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Transport of Interfacial Area Concentration in Two-Phase Flow

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1. Introduction

The accurate prediction of thermal hydraulic behavior of gas-liquid two-phase flow is quite important for the improvement of performance and safety of a nuclear reactor. In order to analyze two-phase flow phenomena, various models such as homogeneous model, slip model, drift flux model and two-fluid model have been proposed. Among these models, the two-fluid model (Ishii (1975), Delhaye (1968)) is considered the most accurate model because this model treats each phase separately considering the phase interactions at gas-liquid interfaces. Therefore, nowadays, two-fluid model is widely adopted in many best estimate codes of nuclear reactor safety. In two-fluid model, averaged conservation equations of mass, momentum and energy are formulated for each phase. The conservation equations of each phase are not independent each other and they are strongly coupled through interfacial transfer terms of mass, momentum and energy through gas-liquid interface. Interfacial transfer terms are characteristic terms in two-fluid model and are given in terms of interfacial area concentration (interfacial area per unit volume of two-phase flow) Therefore, the accurate knowledge of interfacial area concentration is quite essential to the accuracy of the prediction based on two-fluid model and a lot of experimental and analytical studies have been made on interfacial area concentration. In conventional codes based on two-fluid model, interfacial area concentration is given in constitutive equations in terms of Weber number of bubbles or droplets depending upon flow regime of two-phase flow (Ransom et al. (1985), Liles et al. (1984)). However, recently, more accurate and multidimensional predictions of two-phase flows are needed for advanced design of nuclear reactors. To meet such needs for improved prediction, it becomes necessary to give interfacial area concentration itself by solving the transport equation. Therefore, recently, intensive researches have been carried out on the models, analysis and experiments of interfacial area transport throughout the world

In view of above, in this chapter, intensive review on recent developments and present status of interfacial area concentration and its transport model will be carried out.

2. The definition and rigorous formulation of interfacial area concentration

Interfacial area concentration is defined as interfacial area per unit volume of two-phase flow. Therefore, the term "interfacial area concentration" is usually used in the meaning of

volume averaged value and denoted by $\overline{a_i}^V$. For example, one considers the interfacial area concentration in bubbly flow as shown in Fig.1. In this figure, A_i is instantaneous interfacial area included in volume, V. The volume averaged interfacial area concentration is given by

$$\overline{a_i}^V = \frac{A_i}{V} \tag{1}$$

For simplicity, bubbles are sphere of which diameter is d_b, interfacial area concentration is given by

$$\overline{a_i}^V = \frac{\pi N d_b^2}{V} = \frac{6\alpha}{d_b} \tag{2}$$

Here, N is number of bubbles in volume V, and α is void fraction (volumetric fraction of bubbles in volume V).
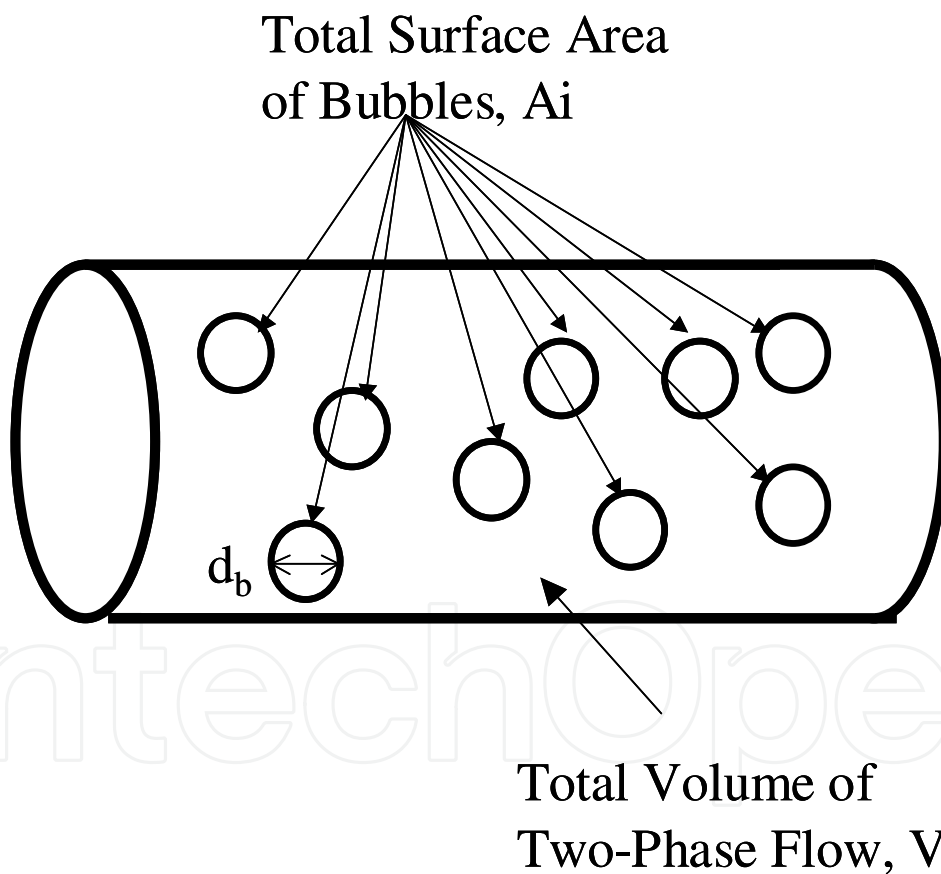

Fig. 1. Interfacial area in bubbly flow

Similarly, time averaged interfacial area concentration, $\overline{a_i}$ and statistical averaged interfacial area concentration, $\overline{a_i}^A$ can be defined. The transport equation of interfacial area concentration is usually given in time averaged form in terms of time averaged interfacial area concentration, $\overline{a_i}$. However, for the derivation of the transport equation, it is desirable to formulate interfacial area concentration and its transport equation in local instant form.

Kataoka et al. (1986), Kataoka (1986) and Morel (2007) derived the local instant formulation of interfacial area concentration as follows.

One considers the one dimensional case where only one plane interface exists at the position of $x=x_0$, as shown in Fig.2. In the control volume which encloses the interface in the width of Δx, as shown in Fig.2, average interfacial area concentration is given by

$$\overline{a_i} = \frac{1}{\Delta x} \tag{3}$$

When one takes the limit of $\Delta x \to 0$, local interfacial area concentration, a_i, is obtained. It takes the value of zero at $x \neq x_0$ and infinity at $x=x_0$. This local interfacial area concentration is given in term of delta function by
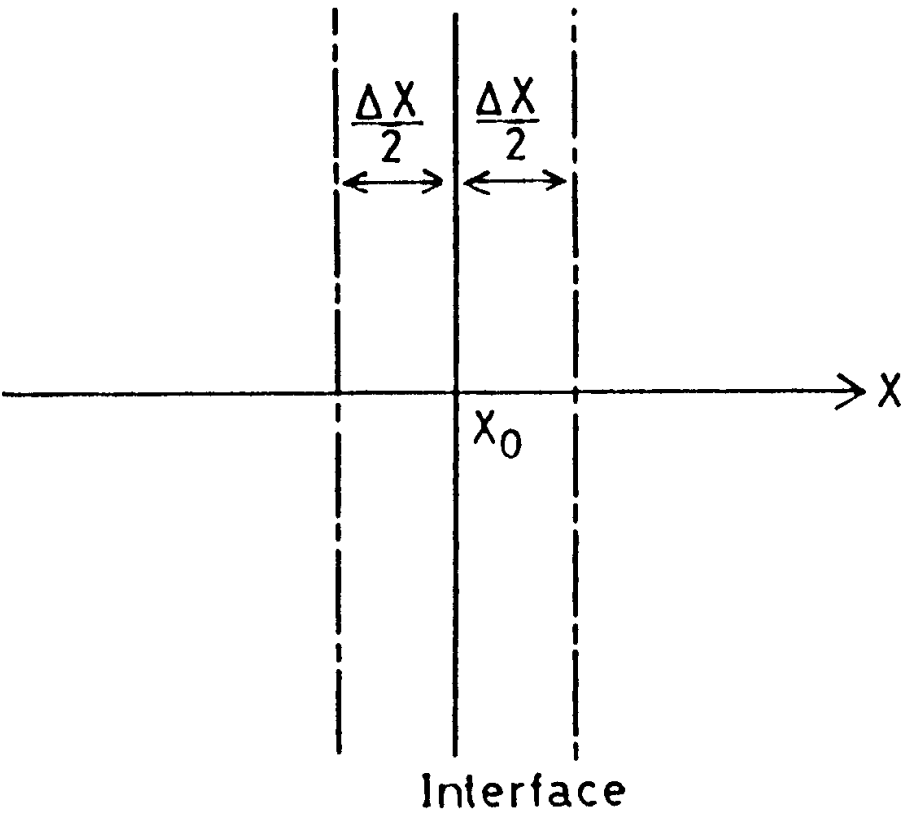
$$a_i = \delta(x - x_0) \tag{4}$$

Fig. 2. Plane interface at $x=x_0$

This formulation can be easily extended to three dimensional case. As Shown in Fig.3, three dimensional interface of gas and liquid is mathematically given by

$$f(x,y,z,t)=0 \tag{5}$$

$$f(x,y,z,t)>0 \ (\text{gas phase}), \quad f(x,y,z,t)<0 \ (\text{liquid phase}) \tag{6}$$

Liquid Phase

$f(x,y,z,t)<0$

Gas Phase

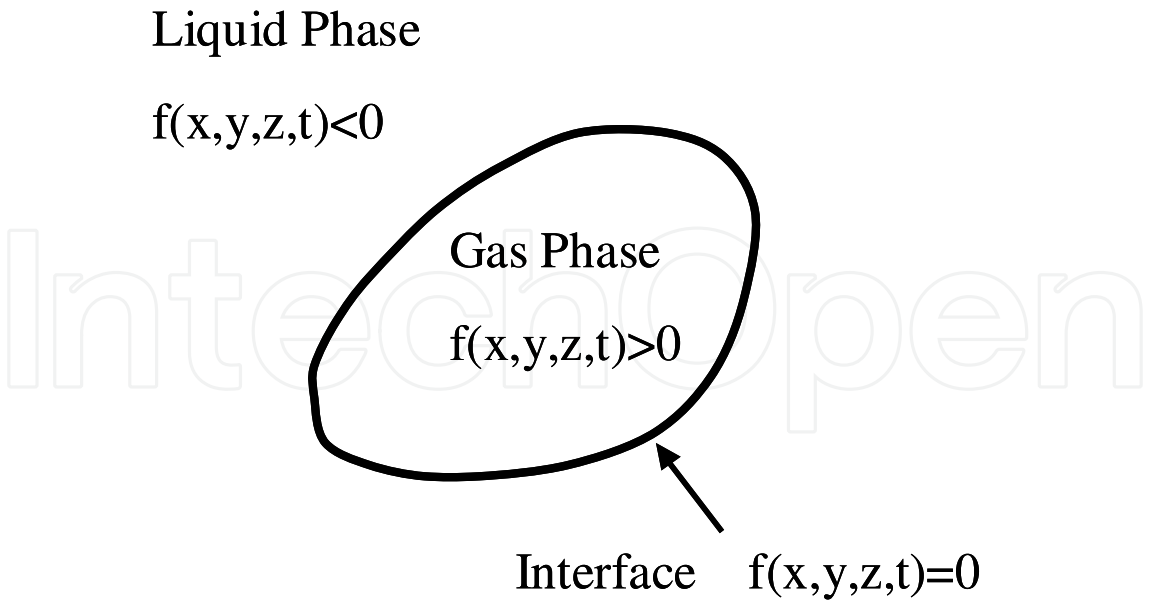$f(x,y,z,t)>0$

Interface $f(x,y,z,t)=0$

Fig. 3. Mathematical representation of three-dimensional interface

As shown in Fig.4, one considers the control volume which encloses the interface by following two surfaces.

