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Downlink Resource Allocation and Frequency Reuse Schemes for WiMAX Networks

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1. Introduction

Throughout this chapter we consider the *downlink* of a cellular WiMAX network where a number of base stations need to communicate simultaneously with their respective active users [1]. Each of these users has typically a certain Quality of Service (QoS) requirement that needs to be satisfied. To that end, base stations dispose of limited wireless resources (subcarriers and transmit powers) that should be shared between users. They also have some amount of Channel State Information (CSI) about users' propagation channels available, if existent, typically via feedback. The problem of determining the subset of subcarriers assigned to each user and the transmit power on each of these subcarriers is commonly referred to as the *resource allocation problem*. This problem should be solved such that all the QoS exigencies are respected.

Of course, the resource allocation problem has several formulations depending on i) the particular QoS-related objective function which we adopt (*e.g.,* achievable rate, transmission error probability . . .) and ii) the channel model that we assume in relation with the available CSI. These CSI-related channel models will be discussed in Section 2 while the different formulations of the resource allocation problem will be covered in Section 4.

Since the set of subcarriers available for the whole WiMAX system is limited, it is typical that some subcarriers are reused at the same time by different base stations. Such base stations will generate multicell interference. Therefore, resource allocation parameters should, in principle, be determined in each cell in such a way that the latter multicell interference does not reach excessive levels. This fact highlights the importance of properly planning the so-called *frequency-reuse scheme* of the network. A frequency-reuse scheme answers the question whether the whole set of subcarriers should be available for allocation in all the cells of the network (meaning better spectral-usage efficiency but higher levels of intercell interference) or whether we should make parts of it exclusive to certain cells (leading thus to less efficiency in spectral usage but to lower levels of interference on the exclusive subcarriers).

Note that frequency-reuse planning is intimately related to resource allocation since it decides the subset of subcarriers that will be available for allocation in each cell of the network. Refer to Sections 4 and 5 for more details.

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[1] We assume that the set of active users in the network is determined in advance by the schedulers at the base stations. We also assume that base stations has for each active user an infinite backlog of data to be transmitted

The rest of the chapter is organized as follows. In Section 2, the different kinds of CSI feedback models are presented and their related channel models are discussed. The issue of frequency reuse planning (which is intimately related to cellular resource allocation) is discussed in Section 3. Each different channel model leads to a different formulation of the resource allocation problem. These formulations are addressed in Section 4. Finally, Section 5 deals with the determination of the so-called *frequency-reuse factor*.

2. Feedback for resource allocation: Channel State Information (CSI) and channel models

Consider the downlink of an OFDMA-based wireless system (such as a WiMAX network) and denote by N, K the total number of subcarriers and of active users, respectively. Assume that the subcarriers are numbered from 1 to N and that the active users of the network are numbered from 1 to K. The network comprises a certain number of cells that are indexed using the notation c. Each cell c consists of a base station communicating with a group of users as shown in Figure 1. The signal received by user k at the nth subcarrier of the mth
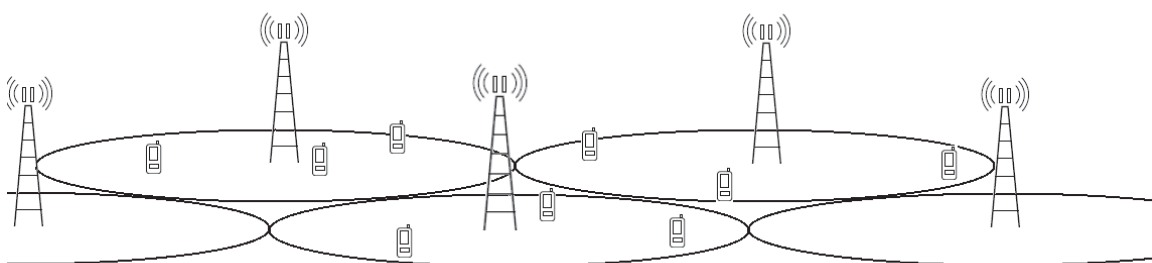

Fig. 1. WiMAX network

OFDM block (m being the time index) is given by

$$y_k(n,m) = H_k^c(n,m)s_k(n,m) + w_k(n,m), \tag{1}$$

where $s_k(n,m)$ is the transmitted symbol and where $w_k(n,m)$ is a random process which is used to model the effects of both thermal noise and intercell interference. Finally, $H_k^c(n,m)$ refers to the (generally complex-valued) coefficient of the propagation channel between the base station of cell c and user k on subcarrier n at time m.

Assume that the duration of transmission is equal to T OFDM symbols *i.e.*, $m \in \{0, 1, \ldots, T-1\}$. Denote by $\mathcal{N}_k$ the subset of subcarriers ($\mathcal{N}_k \subset \{1, 2, \ldots, N\}$) assigned to user k. The codeword destined to user k is thus the $|\mathcal{N}_k| \times T$ matrix

$$\boldsymbol{S}_k = [\boldsymbol{s}_k(0), \boldsymbol{s}_k(1), \ldots, \boldsymbol{s}_k(T-1)], \tag{2}$$

where each $|\mathcal{N}_k| \times 1$ column-vector $\boldsymbol{s}_k(m)$ is composed of the symbols $\{s_k(n,m)\}_{n \in \mathcal{N}_k}$ transmitted during the mth OFDM block on subcarriers $\mathcal{N}_k$.

Depending on the amount of feedback sent from users to the base stations, coefficients $H_k^c(n,m)$ can be modeled either as deterministic or as random variables. As stated in Section I, different formulations of the resource allocation problem exist in the literature, each

formulation being associated with a different model for coefficients $H_k^c(n,m)$. These channels models are summarized in the following subsection.

2.1 Theoretical CSI-related channel models:

The general OFDMA signal model given by (1) does not specify whether the channel coefficients $\{H_k(n,m)\}_{k,n,m}$ associated with each user k are known at the base stations or not. In this chapter, we consider three signal models for these coefficients.

1. **Full CSI: Deterministic channels**.
    In this model, channel coefficients $H_k^c(n,m)$ for each user k are assumed to be perfectly known (thus deterministic) at both the base station side and the receiver side on all the subcarriers $n \in \{0 \ldots N-1\}$. Note that **this assumption implicitly requires that each receiver k feedbacks to the base station the values of the channel coefficients $H_k^c(n,m)$ on all the assigned subcarriers n**.

    For the sake of simplicity, it is also often assumed in the literature that the above deterministic coefficients $H_k^c(n,m)$ remain constant ($H_k^c(n,m) = H_k^c(n)$) during the transmission of a codeword [2] *i.e.,* $\forall m \in \{0,1,\ldots,T-1\}$. Under these assumptions, a transmission to user k at rate R_k nats/sec/Hz is possible from the information-theoretic point of view with negligible probability of error provided that $R_k < C_k$, where C_k denotes the channel capacity associated with user k. If we assume that the noise-plus-interference process $w_k(n,m)$ in (1) is zero-mean Gaussian-distributed[3] with variance σ_k^2, then the channel capacity C_k (in nats/sec/Hz) is given by

    $$C_k = \frac{1}{N} \sum_{n \in \mathcal{N}_k} \log\left(1 + P_{k,n} \frac{|H_k^c(n)|^2}{\sigma_k^2}\right),$$

    where $\mathcal{N}_k$, we recall, is the subset of subcarriers ($\mathcal{N}_k \subset \{0,1,\ldots,N\}$) assigned to user k, and where $P_{n,k}$ is the power transmitted by the base station on subcarrier $n \in \mathcal{N}_k$ *i.e.,* $P_{k,n} = \mathbb{E}\left[|s_{k,n}|^2\right]$.

2. **Statistical CSI: Random ergodic (fast-fading) channels**.
    In this model, we assume that coefficients $H_k^c(n,m)$ associated with each user k on any subcarrier n are time-varying, unknown at the base station side and perfectly known at the receiver side. We can thus think of $\{H_k(n,m)\}_m$ as a random process with a certain statistical distribution *e.g.,* Rayleigh, Rice, Nakagami, etc. We also assume that this process undergo **fast fading** *i.e.,* the coherence time of the channel is much smaller than the duration T of transmission of a codeword. It is thus reasonable to model $\{H_k(n,m)\}_m$ as an independent identically distributed (i.i.d) random ergodic process for each $n \in \{0 \ldots N-1\}$. Finally, **we assume that the parameters of the distribution of this process *i.e.,* its mean, variance ..., are known at the base station, typically via feedback**.

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[2] A codeword typically spans several OFDM blocks *i.e.,* several time indexes m

[3] Eventhough the noise-plus-interference $w_k(n,m)$ is not Gaussian in general, approximating it as a Gaussian process is widely used in the literature (see for instance Gault et al. (2005); S. Plass et al. (2004; 2006)). The reason behind that is twofold: first, the Gaussian approximation provides a lower bound on the mutual information, second it allows us to have an analytical expression for the channel capacity.

Note that since the channel coefficients $\{H_k(n,m)\}_m$ are time-varying in this model, then each single codeword encounters a large number of channel realizations. In this case, it is a well-known result in information theory that transmission to user k at rate R_k nats/sec/Hz is possible with negligible probability of error provided that $R_k < C_k$, where C_k denotes here the channel *ergodic* capacity associated with user k and given (in the case of zero-mean Gaussian distributed noise-plus-interference processes $w_k(n,m)$ with variance σ_k^2) by

$$C_k = \mathbb{E}\left[\log\left(1 + P_{k,n}\frac{|H_k^c(n,m)|^2}{\sigma_k^2}\right)\right].$$

Here, expectation is taken with respect to the distribution of the random channel coefficients $H_k^c(n,m)$.

