frac{m_1\vec{R}_1 + m_2\vec{R}_2 + m_\mu\vec{r}_\mu}{m_1 + m_2 + m_\mu} \tag{8}$$

It is convenient to define Jacobi coordinate *r* and *R* as follow:

$$\vec{r} = \vec{r}_\mu - \frac{\vec{R}_1 + \vec{R}_2}{2} \tag{9}$$

$$\vec{R} = \vec{R}_2 - \vec{R}_1 \tag{10}$$

where *R* is the internuclear coordinate and *r* is the muon coordinate to midpoint between the two nuclei. In these coordinates (*R; r*), the Hamiltonian denoted by equation (7) is change to the following operator:

$$H = -\frac{1}{2M_T}\nabla^2_{R_{CM}} - \frac{1}{2M}(\nabla_R + \lambda/2\nabla_r)^2 - \frac{1}{2m}\nabla^2_r - \frac{z_1 z_\mu}{\left|\vec{r}_\mu - \vec{R}_1\right|} - \frac{z_2 z_\mu}{\left|\vec{r}_\mu - \vec{R}_2\right|} + \frac{z_1 z_2}{\left|\vec{R}_1 - \vec{R}_2\right|} \tag{11}$$

where

$$M_T = m_1 + m_2 + m_\mu \tag{12}$$

$$\frac{1}{M} = \frac{1}{m_1} + \frac{1}{m_2} \tag{13}$$

$$\frac{1}{m} = \frac{1}{m_1 + m_2} + \frac{1}{m_\mu} \tag{14}$$

$$\lambda = \frac{m_1 - m_2}{m_1 + m_2} \tag{15}$$

After separation of variables, the non-relativistic Hamiltonian in units of $e = \hbar = m_\mu = 1$, can be given by

$$H(\hat{o}, R) = -\frac{1}{2M}(\nabla_R + \frac{\lambda}{2}\nabla_r)^2 - H_1 + \frac{z_1 z_2}{R} \tag{16}$$

where

$$H_1 = -\frac{1}{2m}\nabla^2_r - \frac{z_1}{r_1} - \frac{z_2}{r_2} \tag{17}$$

where $\hat{o}$ represent the five dimensional variable. We use the set $\hat{o} = (\Theta, \Phi, \xi, \eta, \varphi)$ where (Θ, Φ, φ) define the Euler rotation specifying the body-fixed frame with its unit vectors to coincide with the principal axes of the inertia tensor of a three-body system. The hyper-spheroidal coordinates ξ and η are easily expressed by the muon-nucleus distances r_1, r_2 and the internuclear distance R,

$$\eta = \frac{r_1 - r_2}{R} \quad (-1 \le \eta \le 1) \tag{18}$$

$$\xi = \frac{r_1 + r_2}{R} \quad (1 \le \xi < \infty) \tag{19}$$

The three-body Hamiltonian (16) commutes with the total angular momentum operator for the three particle system, J, its projection on z-axis, J_z, and the total parity operator, $P(R \to -R, r \to -r)$. Eigenfunctions of the Hamiltonian in the total angular momentum representation reads:

$$\Psi^p_{J,M}(R, r) = \sum_{m=-J}^{J} F^{Jp}_m(R, \xi, \eta) D^{Jp}_{Mm}(\Phi, \Theta, \varphi) \tag{20}$$

Adiabatic expansion of radial function, $F^{Jp}_m(R, \xi, \eta)$ is usually written in the form:

$$F^{Jp}_m(R, \xi, \eta) = \sum_{N=1}^{\infty} \sum_{l=0}^{N-1} \psi_{Nlm}(R; \xi, \eta) \chi^{Jp}_{Nlm}(R) R^{-1} + \sum_{l=0}^{\infty} \int_0^{\infty} dk \psi_{klm}(R; \xi, \eta) \chi^{Jp}_{klm}(R) R^{-1} \tag{21}$$

where $\chi^{Jp}_i(R)$ describe relative motion of the nuclei. Let us consider the Wigner function, $D^{Jp}_{Mm}(\Phi, \Theta, \varphi)$ which is the eigenstates of J^2, J_z and $R.J/R$ with the eigenvalues $J(J + 1)$, M and m (Davydov, 1973). It can be transformed under the inversion as follow:

$$PD^J_{Mm}(\Phi, \Theta, \varphi) = D^J_{Mm}(\Phi + \pi, \pi - \Theta, \pi - \varphi) = (-1)^{J-m} D^J_{M,-m}(\Phi, \Theta, \varphi) \tag{22}$$

If $m \neq 0$, the resultant Wigner functions would be different, and the angular functions consist both even and odd combinations. It is convenient to specify these combinations as follows:

$$D^{Jp}_{Mm}(\Phi,\Theta,\varphi) = \frac{\sqrt{2J+1}}{4\pi}\left[(-1)^m D^{J}_{Mm}(\Phi,\Theta,\varphi) + p(-1)^J D^{J}_{M,-m}(\Phi,\Theta,\varphi)\right] \tag{23}$$

where $p = \pm(-1)^J$ is the eigenvalue of the parity operator:

$$PD^{Jp}_{Mm} = pD^{Jp}_{Mm} \tag{24}$$

The functions presented in equation (23) (in bracket) are satisfying the following orthonormality condition:

$$\int_0^{\pi} \sin\Theta \, d\Theta \int_0^{2\pi} d\Phi \int_0^{2\pi} d\varphi \, [D^{Jp}_{Mm}(\Phi,\Theta,\varphi)]^* D^{J'p'}_{M'm'}(\Phi,\Theta,\varphi) = \delta_{JJ'}\delta_{pp'}\delta_{MM'}\delta_{mm'} \tag{25}$$

If $m = 0$, both the Wigner functions in (22) are reduced to the ordinary spherical function $Y_{JM}(\Theta,\Phi)$ so that the dependence of ' disappears and the angular functions satisfying the conditions (24) and (25) are:

$$D^{Jp}_{M,m=0}(\Phi,\Theta,\varphi) = \frac{Y_{JM}(\Theta,\Phi)}{\sqrt{2\pi}} \tag{26}$$

In this case the parity is unambiguously specified by the quantum number J: $p=+(-1)^J$. So, our basis functions have the following structure:

$$\Psi^{Jp}_{Mjm}(R,\Theta,\Phi,\xi,\eta,\varphi) = D^{Jp}_{Mm}(\Phi,\Theta,\varphi)\psi_{jm}(\xi,\eta;R)\frac{\chi^{Jp}_{jm}(R)}{R} \tag{27}$$

The wave functions $\Psi^{Jp}_{Mjm}(R,\Theta,\Phi,\xi,\eta,\varphi)$ describing reactions $h\mu+h$, h = (p, d, t) can be decomposed over the solutions $\psi_{jm}(\xi,\eta;R)$ of the Coulomb two-center problem. $\psi_{jm}(\xi,\eta;R)$ is the complete set of solutions of the Coulomb two-center problem, therefore

$$H_1\psi_i(\xi,\eta;R)F(\varphi) = E_i(R)\psi_i(\xi,\eta;R)F(\varphi) \tag{28}$$

describing the muon motion around fixed nuclei separated by a distance R. $E_i(R)$ is the energy of a muon in the state i as a function of R. Here we show how to separate the variables through the use of the ellipsoidal (or, prolate spheroidal) coordinates

$$x = \frac{R}{2}\sqrt{(\xi^2-1)(1-\eta^2)}\cos\varphi \tag{29}$$

$$y = \frac{R}{2}\sqrt{(\xi^2-1)(1-\eta^2)}\sin\varphi \tag{30}$$

$$z = \frac{R}{2}\xi\eta \tag{31}$$

Note that the coordinates ξ , η and φ are orthogonal, and we have the first fundamental form

