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# Resonant Effects of Quantum Electrodynamics in the Pulsed Light Field

Sergei P. Roshchupkin, Alexandr A. Lebed', Elena A. Padusenko  
and Alexey I. Voroshilo  
*Institute of Applied Physics, NASU  
Ukraine*

## 1. Introduction

Studying of various aspects of laser field influence on physical processes is one of the most topical problems of modern applied and fundamental physics. Scientific interest is due to numerous unknown before phenomena, which are caused by laser radiation application and make enable coming to the main point of atomic and molecular structure of matter. These phenomena are of great importance over such fields of physics as holography, fiberglass optics, telecommunications, material authority, biophysics, plasma physics, nuclear fusion and so on. The lasers which generate radiation within the range from deep infrared to ultraviolet one and even the soft X-rays region with intensities up to  $10^{22}$  W/cm<sup>2</sup> inclusive are made accessible at present. The sources of laser radiation had been put into practice of modern experiment widespread owing to its unique properties. The laser physics progress is generally concentrated on ever shorter and more powerful laser pulses production and on application of the lasts into various fields of scientific studies. New experimental conditions require continual improvements in computations and development of model of external field description.

Influence of laser field on kinematics and cross-sections of various quantum electrodynamics (QED) processes of the both first and second orders in the fine structure constant has been an object of study over a long period of time already. The characteristic feature of electrodynamics processes of the second order in the fine-structure constant in a laser field is associated with the possibility of their nonresonant and resonant modes. At this rate resonant cross-sections of scattering of particles may exceed the corresponding ones in external field absence in several orders of magnitude. Resonant character relates to the fact that lower-order processes, such as spontaneous emission or one-photon production and annihilation of electron-positron pairs, are allowed in the field of a light wave. Therefore, within a certain range of energy and momentum values a particle in an intermediate state may fall within the mass shell. Then the considered higher-order process effectively decomposes into two consecutive lower-order processes. Occurrence of resonances in a laser field is one of the fundamental problems of QED in strong fields.

Theoretical study of QED processes in an external laser field basis on solutions of the Dirac's equation for an electron in the field of a plane electromagnetic wave namely the Volkov functions (Volkov (1935)). Also one has to use the Green function of an intermediate particle in

a plane wave field when studying processes of the second order in the fine structure constant. The analytical expression of the Green function was obtained (Schwinger (1951); Brown & Kibble (1964)).

Several of significant reviews are already devoted to studying of QED processes in the field of a plane monochromatic wave. The review Nikishov & Ritus (1979) is to be mentioned as one of the earliest works, in which first order processes in the field of a plane electromagnetic wave are studied generally. Processes of an electron scattered by an atom and a multiphoton ionization were considered in Ehlotsky et al. (1998). Theoretical studies of resonant and coherent effects of QED in light field were systematized in the monograph Roshchupkin & Voroshilo (2008) and several QED processes in strong field were reviewed by Ehlotsky et al. (2009).

Detailed consideration of resonant processes in the field of a plane monochromatic wave was fulfilled by Roshchupkin (1996). It is necessary to emphasize, that, when the resonance conditions are satisfied, the amplitude of process of particles scattering in the field of a plane monochromatic wave becomes infinite nominally. The infinity is eliminated by introducing of radiative corrections into Green's function of an intermediate particle according to the Breit-Wigner prescription under consideration as usual. The resonant peak altitude is determined by the lifetime of a particle in the intermediate state (Oleinik (1967)).

Since 1996 experiments of verification of QED in strong fields had been started at SLAC National Accelerator Laboratory (Bula et al. (1996); Burke et al. (1997)) along with theoretical study. The earliest results were related to studying of photon emission by an electron in a collision with laser pulse and photoproduction of electron-positron pairs by a gamma-quantum in the field of a laser. Verification of QED in strong pulsed fields is also expected in the frame of the wide-ranging FAIR project (Darmstadt, Germany). Within the FAIR project the laser facility PHELIX was developed and constructed. It enables to get laser pulses with power up to petawatt range. The earliest experiments at this laser facility have been put into practice (Bagnoud et al. (2009)).

The present paper reviews studies of a number of resonant processes in the field of an intense pulsed laser. The earliest studies on spontaneous bremsstrahlung of an electron in a collision with a laser pulse and photoproduction of electron-positron pairs by a high-energy photon in the pulsed field were performed by Narozhniy & Fofanov (1996). Second order processes in the pulsed fields which allow resonances were analytically studied for the case of moderately strong field (Lebed' & Roshchupkin (2010); Padusenko & Roshchupkin (2010); Lebed' & Roshchupkin (2011); Voroshilo et al. (2011)). These works were performed in recent several years therefore the systematization and generalization of them is definitely significant. It is important to underline that resonant divergences in amplitudes of studied processes are eliminated in a consistent manner due to account of a pulsed character of the external field in mentioned works.

Amplitude of a field strength of intense ultra short laser pulses changes greatly in space and time. Thus, description of the external field as a plane monochromatic wave becomes the problematic one. The external field is modeled usually as a plane quasi-monochromatic wave for description of interaction of particles with a pulsed laser field when the characteristic

number of the external field oscillations in an electromagnetic pulse  $N$  is large:

$$N = \frac{\omega\tau}{2\pi} \gg 1, \quad (1)$$

where  $\omega$  is the characteristic frequency of wave field oscillation,  $\tau$  is the characteristic pulse duration. Quantity  $\tau$  can approach a value of even tens of femtoseconds for fields within the optical range of frequency, thus the condition (1) is satisfied for the majority of modern intense pulsed lasers. Fields are named the quasi-monochromatic ones when the condition (1) is satisfied.

Hereinafter we consider the external electromagnetic pulse as a plane electromagnetic elliptically polarized wave propagating along  $z$ -axis with the four-potential (Narozhniy & Fofanov (1996))

$$A(\varphi) = g\left(\frac{\varphi}{\omega\tau}\right) \cdot \frac{cF_0}{\omega} (e_x \cos \varphi + \delta e_y \sin \varphi), \quad \varphi = (kx) = \omega t - \mathbf{k}\mathbf{x}, \quad (2)$$

where  $\varphi$  is the wave phase;  $c$  is the velocity of light in vacuum,  $F_0$  is the strength of a wave electric field in a pulse peak,  $k = (\omega/c, \mathbf{k})$  is the wave four-vector;  $\delta$  is the wave ellipticity parameter ( $\delta = 0$  corresponds to the linear polarization case,  $\delta = \pm 1$  corresponds to the circular polarization case);  $e_x = (0, \mathbf{e}_x)$ ,  $e_y = (0, \mathbf{e}_y)$  are the wave polarization four-vectors meeting the standard conditions:  $e_x^2 = e_y^2 = -1$ ,  $(e_x k) = (e_y k) = 0$ . The function  $g(\varphi/\omega\tau)$  is the envelope function of the external wave four-potential that allows to take into account the pulsed character of a laser field. Generally it is chosen to be equal to unity in the center of a pulse and to decrease exponentially when  $|\varphi| \gg \omega\tau$ . Thus, in this case it is possible to consider the parameter  $\tau$  as the laser pulse characteristic duration.

Nonlinear effects in the processes of interaction of particles with the field of wave are governed by the classical relativistic-invariant parameter

$$\eta_0 = \frac{eF_0\lambda}{mc^2}. \quad (3)$$

Its value equals to the ratio of work done by the field at the wavelength to the particle rest energy. The parameter (3) is one of the most important characteristics of the external field and means the velocity of particle oscillation in the field of a wave in the case  $\eta_0 \ll 1$ . Multiphoton processes occurring when particles interact in a light field are characterized also by the Bunkin–Fedorov quantum parameter (Bunkin & Fedorov (1966))

$$\gamma_0 = \eta_0 \frac{mvc}{\hbar\omega}. \quad (4)$$

In the Eqs. (3), (4)  $e$  and  $m$  are the electron charge and mass, respectively;  $\lambda = c/\omega$  is the characteristic wavelength,  $v$  is the particle velocity. The multiphoton parameters (3), (4) vary considerably with external field intensity. Thus, within the range of optical frequencies ( $\omega \sim 10^{15} \text{ s}^{-1}$ ) the classical parameter  $\eta_0 \sim 1$  when  $F_0 \sim 10^{10} \div 10^{11} \text{ V/cm}$ , and  $\gamma_0 \sim 1$  when  $F_0 \sim (10^5 \div 10^6) (c/v) \text{ V/cm}$ . Consequently, we study all the processes within the range of moderately strong field when the considered parameters obey the following conditions:

$$\eta_0 \ll 1, \quad \gamma_0 \gtrsim 1. \quad (5)$$

The relativistic system of units, where  $\hbar = c = 1$  and the standard metric for 4-vectors  $(ab) = a_0 b^0 - \mathbf{a} \mathbf{b}$  will be used throughout this paper.

## 2. Resonant spontaneous bremsstrahlung of an electron scattered by a nucleus in the field of a pulsed light wave

We consider in this section the problem of spontaneous bremsstrahlung (SB) of an electron scattered by a nucleus in the external field of a pulsed light wave (see Fig. 1). Studying of SB when an electron is scattered by a nucleus or by an atom in presence of an external electromagnetic field has had a long-standing interest. Analytic expressions for the radiation spectrum of SB in a plane monochromatic wave in the nonrelativistic case have been derived by Karapetian & Fedorov (1978) for any atomic potential field in the Born approximation and by Zhou & Rosenberg (1993) for a short-range potential in the low-frequency approximation. Resonant SB of a nonrelativistic electron scattered by a nucleus in a plane-wave field was studied by Lebedev (1972). Borisov et al. (1980) considered resonant SB, which accompanies collisions of ultrarelativistic electrons for the case of large transferred momenta. In the general relativistic case the problem of electron-nucleus SB in the field of a plane monochromatic wave was studied by Roshchupkin (1985). It should be noted that the theory of SB in presence of an external field is also developed in Lötstedt et al. (2007); Schnez et al. (2007). They contain important numeric calculations for the case of a strong field. Nonresonant SB in a pulsed field was considered by Lebed' & Roshchupkin (2009). Resonant SB of an electron scattered by a nucleus in the field of a pulsed light wave was studied in the general relativistic case (Lebed' & Roshchupkin (2010)).

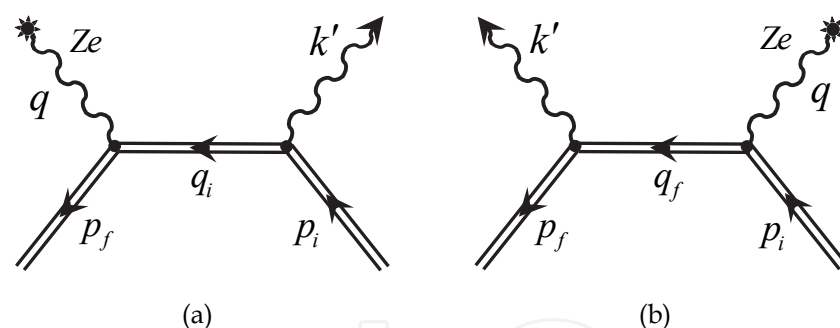


Fig. 1. Feynman diagrams of electron-nucleus SB in the field of a pulsed light wave. Incoming and outgoing double lines correspond to the Volkov functions of an electron in initial and final states; inner lines designate the Green function of an electron in a pulsed field. Wavy lines correspond to four-momenta of spontaneous photon and “pseudophoton” of nucleus recoil.

### 2.1 Amplitude of resonant spontaneous bremsstrahlung

The process of electron-nucleus SB in a pulsed light field (2) in the Born approximation on interaction of an electron with a nucleus, which corresponds to the transition of an electron from an initial state with the four-momentum  $p_i = (E_i, \mathbf{p}_i)$  into a final state with the four-momentum  $p_f = (E_f, \mathbf{p}_f)$ , is described by two Feynman diagrams (Fig. 1).

The  $S$ -matrix element is given by

$$S_{fi} = -ie^2 \int d^4x_1 d^4x_2 \bar{\psi}_f(x_2|A) \left[ \tilde{\gamma}_0 A_0(|\mathbf{x}_2|) G(x_2 x_1|A) \hat{A}'(x_1, k') + \hat{A}'(x_2, k') G(x_2 x_1|A) \tilde{\gamma}_0 A_0(|\mathbf{x}_1|) \right] \psi_i(x_1|A). \quad (6)$$

Here,  $\psi_i(x_1|A)$  and  $\bar{\psi}_f(x_2|A)$  are wave functions of an electron in initial and final states in the field (2), and  $G(x_2 x_1|A)$  is the Green function of an intermediate electron in the field of a pulsed light wave (2). Hereafter, expressions with hats above mean scalar products of corresponding four-vectors with the Dirac  $\tilde{\gamma}$ -matrices. In the amplitude (6)  $A_0(|\mathbf{x}_j|)$  is the Coulomb potential of a nucleus, and  $A'_\mu(x_j, k')$  is the four-potential of a spontaneously radiated photon. They have the following forms

$$A_0(|\mathbf{x}_j|) = \frac{Ze}{|\mathbf{x}_j|}, \quad (7)$$

$$A'_\mu(x_j, k') = \sqrt{\frac{2\pi}{\omega'}} \epsilon_\mu^* \exp(ik'x_j), \quad j = 1, 2. \quad (8)$$

Here,  $\epsilon_\mu^*$  and  $k' = (\omega', \mathbf{k}')$  are the polarization four-vector and the four-momentum of a spontaneous photon,  $k'x_j = \omega't_j - \mathbf{k}'\mathbf{x}_j$ .

The SB amplitude of an electron scattered by a nucleus in the field of a moderately strong pulsed wave (6) in the general relativistic case was derived early (Lebed' & Roshchupkin (2009)). This amplitude may be presented in the following form

$$S_{fi} = \sum_{l=-\infty}^{\infty} S_l, \quad (9)$$

where  $S_l$  is the process partial amplitude with emission or absorption of  $|l|$  laser-wave photons, that is

$$S_l = -i \frac{Ze^3 \sqrt{\pi}}{\sqrt{2\omega' E_f E_i}} \bar{u}_f \left[ B_{li}(\tilde{\gamma}_0, \hat{\epsilon}^*) + B_{lf}(\hat{\epsilon}^*, \tilde{\gamma}_0) \right] u_i. \quad (10)$$

Here, the functions  $B_{li}(\tilde{\gamma}_0, \hat{\epsilon}^*)$  and  $B_{lf}(\hat{\epsilon}^*, \tilde{\gamma}_0)$  correspond to the diagrams of electron-nucleus SB in Fig. 1;  $u_i, \bar{u}_f$  are the Dirac bispinors.

Let us consider the diagram (a):

$$B_{li}(\tilde{\gamma}_0, \hat{\epsilon}^*) = \sum_{r=-\infty}^{\infty} \frac{2\omega\tau^2}{\mathbf{q}^2 + q_0(q_0 - 2q_z)} \int_{-\infty}^{\infty} d\tilde{\xi} \frac{\Lambda_{l+r}(\tilde{\xi}) [\hat{q}_i + m + \tilde{\xi}\hat{k}] \Lambda_{-r}(\tilde{\xi})}{q_i^2 - m^2 + 2\tilde{\xi}(kq_i) + i0}, \quad (11)$$

where the four-vector  $q = (q_0, \mathbf{q})$  is the transferred four-momentum,  $q_i$  is the four-momentum of an intermediate electron for the diagram (a) (Fig. 1)

$$\begin{cases} q = p_f - p_i + k' + lk, \\ q_i = p_i - k' + rk, \\ q_f = p_f + k' + (l+r)k; \end{cases} \quad (12)$$

$q_f$  is the four-momentum of an intermediate electron for the diagram (b) (Fig. 1). The integral functions  $\Lambda_{l+r}$ ,  $\Lambda_{-r}$  are specified as

$$\begin{cases} \Lambda_{l+r}(\xi) = \tilde{\gamma}_0 \int_{-\infty}^{\infty} d\phi \cdot L_{l+r}(\phi) \cdot \exp\{iq_0\tau\phi - i(\xi\omega\tau)\phi\}, \\ \Lambda_{-r}(\xi) = \int_{-\infty}^{\infty} d\phi' \cdot F_{-r}(\phi') \cdot \exp\{i(\xi\omega\tau)\phi'\}. \end{cases} \quad (13)$$

The integration variables in Eqs. (13):

$$\phi = \frac{\varphi}{\omega\tau}, \quad \phi' = \frac{\varphi'}{\omega\tau}. \quad (14)$$

The integral functions  $F_{-r}(\phi')$ ,  $L_{l+r}(\phi)$  in Eqs. (13) are stepless depended on the integration variables (14), and are determined as

$$F_{-r}(\phi') = \hat{\varepsilon}^* \cdot L_{-r}(\chi_{q_i p_i}, \gamma_{q_i p_i}(\phi'), \beta_{q_i p_i}(\phi')) + (e_+ b) \cdot g(\phi') L_{-r+1}(\chi_{q_i p_i}, \gamma_{q_i p_i}(\phi'), \beta_{q_i p_i}(\phi')), \quad (15)$$

where

$$b = \frac{1}{4} \eta_0 m \left( \frac{\hat{\varepsilon}^* \hat{k} \tilde{\gamma}}{(k p_i)} + \frac{\tilde{\gamma} \hat{k} \hat{\varepsilon}^*}{(k q_i)} \right), \quad (16)$$

$$e_+ = e_x + i\delta e_y, \quad (17)$$

$$\begin{aligned} L_{-r}(\chi_{q_i p_i}, \gamma_{q_i p_i}(\phi'), \beta_{q_i p_i}(\phi')) &= \\ &= \frac{1}{2\pi} \int_0^{2\pi} d\varphi \exp\{i[\gamma_{q_i p_i}(\phi') \sin(\varphi - \chi_{q_i p_i}) + \beta_{q_i p_i}(\phi') \sin 2\varphi + r\varphi]\}. \end{aligned} \quad (18)$$

The arguments of functions (18) are defined by the expressions

$$\tan \chi_{q_i p_i} = \delta \frac{(e_y Q_{q_i p_i})}{(e_x Q_{q_i p_i})}, \quad Q_{q_i p_i} = \frac{q_i}{(k q_i)} - \frac{p_i}{(k p_i)}, \quad (19)$$

$$\gamma_{q_i p_i}(\phi') = \eta_0 g(\phi') \cdot m \sqrt{(e_x Q_{q_i p_i})^2 + \delta^2 (e_y Q_{q_i p_i})^2}, \quad (20)$$

$$\beta_{q_i p_i}(\phi') = \frac{1}{8} (1 - \delta^2) \eta_0^2 g^2(\phi') m^2 \left[ \frac{1}{(k q_i)} - \frac{1}{(k p_i)} \right]. \quad (21)$$

Expressions for integral functions  $L_{l+r}(\phi) \equiv L_{l+r}(\chi_{p_f q_i}, \gamma_{p_f q_i}(\phi), \beta_{p_f q_i}(\phi))$  may be easily obtained from the appropriate expressions (18)-(21) after following replacements of indices and four-momenta:  $-r \rightarrow l+r$ ,  $q_i \rightarrow p_f$ ,  $p_i \rightarrow q_i$ .

Functions  $L_n(\chi, \gamma, \beta)$  determine probabilities of multiphoton processes produced by the presence of a strong external field. Note that properties of these functions were studied by Roshchupkin et al. (2000) in detail. Thus, they may be represented as series in integer-order Bessel functions, i.e.

$$L_n(\chi, \gamma, \beta) = \exp(-in\chi) \sum_{s=-\infty}^{\infty} \exp(2is\chi) \cdot J_{n-2s}(\gamma) \cdot J_s(\beta). \quad (22)$$



The form of integral functions (18) is considerably simplified for the case of a circular polarization of an external light wave:

$$L_{-r}(\chi_{q_i p_i}, \gamma_{q_i p_i}(\phi'), 0) = \exp(ir\chi_{q_i p_i}) \cdot J_{-r}(\gamma_{q_i p_i}(\phi')). \quad (23)$$

It is obvious from Eqs. (11), (13) that the essential range of the integration variable  $\xi$  is determined by the condition

$$|\xi| \lesssim \frac{1}{\omega\tau} \ll 1. \quad (24)$$

In view of quick oscillation of the integrand under  $|\xi| \gg (\omega\tau)^{-1}$  the integrals in Eqs. (13) are small. Note that the expression of the amplitude  $B_{lf}(\hat{\epsilon}^*, \tilde{\gamma}_0)$  may be easily obtained from Eqs. (11), (13)-(21), if the replacements  $q_i \rightarrow q_f$ ,  $\tilde{\gamma}_0 \leftrightarrow \hat{\epsilon}^*$  will be performed.

We emphasize, that dependence of the integrand denominator in Eq. (11) on the integration variable expresses consequence of accounting of the field pulsed character. The similar correction is absent in the monochromatic wave case, thus the resonant infinity in the amplitude of SB of an electron scattered by a nucleus in a light field occurs.