3. **Statistical CSI: Random nonergodic (slow-fading) channels**.

   In this case, channel coefficients $H_k(n,m) = H_k(n)$ are assumed to be fixed during the whole transmission of any codeword, but nonetheless random and unknown by the base stations. This case is usually referred to as the **slow fading** case. It arises as the best fitting model for situations where the channel coherence time is larger than the transmission duration. We also **assume that the parameters of the distribution of the random variables $H_k(n)$ i.e., their mean, variance ..., are known at the base station via feedback**.

   In contrast to the ergodic case, there is usually no way for the receiver in the nonergodic case to recover the transmitted information with negligible error probability. Assume that the base station needs to send some information to user k at a data rate R_k nats/sec/Hz. The transmitted message (if the transmitted symbols $s_k(n,m)$ are from a Gaussian codebook) can be decoded by the receiver provided that the required rate R_k is less than the mutual information between the source and the destination *i.e.,* provided that $\frac{1}{N}\sum_{n \in \mathcal{N}_k}\log\left(1 + P_{k,n}\frac{|H_k(n)|^2}{\sigma_k^2}\right) > R_k$. If the channels realization $H_k(n)$ is such that

$$\frac{1}{N}\sum_{n \in \mathcal{N}_k}\log\left(1 + P_{k,n}\frac{|H_k(n)|^2}{\sigma_k^2}\right) \leq R_k,$$

   then the transmitted message cannot be decoded by the receiver. In this case, user k link is said to be in *outage*. The event of outage occurs with the following probability:

$$P_{O,k}(R_k) \triangleq \Pr\left[\frac{1}{N}\sum_{n \in \mathcal{N}_k}\log\left(1 + P_{k,n}\frac{|H_k(n)|^2}{\sigma_k^2}\right) \leq R_k\right]. \tag{3}$$

   Probability $P_{O,k}(R_k)$ is commonly referred to as the *outage probability* associated with user k. In the context of communication over slow fading channels as described above, it is of clear interest to minimize the outage probability associated with each user.

It is worth mentioning that the above distinction between deterministic, ergodic and nonergodic channel was originally done in I. E. Telatar (1999) for Multiple-Input-Multiple-Output (MIMO) channels. We present in the sequel the main existing results on resource allocation for each one of the above signal models.

3. Frequency reuse schemes and the frequency reuse factor: Definition and relation to resource allocation

As we stated earlier, management of multicell interference is one of the major issues in cellular networks design and administration. This management is intimately related to the so-called *frequency-reuse scheme* adopted in the network. Indeed, choosing a frequency-reuse scheme means determining the subset of subcarriers that are available for allocation in each cell (or sector) of the network. In some reuse schemes, the whole set of subcarriers is available for allocation in all the cells of the network, while in others some subsets of subcarriers are made exclusive to certain cells and prohibitted for others.

Many reuse schemes have been proposed in the literature, differing in their complexity and in their **repetitive pattern** *i.e.,* the number of cells (or sectors) beyond which the scheme is repleted. In this chapter, we only **focus on three-cell (or three-sector) based reuse schemes**. Indeed, the level of interference experienced by users in a cellular network is related to the value of parameter α defined as

$$\alpha = \frac{\text{number of subcarriers reused by three adjacent cells}}{N} . \qquad (4)$$

Where N is the total number of subcarriers in the system. In *sectorized networks i.e.,* networks with $120\,^\circ$-directive antennas at their base stations (see Figure 2), the definition of α becomes

$$\alpha = \frac{\text{number of subcarriers reused by three adjacent sectors}}{N} . \qquad (5)$$

Note that in Figure 2, sectors 3,4,5 form the basic pattern of the reuse scheme that is repleted throughout the network.
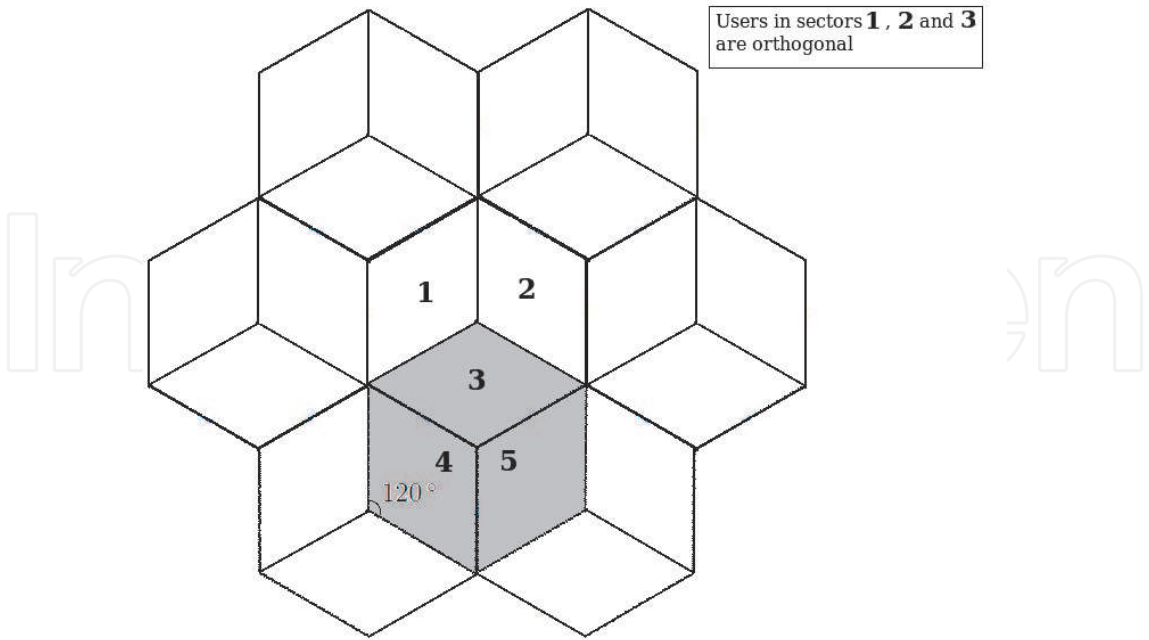

Fig. 2. A sectorized cellular network

Parameter α is called the *frequency reuse factor*. If $\alpha = 1$, then each base station can allocate the totality of the available N subcarriers to its users. This policy is commonly referred to as the *all-reuse scheme* or as the *frequency-reuse-of-one scheme*. Under this policy, all users of the system are subject to multicell interference. If $\alpha = 0$, then no subcarriers are allowed to be used simultaneously in the neighboring cells. This is the case of an *orthogonal reuse scheme*. In such a scheme, users do not experience any multicell interference.

If α is chosen such that $0 < \alpha < 1$, then we obtain the so-called *fractional frequency reuse*, see WiMAX Forum (2006). According to this frequency reuse scheme, the set of available subcarriers is partitioned into two subsets. One subset contains αN subcarriers that can be reused within all the cells (sectors) of the system and is thus subject to multicell interference. The other subset contains the remaining $(1 - \alpha)N$ subcarriers and is divided in an orthogonal way between the different cells (sectors). Such subcarriers are thus protected from interference.

The larger the value of α, the greater the number of available subcarriers for each base station and the higher the level of multicell interference. There is therefore a tradeoff between the number of available subcarriers (which is proportional to α) and the severity of the multicell interference. This tradeoff is illustrated by Figure 3. Generally speaking, the characterization
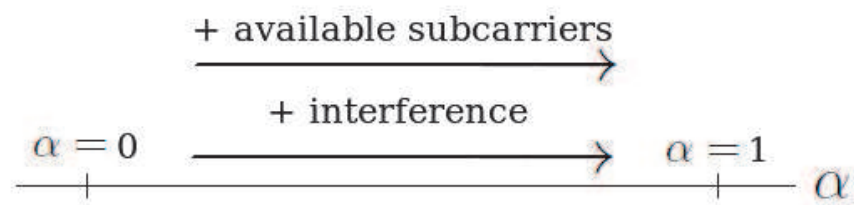

Fig. 3. Tradeoff between interference and number of available subcarriers

of the latter tradeoff is a difficult problem to solve. Most of the approaches used in the literature to tackle this problem were based on numerical simulations. Section 5 is dedicated to the issue of analytically finding the best value of α without resorting to such numerical approaches.

In the sequel, we assume that the frequency-reuse scheme (or the frequency-reuse factor) has already been chosen in advance prior to performing resource allocation. While Section 5 is dedicated to the issue of finding the best value of α.

4. Downlink resource allocation for WiMAX cellular networks

In this section, we give the main existing results on the subject of downlink resource allocation for WiMAX networks. We present the literature on this subject by classifying it with respect to the specific signal models (full-CSI channels, statistical-CSI fast-fading channels, statistical-CSI slow-fading channels).

It is worth noting that many existing works on cellular resource allocation resort to the so-called **single-cell assumption**. Under this simplifying assumption, intercell interference is

considered negligible. The received signal model for some user k in cell c on each subcarrier n can thus be written as

$$y(n,m) = H_k^c(n,m)s_k(n,m) + w_k(n,m),$$

where process $w_k(n,m)$ contains **only thermal noise**. It can thus be modeled in this case as AWGN with distribution $\mathcal{CN}(0,\sigma^2)$. Of course, the single-cell assumption is simplifying and unrealistic in real-world cellular networks where intercell interference prevails. However, in some cases one can manage to use the results of single-cell analysis as a tool to tackle the more interesting and demanding multicell problem (see for example N. Ksairi & Ciblat (2011); N. Ksairi & Hachem (2010a)).