$$ds^2 = dx^2 + dy^2 + dz^2 = h_\xi^2 d\xi^2 + h_\eta^2 d\eta^2 + h_\varphi^2 d\varphi^2 \tag{32}$$

where

$$h_\xi^2 = \left(\frac{\partial x}{\partial \xi}\right)^2 + \left(\frac{\partial y}{\partial \xi}\right)^2 + \left(\frac{\partial z}{\partial \xi}\right)^2 = \frac{R^2}{4}\left(\frac{1-\eta^2}{\xi^2-1}\right) \tag{33}$$

$$h_\eta^2 = \left(\frac{\partial x}{\partial \eta}\right)^2 + \left(\frac{\partial y}{\partial \eta}\right)^2 + \left(\frac{\partial z}{\partial \eta}\right)^2 = \frac{R^2}{4}\left(\frac{\xi^2-1}{1-\eta^2}\right) \tag{34}$$

$$h_\varphi^2 = \left(\frac{\partial x}{\partial \varphi}\right)^2 + \left(\frac{\partial y}{\partial \varphi}\right)^2 + \left(\frac{\partial z}{\partial \varphi}\right)^2 = \frac{R^2}{4}\left(\xi^2-1\right)\left(1-\eta^2\right) \tag{35}$$

Thus

$$\nabla^2 = \frac{1}{h_\xi h_\eta h_\varphi}\left[\frac{\partial}{\partial \xi}\left(\frac{h_\eta h_\varphi}{h_\xi}\frac{\partial}{\partial \xi}\right) + \frac{\partial}{\partial \eta}\left(\frac{h_\xi h_\varphi}{h_\eta}\frac{\partial}{\partial \eta}\right) + \frac{\partial}{\partial \varphi}\left(\frac{h_\xi h_\eta}{h_\varphi}\frac{\partial}{\partial \varphi}\right)\right]$$

$$= \frac{4}{R^2(\xi^2-\eta^2)}\left\{\frac{\partial}{\partial \xi}\left[(\xi^2-1)\frac{\partial}{\partial \xi}\right] + \frac{\partial}{\partial \eta}\left[(1-\eta^2)\frac{\partial}{\partial \eta}\right] + \frac{\xi^2-\eta^2}{(\xi^2-1)(1-\eta^2)}\frac{\partial^2}{\partial \varphi^2}\right\} \tag{36}$$

Note that through the coordinate transformation (29-31), we have

$$r_1 = \frac{R}{2}(\xi + \eta) \tag{37}$$

$$r_2 = \frac{R}{2}(\xi - \eta) \tag{38}$$

Writing the wave function as $\psi_i(\xi,\eta;R)F(\varphi) = G(\xi)H(\eta)F(\varphi)$ and changing the variable to spheroidal coordinates, equation (28) can be separated into following three one-dimensional equations:

$$\frac{d^2F(\varphi)}{d\varphi^2} + m^2F(\varphi) = 0 \tag{39}$$

$$\frac{d}{d\xi}\left[(\xi^2-1)\frac{dG(\xi)}{d\xi}\right] + \left(-A + \alpha q\xi - q^2\xi^2 - \frac{m^2}{\xi^2-1}\right)G(\xi) = 0 \tag{40}$$

$$\frac{d}{d\eta}\left[(1-\eta^2)\frac{dH(\eta)}{d\eta}\right] + \left(A - \beta q\eta - q^2\eta^2 - \frac{m^2}{1-\eta^2}\right)H(\eta) = 0 \tag{41}$$

where

$$\alpha = \frac{R}{q(z_1 + z_2)} \tag{42}$$

$$\beta = \frac{R}{q(z_1 - z_2)} \tag{43}$$

Note that A and q are unknown parameters and should be obtained from (40) and (41) as eigenvalues of the coupled system. Once A and q are obtained, then E can be obtained from q^2=-$R^2E/2$. By substitution of expression (8) into the Schrödinger equation with Hamiltonian (16) and after averaging over spherical angles (Θ,Φ) and the muon state, one obtains the radial equation

$$\frac{1}{2M}\frac{d^2}{dR^2}\chi_i^J(R) + \left[\varepsilon - V_{B-O}(R) - U_i(R) - \frac{J(J+1)}{2MR^2}\right]\chi_i^J(R) = 0 \tag{44}$$

where $\varepsilon = E - E_i(\infty)$ is the collision energy and E is the total energy of the system and $E_i(\infty)$ is the ground state energy of muonic atom. $V_{B-O} = E - E(\infty) + \frac{z_1 z_2}{R}$ is the potential corresponding to the Born-Oppenheimer approximation and $U_i = \left\langle i \middle| (H - H_1 - \frac{z_1 z_2}{R} \middle| i \right\rangle$ is the adiabatic correction. The adiabatic potential $V_{Ad}(R)$ is:

$$V_{Ad}(R) = V_{B-O}(R) + U_i(R) \tag{45}$$

The adiabatic potential $V_{Ad}(R)$ for the $(dd\mu)$ muonic three-body molecule is calculated in the adiabatic expansion method. The adiabatic potential curves and qualitatively similar for each of muonic molecules and are displayed for the $(dd\mu)$ muonic molecule in Figure 1. Results of the calculations of binding energies of the bound states (J,υ) of the $(dd\mu)$ muonic molecule are compared with the results of the other methods used in (Korobov et al., 1992; Kilic, Karr & Hilico, 2004) and are given in Table 1.

States (J,ν)	**Ad** (Pahlavani&Motevalli, 2008)	(Korobov et al., 1992)	(Kilic, Karr & Hilico, 2004)
(0,0)	325.06	325.0735	325.070540
(0,1)	35.79	35.8436	35.844227
(1,0)	226.62	226.6815	226.679792
(1,1)	1.73	1.97475	1.974985
(2,0)	86.20	86.4936	---

Table 1. Binding energies (eV) of the states (J,ν) for the $dd\mu$ muonic molecule.

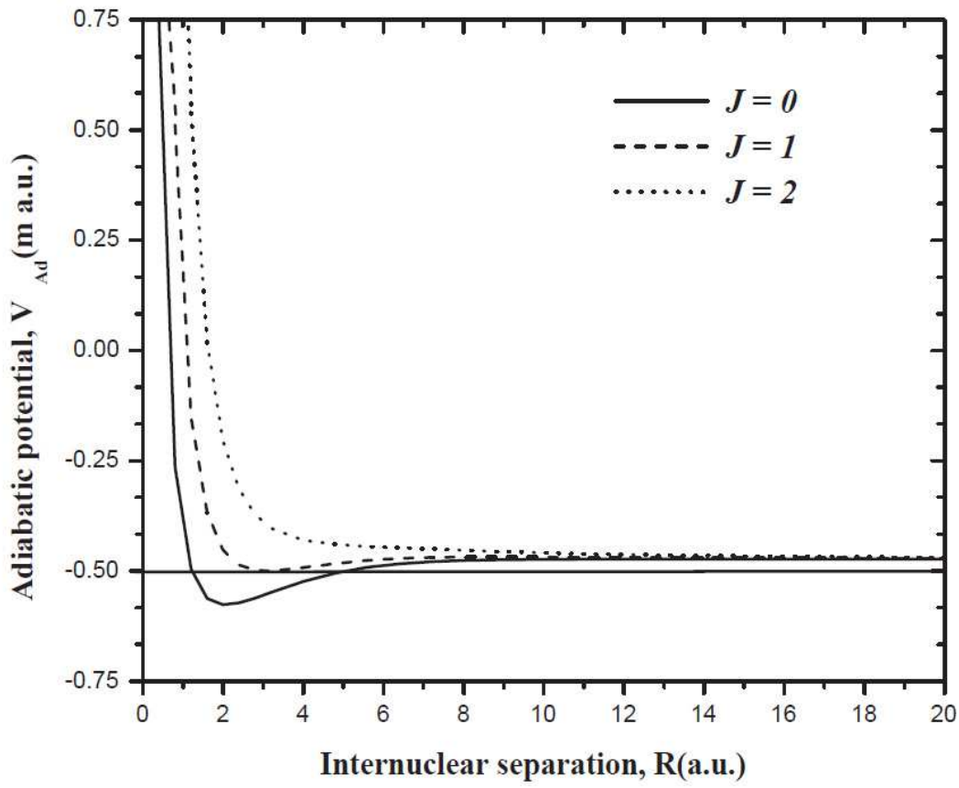

Fig. 1. Adiabatic potential curves, $V_{Ad}(R)$, corresponding to $dd\mu$ system (Pahlavani & Motevalli, 2008).

