6

The Latest Mathematical Models of Earthquake Ground Motion

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1. Introduction

Strong motion instrument networks have enabled creation of a large number of databanks ranging from small to regional and world ones. This data is of a great importance for the investigations aimed at prediction of strong earthquake ground motion parameters by application of empirical mathematical models fitted to the databanks. These mathematical models are referred to as ground motion models or attenuation laws. They define the relationships between ground motion parameters and factors that affect the amplitudes of ground motion as are the released energy, the regional characteristics, the local soil characteristics, the type of fault, the radiation pattern, etc.

Ground motion models are defined by application of the regression analysis method. Regression coefficients and standard deviation are obtained as a result of the regression analysis. Standard deviation is the measure for the dispersion of the data around the computed medium or median value for which a distribution function defined by the probability density function is assumed.

Regression coefficients and standard deviation are the input parameters for the probabilistic seismic hazard analyses (Cornell 1968). Despite the evident results of the progress made in the use of the seismic hazard methodology, there are still uncertainties by which the hazard curves are computed. The mathematical models of ground motion have a big influence upon the results obtained from the seismic hazard analyses that are applied in practice. This justifies the efforts made by a large number of researchers worldwide toward development of mathematical models that will best fit the available databanks obtained from occurred strong earthquakes. As a result, there is a big number of different mathematical models of ground motion.

The presented investigations refer to the latest mathematical models of ground motion during earthquakes. These are: the azimuth dependent mathematical model and the mathematical model based on radius vectors.

2. Azimuth dependent mathematical model

Based on data from records on earthquakes that occurred from the Vranchea focus in Romania, the author has developed an azimuth dependent mathematical model of ground motion. It includes the focal mechanism, the size of the seismic field represented by an ellipse with a shape dependent on the relative relationship of its semi-axes and with a longitudinal axis in the direction of the projection of the fault plain upon the surface as well as the position of the instrument location (Stamatovska, 1996). Presented for this mathematical model are the idea used in defining the mathematical equation for a single earthquake, the general procedure of definition of the azimuth dependent mathematical model for any selected azimuth and its application in the seismic hazard analyses. The detailed description of the procedure of its development is aimed at its easier understanding and use by other researchers. This also contributes to easier understanding of the procedure by which the author has developed a new mathematical model based on radius vectors.

2.1 Mathematical equation

The starting point is a general empirical ground motion model in which ground motion parameter- Y depends on magnitude- M, distance- R and local soil conditions- S. It is given in Equation 1

$$\ln Y = b + b_M M + b_R \ln(R_h + C) + b_S S + \sigma_{\ln Y} P \qquad (1)$$

where,
Y -peak ground acceleration- PGA, or peak ground velocity- PGV or peak ground displacement- PGD; parameter of dynamic response of a linear or nonlinear model of a single degree of freedom system– SDOF, as well as Fourier Amplitude Spectrum- FS
M -magnitude
R_h -hypocentral distance in km
S -parameter that includes the effect of local soil conditions and has values, for example, 0 for rock, 1 for alluvium, 2 for deep alluvium
C -constant by which is defined the shape of the attenuation in the epicentral zone expressed in km
b, b_M, b_R, b_S -regression coefficients
$\sigma_{\ln Y}$ -standard deviation
P -binary variable, which has the value of 0 and 1 for median and median plus one standard deviation, respectively.

The model is based on the following theoretical assumptions: term $e^{b_M M}$ involves the relationship between energy and magnitude; coefficient b_R has a negative value and accounts for the spherical spreading of the seismic wave energy, while term $b_S S$ includes the effect of local soil conditions.

The ground motion model given in Equation 1 is simplified by use of records of occurred strong earthquakes obtained on rock soil type or referent soil with $V_S \geq 700 m/s$, by which the parameter defining the effect of the local soil conditions is omitted. With this, the parameters of ground motion under strong earthquake effect are only a function of distance and magnitude.

2.2 Mathematical equation for a single earthquake

The solution of the mathematical equation of a single earthquake came from the analyses of the records of an earthquake obtained at two locations, i.e., by instruments situated at equal epicentral distance from the earthquake epicenter. For each of the two locations, the epicentral distance and the focal depth are equal. The difference is in their position in respect to the projection of the fault upon the surface, i.e., the angle between the direction of the fault plane and the direction toward the instrument location. Hence, the differences in the recorded amplitudes at these two locations result from the position of the location in respect to the projection of the fault plane and the characteristics of the region in the direction of that location. If the recorded amplitudes, for example, amplitudes of *PGA* with equal value are connected by an isoseismal, then it is clear that, although the two considered locations are at equal epicentral distances, due to the different recorded amplitudes, the two locations will not lie on the same isoseismal. This means that the characteristics of the focus and the region in the direction toward the location perform faster or slower attenuation of the energy of the seismic waves by which they define the form of the isoseismals of equal *PGA*. Since the earthquake depth is the same for both locations, it is clear that the regional characteristics perform correction through the epicentral distances wherefore the form of the seismic field on the surface is not a circle. Therefore, the model of ground motion for each individual earthquake is a function of corrected epicentral distance or epicentral distance divided by a single function, the so called ρ, whose value depends on the form of the isoseismal of equal amplitudes of *PGA* and the angle between the fault plane and the direction of the location, i.e., the radiation pattern.