4.1 Full-CSI resource allocation (deterministic channels)

Although having full (per-subcarrier) CSI at the base stations is quite unrealistic in practice as we argued in Section 2, many existing works on resource allocation for OFDMA systems resorted to this assumption. We give below the main results in the literature on resource allocation in the case of full CSI, mainly for the sake of completeness.

1) Sum rate maximization

Consider the problem of maximizing the sum of all users achievable rates, first in a **single-cell context** (focus for example on cell c). This maximization should be done such that the spent power does not exceed a certain maximum value and such that the OFDMA orthogonal subcarrier assignment constraint (no subcarrier can be assigned to more than one user) is respected. Recall the definition of $P_{k,n} = \mathbb{E}[|s_k(n,m)|^2]$ for any $n \in \mathcal{N}_k$ as the power allocated to user k on subcarrier n. Let $P_{\max}$ designates the maximal power that the base station is allowed to spend. The maximal sum rate should thus be computed under the following constraint:

$$\sum_{k \in c} \sum_{n \in \mathcal{N}_k} P_{k,n} \leq P_{\max}. \tag{6}$$

It is known Tse & Visawanath (2005) that the maximum sum rate is achieved provided that the codeword $\boldsymbol{S}_k = [\boldsymbol{s}_k(0), \boldsymbol{s}_k(1), \ldots, \boldsymbol{s}_k(T-1)]$ of each user is chosen such that

$$\boldsymbol{s}_k(m) \text{ for } m \in \{0, 1, \ldots, T-1\} \text{ is an i.i.d process, and}$$
$$\boldsymbol{s}_k(m) \sim \mathcal{CN}\left(\boldsymbol{0}, \text{diag}\left(\{P_{k,n}\}_{n \in \mathcal{N}_k}\right)\right), \tag{7}$$

where $\boldsymbol{s}_k(m)$, we recall, is the vector composed of the symbols $\{s(n,m)\}_{n \in \mathcal{N}_k}$ transmitted to user k during the mth OFDM symbol. It follows that the maximum sum rate C_{sum} of the downlink OFDMA single cell system can be written as

$$C_{\text{sum}} = \max_{\{\mathcal{N}_k, P_{k,n}\}_{k \in c, n \in \mathcal{N}_k}} \sum_{k \in c} \frac{1}{N} \sum_{n \in \mathcal{N}_k} \log\left(1 + \frac{|H_k^c(n)|^2}{\sigma^2} P_{k,n}\right),$$

subject to subcarrier assignment orthogonality constraint and to (6)

Solving the above optimization problem provides us with the optimal resource allocation which maximizes the sum rate of the system. It is known from Jang & Lee (2003); Tse & Visawanath (2005), that the solution to the above problem is the so-called *multiuser water-filling*. According to this solution, the optimal subcarrier assignment $\{\mathcal{N}_k\}_{k \in c}$ is such that:

Each subcarrier $n \in \{1, 2, \ldots, N\}$ is assigned to the user k_n^* satisfying $k_n^* = \arg\max_k |H_k^c(n)|$.

The powers $\{P_{k_n^*,n}\}_{1 \leq n \leq N}$ can finally be determined by water filling:

$$P_{k_n^*,n} = \left(\frac{1}{\lambda} - \frac{\sigma^2}{|H_{k_n^*}^c(n)|^2} \right)^+ ,$$

where λ is a Lagrange multiplier chosen such that the power constraint (6) is satisfied with equality:

$$\sum_{n=1}^{N} \left(\frac{1}{\lambda} - \frac{\sigma^2}{|H_{k_n^*}^c(n)|^2} \right)^+ = P_{\max} .$$

In a **multicell scenario**, the above problem becomes that of maximizing the sum of data rates that can be achieved by the users of the network subject to a total network-wide power constraint

$$\sum_c \sum_{k \in c} \sum_{n \in \mathcal{N}_k} P_{k,n} \leq P_{\max} . \tag{8}$$

In case the transmitted symbols of all the base stations are from Gaussian codebooks, the sum rate maximization problem can be written as

$$\max_{\{\mathcal{N}_k, P_{k,n}\}_{1 \leq k \leq K, n \in \mathcal{N}_k}} \sum_c \sum_{k \in c} \frac{1}{N} \sum_{n \in \mathcal{N}_k} \log \left(1 + P_{k,n} \frac{|H_k^c(n)|^2}{\sigma_k^2} \right)$$

subject to the OFDMA orthogonality constraint and to (8) , $\tag{9}$

In contrast to the single cell case where the exact solution has been identified, no closed-form solution to Problem (9) exists. An approach to tackle a variant of this problem with per subcarrier peak power constraint ($P_{k,n} \leq P_{\text{peak}}$) has been proposed in Gesbert & Kountouris (2007). The proposed approach consists in performing a decentralized algorithm that maximizes an upperbound on the network sum rate. Interestingly, this upperbound is proved to be tight in the asymptotic regime when the number of users per cell is allowed to grow to infinity. However, the proposed algorithm does not guaranty fairness among the different users.

A heuristic approach to solve the problem of sum rate maximization is adopted in Lengoumbi et al. (2006). The authors propose a centralized iterative allocation scheme allowing to adjust the number of cells reusing each subcarrier. The proposed algorithm promotes allocating subcarriers which are reused by small number of cells to users with bad channel conditions. It also provides an interference limitation procedure in order to reduce the number of users whose rate requirements are unsatisfied.

2) Weighted sum rate maximization

In a wireless system, maximizing the sum rate does not guaranty any fairness between users. Indeed, users with bad channels may not be assigned any subcarriers if the aforementioned multiuser water-filling scheme is applied. Such users may have to wait long durations of time till their channel state is better to be able to communicate with the base station. In order to

ensure some level of fairness among users, one can use the maximization of a weighted sum of users achievable rates as the criterion of optimization of the resource allocation.

In a **single-cell scenario** (focus on cell c), the maximal weighted sum rate $C_{\text{weighted sum}}$ is given by:

$$C_{\text{weighted sum}} = \max \sum_{k \in c} \mu_k R_k \,, \tag{10}$$

where R_k is the data rate achieved by user k, and where the maximization is with respect to the resource allocation parameters and the distribution of the transmit codewords $\mathbf{S}_k$. Weights μ_k in (10) should be chosen in such a way to compensate users with bad channel states. As in the sum rate maximization problem, it can be shown that the weighted sum rate is maximized with random Gaussian codebooks *i.e.*, when (7) holds. The optimal resource allocation parameters can thus be obtained as the solution to the following optimization problem:

$$\max_{\{\mathcal{N}_k, P_{k,n}\}_{k \in c, n \in \mathcal{N}_k}} \sum_{k \in c} \frac{\mu_k}{N} \sum_{n \in \mathcal{N}_k} \log\left(1 + P_{k,n} \frac{|H_k(n)|^2}{\sigma^2}\right),$$
subject to the subcarrier assignment orthogonality constraint and to (6) .

The above optimization problem is of combinatorial nature since it requires finding the optimal set $\mathcal{N}_k$ of subcarriers for each user k. It cannot thus be solved using convex optimization techniques.

For each subchannel assignment $\{\mathcal{N}_k\}_{1 \le k \le K}$, the powers $P_{k,n}$ can be obtained by the so called multilevel water filling Hoo et al. (2004) with a computational cost of the order of $\mathcal{O}(N)$ operations. On the other hand, finding the optimal subcarrier assignment requires an exhaustive search and a computational complexity of the order of K^N operations. The overall computational complexity is therefore $\mathcal{O}(NK^N)$. In order to avoid this exponentially complex solution, the authors of Seong et al. (2006) state that solving the dual of the above problem (by Lagrange dual decomposition for example) entails a negligible duality gap. This idea is inspired by a recent result Yu & Lui (2006) in resource allocation for multicarrier DSL applications.

In a **multicell scenario**, the weighted sum rate maximization problem can be written (in case the transmitted symbols of all the base stations are from Gaussian codebooks) as

$$\max_{\{\mathcal{N}_k, P_{k,n}\}_{c, 1 \le k \le K^c, n \in \mathcal{N}_k}} \sum_c \sum_{k \in c} \frac{\mu_k}{N} \sum_{n \in \mathcal{N}_k} \log\left(1 + P_{k,n} \frac{|H_k^c(n)|^2}{\sigma_k^2}\right),$$
subject to the OFDMA orthogonality constraint and to (8) ,

Here, μ_k is the weight assigned to user k. Since no exact solution has yet been found for the above problem, only suboptimal (with respect to the optimization criterion) approaches exist. The approach proposed in M. Pischella & J.-C. Belfiore (2008) consists in performing resource allocation via two phases: First, the users and subcarriers where the power should be set to zero are identified. This phase is done with the simplifying assumption of uniform power allocation. In the second phase, an iterative distributed algorithm called Dual Asynchronous Distributed Pricing (DADP) J. Huang et al. (2006) is applied for the remaining users under high SINR assumption.