$$f(x,y,z,t)=\Delta f/2 \qquad (7)$$

$$f(x,y,z,t)=-\Delta f/2 \qquad (8)$$

$f(x,y,z,t)=-\Delta f/2$

$\Delta f/|\,\mathrm{grad}\,f\,|$

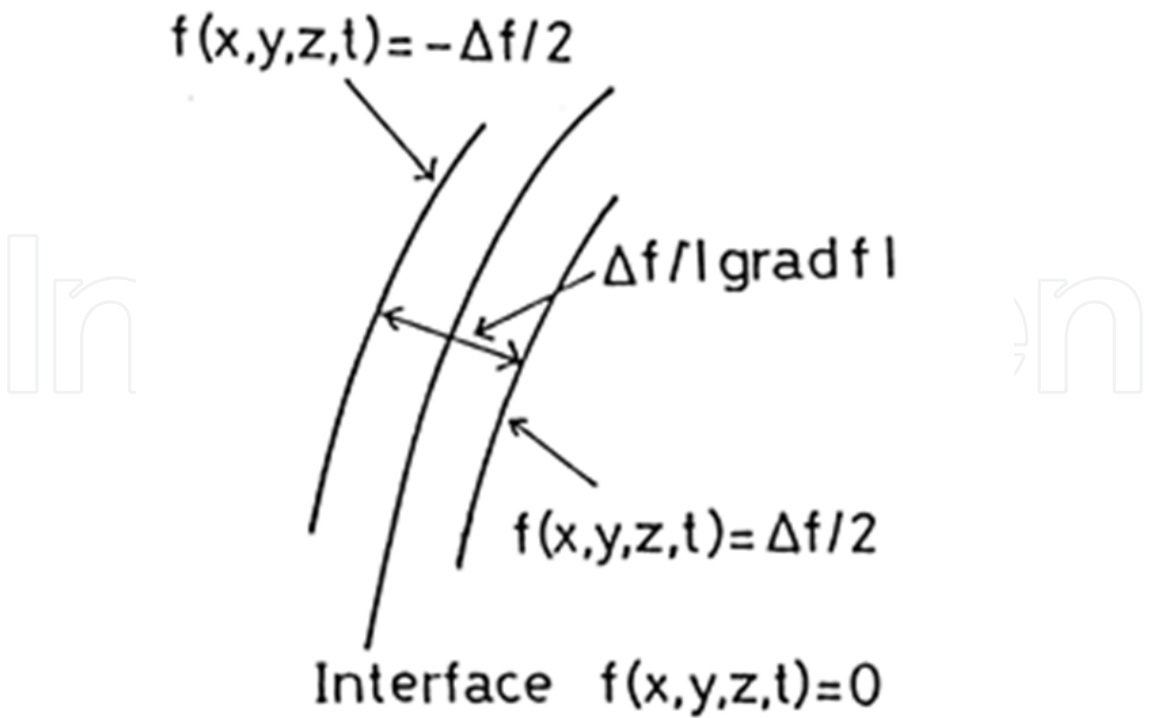$f(x,y,z,t)=\Delta f/2$

Interface $f(x,y,z,t)=0$

Fig. 4. Control volume enclosing three-dimensional interface

By the differential geometry, the width of the control volume is given by

$$\Delta f / |\,\text{grad } f(x,y,z,,t)\,|$$

Then, average interfacial area concentration in this control volume is given by

$$\overline{a_i} = |\text{grad } f(x,y,z,t)| / \Delta f \tag{9}$$

When one takes the limit of $\Delta f \rightarrow 0$, local interfacial area concentration, a_i, is obtained by

$$a_i = |\text{grad } f(x,y,z,t)| \, \delta(f(x,y,z,t)) \tag{10}$$

where $\delta(w)$ is the delta function which is defined by

$$\int_{-\infty}^{\infty} g(w)\delta(w - w_0)dw = g(w_0) \tag{11}$$

where $g(w)$ is an arbitrary continuous function..

In relation to local instant interfacial area concentration, characteristic function of each phase (denoted by ϕ_k) is defined by

$$\phi_G = h(f(x,y,z,t)) \quad \text{(gas phase)} \tag{12a}$$

$$\phi_L = 1 - h(f(x,y,z,t)) \quad \text{(liquid phase)} \tag{12b}$$

where suffixes G and L denote gas and liquid phase respectively. ϕ_k is the local instant void fraction of each phase and takes the value of unity when phase k exists and takes the value of zero when phase k doesn't exist. Here, $h(w)$ is Heaviside function which is defined by

$$\begin{aligned} h(w) &= 1 \ (w>0) \\ &= 0 \ (w<0) \end{aligned} \tag{13}$$

Heaviside function and the delta function are related by

$$\delta(w) = \frac{dh(w)}{dw} \tag{14}$$

Using above equations, the derivatives of characteristic function are related to interfacial area concentration as follows.

$$\text{grad } \phi_k = -n_{ki}a_i \quad (k = G, L) \tag{15}$$

$$\frac{\partial \phi_k}{\partial t} = v_i \bullet n_{ki}a_i \quad (k = G, L) \tag{16}$$

Here, $\mathbf{n}_{ki}$ is unit normal outward vector of phase k as shown in Fig.5 and $\mathbf{v}_i$ is the velocity of interface.

Using above-mentioned relations, it is shown that local instant interfacial area concentration is given in term of correlation function of characteristic function (Kataoka (2008)). As shown in

Eq.(15), local instant interfacial concentration is related to the derivative of characteristic function of each phase. Here, directional differentiation of characteristic function is considered. Spatial coordinate (x,y,z) is denoted by vector **x** and displacement vector of **r** is defined.
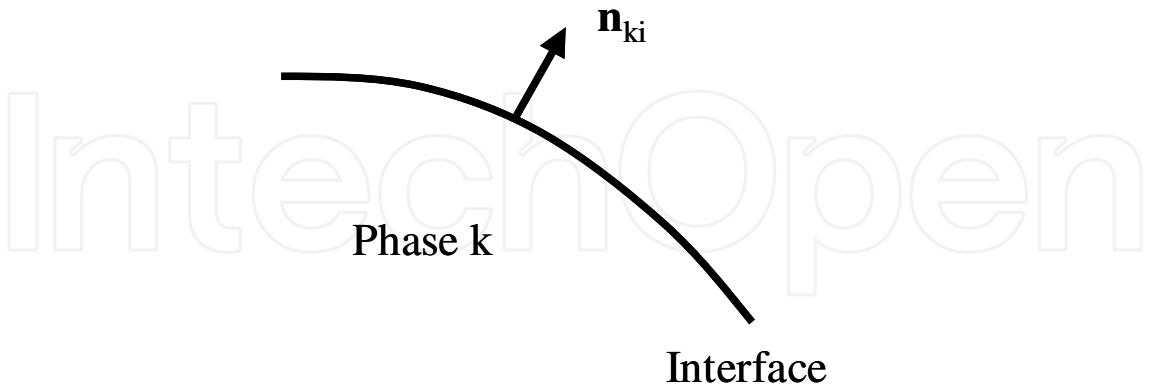

Fig. 5. Unit normal outward vector of phase k

$$x=(x,y,z) \tag{17}$$

$$r=(r_x,r_y,r_z) \tag{18}$$

At position **x**, directional differentiation of characteristic function $\phi_k(\mathbf{x})$ in **r** direction (denoted by $\dfrac{\partial}{\partial r}$) is defined by

$$\frac{\partial}{\partial r}\phi_k(x) = n_r \bullet grad\phi_k(x)$$
$$= -n_r \bullet n_{ki}a_i \tag{19}$$
$$= -\cos\theta\ a_i$$

Here, $\mathbf{n_r}$ is unit vector of **r** direction and θ is the angle between $\mathbf{n_r}$ and $\mathbf{n}_{ki}$ as shown in Fig. 6.
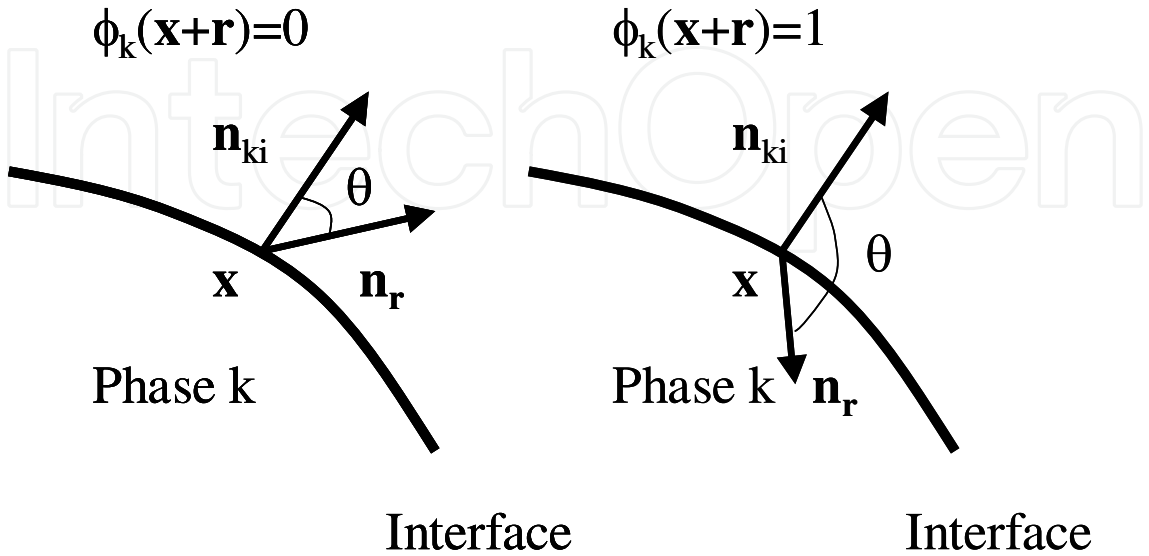

Fig. 6. Configuration of $\mathbf{n_r}$ and $\mathbf{n}_{ki}$

In view of Fig.6, the product of $\phi_k(\mathbf{x+r})$ and Eq(19) is given by

$$\phi_k(x+r)\frac{\partial}{\partial r}\phi_k(x) = 0 \qquad (0 \le \theta \le \pi/2)$$
$$= -\cos\theta \ a_i \quad (\pi/2 \le \theta \le \pi) \tag{20}$$

Equation (19) is rewritten by

$$\phi_k(x+r)\frac{\partial}{\partial r}\phi_k(x) = \frac{1}{2}(-\cos\theta + |\cos\theta|)a_i \tag{21}$$

From Eqs.(19) and (21), one obtains

$$\frac{\partial}{\partial r}\phi_k(x) - 2\phi_k(x+r)\frac{\partial}{\partial r}\phi_k(x) = -|\cos\theta| \ a_i \tag{22}$$

Integrating Eq.(22) for all **r** directions, one obtains

$$\int_0^{2\pi}\int_0^{\pi}\{\frac{\partial}{\partial r}\phi_k(x) - 2\phi_k(x+r)\frac{\partial}{\partial r}\phi_k(x)\}\sin\theta d\theta d\eta = \int_0^{2\pi}\int_0^{\pi} -|\cos\theta| \ a_i\sin\theta d\theta d\eta = -2\pi a_i \tag{23}$$

Rearranging Eq(23), one obtains

$$a_i = -\frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\{\frac{\partial}{\partial r}\phi_k(x) - 2\phi_k(x+r)\frac{\partial}{\partial r}\phi_k(x)\}\sin\theta d\theta d\eta \tag{24}$$

As stated above, $\frac{\partial}{\partial r}$ is directional differentiation of characteristic function $\phi_k(\mathbf{x})$ in **r** direction.

When one approximates the directional differentiation of characteristic function in Eq.(24) in the interval of $|\mathbf{r}|$, one obtains,

$$\frac{\partial}{\partial r}\phi_k(x) \approx \frac{\phi_k(x+r) - \phi_k(x)}{|r|} \tag{25}$$

$$\phi_k(x+r)\frac{\partial}{\partial r}\phi_k(x) \approx \frac{\phi_k(x+r)\phi_k(x+r) - \phi_k(x+r)\phi_k(x)}{|r|} = \frac{\phi_k(x+r) - \phi_k(x+r)\phi_k(x)}{|r|} \tag{26}$$

Then, the integrated function in Eq.(24) can be given by

$$\frac{\partial}{\partial r}\phi_k(x) - 2\phi_k(x+r)\frac{\partial}{\partial r}\phi_k(x) \approx \frac{\phi_k(x)\{\phi_k(x+r)-1\} + \phi_k(x+r)\{\phi_k(x)-1\}}{|r|} \tag{27}$$

$$\left(a_i|\cos\theta| \approx \frac{\phi_k(\mathbf{x})\{1-\phi_k(\mathbf{x+r})\} + \phi_k(\mathbf{x+r})\{1-\phi_k(\mathbf{x})\}}{|\mathbf{r}|}\right)$$

Using this relation , Eq.(24) can be rewritten by

$$a_i = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\pi} 1 \lim_{|r| \to 0} \left\{ \frac{\phi_k(x)\{1 - \phi_k(x+r)\} + \phi_k(x+r)\{1 - \phi_k(x)\}}{|r|} \right\} \sin\theta d\theta d\eta \qquad (28)$$

Averaging Eq.(28), one obtains,

$$\overline{a_i} = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\pi} \lim_{|r| \to 0} \left\{ \frac{\overline{\phi_k(x)\{1 - \phi_k(x+r)\}} + \overline{\phi_k(x+r)\{1 - \phi_k(x)\}}}{|r|} \right\} \sin\theta d\theta d\eta \qquad (29)$$

In the right hand side of Eq.(29), the term

$$\overline{\phi_k(x)\{1 - \phi_k(x+r)\}} + \overline{\phi_k(x+r)\{1 - \phi_k(x)\}}$$

represents the probability where gas-liquid interface exists between x and x+r as shown in Fig.7.
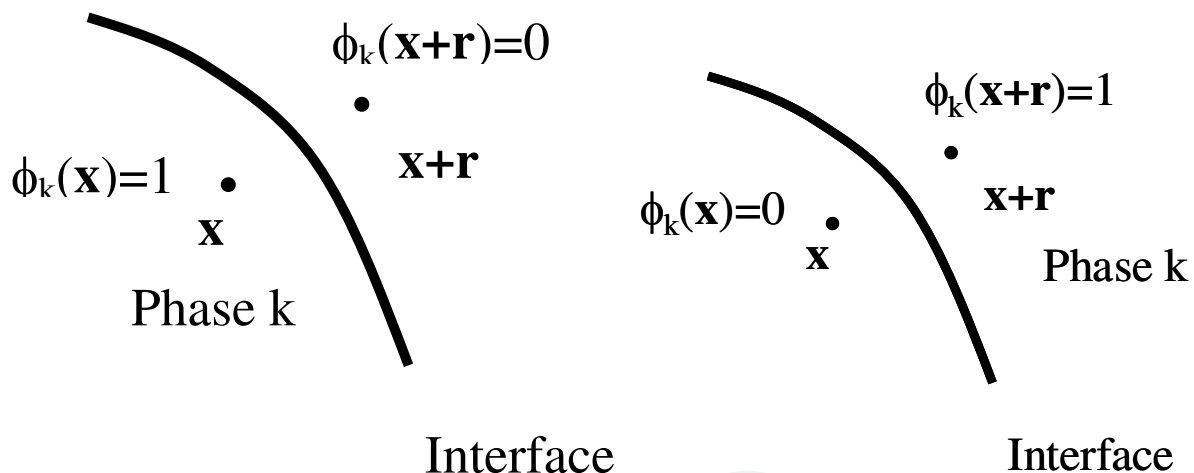
Fig. 7. The case where interface exists between **x** and **x+r**.