## 2.2 Resonance conditions

Fulfillment of the energy-momentum conservation law for components of a process of the second order caused a phenomenon when a virtual intermediate particle becomes real – that is, on-shell. Thus, the resonant divergence occurs in the process's amplitude. The energy-momentum conservation law for QED processes in a pulsed light field does not fulfill strictly. This peculiarity is inessential when nonresonant processes are studied. On the contrary, the energy-momentum nonconservation in the case of resonant SB of an electron scattered by a nucleus in a pulsed light field results following resonance conditions

$$q_j^2 - m^2 \lesssim \frac{(kq_j)}{\omega\tau}, \quad j = i, f. \quad (25)$$

(it follows from consideration of Eqs. (11), (24)). Therefore, the four-momentum of an intermediate electron occurs near the mass shell.

It is convenient to set down expressions which determine  $q_{i,f}$  and  $q$  (12) in following form for the both amplitudes (a) and (b) (Fig. 1), respectively

$$\begin{cases} p_i + rk = q_i + k', \\ q = p_f - q_i + (l + r)k; \end{cases} \quad (26)$$

$$\begin{cases} q_f + rk = p_f + k', \\ q = q_f - p_f + (l + r)k. \end{cases} \quad (27)$$

Eqs. (26)-(27) represent the four-momentum conservation laws for the diagrams' vertices. These laws are fulfilled for only the values  $r > 0$  under the conditions (25).

It is easy to ascertain that if a spontaneous photon propagates in the same direction as a photon of an external field, the conditions (25) cannot be satisfied simultaneously with the conservation laws (26) or (27) because the transit amplitude equals zero in this case. Therefore, resonances occur only when these photons propagate nonparallel to each other.



Taking Eq. (25) into account, we can use Eqs. (26), (27) for a moderately strong field (5) to find the frequency of a spontaneous photon in the resonance (the resonant frequency) for the both direct and exchange amplitudes (Figs. 1(a) and 1(b), respectively). Within zeroth order with respect to the small parameter  $(\omega\tau)^{-1}$  the resonant frequency is specified:

$$\omega'_{res} \equiv \omega'_j = r\omega_j \frac{1}{1 \pm d_j}, \quad j = i, f, \quad (28)$$

where the signs “+” and “−” correspond to index values  $i$  and  $f$ , respectively,

$$\omega_j = \omega \cdot \frac{\kappa_j}{\kappa'_j}, \quad d_j = r(nn') \cdot \frac{\omega}{\kappa'_j}, \quad (29)$$

$$\kappa_j = E_j - \mathbf{n}\mathbf{p}_j, \quad \kappa'_j = E_j - \mathbf{n}'\mathbf{p}_j, \quad (30)$$

$$n = \frac{k}{\omega} = (1, \mathbf{n}), \quad n' = \frac{k'}{\omega} = (1, \mathbf{n}'). \quad (31)$$

It is obvious from Eq. (29), that within a rather broad range of electron energies and scattering angles we have  $d_j \ll 1$  (except an ultrarelativistic electron with the energy  $\sim m^2/\omega$ , moving within a narrow cone close to the direction of the momentum of a spontaneous photon). Therefore, resonances are mainly observed when the frequency of a spontaneous photon is multiple to  $\omega_j$  (29).

Eqs. (28)-(31) for the resonant frequency imply that we may separate four characteristic domains of the frequency  $\omega_j$ : the nonrelativistic case,  $\omega_j \cong \omega$ ; the limiting case of ultrarelativistic energies, when an electron moves within a narrow cone related to a photon of an external field  $\omega_j \ll \omega$ ; an ultrarelativistic electron moves within a narrow cone with a spontaneous photon,  $\omega_j \gg \omega$ ; otherwise,  $\omega_j \sim \omega$ . Here, we consider resonant frequencies in detail.

The four-momentum conservation law (26) and the function  $F_{-r}$  explicit form (15) result that this function represents the amplitude of such process: an electron with the four-momentum  $p_i$  absorbs  $r$  photons of the external wave and emits a photon with four-momentum  $k'$ . This process was considered by Nikishov & Ritus (1979) in the case of a plane monochromatic wave, and by Narozhniy & Fofanov (1996) in the case of a pulsed light wave. The expression for the transferred four-momentum  $q$  (see the second equality in Eq. (26)) shows that the quantity  $L_{l+r}(\chi_{p_f q_i}, \gamma_{p_f q_i}(\phi), \beta_{p_f q_i}(\phi))$  defines the amplitude of scattering of an intermediate electron with the four-momentum  $q_i$  by a nucleus in the field of a light wave with absorption or emission of  $|l+r|$  wave photons. In the nonrelativistic limiting case this process was studied by Bunkin & Fedorov (1966). Denisov & Fedorov (1967) considered this process in the general relativistic case. The process when an electron scattered by a nucleus in a pulsed light wave was studied by Lebed' & Roshchupkin (2008).

Consequently, if the interference between the direct and the exchange amplitudes is absent, the process of resonant electron-nucleus SB in the field of a light wave effectively decomposes into two consecutive processes of the first order: emission of a photon with the four-momentum  $k'$  by an electron in a pulsed light wave and scattering of an electron by a nucleus in a pulsed light wave (see Fig. 2).

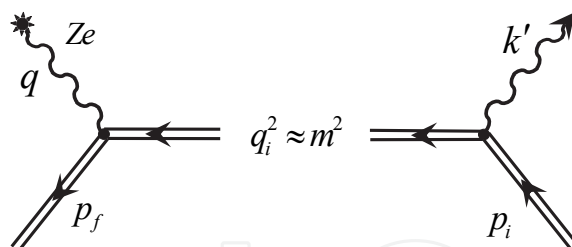


Fig. 2. Resonant electron-nucleus SB in the field of a pulsed light wave.

The difference for the other diagram (Fig. 1(b)) is concluded in the both replacement of the intermediate electron four-momentum ( $q_i \rightarrow q_f$ ) and interchange of sequence of first order processes. Thus, an electron is scattered by a nucleus with absorption or emission of  $r$  wave photons, and then it spontaneously emits a photon with the four-momentum  $k'$  with  $|l + r|$  wave photons absorption.

As it was pointed above, the integral functions (18) are determined by the integer-order Bessel functions (23) for the case of a circularly polarized external wave. It is not difficult to verify that for given type polarization under the resonance conditions the arguments of the Bessel functions (20) may be represented as

$$\gamma_{q_i p_i}(\phi') = 2r \cdot \eta_0 g(\phi') \cdot \sqrt{\frac{u}{u_r} \cdot \left(1 - \frac{u}{u_r}\right)}. \quad (32)$$

Here,  $u$ ,  $u_r$  are the relativistic invariant parameters

$$u = \frac{(kk')}{(kq_i)}, \quad u_r = 2r \cdot \frac{(kp_i)}{m^2}. \quad (33)$$

Equations (32)-(33) imply

$$\gamma_{q_i p_i}(\phi') \sim \eta_0 \ll 1. \quad (34)$$

Consequently, within the range of fields specified by Eq. (5) the first resonance, that is, the resonance with  $r = 1$ , provides the main contribution to the resonant cross section, when the Bessel function has the largest value. This implies that the Compton scattering of a light wave by an initial electron is mainly due to absorption of one photon of an external field. However, the argument of the Bessel function  $J_{l+r}(\gamma_{p_f q_i}(\phi))$  is of the order of magnitude:  $\gamma_{p_f q_i}(\phi) \sim \gamma_0 \gtrsim 1$ . Thus, scattering of an intermediate electron by a nucleus in a pulsed wave field under these conditions is generally a multiphoton process.

Interference of the resonant amplitudes (which correspond to direct and exchange diagrams) implies the equality of their resonant frequencies, i.e.  $\omega'_i = \omega'_f$ . Within the field range specified by Eq. (5) the condition of interference between direct and exchange resonant amplitudes is written as:

$$(\mathbf{v}_f - \mathbf{v}_i)(\mathbf{n} - \mathbf{n}') + (\mathbf{v}_f \times \mathbf{v}_i)(\mathbf{n}' \times \mathbf{n}) = (\mathbf{n}'\mathbf{n}) \cdot \frac{r\omega \cdot (\kappa_i + \kappa_f)}{E_i E_f}. \quad (35)$$

Here,  $\mathbf{v}_j = \mathbf{p}_j/E_j$  is the electron velocity before ( $j = i$ ) and after ( $j = f$ ) scattering. The quantity involved in the right-hand side of Eq. (35) is small compared with the unity. Therefore, this equality is satisfied when directions of motion of photons (a spontaneous photon and a photon of an external field) or electrons (before and after scattering) are close to each other. It follows from Eq. (35) and from the fact that resonances vanish, when direction of spontaneous photon motion is close to direction of external field photon motion, that resonant amplitudes, which correspond to the processes shown on Figs. 1(a) and 1(b), interfere when an electron is scattered on the small angles, i.e.

$$\theta = \angle(\mathbf{v}_i, \mathbf{v}_f) \sim (1 - \mathbf{n}\mathbf{v}_i) \cdot (\omega/|\mathbf{v}_i| E_i) \ll 1. \quad (36)$$

Hereinafter, we consider the resonance of one diagram. We assume that the spontaneous photon frequency is equal

$$\omega' \approx \omega'_{res} = \omega'_i. \quad (37)$$

### 2.3 Amplitude integration

Let us study the process of resonant SB of an electron scattered by a nucleus in a pulsed light field at the expense of only one photon absorption, i.e.  $r = 1$ . The condition (24) allows to simplify the integration in Eq. (11)

$$\int_{-\infty}^{\infty} d\xi \frac{\exp\{i\xi\omega\tau(\phi' - \phi)\}}{q_i^2 - m^2 + 2\xi(kq_i) + i0} = \frac{\exp\{-2i\beta(\phi' - \phi)\}}{2(kq_i)} i\pi (\text{sgn}(\phi' - \phi) - 1). \quad (38)$$

Eq. (38) contains the relevant parameter, which determines resonant electron-nucleus SB in the field of a pulsed light wave:

$$\beta = \frac{q_i^2 - m^2}{4(kq_i)} \omega\tau. \quad (39)$$

As it can be seen from Eq. (39), values of the parameter  $\beta$  are defined by process kinematics and external pulsed-wave properties. This parameter specifies how closely the four-momentum of an intermediate electron coincides with the value on the mass shell under resonance conditions for electron-nucleus SB in the field of a pulsed light wave.

The subsequent analysis will be performed for the particular form of the envelope function of the pulsed light wave four-potential. We choose the Gaussian function:

$$g\left(\frac{\varphi}{\omega\tau}\right) = \exp\left\{-\left(\frac{2\varphi}{\omega\tau}\right)^2\right\} = \exp\left\{-(2\phi)^2\right\}. \quad (40)$$

Under the condition (34) the function  $F_{-r}(\phi')$  (15) in the amplitude may be expanded in the Taylor series. We may keep only linear terms with respect to the parameter  $\eta_0$ . For the envelope function (40), after simple computation we obtain the amplitude of resonant SB of an electron scattered by a nucleus in a pulsed light field:

$$B_{li}(\tilde{\gamma}_0, \hat{\varepsilon}^*) = \frac{2\pi \cdot \tilde{\gamma}_0 (\hat{q}_i + m) \hat{F}}{\mathbf{q}^2 + q_0(q_0 - 2q_z)} \cdot \frac{-i\omega\tau\sqrt{\pi}}{4(kq_i)} \exp\left\{-\frac{\beta^2}{4}\right\} \cdot I(q_0, \beta), \quad (41)$$

$$\hat{F} = -\frac{1}{2} \exp\{i\chi_{q_i p_i}\} \cdot \gamma_{q_i p_i}(0) \cdot \hat{\varepsilon}^* + (e_+ b). \quad (42)$$

$$I(q_0, \beta) = \tau \int_{-\infty}^{\infty} d\phi \cdot J_{l+1}(\phi) \exp\{i(q_0 \tau + 2\beta)\phi\} \left( \operatorname{erf}\left(2\phi + \frac{i\beta}{2}\right) + 1 \right). \quad (43)$$

Here, the function  $\operatorname{erf}(2\phi + i\beta/2)$  is the error function.

## 2.4 Cross-section of spontaneous bremsstrahlung

Let us calculate the differential probability during the entire time of electron-nucleus SB in a pulsed light field from the amplitude, Eqs. (9)-(10), (41)-(43) in standard manner (see Berestetskii et al. (1982)) for the spontaneous photon frequency (37):

$$dw = \sum_{l=-\infty}^{\infty} dw_l, \quad (44)$$

$$dw_l = \frac{Z^2 e^6 \pi}{2\omega' E_f E_i} \cdot \left| \bar{u}_f B_{li} u_i \right|^2 \cdot \frac{d^3 p_f d^3 k'}{T (2\pi)^6}. \quad (45)$$

Here,  $T$  is some comparatively large ( $T \gtrsim \tau$ ) interval of the observation time. The energy-momentum conservation law for SB of an electron scattered by a nucleus in a pulsed light field does not fulfill strictly, however, under the condition (1) the essential range of integration is converged. Energies of a final electron are negligibly differ from the values, which are specified by the strict energy conservation law. We exclude small scattering angles from the consideration:

$$\theta = \angle(\mathbf{p}_i, \mathbf{p}_f) \gg \sqrt{\frac{\omega}{|\mathbf{p}_i|(\omega\tau)}} \ll 1. \quad (46)$$

The differential cross section of SB of an electron scattered by a nucleus in the field of a pulsed light wave is obtained by means of division of the probability per unit time per a flux density of scattered particles  $v_i = |\mathbf{p}_i|/E_i$ . Thus, we derive

$$d\sigma = \sum_{l=-\infty}^{\infty} d\sigma_l, \quad (47)$$

where  $d\sigma_l$  is the partial cross section of a process with a spontaneous photon in the frequency interval  $d\omega'$  within the solid angle  $d\Omega'$ , and a final electron within the solid angle  $d\Omega_f$  with emission ( $l > 0$ ) or absorption ( $l < 0$ ) of wave photons. It may be written in the form

$$\begin{aligned} \frac{d\sigma_l}{d\omega' d\Omega' d\Omega_f} &= \frac{Z^2 e^6 \omega' \pi (\omega\tau)^2}{(2\pi)^2 \mathbf{q}^4} \frac{|\mathbf{p}_f|}{|\mathbf{p}_i|} \frac{\exp\{-\beta^2/2\}}{64 (kq_i)^2} \times \\ &\times \left| \bar{u}_f M_i u_i \right|^2 \cdot \frac{\tau}{T} \int_{-T/2\tau}^{T/2\tau} d\phi \cdot J_{l+1}^2(\phi) \left| \operatorname{erf}\left(2\phi + \frac{i\beta}{2}\right) + 1 \right|^2, \end{aligned} \quad (48)$$

$$M_i = \tilde{\gamma}_0 (\hat{q}_i + m) \hat{F}. \quad (49)$$

It is taken into account that  $d^3p_f = |\mathbf{p}_f| E_f dE_f d\Omega_f$  and  $d^3k' = \omega'^2 d\omega' d\Omega'$ . It is important to note that the main contribution into the sum (47) is given by the processes with emission (absorption) of  $|l| \lesssim \gamma_0$  number of wave photons within the range of a moderately strong field for electron relativistic energies ( $E_{i,f} \gtrsim m$ ). Therefore, the energy contribution of stimulated photons may be neglected ( $|l| \omega / E_{i,f} \lesssim \eta_0 m / E_{i,f} \ll 1$ ) in Eq. (12). Thus, it is easy to sum over all possible partial processes of electron scattering by a nucleus (47).

If polarization effects are not of interest, then averaging over polarizations of an initial electron and summation over polarizations of a final electron and a spontaneous photon are to be done. Performing the relevant procedures of averaging and summation, we derive the general relativistic expression for the resonant differential cross section of electron-nucleus SB in a pulsed light field in the case of electron large-angle scattering (46)

$$\frac{d\sigma_{res}}{d\Omega'} = \frac{1}{\pi^2} \cdot \frac{E_i \kappa_i^2 |\mathbf{q}_i| u}{(nn')^2 |\mathbf{p}_i| (1+u)} \cdot P_{res} \cdot d\sigma_s dW^{(1)}. \quad (50)$$

Here,

$$d\sigma_s = 2Z^2 r_e^2 \frac{|\mathbf{p}_f| m^2}{|\mathbf{q}_i| q^4} \left( m^2 + E_f q_{i0} + \mathbf{p}_f \mathbf{q}_i \right) d\Omega_f \quad (51)$$

is the differential cross section of scattering of an intermediate electron with the four-momentum  $q_i$  by a nucleus in a wave field;  $r_e$  is the classical electron radius.

$$dW^{(1)} = \frac{\alpha \eta_0^2 m^2}{4E_i} \left\{ 2 + \frac{u^2}{1+u} - \frac{4u}{u_1} \left( 1 - \frac{u}{u_1} \right) \right\} \cdot \frac{du}{(1+u)^2} \quad (52)$$

is the probability that an electron with the four-momentum  $p_i$  absorbs one photon from an external field and emits a photon with the four-momentum  $k'$ . The function  $P_{res}$  in Eq. (50) has the form

$$P_{res} = \frac{\pi (\omega \tau)^2}{64 (k q_i)^2} \cdot P_{res}^\beta, \quad (53)$$

$$P_{res}^\beta = \exp\{-\beta^2/2\} \cdot \frac{1}{2\rho} \int_{-\rho}^{\rho} d\phi \cdot \left| \operatorname{erf}\left(\phi + \frac{i\beta}{2}\right) + 1 \right|^2, \quad (54)$$

$$\rho = T/\tau. \quad (55)$$

Here, the parameter  $\rho$  is the relation between the observation time and the pulse duration, its value is determined by conditions of the concrete experiment. Thus, if an external field is represented as electromagnetic pulses abiding one by one, then the parameter  $\rho$  assumes sense of the ratio of a distance between the nearest-neighbor pulses to the characteristic pulse duration. Dependence of the function  $P_{res}$  (53) on the parameter  $\beta$  (39) defines magnitude and shape of the resonant peak in the cross section of an electron-nucleus SB process in a pulsed light field (see Fig. 7).

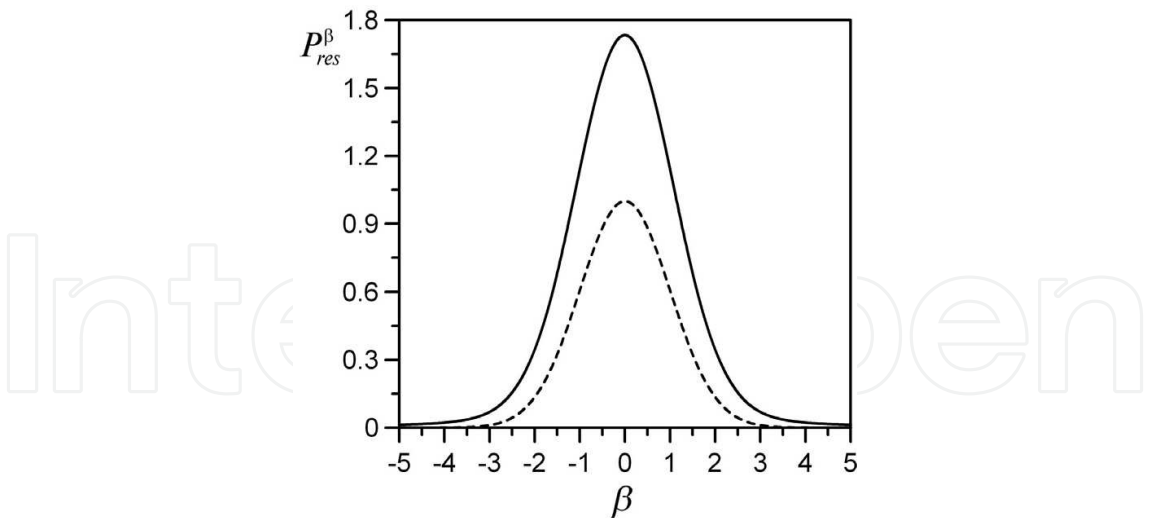


Fig. 3. Shape of the first resonant peak in the cross section of electron-nucleus SB in a pulsed light field ( $\rho = 3$ ). The dashed line represents the Gaussian function:  $\exp(-\beta^2/2)$ .