3) Power minimization with individual rate constraints

Now assume that each user k has a data rate requirement equal to R_k in a **signel-cell scenario** (we focus on cell c). The subcarriers $\mathcal{N}_k$ and the transmit powers $\{P_{k,n}\}_{n \in \mathcal{N}_k}$ assigned to user k should thus be chosen such that the following constraint is satisfied:

$$R_k < C_k = \frac{1}{N} \sum_{n \in \mathcal{N}_k} \log\left(1 + P_{k,n}\frac{|H_k(n)|^2}{\sigma^2}\right), \tag{11}$$

and such that the total transmit power is minimal. Here, C_k is the maximal rate per channel use that can be achieved by user k when assigned $\mathcal{N}_k$ and $\{P_{k,n}\}_{n \in \mathcal{N}_k}$. This maximal rate is achieved for each user by using random Gaussian codebooks as in (7). The resource allocation problem can be formulated in this case as follows:

$$\min_{\{\mathcal{N}_k, P_{k,n}\}_{k \in c, n \in \mathcal{N}_k}} \sum_{k \in c} \sum_{n \in \mathcal{N}_k} P_{k,n}$$

such that the subcarrier assignment orthogonality constraint and (11) are satisfied.

Some approaches to solve this combinatorial optimization problem can be found in Kivanc et al. (2003). However, these approaches are heuristic and result in suboptimal solutions to the above problem.

In order to avoid the high computational complexity required for solving combinatorial optimization problems, one alternative consists in relaxing the subcarrier assignment constraint by introducing the notion of subcarrier time-sharing as in Wong et al. (1999). According to this notion, each subcarrier n can be orthogonally time-shared by more than one user, with each user k modulating the subcarrier during an amount of time proportional to $\gamma_{k,n}$. Here, $\{\gamma_{k,n}\}_{k,n}$ are real number from the interval $[0,1]$ satisfying

$$\forall n \in \{1, 2, \ldots, N\}, \sum_{k=1}^{K} \gamma_{k,n} \le 1. \tag{12}$$

The rate constraint of user k becomes

$$R_k < \sum_{n=1}^{N} \gamma_{k,n} \log\left(1 + P_{k,n}\frac{|H_k(n)|^2}{\sigma^2}\right). \tag{13}$$

The optimal value of the new resource allocation parameters can be obtained as the solution to the following optimization problem:

$$\min_{\{\gamma_{k,n}, P_{k,n}\}_{1 \le k \le K, 1 \le n \le N}} \sum_{k=1}^{K} \sum_{n=1}^{N} \gamma_{k,n} P_{k,n}$$
such that constraints (12) and (13) are satisfied.

The above problem can be easily transformed into a convex optimization problem by a simple change of variables. One can therefore use usual convex optimization tools to find its solution.

Remark 1. *It is worth mentioning here that the assumption of per-subcarrier full CSI at the transmitters is quite unrealistic in practice. First of all, it requires large amounts of feedback messages from the different users to their respective base stations, which is not practically possible in most*

real-world wireless communication systems. Even if the wireless system allows that amount of feedback, it is not clear yet whether the benefit obtained by this additional complexity would outweigh the additional costs due to the resulting control traffic Stańczak et al. (2009). For these reasons, most of the above mentioned resource allocation techniques which assume perfect CSI have not been adopted in practice.

Remark 2. *So far, it was assumed throughout the previous subsection that all the subcarriers $\{1, 2, \ldots, N\}$ are available to the users of each cell* i.e., *a frequency reuse of one is assumed. Resource allocation under fractional frequency reuse is addressed in the next subsection.*

4.2 Average-rate multicell resource allocation in the case of (statistical-CSI fast-fading channels)

Several works such as Brah et al. (2007; 2008); I.C. Wong & B.L. Evans (2009); Wong & Evans (2007) consider the problem of ergodic sum-rate and ergodic weighted sum-rate maximization in WiMAX-like networks. However, these works do not provide analytical solutions to these optimization problems. Instead, they resort to suboptimal (and rather computationally-complex) *duality techniques*. This is why we focus in the sequel on a **special case** of the average-rate resource allocation problem where closed-form characterization of the *optimal* solution has been provided in N. Ksairi & Ciblat (2011); N. Ksairi & Hachem (2010a;b). In particular, we highlight the methodology adopted in these recent works and which consists in using the single-cell results as a tool to solve the more involved multicell allocation problem.

Consider the the **downlink of a sectorized WiMAX cellular system composed of hexagonal cells** as shown in Figure 2. Assume that the **fractional frequency reuse (FFR) scheme** illustrated in Figure 4 is adopted. Due to this scheme, a certain subset of subcarriers $\mathcal{I} \subset \{1, 2, \ldots, N\}$ ($\mathcal{I}$ as in $\mathcal{I}$nterference) is reused in the three cells. If user k modulates such a subcarrier $n \in \mathcal{I}$, process $w_k(n, m)$ will contain both thermal noise and multicell interference. Recall the definition of the reuse factor α given by (5) as the ratio between the number of
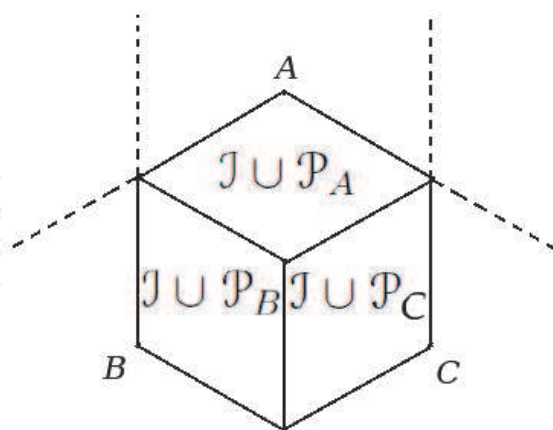

Fig. 4. Frequency reuse scheme

reused subcarriers and the total number of available subcarriers:

$$\alpha = \frac{\text{card}(\mathcal{I})}{N} .$$

Note that $\mathfrak{I}$ contains αN subcarriers. The remaining $(1 - \alpha)N$ subcarriers are shared by the three sectors in an orthogonal way, such that each base stations c has at its disposal a subset $\mathcal{P}_c$ ($\mathcal{P}$ as in $\mathcal{P}$rotected) of cardinality $\frac{1-\alpha}{3}N$. If user k modulates a subcarrier $n \in \mathcal{P}_c$, then process $w_k(n, m)$ will contain only thermal noise with variance σ^2. Finally,

$$\mathfrak{I} \cup \mathcal{P}_A \cup \mathcal{P}_B \cup \mathcal{P}_C = \{0, 1, \dots, N-1\}\,.$$

Also assume that **channel coefficients** $\{H_k^c(n, m)\}_{n \in \mathcal{N}_k}$ **are Rayleigh distributed and have the same variance** $\rho_k^c = \mathbb{E}\left[|H_k^c(n, m)|^2\right], \forall n \in \mathcal{N}_k$. This assumption is realistic in cases where the propagation environment is **highly scattering**, leading to decorrelated Gaussian-distributed time-domain channel taps. Under all the aforementioned assumptions, it can be shown that the ergodic capacity associated with each user k **only** depends on the **number of subcarriers** assigned to user k in subsets $\mathfrak{I}$ and $\mathcal{P}_c$ respectively, rather than on the **specific** subcarriers assigned to k.

The resource allocation parameters for user k are thus:

i) The sharing factors $\gamma_{k,\mathfrak{I}}$, $\gamma_{k,\mathcal{P}}$ defined by

$$\gamma_{k,\mathfrak{I}}^c = \text{card}(\mathfrak{I} \cap \mathcal{N}_k)/N \qquad \gamma_{k,\mathcal{P}}^c = \text{card}(\mathcal{P}_c \cap \mathcal{N}_k)/N\,. \tag{14}$$

ii) The powers $P_{k,\mathfrak{I}}$, $P_{k,\mathcal{P}}$ transmitted on the subcarriers assigned to user k in $\mathfrak{I}$ and $\mathcal{P}_c$ respectively.

We assume from now on that $\gamma_{k,\mathfrak{I}}$ and $\gamma_{k,\mathcal{P}}$ can take on any value in the interval $[0,1]$ (not necessarily integer multiples of $1/N$).

Remark 3. *Even though the sharing factors in our model are not necessarily integer multiples of $1/N$, it is still possible to practically achieve the exact values of $\gamma_{k,\mathfrak{I}}$, $\gamma_{k,\mathcal{P}}$ by simply exploiting the time dimension. Indeed, the number of subcarriers assigned to user k can be chosen to vary from one OFDM symbol to another in such a way that the* average *number of subcarriers in subsets $\mathfrak{I}$, $\mathcal{P}_c$ is equal to $\gamma_{k,\mathfrak{I}}N$, $\gamma_{k,\mathcal{P}}N$ respectively. Thus the fact that $\gamma_{k,\mathfrak{I}}$, $\gamma_{k,\mathcal{P}}$ are not strictly integer multiples of $1/N$ is not restrictive, provided that the system is able to grasp the benefits of the time dimension. The particular case where the number of subcarriers is restricted to be the same in each OFDM block is addressed in N. Ksairi & Ciblat (2011).*

The sharing factors of the different users should be selected such that

$$\sum_{k \in c} \gamma_{k,\mathfrak{I}} \leq \alpha \qquad \sum_{k \in c} \gamma_{k,\mathcal{P}} \leq \frac{1-\alpha}{3}\,. \tag{15}$$