3. Basic transport equations of interfacial area concentration

Based on the rigorous formulation of interfacial area concentration, one can derive transport equation of interfacial area concentration. The transport equations of interfacial area concentration consist of two equations. One is the conservation equation of interfacial area concentration and the other is the conservation equation of interfacial velocity (velocity of interface), V_i.

Kataoka (2008) derived the local instant conservation equation of interfacial area concentration based on the formulation given by Eq.(24). In order to obtain the local instant conservation equation of interfacial area concentration, characteristic function of each phase

(denoted by ϕ_k) given by Eqs.(11) or (12) is needed. Kataoka(1986) also derived local instant formulation of two-phase flow which gives the local instant conservation equations of mass momentum and energy in each phase. The conservation equation of characteristic function of each phase (denoted by ϕ_k) given by

$$\frac{\partial}{\partial t}(\phi_k \rho_k) + \text{div}(\phi_k \rho_k v_k) = -\rho_{ki}(v_{ki} - V_i) \bullet n_{ki} a_i \quad (k = G, L) \tag{30}$$

Here, ρ_k, $\mathbf{v}_k$ are density, velocity of each phase. Suffix ki is value of phase k at interface. Using Eqs.(24) and (30), Local instant conservation equation of interfacial area concentration is given by

$$\frac{\partial}{\partial t}(a_i) + \text{grad}(a_i V_i) = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\pi} \{\frac{\partial v_k}{\partial r} \text{grad}(\varphi_k) - 2\phi_k(x+r)\frac{\partial v_k}{\partial r} \text{grad}(\varphi_k)\} \sin\theta d\theta d\eta \tag{31}$$

Averaging Eq.(31), one obtains the conservation equation of averaged interfacial area concentration by

$$\frac{\partial}{\partial t}(\overline{a_i}) + \text{grad}(\overline{a_i \overline{V_i}}) = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\pi} \overline{\{\frac{\partial v_k}{\partial r} \text{grad}(\phi_k) - 2\phi_k(x+r)\frac{\partial v_k}{\partial r} \text{grad}(\phi_k)\}} \sin\theta d\theta d\eta \tag{32}$$

where averaged interfacial velocity $\overline{V_i}$ is defined by

$$\overline{V_i} = (\overline{a_i V_i}) / \overline{a_i} \tag{33}$$

The right hand side of Eqs.(31) and(32) represent the source term of interfacial area concentration due to the deformation of interface. In the dispersed flows such as bubbly flow and droplet flow, this term correspond breakup or coalescence of bubbles and droplets.

Morel (2007) derived the conservation equation of averaged interfacial area concentration based on the detailed geometrical consideration of interface.

$$\frac{\partial \overline{a_i}}{\partial t} + \nabla \bullet \overline{a_i \overline{V_i}} = \overline{a_i(V_i \bullet n_{Gi})\nabla \bullet n_{Gi}} \tag{34}$$

Here, $\overline{}$ denotes time averaging and $\overline{V_i}$ is the time averaged velocity of interface which is given by

$$\overline{V_i} = \overline{a_i(V_i \bullet n_{Gi})n_{Gi}} / \overline{a_i} \tag{35}$$

The research group directed by Prof. Ishii in Purdue university derived the transport equation of interfacial area concentration of time averaged interfacial area concentration based on the transport equation of number density function of bubbles (Kocamustafaogullari and Ishii (1995), Hibiki and Ishii (2000a)) . It is given by

$$\frac{\partial \overline{a_i}}{\partial t} + \nabla \bullet \overline{a_i \overline{V_i}} = \sum_{j=1}^{4} \phi_j + \phi_{ph} \tag{36}$$

Here, the first term in right hand side of Eq.(36) represent the source and sink terms due to bubble coalescence and break up. Interfacial area decreases when bubbles coalescence and increases when bubbles break up. This term is quite important in interfacial area transport. Therefore, the constitutive equations of this term are given by Hibiki and Ishii (2000a, 2000b) and Ishii and Kim (2004) based on detailed mechanistic modeling. The second term in right hand side of Eq.(36) represent the source and sink terms due to phase change. Equation (36) is practical transport equation of interfacial area concentration.

As for the conservation equation of interfacial velocity, Kataoka et al. (2010,2011a) have derived rigorous formulation based on the local instant formulation of interfacial area concentration and interfacial velocity, which is shown below. Since interface has no mass, momentum equation of interface cannot be formulated. Therefore, the conservation equation of interfacial velocity (or governing equation of interfacial velocity) has to be derived in collaboration with the momentum equation of each phase. Since interfacial velocity is only defined at interface, local instant formulation of interfacial velocity must be expressed in the form of

$$a_i V_i$$

Using Eq.(22), interfacial velocity is expressed by

$$a_i V_i |\cos\theta| = -V_i \frac{\partial}{\partial r} \phi_k(x) + 2V_i \phi_k(x+r)\frac{\partial}{\partial r} \phi_k(x) \tag{37}$$

Considering Fig.7, interfacial velocity is approximated by velocity of each phase without phase change.

$$V_i \approx \phi_k(x)\{1 - \phi_k(x+r)\}v_k(x) + \phi_k(x+r)\{1 - \phi_k(x)\}v_k(x+r) \tag{38}$$

From Eqs.(27) and (38) with some rearrangements, one obtains following approximate expression.

$$a_i V_i |\cos\theta| \approx \frac{\phi_k(x)\{1 - \phi_k(x+r)\}v_k(x) + \phi_k(x+r)\{1 - \phi_k(x)\}v_k(x+r)}{|r|} \tag{39}$$

When one takes the limit of $|r| \to 0$, one obtains

$$a_i V_i |\cos\theta| = \lim_{|r| \to 0} \frac{\phi_k(x)\{1 - \phi_k(x+r)\}v_k(x) + \phi_k(x+r)\{1 - \phi_k(x)\}v_k(x+r)}{|r|} \tag{40}$$

Integrating Eq.(40) in all direction, one finally obtains

$$a_i V_i = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\pi} \lim_{|r| \to 0} \frac{\phi_k(x)\{1 - \phi_k(x+r)\}v_k(x) + \{1 - \phi_k(x)\}\phi_k(x+r)v_k(x+r)}{|r|} \sin\theta d\theta d\phi \tag{41}$$

Averaging Eq.(41), averaged interfacial velocity is given by

$$\overline{V_i a_i} = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\pi} \lim_{|r| \to 0} \frac{\overline{\phi_k(x)\{1 - \phi_k(x+r)\}v_k(x)} + \overline{\{1 - \phi_k(x)\}\phi_k(x+r)v_k(x+r)}}{|r|} \sin\theta d\theta d\phi \tag{42}$$

On the other hand, using Eq.(29), following relation can be obtained for averaged velocity of each phase, $\overline{\overline{v_k}}$ by

$$\overline{\overline{v_k}}\,\overline{a_i} = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\lim_{|r|\to 0}\frac{\overline{\phi_k(x)\{1-\phi_k(x+r)\}\overline{\overline{v_k}}(x)} + \overline{\{1-\phi_k(x)\}\phi_k(x+r)\overline{\overline{v_k}}(x)}}{|r|}\sin\theta d\theta d\phi \quad (43)$$

where $\overline{\overline{v_k}}$ is defined by

$$\overline{\overline{v_k}} = (\overline{\phi_k v_k}) / \overline{\phi_k} \quad (44)$$

Using, Eqs.(42) and (43), the difference between time averaged interfacial velocity, $\overline{V_i}$ and time average velocity of phase k, $\overline{\overline{v_k}}$ is given by

$$\begin{aligned}(\overline{V_i} - \overline{\overline{v_k}})\overline{a_i} \\ = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\lim_{|r|\to 0}\frac{\overline{\phi_k(x)\{1-\phi_k(x+r)\}\{v_k(x)-\overline{\overline{v_k}}(x)\}} + \overline{\{1-\phi_k(x)\}\phi_k(x+r)\{v_k(x+r)-\overline{\overline{v_k}}(x)\}}}{|r|}\sin\theta d\theta d\phi\end{aligned} \quad (45)$$

Rearranging the term in integration in the right hand side of Eq.(45) one obtains

$$\begin{aligned}\lim_{|r|\to 0}&\frac{\overline{\phi_k(x)\{1-\phi_k(x+r)\}\{v_k(x)-\overline{\overline{v_k}}(x)\}} + \overline{\{1-\phi_k(x)\}\phi_k(x+r)\{v_k(x+r)-\overline{\overline{v_k}}(x)\}}}{|r|} \\ &= -\lim_{|r|\to 0}\frac{\overline{\phi_k{}'(x)\phi_k(x+r)v_k{}'(x+r)} + \overline{\phi_k(x)\phi_k{}'(x+r)v_k{}'(x)}}{|r|}\end{aligned} \quad (46)$$

Here, $\phi_k{}'$ and $v_k{}'$ are fluctuating terms of local instant volume fraction and velocity of phase k which are given by

$$v_k{}' = v_k - \overline{\overline{v_k}} \quad (47)$$

$$\phi_k{}' = \phi_k - \overline{\phi_k} \quad (48)$$

Equations (45) and (46) indicate that the difference between time averaged interfacial velocity, $\overline{V_i}$ and time averaged velocity of phase k, $\overline{\overline{v_k}}$ is given in terms of correlations between fluctuating terms of local instant volume fraction and velocity of phase k which are related to turbulence terms of phase k.

Then, it is important to derive the governing equation of the correlation term given by Eq.(46). In what follows, one derives the governing equation based on the local instant basic equations of mass conservation and momentum conservation of phase k which are given below (Kataoka (1986)). In these conservation equations, tensor representation is used. Einstein abbreviation rule is also applied. When the same suffix appear, summation for that suffix is carried out except for the suffix k denoting gas and liquid phases.

(Mass conservation)

$$\phi_k \frac{\partial v_{k\beta}}{\partial x_\beta} = 0 \tag{49}$$

(Momentum conservation)

$$\phi_k \frac{\partial v_{k\alpha}}{\partial t} + \phi_k \frac{\partial}{\partial x_\beta}(v_{k\alpha}v_{k\beta}) = -\phi_k \frac{1}{\rho_k}\frac{\partial P_k}{\partial x_\alpha} + \phi_k \frac{1}{\rho_k}\frac{\partial \tau_{k\alpha\beta}}{\partial x_\beta} + \phi_k F_{k\alpha} \tag{50}$$

Averaging Eqs.(49) and (50) , one obtains time averaged conservation equation of mass and momentum conservation of phase k.

(Time averaged mass conservation)

$$\frac{\overline{\partial \overline{v_{k\beta}}}}{\partial x_\beta} = -\frac{1}{\overline{\phi_k}}\overline{v'_{k\beta i}n_{k\beta i}a_i} \tag{51}$$

(Time averaged momentum conservation)

$$\frac{\partial \overline{\overline{v_{k\alpha}}}}{\partial t} + \frac{\partial}{\partial x_\beta}(\overline{\overline{v_{k\alpha}}\,\overline{v_{k\beta}}}) = -\frac{1}{\rho_k}\frac{\partial \overline{\overline{P_k}}}{\partial x_\alpha} + \frac{1}{\rho_k}\frac{\partial}{\partial x_\beta}\left(\overline{\overline{\tau_{k\alpha\beta}}} - \rho_k \overline{\overline{v'_{k\alpha}v'_{k\beta}}}\right) + \overline{\overline{F_{k\alpha}}} - \frac{\overline{\overline{v_{k\alpha}}}}{\overline{\phi_k}}\overline{v'_{k\beta i}n_{k\beta i}a_i}$$
$$- \frac{1}{\overline{\phi_k}}\frac{1}{\rho_k}\overline{P'_{ki}n_{k\alpha i}a_i} + \frac{1}{\overline{\phi_k}}\frac{1}{\rho_k}\overline{\tau'_{k\alpha\beta i}n_{k\beta i}a_i} + \frac{1}{\overline{\phi_k}}\overline{\overline{v'_{k\alpha}v'_{k\beta}}n_{k\beta i}a_i} \tag{52}$$

Subtracting Eqs(51) and (52) from Eqs.(49) and (50), the conservation equations of fluctuating terms are obtained.