2.4.1 Resonant width

In the frame of subsequent analysis we are to estimate the magnitude of the resonant width. For this purpose we consider the case when the four-momentum of an intermediate photon occurs near the mass shell:

$$\beta = \frac{(q_i^2 - m^2)}{4(kq_i)}\omega\tau \ll 1. \tag{56}$$

Thus, we can easily write

$$P_{res} \approx \frac{\pi}{4} \cdot \frac{(a_1/a_2)}{(q_i^2 - m^2)^2 + 4m^2\Gamma_\tau^2}, \tag{57}$$

where the width  $\Gamma_\tau$ , caused by the pulsed character of an external wave, is equal to:

$$\Gamma_\tau = \frac{2}{\sqrt{a_2}} \frac{(kq_i)}{m} \frac{1}{\omega\tau}, \tag{58}$$

and the coefficients  $a_1$  and  $a_2$  are specified by

$$a_1 = \frac{1}{2\rho} \int_{-\rho}^{\rho} (\operatorname{erf}(\phi) + 1)^2 d\phi, \tag{59}$$

$$a_2 = \frac{1}{2} - \frac{1}{4\sqrt{\pi}a_1\rho} \left( \sqrt{2}\operatorname{erf}(\sqrt{2}\rho) + \int_{-\rho}^{\rho} \phi \exp(-\phi^2) (\operatorname{erf}(\phi) + 1) d\phi \right). \tag{60}$$

It is important to underline that the relationship for the function (53) under the condition (56) turns into the standard resonant expression (57), which is usually used in the Breit-Wigner prescription. It is convenient to represent the resonant peak profile  $P_{res}$  in the form (57) to compare obtained results with corresponding ones for the case of a monochromatic wave. Note, that in the monochromatic wave case the resonant infinity in the cross section is eliminated by radiative corrections introducing into the Green function. The Breit-Wigner



broadening prescription is concluded in addition of the imaginary part of the electron mass, that is  $m \rightarrow m - i\Gamma_R$ . Here, the radiation width is specified

$$\Gamma_R = \frac{1}{3} \alpha \eta_0^2 \frac{\sigma_c(q_i)}{\sigma_T} \cdot \frac{(kq_i)}{m}, \quad (61)$$

where  $\sigma_c(q_i)$  is the total cross section of the Compton scattering of an external field photon by an intermediate electron with the four-momentum  $q_i$  (it is the most probable way out of an electron from an intermediate state),  $\sigma_T$  is the cross section of the Thomson scattering.

The resonant width (58) providing by finite time of particle-field interaction is so-called transit width. In real experiments the transit width value is generally determined by geometry of an experiment and linear sizes of space where a particle interacts with an external field. It can be seen from Eq. (58) that the transit width is specified by the pulse duration and process kinematics. Influence of the pulse duration on the resonant behavior of the electron-nucleus SB cross section was discussed by Schnez et al. (2007). The electromagnetic pulse duration has to be longer than the lifetime of an intermediate electron state. Otherwise, an electron will not have enough time to interact with a wave. Thus,

$$\tau \gtrsim \frac{1}{\Gamma_R}. \quad (62)$$

Requirements (62), (58) implies that values of the quantity  $\omega\tau$  have to satisfy the following condition:

$$\omega\tau \gtrsim \frac{1}{\alpha \eta_0^2} \frac{\omega m}{(kq_i)}. \quad (63)$$

Comparison of the resonance widths for the pulse duration values (63) implies that  $\Gamma_\tau \sim \Gamma_R$  within a sufficiently broad range of electron energies and scattering angles. Consequently, the both radiation and transit widths have to be simultaneously considered in resonant SB study. An exception is the case of ultrarelativistic energies when

$$\frac{1}{\alpha \eta_0^2} \frac{\omega m}{(kq_i)} \lesssim \omega\tau \ll \frac{1}{\alpha \eta_0^2}. \quad (64)$$

In this case  $\Gamma_\tau \gg \Gamma_R$  and the expressions for the resonant differential cross section of electron-nucleus SB in a pulsed field (50)-(52), (57)-(60) are correct without radiation width accounting.

It should be pointed that we excluded other causes of the resonant peak widening from consideration. Thus, we assume that the Doppler broadening of the resonance (the real electron bunch is not monochromatic) and broadening caused by collisions of electrons are negligible.

#### 2.4.2 Range of relativistic energies

In this section we consider the range of electron relativistic energies:  $E_i \gtrsim m$ . Here we eliminate the case when ultrarelativistic electrons are moving within a narrow cone with a spontaneous photon or an external field photon from consideration. Then  $|d_i| \ll 1$  (it follows from Eq. (29)). Therefore, the resonant frequency  $\omega'_i$  (28) in this case is of the order of the external field frequency. Depending on the spontaneous photon emission angle with respect

to direction of the initial electron momentum the resonant frequency falls within the interval:

$$\omega \cdot \frac{\kappa_i}{E_i + |\mathbf{p}_i|} \leq \omega'_i \leq \omega \cdot \frac{\kappa_i}{E_i - |\mathbf{p}_i|}. \quad (65)$$

This frequency reaches its minimum and maximum when a spontaneous photon is emitted along direction of the initial electron motion and in opposite direction, respectively.

The invariant parameters (33) assume the form

$$u_r = 2r \cdot \frac{\omega \kappa_i}{m^2}, \quad u \cong (nn') \cdot \frac{\omega'}{\kappa_i} \ll 1. \quad (66)$$

Taking the radiation width into account, we may represent the resonant denominator (57) as

$$\left(q_i^2 - (m - i\Gamma_R)^2\right)^2 + (2m\Gamma_\tau)^2 = (2\omega' |\mathbf{p}_i|)^2 \cdot \left[(\cos \theta'_i - \cos \theta'_{res})^2 + C_\tau^2\right]. \quad (67)$$

Here we introduced the notations

$$\theta'_{i,f} = \angle(\mathbf{k}', \mathbf{p}_{i,f}), \quad \theta_{i,f} = \angle(\mathbf{k}, \mathbf{p}_{i,f}), \quad (68)$$

$$\cos \theta'_{res} = \frac{E_i - (\omega/\omega') \cdot \kappa_i}{|\mathbf{p}_i|}, \quad C_\tau = \frac{m\Gamma_\tau \sqrt{1 + \mu_\tau^2}}{\omega' |\mathbf{p}_i|}, \quad (69)$$

$$\mu_\tau = \frac{\Gamma_R}{\Gamma_\tau} = \frac{\sqrt{a_2}}{6} \alpha \eta_0^2 \omega \tau. \quad (70)$$

For the resonant angles that are not too close to zero and  $\pi$  we can expand  $\cos \theta'_i$  in Eq. (67) into the Taylor series near the resonant angle  $\theta'_{res}$  with an accuracy up to the term of the first order with respect to  $t = \theta'_i - \theta'_{res}$ . The solid angle which corresponds to spontaneous photon emission is written as  $d\Omega' = \sin \theta'_{res} d\varphi dt$ . Then the resonant cross section (50) assumes the following form

$$d\sigma_{res} = \frac{1}{4\pi^2} \cdot \frac{d\varphi \cdot d(t/y)}{1 + (t/y)^2} \cdot \frac{E_i \kappa_i}{(nn') |\mathbf{p}_i| \Gamma_\tau \sqrt{1 + \mu_\tau^2} m} \cdot dW^{(1)} d\sigma_S(q_i). \quad (71)$$

Here,  $y = m\Gamma_\tau (1 + \mu_\tau) / (\omega' |\mathbf{p}_i| \sin \theta'_{res}) \sim (\omega\tau)^{-1} \ll 1$ . Since the resonance angular width is very small, we may integrate the expression (71) with respect to the azimuthal angle  $d\varphi$ , and with respect to  $d(t/y)$  within the limits from zero to  $+\infty$  (we extend the integration limits to infinity because of integral fast convergence). Finally, we derive

$$d\sigma_{res} = \frac{E_i \kappa_i}{2 (nn') m |\mathbf{p}_i| \Gamma_\tau \sqrt{1 + \mu_\tau^2}} \cdot dW^{(1)} d\sigma_S(q_i), \quad (72)$$

where

$$dW^{(1)} = \alpha \eta_0^2 (nn') \frac{m^2}{2E_i \kappa_i} \left\{ 1 - \frac{2u}{u_1} \cdot \left( 1 - \frac{u}{u_1} \right) \right\} d\omega'. \quad (73)$$

Derived expressions (72)-(73) for the resonant cross section are valid within the range of field intensities (5) when an electron scatters into the large angles  $\theta \gg \omega/|\mathbf{p}_i|$ . Spontaneous

photon frequency and emission angle with respect to the initial electron momentum are unambiguously related to each other by Eq. (69), where the spontaneous photon frequency is chosen from the interval (65).

Note, that the conventional cross section  $d\sigma_*$  of electron-nucleus bremsstrahlung (in external field absence) may be factorized as a product of the cross section  $d\sigma_S(p_i)$  of electron-nucleus elastic scattering (see (51)) and the probability  $dW_\gamma$  of photon emission

$$d\sigma_* = d\sigma_S \cdot dW_\gamma, \quad (74)$$

$$dW_\gamma = \frac{\alpha}{4\pi^2} \cdot \left\{ \mathbf{q}^2 - (\mathbf{n}' \mathbf{q})^2 \cdot \frac{m^2}{\kappa'_i \kappa'_f} \right\} \cdot \frac{d\omega'}{\omega' \kappa'_i \kappa'_f} \cdot d\Omega', \quad \mathbf{q} = \mathbf{p}_f - \mathbf{p}_i. \quad (75)$$

Let us calculate the ratio of the resonant cross section (72) to the conventional cross section of electron-nucleus bremsstrahlung (74) (in absence of an external field). At that we take into account the resonant relation (69) between spontaneous photon frequency and emission angle

$$R_{res} = \frac{d\sigma_{res}}{d\sigma_*/d\Omega'} = f_1 \cdot \pi^2 \eta_0^2 \frac{\omega\tau}{\sqrt{1 + \mu_\tau^2}} \left( \frac{m}{|\mathbf{p}_i|} \right)^2, \quad (76)$$

where the function  $f_1 \sim 1$  and has a rather cumbersome form:

$$f_1 = \frac{\sqrt{a_2} \kappa'_f}{2 |\mathbf{p}_i|} \frac{1 - (nn') \frac{m^2}{\kappa_i \kappa'_i} \left( 1 - (nn') \frac{m^2}{2\kappa_i \kappa'_i} \right)}{4 \sin^2 \left( \frac{\theta}{2} \right) - \left( \cos \theta'_f - \cos \theta'_i \right)^2 \frac{m^2}{\kappa'_i \kappa'_f}}. \quad (77)$$

Let us choose for calculation the laser field characteristic according to SLAC experiments (Bula et al. (1996)): laser-wave frequency,  $\omega = 2.35$  eV; laser pulsewidth,  $\tau = 1.5$  ps; field strength in a pulse peak,  $F_0 = 6 \cdot 10^9$  V/cm; ratio between observation time and laser pulse width,  $\rho = 5$ . Fig. 4 displays the ratio of the resonant differential cross-section of electron-nucleus SB to the cross section of bremsstrahlung in absence of an external field (76) as a function of the electron velocity.

Eq. (76) and Fig. 4 show that within the range of electron relativistic energies the resonant differential cross section of electron-nucleus SB, when the scattered electron ejection angle is detected simultaneously with the spontaneous photon emission angle, may be five orders of magnitude greater than the corresponding cross section in external field absence. Within the range of electron ultrarelativistic energies this ratio decreases drastically:  $R_{res} \sim (m/E_i)^2 \rightarrow 0$ .

The ratio (76) as a function of the spontaneous photon azimuthal angle is of interest from a perspective of experimental testing of obtained results. In the actual experiment usually the radiation detection over the azimuthal angle is technically implemented significantly easier than over the polar angle. Fig. 5 displays  $\lg R_{res}$  (76) as a function of the spontaneous photon azimuthal angle.

Fig. 5 shows that the ratio (76) may change its order of magnitude with the azimuthal angle value. This dependence is characterized by presence of two maxima in distribution over the azimuthal angle. Thus, when the final electron azimuthal angle coincides with the initial electron angle (it is scattering in the plane of the vectors  $(\mathbf{k}, \mathbf{p}_i)$ ) the maxima in distribution

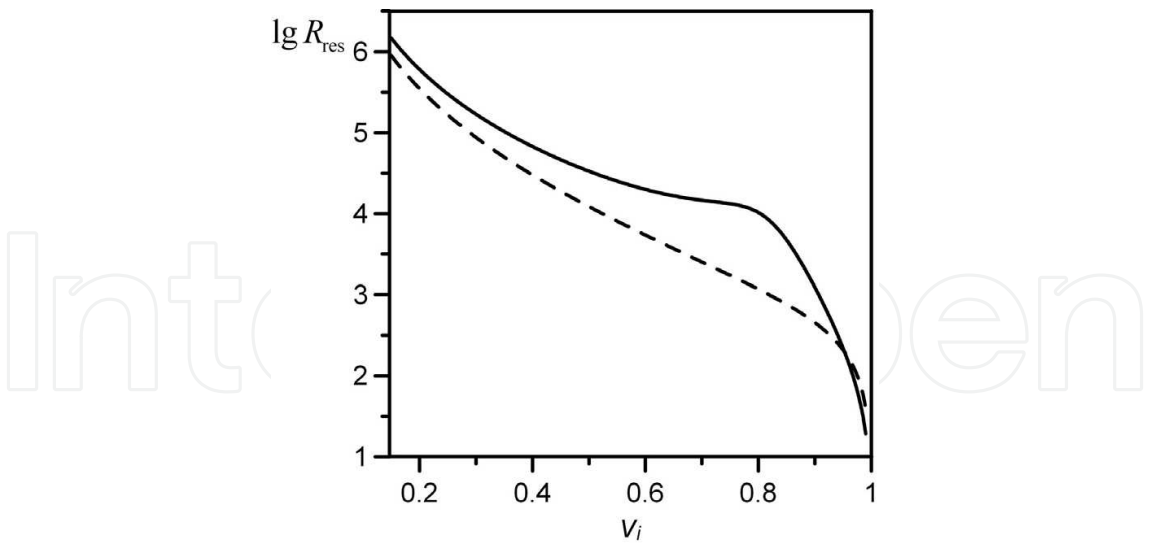


Fig. 4. Ratio  $R_{res}$  (76) as a function of the initial velocity for electron momentum preset orientations in initial ( $\theta_i = 163^\circ, \varphi_i = 0^\circ$ ) and final ( $\theta_f = 150^\circ, \varphi_f = 0^\circ$ ) states and spontaneous photon fixed orientation: solid line,  $\theta' = 120^\circ, \varphi' = 10^\circ$ ; dashed line,  $\theta' = 120^\circ, \varphi' = 60^\circ$ .

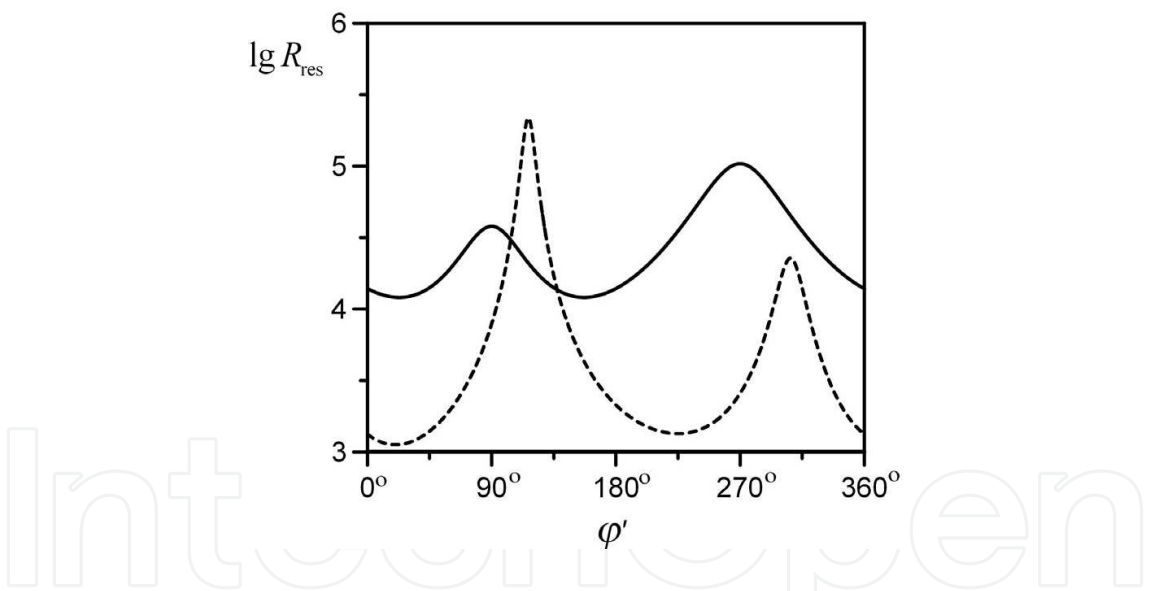


Fig. 5. Ratio  $R_{res}$  (76) as a function of the azimuthal angle of a spontaneous photon for electron momentum preset orientations in initial and final states and the spontaneous photon fixed polar angle:  $\theta_i = 163^\circ, \theta_f = 150^\circ, \theta' = 120^\circ$ . Solid line,  $\varphi_i = \varphi_f = 90^\circ$ ; dashed line,  $\varphi_i = 90^\circ, \varphi_f = 320^\circ$ .

correspond to spontaneous photons emission just within this plane (solid line). In the case when a final electron scatters in another way the peak position in distribution over the azimuthal angle is specified by both initial and final azimuthal angles. The value of the ratio of the resonant differential cross section of electron-nucleus SB to the ordinary bremsstrahlung cross section as a function of the azimuthal angle may be changed in two orders of magnitude.

### 2.4.3 Range of nonrelativistic electron energies

In this section we assume that initial and final electron energies are small in comparison with the light speed:  $Z\alpha \ll v_{i,f} \ll 1$ . It follows from Eqs. (28)-(31) that resonant frequencies for nonrelativistic electrons are given by

$$\omega'_{i,f} = r\omega \left( 1 + \mathbf{v}_{i,f} \cdot (\mathbf{n}' - \mathbf{n}) \right) \cong r\omega. \quad (78)$$

Thus, resonances occur when the spontaneous photon frequency is multiple to the external field frequency. The condition of interference between direct and exchange resonant amplitudes (35) is written as

$$(\mathbf{v}_f - \mathbf{v}_i) \cdot (\mathbf{n} - \mathbf{n}') = 2r \cdot (nn') \cdot \frac{\omega}{m} \ll 1, \quad (79)$$

and, consequently, interference appears when an electron scatters into the small angles  $\theta \sim \omega/mv_i \ll 1$ .

The resonant cross section in the case when a nonrelativistic electron scatters into the large angles is obtained from Eq. (50):

$$d\sigma_{res} = \frac{1}{2(nn') v_i \Gamma_\tau \sqrt{1 + \mu_\tau^2}} dW^{(1)} d\sigma_s, \quad (80)$$

where

$$dW^{(1)} = \frac{1}{2} \alpha \eta^2 (nn') \cdot \left\{ 1 - \frac{2u}{u_1} \cdot \left( 1 - \frac{u}{u_1} \right) \right\} d\omega', \quad (81)$$

$$\frac{u}{u_1} = (nn') \frac{\omega'}{2\omega},$$

$$d\sigma_s = (2Z)^2 r_e^2 \frac{|\mathbf{p}_f|}{|\mathbf{q}_i|} \frac{m^4}{\mathbf{q}^4} d\Omega_f. \quad (82)$$

The resonant frequency of a spontaneous photon depends on the emission angle of this photon with respect to the initial electron momentum and lies within a narrow interval:

$$\omega \left( 1 - 2v_i \cos^2 (\theta_i/2) \right) \leq \omega'_{res} \leq \omega \left( 1 + 2v_i \sin^2 (\theta_i/2) \right). \quad (83)$$

The transit width  $\Gamma_\tau$  (58) and the radiation width  $\Gamma_R$  (61) in the nonrelativistic limit are given by

$$\Gamma_\tau = \frac{2}{\sqrt{a_2}} \frac{1}{\tau}, \quad \Gamma_R = \frac{1}{3} \alpha \eta^2 \omega. \quad (84)$$

We may write the ratio of the resonant cross section (50) to the corresponding conventional nonrelativistic cross section of electron-nucleus bremsstrahlung as

$$R_{res} = f_2 \cdot \pi^2 \eta_0^2 \frac{\omega \tau}{\sqrt{1 + \mu_\tau^2}} v_i^{-3}, \quad (85)$$

where the function  $f_2 \sim 1$  and has the form

$$f_2 = \frac{\sqrt{a_2}}{2} \frac{1 - (1/2) \sin^2 \theta'}{4 \sin^2 (\theta/2) - (\cos \theta'_f - \cos \theta'_i)^2}. \quad (86)$$

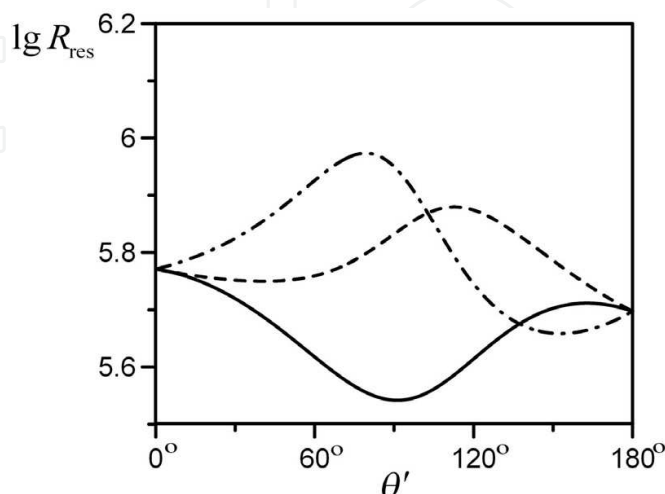


Fig. 6. Dependence of  $R_{res}$  (85) on the polar angle of a spontaneous photon for a nonrelativistic electron with the initial velocity  $v_i = 0.1$ . Orientations of the electron momentum in initial ( $\theta_i = 163^\circ$ ,  $\varphi_i = 90^\circ$ ) and final ( $\theta_f = 150^\circ$ ,  $\varphi_f = 320^\circ$ ) states are fixed. The azimuthal angle, corresponding to emission of a spontaneous photon, is (solid line)  $\varphi' = 60^\circ$ , (dashed line)  $\varphi' = 160^\circ$ , and (dash-dotted line)  $\varphi' = 270^\circ$ .