We now describe the adopted model for the multicell interference. Consider one of the non protected subcarriers n assigned to user k of cell A in subset $\mathfrak{I}$. Denote by σ_k^2 the variance of the additive noise process $w_k(n, m)$ in this case. This variance is assumed to be constant w.r.t both n and m. It only depends on the position of user k and the average powers [4] $Q_{B,\mathfrak{I}} = \sum_{k \in B} \gamma_{k,\mathfrak{I}} P_{k,\mathfrak{I}}$ and $Q_{C,\mathfrak{I}} = \sum_{k \in C} \gamma_{k,\mathfrak{I}} P_{k,\mathfrak{I}}$ transmitted respectively by base stations B and C in $\mathfrak{I}$. This assumption is valid in OFDMA systems that adopt random subcarrier assignment

---

[4] The dependence of interference power on only the average powers transmitted by the interfering cells rather than on the power of each single user in these cells is called *interference averaging*

or frequency hopping (which are both supported in the WiMAX standard [5]). Finally, let σ^2 designate the variance of the thermal noise. Putting all pieces together:

$$
\mathbb{E}\left[|w_k(n,m)|^2\right] =
\begin{cases}
\sigma^2 & \text{if } n \in \mathcal{P}_c \\
\sigma_k^2 = \sigma^2 + \mathbb{E}\left[|H_k^B(n,m)|^2\right] Q_1^B + \mathbb{E}\left[|H_k^C(n,m)|^2\right] Q_1^C & \text{if } n \in \mathcal{I}
\end{cases}
\tag{16}
$$

where $H_k^B(n,m)$ (resp. $H_k^C(n,m)$) represents the channel between base station B (resp. C) and user k of cell A at subcarrier n and OFDM block m. Of course, the average channel gains $\mathbb{E}\left[|H_k^A(n,m)|^2\right]$, $\mathbb{E}\left[|H_k^B(n,m)|^2\right]$ and $\mathbb{E}\left[|H_k^C(n,m)|^2\right]$ depend on the position of user k via the path loss model.

Now, let $g_{k,\mathcal{I}}$ (resp. $g_{k,\mathcal{P}}$) be the channel Gain-to-Noise Ratio (GNR) for user k in band $\mathcal{I}$ (resp. $\mathcal{P}_c$), namely

$$
g_{k,\mathcal{I}}(Q_{B,\mathcal{I}}, Q_{C,\mathcal{I}}) = \frac{\rho_k}{\sigma_k^2(Q_{B,\mathcal{I}}, Q_{C,\mathcal{I}})} \qquad g_{k,\mathcal{P}} = \frac{\rho_k}{\sigma^2} ,
$$

where $\sigma_k^2(Q_{B,\mathcal{I}}, Q_{C,\mathcal{I}})$ is the variance of the noise-plus-interference process associated with user k given th interference levels generated by base stations B, C are equal to $Q_{B,\mathcal{I}}$, $Q_{C,\mathcal{I}}$ respectively.

The ergodic capacity associated with k in the whole band is equal to the sum of the ergodic capacities corresponding to both bands $\mathcal{I}$ and $\mathcal{P}_A$. For instance, the part of the capacity corresponding to the protected band $\mathcal{P}_A$ is equal to

$$
\gamma_{k,\mathcal{P}}\mathbb{E}\left[\log\left(1 + P_{k,\mathcal{P}}\frac{|H_k^A(n,m)|^2}{\sigma^2}\right)\right] ,
$$

where factor $\gamma_{k,\mathcal{P}}$ traduces the fact that the capacity increases with the number of subcarriers which are modulated by user k. In the latter expression, the expectation is calculated with respect to random variable $\frac{|H_k^A(m,n)|^2}{\sigma^2}$. Now, $\frac{|H_k^A(m,n)|^2}{\sigma^2}$ has the same distribution as $\frac{\rho_k}{\sigma^2}Z = g_{k,\mathcal{P}}Z$, where Z is a standard exponentially-distributed random variable. Finally, the ergodic capacity in the whole bandwidth is equal to

$$
\begin{aligned}
C_k(\gamma_{k,\mathcal{I}}, \gamma_{k,\mathcal{P}}, P_{k,\mathcal{I}}, P_{k,\mathcal{P}}, Q_{B,\mathcal{I}}, Q_{C,\mathcal{I}}) = \\
\gamma_{k,\mathcal{I}}\mathbb{E}\left[\log\left(1 + g_{k,\mathcal{I}}(Q_{B,\mathcal{I}}, Q_{C,\mathcal{I}})P_{k,\mathcal{I}}Z\right)\right] + \gamma_{k,\mathcal{P}}\mathbb{E}\left[\log\left(1 + g_{k,\mathcal{P}}P_{k,\mathcal{P}}Z\right)\right] .
\end{aligned}
\tag{17}
$$

Assume that user k has an *average* rate requirement R_k (nats/s/Hz). This requirement is satisfied provided that R_k is less that the ergodic capacity C_k i.e.,

$$
R_k < C_k(\gamma_{k,\mathcal{I}}, \gamma_{k,\mathcal{P}}, P_{k,\mathcal{I}}, P_{k,\mathcal{P}}, Q_{B,\mathcal{I}}, Q_{C,\mathcal{I}}) .
\tag{18}
$$

Finally, the quantity Q_c defined by

$$
Q_c = \sum_{k \in c}(\gamma_{k,\mathcal{I}}P_{k,\mathcal{I}} + \gamma_{k,\mathcal{P}}P_{k,\mathcal{P}})
\tag{19}
$$

---

[5] In WiMAX, one of the types of subchannelization *i.e.*, grouping subcarriers to form a subchannel, is *diversirty permutation*. This method draws subcarriers pseudorandomly, thereby resulting in interference averaging as explained in Byeong Gi Lee & Sunghyun Choi (2008)

denotes the average power spent by base station c during one OFDM block.

$\mathfrak{I}$	subset of reused subcarriers that are subject to multicell interference
$\mathcal{P}_c$	subset of interference-free subcarriers that are exclusively reserved for cell c
R_k	rate requirement of user k in nats/s/Hz
C_k	ergodic capacity associated with user k
$g_{k,\mathfrak{I}}, g_{k,\mathcal{P}}$	GNR of user k in bands $\mathfrak{I}$, $\mathcal{P}_A$ resp.
$\gamma_{k,\mathfrak{I}}, \gamma_{k,\mathcal{P}}$	sharing factors of user k in bands $\mathfrak{I}$, $\mathcal{P}_c$ resp.
$P_{k,\mathfrak{I}}, P_{k,\mathcal{P}}$	power allocated to user k in bands $\mathfrak{I}$, $\mathcal{P}_A$ resp.
$Q_{c,\mathfrak{I}}, Q_{c,\mathcal{P}}$	power transmitted by base station c in bands $\mathfrak{I}$, $\mathcal{P}_A$ resp.
Q_c	total power transmitted by base station c

Table 1. Some notations for cell c

Optimization problem

The joint resource allocation problem that we consider consists in minimizing the power that should be spent by the three base stations A, B, C in order to satisfy all users' rate requirements:

$$\min_{\{\gamma_{k,\mathfrak{I}},\gamma_{k,\mathcal{P}},P_{k,\mathfrak{I}},P_{k,\mathcal{P}}\}_{k=1\ldots K}} \sum_{c=A,B,C} \sum_{k\in c} \gamma_{k,\mathfrak{I}}P_{k,\mathfrak{I}} + \gamma_{k,\mathcal{P}}P_{k,\mathcal{P}}$$

$$\text{subject to constraints (15) and (18)} . \qquad (20)$$

This problem is not convex with repsect to the resource allocation parameters. It cannot thus be solved using convex optimization tools. Fortunately, it has been shown in N. Ksairi & Ciblat (2011) that a resource allocation algorithm can be proposed that is **asymptotically optimal** *i.e.*, the transmit power it requires to satisfy users' rate requirements is equal to the transmit power of an **optimal** solution to the above problem **in the limit of large numbers of users**. We present in the sequel this allocation algorithm, and we show that it can be implemented in a distributed fashion and that it has relatively low computational complexity.

Practical resource allocation scheme

In the proposed scheme we force the users near the cell's borders (who are normally subject to sever fading conditions and to high levels of multicell interference) to modulate uniquely the subcarriers in the protected subset $\mathcal{P}_c$, while we require that the users in the interior of the cell (who are closer to the base station and suffer relatively low levels on intercell interference) to modulate uniquely subcarriers in the interference subset $\mathfrak{I}$.