During mathematical modelling, particular importance is given to idealization of the form of the seismic field on the surface. For the azimuth dependent mathematical model developed by the author, it is assumed that this form may range from a circle to any shape of an ellipse with a longitudinal axis in the direction of the projection of the fault plane upon the surface (Figure 1). The shape of the ellipse is defined by the ratio of the semi-axes $a:b$, whereas the position of any two points M and M_i lying on it, is defined by radius vectors $\overrightarrow{\rho}$ and $\overrightarrow{\rho_i}$, whose moduli are equal to ρ and ρ_i.

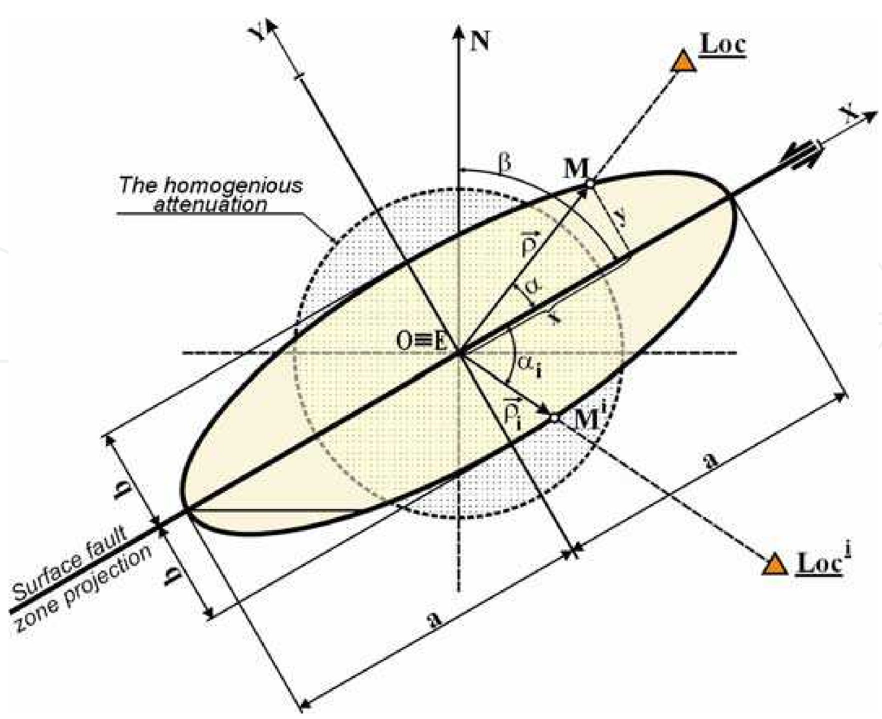


Fig. 1. Function ρ

$$\rho = \left|\vec{\rho}\right| = \sqrt{x^2 + y^2} \tag{2}$$

$$tg\alpha = y/x \tag{3}$$

$$x^2 / a^2 + y^2 / b^2 = 1 \tag{4}$$

$$\rho = \sqrt{\frac{1 + tg^2\alpha}{a^{-2} + tg^2\alpha}} \tag{5}$$

$$\beta_i^L \pm \alpha_i = \beta \tag{6}$$

So the mathematical equation for the *PGA* of an earthquake acquires a form dependent on the corrected epicentral distance $\frac{R_e}{\rho}$:

$$PGA = b_0 (\frac{R_e}{\rho})^{b_1} e^{\sigma_{\ln PGA}} \tag{7}$$

where,
b_0 and b_1 are regression coefficients
$\frac{R_e}{\rho}$ - corrected epicentral distance, and
$\sigma_{\ln PGA}$ - standard deviation

2.3 Regression analysis method

The exploration through analysis of a large number of published ground motion models (Joyner & Boore, 1981; 1988; Boore & Joyner 1982; Ambraseys & Bommer, 1992; Ambraseys et al., 1996; Boore et al., 1993; Sabetta & Pugliese, 1987, 1996; Idriss, 1991; Sadigh, 1993; Sadigh at al., 1993; Campbell, 1981) has pointed out the primary importance of the empirical model developed by application of the double regression method. This method (Joyner & Boore, 1981) involves the mode in which earthquakes occur in nature, one at a time, which is encompassed by the first step. Their connection is the objective of the second step. Accordingly, the regression analysis method is carried out in two steps as follows:

First step: Definition of ground motion models for each occurred earthquake taken separately, and,

Second step: Connection of all occurred earthquakes, i.e., different magnitudes and focal depths.

2.3.1 First step of regression analysis

The first step of the regression analysis involves definition of regression coefficients b_0 and b_1, and standard deviation $\sigma_{\ln PGA}$. To carry out the first step, it is necessary to perform parametric analysis in which the value of the parameters affecting function ρ will vary.

These are: the azimuth of the projection of the fault plane upon the surface β and the ratio of the semi-axes of the ellipse of the seismic field $a:b$.

The procedure itself is reduced to the following:

1. An initial value for the azimuth of the projection of the fault plane on the surface- β (Figure 2a) is selected;
2. The $a:b$ ratio is defined for value of $b=1.$, by which the relative ratio of the semi-axes of the seismic field is $a:1=a$ (Figure 2a)
3. An initial value of the relative ratio $a=1$. (Figure 2a) is defined;
4. The values of function ρ for all instrument locations and the values of the corrected epicentral distances $\frac{R_e}{\rho}$ are computed;
5. Linear regression is carried out for dependent random variable PGA and independent random variable a $\frac{R_e}{\rho}$. Then, the regression coefficients b_0 and b_1 and the standard deviation $\sigma_{\ln PGA}$ from the first step is computed.
6. The value of the relative ratio a is changed for an increase of Δa and the procedure from item 4. (Figure 2b) is repeated;
7. A new value of azimuth β with an increase $\Delta\beta$ is selected and the procedure pursuant to 1 (Figure 2c) is repeated.