Fig. 6 shows the dependence of quantity  $R_{res}$  (85) on the polar angle of spontaneous photon emission for a nonrelativistic electron with the initial velocity  $v_i = 0.1$ . Fig. 6 shows that for the case of electron kinetic energies of several kiloelectronvolts the resonant differential SB cross section may be 5–6 orders of magnitude greater than the corresponding cross section of bremsstrahlung in external field absence when the angle of spontaneous photon emission is detected simultaneously with the ejection angle of an electron scattered into the large angle.

#### 2.4.4 Range of ultrarelativistic energies of electrons moving within a narrow cone with a photon from the wave

In this section we consider an ultrarelativistic electron that moves (in initial or final states) within the narrow cone related to an external field photon. Therefore, the quantities  $\kappa_{i,f}$  (30) in Eqs. (28)–(31) may be written as

$$\kappa_{i,f} = \left(1 + \delta_{i,f}^2\right) \cdot m^2 / 2E_{i,f}, \quad \delta_{i,f} = \theta_{i,f} \cdot E_{i,f} / m. \quad (87)$$

Taking these relations into account and using Eqs. (28)–(31) we find that the resonant frequencies are much less than the external field frequency. They are given by:

$$\omega'_{i,f} = r\omega_{i,f}, \quad \omega_{i,f} = \frac{\left(1 + \delta_{i,f}^2\right)}{2(nn')} \cdot \left(\frac{m}{E_{i,f}}\right)^2 \cdot \omega \ll \omega. \quad (88)$$



From Eq. (88) follows that the condition of interference between direct and exchange resonant amplitudes implies that  $\delta_i = \delta_f$  and  $\theta_i \cong \theta_f$ , that is, initial and final electrons form the equal angles with the external field photon momentum and are located on different sides of this photon momentum. Also, it can be seen from (35) that  $\theta_i \sim \omega m^2/E_i^3 \ll 1$ . When an ultrarelativistic initial electron moves within the narrow cone with an external field photon and scatters into the large angle  $\theta_i \gg \omega m^2/E_i^3$  the resonant cross section is derived from Eq. (50) under the condition (87):

$$d\sigma_{res} = \frac{(1 + \delta_i^2) m}{4 (nn') E_i \Gamma_\tau \sqrt{1 + \mu_\tau^2}} \cdot dW^{(1)} \cdot d\sigma_S(q_i). \quad (89)$$

Here, the spontaneous photon resonant frequency is given by Eq. (88) with value  $r = 1$ , and the angle of spontaneous photon emission is not close to direction of initial electron motion. Ratio of the resonant cross section (89) to the conventional cross section of electron-nucleus bremsstrahlung may be derived from Eq. (76) with respect to Eq. (87):

$$R_{res} = f_3 \cdot \pi^2 \eta_0^2 \frac{\omega \tau}{\sqrt{1 + \mu_\tau^2}} \left( \frac{m}{E_i} \right)^2, \quad (90)$$

where the function  $f_3 \sim 1$  and has a rather cumbersome form.

It may be easily estimated that for the pulsed field parameters  $\omega = 2.35$  eV,  $\tau = 1.5$  ps,  $F_0 = 6 \cdot 10^9$  V/cm,  $\rho = 5$  and the electron energy  $E_i = 5$  MeV the resonant cross section is of the order of the ordinary cross section when the angle of spontaneous photon emission is detected simultaneously with the ejection angle of an electron scattered on the large angle.

#### 2.4.5 Range of ultrarelativistic energies of electrons moving within a narrow cone with a spontaneous photon

We suppose that an ultrarelativistic electron (an initial or a final one) moves within the narrow cone with a spontaneous photon. Then the quantities  $\kappa'_{i,f}$  (30) may be written in an analogous to Eq. (87) form, where

$$\delta_{i,f} \rightarrow \delta'_{i,f} = \theta'_{i,f} \cdot E_{i,f}/m. \quad (91)$$

Here, depending on the electron energy we may deal with one of two possible situations. It is provided that  $m \ll E_{i,f} \ll m^2/\omega$ , then resonant frequencies fall within the interval  $\omega \ll \omega'_{i,f} \ll E_{i,f}$  and are given by

$$\omega'_{i,f} = r\omega_{i,f}, \quad \omega_{i,f} = \frac{2 (nn')}{(1 + \delta'^2_{i,f})} \cdot \left( \frac{E_{i,f}}{m} \right)^2 \cdot \omega. \quad (92)$$

It was demonstrated by Roshchupkin (1985) that resonances do not occur for energies  $E_{i,f} \gg m^2/\omega$ . It is obviously that direct and exchange resonant amplitudes may interfere with each other only when initial and final electrons move within the narrow cone with a spontaneous photon, so  $\delta'_i = \delta'_f$ .

When an ultrarelativistic initial electron moves within the narrow cone with a spontaneous photon and scatters on the large angle  $\theta \gg \omega/E_i$  we may use Eqs. (50)-(52) to find the resonant cross section. In this case, it is convenient to represent the resonant denominator in the following form

$$\left(q_i^2 - (m - i\Gamma_R)^2\right)^2 + (2m\Gamma_\tau)^2 = m^4 \left[ \left(x - \delta_i'^2\right)^2 + y^2 \right] \cdot \frac{u^2}{(1+u)^2}, \quad (93)$$

where

$$x = \frac{u_1}{u} + \frac{(1+u)\Gamma_\tau^2(1+\mu_\tau^2)}{u \cdot m^2} - 1, \quad y = \frac{2(1+u)\Gamma_\tau\sqrt{1+\mu_\tau^2}}{u \cdot m}. \quad (94)$$

Here, the invariant parameters  $u, u_1$  are given by

$$u \cong \frac{\omega'}{E_i - \omega'}, \quad u_1 = 2(nn') \cdot \frac{\omega E_i}{m^2}. \quad (95)$$

Now Eqs. (50)-(52), (93) are to be taken into account, the solid angle is to be written as  $d\Omega' = (m^2/2E_i^2) d\varphi d\delta_i'^2$ , and integration should be performed with respect to the azimuthal angle, and  $\delta_i'^2$  within the limits from zero to  $+\infty$ . Thus, we derive the following expression for the resonant cross section:

$$d\sigma_{res} = Y(xy) \cdot \frac{q_{i0}}{m\Gamma_\tau\sqrt{1+\mu_\tau^2}} \cdot dW^{(1)} \cdot d\sigma_S(q_i). \quad (96)$$

Here,

$$Y(xy) = \frac{1}{\pi} \int_0^\infty \frac{d\delta_i'^2}{\left(x - \delta_i'^2\right)^2 + y^2} = \frac{1}{2} + \frac{1}{\pi} \arctg\left(\frac{x}{y}\right) \quad (97)$$

is a smoothed step function. In regions far from the resonance  $|u_1 - u| \gg 2(1+u)(\Gamma_\tau/m)$  and at the resonance point  $u_1 = u$  this function takes the following limiting values:

$$Y(xy) = \begin{cases} 1, & \text{if } u < u_1, \\ 0.5, & \text{if } u = u_1, \\ y \cdot u / \pi(u - u_1), & \text{if } u > u_1. \end{cases} \quad (98)$$

The probability is given by

$$dW^{(1)} = \alpha\eta^2 \cdot \frac{m^2}{4E_i} \cdot \left\{ 2 + \frac{u^2}{1+u} - \frac{4u}{u_1} \left( 1 - \frac{u}{u_1} \right) \right\} \cdot \frac{du}{(1+u)^2}. \quad (99)$$

We consider ratio of the resonant cross section (96) to the conventional cross section of electron-nucleus bremsstrahlung in the case when an ultrarelativistic electron moves within the narrow cone with a photon produced in bremsstrahlung and scatters on the large angles. Using the results obtained by Baier et al. (1973) we may write the following expression:

$$R_{res} = \frac{d\sigma_{res}^{(1)}}{d\sigma_a} = Y(xy) \cdot \frac{E_i}{m\Gamma_\tau\sqrt{1+\mu_\tau^2}} \cdot \frac{dW^{(1)}}{dW_{p_i}(k')}. \quad (100)$$

Here,  $dW_{p_i}(k')$  is the probability that an electron with the four-momentum  $p_i$  emits a photon with the four-momentum  $k'$ . For electron energies  $m \ll E_i \ll m^2/\omega$  the expression (100) may be written as

$$R_{res} = \frac{\sqrt{a_2}}{8} \pi \frac{\eta_0^2 \omega \tau}{\sqrt{1 + \mu_\tau^2}} \cdot \frac{1}{\ln(E_i/m)}. \quad (101)$$

If the considered process characteristics satisfy the conditions (64), than the parameter  $\mu_\tau \ll 1$  (70) and the resonant shape is specified by the laser pulse duration. Eq. (101) implies, when the ultrarelativistic electron energy grows, the resonant cross section decreases drastically.

### 3. Resonant photoproduction of an electron-positron pair on a nucleus in the field of a pulsed light wave

The most general computations of the resonant Coulomb electron-positron pair photoproduction (CPP) on a nucleus in the field of an electromagnetic plane wave was performed by Roshchupkin (1983). Borisov et al. (1981) studied the resonant CPP in the special case of ultrarelativistic electron and positron energies where the incident photon and the wave photon fly toward each other. The work of Lötstedt et al. (2008) in which resonant cross sections were calculated for strong external fields should also be noted. The resonant CPP in the pulsed light wave was studied in detail in the work of Lebed' & Roshchupkin (2011).

We consider the photoproduction of an electron-positron pair on a nucleus in a pulsed light field (2). The interaction of an electron and positron with a nucleus is considered in the first order of the perturbation theory (the Born approximation). Note that CPP is a crossed channel of bremsstrahlung due to electron scattering by a nucleus. Spontaneous bremsstrahlung of an electron scattered by a nucleus in a pulsed light field was studied early. In consideration of the known calculation procedure we may obtain the amplitude of CPP process on a nucleus in the field of a moderately strong pulsed wave from the expressions (9)-(18) by the following replacement:

$$p_- \rightarrow p_f, \quad p_+ \rightarrow -p_i, \quad k_i \rightarrow -k', \quad (102)$$

where  $p_-$ ,  $p_+$ ,  $k_i$  are the four-momenta of an electron, a positron and an initial photon, respectively. For CPP on a nucleus  $q = (q_0, \mathbf{q})$  is the four-vector is the transferred momentum,  $q_-$  and  $q_+$  are the four-momenta of an intermediate electron and an intermediate positron (for the diagrams on Fig. 7 (a) and (b), respectively). These quantities are expressed by the relationships:

$$\begin{cases} q = p_- + p_+ - k_i + lk, \\ q_- = k_i + rk - p_+, \\ q_+ = k_i + rk - p_-. \end{cases} \quad (103)$$

#### 3.1 Resonance conditions

Let us consider the resonances that occur when an intermediate particle reaches the mass shell. The conditions of resonant CPP on a nucleus in a pulsed light field is determined by the relationship

$$q_\pm^2 - m^2 \lesssim \frac{(kq_\pm)}{\omega\tau}. \quad (104)$$

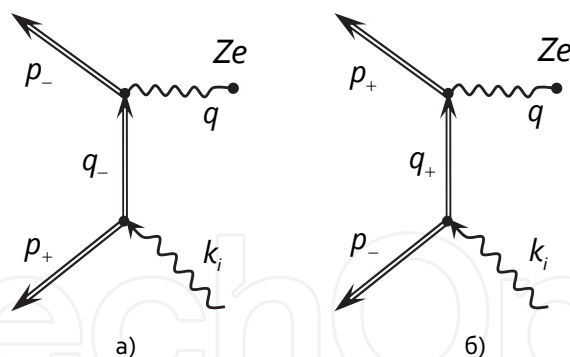


Fig. 7. Photoproduction of an electron–positron pair on a nucleus in a pulsed light wave.

Consequently, the four-momentum of an intermediate particle appears near the mass surface under the resonant conditions.

It is convenient to write Eqs. (103), which define the four-momenta  $q$  and  $q_{\pm}$ , for amplitudes (a) and (b) in Fig. 7, respectively, as

$$\begin{cases} k_i + rk = q_- + p_+, \\ q = p_- - q_- + (l + r)k; \end{cases} \quad (105)$$

$$\begin{cases} k_i + rk = p_- + q_+, \\ q = p_+ - q_+ + (l + r)k. \end{cases} \quad (106)$$

Eqs. (105)–(106) represent the four-momentum conservation laws for the diagrams vertices (Fig. 7) that, in view of the condition (104), hold only for  $r > 0$ .

Taking into account the condition (104) we will obtain the initial photon frequency  $\omega_i^{res}$  for which a resonance can be observed (the resonant frequency) from the Eq. (105). Within the zeroth order with respect to the small parameter  $(\omega\tau)^{-1}$  for the diagrams (a) and (b) (see Fig. 7), we obtain

$$\omega_i^{res} = \omega_i^{\pm} \equiv r\omega \cdot \frac{(np_{\mp})}{(n_i q_{\pm})}, \quad (107)$$

$$n = k/\omega = (1, \mathbf{n}), \quad n_i = k_i/\omega_i = (1, \mathbf{n}_i). \quad (108)$$

Within the region of moderately strong fields (5) the energy conservation law ( $q_0 \approx 0$ ) may be written as

$$\omega_i \approx E_- + E_+. \quad (109)$$

Therefore, it follows from Eq. (107) that within the moderately strong fields region resonances are possible only for ultrarelativistic positron  $p_+$  (diagram (a), Fig. 7) and electron  $p_-$  (diagram (b), Fig. 7), if they move within a narrow cone with the incident  $\gamma$ -ray photon  $k_i$ . In this case resonant frequencies (107) take the form

$$\omega_i^{\pm} = \frac{E_{\pm}}{1 - W_{\pm}/E_{\pm}}, \quad W_{\pm} = \frac{m^2}{r\omega} \cdot \frac{(1 + \delta_{i\pm}^2)}{2(nn_i)}, \quad (110)$$

where

$$\delta_{i\pm} = \theta_{i\pm} \cdot (E_{\pm}/m), \quad \theta_{i\pm} = \angle(\mathbf{k}_i, \mathbf{p}_{\pm}) \ll 1. \quad (111)$$

Hence the resonances are possible only for the electron (positron) energies above some threshold value  $W_{\pm}$ :  $E_{\pm} > W_{\pm} \sim m^2/\omega$ .

Using Eqs. (110) it is easy to obtain the positron energy at resonance:

$$E_+ = \frac{1}{2} \left\{ 1 \pm \sqrt{1 - \frac{\omega_i^{th}}{\omega_i}} \right\} \cdot \omega_i, \quad (112)$$

where  $\omega_i^{th}$  is the threshold frequency of an incident  $\gamma$ -ray photon,

$$\omega_i^{th} = \frac{2m^2}{\omega(1 - \cos \theta_i)}, \quad \theta_i = \angle(\mathbf{k}, \mathbf{k}_i). \quad (113)$$

As we see from Eq. (113), the threshold energy of an initial photon appreciably depends on its orientation relative to wave propagation direction. Thus, the threshold energy is minimal when an incident photon propagates towards the wave. In the opposite case, when an initial photon moves parallel to external field photons, no resonances are observed. Note that the electron energy can be obtained from Eq. (112) by reversing the sign in front of the square root. It follows from Eq. (112), that the energies of produced electron and positron near the threshold ( $\omega_i - \omega_i^{th} \ll \omega_i^{th}$ ) are equal  $E_+ = E_- \cong \omega_i^{th}/2$ . If, alternatively, the frequency of an incident  $\gamma$ -ray photon is great ( $\omega_i \gg \omega_i^{th}$ ) then electron and positron energies differ considerably ( $E_+ = \omega_i - \omega_i^{th}/4 \approx \omega_i$ ,  $E_- \approx \omega_i^{th}/4$ ).

The condition of interference of resonant amplitudes, that is  $\omega_i^+ = \omega_i^-$ , assumes the form

$$(np_-)(n_i q_-) = (np_+)(n_i q_+). \quad (114)$$

Using the energy conservation law (109) and Eq. (110) we derive that the interference of resonant amplitudes appears when an electron  $p_-$  and a positron  $p_+$  propagate within a narrow cone with an incident  $\gamma$ -ray photon  $k_i$ , with  $\delta_{i-} = \delta_{i+}$  and  $\theta_{i-} \sim \omega/E_-$ .

Below, we will consider the resonance of one diagram. We will assume that the initial photon frequency is

$$\omega_i \approx \omega_i^{res} = \omega_i^-. \quad (115)$$

### 3.2 Resonant amplitude

The amplitude of CPP on a nucleus in a pulsed light field under resonance conditions (107) has the form

$$S^{(\pm)} = \sum_{l=-\infty}^{\infty} S_l^{(\pm)}, \quad (116)$$

where  $S_l^{(\pm)}$  is the partial amplitude, which corresponds to processes with emission ( $l > 0$ ) or absorption ( $l < 0$ ) of laser-wave  $|l|$  photons

$$S_l^{(\pm)} = -i \frac{Ze^3 \sqrt{\pi}}{\sqrt{2\omega_i E_- E_+}} \bar{u}_- [B_{l-}(\tilde{\gamma}_0, \hat{\epsilon}_i) + B_{l+}(\hat{\epsilon}_i, \tilde{\gamma}_0)] u_+. \quad (117)$$

Here, the functions  $B_{l-}(\tilde{\gamma}_0, \hat{\varepsilon}_i)$  and  $B_{l+}(\hat{\varepsilon}_i, \tilde{\gamma}_0)$  correspond to the CPP diagrams in Figs. 7(a) and 7(b), respectively

$$B_{l-}(\tilde{\gamma}_0, \hat{\varepsilon}_i) = \sum_{r=-\infty}^{\infty} \frac{2\omega\tau^2}{\mathbf{q}^2 + q_0(q_0 - 2q_z)} \cdot \frac{i\pi}{2(kq_-)} \times \\ \times \int_{-\infty}^{\infty} d\phi L_{l+r}(\phi) \exp\{i(q_0\tau + 2\beta)\phi\} \cdot \tilde{\gamma}_0(\hat{q}_- + m) \times \\ \times \int_{-\infty}^{\infty} d\phi' F_{-r}(\phi') \exp\{-2i\beta\phi'\} (\text{sgn}(\phi' - \phi) - 1), \quad (118)$$

$$\beta = \frac{q_-^2 - m^2}{4(kq_-)} \omega\tau. \quad (119)$$

Here, functions  $F_{-r}(\phi')$  and  $L_{l+r}(\phi)$  are defined by relations (15)-(21) with the replacement (102).

With allowance of the four-momentum conservation law (that is the first Eq. in (105)), the matrix function  $F_{-r}(\chi_{q-p_+}, \gamma_{q-p_+}(\phi'), \beta_{q-p_+}(\phi'))$  (15) under resonance conditions defines the amplitude of the production of an electron-positron pair with the four-momenta  $q_-$  and  $p_+$  by a photon with the four-momentum  $k_i$  in a pulsed light field through  $r$  wave photons absorption. This process was considered by Nikishov & Ritus (1979) in the case of a plane monochromatic wave, and by Narozhny & Fofanov (1997) in the case of a pulsed light wave. With allowance of the transferred four-momentum  $q$  (see the second equality in (105)) the quantity  $L_{l+r}(\chi_{p-q_-}, \gamma_{p-q_-}(\phi), \beta_{p-q_-}(\phi)) \tilde{\gamma}_0$  defines the amplitude of scattering of an intermediate electron with the four-momentum  $q_-$  by a nucleus in a pulsed light field with absorption or emission of  $|l+r|$  photons of the wave (Lebed' & Roshchupkin (2008)).

Consequently, if the interference between direct and exchange amplitudes is absent, the process of resonant CPP on a nucleus in a pulsed light field effectively decomposes into two consecutive processes of the first order. The distinction for the diagram (b) on Fig. 7 is concluded in replacement of the four-momentum of an intermediate electron  $q_- \rightarrow -q_+$  and change of sequence of first order processes.