Of course, we still need to define a **separating curve** that split the users of the cell into these two groups of interior and exterior users. For that sake, we define on $\mathbb{R}_+^5 \times \mathbb{R}$ the function

$$(\theta, x) \mapsto d_\theta(x)$$

where $x \in \mathbb{R}$ and where θ is a set of parameters[6]. We use this function to define the separation curves d_{θ^A}, d_{θ^B} and d_{θ^C} for cells A, B and C respectively. The determination of parameters θ^A, θ^B and θ^C is discussed later on. Without any loss of generality, let us now focus on cell A. For

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[6] The closed-form expression of function $d_\theta(x)$ is provided in N. Ksairi & Ciblat (2011).

a given user k in this cell, we designate by (x_k, y_k) its coordinates in the Cartesian coordinate system whose origin is at the position of base station A and which is illustrated in Figure 5. In
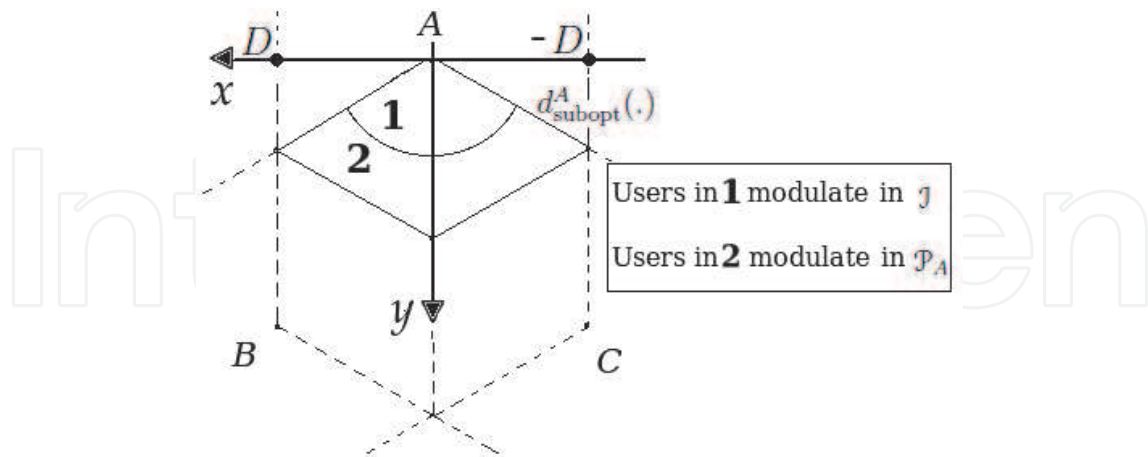

Fig. 5. Separation curve in cell A

the proposed allocation scheme, user k modulates in the interference subset $\mathfrak{I}$ if and only if

$$y_k < d_{\theta^A}(x_k).$$

Inversely, the user modulates in the interference-free subset $\mathcal{P}_A$ if and only if

$$y_k \geq d_{\theta^A}(x_k)$$

Therefore, we have defined in each sector two geographical regions: the first is around the base station and its users are subject to multicell interference; the second is near the border of the cell and its users are protected from multicell interference.

The resource allocation parameters $\{\gamma_{k,\mathcal{P}}, P_{k,\mathcal{P}}\}$ for the users of the three protected regions can be easily determined by solving three **independent** convex resource allocation problems. In solving these problems, there is no interaction between the three sectors thanks to the absence of multicell interference for the protected regions. The closed-form solution to these problems is given in N. Ksairi & Ciblat (2011).

However, the resource allocation parameters $\{\gamma_{k,\mathfrak{I}}, P_{k,\mathfrak{I}}\}$ of users of the non-protected interior regions should be **jointly** optimized in the three sectors. Fortunately, a **distributed iterative** algorithm is proposed in N. Ksairi & Ciblat (2011) to solve this joint optimization problem. This iterative algorithm belongs to the family of **best dynamic response** algorithms. At each iteration, we solve in each sector a single-cell allocation problem given a fixed level of multicell interference generated by the other two sectors in the previous iteration. The mild conditions for the convergence of this algorithm are provided in N. Ksairi & Ciblat (2011). Indeed, it is shown that the algorithm converges for all realistic average data rate requirements provided that the separating curves are carefully chosen as will be discussed later on.

Determination of the separation curves and asymptotic optimality of the proposed scheme

It is obvious that the above proposed resource allocation algorithm is **suboptimal** since it forces a "binary" separation of users into protected and non-protected groups. Nonetheless, it has been proved in N. Ksairi & Ciblat (2011) that this binary separation is **asymptotically**

optimal in the sense that follows. Denote by $Q_{\text{subop}}^{(K)}$ the total power spent by the three base stations if this algorithm is applied. Also define $Q_T^{(K)}$ as the total transmit power of an optimal solution to the original joint resource allocation problem. The suboptimality of the proposed resource allocation scheme trivially implies

$$Q_{\text{subop}}^{(K)} \geq Q_T^{(K)}$$

The asymptotic behaviour of both $Q_{\text{subop}}^{(K)}$ and $Q_T^{(K)}$ as $K \to \infty$ has been studied[7] in N. Ksairi & Ciblat (2011). In the asymptotic regime, it can be shown that the configuration of the network, as far as resource allocation is concerned, is completely determined by i) the **average** (as opposed to **individual**) data rate requirement $\bar{r}$ and ii) a function $\lambda(x,y)$ that characterizes the asymptotic "density" of users' geographical positions in the coordination system (x,y) of their respective sectors. To better understand the physical meaning of the density function $\lambda(x,y)$, note that it is a constant function in the case of uniform distribution of users in the cell area..

Interestingly, one can find values for parameters θ^A, θ^B and θ^C (characterizing the separatin curves d_{θ^A}, d_{θ^B}, and d_{θ^C} respectively) that i) depend only on the average rate requirement $\bar{r}$ and on the asymptotic geographical density of users and ii) which satisfy

$$\lim_{K \to \infty} Q_{\text{subopt}}^{(K)} = \lim_{K \to \infty} Q_T^{(K)} \stackrel{(\text{def})}{=} Q_T .$$

In other words, one can find separating curves d_{θ^A}, d_{θ^B}, and d_{θ^C} such that the proposed suboptimal allocation algorithm is **asymptotically optimal** in the limit of large numbers of users. We plot in Figure 6 these asymptotically optimal separating curves for several values of the average data rate requirement [8]. The performance of the proposed algorithm *i.e.,* its total
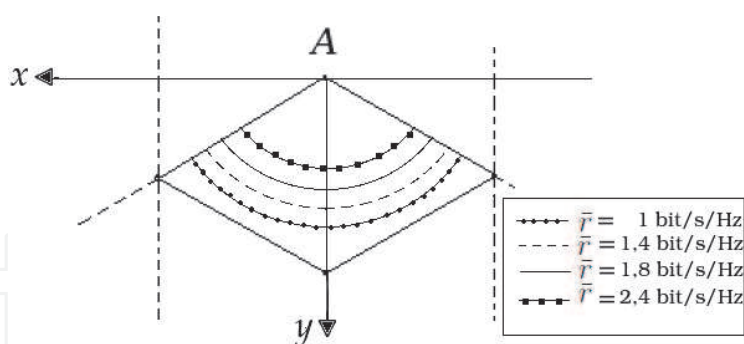

Fig. 6. Asymptotically optimal separating curves

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[7] In this asymptotic analysis, a technical detail requires that we also let the total bandwidth B (Hz) occupied by the system tend to infinity in order to satisfy the sum of users' rate requirements $\sum_{k=1}^{K} r_k$ which grows to infinity as $K \to \infty$. Moreover, in order to obtain relevant results, we assume that as K, B tends to infinity, their ratio B/K remains constant

[8] In all the given numerical and graphical results, it has been assumed that the radius of the cells is equal to $D = 500$m. The path loss model follows a Free Space Loss model (FSL) characterized by a path loss exponent $s = 2$. The carrier frequency is $f_0 = 2.4GHz$. At this frequency, path loss in dB is given by $\rho_{dB}(x) = 20\log_{10}(x) + 100.04$, where x is the distance in kilometers between the BS and the user. The signal bandwidth B is equal to 5 MHz and the thermal noise power spectral density is equal to $N_0 = -170$ dBm/Hz.

transmit power when the asymptotically optimal separating curves are used, is compared in Figure 7 to the performance of an all-reuse scheme ($\alpha = 1$) that has been proposed in Thanabalasingham et al. (2006). It is worth mentioning that the reuse factor α assumed for our algorithm in Figure 7 has been obtained using the procedure described in Section 5. It is clear from the figure that a significant gain in performance can be obtained from applying a carefully designed FFR allocation algorithm (such as ours) as compared to an all-reuse scheme. The above comparison and performance analysis is done assuming a 3-sector network. This
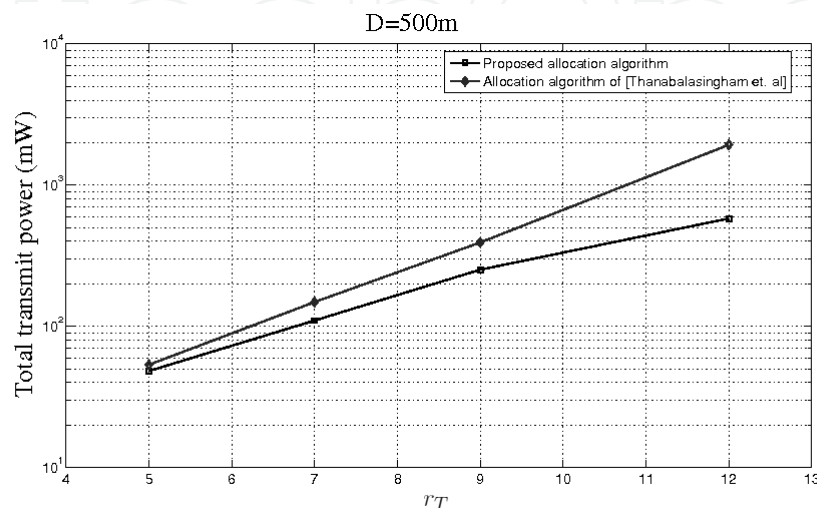
Fig. 7. Performance of the proposed algorithm vs total rate requirement per sector compared to the all-reuse scheme of Thanabalasingham et al. (2006)

assumption is valid provided that the intercell interference in one sector is mainly due to only the two nearest base stations. If this assumption is not valid (as in the 21-sector network of Figure 8), the performance of the proposed scheme will of course deteriorates as can be seen in Figure 9. The same figure shows that the proposed scheme still performs better than an all-reuse scheme, especially at high data rate requirements.