A number of solutions is obtained. Out of these, the one for which the standard deviation has the least value is selected. With this, the ground motion model due to an earthquake is defined. In the same way, the ground motion models are defined for all occurred earthquakes originating from a single focus.

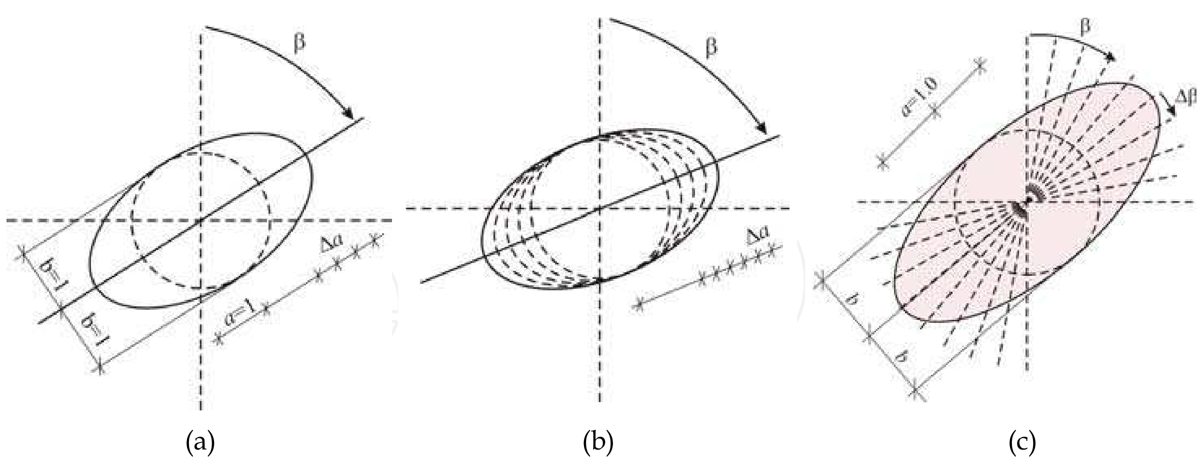


Fig. 2. Procedure referring to the first step of the regression analysis

2.3.2 Second step of regression analysis

In the second step of the regression analysis, all the occurred earthquakes originating from the same focus are connected and regression coefficients b, b_R and b_M and the standard deviation $\sigma_{\ln PGA}$ are computed. The data used in the second step of the regression analysis are: earthquake magnitude- M and hypocentral distance- R_h as independent variables and

PGA as dependent variable (Equation 1). Hypocentral distance is computed according to the following formula:

$$R_h{}^2 = \left(\frac{R_e}{\rho}\right)^2 + h^2 \tag{8}$$

while value $\dfrac{R_e}{\rho}$ is computed separately for each occurred earthquake and for all the instrument locations on which the records from that earthquake are obtained.

A key issue in the second step of the regression analysis is the connection of all the earthquakes (Figure 3) and definition of the ground motion model given by Equation 1.

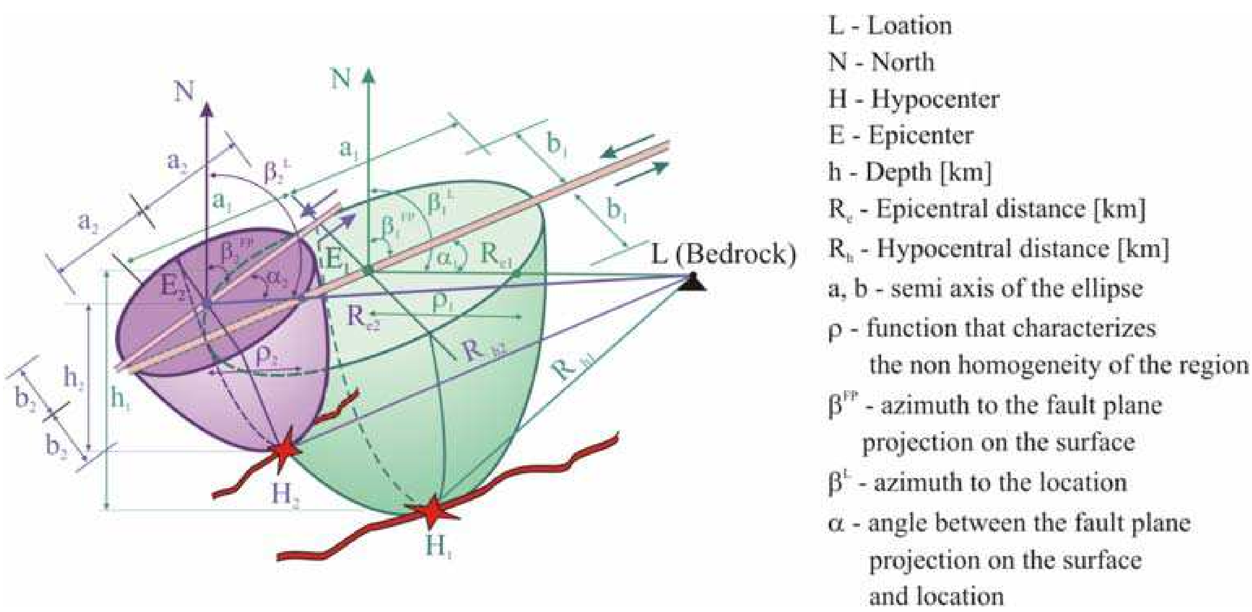
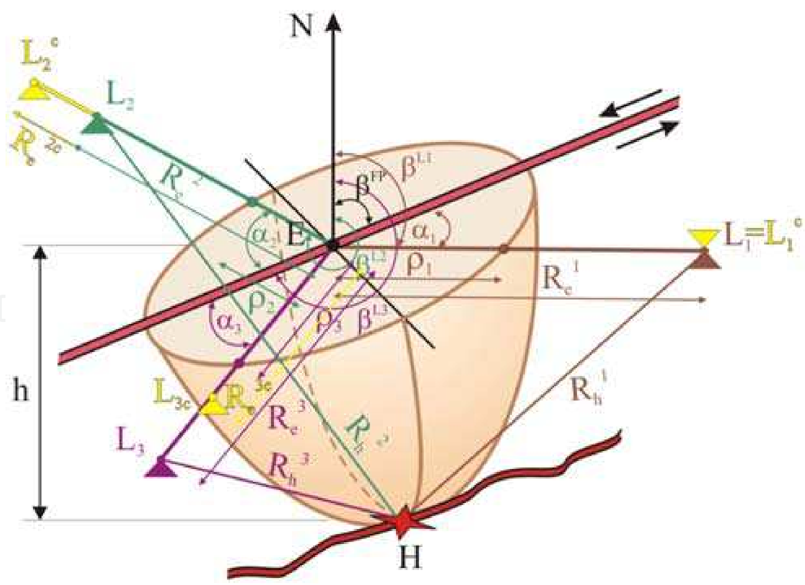