The calculated binding energies are in good agreement with the previous calculations by other authors using different methods.

3. Charge-asymmetric three-body system in hyper-spherical elliptic coordinate system

Study of the nuclear synthesis reaction $d-^3He$ at low collision energies (below $1\ keV$) is of interest for its applications in nuclear and astrophysics (Belyaev et al.,1995). The relatively large energy gain as well as the lack of tritons in the initial and neutrons in the final channel makes this reaction a very attractive source of thermonuclear fusion energy.

The negatively charged energetic muons, after stopping in the $D-^3He$ mixture, fuse to d or ^3He in order to form the mesic atoms in excited states. After a sequence of cascade transitions lasting about $10^{-11}\ sec$ at Liquid Hydrogen Density (LHD), mesic atoms are formed in the ground state (Ponomarev, 1991; Breunlich et al., 1989; Czaplinski et al., 1996). The three-body molecules, $\left(^3He\mu d\right)_J^{++}$, are formed in collision of $(d\mu)$ atoms in ground state with helium atoms via so-called electron conversion process,

$$(d\mu)_{1s} + {}^3He \xrightarrow{\lambda_m^J} \left[\left(^3He\mu d\right)_{J\nu}^{++}, e\right]^+ + e \tag{46}$$

The molecule dissociates quickly with a rate of about $10^{12}\ sec^{-1}$ to the unbound ground state either by a well-known predissociation mechanism, via Auger transition or γ-emission processes

$$\left({}^3He\mu d\right)_J \xrightarrow{\lambda_p^J} d + \left(He\mu\right)_{1s} \tag{47}$$

$$\left({}^3He\mu d\right)_J \xrightarrow{\lambda_A^J} d + \left({}^3He\mu\right)_{1s} + e \tag{48}$$

$$\left({}^3He\mu d\right)_J \xrightarrow{\lambda_\gamma^J} d + \left({}^3He\mu\right)_{1s} + \gamma \tag{49}$$

resulting a hydrogen nucleus and a mesic helium atom. This mechanism leads to transfer rates of the order $10^8\ sec^{-1}$. The asymmetric-charged ${}^3He\mu d$ molecule undergo nuclear fusion via two different channels,

$$\left({}^3He\mu d\right)_J \rightarrow \alpha + p + \mu + 14.64\ MeV \tag{50}$$

$$\left({}^3He\mu d\right)_J \rightarrow {}^5Li\mu + \gamma + 16.7\ MeV \tag{51}$$

The muon is released after the fusion and can proceed to cause another fusion. Thus the muon works as a catalyst and this cycle can be repeated many times during its lifetime.

To test the stability of the mesic ${}^3He\mu d$ system, we consider only Coulomb interaction between particles. As a starting point, we employ the aim of hyperspherical method to solve the multi-dimensional Schrödinger equation numerically for this three-body system.

$$H\Psi = (T + V)\Psi = E\Psi \tag{52}$$

The wave function, Ψ , can be constructed explicitly by exploiting a specific representation, namely, the hyper-spherical adiabatic expansion method. Here, T is the kinetic energy in its enter-of-mass coordinate frame, V is the potential energy, and E is the total energy of the system. We briefly discuss the general structure of the method and formulate its basic equations for a three-body system in hyper-spherical elliptic coordinates. The Hamiltonian of this molecule in Jacobian coordinates *(R, r)* can be shown by the following equation (Gusev et al., 1990; Stuchi et al., 2000)

$$H = T + V = \left[\frac{1}{2M_i}\Delta(\vec{R}_i) - \frac{1}{2m_i}\Delta(\vec{r}_i)\right] + V \tag{53}$$

where M_i and m_i are reduced masses. It is convenient to define mass-scaled Jacobian vectors, ($\vec{x}_i$ and $\vec{y}_i$),

$$\vec{x}_i = \sqrt{\frac{M_i}{\mu}}\vec{R}_i \tag{54}$$

$$\vec{y}_i = \sqrt{\frac{m_i}{\mu}}\vec{r}_i \tag{55}$$

Therefore the kinetic energy of the system can be rewritten as,

$$T = -\frac{1}{2\mu}\left[\Delta(\vec{x}_i) - \Delta(\vec{y}_i)\right] \tag{56}$$

In this relation, $\mu = (m_d\, m_{^3He}\, m_\mu / m_{tot})^{1/2}$ is an arbitrary coefficient with dimension of mass and index *i=1,2,3* refer to a set of Jacobian coordinates ($\vec{x}_i$, $\vec{y}_i$). The transformation of Jacobian coordinates (e.g. ($\vec{x}_1$, $\vec{y}_1$)) in to another set (e.g. ($\vec{x}_2$, $\vec{y}_2$)), can be done as follows:

$$\vec{x}_2 = -\vec{x}_1 \cos\gamma - \vec{y}_1 \sin\gamma \tag{57}$$

$$\vec{y}_2 = \vec{x}_1 \sin\gamma - \vec{y}_1 \cos\gamma \tag{58}$$

where can be regarded as a rotational parameter which is shown by,

$$\gamma = Arc\tan\left[\sqrt{\frac{m_\mu(m_d + m_{^3He} + m_\mu)}{m_d\, m_{^3He}}}\right]; \qquad 0 \le \gamma \le \frac{\pi}{2} \tag{59}$$

It is convenient to calculate these sets of Jacobian coordinates ($\vec{x}_i$, $\vec{y}_i$) for mesic three-body, $^3He\mu d$ system. These three sets should be used as coordinates in configuration space. Therefore this system contains six dimensions (*d=6*). In hyperspherical coordinates, (ρ,Ω), $\rho = \sqrt{x^2 + y^2}$ represents the size of the system and $\Omega = (\Omega_s, \Omega_0)$, consist of five variables, where Ω_s denote a set of two angles defining the shape of the system and Ω_0 refer to a set of three angles defining the overall orientation of the three-body system. The Hamiltonian, in this coordinate system, will take the following form:

$$H = -\frac{1}{2\mu}\left[\left(\rho^{-5}\frac{\partial}{\partial\rho}\rho^5\frac{\partial}{\partial\rho}\right) - \frac{\Lambda^2}{\rho^2}\right] + V \tag{60}$$

where Λ^2 is regarded as the square of general angular momentum operator. Our aim is to solve the eigenvalue equation $H\Psi(\rho,\Omega) = E\Psi(\rho,\Omega)$ in the adiabatic expansion method. The idea of adiabatic separability between the hyper-radius ρ and the hyper-angular variables Ω in three-body systems was first exploited by Macek (Macek, 1968) for studying doubly excited states of the Helium atom. The wave equation of the system in this method can be defined by:

$$\Psi(\rho,\Omega) = \rho^{-\frac{5}{2}}\sum_\upsilon F_\upsilon(\rho)\Phi_\upsilon(\rho,\Omega) \tag{61}$$