Integral functions (18) are determined by the integer-order Bessel functions (23) for the case of a circularly polarized external wave. For circular polarization of a wave under resonance conditions the arguments of the Bessel functions (20) for CPP on a nucleus may be represented as

$$\gamma_{q-p_+}(\phi') = 2r \cdot \eta_0 g(\phi') \cdot \frac{1+z_+}{z_+z_r} \sqrt{z_+z_r - (1+z_+)^2}, \quad (120)$$

where the invariant parameters  $z_+$  and  $z_r$  are defined by

$$z_+ = \frac{(kp_+)}{(kq_-)} \approx \frac{E_+}{\omega_i - E_+}, \quad z_r = 2r \cdot \frac{(kk_i)}{m^2}. \quad (121)$$

It was expected for this part of the amplitude that the Bunkin-Fedorov quantum parameter becomes a classical one under resonance conditions (see Eqs. (32)-(33)).

$$\gamma_{q-p_+}(\phi') \sim \eta_0 \ll 1. \quad (122)$$



Consequently within the field range, specified by Eq. (5), the first resonance, that is, the resonance with  $r = 1$ , provides the main contribution to the resonant cross section, when the Bessel function has the largest value. This implies that the single-photon production of an electron-positron pair in a pulsed field proceeds mainly through absorption of one external field photon. However, the argument of the Bessel function  $J_{l+r}(\gamma_{p-q_-}(\phi))$  is of the order of a magnitude  $\gamma_{p-q_-}(\phi) \sim \gamma_0 \gtrsim 1$ , i.e. it saves the quantum nature. Thus, scattering of an intermediate electron by a nucleus in the field of a moderately strong pulsed wave is a multiquantum process.

We perform the subsequent analysis for the case of wave circularly polarization ( $\delta = \pm 1$ ) at expense of one wave photon absorption, i.e.  $r = 1$ . In view of the envelope function (40), after simple manipulations we obtain the amplitude (118) in the form

$$B_{l-}(\tilde{\gamma}_0, \hat{\epsilon}_i) = \frac{2\pi \cdot \tilde{\gamma}_0 (\hat{q}_- + m) \hat{F}}{\mathbf{q}^2 + q_0 (q_0 - 2q_z)} \cdot \frac{-i\omega\tau^2\sqrt{\pi}}{4(kq_-)} \exp\left\{-\frac{\beta^2}{4}\right\} \cdot I(q_0, \beta), \quad (123)$$

$$\hat{F} = -\frac{1}{2} \exp\{i\chi_{q-p_+}\} \cdot \gamma_{q-p_+}(0) \cdot \hat{\epsilon}_i + ((e_x + i\delta e_y) b), \quad (124)$$

$$I(q_0, \beta) = \int_{-\infty}^{\infty} d\phi \cdot J_{l+1}(\phi) \exp\{i(q_0\tau + 2\beta)\phi\} \left(\operatorname{erf}\left(2\phi + \frac{i\beta}{2}\right) + 1\right). \quad (125)$$

Here,  $\operatorname{erf}(2\phi + i\beta/2)$  is the error function.

### 3.3 Resonant cross section

The differential cross section of CPP on a nucleus in a pulsed light field may be easily obtained by standard mode (Berestetskii et al. (1982)) from the amplitude, Eqs. (116)-(117), (123)-(125)

$$d\sigma^{(\pm)} = \sum_{l=-\infty}^{\infty} d\sigma_l^{(\pm)}, \quad (126)$$

where  $d\sigma_l^{(\pm)}$  is the partial cross section of CPP on a nucleus in a pulsed light field with emission ( $l > 0$ ) or absorption ( $l < 0$ ) of  $|l|$  wave photons.

Under resonance conditions and for ultrarelativistic electron and positron energies, the energy contribution from external pulsed field photons may be neglected. Therefore, the resonant cross section (126) may be summed over all possible partial processes. Thus, the differential cross section of CPP on a nucleus in a pulsed light field with the positron energy in the interval  $[E_+, E_+ + dE_+]$  within the solid angle  $[\Omega_+, \Omega_+ + d\Omega_+]$  and the final electron within the solid angle  $[\Omega_-, \Omega_- + d\Omega_-]$  assumes the form

$$\frac{d\sigma_{1res}^{(\pm)}}{dE_+ d\Omega_+ d\Omega_-} = \frac{Z^2 e^6}{(2\pi)^2} \frac{|\mathbf{p}_-| |\mathbf{p}_+|}{\omega_i \mathbf{q}^4} |\bar{u}_- M_- u_+|^2 \cdot P_{res}, \quad (127)$$

$$M_- = \tilde{\gamma}_0 (\hat{q}_- + m) \hat{F}. \quad (128)$$

In Eq. (127) the function  $P_{res}$  is defined by the expression (53), where the replacement  $q_i \rightarrow q_-$  has to be performed. We don't take polarization effects into consideration. After

performing of corresponding averaging and summation procedures and considering that  $d\Omega_+ = (m^2/2E_+^2) d\delta_{i+}^2 d\varphi_{az}$  we derive

$$d\sigma_{1res}^{(\pm)} = \frac{1}{2\pi^2} \cdot \frac{m^2\omega_i}{z_+} \cdot P_{res} \cdot d\sigma_s(q_-) dW_{pair}^{(1)} d\delta_{i+}^2 d\varphi_{az}. \quad (129)$$

Here,

$$d\sigma_s(q_-) = 2Z^2 r_e^2 \frac{|\mathbf{p}_-| m^2}{|\mathbf{q}_-|^4} \left( m^2 + E_- q_{0-} + \mathbf{p}_- \cdot \mathbf{q}_- \right) d\Omega_- \quad (130)$$

is the differential cross section of scattering of an intermediate electron with the four-momentum  $q_-$  by a nucleus, and

$$dW_{pair}^{(1)} = \alpha \frac{\eta_0^2 m^2}{4\omega_i} \left\{ \frac{4(1+z_+)^2}{z_+ z_1} \left( 1 - \frac{(1+z_+)^2}{z_+ z_1} \right) - 2 + \frac{(1+z_+)^2}{z_+} \right\} \cdot \frac{dz_+}{(1+z_+)^2} \quad (131)$$

is the probability of production of an electron-positron pair with the four-momenta  $q_-$  and  $p_+$  by the an incident photon with the four-momentum  $k_i$  at the expense of one wave photon absorption. We can perform integration in Eq. (129) over the azimuthal angle  $d\varphi_{az}$  and  $d\delta_{i+}^2$ . At that replacement  $d\delta_{i+}^2 \rightarrow d\beta$  is to be carried out. The parameter  $\beta$  (119) under resonance conditions assumes the form

$$\beta = \frac{\omega\tau}{2} \left[ 1 - \frac{(1+z_+)^2}{z_+ z_1} \left( 1 + \delta_{i+}^2 \right) \right]. \quad (132)$$

We derive consequently

$$d\sigma_{1res}^{(\pm)} = \sqrt{\frac{\pi}{2}} \cdot \frac{\omega\tau}{2} \cdot \frac{\omega_i}{m^2 z_1} \cdot d\sigma_s(q_-) dW_{pair}^{(1)}. \quad (133)$$

Within the kinematical region of resonance, CPP on a nucleus in external field absence was investigated by Baier et al. (1973). It was concluded that amplitudes (a) and (b) (see Fig. 7) have poles within different regions of pair emission angles, therefore, they do not interfere. At that, the cross section is factorized, i.e.

$$d\sigma_{pair} = dW_{k_i}(p_+, q_-) \cdot d\sigma_s(q_-), \quad (134)$$

where  $q_- = k_i - p_+$ ;  $dW_{k_i}(p_+, q_-)$  is the probability of production of an electron-positron pair  $(p_+, q_-)$  by an incident  $\gamma$ -ray photon with the four-momentum  $k_i$ . We express the resonant cross section (133) in terms of ordinary one (134),

$$R_{res} = \frac{d\sigma_{1res}^{(\pm)}}{d\sigma_{pair}} = \frac{\omega_i}{4m\Gamma_\tau(1+z_+)} \cdot \frac{dW_{pair}^{(1)}}{dW_{k_i}(p_+, q_-)}. \quad (135)$$

The transit width  $\Gamma_\tau$  of the resonance was introduced here. It has the form

$$\Gamma_\tau = \sqrt{\frac{2}{\pi}} \cdot \frac{1}{\omega\tau} \cdot \frac{(kq_-)}{m}. \quad (136)$$

It is obvious from Eq. (136) that the transit width is specified by the pulsed field frequency and duration as well as by the particle energy and process kinematics. We underline that when CPP on a nucleus in the field of a plane monochromatic wave is studied the divergence in the differential cross section is eliminated by introducing of radiative corrections into the Green function of an intermediate particle according to the Breit-Wigner prescription as usual. It is concluded in addition of the imaginary part of the electron or positron mass:  $m \rightarrow m - i\Gamma_R$ . Here, the radiation width of resonance  $\Gamma_R$  is introduced phenomenologically. It has the form

$$\Gamma_R = \frac{1}{3} \alpha \eta_0^2 \cdot \frac{\sigma_c(q_-)}{\sigma_T} \cdot \frac{(kq_-)}{m}, \quad (137)$$

where  $\sigma_c(q_-)$  is the total cross section of the Compton scattering of an external field photon by an intermediate electron with the four-momentum  $q_-$  (it is the most probable channel of an electron escape from an intermediate state), and  $\sigma_T$  is the Thompson cross section. Comparison of resonant widths (136) and (137) ascertains that the transit width exceeds the radiation one if laser pulse parameters satisfy the condition

$$\omega\tau < \frac{3}{\alpha \eta_0^2} \cdot \frac{\sigma_c(q_-)}{\sigma_T}. \quad (138)$$

Moderately strong fields of optical frequencies and the picosecond range of widths meet the inequality (138). The titanium-sapphire laser (Ti:Sapphire) or the solid-state laser based on aluminum-yttrium garnet  $Y_2Al_5O_{12}$  with neodymium Nd admixtures (Nd:YAG) can be used as sources of such pulsed fields. Titanium-sapphire lasers have a broad lasing band (700-1100 nm) and a wide range of pulse duration (10 ps –10 fs) due to various choices of pulse compression. The PICAR picosecond Nd:YAG laser (designed at the International Educational-Scientific Laser Center of the Moscow State University named by M.V. Lomonosov) appropriate field characteristics to be achieved through the combined action of active-passive mode locking and a negative feedback (Gorbunkov et al. (2005)).

Ratio of cross-sections (135) is simplified considerably in the logarithmic approximation:

$$R_{res} = \frac{\pi}{8} \sqrt{\frac{\pi}{2}} \cdot \eta_0^2 \omega \tau \cdot \left[ \ln \frac{E_+}{m} \right]^{-1}. \quad (139)$$

Let us estimate the ratio of the cross sections (139) for PICAR picosecond Nd:YAG laser with additional amplifiers with parameters  $\eta_0 \approx 0.1$ ,  $\lambda = 1064$  nm ( $\omega = 1.17$  eV),  $\tau = 25$  ps. An incident  $\gamma$ -ray photon with an energy near the threshold value (113)  $\omega_i = 5 \cdot 10^5 m = 255$  GeV propagates towards the pulsed laser wave. We obtain the following ratio of cross-sections:  $R_{res} \approx 40$ . Consequently, the resonant cross-section of CPP on a nucleus in a pulsed light field may exceed the corresponding one in external field absence by an order of magnitude.

#### 4. Resonant scattering of a lepton by a lepton in the pulsed light field

Study of various processes of leptons scattering in an external electromagnetic fields is one of the fundamental directions of QED. Cross sections of basic scattering processes in the external field absence were obtained in the middle of the twentieth century. Thus, the scattering of an electron by an electron was considered by Möller (1932), the scattering of an electron by

a positron - Bhabha (1938), the scattering of an electron by a muon - by Bhabha (1938) and Massey & Corben (1939). The detailed consideration of nonresonant scattering of an electron by a muon in a pulsed light field was performed by Padusenko et al. (2009).

We underline that the Bunkin–Fedorov quantum parameter  $\gamma_0$  (4) is the main one which determines multiphoton processes in leptons nonresonant scattering. However in the case of leptons resonant scattering the influence of the quantum parameter  $\gamma_0$  does not appear (it becomes a classical one due to resonance conditions and possess the values in order to  $\eta_0$ ), thus the classical parameter  $\eta_0$  (3) determines multiphoton processes. Therefore study of lepton by a lepton resonant scattering is carried out within the intensity range (5), that is within the framework of the first order of the perturbation theory with respect to an external laser field.

The electron mass  $m_e$  is considerably less than the muon one  $m_\mu$  ( $m_e \ll m_\mu$ ), therefore the corresponding classical parameters (3) satisfy the following condition as well

$$\eta_{0\mu} \ll \eta_{0e}. \quad (140)$$

The classical parameters  $\eta_{0\mu}$  and  $\eta_{0e}$  are defined by Eqs. (3), where replacements  $m \rightarrow m_\mu$  and  $m \rightarrow m_e$  are to be performed. Hereinafter we consider resonances for direct Feynman diagrams of scattering type exceptionally (Fig. 8). Exchange diagrams for identical leptons and annihilation diagrams of scattering of a lepton by an antilepton are outside of attention. Such a problem statement is possible due to fact that resonances for direct diagrams of scattering type and resonances for exchange (annihilation) diagrams within the intensity range (5) occur within essentially different nonoverlapping kinematical regions (Roshchupkin & Voroshilo (2008)). For direct scattering amplitude within the fields range (5) the process of lepton by a lepton resonant scattering occurs when leptons scatter forwards into the small angles in the frame of the reference related to the center of inertia of initial particles and effectively decomposes into two processes of the first order similar to the Compton scattering of a wave by a lepton.

The S-matrix element for a direct amplitude (see Fig. 8) is given by

$$S = ie^2 \int d^4x_1 d^4x_2 D_{\mu\mu'}(x_1 - x_2) \times \\ \times \left[ \bar{\psi}_{p'_1}(x_1|A) \tilde{\gamma}^\mu \psi_{p_1}(x_1|A) \right] \left[ \bar{\psi}_{p'_2}(x_2|A) \tilde{\gamma}^{\mu'} \psi_{p_2}(x_2|A) \right]. \quad (141)$$

Here,  $D_{\mu\mu'}(x_1 - x_2)$  is the Green function of an intermediate free photon;  $\psi_{p_j}(x|A)$  and  $\bar{\psi}_{p'_j}(x|A)$  are the wave functions of initial and final leptons in the field of a pulsed light wave (2), respectively ( $j = 1, 2$ ).

The amplitude of scattering of a lepton  $l_1$  (with the mass  $m_1$  and the four-momentum  $p_1$ ) by a lepton  $l_2$  (with the mass  $m_2$  and the four-momentum  $p_2$ ) in a pulsed light field may be represented as a sum of partial components with emission ( $l > 0$ ) and absorption ( $l < 0$ )  $|l|$  wave photons:

$$S = \sum_{l=-\infty}^{\infty} S_l, \quad (142)$$

$$S_l = \frac{(2\pi)^4 ie^2}{2\sqrt{E_1 E_2 E'_1 E'_2}} \delta(q_x) \delta(q_y) \delta(q_0 - q_z) D_{ls}. \quad (143)$$

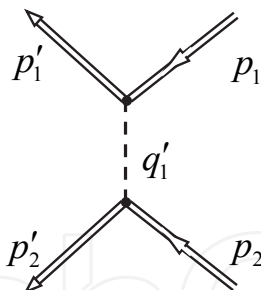


Fig. 8. The Feynman diagram of direct amplitude of scattering of a lepton  $l_1$  by a lepton  $l_2$  in the field of a pulsed light wave. External incoming and outgoing double lines correspond to the wave functions of leptons in initial and final states in the field of a plane wave (the Volkov functions), and an inner dashed line corresponds to a Green function of a free photon.

Here, the arguments of delta-functions are the four-vector  $q = (q_0, \mathbf{q})$  components

$$q = p'_1 + p'_2 - p_1 - p_2 + lk. \quad (144)$$

The function  $D_{ls}$  in Eq. (143) has the form

$$D_{ls} = \sum_{s=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(\bar{u}_{p'_1} \Lambda_{l-s}^{\nu}(\zeta) u_{p_1}) (\bar{u}_{p'_2} \Lambda_{sv}(\zeta) u_{p_2})}{q'^2_1 + 2\zeta(kq'_1) + i0} d\zeta. \quad (145)$$

Here,  $q'_1$  is the four-vector of an intermediate photon

$$q'_1 = p'_2 - p_2 + sk = p_1 - p'_1 + (l-s)k, \quad (146)$$

and functions  $\Lambda_{l-s}^{\nu}(\zeta)$ ,  $\Lambda_{sv}(\zeta)$  are represented by

$$\begin{cases} \Lambda_{l-s}^{\nu}(\zeta) = \tau \int_{-\infty}^{\infty} d\phi_1 \cdot G_{l-s}^{\nu}(\phi_1) \cdot \exp \left\{ i \frac{q_0 + q_z}{2} \tau \phi_1 \right\} \cdot \exp \{ -i(\zeta \omega \tau) \phi_1 \}, \\ \Lambda_{sv}(\zeta) = \tau \int_{-\infty}^{\infty} d\phi_2 \cdot G_{sv}(\phi_2) \cdot \exp \{ i(\zeta \omega \tau) \phi_2 \}. \end{cases} \quad (147)$$

Functions  $G_{l-s}^{\nu}(\phi_1)$  in Eq. (147) have the form

$$\begin{aligned} G_{l-s}^{\nu}(\phi_1) = & a^{\nu} L_{l-s}(\phi_1) + \eta_{01}(\phi_1) \frac{m_1}{4\omega\kappa_1} \tilde{\gamma}^{\nu} \hat{k} [\hat{\varepsilon}_{-} L_{l-s+1}(\phi_1) + \hat{\varepsilon}_{+} L_{l-s-1}(\phi_1)] + \\ & + \eta_{01}(\phi_1) \frac{m_1}{4\omega\kappa'_1} [\hat{\varepsilon}_{-} L_{l-s+1}(\phi_1) + \hat{\varepsilon}_{+} L_{l-s-1}(\phi_1)] \hat{k} \tilde{\gamma}^{\nu} + \\ & + (1 - \delta^2) \eta_{01}^2(\phi_1) \frac{m_1^2}{8\omega^2\kappa_1\kappa'_1} k^{\nu} \hat{k} [(L_{l-s+2}(\phi_1) + L_{l-s+2}(\phi_1))], \end{aligned} \quad (148)$$

$$a^{\nu} = \tilde{\gamma}^{\nu} + (1 + \delta^2) \eta_{01}^2(\phi_1) \frac{m_1^2}{4\omega^2\kappa_1\kappa'_1} k^{\nu} \hat{k}, \quad (149)$$

$$\eta_{0j}(\phi_j) = \eta_{0j} \cdot g(\phi_j). \quad (150)$$

Here,  $\hat{\varepsilon}_{\pm}$  is the compression of four-vectors  $\varepsilon_{\pm} = e_x \pm i\delta e_y$  with the Dirac's  $\tilde{\gamma}^{\nu}$ -matrices. The expression for the function  $G_{sv}(\varphi_2)$  is ensued from Eqs. (148)-(149) by following indices replacement:  $1 \rightarrow 2, l-s \rightarrow s, l-s \pm 1 \rightarrow s \pm 1, l-s \pm 2 \rightarrow s \pm 2$  and by the index  $\nu$  omission also. By means of  $\kappa_j$  and  $\kappa'_j$  in functions  $G_{l-s}^{\nu}(\varphi_1)$  and  $G_{sv}(\varphi_2)$  the following expressions are denoted

$$\begin{cases} \kappa_j = E_j - \mathbf{n} \mathbf{p}_j, \\ \kappa'_j = E'_j - \mathbf{n} \mathbf{p}'_j. \end{cases} \quad (151)$$

Here,  $\mathbf{n}$  is the unit vector along the direction of external wave propagation

$$\mathbf{n} = \frac{\mathbf{k}}{|\mathbf{k}|}. \quad (152)$$

There are the integral functions  $L_{l-s}(\varphi_1)$ ,  $L_s(\varphi_2)$  which determine probability of emission and absorption of external wave photons in Eqs. for  $G_{l-s}^{\nu}(\varphi_1)$ ,  $G_{sv}(\varphi_2)$ . They have the form

$$\begin{aligned} L_n(\varphi_j) &\equiv L_n(\chi_j, \gamma_{0j}(\varphi_j), \beta_j(\varphi_j)) = \\ &= \frac{1}{2\pi} \int_0^{2\pi} d\varphi'_j \exp \left\{ i \left[ \gamma_{0j}(\varphi_j) \sin(\varphi'_j - \chi_j) + \beta_j(\varphi_j) \sin 2\varphi'_j - n\varphi'_j \right] \right\} \end{aligned} \quad (153)$$

( $j = 1$  for  $n = l-s$ ,  $j = 2$  for  $n = s$ ) with the arguments

$$\gamma_{0j}(\varphi_j) = \eta_{0j}(\varphi_j) \cdot \frac{m_j}{\omega} \sqrt{(e_x g_j)^2 + \delta^2 (e_y g_j)^2}, \quad (154)$$

$$\text{tg} \chi_j = \delta \frac{(e_y g_j)}{(e_x g_j)}, \quad g_j = \frac{p'_j}{\kappa'_j} - \frac{p_j}{\kappa_j}, \quad (155)$$

$$\beta_j(\varphi_j) = (1 - \delta^2) \eta_j^2(\varphi_j) \frac{m_j^2}{8\omega} \left[ \frac{1}{\kappa'_j} - \frac{1}{\kappa_j} \right]. \quad (156)$$

Before performing of integration of the function  $D_{ls}$  (145) over the variable  $\zeta$  we remind that the subject of studying is the resonant character of amplitude behavior caused by quasi discrete structure: charged particle + plane electromagnetic wave. It is obvious that the resonant character of lepton-lepton scattering occurs when the denominator of the function  $D_{ls}$  approaches zero. We should underline that the possibility of lepton-lepton resonant scattering in a pulsed light field is provided by the both energy (with accuracy  $q_0 \lesssim 1/\tau \ll \omega$ ) and momentum conservation laws fulfillment. Thus, the squared four-momentum of an intermediate photon  $q_1'$  vanishes. It implies that the considered particle falls within the mass shell, i.e. an intermediate virtual photon becomes a real one. In this case the correction to the intermediate photon squared four-momentum in the denominator of the expression (145) is caused by the external field pulsed character and is essential through integration of the function  $D_{ls}$  (145) over the variable  $\zeta$ . Hence, the following correlation is valid

$$q_1'^2 \lesssim \frac{(kq_1')}{\omega\tau}. \quad (157)$$



The condition (157) determines such a kinematical region, which is accepted to name the resonant one. In the case of external field modeling as a plane monochromatic wave there is the intermediate particle squared four-momentum alone in a process amplitude denominator. Therefore, when a denominator is equal zero the resonant divergence occurs. It is eliminated by radiative corrections introducing into the Green function according the Breit–Wigner prescription. But now there is an addition in a denominator, caused by the laser wave pulsed character. Thus, the divergence in the process amplitude disappears.