Fig. 3. Connection of earthquakes – second step of regression analysis

The solution is possible only if a ground motion model is defined for a direction toward a location, in which case it is necessary to perform normalization of value $\dfrac{R_e}{\rho}$. The normalization is performed separately for each occurred earthquake with value ρ_i defined for the direction toward the selected location by use of the ground motion model computed in the first step of the regression analysis performed for that earthquake (Figure 4). All the normalized values are used in the second step of the regression analysis.

It is possible to compute ground motion models for different directions (azimuths according to locations) in which case it is necessary to perform normalization of $\dfrac{R_e}{\rho}$ for each selected direction, separately.

The value of constant C is defined by its variation (for example, from 0 km to 200 km, by a step of 1, or 2, or more km) and execution of the second step of the regression analysis for each of its values. A number of solutions is obtained out of which the one for which the standard deviation in the second step of the regression analysis is minimal, is selected.



Normalization according to the azimuth's location

$$\text{Location } L_1\text{: } \frac{R_e^1}{\rho_1}\rho_1;\ \frac{R_e^2}{\rho_2}\rho_1;\ \frac{R_e^3}{\rho_3}\rho_1;$$

$$\text{Location } L_2\text{: } \frac{R_e^1}{\rho_1}\rho_2;\ \frac{R_e^2}{\rho_2}\rho_2;\ \frac{R_e^3}{\rho_3}\rho_2;$$

$$\text{Location } L_3\text{: } \frac{R_e^1}{\rho_1}\rho_3;\ \frac{R_e^2}{\rho_2}\rho_3;\ \frac{R_e^3}{\rho_3}\rho_3;$$

Fig. 4. Normalization over selected azimuth

2.4 Advantages

The advantages of the azimuth dependent ground motion model are:

- Definition of separate ground motion models for different directions
- The mathematical form of the azimuth dependent ground motion model (Equation 1) is applicable in seismic hazard methodology;
- Application in definition of ground motion models for spectral characteristics of ground motion expressed by response spectra and the Fourier Amplitude Spectrum.
  In this case, the results from the first step of the regression analysis (Stamatovska, 2008) (β, a , b_0, b_1 and $\sigma_{\ln PGA}$ from the first step) defined for PGA are used, and it is only in the second step that the PGA value is replaced by the value of the spectral characteristic of the earthquake, as for example, the spectrum of the linear model of SDOF (absolute acceleration– SA, relative velocity- SV, relative displacement SD), the Fourier Amplitude Spectrum- FS and the spectrum of the nonlinear model of SDOF (acceleration spectrum, displacement spectrum, ductility factor and alike);
- In case of a new earthquake, only the ground motion model for the new earthquake is defined in the first step. All the previous results from the first step obtained for the preceding earthquakes are used (preceding earthquakes + the new earthquake) and the second step of the regression analysis is carried out;

- Improvement of the azimuth dependent ground motion model is possible through idealization of the seismic field upon the surface via including irregular forms defined by radius vectors.

2.5 Application in probabilistic seismic hazard analyses - PSHA

The application of the azimuth dependent ground motion model in PSHA is based on the following two steps:

- Definition of azimuth dependent ground motion models for different azimuth directions;
- Definition of sub-sources in a seismic source.

To define the ground motion model for any azimuth direction of a seismic source, it is necessary to pre-define ground motion models for each occurred earthquake from that source by application of the first step of the regression analysis of the azimuth-dependent empirical mathematical model (Stamatovska, 1996, 2002, 2006, 2008; Stamatovska & Petrovski, 1996, 1997) presented by Equation 1.

Important parameters from the first step of the regression analysis for each occurred earthquake are: the azimuth of the projection of the fault upon the surface- β and the value of the relative ratio a. By using these parameters, the value of function ρ_i can be computed for each selected direction **i** defined by azimuth- β_i. In doing so, angle- α_i, as an angle between the azimuth of the projection of the fault plane upon the surface- β and the selected azimuth- β_i is defined by using Equation 6.