(Conservation equation of mass fluctuation)

$$\phi_k \frac{\partial v'_{k\beta}}{\partial x_\beta} = \frac{\phi_k}{\overline{\phi_k}}\overline{v'_{k\beta i}n_{k\beta i}a_i} \tag{53}$$

(Conservation equation of momentum fluctuation)

$$\phi_k \frac{\partial v'_{k\alpha}}{\partial t} + \phi_k \frac{\partial}{\partial x_\beta}(v'_{k\alpha}v'_{k\beta} + v'_{k\alpha}\overline{\overline{v_{k\beta}}} + v'_{k\beta}\overline{\overline{v_{k\alpha}}}) = -\phi_k \frac{1}{\rho_k}\frac{\partial P'_k}{\partial x_\alpha} + \phi_k \frac{1}{\rho_k}\frac{\partial}{\partial x_\beta}(\tau'_{k\alpha\beta} + \rho_k \overline{v'_{k\alpha}v'_{k\beta}})$$
$$+ \phi_k F_{k\alpha}' + \frac{\phi_k}{\overline{\phi_k}}\overline{\overline{v_{k\alpha}}}\overline{v'_{k\beta i}n_{k\beta i}a_i} + \frac{\phi_k}{\overline{\phi_k}}\frac{1}{\rho_k}\overline{P'_{ki}n_{k\alpha i}a_i} - \frac{\phi_k}{\overline{\phi_k}}\frac{1}{\rho_k}\tau'_{k\alpha\beta i}n_{k\beta i}a_i - \frac{\phi_k}{\overline{\phi_k}}\overline{v'_{k\alpha}v'_{k\beta}}n_{k\beta i}a_i \tag{54}$$

Using Eqs(53) and (54), one can derive conservation equation of

$$\overline{\phi_k'(x)\phi_k(x+r)v_k'(x+r)} + \overline{\phi_k(x)\phi_k'(x+r)v_k'(x)}$$

Then, conservation equation of the difference between interfacial velocity and averaged velocity of each phase is derived. The result is given by

$$\frac{\partial}{\partial t}\{\left(\overline{V_i} - \overline{\overline{v_k}}\right)\overline{a_i}\} + \frac{\partial}{\partial x_\beta}\left(\left(\overline{V_i} - \overline{\overline{v_k}}\right)\overline{a_i}\overline{\overline{v_{k\beta}}}\right)$$

$$= \frac{1}{\rho_k}\frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\lim_{|r|\to 0}\frac{1}{|r|}\overline{(\phi'_k\phi_{kr}\frac{\partial P'_{kr}}{\partial x_\alpha} + \overline{\varphi_k\varphi'_{kr}\frac{\partial P'_k}{\partial x_\alpha}})}\sin\theta d\theta d\phi$$

$$- \frac{1}{\rho_k}\frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\lim_{|r|\to 0}\frac{1}{|r|}\{\overline{\phi'_k\phi_{kr}\frac{\partial}{\partial x_\beta}(\tau'_{k\alpha\beta r} + \rho_k\overline{\overline{v'_{k\alpha r}v'_{k\beta r}}})} +$$

$$+ \overline{\phi_k\phi'_{kr}\frac{\partial}{\partial x_\beta}(\tau'_{k\alpha\beta} + \rho_k\overline{\overline{v'_{k\alpha}v'_{k\beta}}})}\}\sin\theta d\theta d\phi$$

$$- \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\lim_{|r|\to 0}\frac{1}{|r|}\overline{(\phi'_k\phi_{kr}F'_{k\alpha r} + \overline{\phi_k\phi'_{kr}F'_{k\alpha}})}\sin\theta d\theta d\phi$$

$$- \frac{1}{2\pi}\int_0^{\pi}\int_0^{2\rho}\lim_{|r|\to 0}\frac{1}{|r|}(\frac{\overline{\phi'_k\phi_{kr}}}{\overline{\varphi_{kr}}}\frac{1}{\rho_k}\overline{P'_{ki}n_{k\alpha i}a_{ir}} + \frac{\overline{\phi_k\phi'_{kr}}}{\overline{\varphi_k}}\frac{1}{\rho_k}\overline{P'_{ki}n_{k\alpha i}a_i})\sin\theta d\theta d\phi$$

$$+ \frac{1}{2\pi}\int_0^{\pi}\int_0^{2\rho}\lim_{|r|\to 0}\frac{1}{|r|}(\frac{\overline{\phi'_k\phi_{kr}}}{\overline{\varphi_{kr}}}\frac{1}{\rho_k}\overline{\tau'_{ki}n_{k\alpha i}a_{ir}} + \frac{\overline{\phi_k\phi'_{kr}}}{\overline{\varphi_k}}\frac{1}{\rho_k}\overline{\tau'_{ki}n_{k\alpha i}a_i})\sin\theta d\theta d\phi$$

$$+ \frac{1}{2\pi}\int_0^{\pi}\int_0^{2\rho}\lim_{|r|\to 0}\frac{1}{|r|}(\frac{\overline{\phi'_k\phi_{kr}}}{\overline{\varphi_{kr}}}\overline{\overline{v'_{k\alpha r}v'_{k\beta r}}}n_{k\alpha i}a_{ir} + \frac{\overline{\phi_k\phi'_{kr}}}{\overline{\varphi_k}}\overline{\overline{v'_{k\alpha}v'_{k\beta}}}n_{k\alpha i}a_i)\sin\theta d\theta d\phi$$

$$+ \frac{1}{2\pi}\int_0^{\pi}\int_0^{2\rho}\lim_{|r|\to 0}\frac{1}{|r|}\{\frac{\partial}{\partial x_\beta}(\overline{\phi'_k\phi_{kr}v'_{k\alpha r}v'_{k\beta r}}) + \frac{\partial}{\partial x_\beta}(\overline{\phi_k\phi'_{kr}v'_{kr}v'_{k\beta}})\}\sin\theta d\theta d\phi$$

$$+ \frac{1}{2\pi}\int_0^{\pi}\int_0^{2\rho}\lim_{|r|\to 0}\frac{1}{|r|}(\overline{\phi'_k\phi_{kr}v'_{k\beta r}\frac{\partial\overline{\overline{v_{k\beta r}}}}{\partial x_\beta}} + \overline{\phi_k\phi'_{kr}v'_{k\beta}\frac{\partial\overline{\overline{v_{k\beta}}}}{\partial x_\beta}})\sin\theta d\theta d\phi$$

$$- \frac{1}{2\pi}\int_0^{\pi}\int_0^{2\rho}\lim_{|r|\to 0}\frac{1}{|r|}\{\overline{v'_{k\alpha r}(\overline{\overline{v_{k\beta r}}} + v'_{k\beta r})\phi_{kr}\frac{\partial}{\partial x_\beta}\phi'_k} +$$

$$+ \{\overline{v'_{k\alpha}(\overline{\overline{v_{k\beta}}} + v'_{k\beta})\phi_k\frac{\partial}{\partial x_\beta}\phi'_{kr}}\}\sin\theta d\theta d\phi \tag{55}$$

$$+ \frac{1}{2\pi}\int_0^{\pi}\int_0^{2\rho}\lim_{|r|\to 0}\frac{1}{|r|}\{\overline{v'_{k\alpha r}\phi_{kr}\frac{\partial}{\partial x_\beta}(\phi_r v'_{k\beta} + \phi'_k\overline{\overline{v_{k\beta r}}})} +$$

$$+ \overline{v'_{k\alpha}\phi_k\frac{\partial}{\partial x_\beta}(\phi_{kr}v'_{k\beta r} + \phi'_{kr}\overline{\overline{v_{k\beta}}})}\}\sin\theta d\theta d\phi$$

As shown above, the formulation of governing equation of interfacial velocity is derived. Then, the most strict formulation of transport equations of interfacial area concentration is given by conservation equation of interfacial area concentration (Eq.(32) , Eq(34), or Eq.(36)) and conservation equation of interfacial velocity (Eq.(55)). As shown in Eq.(55), the conservation equation of interfacial velocity consists of various correlation terms of fluctuating terms of velocity and local instant volume fractions. These correlation terms represent the turbulent transport of interfacial area, which reflects the interactions between gas liquid interface and turbulence of gas and liquid phases. Equation (55) represents such turbulence transport terms of interfacial area concentration. Accurate predictions of interfacial area transport can be possible by solving the transport equations derived here. However, Eq.(55)

consists of complicated correlation terms of fluctuating terms of local instant volume fraction, velocity, pressure and shear stress. The detailed knowledge of these correlation terms is not available. Therefore, solving Eq.(55) together with basic equations of two-fluid model is difficult at present. More detailed analytical and experimental works on turbulence transport terms of interfacial area concentration are necessary for solving practically Eq.(55).

4. Constitutive equations of transport equations of interfacial area concentration. Source and sink terms, diffusion term, turbulence transport term

As shown in the previous section, the rigorous formulation of transport equation of interfacial area concentration are given by conservation equation of interfacial area concentration (Eq.(32), Eq.(34) or Eq(36)) and conservation equation of interfacial velocity (Eq.(55)). However, Eq.(55) consists of complicated correlation terms of fluctuating terms of local instant volume fraction, velocity, pressure and shear stress. The detailed knowledge of these correlation terms is not available. Therefore, solving Eq.(55) together with basic equations of two-fluid model is difficult at present. More detailed analytical and experimental works on turbulence transport terms of interfacial area concentration are necessary for solving practically Eq.(55). From Eqs.(45) and (46), interfacial velocity is related to averaged velocity of phase k (gas phase or liquid phase)by following equation.

$$\overline{V_i a_i} = \overline{\overline{v_k} a_i} - \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\pi} \lim_{|r| \to 0} \frac{1}{|r|} (\overline{\phi'_k \varphi_{kr} v'_{kr}} + \overline{\phi_k \phi'_{kr} v'_k}) \sin\theta d\theta d\phi \tag{56}$$

When one considers bubbly flow and phase k is gas phase, Eq.(56) can be rewritten by

$$\overline{V_i a_i} = \overline{\overline{v_G} a_i} - \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\pi} \lim_{|r| \to 0} \frac{1}{|r|} (\overline{\phi'_G \phi_{Gr} v'_{Gr}} + \overline{\phi_G \phi'_{Gr} v'_G}) \sin\theta d\theta d\phi \tag{57}$$

From Eqs.(28) and (29) , following relation is derived.

$$a'_i = a_i - \overline{a_i} = -\frac{1}{2\pi} \int_0^{2\pi} \int_0^{\pi} \lim_{|r| \to 0} \frac{1}{|r|} \left\{ \phi'_k(1 - 2\overline{\phi_{kr}}) + \phi'_{kr}(1 - 2\overline{\phi_k}) - 2(\phi'_k \phi'_{kr} - \overline{\phi'_k \phi'_{kr}}) \right\} \sin\theta d\theta d\phi \tag{58}$$

In Eq.(57), the terms, $\phi'_G / |r|$ and $\phi'_{Gr} / |r|$ are related to the fluctuating term of interfacial area concentration. On the other hand, the terms, $\phi_{Gr} v'_{Gr}$ and $\phi_G v'_G$ are the fluctuating term of gas phase velocity at the location, **x+r.** and **x.** Therefore, the second term of right hand side of Eq.(57) is considered to correspond to turbulent transport term due to the turbulent velocity fluctuation. In analogous to the turbulent transport of momentum, energy (temperature) and mass, the correlation term described above is assumed to be proportional to the gradient of interfacial area concentration which is transported by turbulence (diffusion model). Then, one can assume following relation.

$$-\frac{1}{2\pi} \int_0^{2\pi} \int_0^{\pi} \lim_{|r| \to 0} \frac{1}{|r|} (\overline{\phi'_G \phi_{Gr} v'_{Gr}} + \overline{\phi_G \phi'_{Gr} v'_G}) \sin\theta d\theta d\phi = -D_{ai} \text{grad} \overline{a_i} \tag{59}$$

Here, the coefficient, D_{ai} is considered to correspond to turbulent diffusion coefficient of interfacial area concentration. In analogy to the turbulent transport of momentum, energy (temperature) and mass, this coefficient is assumed to be given by

$$D_{ai} \propto |v'_G| L \tag{60}$$

Here, L is the length scale of turbulent mixing of gas liquid interface and $|v'_G|$ is the turbulent velocity of gas phase. In bubbly flow, it is considered that turbulent mixing of gas liquid interface is proportional to bubble diameter, d_B and the turbulent velocity of gas phase is proportional to the turbulent velocity of liquid phase. These assumptions were confirmed by experiment and analysis of turbulent diffusion of bubbles in bubbly flow (Kataoka and Serizawa (1991a)). Therefore, turbulent diffusion coefficient of interfacial area concentration is assumed by following equation.

$$D_{ai} = K_1 |v'_L| d_B = 6K_1 \frac{\alpha}{a_i} |v'_L| \tag{61}$$

Here, α is the averaged void fraction and $|v'_L|$ is the turbulent velocity of liquid phase. K_1 is empirical coefficient. For the case of turbulent diffusion of bubble, experimental data were well predicted assuming $K_1 = 1/3$ For the case of turbulent diffusion of interfacial area concentration, there are no direct experimental data of turbulent diffusion. However, the diffusion of bubble is closely related to the diffusion of interfacial area (surface area of bubble). Therefore, as first approximation, the value of K_1 for bubble diffusion can be applied to diffusion of interfacial area concentration in bubbly flow.

Equations (61) is based on the model of turbulent diffusion of interfacial area concentration. In this model, it is assumed that turbulence is isotropic. However, in the practical two-phase flow in the flow passages turbulence is not isotropic and averaged velocities and turbulent velocity have distribution in the radial direction of flow passage. In such non-isotropic turbulence, the correlation terms of turbulent fluctuation of velocity and interfacial area concentration given by Eq.(57) is largely dependent on anisotropy of turbulence field. Such non-isotropic turbulence is related to the various terms consisting of turbulent stress which appear in the right hand side of Eq.(55). Assuming that turbulent stress of gas phase is proportional to that of liquid phase and turbulence model in single phase flow, turbulent stress is given by

$$\overline{v'_L v'_L} = -\varepsilon_{LTP} \{ \overline{\nabla \overline{v_L}} + {}^t(\overline{\nabla \overline{v_L}}) \} + \frac{2}{3} k \delta_{ij} \tag{62}$$

Here, ε_{LTP} is the turbulent diffusivity of momentum in gas-liquid two-phase flow. For bubbly flow, this turbulent diffusivity is given by various researchers (Kataoka and Serizawa (1991b,1993)).

$$\varepsilon_{LTP} = \frac{1}{3} \alpha d_B |v'_L| \tag{63}$$

Based on the model of turbulent stress in gas-liquid two-phase flow and Eq.(55), it is assumed that turbulent diffusion of interfacial area concentration due to non-isotropic turbulence is proportional to the velocity gradient of liquid phase. For the diffusion of bubble due to non-isotropic turbulence in bubbly flow in pipe, Kataoka and Serizawa (1991b,1993) proposed the following correlation based on the analysis of radial distributions of void fraction and bubble number density.

$$J_B = K_2 \alpha d_B n_B \frac{\partial \overline{\overline{v_L}}}{\partial y} \tag{64}$$

Here, J_B is the bubble flux in radial direction and n_B is the number density of bubble. y is radial distance from wall of flow passage. K_2 is empirical coefficient and experimental data were well predicted assuming $K_2=10$. In analogous to Eq.(64), it is assumed that turbulent diffusion of interfacial area concentration due to non-isotropic turbulence is given by following equation.

$$J_{ai} = K_2 \alpha d_B a_i \frac{\partial \overline{\overline{v_L}}}{\partial y} \tag{65}$$

Here, J_{ai} is the flux of interfacial area concentration in radial direction. Equation (64) can be interpreted as equation of bubble flux due to the lift force due to liquid velocity gradient.