Finally, the function  $D_{ls}$  (145) assumes the form:

$$D_{ls} = \sum_{s=-\infty}^{\infty} \frac{i\pi\omega\tau^2}{(kq'_1)} \left( \bar{u}_{p'_\mu} \Delta_{l-s}^\nu u_{p_\mu} \right) \left( \bar{u}_{p'_e} \Delta_{sv} u_{p_e} \right), \quad (158)$$

where integral functions  $\Delta_{l-s}^\nu, \Delta_{sv}$  are defined by following expressions

$$\begin{cases} \Delta_{l-s}^\nu = \int_{-\infty}^{\infty} d\phi_1 \cdot G_{l-s}^\nu(\phi_1) \cdot \exp \left\{ i \left( \frac{q_0 + q_z}{2} \tau + 2\beta_\tau \right) \phi_1 \right\}, \\ \Delta_{sv} = \int_{-\infty}^{\infty} d\phi_2 \cdot G_{sv}(\phi_2) \cdot \exp \{ -2i\beta_\tau \phi_2 \} (sgn(\phi_1 - \phi_2) - 1); \end{cases} \quad (159)$$

$$\beta_\tau \equiv \frac{q_1'^2}{4(kq'_1)} \omega\tau. \quad (160)$$

Here  $\beta_\tau$  is the relevant parameter which is defined by the both resonant scattering kinematics and external pulsed wave characteristics.

#### 4.1 Resonance conditions

In this section we analyze in detail the case when an intermediate photon falls within the mass shell. Inner line discontinuity at the Feynman diagram appears and the studying process is effectively decomposes into two consecutive processes of the first order: a lepton  $l_1$  with the four-momentum  $p_1$  emits a real photon with the four-momentum  $q'_1$  at the expense of external wave photons absorption, then a real photon is absorbed by a lepton  $l_2$  with external wave photons emission or vice versa.

Generally speaking owing to condition (157) the squared four-momentum of an intermediate photon is founded within the very narrow region near zero. We will show below that this region depends on initial four-momenta of scattered particles and their scattering angles. However, the given region has to be taken into consideration in the denominator of the resonant amplitude exceptionally (145). Thus, the four-momentum conservation laws for resonant diagram vertexes may be written as two equalities:

$$p_1 + |s|k = p'_1 + q'_1, \quad (161)$$

$$p_2 + q'_1 = p'_2 + s' \cdot k. \quad (162)$$

The equality (161) expresses the four-momentum conservation law in the process when an intermediate real photon is emitted by a lepton  $l_1$  at the expense of  $|s|$  external wave photons absorption. The equality (162) corresponds to the four-momentum conservation law in the

process when an intermediate real photon is absorbed by a lepton  $l_2$  with  $s' = |s| + l$  external wave photons emission.

Remind that integral functions (153) are determined by the integer-order Bessel functions for the case of a circularly polarized external wave. It is not difficult to verify that for this polarization under the resonance conditions (157) the arguments of the Bessel functions (154) may be represented as

$$\gamma_{0j}(\phi_j) = 2s' \cdot \eta_{0j}(\phi_j) \sqrt{\frac{u}{u_{s'}} \cdot \left(1 - \frac{u}{u_{s'}}\right)}. \quad (163)$$

$$u \equiv \frac{(kp_j)}{(kp'_j)} - 1 = \frac{\kappa_j}{\kappa'_j} - 1, \quad u_{s'} \equiv 2s' \cdot \frac{\omega \kappa_j}{m_j^2}. \quad (164)$$

It was expected, that for processes of resonant lepton-lepton scattering the influence of the Bunkin-Fedorov quantum parameter does not reveal, in opposite the nonresonant case. Since  $\gamma_{0e} \sim \eta_{0e} \ll 1$  (see Eq. (163)), then the most probable case when a lepton  $l_1$  absorbs and a lepton  $l_2$  emits equally the only one external wave photon is realized, i.e.:

$$s' = |s| = 1, \quad l = s + s' = 0. \quad (165)$$

The region of resonant scattering is to be defined. We use the frame of reference related to a center of initial particles inertia, that is  $\mathbf{p}_1 + \mathbf{p}_2 = 0$ . In this frame the particle relative momentum  $\mathbf{p} = \mathbf{p}_1 = -\mathbf{p}_2$  and after scattering changes only the direction:  $|\mathbf{p}'| = |\mathbf{p}|$ . We introduce also the unit vectors along the directions of initial and final momenta  $\mathbf{n}_f$  and  $\mathbf{n}_i$

$$\mathbf{n}_f = \frac{\mathbf{p}'}{|\mathbf{p}'|}, \quad \mathbf{n}_i = \frac{\mathbf{p}}{|\mathbf{p}|}. \quad (166)$$

With expressions (157) consideration it is easy to verify that in view of chosen direction of intermediate photon motion the resonance occurs if leptons scatter into the small angles in the frame of reference related to a center of inertia:

$$\theta = \angle(\mathbf{n}_f, \mathbf{n}_i) = \theta_{res} = 2 \frac{\omega}{|\mathbf{p}|} \sin \theta_i \ll 1, \quad (167)$$

where  $\theta_i = \angle(\mathbf{n}, \mathbf{n}_i)$  is the angle between the directions of wave propagation and the initial relative momentum  $p$ .

Meanwhile the resonance for exchange (annihilation) amplitude occurs in the essentially different kinematical region (see Roshchupkin & Voroshilo (2008)).

Thus, we expand the Bessel functions (148) as series in order of  $\gamma_{0j} \sim \eta_{0j} \ll 1$  and keep the summands proportional to the first order of the parameter  $\eta_{0j}$ . Under the condition (165) we obtain:

$$G_{l-s}^{\nu}(\phi_1) = g(\phi_1) \cdot G_1^{\nu}, \quad (168)$$

$$G_{sv}(\phi_2) = g(\phi_2) \cdot G_{1\nu}. \quad (169)$$

where the matrices  $G_1^\nu$  and  $G_{1\nu}$  have the following form

$$G_1^\nu = (-1) \frac{\gamma_{01}}{2} \exp(i\chi_1) \tilde{\gamma}^\nu + \frac{\eta_{01}m_1}{2\omega\kappa_1} [k^\nu \hat{\varepsilon}_- - \varepsilon_-^\nu \hat{k}] + \frac{\eta_{01}m_1}{4\omega} \left( \frac{1}{\kappa'_1} - \frac{1}{\kappa_1} \right) \hat{\varepsilon}_- \hat{k} \tilde{\gamma}^\nu, \quad (170)$$

$$G_{1\nu} = \frac{\gamma_{02}}{2} \exp(-i\chi_2) \tilde{\gamma}_\nu + \frac{\eta_{02}m_2}{2\omega\kappa_2} [k_\nu \hat{\varepsilon}_+ - \varepsilon_+^\nu \hat{k}] + \frac{\eta_{02}m_2}{4\omega} \left( \frac{1}{\kappa'_2} - \frac{1}{\kappa_2} \right) \hat{\varepsilon}_+ \hat{k} \tilde{\gamma}_\nu. \quad (171)$$

The resonant region of scattering angles in the frame of reference related to a center of inertia is determined as

$$|\theta - \theta_{res}| \lesssim \frac{\theta_{res}}{\omega\tau} \ll \theta_{res}, \quad (172)$$

and expressions for the parameter  $\beta_\tau$  (160) assumes the form

$$\beta_\tau = \frac{1}{2} \omega\tau \left( 1 - \frac{\theta}{\theta_{res}} \right) \lesssim 1. \quad (173)$$

Finally, the resonant amplitude of a lepton  $l_1$  scattered by a lepton  $l_2$  in the field of a pulsed electromagnetic moderately strong wave of a circularly polarization in the frame of reference related to a center of inertia takes the form

$$S = S_0 \cdot Y_\tau, \quad (174)$$

where

$$S_0 = \frac{i\pi^{3/2}e^2\hat{M}}{\mathbf{p}^2\sqrt{E_1E_2E'_1E'_2}} \delta(P'_x) \delta(P'_y) \delta(E' - E - P'_z), \quad (175)$$

$$\hat{M} = (\bar{u}_{p'_1} G_1^\nu u_{p_1}) (\bar{u}_{p'_2} G_{1\nu} u_{p_2}). \quad (176)$$

The function  $Y_\tau$  in Eq. (174) is represented by

$$Y_\tau = \frac{\omega\tau}{\theta \cdot \theta_{res}} \exp\left\{-\frac{\beta_\tau^2}{4}\right\} \cdot I_\tau(q_+\tau). \quad (177)$$

Here,  $I_\tau(q_+)$  is the integral function:

$$I_\tau(q_+\tau) = \tau \int_{-\infty}^{\infty} d\phi \cdot g(\phi) \cdot \exp\left\{i\left(\frac{q_0 + q_z}{2}\tau + 2\beta_\tau\right)\phi\right\} \cdot \left[\operatorname{erf}\left(2\phi + \frac{i\beta_\tau}{2}\right) + 1\right]. \quad (178)$$

In Eqs. (177) and (178) the parameter  $\beta_\tau$  is determined by the expression (173). We underline, that presence of three delta-functions in the resonant amplitude (174)-(178) is considered as realizing of three following conservation laws:

$$P'_x = 0, \quad P'_y = 0, \quad E' - E = P'_z, \quad (179)$$

where  $\mathbf{P}' = (P'_x, P'_y, P'_z)$  is the momentum of the inertia center after scattering,  $E$  and  $E'$  are particle total energies before and after scattering, correspondingly.

## 4.2 Resonant cross-section

In view of finite duration of an external pulsed light field there is a sense to define the differential probability over all the observation time  $T$  in the process of scattering of a lepton  $l_1$  by a lepton  $l_2$ . Thus,

$$dW = |S|^2 \frac{d^3 p'}{(2\pi)^3} \frac{d^3 P'}{(2\pi)^3}. \quad (180)$$

Using the expressions for the amplitude (174)-(178) and performing uncomplicated computations we obtain the differential probability per time unit and per volume unit:

$$\begin{aligned} \frac{dW}{T} = dw = & \frac{e^4}{2(2\pi)^3 p^4 E_1 E_2 E'_1 E'_2 T} \cdot \frac{(\omega\tau)^2}{\theta^2 \theta_{res}^2} \cdot \left| \left( \bar{u}_{p'_1} G_1^\nu u_{p_1} \right) \left( \bar{u}_{p'_2} G_{1\nu} u_{p_2} \right) \right|^2 \times \\ & \times \exp \left( -\beta_\tau^2 / 2 \right) \cdot |I_\tau(q_+ \tau)|^2 \delta(P'_x) \delta(P'_y) \delta(q_0 - P'_z) d^3 p' d^3 P'. \end{aligned} \quad (181)$$

The differential cross section we obtain from Eq. (181) by division by a density of the scattered particles flux  $j = |\mathbf{p}|/E$ . The integration of the differential cross section over  $d^3 P'$  should be performed via the delta-functions. We present  $d^3 p'$  as

$$d^3 p' = E'_1 E'_2 |\mathbf{p}'| d\Omega' \frac{dE'}{E'}, \quad (182)$$

where  $d\Omega'$  is the elementary solid angle of particles scattering, and introduce a new integration dimensionless variable:  $dE' \rightarrow d\xi$  ( $\xi = q_0/\omega$ ,  $E' = \xi\omega + E$ ,  $dE' = \omega d\xi$ ). After simple transformations we derive

$$\frac{d\sigma_{res}^{l_1 l_2}}{d\Omega'} = \frac{e^4 E}{2p^4 E_1 E_2} \cdot \frac{\omega (\omega\tau)^2}{\theta^2 \theta_{res}^2} \cdot \left| \left( \bar{u}_{p'_1} G_1^\nu u_{p_1} \right) \left( \bar{u}_{p'_2} G_{1\nu} u_{p_2} \right) \right|^2 \exp \left( -\beta_\tau^2 / 2 \right) \cdot H. \quad (183)$$

Here, the function  $H$  has the form

$$\begin{aligned} H = & \int_{-\infty}^{\infty} d\xi \frac{|I_\tau(\xi\omega\tau)|^2}{\xi\omega + E} = \tau^2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d\xi d\phi d\phi'}{\xi\omega + E} g(\phi) \exp \{ i(\xi\omega\tau + 2\beta_\tau) \phi \} \times \\ & \times \left( \operatorname{erf} \left( 2\phi + \frac{i\beta_\tau}{2} \right) + 1 \right) g(\phi') \exp \{ -i(\xi\omega\tau + 2\beta_\tau) \phi' \} \left( \operatorname{erf} \left( 2\phi' - \frac{i\beta_\tau}{2} \right) + 1 \right). \end{aligned} \quad (184)$$

The differential cross section of resonant scattering of nonpolarized leptons in the field of a pulsed light wave into the elementary solid angle may be represented as

$$\frac{d\sigma_{res}^{l_1 l_2}}{d\Omega'} = r_e^2 \frac{4\pi m_e^2 m_1^2 m_2^2}{p^4 E_1 E_2} \eta_{01}^2 \eta_{02}^2 \cdot f_0 \cdot f_{res}. \quad (185)$$

Here, the function  $f_0$  is determined by

$$\begin{aligned} f_0 = & \left[ \frac{2d_{fi} h_{fi} |\mathbf{p}| (E_1 + E_2)}{(E_1 - |\mathbf{p}| \cos \theta_i) (E_2 + |\mathbf{p}| \cos \theta_i)} \right]^2 + \left[ 2 + \frac{d_{fi}^2 (E_1 E_2 + \mathbf{p}^2)}{(E_1 - |\mathbf{p}| \cos \theta_i) (E_2 + |\mathbf{p}| \cos \theta_i)} \right] \times \\ & \times \left[ 2 + \frac{d_{fi}^2 (E_1 E_2 + \mathbf{p}^2) + 4d_{fi} h_{fi} |\mathbf{p}| (E_1 + E_2)}{(E_1 - |\mathbf{p}| \cos \theta_i) (E_2 + |\mathbf{p}| \cos \theta_i)} \right]. \end{aligned} \quad (186)$$

The following designations are used in Eq. (186)

$$h_{fi} = (\mathbf{e}_x \mathbf{n}_i) \cos \chi_{fi} + \delta (\mathbf{e}_y \mathbf{n}_i) \sin \chi_{fi}, \quad (187)$$

$$d_{fi} = 2 (\mathbf{n} \tau_{fi}) \sqrt{(\mathbf{e}_x \tau_{fi})^2 + (\mathbf{e}_y \tau_{fi})^2}, \quad (188)$$

$$tg \chi_{fi} = \delta \frac{(\mathbf{e}_y \cdot \tau_{fi})}{(\mathbf{e}_x \cdot \tau_{fi})}, \quad (189)$$

$$\tau_{fi} = \frac{\mathbf{n}_f - \mathbf{n}_i}{|\mathbf{n}_f - \mathbf{n}_i|}. \quad (190)$$

The function  $f_{res}$  in Eq. (185) has the form

$$f_{res} = \left( \frac{\omega \tau}{\theta_{res}^2} \right)^2 \cdot f(\rho, \beta_\tau), \quad (191)$$

$$f(\rho, \beta_\tau) = \exp(-\beta_\tau^2/2) \cdot \frac{1}{\rho} \int_{-\rho}^{\rho} d\phi g^2(\phi) \left| \operatorname{erf} \left( \phi + \frac{i\beta_\tau}{2} \right) + 1 \right|^2. \quad (192)$$

We underline that the dependence of the function  $f_{res}$  on the parameter  $\beta_\tau$  (173) determines resonant peak magnitude and shape. It is easy to notice that when leptons scatter into the resonant angle  $\theta \simeq \theta_{res}$  than the parameter  $\beta_\tau$  becomes equal zero (see Eq. (173)). At that the function  $f_{res}$  (191) possesses the finite value as opposed to the plane monochromatic wave case when  $f_{res} \rightarrow \infty$  is correct.

The significant issue is the influence of the pulse finite duration on the cross section resonant behavior. The pulse duration has to exceed the time required for the Compton scattering of an external field photon by each of leptons  $l_1$  and  $l_2$ . If this condition is not satisfied than particles do not have time to interact with a wave under the resonance conditions. Consequently, the following correlation for the pulse duration is valid:

$$\omega \tau \gtrsim \frac{1}{\alpha \eta_{0j}^2} \frac{E_j}{\kappa_j}. \quad (193)$$

Thus, experimental treatment of resonant scattering of a lepton by a lepton may be verified in the fields created by picosecond pulsed lasers which generate the radiation within the optical frequencies range. Such scientific facilities are employed in SLAC National Accelerator Laboratory (Bula et al. (1996); Burke et al. (1997)) research centers and also in the frame of the FAIR project (Bagnoud et al. (2009)).

We can integrate the differential cross section (185) within the narrow range of scattering angles near the resonance (172). Under the resonance conditions the vector  $\tau_{fi}$  (190) may be represented as

$$\tau_{fi} \approx \frac{1}{\theta_{res}} (\mathbf{n}_f - \mathbf{n}_i), \quad (194)$$

and  $(\mathbf{n}_i \tau_{fi}) \simeq 0$ . We perform the integration over the parameter  $\beta_\tau$  (173) (instead the scattering angle  $\theta$ ), and finally derive

$$\frac{d\sigma_{res}^{l_1 l_2}}{d\varphi_f} = 16\pi r_e^2 \eta_{01}^2 \eta_{02}^2 \frac{m_e^2 m_1^2 m_2^2}{E_1 E_2 |p|^4} \frac{(\omega\tau)}{\theta_{res}^2} \cdot f_0 \cdot F(\rho), \quad (195)$$

where function  $F(\rho)$  is determined by

$$F(\rho) = \int_{-\infty}^{\infty} d\beta_\tau \cdot f(\rho, \beta_\tau). \quad (196)$$

Here, the function  $f(\rho, \beta_\tau)$  is specified by Eq. (192). The limits of the integration in Eq. (196) are extended over the infinity owing to the integral quick convergence (though the values of the parameter  $\beta_\tau \lesssim 1$  within the resonant region).

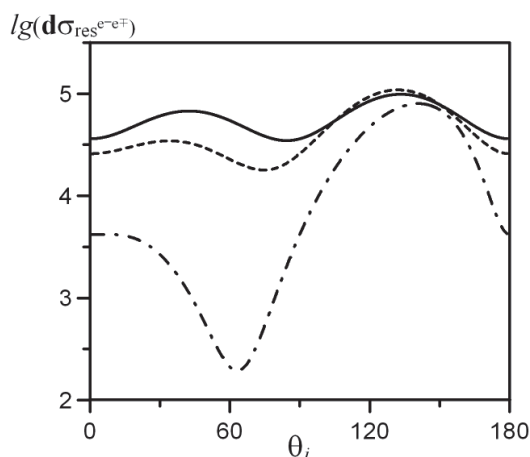


Fig. 9. The dependence of the differential cross-section of scattering of an electron by an electron (an electron by a positron) in a pulsed light field (195) (in units of respective cross-sections in an external field absence) on the initial polar angle when an azimuthal angle is fixed  $\varphi_i = \pi/4$  and value of the parameter  $\rho = 2$ . The external laser wave frequency amounts to the value  $\omega = 2.35$  eV, the pulse duration is equal to  $\tau = 1.5$  ps, the field strength in a pulse peak  $F_0 = 6 \cdot 10^9$  V/cm. The cases of particles relative velocities  $V = 0.2$  (solid line),  $V = 0.6$  (dotted line), and  $V = 0.9$  (dash-dotted line) are represented.