With the value of function ρ_i normalization for the selected azimuth is performed. Each corrected epicentral distance $\frac{R_e}{\rho}$ in which ρ is the value computed for the azimuth of the instrument location, is multiplied by ρ_i.

This procedure is iterated separately for each occurred earthquake originating from the investigated seismic focus (for example, if four strong earthquakes took place, it is iterated 4 times). All the normalized values are used in the second step of the regression analysis and the regression coefficients b, b_M and b_R as well as the standard deviation $\sigma_{\ln Y}$ are computed. With this, the ground motion model for that azimuth is defined. By selection of a new azimuth (new location) and iteration of the entire procedure described in this part, ground motion models for different azimuth directions are obtained. This step is schematically presented in Figure 5.

The computed ground motion models can directly be applied in analyses of seismic hazard for all the software packages in which the ground motion model is assigned or reduced to the mathematical form presented in Equation 1 in the case of a point seismic source. In all other cases of seismic sources, it is necessary to model sub-sources.

2.5.1 Definition of sub-sources in seismic source

In the methods for computation of seismic hazard (Cornell, 1968), the seismic source is modelled as point, line or area source. Each point of the seismic source, defined by

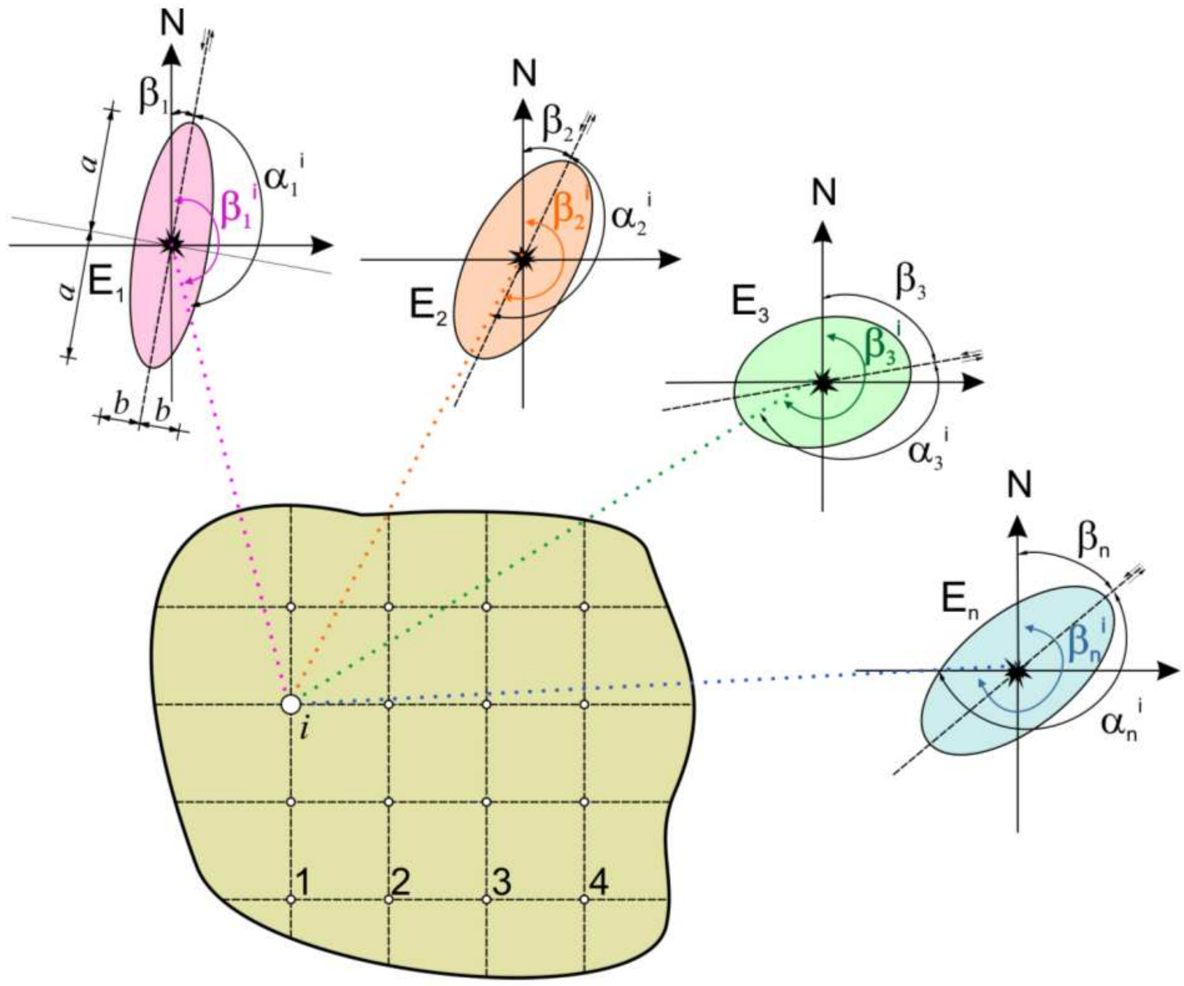


Fig. 5. Application of the results obtained in the first step of the regression analysis for definition of the model of ground motion at a selected location

coordinates (x, y) where x is east longitude, while y – north latitude, is a potential epicenter of a future earthquake from that focus. The possibility that the model of the seismic source be represented by a point (in the case of a point seismic source), or a number of points (in the case of a linear or an area model of seismic source) facilitates the procedure to be applied if a software package is developed for the purpose of avoiding a large number of computations. Then, the area of the seismic source is modelled by sub-sources with very small areas $\Delta S = \Delta x \Delta y$, to be harmonized with the computed ground motion models for different azimuths (Figure 6).