As shown above, turbulent diffusion of interfacial area concentration due to non-isotropic turbulence is related to the gradient of averaged velocity of liquid phase and using analogy to the lift force of bubble, Eq.(58) can be rewritten in three dimensional form by

$$-\frac{1}{2\pi} \int_0^{2\pi} \int_0^{\pi} \lim_{|r| \to 0} \frac{1}{|r|} (\overline{\phi'_G \phi_{Gr} v'_{Gr}} + \overline{\phi_G \phi'_{Gr} v'_G}) \sin\theta d\theta d\phi =$$
$$-D_{ai} \mathrm{grad} \overline{a_i} + C\alpha a_i (\overline{\overline{v_G}} - \overline{\overline{v_L}}) \times \mathrm{rot}(\overline{\overline{v_L}}) \tag{66}$$

Empirical coefficient C in the right hand side of Eq.(66) should be determined based on the experimental data of spatial distribution of interfacial area concentration and averaged velocity of each phase. However, at present, there are not sufficient experimental data. Therefore, as first approximation, the value of coefficient C can be given by Eq.(67)

$$C = K_2 d_B / u_R \ \text{(based on Eq.(65))} \tag{67}$$

Using Eqs(55) and (66), transport equation of interfacial area concentration (Eq.(32),(34) or (36)) can be given by following equation for gas-liquid two-phase flow where gas phase is dispersed in liquid phase for bubbly flow.

$$\frac{\partial \overline{a_i}}{\partial t} + \mathrm{div}(\overline{a_i} \overline{\overline{v_G}}) = \mathrm{div}(D_{ai} \mathrm{grad} \overline{a_i}) - \mathrm{div}\{C\alpha a_i (\overline{\overline{v_G}} - \overline{\overline{v_L}}) \times \mathrm{rot}(\overline{\overline{v_L}})\} +$$
$$+ \frac{2}{3} \frac{\overline{a_i}}{\alpha \rho_G} \left(\Gamma_G - \alpha \frac{D\rho_G}{Dt} \right) + \phi_{CO} + \phi_{BK} \tag{68}$$

Here, D/Dt denotes material derivative following the gas phase motion and turbulent diffusion coefficient of interfacial area concentration, D_{ai} is given by Eq.(61). Coefficient of turbulent diffusion of interfacial area concentration due to non-isotropic turbulence, C is given by Eq.(67). The third term in the right hand side of Eq.(68) is source term of interfacial area concentration due to phase change and density change of gas phase due to pressure change. Γ_G is the mass generation rate of gas phase per unit volume of two-phase flow due to evaporation. ϕ_{CO} and ϕ_{Bk} are sink and source term due to bubble coalescence and break up

Similarly, the transport equation of interfacial area concentration for droplet flow is given by

$$
\begin{aligned}
\frac{\partial \overline{a_i}}{\partial t} + \mathrm{div}(\overline{a_i}\,\overline{\overline{v_L}}) = \\
\mathrm{div}(D_{ai}\mathrm{garda}\overline{a_i}) - \mathrm{div}\{C(1-\alpha)\overline{a_i}(\overline{\overline{v_L}} - \overline{\overline{v_G}}) \times \mathrm{rot}(\overline{\overline{v_G}})\} + \\
+ \frac{2}{3}\frac{\overline{a_i}}{(1-\alpha)\rho_L}\left(\Gamma_L - (1-\alpha)\frac{D\rho_L}{Dt}\right) + \phi_{CO} + \phi_{BK}
\end{aligned}
\tag{69}
$$

Here, D/Dt denotes material derivative following the liquid phase motion and ϕ_{CO} and ϕ_{Bk} are sink and source terms due to droplet coalescence and break up and Γ_L is the mass generation rate of liquid phase per unit volume of two-phase flow due to condensation. Here, turbulent diffusion coefficient of interfacial area concentration is approximated by turbulent diffusion coefficient of droplet (Cousins and Hewitt (1968)) as first approximation. The coefficient C for turbulent diffusion of interfacial area concentration due to non-isotropic turbulence (or lift force term) can be approximated by lift force coefficient of solid sphere as first approximation.

The research and development of source and sink terms in transport equation of interfacial area concentration have been carried out mainly for the bubbly flow based on detailed analysis and experiment of interfacial area concentration which is shown below.

Hibiki and Ishii(2000a,2002) developed the transport equation of interfacial area mentioned above and carried out detailed modeling of source and sink terms of interfacial area concentration. They assumed that the sink term of interfacial area concentration is mainly due to the coalescence of bubble. On the other hand, they assumed the source term is mainly contributed by the break up of bubble due to liquid phase turbulence. Based on detailed mechanistic modeling of bubble liquid interactions, they finally obtained the constitutive equations for sink and source terms of interfacial area transport.

The sink term of interfacial area concentration due to the coalescence of bubbles, ϕ_{CO} is composed of number of collisions of bubbles per unit volume and the probability of coalescence at collision and given by

$$
\phi_{CO} = -\left(\frac{\alpha}{\overline{\overline{a_i}}}\right)^2 \frac{\Gamma_C\alpha^2\varepsilon^{1/3}}{d_b^{11/3}(\alpha_{max}-\alpha)}\exp\left(-K_C\sqrt[6]{\frac{d_b^5\rho_L^3\varepsilon^2}{\sigma^3}}\right)
\tag{70}
$$

where the term $\dfrac{\Gamma_C\alpha^2\varepsilon^{1/3}}{d_b^{11/3}(\alpha_{max}-\alpha)}$ represents the number of collisions of bubbles per unit volume and the term $\exp\left(-K_C\sqrt[6]{\dfrac{d_b^5\rho_L^3\varepsilon^2}{\sigma^3}}\right)$ represents the probability of coalescence at collision. d_b, ε and σ are bubble diameter, turbulent dissipation and surface tension. α_{max} is maximum permissible void fraction in bubbly flow and assumed to be 0.52. Γ_C and K_C are empirical constants and following values are given

$$\Gamma_C = 0.188, \; K_C = 1.29 \tag{71}$$

As for the source term due to the break up of bubble, it is assumed that bubble break up mainly occurs due to the collision between bubble and turbulence eddy of liquid phase. The constitutive equation is given based on the detailed mechanistic modeling of this phenomenon as

$$\phi_{BK} = \left(\frac{\alpha}{\overline{\overline{a_i}}}\right)^2 \frac{\Gamma_B \alpha(1-\alpha)\varepsilon^{1/3}}{d_b^{11/3}(\alpha_{max}-\alpha)} \exp\left(-\frac{K_B \sigma}{\rho_L d_b^{5/3}\varepsilon^{2/3}}\right) \tag{72}$$

where the term $\dfrac{\Gamma_B \alpha(1-\alpha)\varepsilon^{1/3}}{d_b^{11/3}(\alpha_{max}-\alpha)}$ represents the number of collisions of bubble and

turbulence eddy per unit volume and the term $\exp\left(-\dfrac{K_B \sigma}{\rho_L d_b^{5/3}\varepsilon^{2/3}}\right)$ represents the

probability of break up at collision. Γ_B and K_B are empirical constants and following values are given.

$$\Gamma_B = 0.264, \; K_B = 1.37 \tag{73}$$

The validity of transport equation of interfacial area concentration (Eq.(68)) and constitutive equations for sink term due to bubble coalescence (Eq.(70)) and source term due to bubble break up (Eq.(72)) are confirmed by experimental data as will be described later in details.

Hibiki and Ishii (2000b) further modified their model of interfacial area transport and applied to bubbly-to-slug flow transition. In bubbly-to-slug flow transition, bubbles are classified into two groups that are small spherical/distorted bubble (group I) and large cap/slug bubble (group II). They derived transport equations of interfacial area concentration and constitutive equations for sink and source terms for group I and group II bubbles based on the transport equation and constitutive equations for bubbly flow mentioned above.

Yao and More (2004) developed more practical transport equation of interfacial area concentration and constitutive correlations of source terms. They derived these equations based on the basic transport equation developed in CEA and models of source terms developed at Purdue University (Ishii 's group). They also developed sink term due to coalescence of bubbles which is given by

$$\phi_{CO} = -107.8\left(\frac{\alpha}{\overline{\overline{a_i}}}\right)^2 \frac{\alpha^2 \varepsilon^{1/3}}{d_b^{11/3}} \frac{1}{g(\alpha)+1.922\alpha\sqrt{We/1.24}} \exp\left(-1.017\sqrt{\frac{We}{1.24}}\right) \tag{74}$$

where $g(\alpha)$ and We is given by

$$g(\alpha) = \frac{(\alpha_{max}^{1/3}-\alpha^{1/3})}{\alpha_{max}^{1/3}} \qquad (\alpha_{max}=0.52) \tag{75}$$

$$We = \frac{2\rho_L(\epsilon d_b)^{2/3} d_b}{\sigma} \tag{76}$$

On the other hand, source term due to break up of bubble by liquid phase turbulence is given by

$$\phi_{BK} = 60.3\left(\frac{\alpha}{a_i}\right)^2 \frac{\epsilon^{1/3}\alpha(1-\alpha)}{d_b^{11/3}} \frac{1}{1+0.42(1-\alpha)\sqrt{We/1.24}} \exp\left(-\sqrt{\frac{1.24}{We}}\right) \tag{77}$$

The transport equation of interfacial area concentration and constitutive equations of source terms described above are implemented to CATHARE code which is developed at CEA using three-dimensional two-fluid model and k-ε turbulence model. Predictions were carried out for thermal hydrodynamic structure of boiling and non-boiling (air-water) two-phase flow including of interfacial area concentration. Comparisons were made with experimental data of DEBORA experiment which is boiling experiment using R-12 carried out at CEA and DEDALE experiment which is air-water experiment carried out at EDF, Electricite de France. The predictions reasonably agreed with experimental data of boiling and non-boiling two-phase flow for distribution of void fraction, velocities of gas and liquid phase, turbulent velocity and interfacial area concentration and the validity of transport equation and constitutive equations described above was confirmed.

5. Experimental researches on interfacial area concentration

The measurements of interfacial area have been carried out earlier in the field of chemical engineering using chemical reaction and/or chemical absorption at gas-liquid interface (Sharma and Danckwerts (1970)). A lot of experimental studies have been reported and reviewed (Ishii et al.,(1982), Kocamustafaogullari and Ishii (1983)). However, in this method, measured quantity is the product of interfacial area concentration and mass transfer coefficient. Light attenuation method and photographic method were also developed and measurement of interfacial area concentration was carried out. However, the measured interfacial area concentration using these methods is volumetric averaged value and measurement of local interfacial area concentration is impossible. In the detailed analysis of multidimensional two-phase flow, measurements of distribution of local interfacial area concentration are indispensable for the validation of interfacial area transport model. Therefore, the establishment of the measurement method of local interfacial area concentration was strongly required. Ishii (1975) and Delhaye (1968) derived following relation among time averaged interfacial area concentration, number of interfaces and velocity of interface. They pointed out local interfacial area concentration can be measured using two or three sensor probe based on this relation.

$$\overline{a_i} = \frac{1}{T}\sum_{j=1}^{N} \frac{1}{|\mathbf{n}_{ij} \bullet \mathbf{v}_{ij}|} \tag{78}$$