Here the quantum number, *j* characterizes a channel function, the radial functions, $F_j(\rho)$ satisfy the system of coupled ordinary differential equations and $\phi_j(\Omega;\rho)$ are angular functions. For any value of ρ, these functions form a set of complete orthogonal basis which satisfy the following relation:

$$\left[H_{ad} - U_\nu(\rho)\right]\phi_\nu(\Omega;\rho) = 0 \tag{62}$$

In this relation, H_{ad} is the adiabatic Hamiltonian which is defined by,

$$H_{ad} = \frac{1}{2}\Lambda^2 + \rho C \tag{63}$$

where $C = \rho V$ is the effective charge of the system. In the first step we try to solve the differential equation (62), which contains coordinate ρ as a parameter. The hyper-spherical elliptic coordinates (η,ξ) on S (projection of the hyper-sphere $\rho = const.$ onto shape space) are induced by conical coordinates on its 3D image. The hyper-spherical elliptic coordinates (η,ξ) are defined in the following intervals (Tolstikhin et al., 1995)

$$-2\gamma \le \eta \le 2\gamma$$

$$2\gamma \le \zeta \le 2\pi - 2\gamma \tag{64}$$

The definition of (η,ξ) resembles the representation of plane elliptic coordinates. In order to rewrite Eq. (62) in a new set of coordinates, (η,ξ), it is necessary to define the square of general angular momentum operator, Λ^2 in this set of coordinates,

$$\Lambda^2 = \frac{-16}{\cos\eta - \cos\xi}\left[\frac{\partial}{\partial\eta}\left(\cos\eta - \cos 2\gamma\right)\frac{\partial}{\partial\eta} + \frac{\partial}{\partial\xi}\left(\cos 2\gamma - \cos\xi\right)\frac{\partial}{\partial\xi}\right]$$

$$+\frac{4m^2\sin^2 2\gamma}{\cos\eta - \cos\xi}\left[\frac{1}{\cos\eta - \cos 2\gamma} + \frac{1}{\cos 2\gamma - \cos\xi}\right] \tag{65}$$

where m is azimuthal quantum number which is the projection of the general angular momentum along body-fixed axis. The potential energy, V, of the system is the sum of three inter-particle Coulombian interactions potential,

$$V = \frac{Z_1 Z_2 e^2}{r_{12}} + \frac{Z_2 Z_3 e^2}{r_{23}} + \frac{Z_3 Z_1 e^2}{r_{31}} = \frac{C(\eta,\xi)}{\rho} \tag{66}$$

As it was mentioned earlier, $C(\eta,\xi) = \rho V$, is the effective charge. The inter-particle distances r_{12}, r_{23} and r_{31} are simply defined by the following relations:

$$r_{12} = \frac{\rho}{\sqrt{2\mu_3}}\sqrt{1 + d^+\cos\left(\frac{\eta}{2}\right)\cos\left(\frac{\xi}{2}\right) - d^-\sin\left(\frac{\eta}{2}\right)\sin\left(\frac{\xi}{2}\right)}$$

$$r_{23} = \frac{\rho}{\sqrt{\mu_1}}\sin\left(\frac{\eta+\xi}{4}\right) \tag{67}$$

$$r_{31} = \frac{\rho}{\sqrt{\mu_2}}\sin\left(\frac{\eta-\xi}{4}\right)$$

where $d^{+}=1+\frac{2m_{\mu}}{m_{d}+m_{^{3}He}}$ and $d^{-}=\frac{m_{d}-m_{^{3}He}}{m_{d}+m_{^{3}He}}$ are mass related constants respectively. The effective charge *C* of $^{3}He\mu d$ molecule as a function of variables η and ξ is shown in Fig. 2. The steep spike at $(\eta,\xi)=(-2\gamma,2\gamma),(2\gamma,2\gamma)$, corresponds to the strong attractive coulomb singularities of effective charge *C* and associated to collisions in the pairs $d\mu$ and $^{3}He\mu$, when muon is very close to the nucleus. The singular Coulomb repulsion between two positively charged particles, are represented by the repulsive wall at the neighborhoods of the $(\eta,\xi)=(0,2\pi-2\gamma)$.

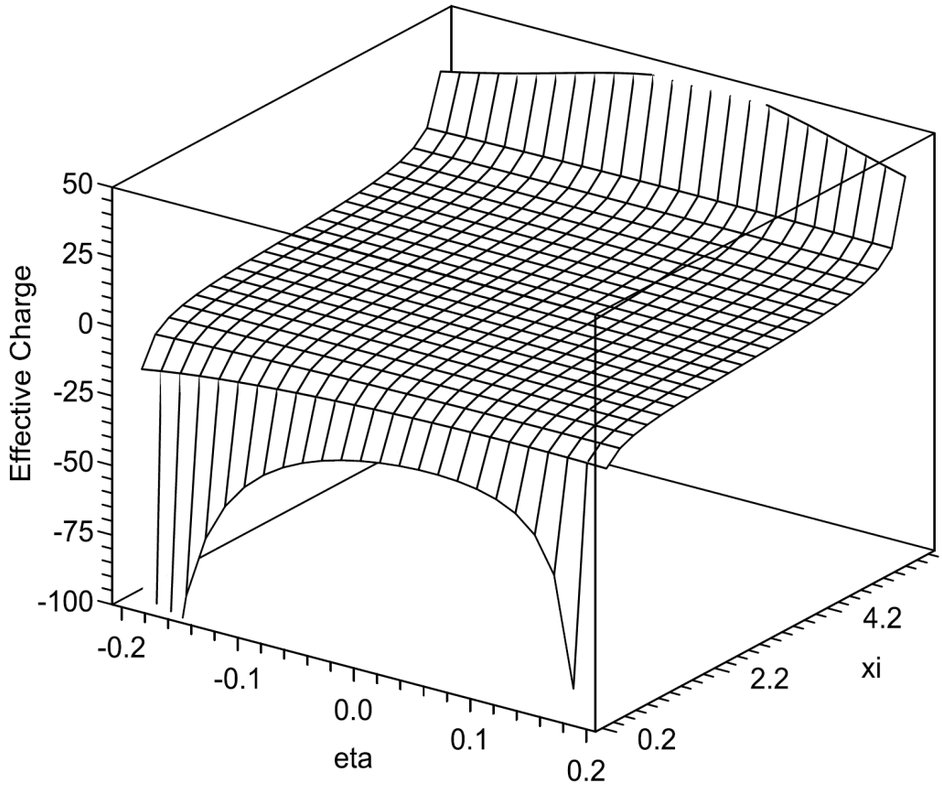

Fig. 2. Variation of effective charge *C* as a function of hyper-angular variables $-2\gamma\le\eta\le2\gamma$ and $2\gamma\le\zeta\le2\pi-2\gamma$ for the $^{3}He\mu d$ molecule (Pahlavani, Sadeghi, Motevalli & Aqabaei, 2010).