The above means that the azimuths of the end points of the small seismic sub-source computed in respect to a single point in region-**i** for which the seismic hazard is computed should tend to a single azimuth value. This is possible in all cases where the seismic hazard is computed for a point in the region that is sufficiently distant to reach an azimuth (Figure 6, point 1). However, particular attention should be paid to a point of the region that is very close to the seismic source (Figure 6, point 2) when the azimuth of the end points of the small seismic sub-source do not tend to an azimuth but there is a considerable difference among them. It is further necessary to reduce the area of the seismic sub-source $\Delta S_1 \langle \Delta S$, or $\Delta \beta_1 \langle \Delta \beta$ (Figure 6).

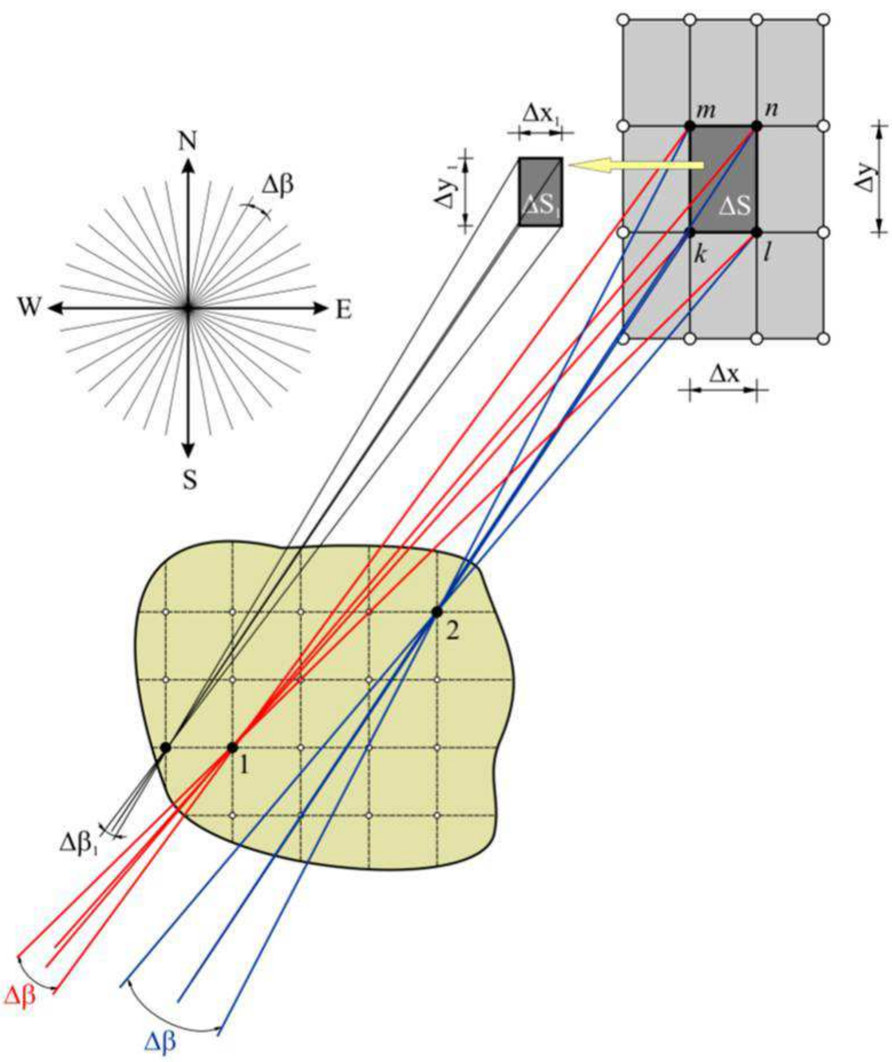


Fig. 6. Effect of modelling of seismic source and epicentral distance upon the extent of deviation from an azimuth

3. Mathematical model based on radius vectors

The mathematical model based on radius vectors represents an advanced azimuth dependent mathematical model. It is developed as an azimuth dependent model of a random shape of a seismic field defined by radius vectors in different azimuth directions.

3.1 Theoretical background

The ground motion model defined on the basis of radius vectors has the same mathematical form as the azimuth dependent model, or,

$$\ln Y = b + b_M M + b_R \ln(R_h + C) + \varepsilon \tag{9}$$

$$R_h{}^2 = (R_e^c)^2 + h^2 \quad R_e{}^c = R_e\left(\frac{\rho_L}{\rho_i}\right) \quad \frac{\rho_L}{\rho_i} = \frac{\left|\overrightarrow{\rho_L}\right|}{\left|\overrightarrow{\rho_i}\right|} \tag{10}$$

where: Y is the ground motion parameter (peak acceleration, velocity, displacement, horizontal vector, spectral amplitude, etc.), ρ_i is the modulus of the radius vector in respect to any instrument location, whereas ρ_L is the modulus of the radius vector in respect to the location/or the direction for which the ground motion model is defined. The effect of the local soil conditions is not included in this mathematical model due to usage of records obtained on one type of local soil conditions (for example, rock with $V_s \geq 700m/s$).