Here, T and N are time interval of measurement and number of interfaces passing a measuring point during time interval T. $\mathbf{n}_{ij}$ and $\mathbf{v}_{ij}$ are unit normal vector and interfacial velocity of j-th interface. For bubbly flow, assuming that shape of bubble is spherical and sensor of probe passes any part of bubble with equal probability, Eq.(78) can be simplified to

$$\overline{a_i} = 4\frac{N}{T}\overline{\frac{1}{|v_{sz}|}} \tag{79}$$

Here, v_{sz} is the z directional (flow directional) component of velocity of interface measured by double sensor probe as shown in Fig.8. v_{sz} is obtained by

$$v_{sz} = \frac{\Delta s}{\Delta t} \tag{80}$$

where Δs is spacing of two sensors (Fig.8) and Δt is the time interval where interface passes upstream sensor and downstream sensor.
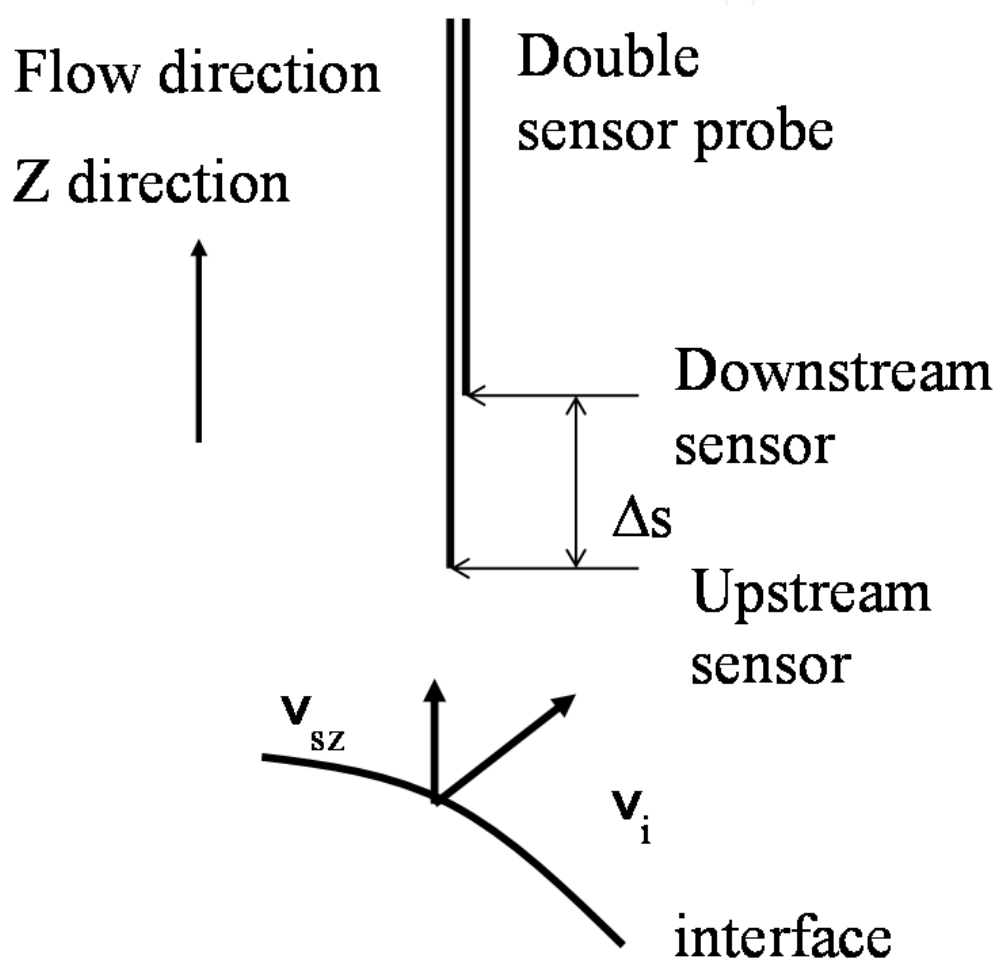

Fig. 8. Double sensor probe and velocity of interface

Later, based on local instant formulation of interfacial area concentration, Kataoka et al. (1986) proposed three double sensor probe method (four sensor probe method) as shown in Fig.9. Using this method, time averaged interfacial area is measured without assuming spherical bubble and statistical behavior of bubbles. The passing velocities measured by each double sensor probe are denoted by v_{sk} which are given by

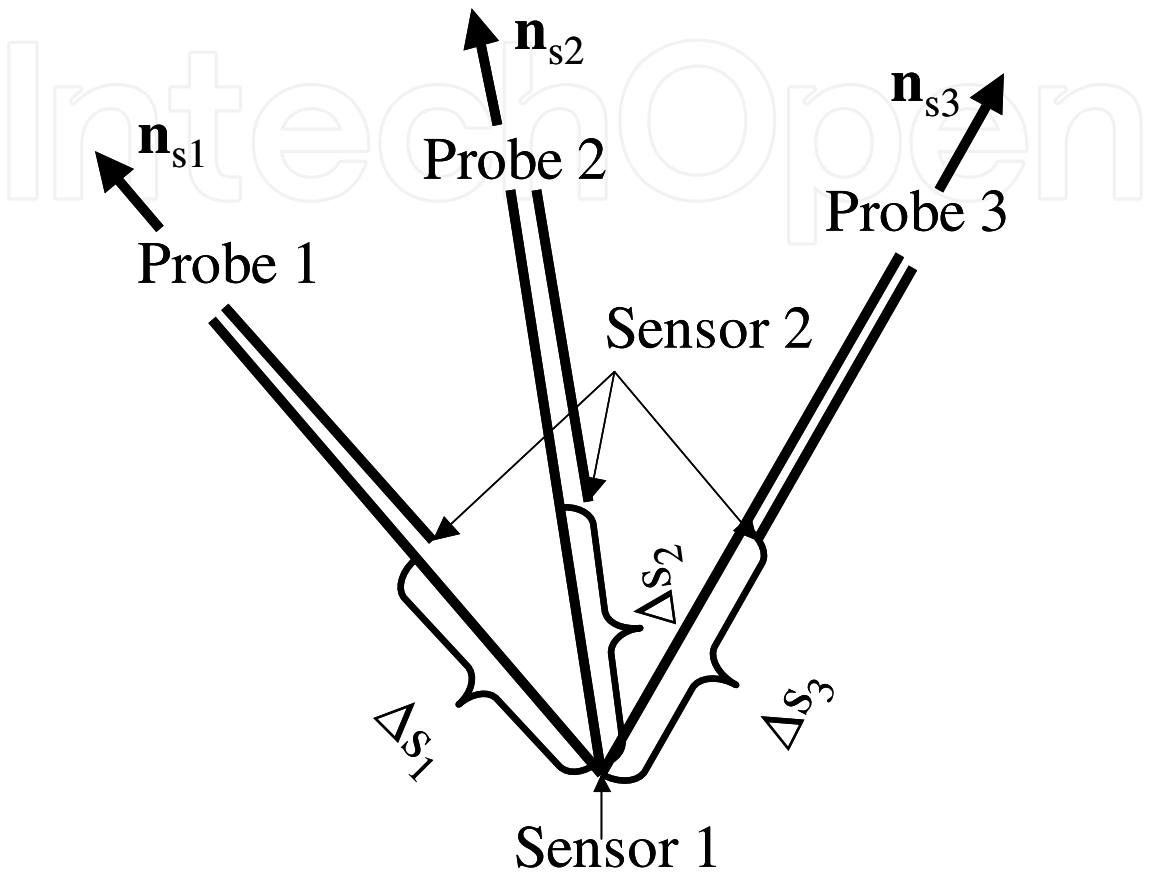$$v_{sk} = \frac{\Delta s_k}{\Delta t_k} \quad (k=1,2,3) \tag{81}$$

Fig. 9. Three double sensor probe (four sensor probe)

The direction cosines of unit vector of each double sensor probe ($\mathbf{n}_{sk}$, as shown in Fig.9) are denoted by $\cos\eta_{xk}$, $\cos\eta_{yk}$, $\cos\eta_{zk}$. Then, the inverse of product of interfacial velocity and unit normal vector of interface which appears in Eq.(78) is given by

$$\frac{1}{\left|n_i \bullet v_i\right|} = \frac{\sqrt{\left|A_1\right|^2 + \left|A_2\right|^2 + \left|A_3\right|^2}}{\sqrt{\left|A_0\right|^2}} \tag{82}$$

Here, $|A_0|$, $|A_1|$, $|A_2|$ and $|A_3|$ are given by

$$\left|A_0\right| = \begin{vmatrix} \cos\eta_{x1} & \cos\eta_{y1} & \cos\eta_{z1} \\ \cos\eta_{x2} & \cos\eta_{y2} & \cos\eta_{z2} \\ \cos\eta_{x3} & \cos\eta_{y3} & \cos\eta_{z3} \end{vmatrix} \tag{83}$$

$$|A_1| = \begin{vmatrix} 1/v_{s1} & \cos\eta_{y1} & \cos\eta_{z1} \\ 1/v_{s2} & \cos\eta_{y2} & \cos\eta_{z2} \\ 1/v_{s3} & \cos\eta_{y3} & \cos\eta_{z3} \end{vmatrix} \tag{84}$$

$$|A_2| = \begin{vmatrix} \cos\eta_{x1} & 1/v_{s1} & \cos\eta_{z1} \\ \cos\eta_{x2} & 1/v_{s2} & \cos\eta_{z2} \\ \cos\eta_{x3} & 1/v_{s3} & \cos\eta_{z3} \end{vmatrix} \tag{85}$$

$$|A_3| = \begin{vmatrix} \cos\eta_{x1} & \cos\eta_{y1} & 1/v_{s1} \\ \cos\eta_{x2} & \cos\eta_{y2} & 1/v_{s2} \\ \cos\eta_{x3} & \cos\eta_{y3} & 1/v_{s3} \end{vmatrix} \tag{86}$$

When three double sensor probes as shown in Fig.9 are orthogonal (perpendicular to each other), Eq.(82) is simply given by

$$\frac{1}{|n_i \bullet v_i|} = \sqrt{\left(\frac{1}{v_{s1}}\right)^2 + \left(\frac{1}{v_{s2}}\right)^2 + \left(\frac{1}{v_{s3}}\right)^2} \tag{87}$$

Then time averaged interfacial area concentration is given by

$$\overline{a_i} = \frac{1}{T}\sum_{j=1}^{N}\sqrt{\left(\frac{1}{v_{s1j}}\right)^2 + \left(\frac{1}{v_{s2j}}\right)^2 + \left(\frac{1}{v_{s3j}}\right)^2} \tag{88}$$

Most of recent experimental works of local interfacial area measurement are carried out by double sensor probe or three double sensor probe (four sensor probe) using electrical resistivity probe or optical probe.

For practical application, Kataoka et al.(1986) further proposed a simplified expression of Eq.(88) for double sensor probe which is given by

$$\overline{a_i} = 4\frac{1}{T}\sum_{j=1}^{N}\sqrt{\left(\frac{1}{v_{szj}}\right)^2}\ \frac{1}{1 - \cot\frac{1}{2}\alpha_0 \ln(\cos\frac{1}{2}\alpha_0) - \tan\frac{1}{2}\alpha_0 \ln(\sin\frac{1}{2}\alpha_0)} \tag{89}$$

where α_0 is given by

$$\frac{\sin 2\alpha_0}{2\alpha_0} = \frac{1 - (\sigma_z^2 / |\overline{v_{iz}}|^2)}{1 + 3(\sigma_z^2 / |\overline{v_{iz}}|^2)} \tag{90}$$

Here, $|\overline{v_{iz}}|$ and σ_z are the mean value and fluctuation of the z component interfacial velocity.

Hibiki, Hognet and Ishii (1998) carried out more detailed analysis of configuration of gas-liquid interface and double sensor probe and proposed more accurate formulation of interfacial area concentration measurement using double sensor probe. It is given by

$$\overline{a_i} = 2\frac{1}{T}\sum_{j=1}^{N}\sqrt{\left(\frac{1}{v_{szj}}\right)^2}\ I(\omega_0)\frac{\omega_0^{\ 3}}{3(\omega_0 - \sin\omega_0)} \tag{91}$$

Here ω_0 is given by

$$\frac{3}{2\omega_0^{\ 2}}\left(1 - \frac{\sin 2\omega_0}{2\omega_0}\right) = \frac{1 - (\sigma_z^{\ 2}/\left|\overline{v_{iz}}\right|^2)}{1 + 3(\sigma_z^{\ 2}/\left|\overline{v_{iz}}\right|^2)} \tag{92}$$

Double sensor probe or three double sensor probe (four sensor probe) has finite spacing between sensors. In relation to sensor spacing and size of bubble, some measurement errors are inevitable. In order to evaluate such measurement errors, a numerical simulation method using Monte Carlo approach is proposed (Kataoka et al., (1994), Wu and Ishii (1999)) for sensitivity analysis of measurement errors of double sensor probe or three double sensor probe. Using this method, Wu and Ishii (1999) carried out comprehensive analysis of accuracy of interfacial area measurement using double sensor probe including the probability of missing bubbles. They obtained formulation of interfacial area concentration measurement similar to Eqs.(91) and (92). The method using Eqs.(89) and (90) underestimated the interfacial area concentration up to 50%.

For adiabatic two-phase flow, many research groups all over the world, carried out measurements of interfacial area concentration mainly using double sensor or four sensor electrical resistivity probes. Most of experiments were carried out for vertical upward air-water two-phase flow in pipe. Some data were reported in annulus or downward flow. Flow regime covers bubbly flow to bubbly-to-slug transition. Some data are reported for annular flow. The experimental database of interfacial area concentration for non-boiling system described above is summarized in Table 1 (Kataoka (2010)).

Measurement of interfacial area concentration in boiling two-phase flow is quite important in view of practical application to nuclear reactor technology. However, in boiling two-phase flow, measurement of interfacial area is much more difficult compared with the measurement in non-boiling two-phase flow because of the durability of electrical resistivity and optical probes in high temperature liquid. Therefore, the accumulation of experimental data in boiling system was not sufficient compared with those in non-boiling system. However, recently, based on the establishment of measurement method of interfacial area as described above and improvement of electrical resistivity and optical probes, detailed measurements of interfacial area concentration become possible and experimental works have been carried out by various research groups. Most of experiments are carried out in annulus test section where inner pipe is heated. However, recently, some experimental studies are reported in rod bundle geometry. The experimental database of interfacial area concentration for boiling system described above is summarized in Table 2 (Kataoka (2010)).