By substituting Eqs. (65) and (66) for Λ^{2} and *V* into Eq. (62), we obtain a differential equation for adiabatic Hamiltonian that should be solve with appropriate boundary conditions. In the case, for infinite small values of ρ, the solutions of adiabatic Hamiltonian (62) can be constructed in the following form (Pahlavani & Motevalli, 2008):

$$\Phi(\eta,\xi)=N\,X(\eta)Y(\xi) \tag{68}$$

where *N* is the normalization parameter. With some mathematical simplification, one obtains the following set of ordinary differential equations:

$$\left[L^{\eta}-\frac{2m^2\cos^2 2\gamma}{\cos\gamma-\cos 2\gamma}+U(\cos\eta-\cos 2\gamma)+A\right]+X(\eta)=0 \tag{69}$$

$$\left[L^{\xi}-\frac{2m^2\cos^2 2\gamma}{\cos 2\gamma-\cos\xi}+U(\cos 2\eta-\cos\xi)-A\right]+Y(\xi)=0 \tag{70}$$

where A is the separation constant and the one-dimensional derivative operators, L^{η} and L^{ξ} are defined as,

$$L^{\eta}=8\frac{d}{d\eta}(\cos\eta-\cos 2\gamma)\frac{d}{d\eta} \tag{71}$$

$$L^{\xi}=8\frac{d}{d\xi}(\cos 2\gamma-\cos\xi)\frac{d}{d\xi} \tag{72}$$

The resultant equations are subject to the regularity of boundary conditions and can be satisfied only for certain values of A and U. The method for solving these set of differential equations, are very similar to those equations which we presented in our previous work (Pahlavani & Motevalli, 2008), when we have studied the motion of muon in the two-center Coulomb problem in prolate spheroidal coordinate system for the symmetric mesic system $dd\mu$. By solving these equations, one obtains the functions $\Phi_{n(\eta)n(\xi)}(\eta,\xi)$, where $n(\eta)$ and $n(\xi)$ are quantum numbers correspond to number of zeros of the functions $X(\eta)$ and $Y(\xi)$ that appeared in Eqs. (69) and (70). These functions form a set of complete bases and satisfy the following normalization condition,

$$\int_{-2\gamma}^{2\gamma}\int_{2\gamma}^{2\pi-2\gamma}\Phi^{*}{}_{n(\eta)n(\xi)}(\eta,\xi)\Phi_{n'(\eta)n'(\xi)}(\eta,\xi)\,ds=\delta_{n(\eta)n(\xi)}\delta_{n'(\eta)n'(\xi)} \tag{73}$$

where ds is the surface element that can be defined by,

$$ds=\frac{\pi^2}{4\cos 2\gamma}(\cos\eta-\cos\xi)\,d\eta\,d\xi \tag{74}$$

The results of the calculations are displayed graphically in Fig. 3. The normalization factor $N_{n(\eta)n(\xi)}$, is a function of the rotational parameter, at different quantum numbers $n(\eta)$ and $n(\xi)$. Calculated values of adiabatic potential $U(\rho)$ as a function of hyper-radius ρ have been shown in Fig. 4. By substituting Eq. (61) into Eq. (52), one can obtain the following set of ordinary differential equations for radial functions $F_j(\rho)$:

$$\left\{\frac{d^2}{d\rho^2}+2\mu\left[E-U(\rho)-\frac{15}{8\mu\rho^2}\right]\right\}F_{\nu}(\rho)+\sum_{\kappa}W_{\nu\kappa}F_{\kappa}(\rho)=0 \tag{75}$$

where $U_j(\rho)$ is the adiabatic potential and the operator $W_{j\kappa}(\rho)$ has the following form:

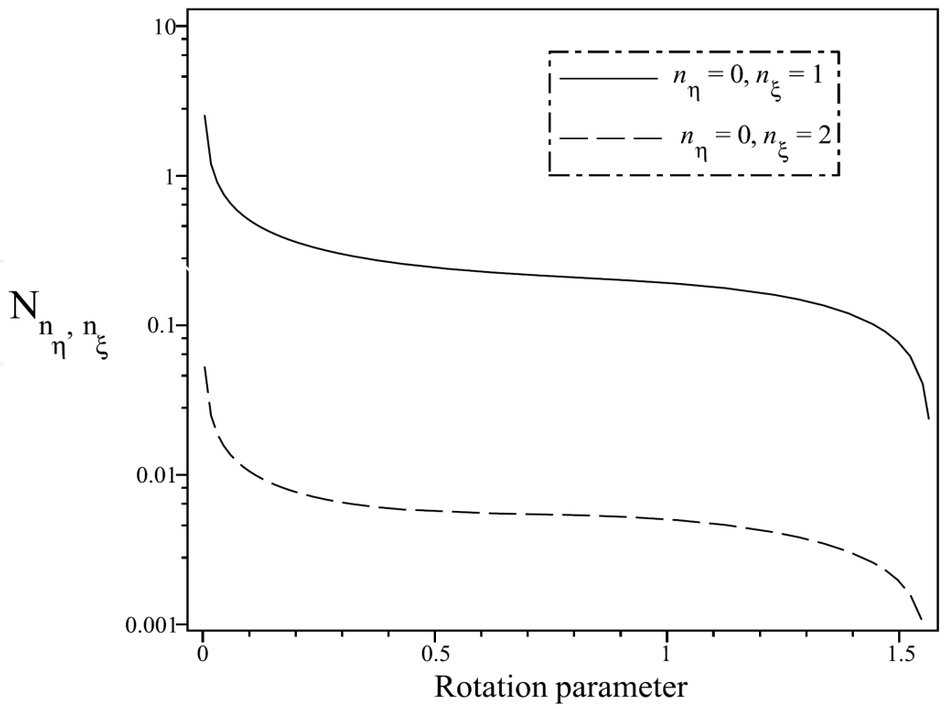

Fig. 3. Variation of normalization parameter $N_{n(\eta)n(\xi)}$ for the cases: $(n_\eta, n_\eta) = (0,1)$ and $(n_\eta, n_\eta) = (0,2)$ for the $^3He\mu d$ molecule (Pahlavani, Sadeghi, Motevalli & Aqabaei, 2010).

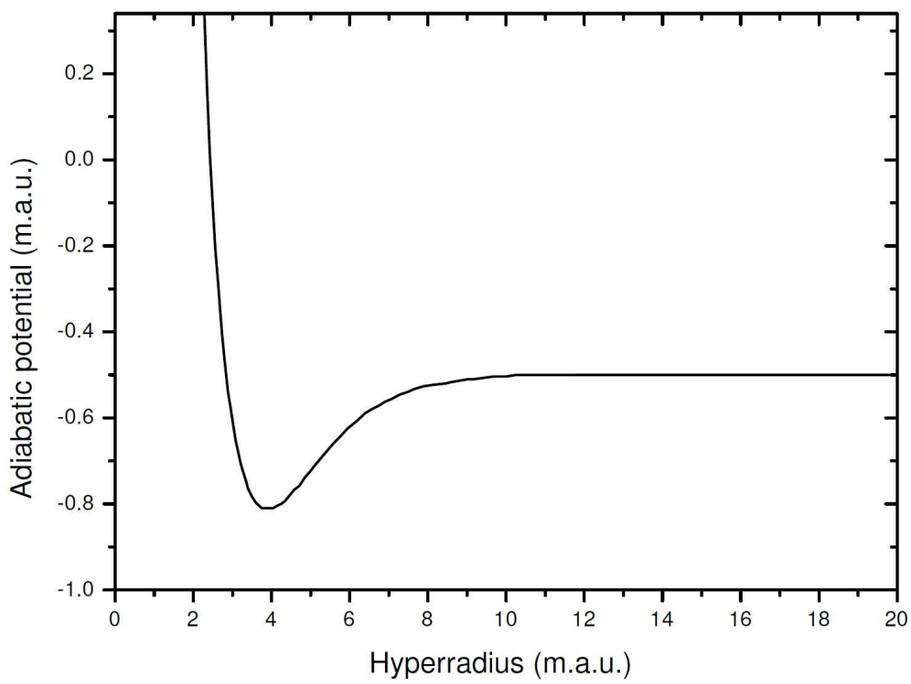

Fig. 4. The adiabatic potential as a function of hyper-radius coordinate for the $^3He\mu d$ molecule (Pahlavani, Sadeghi, Motevalli & Aqabaei, 2010).