Let us consider the ratio of the derived resonant differential cross section (195) to the differential cross section of scattering of the same leptons in an external field absence for such processes: scattering of an electron by an electron, scattering of an electron by a positron, scattering of an electron by a muon. Figs. 9, 10 show the dependencies of the considered ratio on the initial polar angle  $\theta_i$ . We should underline that under scattering of both an electron by an electron and an electron by a positron within the small angles range (172) the respective cross-sections coincide each with other.

In accordance with the Figs. 9, 10 we consider that within the broad range of particles velocities the resonant cross sections of scattering of an electron by an electron (an electron by a positron, an electron by a muon) in a pulsed light field exceed the corresponding differential

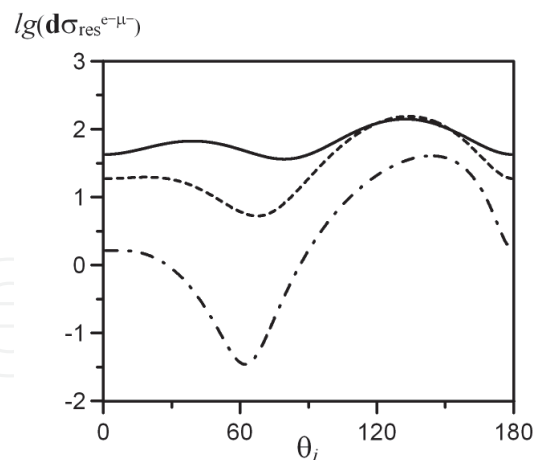


Fig. 10. The dependence of the differential cross-section of scattering of an electron by muon in a pulsed light field (195) (in units of respective cross-sections in an external field absence) on the initial polar angle when an azimuthal angle is fixed  $\varphi_i = \pi/4$  and the value of the parameter  $\rho = 2$ . The cases of particles relative velocities  $V = 0.2$  (solid line),  $V = 0.6$  (dotted line) and  $V = 0.9$  (dash-dotted Line) are represented.

cross sections in an external field absence within the whole polar angles range. Hereby, the greatest exceeding appears for the case of particles small relative velocities ( $V = 0.2$ ), at that the exceeding reaches into five orders of the magnitude (for scattering of an electron by an electron (positron)), and two orders for scattering of an electron by a muon. Also there is a suppression of the resonant cross section in the case of leptons high relative velocities within the range of the initial polar angles  $\theta_i \approx 60^\circ$ .

5. Resonant scattering of a photon by an electron in the pulsed laser field

Oleinik (1967) specified resonances in the Compton effect in the field of a plane monochromatic wave for the first time, but his studies had a rather fragmentary form (see also Belousov (1977)). The resonance of direct and exchange diagrams in the general relativistic case for the field of a weakly intensive plane monochromatic electromagnetic wave was considered by Voroshilo & Roshchupkin (2005). Scattering of a photon by an electron in a pulsed light field for the direct diagram resonance in the range of weak fields (5) was studied in work Voroshilo et al. (2011).

5.1 Process amplitude

The amplitude of scattering of a photon with the four momentum  $k_i = (\omega_i, \mathbf{k}_i)$  by an electron with the four momentum  $p_i = (E_i, \mathbf{p}_i)$  in an pulsed field (2) (Fig. 11) is given by the expression

$$S_{fi} = S_{fi}^{(d)} + S_{fi}^{(e)}, \tag{197}$$

$$S_{fi}^{(d)} = -ie^2 \int d^4r d^4r' \bar{\Psi}_{p_f}(r) \tilde{\gamma}^\mu G(r, r') \tilde{\gamma}^\nu \Psi_{p_i}(r') A_\mu^*(k_f r) A_\nu(k_i r'), \quad S_{fi}^{(e)} = S_{fi}^{(d)}(k_f \leftrightarrow -k_i), \tag{198}$$



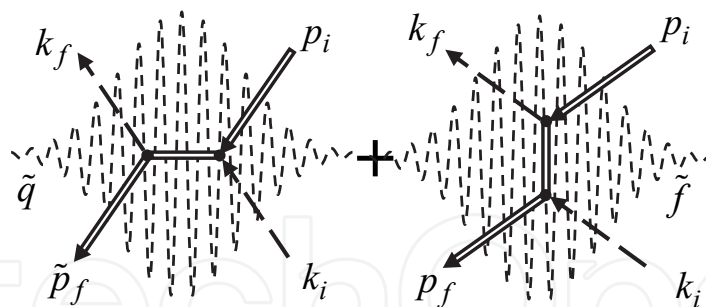


Fig. 11. The Feynman diagram of the Compton effect in the field of a pulsed light wave.

where  $p_f = (E_f, \mathbf{p}_f)$  and  $k_f = (\omega_f, \mathbf{k}_f)$  are four momenta of an outgoing electron and a photon;  $\tilde{\gamma}^\nu$  ( $\nu = 0, 1, 2, 3$ ) are the Dirac matrices;  $A_\mu(k_i r')$  is the wave function of a photon (8);  $e_\mu$  is the photon polarization four-vector;  $G(r, r')$  is the Green function of an electron in the field (2).

The case when a laser field intensity meets the following condition

$$\eta_0^2 \lesssim \varphi_0^{-1} \ll 1, \quad \varphi_0 = \omega\tau, \quad (199)$$

is considered through this section. This condition allows both to carry out the decomposition with respect to the small parameter and to neglect the interference of contributions of the pulsed wave anterior and posticous parts.

The amplitude (197) accurate within terms  $\sim \eta_0^2$  assumes the form

$$S_{fi} \approx B \delta^{(2)}(\mathbf{p}_{i,\perp} + \mathbf{k}_{i,\perp} - \mathbf{p}_{f,\perp} - \mathbf{k}_{f,\perp}) \delta(p_{i,-} + k_{i,-} - p_{f,-} - k_{f,-}) e'^*_{\nu} e_{\mu} \cdot \bar{u}_{p_f} T_{fi}^{\nu\mu} u_{p_i}, \quad (200)$$

$$T_{fi}^{\nu\mu} = \sum_j \left( T_{0,0}^{(j)\nu\mu} + \eta_0 \sum_{l,l' \in (|l-l'|+|l'|=1)} T_{l-l',l'}^{(j)\nu\mu} + \eta_0^2 \left( T_{0,0}^{(j)\nu\mu} + \sum_{l,l' \in (|l-l'|+|l'|=2)} T_{l-l',l'}^{(j)\nu\mu} \right) \right), \quad (201)$$

where  $j = e, d$ ; indices  $d, e$  are concerned to direct and exchange diagrams;  $B$  is the normalization factor;  $\mathbf{p}_{i,\perp}, \mathbf{k}_{i,\perp}, \mathbf{p}_{f,\perp}, \mathbf{k}_{f,\perp}$  are the projections of corresponding vectors on the wave polarization plane;  $p_{i,-} = E_i - p_{i,z}$ ,  $k_{i,-} = \omega_i - k_{i,z}$ ,  $p_{f,-} = E_f - p_{f,z}$ ,  $k_{f,-} = \omega_f - k_{f,z}$  are differences between zeroth components of the corresponding four momentum and its projection on direction of wave propagation;  $q, f$  are four momenta of an intermediate particle, which conform to direct and exchange diagrams on Fig. 11, at that under the four momenta conservation laws we have

$$\mathbf{q}_{\perp} = \mathbf{p}_{i,\perp} + \mathbf{k}_{i,\perp}, \quad q_{-} = p_{i,-} + k_{i,-}; \quad \mathbf{f}_{\perp} = \mathbf{p}_{i,\perp} - \mathbf{k}_{f,\perp}, \quad f_{-} = p_{i,-} - k_{f,-}. \quad (202)$$

The summands in Eq. (201), proportional to the zeroth degree of  $\eta_0$  determine the amplitude of the Compton effect in external field absence (Klein & Nishina (1929)). The summands, proportional to the first degree of the parameter  $\eta_0$ , determine the corrections (for them  $|l-l'| + |l'| = 1$  is valid) specified by participation of one wave photon in the process. The summands, proportional to the second degree of the parameter  $\eta_0$ , determine the corrections

specified by participation of two wave photons in the process (for them  $|l - l'| + |l'| = 2$  is valid).

In the case of a plane monochromatic wave the resonance is associated with the fact that an intermediate particle falls within the mass shell:  $q^2 = m^2$ ,  $f^2 = m^2$ . The corrections to the Compton effect probability, which are specified by processes with one wave photon participation, are the nonresonant. They are proportional to the second order of the parameter  $\eta_0$  and, therefore, are small in comparison with the Compton effect probability. But among processes with two wave photons participation there are such ones, which may have the resonant behavior. The both resonance of direct diagram through an electron intermediate state and resonance of exchange diagram through a positron intermediate state permit the processes with  $l' = -1$ ,  $l = 0$ . The resonance of the exchange diagram through an electron intermediate state permits the process with  $l' = 1$ ,  $l = 0$ . These processes may have resonant character in the case of a pulsed field (2) (Voroshilo et al. (2011)).

The expressions for  $T_{l-l',l'}^{(d,e)\nu\mu}$  in Eq. (201) for resonant processes have the form:

$$T_{1,-1}^{(d)\nu\mu} \approx \frac{\pi\omega}{8(kq_{-1})} \cdot I(\beta_{-1}(q_{-1}), l_*) \left[ M_1^\nu(p_f, q_{-1}) (\hat{q}_{-1} + m) M_{-1}^\mu(q_{-1}, p_i) \right], \quad (203)$$

$$T_{\mp 1, \pm 1}^{(e)\nu\mu} \approx \frac{\pi\omega}{8(kf_{\mp 1})} \cdot I(\beta_{\mp 1}(f_{\mp 1}), l_*) \left[ M_{\pm 1}^\nu(p_f, f_{\mp 1}) (\hat{f}_{\mp 1} + m) M_{\mp 1}^\mu(f_{\mp 1}, p_i) \right]. \quad (204)$$

Here

$$I(\beta_{l'}, l_*) = \frac{\pi}{4(kq_{l'})} \left( \operatorname{erfi} \left( \frac{\sqrt{2}}{2} \left( \beta_{l'} - \frac{l_* \varphi_0}{4} \right) \right) + i \right) \exp \left\{ -\frac{\varphi_0^2 l_*^2 + 8(\beta_{l'} - l_* \varphi_0/4)^2}{16} \right\}, \quad (205)$$

where  $\operatorname{erfi}(z)$  is the error function of imaginary argument;  $\beta_{l'}$  is the resonant parameter:

$$\beta_{l'}(q_{l'}) = \frac{q_{l'}^2 - m^2}{4(kq_{l'})} \omega \tau. \quad (206)$$

Exactly the parameter  $\beta_{l'}$  determines the process behavior character. Thus, the values  $\beta_{l'} \lesssim 1$  correspond to the resonant behavior. The opposite case  $\beta_{l'} \gg 1$  corresponds to the nonresonant one. Under the values  $\beta_{l'} \gg 1$  the function  $I(\beta_{l'}, a)$  has the following asymptotic form:

$$I(\beta_{l'}, l_*) \approx \sqrt{\frac{\pi}{2}} \frac{1}{\beta_{l'}} \exp \left\{ -\frac{1}{32} l_*^2 \varphi_0^2 \right\}. \quad (207)$$

In Eqs. (203)-(206) the quantities  $q_{-1} = p_i + k_i - k$ ,  $f_{\mp 1} = p_i - k_f \pm k$  correspond to the "strict" four momentum conservation law (like the monochromatic wave case, when summands  $\sim \eta_0^2$  are neglected); the quantity  $l_*$  are the invariant parameter which are determined from the following equation:

$$p_i + k_i + l_* k = p_f + k_f. \quad (208)$$

It follows from Eq. (205) that  $|l_*| \sim \varphi_0^{-1}$ . Consequently, in the zero-order approximation with respect to the parameter  $\varphi_0^{-1}$  the frequency of a scattered photon is amount:

$$\omega_f \approx \omega_f^{(0)}, \quad \omega_f^{(0)} = \frac{(p_i k_i)}{E_i + \omega_i - ([\mathbf{p}_i + \mathbf{k}_i] \mathbf{n}_f)}, \quad (209)$$

where  $\mathbf{n}_f$  is the directive unit vector for the final photon emission.

Bispinor matrices  $M_{\pm 1}^\nu$  in (203), (204) are determined by:

$$M_{\pm 1}^\nu(p_f, q_{-1}) = \pm \frac{y_0(p_f, q_{-1})}{2} e^{\mp i\chi} \cdot \gamma^\nu + \frac{m}{4} \left( \frac{2}{(kq_{-1})} [\hat{\varepsilon}^{(\mp)} k^\nu - \hat{k} \varepsilon^{(\mp)\nu}] + \left[ \frac{1}{(kp_f)} - \frac{1}{(kq_{-1})} \right] \hat{\varepsilon}^{(\mp)} \hat{k} \gamma^\nu \right). \quad (210)$$

Here, quantities  $\varepsilon^{(\pm)} = e_x \pm i\delta e_y$ ;  $y_0(p_f, q)$ ,  $\chi \equiv \chi(p_f, q)$  are the kinematical parameters

$$y_0(p_f, q) = m\eta \sqrt{-g^2(p_f, q)}, \quad \tan \chi = \frac{\delta(g e_y)}{(g e_x)}, \quad g \equiv g(p_f, q) = \frac{p_f}{(kp_f)} - \frac{q}{(kq)}. \quad (211)$$

## 5.2 Resonant kinematics

### 5.2.1 Resonance conditions for the direct diagram

The parameter  $\beta \equiv \beta(q_{-1})$  which corresponds to the resonant process with  $l' = -1$ ,  $l = 0$  (one field photon emits in the beginning, and one photon absorbs at the end of the process) may be written in the form:

$$\frac{\beta}{\varphi_0} = \frac{1}{2} \frac{1 - \tilde{u}}{[1 + \tilde{u}(\omega_i/\omega_{i,\text{res}} - 1)]} \left( \frac{\omega_i}{\omega_{i,\text{res}}} - 1 \right). \quad (212)$$

Here, the invariant parameter  $\tilde{u}$  and the frequency  $\omega_{i,\text{res}}$ , which corresponds to the resonant maximum, are determined by:

$$\tilde{u} = \frac{(kk_i)}{(p_i k_i)}, \quad 0 \leq \tilde{u} \leq u_1, \quad u_1 = \frac{2(kp_i)}{m^2}, \quad (213)$$

$$\omega_{i,\text{res}} = \frac{(kp_i)}{E_i - \omega - ([\mathbf{p}_i - \mathbf{k}]\mathbf{n}_i)}, \quad (214)$$

where  $\mathbf{n}_i$  is the unit vector along the propagation direction of incident photon. We rewrite this expression as

$$\omega_{i,\text{res}} = \frac{mu_1}{2} \frac{(E_i - \omega)/m + \sqrt{(E_i - \omega)^2/m^2 + u_1 - 1 \cdot \cos \tilde{\theta}_{\mathbf{S}}}}{1 - u_1 + ((E_i - \omega)^2/m^2 + u_1 - 1) \sin^2 \tilde{\theta}_{\mathbf{S}}}, \quad (215)$$

where

$$\tilde{\theta}_{\mathbf{S}} = \angle(\mathbf{S}, \mathbf{n}_i), \quad \mathbf{S} = \mathbf{p}_i - \mathbf{k}. \quad (216)$$

We consider that the correlation  $\omega \ll m$  is valid in the region  $v_i = |\mathbf{p}_i|/E_i \ll \omega/m \ll 1$  (it is the nonrelativistic case, which also corresponds to the rest frame of an final electron) and obtain:

$$\omega_{i,\text{res}} = \frac{\omega}{1 - \omega/m(1 - \cos \tilde{\theta})} \approx \omega \left( 1 + \frac{\omega}{m}(1 - \cos \tilde{\theta}) \right), \quad \tilde{\theta} = \angle(\mathbf{k}, \mathbf{n}_i) \approx \pi - \tilde{\theta}_{\mathbf{S}}. \quad (217)$$

Therefore, in this case the resonant frequency is closely approximated to the laser field frequency.

In range where the correlation  $\omega / m \ll v_i < 1$  is valid (it is the ultrarelativistic case) we derive:

$$\omega_{i,\text{res}} \approx \frac{mu_1}{2} \frac{E_i/m + \sqrt{E_i^2/m^2 - 1} \cos \tilde{\theta}_S}{1 - u_1 + (E_i^2/m^2 - 1) \sin^2 \tilde{\theta}_S}.$$

In the ultrarelativistic case ( $u_1 > 1, E_i/m > m/\omega \gg 1$ ) under  $(m/\omega) \sqrt{u_1 - 1} < \tilde{\theta}_S \ll 1$  ( $\tilde{\theta}_S \approx \tilde{\theta}_{\mathbf{p}_i} = \angle(\mathbf{p}_i, \mathbf{n}_i)$ ) we obtain:

$$\omega_{i,\text{res}} \approx \frac{u_1 E_i}{1 - u_1 + (E_i/m)^2 \tilde{\theta}_S^2}. \tag{218}$$

Fig. 12 demonstrated dependence of the resonant frequency on the angle  $\tilde{\theta}_S$  for different energies of an electron.

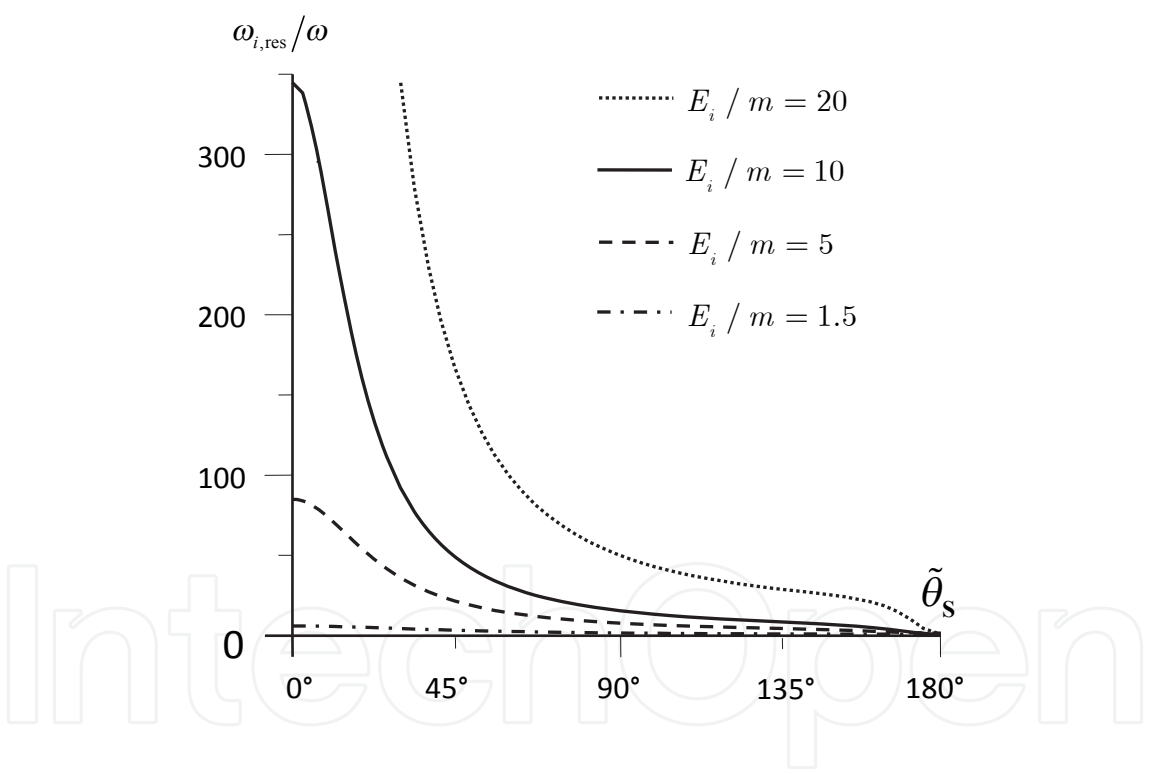


Fig. 12. The dependence of ratio of the resonant frequency of an ingoing photon to the laser field frequency  $\omega_{i,\text{res}}/\omega$  (215) from the angle  $\tilde{\theta}_S$  (216) under  $\omega/m = 10^{-5}$  for different energies of an ingoing electron.