Serizawa et al.1975,1992
Air-Water   Pipe, D=60mm, 30mm j_L=0.5 – 5.0 m/s   Double sensor,
Vertical up   L=1800mm,2500mm j_G=0.047 –0.54 m/s   electrical resistivity

Grossetete 1995
Air-Water   Pipe, D=38.1mm j_L=0.526 – 0.877 m/s   Double sensor
                                                                                 optical fiber
Vertical up   L=5906 mm j_G=0.0588 –0.322 m/s

Hibiki et al 1998, 1999,2001.
Air-Water   Pipe, D=25.4mm, 50.8mm j_L=0.292 – 5.0m/s   Double sensor,
Vertical up   L=3150 mm j_G=0.0162 –3.9 m/s   electrical resistivity

Hibiki et al. 2003a,20003b
Air-Water   Annulus, Di=19.2mm
                     Do=38.1mm j_L=0.272 – 2.08 m/s   Double & four sensor
Vertical up   L=4730 mm j_G=0.0313 –3.8 m/s   electrical resistivity

Hibiki et a l2004,2005.
Air-Water   Pipe, D=25.4mm, 50.8mm j_L=-0.62 – -3.11 m/s   Four sensor
Vertical down   L=3400 mm j_G=-0.00427 – -0.486 m/s   electrical resistivity

Takamasa et al.2003a,2003b
Air-Water   Pipe, D=9mm j_L=0.138 – 1.0m/s   Stereo image processing
Nitrogen-Water   L=819 mm j_G=0.0084 – 0.052m/s   Micro & normal gravity
Vertical up

Hazuku et al. 2007
Air-Water   Pipe, D=11mm j_L=0.0878 – 0.790m/s   Laser focus
 Vertical up   L=2750 mm j_G=39.5 – 73.0 m/s   displacement meter

Shen et al . 2005
Air-Water   Pipe, D=200mm j_L=0.035 – 0.277 m/s   Double sensor,
Vertical up   L=22600mm j_G=0.186 – 0.372 m/s   electrical resistivity

Ohuki and Akimoto 2000
Air-Water   Pipe, D=200mm j_L==0.06 – 1.06 m/s   Double sensor,
Vertical up   L=12000mm j_G=0.03 – 4.7 m/s   electrical resistivity
                                                                                 Optical fiber probe
Shawkat et al. 2008
Air-Water   Pipe, D=200mm j_L==0.2 – 0.68 m/s   X type anemometer
Vertical up   L=8400mm j_G=0.005 – 0.18 m/s

Prasser 2007
Air-Water   Pipe, D=51.2, 195.3 mm j_L==1.02 m/s   Wire mesh sensor
Vertical up   L=3072, 7812mm j_G=0.0094 – 0.53 m/s

Table 1. Summary of Experimental Database of Interfacial Area Concentration for Non-Boiling System

Roy and et al. 1994

R-113   Annulus, Di=19.2mm Do=38.1mm   G=579 - 801Kg/m^2/s   Double sensor optical fiber
Vertical up   L=2750mm   q= 0.79 – 116kW/m^2 0.269 MPa
T$_L$=43.0 – 50.3 C, ΔTsub=37.1 – 29.8 C

Zeitoun et al. 1994, 1996

Water   Annulus, Di=12.7mm Do=25.4mm   G=151.4 – 411.7Kg/m^2/s High speed video camera
Vertical up   L=306mm   q= 287 – 796kW/m^2 0.117 – 0.166 MPa
T$_L$ =11.6 – 31.1 C

Situ et al. 2004a,2004b

Water   Annulus, Di=19.0mm Do=38.1mm   j$_L$==0.5 – 2.02m/s Double sensor electrical resistivity
Vertical up   L=1700mm   q= 5 – 200kW/m^2 0.1292 – 0.1481 MPa
T$_L$ =95.0– 99.0 C

Bae et al. 2008

Water   Annulus, Di=21.0mm Do=40.0mm   G=340 – 674Kg/m^2/s   Double sensor
                                                                           electrical resistivity
Vertical up   L=1870mm   q= 97 – 359kW/m^2 0.121 –0.142 MPa
T$_L$ =95.0– 99.0 C

Yun et al. 2008

Water   3x3 rod bundle pitch: 16.6mm, diameter 8.2 mm
                        G=250 – 552Kg/m^2/s   Double sensor electrical resistivity
Vertical up   L=1700mm   q= 25 – 185kW/m^2 0.12 MPa
T$_L$ =96.0– 104.9 C

Lee et al. 2002,2008

Water   Annulus, Di=19mm Do=37.5mm   G=478 – 1049.5Kg/m^2/s   Double sensor
                                                                           electrical resistivity
Vertical up   L=1670mm   q= 88 – 359kW/m^2 0.01147 – 0.1698 MPa
T$_L$ =84.3– 100.4 C

Table 2. Summary of Experimental Database of Interfacial Area Concentration for Boiling System

6. Validation of interfacial area transport models by experimental data

In order to confirm the validity of transport equation of interfacial area, comparisons with experimental data were carried out mainly for bubbly flow and churn flow. The transport equation for bubbly flow is given by Eq.(68). This equation includes turbulent diffusion term

of interfacial area, turbulent diffusion term due to non-isotropic turbulence, sink term due to bubble coalescence and source term due to bubble break up. Each term is separately validated by experimental data.

Kataoka et al. (2011b, 2011c) carried out the validation of turbulent diffusion term of interfacial area, turbulent diffusion term due to non-isotropic turbulence using experimental data of radial distributions in air-water two-phase flow in round pipe under developed region. Under steady state and developed region without phase change, coalescence and break up of bubbles are negligible. Under such assumptions, transport equation of interfacial area concentration based on turbulent transport model, Eq.(68) can be simplified and given by following equation.

$$K_1 d_B |v'_L| \frac{1}{R-y} \frac{\partial}{\partial y}\left((R-y)\frac{\partial \overline{a_i}}{\partial y}\right) + K_2 \alpha d_B \overline{a_i} \frac{1}{R-y} \frac{\partial}{\partial y}\left((R-y)\frac{\partial \overline{\overline{V_L}}}{\partial y}\right) = 0 \qquad (93)$$

Here, R is pipe radius and y is distance from pipe wall. Kataoka's model for turbulent diffusion of interfacial area concentration, (Eqs.(61) , (65) and (67)) was used.

Kataoka et al. (2011c) further developed the model of turbulent diffusion term due to non-isotropic turbulence for churn flow. In the churn flow, additional turbulence void transport terms appear due to the wake of large babble as schematically shown in Fig.10.
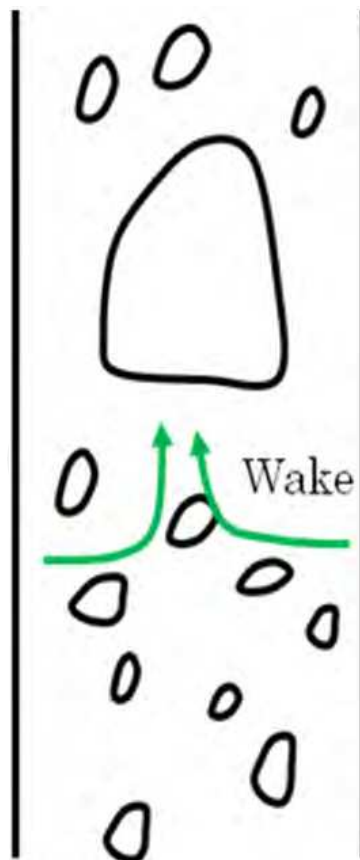

Fig. 10. Wake in Churn Flow Regime

For interfacial area transport due to wake of churn bubble, interfacial area is transported toward the center of pipe. The flux of interfacial area concentration in radial direction J_{ai}, due to churn bubble is related to the terminal velocity of churn bubble. The flux of interfacial area concentration toward the center of pipe is large at near wall and small at the center of pipe. Then, it is simply assumed to be proportional to the distance from pipe center. Finally, the flux of interfacial area concentration in radial direction, J_{ai} due to churn bubble is assumed to be given by

$$J_{ai} = K_{Cai} \frac{R-y}{R} \{0.35\sqrt{gD}\}\overline{a_i} \tag{94}$$

Then, transport equation of interfacial area concentration based on turbulent transport model in churn flow is given by

$$\frac{1}{R-y} \frac{\partial}{\partial y}\left((R-y)K_1 d_B \left|v_L'\right| \frac{\partial \overline{a_i}}{\partial y} \right) + \frac{1}{R-y} \frac{\partial}{\partial y}\left((R-y)^2 K_{Cai} \frac{0.35\sqrt{gD}}{R}\overline{a_i} \right) = 0 \tag{95}$$

In order to predict radial distribution of interfacial area concentration using Eq.(93) or Eq.(95), radial distributions of void fraction, averaged liquid velocity and turbulent liquid velocity are needed. These distributions were already predicted based on the turbulence model of two-phase flow for bubbly flow and churn flow (Kataoka et al. (2011d)).

Using transport equation of interfacial area concentration for bubbly flow (Eq.(93) and churn flow (Eq.(95)), the radial distributions of interfacial area concentration are predicted and compared with experimental data. Serizawa et al. (1975, 1992) measured distributions of void fraction, interfacial area concentration, averaged liquid velocity and turbulent liquid velocity for vertical upward air-water two-phase flow in bubbly and churn flow regimes in round tube of 60mm diameter. Void fraction and interfacial area were measured by electrical resistivity probe and averaged liquid velocity and turbulent liquid velocity were measured by anemometer using conical type film probe with quartz coating. Their experimental conditions are

Liquid flux, J_L: 0.44 - 1.03 m/s

Gas flux, J_G: 0 - 0.403 m/s

For empirical coefficient, K_{cai} is assumed to be 0.01 based on experimental data. The condition of flow regime transition from bubbly to churn flow is given in terms of area averaged void fraction, $\overline{\alpha}$ based on experimental results which is given by

$$\overline{\alpha} = 0.2 \tag{96}$$

Figures 11 and 12 show some examples of the comparison between experimental data and prediction of radial distributions of interfacial area concentration in bubbly flow and churn flow. In bubbly flow regime, distributions of interfacial area concentration show wall peak of which magnitude is larger for larger liquid flux whereas distributions interfacial area concentration in churn flow show core peak. The prediction based on the present model well reproduces the experimental data.
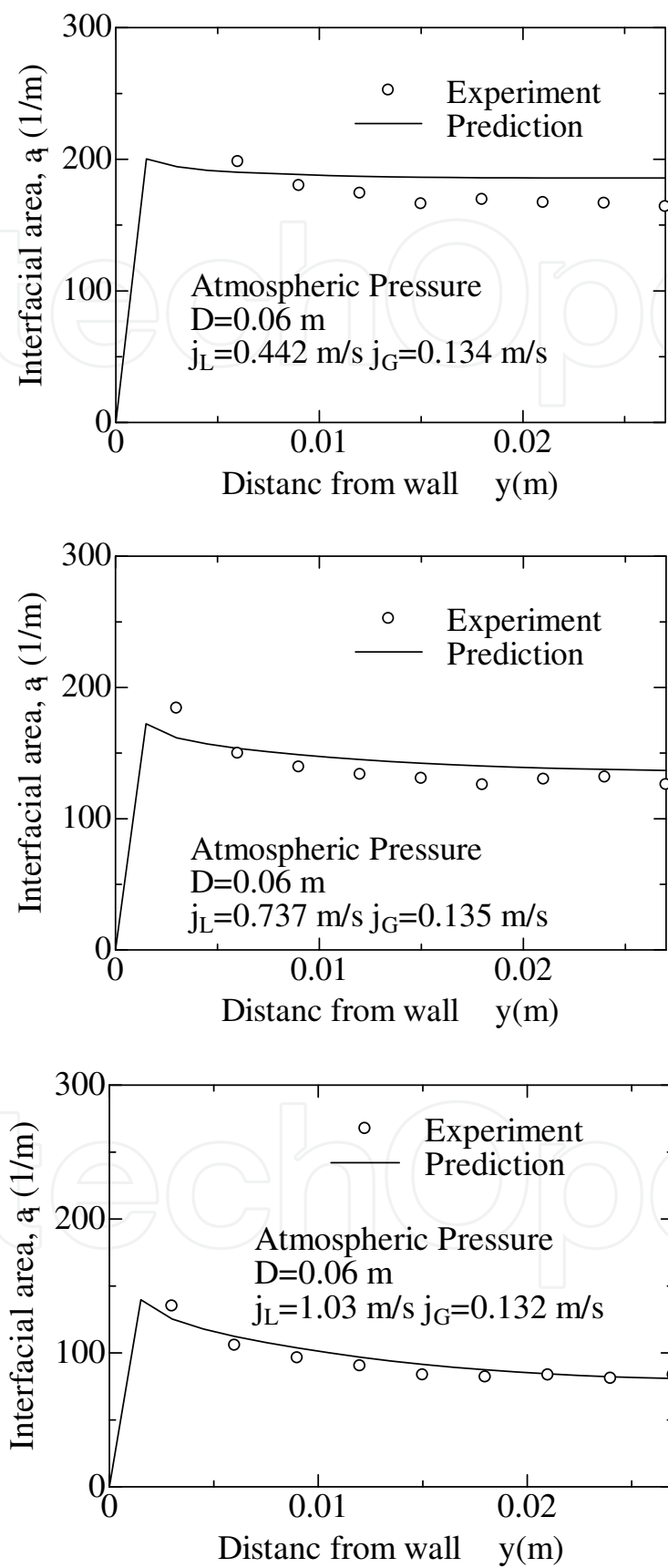
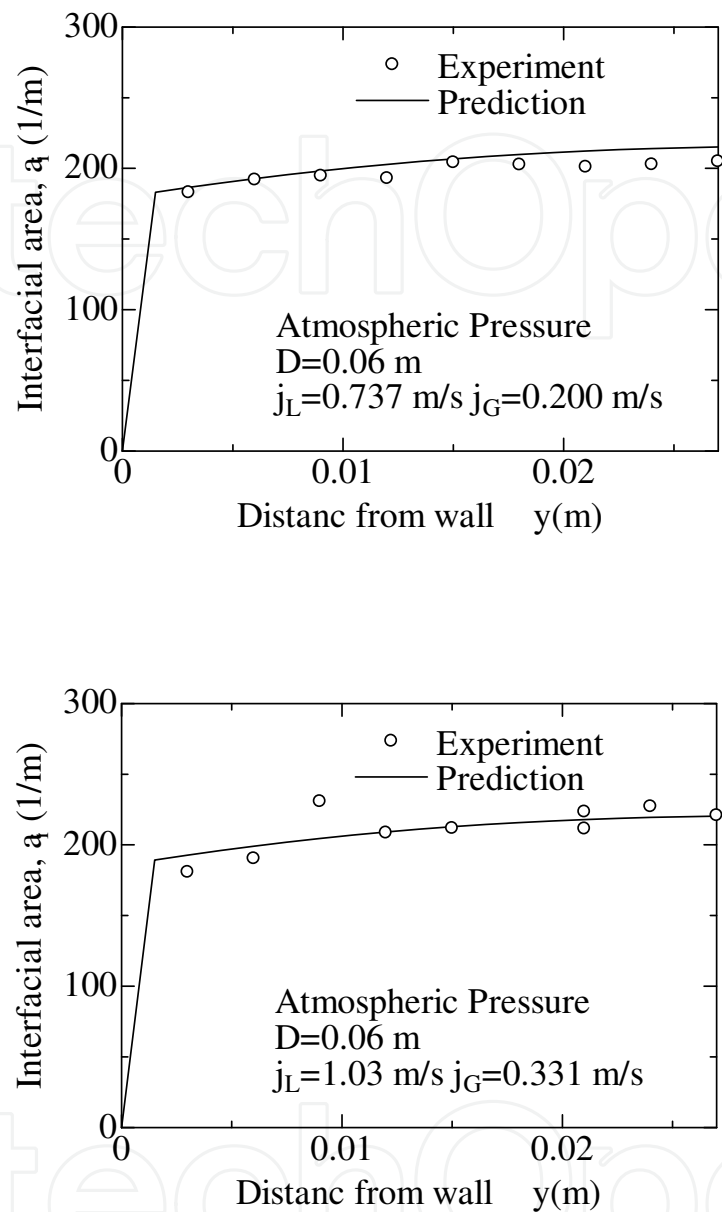Fig. 11. Distributions of Interfacial Area Concentration for Bubbly Flow