3.2 Method

The method for definition of this model consists of two parts. The first part involves preparation of data to be used in the regression analysis. In this part, the shape of the recorded seismic field defined by radius vectors (Fig. 7) is established. Each radius vector begins at the earthquake epicentre and runs in the direction from the epicentre to the instrument location. Its modulus is equal to the absolute value of peak acceleration /or velocity/ or displacement/ of ground or vector defined for horizontal direction under the earthquake effect. Applying the normalized seismic field for a selected azimuth/ or direction toward a selected location, the value of the relative relationship of $\frac{\rho_L}{\rho_i}$ or $\frac{\rho_i}{\rho_L}$ moduli (Fig. 8) is defined. This relationship is a dimensionless number and enables obtaining the regional characteristics in different directions. It is used to correct the epicentral distances. This is carried out separately for each earthquake that has occurred from a single seismic focus.

In the second part, the multi linear regression analysis method is used. The data for the regression analysis are: PGA - dependent variable, M and R_h - independent variables. Each regression analysis results in regression coefficients b, b_M, b_R and standard deviation - $\sigma_{\ln Y}$. The number of regression analyses depends on the number of variations of constant C (for example, 27 analyses with variable C ranging from 0 to 130 km, with a step of 5 km). From the multitude solutions, the one for which the standard deviation is minimal is selected.

The second part is equal to the second step of the regression analysis applied in the azimuth dependent model. In this way, the simplest mathematical model for prediction of characteristics of future earthquakes from a single seismic focus is obtained. According to the author, this model is the closest to the physical model since it includes a realistically occurred seismic field recorded by strong motion instruments.

The described procedure is based on the idea that the amplitudes of ground motion obtained for different epicentral distances and different azimuths result from the effect of the amount of the energy released by the earthquake, the focal mechanism and the regional characteristics at different azimuths from the earthquake hypocenter.

3.3 Method verification

The method verification has been performed on the basis of the created data bank of available three-component records of strong earthquakes that occurred on March 4, 1977 (epicenter 45.8N and 26.8E, M=7.2, h=109 km), August 30, 1986 (epicenter 45.52N and 26.49E, M=7.0, h=131 km), May 30, 1990 (epicenter 45.872N and 26.885E, M=6.7, h=99.1 km) and

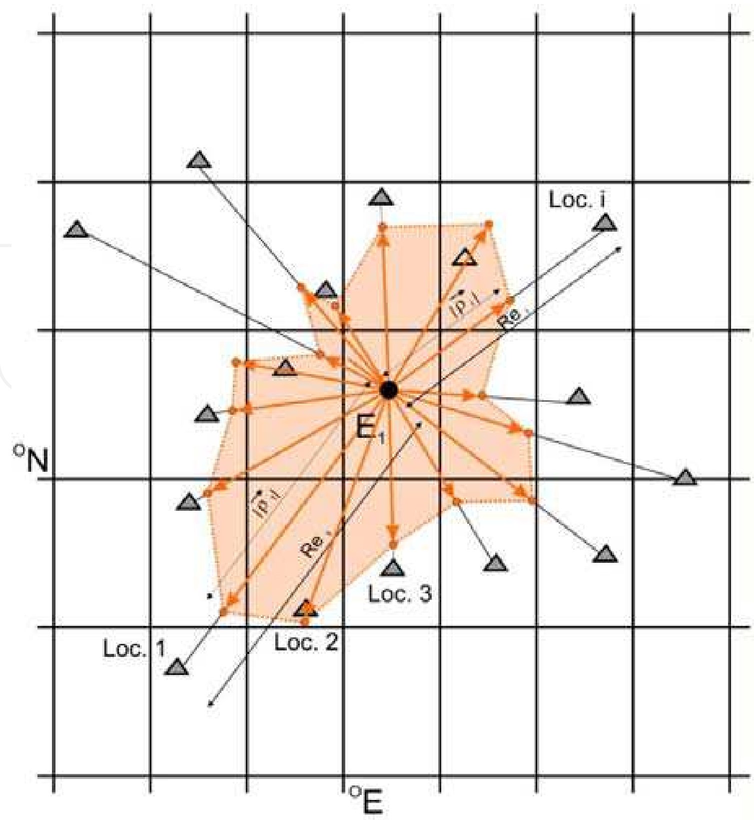


Fig. 7. Recorded seismic field of PGA at rock

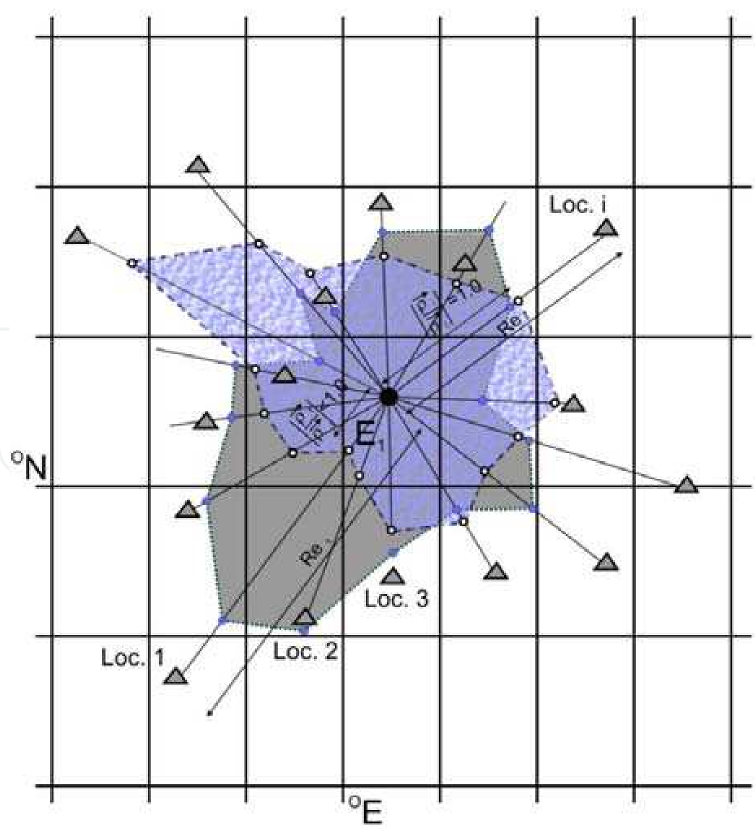


Fig. 8. Normalized seismic field for the azimuth toward location i

May 31, 1990 (epicenter 45.852N, 26.882E, M=6.1, h=89.1 km). The data bank includes data from records of occurred deep earthquakes at the Vranchea focus (Romania) obtained by the instruments of the Romanian, Bulgarian and Former Yugoslav strong motion networks.