The resonance of the amplitude, which corresponds to the direct diagram, is feasible only when the condition  $\tilde{u} < 1$  is satisfied, so that for the values  $u_1 > 1$  the angle  $\tilde{\theta}_S$  is restricted by the interval:

$$\alpha_0 < \tilde{\theta}_S < \pi, \quad \alpha_0 = \arccos \frac{E_i - \omega}{|\mathbf{S}|}. \tag{219}$$

The cases close to realization of the condition  $\tilde{u} = 1$  ( $\tilde{\theta}_S = \alpha_0$ ) also have to be excluded, because the frequency of a resonant photon in these cases has to be infinite, but it is impossible to put into practice. Therefore, the condition of the direct diagram resonance is determined by:

$$\left| 1 - \frac{\omega_i}{\omega_{i,\text{res}}} \right| \sim \frac{1}{\varphi_0} \ll 1. \quad (220)$$

Under the condition (220) the resonant parameter assumes the form:

$$\frac{\beta}{\varphi_0} \approx \frac{1}{2} (1 - \tilde{u}) \left( \frac{\omega_i}{\omega_{i,\text{res}}} - 1 \right), \quad 0 < \tilde{u} < \begin{cases} u_1, & u_1 < 1; \\ 1, & u_1 > 1. \end{cases} \quad (221)$$

### 5.2.2 Resonance conditions for the exchange diagram

For the processes with  $l' = \pm 1$ ,  $l = 0$  which permit the resonance of the exchange diagram through an electron ( $l' = 1$ ) and a positron ( $l' = -1$ ) intermediate states the resonant parameters  $\beta_{\mp} \equiv \beta(f_{\pm 1})$  have the form:

$$\frac{\beta_{\mp}}{\varphi_0} = \frac{1}{2} \frac{v' \pm 1}{\left[ v' \left( 1 - \omega_f / \omega_{f,\text{res}}^{(\mp)} \right) \pm 1 \right]} \left( 1 - \frac{\omega_f}{\omega_{f,\text{res}}^{(\mp)}} \right), \quad (222)$$

where the upper sign is concerned to an electron intermediate state, the lower sign is concerned to a positron one; the invariant parameter  $v'$  and the frequencies  $\omega_{f,\text{res}}^{(\mp)}$  of a final photon, which correspond to the resonant maximum, are defined by:

$$v' = \frac{(kk_f)}{(p_ik_f)}, \quad \omega_{f,\text{res}}^{(\mp)} = \frac{(kp_i)}{(E_i - \mathbf{p}_i \mathbf{n}_f)(v' \pm 1)}. \quad (223)$$

It follows from Eq. (223) that the resonance via positron intermediate state can be observed under limitations on parameter  $v'$  and, hence, angle  $\tilde{\theta}'_S = \angle(\mathbf{S}, \mathbf{n}_f)$ :

$$v' > 1(u_1 > 1) \quad 0 \leq \tilde{\theta}'_S \leq \alpha_0 \quad \text{and} \quad \pi - \alpha_0 \leq \tilde{\theta}'_S \leq \pi. \quad (224)$$

Equating the expressions (209), (223) we obtain that under the exchange diagram resonance directions of a scattered photon correspond to the condition of the resonant maximum; these directions lie on the surface of a cone (see Fig. 13); axis of the cone coincides with the vector  $\mathbf{h}^{\mp}$  and the opening angle  $\theta'_{\mathbf{h}^{\mp}} = \angle(\mathbf{h}^{\mp}, \mathbf{n}_f)$ :

$$\cos \theta'_{\mathbf{h}^{\mp}} = h_0^{\mp} / |\mathbf{h}^{\mp}|, \quad h^{\mp} = (h_0^{\mp}, \mathbf{h}^{\mp}) = (kp_i) [p_i + k_i] - (p_i k_i) [k \pm p_i]. \quad (225)$$

Thus, the four vector has to be a spatially similar one  $(h^{\mp})^2 \leq 0$ , i.e. the inequality

$$\tilde{u}_1^2 (1 - \tilde{u} u_1) \mp 2 \tilde{u}_1 u_1 + u_1^2 \leq 0, \quad (226)$$

has to be met.

The invariant parameter  $\tilde{u}_1$  is equal to:

$$\tilde{u}_1 = \frac{2(pk_i)}{m^2}. \quad (227)$$

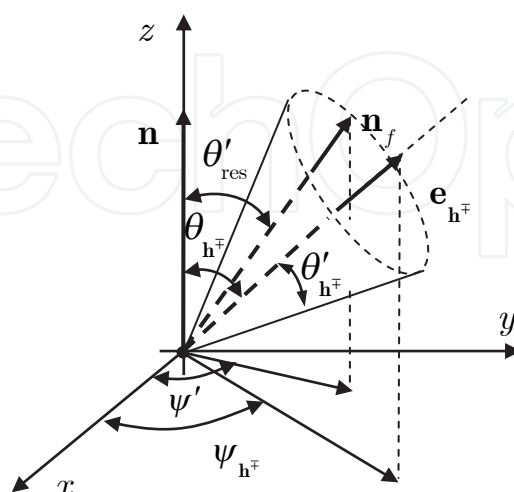


Fig. 13. Geometry of emission of an outgoing photon in the case of occurrence of the exchange diagram resonance.

From the inequality (226) we derive the following condition on the initial photon frequency; at this frequency the exchange diagram resonance through the electron intermediate state occurs:

$$\begin{cases} \frac{\omega_f}{1 + \sqrt{u_1 \tilde{u}}} \leq \omega_{i,\text{res}} \leq \frac{\omega_f}{1 - \sqrt{u_1 \tilde{u}}}, & \tilde{u} < u_1^{-1}; \\ \omega_{i,\text{res}} \geq \frac{\omega_f}{1 + \sqrt{u_1 \tilde{u}}}, & u_1^{-1} < \tilde{u} < u_1. \end{cases} \quad (228)$$

Here, the function  $f$  has the form

$$f = \frac{1 - v_i \cos \theta}{1 - v_i \cos \tilde{\theta}}, \quad (229)$$

where  $\theta = \angle(\mathbf{k}, \mathbf{k}_i)$ ,  $\tilde{\theta} = \angle(\mathbf{k}, \mathbf{p}_i)$ .

For a positron intermediate state the resonance occur under the condition that the initial photon frequency exceeds a certain threshold value:

$$\omega_{i,\text{res}} \geq \frac{\omega_f}{\sqrt{u_1 \tilde{u}} - 1}, \quad u_1^{-1} < \tilde{u} < u_1. \quad (230)$$

Values of initial photon frequencies meet the condition of the direct diagram resonance  $\omega_i = \omega_{i,\text{res}}$  (214). They are founded within the frequencies interval (228); the exchange diagram resonance through an electron intermediate state occurs under these frequencies. Consequently, the direct diagram resonance is always accompanied by the exchange diagram resonance through an electron intermediate state, and within the region

$$u_1 > 1, \quad 1/u_1 < \tilde{u} < 1 \quad (231)$$

through a positron intermediate state also.

Fig. 14 shows the resonant region of final photon frequency values  $\omega_i$  (in units of the initial electron energy  $E_i$ ), which is determined by the system of the equations and the inequalities (215), (228), (230), as a function of the parameter  $\alpha = (\theta - \tilde{\theta})(m/E)$  when  $\omega = 2.36$  eV,  $E_i = 48.0$  GeV (since  $E_i \gg \omega$ , then  $\tilde{\theta}_S \approx \tilde{\theta}$ ),  $\theta = 163^\circ$ .

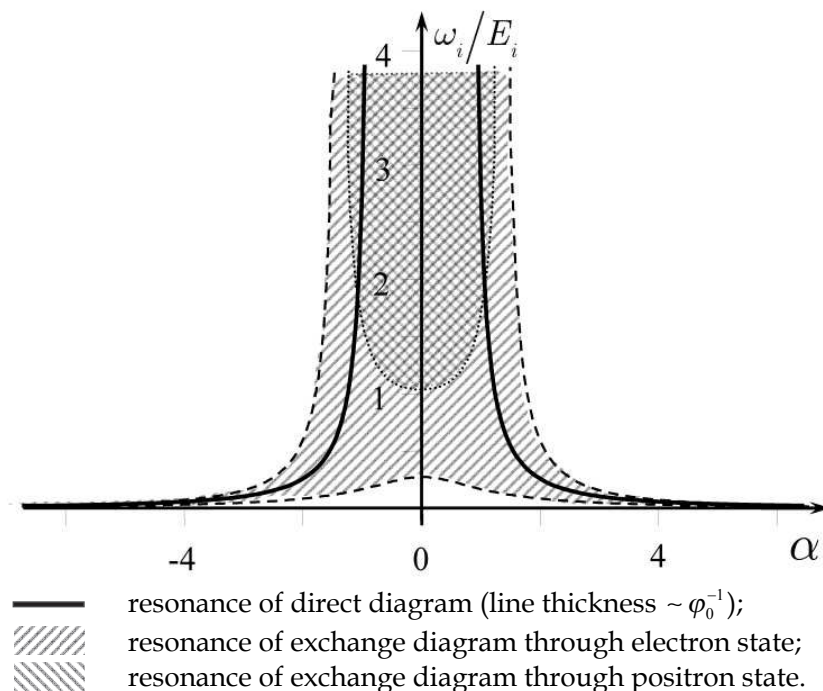


Fig. 14. Resonant region of frequencies  $\omega_i(\alpha)$  of an ingoing photon (in units of the ingoing electron energy  $E_i$ ), which is determined by the system of equations and inequalities (215), (228), (230).

### 5.3 Resonant probability for the direct diagram

We consider the case when the conditions of the direct diagram resonance (220) are realized. Though it is accompanied by the exchange diagram resonance, but its contribution may be neglected in the following cases:

1. when an initial photon is emitted out of the strictly defined and narrow region of an initial photon directions when the exchange diagram resonance occurs (see Fig. 14);
2. when the total probability is obtained, since the contribution to the total probability from the exchange diagram is  $\sim (\omega\tau)^{-1} \ll 1$  and, therefore, it may be neglected.

The differential probability is obtained by standard mode (Berestetskii et al. (1982)). After averaging over initial particle polarizations and summation over final particle polarizations and also the integration over frequencies  $\omega_f$  and the azimuthal angle  $\psi' = \angle(\mathbf{e}_x, \mathbf{k}_{f,\perp})$  of a final photon emission we obtain the differential probability:

$$dW_{fi}^{res} \approx \frac{2e^4\eta_0^4 m^2}{\pi\omega_i E_i V \tilde{u}_1} (\omega\tau)^2 P_{res}(\beta) [f(u', \tilde{u}_1)f(u, \tilde{u}_1) - g(u', \tilde{u}_1)g(u, \tilde{u}_1)] \frac{du'}{(1+u')^2} \tau. \quad (232)$$



Here,

$$\begin{aligned} u &= \frac{(kk_i)}{(qk_i)}, \\ u' &= \frac{(kk_f)}{(qk_f)}, \end{aligned} \quad (233)$$

$$\begin{aligned} \tilde{u}_1 &= \frac{u_1}{1 - \tilde{u}'}, \\ u_{\text{res}} &= \frac{\tilde{u}}{1 - \tilde{u}'}, \end{aligned}$$

at that  $0 \leq u \leq \tilde{u}_1$ ,  $0 \leq u' \leq \tilde{u}_1$ ,  $0 \leq \tilde{u}' \leq \tilde{u}_1$ . In Eq. (232)  $P_{\text{res}}(\beta)$  is the function, which determines the resonant profile (see Fig. 15). It is obtained by

$$P_{\text{res}}(\beta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |I_1(\beta, l_*)|^2 d(\varphi_0 l_*). \quad (234)$$

We determine the resonance width at a half of the probability maximum (see Fig. 15). The

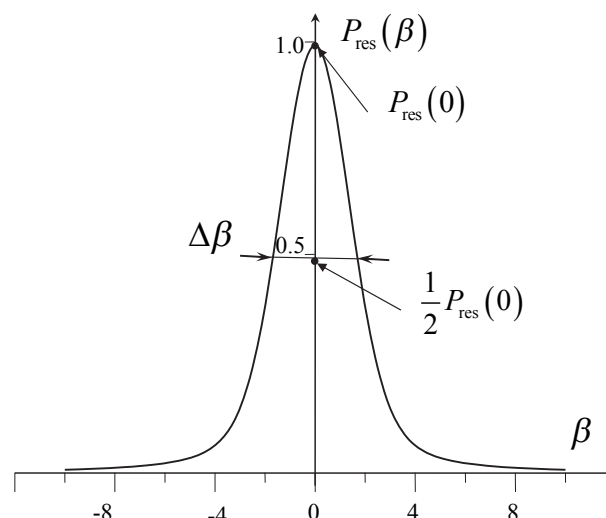


Fig. 15. Dependence of the function  $P_{\text{res}}$  (234), which determines the resonant profile, on the resonant parameter  $\beta$  (221).

width which corresponds to the resonant parameter  $\beta$  is equal to  $\Delta\beta \approx 3.40$ . Therefore, the width specified by the field pulsed character is obtained by

$$\Gamma_{\text{imp}} = \frac{\Delta(q^2 - m^2)}{4m} = \Delta\beta \frac{\tilde{u}_1}{2} \frac{m}{\varphi_0} \approx 1.70 \frac{m\tilde{u}_1}{\varphi_0}. \quad (235)$$

We compare the resonance width specified by the field pulsed character (235) with the radiation width:

$$\Gamma_R = \frac{q_0}{m} W_1 = \frac{e^2 m}{4\sqrt{\pi}} \eta_0^2 F(\tilde{u}_1), \quad (236)$$

where  $W_1$  is the total probability of the intermediate state decay in a weakly intensive field; the function  $F(\tilde{u}_1)$  is defined by

$$F(\tilde{u}_1) = \left(1 - \frac{4}{\tilde{u}_1} - \frac{8}{\tilde{u}_1^2}\right) \ln(1 + \tilde{u}_1) + \frac{1}{2} + \frac{8}{\tilde{u}_1} - \frac{1}{2(1 + \tilde{u}_1)^2}. \quad (237)$$

This ratio equals

$$\frac{\Gamma_{\text{imp}}}{\Gamma_R} \approx \frac{8.51}{e^2 \eta_0^2 (\omega \tau)} \frac{\tilde{u}_1}{F(\tilde{u}_1)}. \quad (238)$$

When the condition (199) is met the appraisal value of ratio is equal to  $\Gamma_{\text{imp}}/\Gamma_R \geq 10^3 \gg 1$ . Therefore, the width specified by the field pulsed character is the major one and the radiation widening may be neglected.

After the integration over the invariant parameter  $u'$  we derive the total probability of photon-electron scattering under the direct diagram resonance

$$W_{fi}^{\text{res}} \approx \frac{2e^4 \eta_0^4 m^2}{\pi \omega_i E_i V \tilde{u}_1} (\omega \tau)^2 P_{\text{res}}(\beta) [F(\tilde{u}_1) f(u, \tilde{u}_1) - G(\tilde{u}_1) g(u, \tilde{u}_1)] \tau, \quad (239)$$

$$\begin{aligned} G(u', \tilde{u}_1) &= \int_0^{\tilde{u}_1} g(u', \tilde{u}_1) \frac{du'}{(1 + u')^2} = \\ &= \frac{1}{4\tilde{u}_1(1 + \tilde{u}_1)^2} \left( -4\tilde{u}_1 - 8\tilde{u}_1^2 - 5\tilde{u}_1^3 + (4 + 10\tilde{u}_1 + 8\tilde{u}_1^2 + 2\tilde{u}_1^3) \ln(1 + \tilde{u}_1) \right). \end{aligned} \quad (240)$$

Ratio of the total probability (239) to the total probability of the Compton effect in external field absence is expressed as

$$\frac{W_{fi}^{\text{res}}}{W_{\text{Compt}}} \approx \frac{\tau}{T} P_{\text{res}}(\beta) \cdot R(u, \tilde{u}_1), \quad R(\tilde{u}, u_1) = \frac{2\eta_0^4 (\omega \tau)^2}{\pi^2} \frac{[F(\tilde{u}_1) f(u, \tilde{u}_1) - G(\tilde{u}_1) g(u, \tilde{u}_1)]}{\tilde{u}_1 F(\tilde{u}_1)}, \quad (241)$$

where  $T$  is the observation time ( $T \gtrsim \tau$ ), which is determined by conditions of the concrete experiment.

When  $u_1 \ll 1$  we derive

$$R(\tilde{u}, u_1) \approx \frac{4\eta_0^4 (\omega \tau)^2}{\pi^2} \frac{(1 - \tilde{u})}{u_1} \left( 1 - 2 \frac{\tilde{u}}{u_1} \left( 1 - \frac{\tilde{u}}{u_1} \right) \right). \quad (242)$$

Fig. 16 demonstrates the ratio of the resonant probability of scattering of a photon by an electron in the field of a pulsed wave to probability of the Compton effect as a function of parameters  $\tilde{u}$ ,  $u_1$  within the resonant peak ( $\beta = 0$ ) under  $\tau/T = 1$ ,  $\eta_0 = 0.05$ . It can be seen from Fig. 16 that the resonant probability may exceed considerably the probability of the Compton effect in external field absence. This fact becomes apparent particularly in the case  $u_1 \ll 1$  (but it should be noticed here, that in view of infrared divergence the formulae (239), (241) are correct within the region  $u_1 \gtrsim \eta_0^2$ ). Within the region  $u_1 \gtrsim 1$  this effect disappears. Under conditions  $\eta_0^2 \sim u_1 \ll 1$  within the range of optical frequencies  $E_i/m \ll m/\omega \sim 10^5$  for the ratio of probabilities is correct  $R \sim 10^3$ .

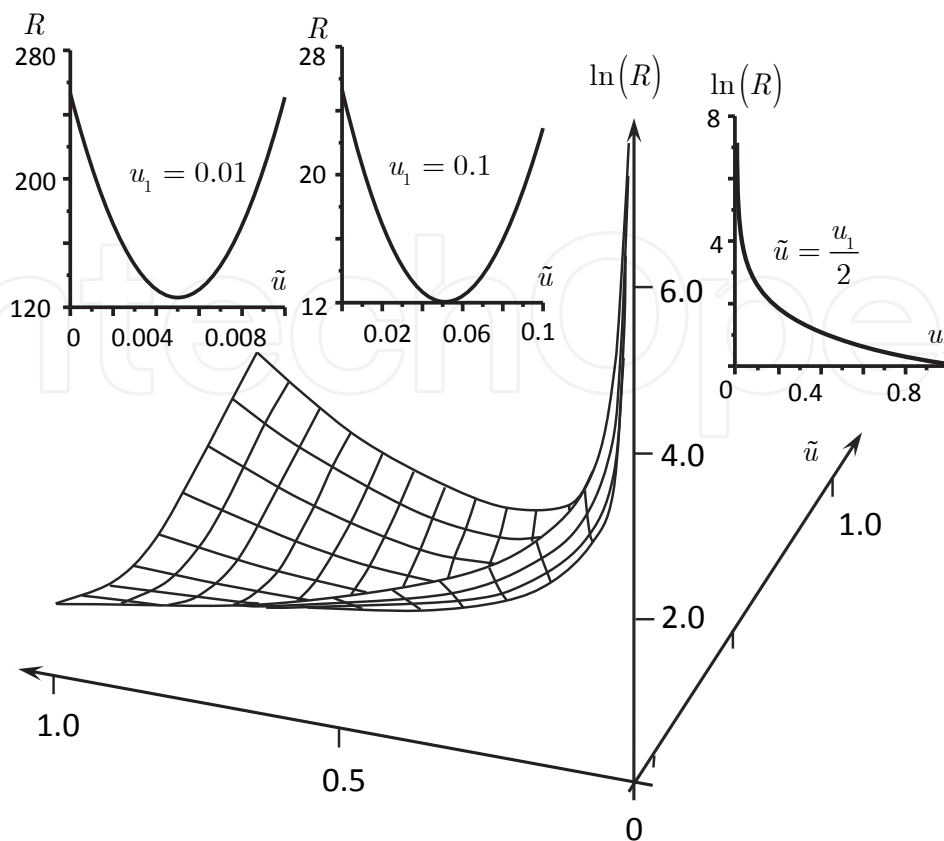


Fig. 16. Ratio of the resonant probability of scattering of a photon by an electron in the field of a pulsed wave to the probability of the Compton effect in external field absence (241) as a function of the parameter  $u_1$  (213) in the resonance peak ( $\beta = 0$ ) under  $\tau_{\text{imp}}/T = 1$ ,  $\eta = 0.05$ .

## 6. Conclusions

Performed studies of resonant QED processes in a pulsed light field result:

1. The QED processes of the second order in a pulsed light field may occur under resonant conditions when the four-momentum of an intermediate particle lies near the mass surface.
2. The resonant behavior of the cross-section is specified by characteristics of the laser pulse. The resonant infinity in the process amplitude is eliminated by accounting for the pulsed character of an external field.
3. The differential cross section of the resonant process may be several orders of magnitude higher than the corresponding cross section in external field absence.

The results can be tested in the experiments on verification of quantum electrodynamics in presence of strong fields (SLAC and FAIR).

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## **Quantum Optics and Laser Experiments**

Edited by Dr. Sergiy Lyagushyn

ISBN 978-953-307-937-0

Hard cover, 180 pages

**Publisher** InTech

**Published online** 20, January, 2012

**Published in print edition** January, 2012

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In order to correctly reference this scholarly work, feel free to copy and paste the following:

Sergei P. Roshchupkin, Alexandr A. Lebed', Elena A. Padusenko and Alexey I. Voroshilo (2012). Resonant Effects of Quantum Electrodynamics in the Pulsed Light Field, Quantum Optics and Laser Experiments, Dr. Sergiy Lyagushyn (Ed.), ISBN: 978-953-307-937-0, InTech, Available from:  
<http://www.intechopen.com/books/quantum-optics-and-laser-experiments/resonant-effects-of-quantum-electrodynamics-in-the-pulsed-light-field>

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Phone: +86-21-62489820  
Fax: +86-21-62489821

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