Fig. 12. Distributions of Interfacial Area Concentration for Churn Flow

Hibiki and Ishii (2000a) carried out the validation of their own correlations of sink term due to bubble coalescence (Eq.(72)) and source term due to bubble break up (Eq.(70)) using experimental data. They carried out experiments in vertical upward air water two-phase flow in pipe under atmospheric pressure. In order to validate their interfacial transport model, evolutions of radial distributions of interfacial area concentration in the flow direction were systematically measured. Experimental conditions are as follows.

Condition I

Pipe diameter D: 25.4mm,   Measuring positions z from inlet: (z/D=12, 65,125),

Liquid flux j_L=0.292 – 3.49 m/s, Gas flux j_G=0.05098 –0.0931 m/s

Condition II

Pipe diameter D: 50.8mm,   Measuring positions z from inlet: (z/D=6,30.3, 53.5)

Liquid flux j_L=0.491 – 5.0 m/s, Gas flux j_G=0.0556 –3.9 m/s

Figures 13 and 14 show the result of comparison between experimental data and prediction using transport equation of interfacial area with sink term due to bubble coalescence (Eq.(70)) and source term due to bubble break up (Eq.(72)). Predictions agree with experimental data within 10% accuracy.
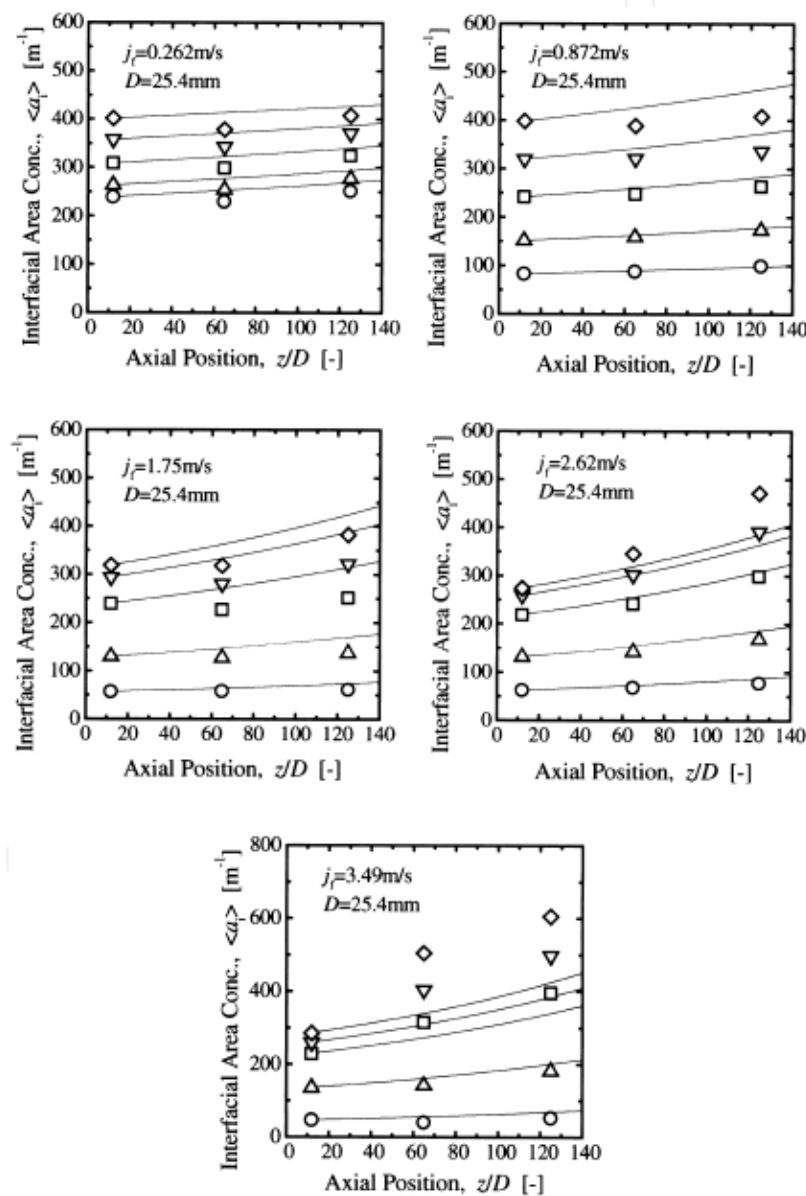
Fig. 13. Comparison between Experimental data and prediction for the variation of interfacial area concentration along flow direction for 25.4mm diameter pipe (Hibiki, T. and Ishii, M. 2000a One-Group Interfacial Area Transport of Bubbly Flows in Vertical Round Tubes, International Journal of Heat and Mass Transfer, 43, 2711-2726.Fig.8)
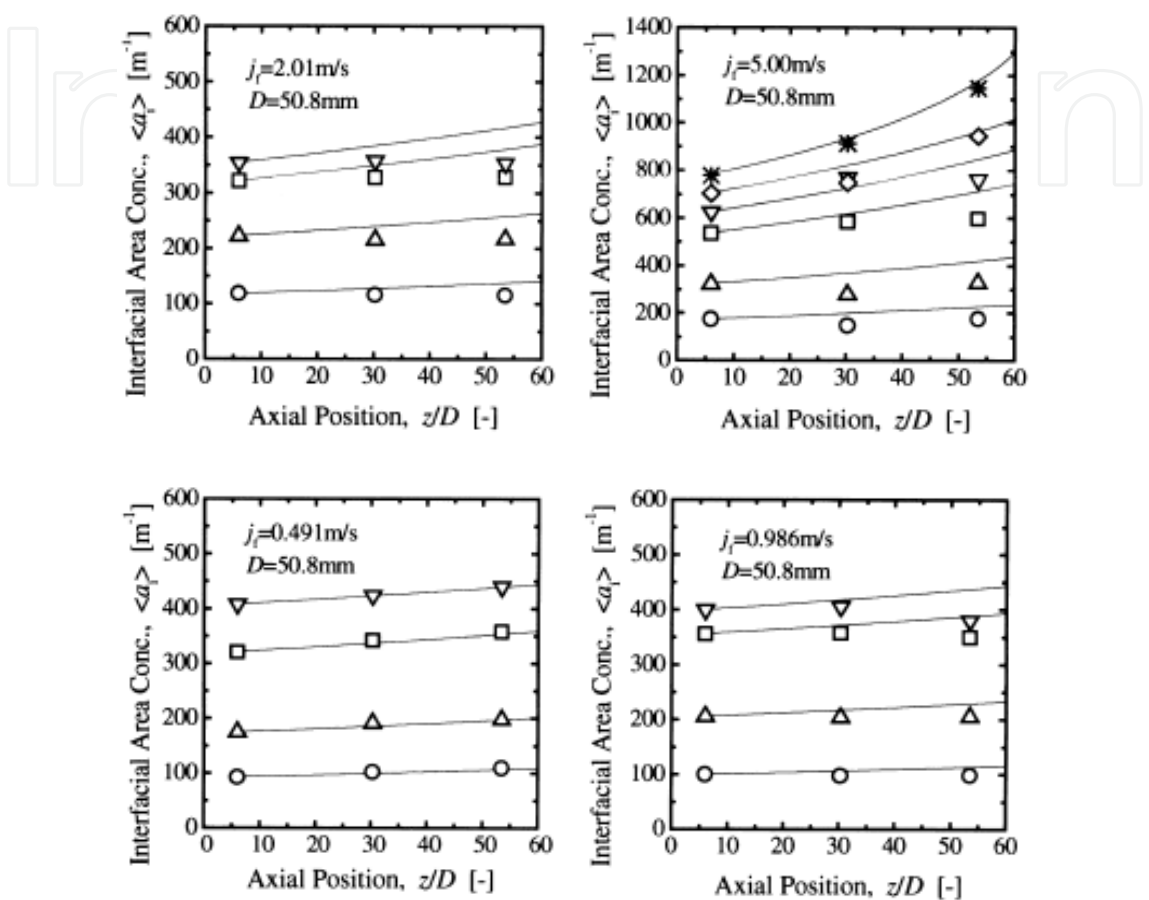
Fig. 14. Comparison between Experimental data and prediction for the variation of interfacial area concentration along flow direction for 50.8mm diameter pipe (Hibiki, T. and Ishii, M. 2000a One-Group Interfacial Area Transport of Bubbly Flows in Vertical Round Tubes, International Journal of Heat and Mass Transfer, 43, 2711-2726.Fig.9)

7. Conclusion

In this chapter, intensive review on recent developments and present status of interfacial area concentration and its transport model was carried out. Definition of interfacial area and rigorous formulation of local instant interfacial area concentration was introduced. Using this formulation, transport equations of interfacial area concentration were derived in details. Transport equations of interfacial area concentration consist of conservation

equation of interfacial area concentration and conservation equation of interfacial velocity. For practical application, simplified transport equation of interfacial area concentration was derived with appropriate constitutive correlations. For bubbly flow, constitutive correlations of turbulent diffusion, turbulent diffusion due to non-isotropic turbulence, sink term due to bubble coalescence and source term due to bubble break up were developed. Measurement methods on interfacial area concentration were reviewed and experiments of interfacial area concentration for non-boiling system and boiling system were reviewed. Validation of transport equations of interfacial area concentration was carried out for bubbly and churn flow with satisfactory agreement with experimental data. At present, transport equations of interfacial area concentration can be applied to analysis of two-phase flow with considerable accuracy. However, the developments of constitutive correlations are limited to bubbly and churn flow regimes. Much more researches are needed for more systematic developments of transport equations of interfacial area concentration.

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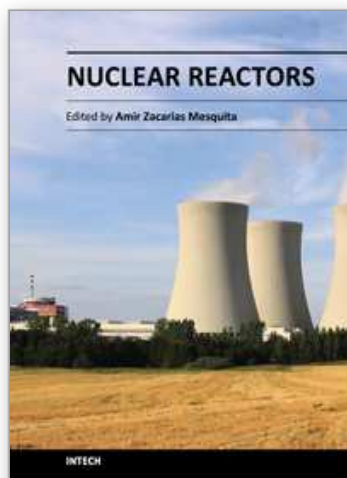
Nuclear Reactors

Edited by Prof. Amir Mesquita

This book presents a comprehensive review of studies in nuclear reactors technology from authors across the globe. Topics discussed in this compilation include: thermal hydraulic investigation of TRIGA type research reactor, materials testing reactor and high temperature gas-cooled reactor; the use of radiogenic lead recovered from ores as a coolant for fast reactors; decay heat in reactors and spent-fuel pools; present status of two-phase flow studies in reactor components; thermal aspects of conventional and alternative fuels in supercritical water?cooled reactor; two-phase flow coolant behavior in boiling water reactors under earthquake condition; simulation of nuclear reactors core; fuel life control in light-water reactors; methods for monitoring and controlling power in nuclear reactors; structural materials modeling for the next generation of nuclear reactors; application of the results of finite group theory in reactor physics; and the usability of vermiculite as a shield for nuclear reactor.

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