$$W_{\nu\kappa}(\rho) = 2R_{\nu\kappa}(\rho)\frac{d}{d\rho} + S_{\nu\kappa}(\rho) \tag{76}$$

The above equation is a set of differential equations coupled by the following nonadiabatic terms:

$$R_{\nu\kappa}(\rho) = \left\langle \phi_\nu(\Omega;\rho) \middle| \frac{\partial}{\partial\rho}\phi_\kappa(\Omega;\rho) \right\rangle \tag{77}$$

$$S_{\nu\kappa}(\rho) = \left\langle \phi_\nu(\Omega;\rho) \middle| \frac{\partial^2}{\partial\rho^2}\phi_\kappa(\Omega;\rho) \right\rangle \tag{78}$$

where the brackets represent integration over the angular variables Ω. The hyperspherical adiabatic approximation amounts to retaining only one term in Eq. (61) (Macek, 1968). Then the radial function, $F_j(\rho)$ satisfies the following differential equation

$$\left\{ \frac{d^2}{d\rho^2} + 2\mu\left[E - U(\rho) - \frac{15}{8\mu\rho^2} \right] + W_{\nu\nu} \right\} F_\nu(\rho) = 0 \tag{79}$$

This approximation turns out to be surprisingly accurate in the sense that in many situations the non adiabatic couplings in Eq. (75) are rather weak. Subject to this reality and considering appropriate boundary conditions, we obtain solutions of the differential equation (79) numerically. Finally, the calculated values of the binding energies of the bound states (J,υ) for the $^3He\mu d$ system are compared with available data obtained using other methods in Table 2.

States (J,ν)	(Gershtein & Gusev, 1993)	(Kravtsov et al., 1993)	(Hara & Ishihara, 1989)	(Pahlavani et al., 2010)
(0,0)	69.96	70.6	70.74	70.879
(1,0)	46.75	48.2	47.90	48.391
(2,0)	---	9.6	---	9.346

Table 2. Binding energy E_B *(eV)* of the bound states (J,ν) (the quantum numbers of rotational-vibrational state) for the $^3He\mu d$ molecule.

The Born–Oppenheimer approach assumes the nuclei to be infinitely heavy with respect to the negatively charged particle. It should be kept in mind that Born–Oppenheimer approach is the simplest solution to the three-body Coulomb system. Usually, the most accurate results for the ground state energy levels of mesic three-body molecule were obtained from variational calculations. Comparison of our result for *J = 0* with the ones obtained by the available variational calculation (Bogdanova et al., 1982) indicates difference that dose not exceed *0.2%*. One can conclude that this fact supports the validity of adiabatic expansion in hyper-spherical elliptic coordinates method which have been used.

4. Muon stripping in the muon catalyzed fusion

The sticking of muons to alpha particles after fusion is an unwanted process and eliminates muons from the chain of fusion reactions. This process is the main loss mechanism in the μCF. The probability of forming a muonic helium ion is called initial sticking probability $\omega_S^0 (= 0.912\%)$ (Hu, Hale & Cohen, 1994). Where muonic helium ions are formed with an energy of $E_{\alpha\mu}^{in} = 3.47\,MeV$ $(v_{\alpha\mu}^{in} = 5.83\,a.u.)$ then are slowed down toward thermal energy by collision with the surrounding D_2 and DT molecules (Jones, 1986). During the same time, as long as the kinetic energy exceeds the appropriate threshold $(E_{\alpha\mu}^{th} \approx 10\,KeV)$, the $a\mu$ ion can be stripped as a result of collisions. This process is referred to as reactivation and final sticking fraction, ω_s that conventionally related to the initial sticking fraction by $\omega_S = \omega_S^0 (1 - R)$. The reactivation coefficient, R depends upon the stopping power of the media and several important cross sections. Stripping process can occur through several channels. Collisions of the $(a\mu)_{1s}$ ions with the surrounding D_2 and DT molecules during the slowing down process can result in $a\mu$ charge transfer, ionization or excitation of the discrete $a\mu$ levels. Stripping (charge transfer plus ionization) can also happen from the $a\mu$ which is the results of the sticking or collisional excitation processes.

The kinetic of reactivation is described by the various rates in a set of coupled differential equations. The fraction of stripped muonic helium ions in terms of population probabilities can be written as

$$\frac{dP_{strip}(t)}{dt} = \sum_i \lambda_{strip}^{(i)}\left(v(t)\right) P_i(t)\,. \tag{80}$$

where $\lambda_{strip}^{(i)}\left(v(t)\right)$ are velocity-dependent stripping rates from the individual energy levels and $P_i(t)$ are the time dependent population probabilities for the state i of muonic helium ion. The time-dependent population probabilities for the state i of the muonic helium ion are determined by

$$\frac{dP_i(t)}{dt} = \lambda_{pop}^{(i)} - P_i(t)\,\lambda_{depop}^{(i)} \tag{81}$$

where $\lambda_{pop}^{(i)}$ and $\lambda_{depop}^{(i)}$ are the rates of populating and de-populating probability of state i, respectively. These rates can be given by the following relations:

$$\begin{aligned}\lambda_{pop}^{(i)} = &\sum_{i'(n_{i'}>n_i)} \left(\lambda_{Au}^{(i'\to i)} + \lambda_{ra}^{(i'\to i)} + \lambda_{de-ex}^{(i'\to i)}\right) P_{i'}(t) \\ &+ \sum_{i'(n_{i'}<n_i)} \lambda_{ex}^{(i'\to i)} P_{i'}(t) + \sum_{i'(n_{i'}=n_i)} \lambda_{Stark}^{(i'\to i)} P_{i'}(t)\end{aligned} \tag{82}$$

$$\lambda_{depop}^{(i)} = \lambda_{strip}^{(i)} + \sum_{i'(n_{i'}<n_i)} \left(\lambda_{Au}^{(i\to i')} + \lambda_{ra}^{(i\to i')} + \lambda_{de-ex}^{(i\to i')}\right) + \sum_{i'(n_{i'}>n_i)} \lambda_{ex}^{(i\to i')} + \sum_{i'(n_{i'}=n_i)} \lambda_{Stark}^{(i\to i')} \tag{83}$$

where λ_{Au}, λ_{ra}, λ_{de-ex}, λ_{ex}, λ_{Stark} and λ_{strip} are the Auger de-excitation, radiative, Coulomb de-excitation, Coulomb excitation, Stark mixing and striping rates, respectively. In general, λ is given by

$$\lambda = N\sigma v \quad (\sec^{-1}) \tag{84}$$

where N, v and σ are density of surrounded media, relative velocity and cross section for all processes under consideration, respectively. The time and velocity dependence in Eq. (80) are coupled through the energy-loss equation for muonic helium ion given by

$$\frac{dE_{\alpha\mu}}{dx} = -v_{\alpha\mu} S\left(E_{\alpha\mu}\right) = -\left(\frac{2E_{\alpha\mu}}{m_{\alpha\mu}}\right)^{1/2} S\left(E_{\alpha\mu}\right) \tag{85}$$

where $S = -dE/dx$ is the stopping power of the surrounding media and $m_{\alpha\mu}$ is the mass of muonic helium ion. The initial conditions are: $E_{\alpha\mu}(0) = E_{\alpha\mu}^{in} = 3.47\,MeV$, $P_{st}(0) = 0$ and the initial values of populated levels are determined by the initial sticking, $P_i(0) = \omega_s^0(i) / \omega_s^0$. The populations $P_i(t)$ for $n = 1,2,...,6$ and the l sublevels are treated in detail for $n < 4$. The reactivation coefficient R is equivalent to the stripping fraction $P_{st}(t)$ at $t \to \infty$. The intensity of X-ray transition in muonic helium ion is another quantity which can be measured experimentally and calculated along with reactivation coefficient (R). Muons in excited levels of the $\alpha\mu^+$ may de-excite under X-ray emission. The X-ray spectrum depends not only on the initial sticking in the atomic levels and the reactivation of the muon but also on intra-atomic transitions due to inelastic collisions, internal and external Auger effect and Stark mixing. The photon intensity per sticking event is calculated using

$$\frac{d\gamma_{n'\to n}}{dt} = \sum_{i'(n_{i'}=n')} \sum_{i(n_i=n)} \lambda_{ra}^{(i'\to i)} P_{i'}(t) \tag{86}$$