4.3 Outage-based resource allocation (statistical-CSI slow-fading channels)

Recall from Section 2 that the relevant performance metric in the case of slow-fading channels is the outage probability $P_{O,k}$ given by (3) (in the case of Gaussian codebooks and Gaussian-distributed noise-plus-interference process) as

$$P_{O,k}(R_k) \triangleq \Pr\left[\frac{1}{N}\sum_{n\in\mathcal{N}_k} \log\left(1 + P_{k,n}\frac{|H_k^c(n)|^2}{\sigma_k^2}\right) \leq R_k\right].$$

Where R_k is the rate (in nats/s/Hz) at which data is transmitted to user k. Unfortunately, no closed-form expression exists for $P_{O,k}(R_k)$. The few works on outage-based resource allocation for OFDMA resorted to **approximations** of the probability $P_{O,k}(R_k)$.

For example, consider the problem of maximizing the sum of users' data rates R_k under a total power constraint $P_{\max}$ such that the outage probability of each user k does not exceed a certain threshold ϵ_k:
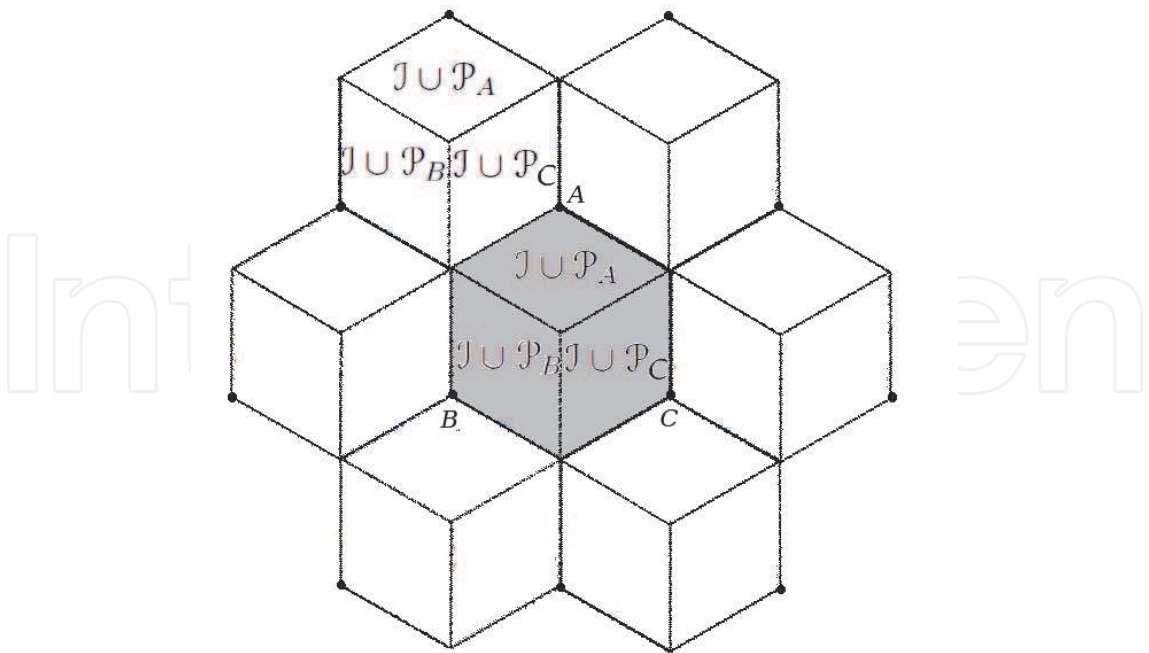
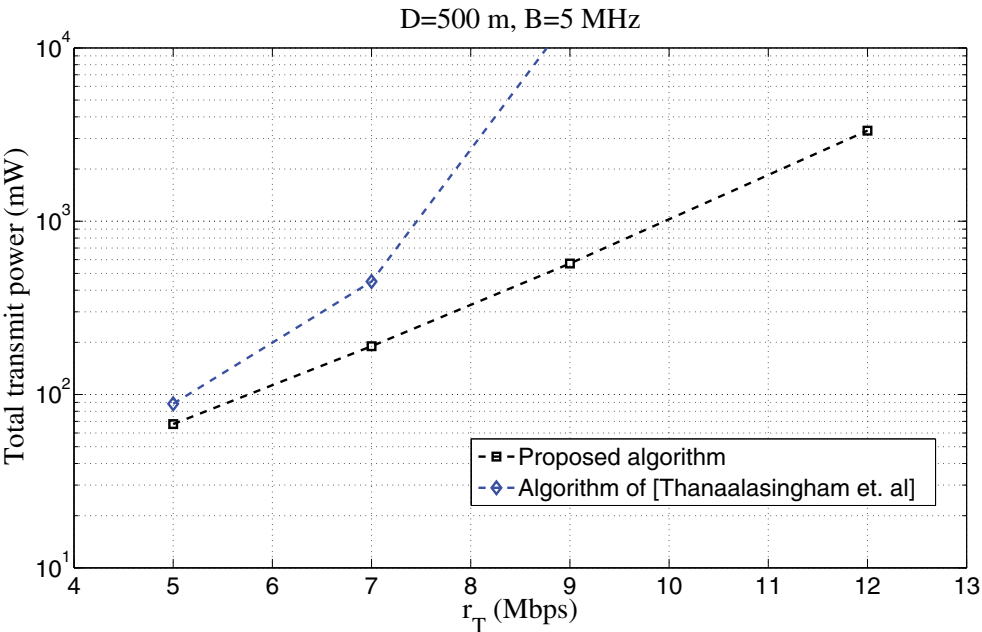Fig. 8. 21-sector system model and the frequency reuse scheme

Fig. 9. Comparison between the proposed allocation algorithm and the all-reuse scheme of Thanabalasingham et al. (2006) in the case of 21 sectors (25 users per sector) vs the total rate requirement per sector

$$\max_{\{\mathcal{N}_k, P_{k,n}\}_{1 \le k \le K, n \in \mathcal{N}_k}} \sum_c \sum_{k \in c} R_k$$

subject to the OFDMA orthogonality constraint and to (8) and $P_{O,k}(R_k) \le \epsilon_k$.   (21)

In M. Pischella & J.-C. Belfiore (2009), the problem is tackled in the context of MIMO-OFDMA systems where both the base stations and the users' terminals have multiple antennas. In the approach proposed by the authors to solve this problem, **the outage probability is replaced with an approximating function**. Moreover, subcarrier assignment is performed **independently** (and thus suboptimally) in each cell assuming equal power allocation and equal interference level on all subcarriers. Once the subcarrier assignment is determined, multicell power allocation *i.e.,* the determination of $P_{k,n}$ for each user k is done thanks to an iterative allocation algorithm. Each iteration of this algorithm consists in solving the power allocation problem separately in each cell based on the current level of multicell interference. The result of each iteration is then used to update the value of multicell interference for the next iteration of the algorithm. The convergence of this iterative algorithm is also studied by the authors. A solution to Problem (21) which performs **joint** optimization of subcarrier assignment and power allocation is yet to be provided.

In S. V. Hanly et al. (2009), a min-max outage-based multicell resource allocation problem is solved assuming that there exists a genie who can instantly return the outage probability of any user as a function of the power levels and subcarrier allocations in the network. When this restricting assumption is lifted, only a suboptimal solution is provided by the authors.

4.4 Resource allocation for real-world WiMAX networks: Practical considerations

- All the resource allocation schemes presented in this chapter assume that the transmit symbols are from Gaussian codebooks. This assumption is widely made in the literature, mainly for tractability reasons. In real-world WiMAX systems, Gaussian codebooks are not practical. Instead, discrete modulation (*e.g.* QPSK,16-QAM,64-QAM) is used. The adaptation of the presented resource allocation schemes to the case of dynamic Modulation and Coding Schemes (MCS) supported by WiMAX is still an open area of research that has been addressed, for example, in D. Hui & V. Lau (2009); G. Song & Y. Li (2005); J. Huany et al. (2005); R. Aggarwal et al. (2011).

- The WiMAX standard provides the necessary signalling channels (such as the CSI feedback messages (CQICH, REP-REQ and REP-RSP) and the control messages DL-MAP and DCD) that can be used for resource allocation, as explained in Byeong Gi Lee & Sunghyun Choi (2008), but does not oblige the use of any specific resource allocation scheme.

- The smallest unity of band allocation in WiMAX is **subchannels** (A subchannel is a group of subcarriers) **not** subcarriers. Moreover, WiMAX supports transmitting with different powers and different rates (MCS schemes) on different subchannels as explained in Byeong Gi Lee & Sunghyun Choi (2008). This implies that the per-subcarrier full-CSI schemes presented in Subsection 4.1 are not well adapted for WiMAX systems. They should thus be first modified to per-subchannel schemes before use in real-world WiMAX networks. However, the average-rate statistical-CSI schemes of Subsection 4.2 are compatible with the subchannel-based assignment capabilities of WiMAX.

5. Optimization of the reuse factor for WiMAX networks

The selection of the frequency reuse scheme is of crucial importance as far as cellular network design is concerned. Among the schemes mentioned in Section 3, fractional frequency reuse (FFR) has gained considerable interest in the literature and has been explicitly recommended

for WiMAX in WiMAX Forum (2006), mostly for its simplicity and for its promising gains. For these reasons, we give special focus in this chapter to this reuse scheme.