The number of X-ray photons emitted per fusion is the most useful quantity that can be measured experimentally. The X-ray yields for the $n' \to n$ transition is given by

$$Y(n' \to n) = \gamma_{n'\to n}\omega_s^{\circ} \tag{87}$$

The calculation for muon stripping probability from $a\mu^+$ and the intensity of X-ray transitions have been done by solving a set of coupled differential equations numerically. The time-dependent population probabilities $P_i(t)$ for *1s, 2s, 2p, 3s, 3p, 3d* are shown in Fig. 5 for a deuterium–tritium target at density φ=*1.2 L.H.D* (*L.H.D*≡ *Liquid Hydrogen Density* = *4.25* × 10^{22} *atoms/cm*^{3}). The initial populations of all excited states are seen to drop to *0* during the stopping time, and only *1s* orbital stays occupied.

The time-dependent stripping fraction, $P_{st}(t)$ and surviving fraction of the initial kinetic energy, E/E_0 are shown in Fig. 6. Slowing down of $a\mu^+$ from $v_{\alpha\mu} = 5.83\,a.u.$ to $v_{\alpha\mu} \approx 1\,a.u.$ takes about $t_{stop} \approx 4\times10^{-11}$ (sec) . This time is longer than the lifetime of the excited $a\mu^+$ states so that the cascade of $a\mu^+$ actually takes place during the slowing down process. The calculated reactivation coefficient, final sticking and the average number of X-

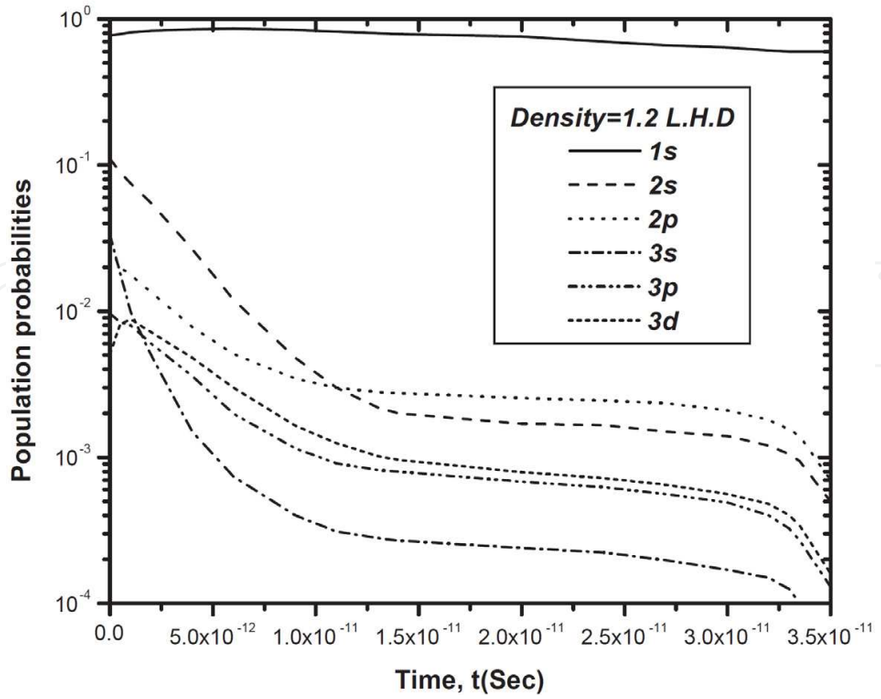

Fig. 5. The population probabilities $P_i(t)$ as a function of time in a D–T target at density φ=*1.2 L.H.D* (Pahlavani & Motevalli, 2008).

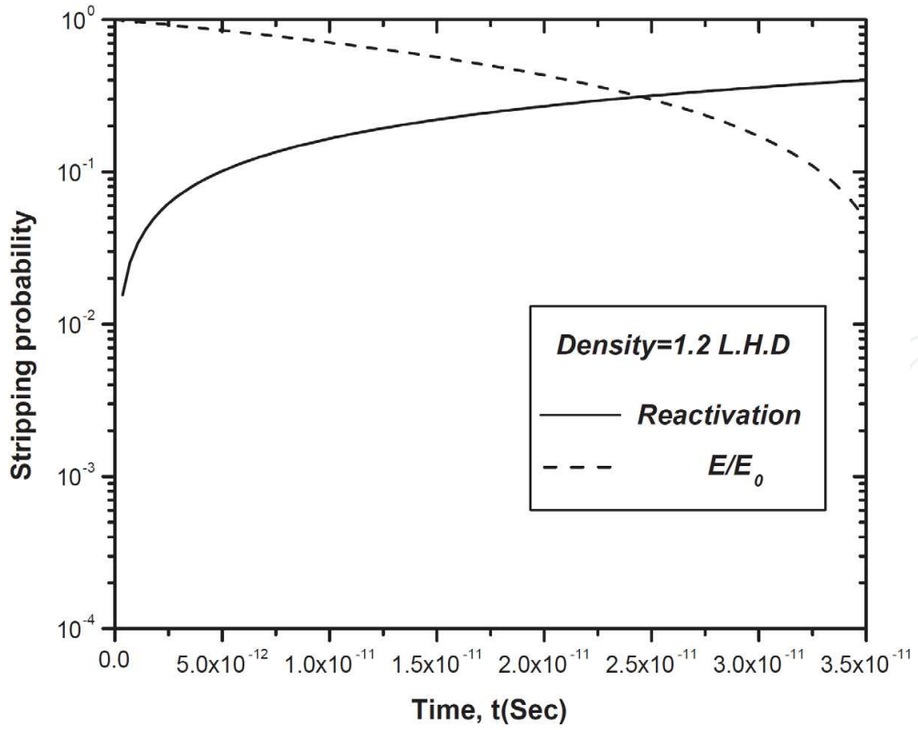

Fig. 6. Stripping fraction, R (heavy solid curve), surviving fraction of initial kinetic energy, E/E_0 (dashed curve) in a D–T target at density φ = *1.2 L.H.D* (Pahlavani & Motevalli, 2008).

rays per sticking $(K_\alpha, K_\beta, K_\gamma)$ as a function of density are shown in Fig. 7 for $\phi < 4LHD$. The most K_α radiation actually emitted by $a\mu^+$ atoms that formed in the ground state. If $a\mu^+$ is formed in the $2p$ state more than one $K_\alpha(2p \to 1s)$ X-ray expected per sticking. Our theoretical results for stripping are compared in Table 3 with other theoretical and experimental data. It is evident that experimental results of the effective sticking probability are smaller than the theoretical calculations, however, our results agree well with experiment.