Recall from Section 3 that the principal parameter characterizing FFR is the *frequency reuse factor α*. The determination of a relevant value *α* for the this factor is thus a key step in optimizing the network performance. The definition of an *optimal* reuse factor requires however some care. For instance, the reuse factor should be fixed in practice prior to the resource allocation process and its value should be independent of the particular network configuration (such as the changing users' locations, individual QoS requirements, etc).

A solution adopted by several works in the literature consists in performing system level *simulations* and choosing the corresponding value of *α* that results in the best average performance. In this context, we cite M. Maqbool et al. (2008), H. Jia et al. (2007) and F. Wang et al. (2007) without being exclusive. A more interesting option would be to provide *analytical* methods that permit to choose a relevant value of the reuse factor.

In this context, A promising analytical approach adopted in recent research works such as Gault et al. (2005); N. Ksairi & Ciblat (2011); N. Ksairi & Hachem (2010b) is to resort to **asymptotic analysis** of the network in the limit of **large number of users**. The aim of this approach is to obtain optimal values of the reuse factor that no longer depend on the particular configuration of the network *e.g.,* the exact positions of users, their single QoS requirements, etc, but rather on an asymptotic, or "average", state of the network *e.g.,* density of users' geographical distribution, average rate requirement of users, etc.

In order to illustrate this concept of *asymptotically optimal* values of the reuse factor, we give the following example that is taken from N. Ksairi & Ciblat (2011); N. Ksairi & Hachem (2010b). Consider the resource allocation problem presented in Section 4.2 and which consists in minimizing the total transmit power that should be spent in a 3-sector [9] WiMAX network using the FFR scheme with reuse factor *α* such that all users' *average* (i.e. *ergodic*) *rate* requirements r_k (nats/s) are satisfied (see Figure 10). Denote by $Q_T^{(K)}$ the total transmit power spent by the three base stations of the network when the **optimal** solution (see Subsection 4.2) to the above problem is applied. We want to study the behaviour of $Q_T^{(K)}$ as the number *K* of users tends to infinity [10]. As we already stated, the following holds under mild assumptions:

1. the asymptotic configuration of the network, as far as resource allocation is concerned, is completely characterized by i) the **average** (as opposed to **individual**) data rate requirements $\bar{r}$ and ii) a function $\lambda(x, y)$ that characterizes the asymptotic density of users' geographical positions in the coordination system (x, y) of their respective cells.

2. the optimal total transmit power $Q_T^{(K)}$ tends as $K \to \infty$ to a value Q_T that is given in closed form in N. Ksairi & Ciblat (2011):

$$\lim_{K \to \infty} Q_T^{(K)} \overset{(\text{def})}{=} Q_T \,.$$

---

[9] The restriction of the model to a network composed of only 3 neighboring cells is for tractability reasons. This simplification is justified provided that multicell interference can be considered as mainly due to the two nearest neighboring base stations.

[10] As stated earlier, we also let the total bandwidth *B* (Hz) occupied by the system tend to infinity such that the ratio B/K remain constant
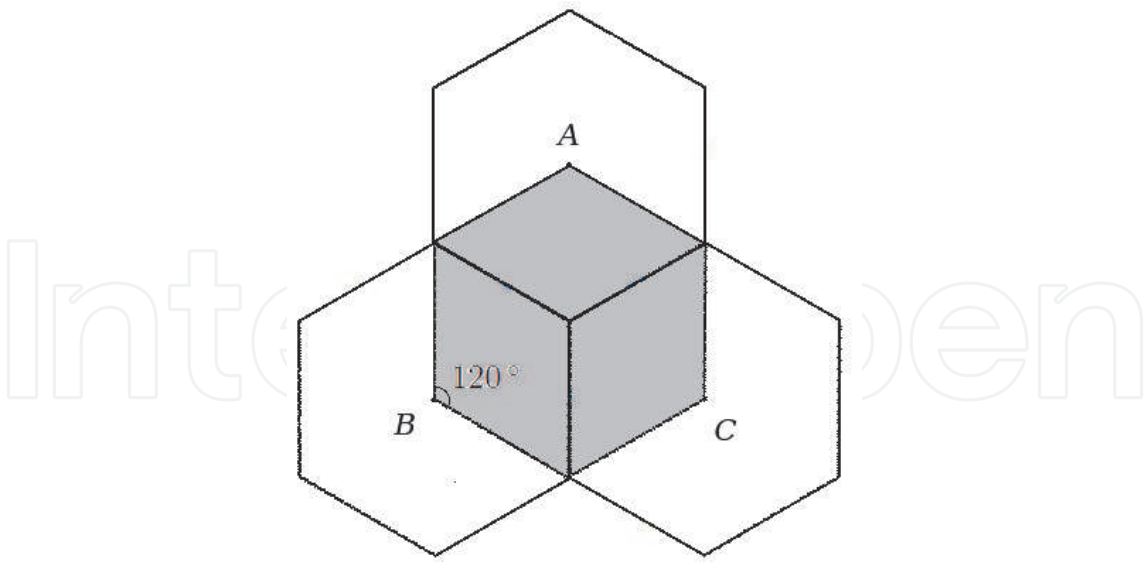
Fig. 10. 3-sectors system model

It is worth noting that the limit value Q_T **only** depends on i) the above-mentioned asymptotic state of the network *i.e.*, on the average rate $\bar{r}$ and on the asymptotic geographical density λ and ii) on the value of the reuse factor α.

It is thus reasonable to select the value α_{opt} of the reuse factor as

$$\alpha_{opt} = \arg\min_{\alpha} \lim_{K\to\infty} Q_T^{(K)}(\alpha).$$

In practice, we propose to compute the value of $Q_T = Q_T(\alpha)$ for several values of α on a grid in the interval $[0,1]$. In Figure 11, α_{opt} is plotted as function of the average data rate requirement $\bar{r}$ for the case of a network composed of cells with radius $D = 500m$ assuming uniformly distributed users' positions. Also note that complexity issues are of few importance, as
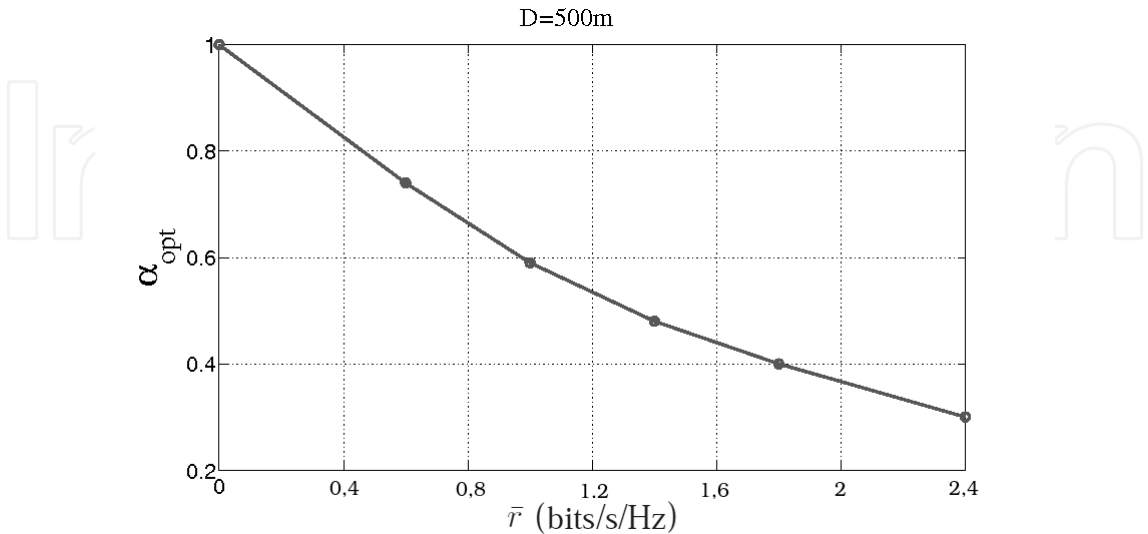
Fig. 11. Asymptotically optimal reuse factor vs average rate requirement. Source:N. Ksairi & Ciblat (2011)

the optimization is done prior to the resource allocation process. It does not affect the complexity of the global resource allocation procedure. It has been shown in N. Ksairi & Ciblat (2011); N. Ksairi & Hachem (2010b) that significant gains are obtained when using the asymptotically-optimal value of the reuse factor instead of an arbitrary value, even for moderate numbers of users.

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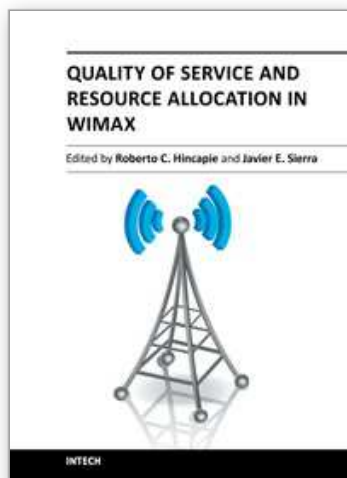
Quality of Service and Resource Allocation in WiMAX

Edited by Dr. Roberto Hincapie

This book has been prepared to present state of the art on WiMAX Technology. It has been constructed with the support of many researchers around the world, working on resource allocation, quality of service and WiMAX applications. Such many different works on WiMAX, show the great worldwide importance of WiMAX as a wireless broadband access technology. This book is intended for readers interested in resource allocation and quality of service in wireless environments, which is known to be a complex problem. All chapters include both theoretical and technical information, which provides an in depth review of the most recent advances in the field for engineers and researchers, and other readers interested in WiMAX.

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