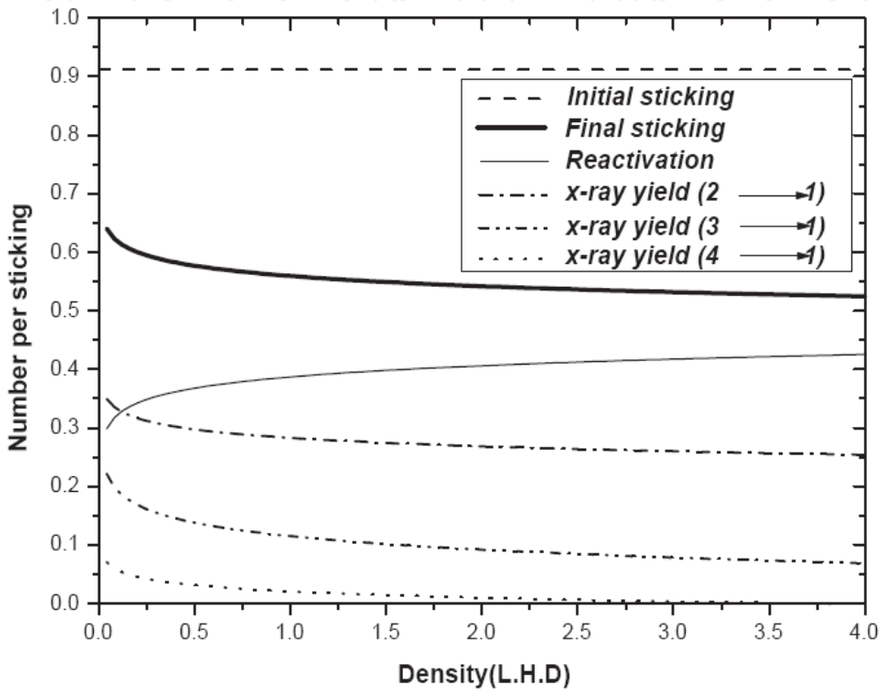

Fig. 7. The density dependence of initial sticking, $\omega_s^0(\%)$, final sticking, $\omega_s(\%)$, reactivation coefficient, *R* and *K*-series X-ray per sticking $(K_\alpha, K_\beta, K_\gamma)$ for dtμ fusion (K_β and K_γ multiplied by factor 3) (Pahlavani, Motevalli, 2008).

The density dependence of probability of muon reactivation, final sticking coefficient and intensity of X-rays emitted by muonic helium ion have been studied numerically. In order to do this, we consider all reactions that separate muon from muonic helium ion, namely coulomb excitation and de-excitation, ionization, charge transfer, Stark mixing, radiative transitions and Auger de-excitation. Using a set of coupled differential equations, the time dependence of muon reactivation coefficient (*R*) and surviving fraction of the initial kinematic energy of $a\mu^+$ (E/E_0) in the *D*–*T* mixture for different fuel density have been calculated. The measurement of muonic helium ion X-ray provides an independent method to test our knowledge about muon reactivation and sticking. Results based on our calculation shown that the muon reactivation increases when the average number of X-rays per sticking reduces with increasing density. Our calculated results are in good agreement with available experimental data (Ishida, Nagamine et al., 1999; Petitjean et al., 1993; Breunlich et al., 1987; Bossy et al., 1987; Jones, Taylor & Andeson, 1993; Nagamine et al., 1993; Ishida et al., 2001; Petitjean, 2001) at all.

Source	Reactivation	Final sticking (%)
Density = 1.2 *LHD*		
(Pahlavani & Motevalli, 2008)	0.391	0.555
(Markushin, 1988)	---	0.57 ± 0.07
(Refelski et al.,1989)	0.36	0.57
(Takahashi, 1987)	0.248	0.664
(Cohen, Hale, Hu, 1996)	---	0.59
Experimets		
PSI (Bossy et al., 1987)	---	0.39 ± 0.10
PSI (Breunlich et al., 1987)	---	0.45 ± 0.05
PSI (Petitjean et al., 1993)	---	$0.48 \pm 0.02 \pm 0.04$
LAMPF (Jones, Taylor & Andeson, 1993)	---	$0.43 \pm 0.05 \pm 0.06$
KEK (Nagamine et al., 1993)	---	0.51 ± 0.004
RIKEN-RAL, Liquid (Ishida, Nagamine et al., 1999)	---	0.434 ± 0.030
RIKEN-RAL, Solid (Ishida, Nagamine et al., 1999)		0.421 ± 0.030
RIKEN (Ishida et al., 2001)	---	0.532 ± 0.030
Density = 1.45 *LHD*		
(Pahlavani, Motevalli, 2008)	0.395	0.551
Experiment		
PSI (Petitjean, 2001)	---	0.505 ± 0.029

Table 3. The reactivation coefficient, R and final sticking, $\omega_s(\%)$ for muonic helium ion in different densities.

5. Conclusions

The quantum-mechanical three-body problem plays an important role in modern physics by providing an appropriate description of three-particle systems in presence of Coulomb and nuclear forces. Developed methods in this field are based on detailed three-body equations which provide a correct description of the quantum mechanical three-body systems (Takahashi & Takatsuka, 2006; Kilic, Karr & Hilico, 2004; Nielsen et al., 2001; Pahlavani, 2010). Theoretical study of muonic three-body system comprises different theoretical methods, e.g. variational methods (Viviani et al. 1998; Frolov, 1993), Born-Oppenheimer approximation (Beckel et al., 1970; Kilic et al., 2004) and adiabatic expansion (Fano, 1981; Lin, 1995). In this investigation, we presented an appropriate method that enables us to study the solutions of Schrodinger equation for $^3He\mu d$ system. The adiabatic expansion in hyper-spherical elliptic coordinates has shown a good approach for calculating the adiabatic potential. Fast convergent of this method led us to obtain precise results for the existence of the bound states in $^3He\mu d$ three-body molecule. The obtained results for the adiabatic potential of this system are comparable with results gathered from other approximation methods.

The corresponding eigenvalue problem has been solved and the binding energy of this system is calculated. The obtained results agreed with the expected values of various theoretical methods. This approach can be applied for other three-body systems with variety of masses and charges. The obtained results are of significant importance for experimental and theoretical investigation of $d-{}^3He$ nuclear fusion especially at low collision energies.

In section 4, the obtained results show that the muon cycle coefficient increases almost slowly with the density of deuterium and tritium mixture. The energy required to produce a muon estimated to be about *5000 MeV*. Since each deuterium and tritium fusion generates *17.6 MeV*, we see that the number of catalysis reactions by a muon should be about 285 to reach the scientific break-even (*1/3* of the commercial break-even). The break-even point is reached when the fusion process generates as much energy as was initially put in (i.e., the energy output equals the energy input). The output energy of the number of catalysis reactions by a muon in it's lifetime (τ_μ = *2.197 μsec*), is much smaller than the input energy required to produce a muon. Therefore, a fusion energy system based on the muon catalyzed fusion in deuterium and tritium fuel seems to be viable at plasma conditions with fuel densities about *100* times of L.H.D.

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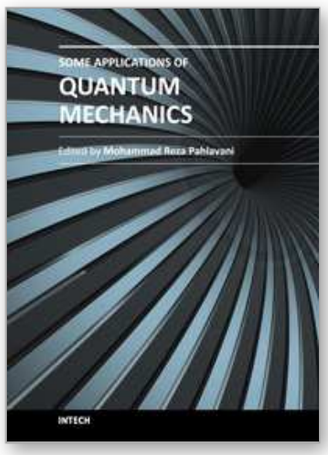

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Quantum mechanics, shortly after invention, obtained applications in different area of human knowledge. Perhaps, the most attractive feature of quantum mechanics is its applications in such diverse area as, astrophysics, nuclear physics, atomic and molecular spectroscopy, solid state physics and nanotechnology, crystallography, chemistry, biotechnology, information theory, electronic engineering... This book is the result of an international attempt written by invited authors from over the world to response daily growing needs in this area. We do not believe that this book can cover all area of application of quantum mechanics but wish to be a good reference for graduate students and researchers.

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