The isoseismals of the recorded PGA seismic field (in cm/s^2 for $V_S \geq 700m/s$) referring to the earthquakes that occurred at the Vranchea focus are given in figures 9, 10 and 11.

Two separate investigations have been performed. In the first one, the ground motion parameter are the peak ground accelerations from the two horizontal components, while in the second investigation, the ground motion parameter is the higher value of the two horizontal components of the peak ground acceleration. Mathematical models of ground motion have been defined for seven azimuths toward the following instrument locations: BUC (Bucharest), CFR (Carcaliu), CVD (Chernavoda), IASI (Iasi), VLM (Valeni de Munte) and VRI (Vrincioaia). For all these, the regression coefficients and standard deviations are given (Tables 1 and 2). The results shown in Table 1 refer to two horizontal components, whereas those in Table 2 refer to the larger component of the two horizontal components. The March 4, 1977 earthquake is included only for an azimuth toward the INC (INCERC-Bucharest) location.

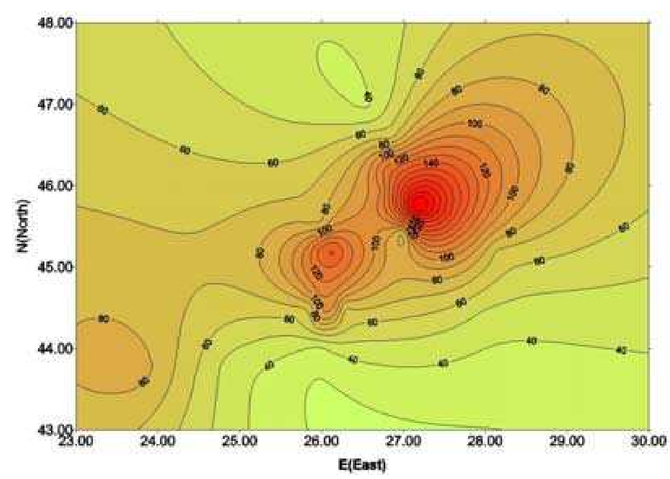


Fig. 9. The earthquake of 30th August 1986 – recorded PGA seismic field

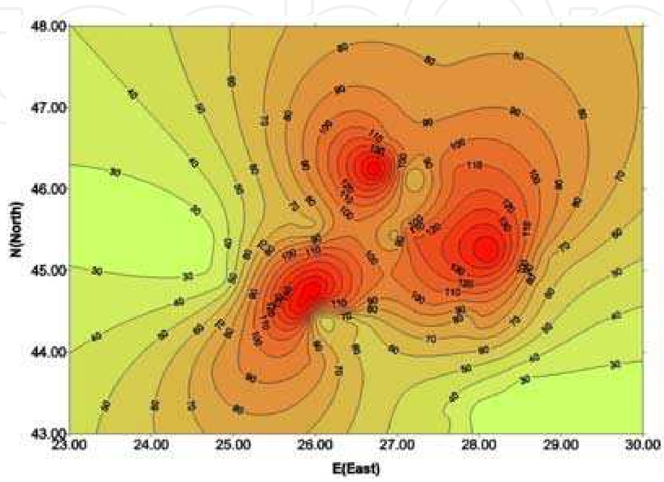


Fig. 10. The earthquake of 30th May 1990 – recorded PGA seismic field

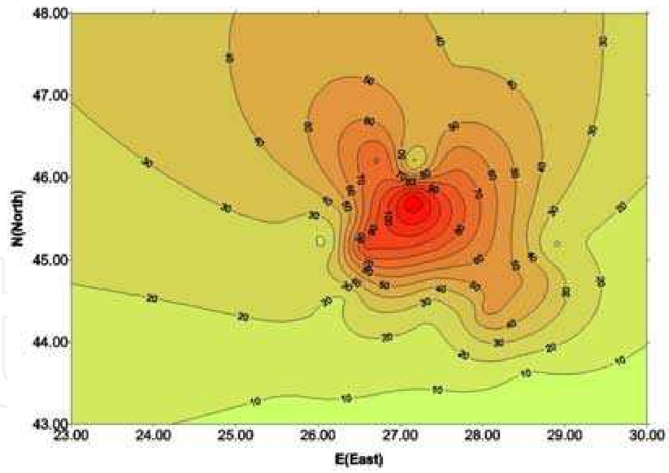


Fig. 11. The earthquake of 31st May 1990 – Recorded PGA seismic field

The data used for definition of the mathematical model based on radius vectors for the MLR azimuth based on the larger of the two horizontal components (a total of 95 PGA) are given in Table A1 (Appendix A). The isoseismals of the normalized seismic field $|\rho_{VLM} / \rho_i|$ for the VLM azimuth are given in figures 12, 13 and 14.

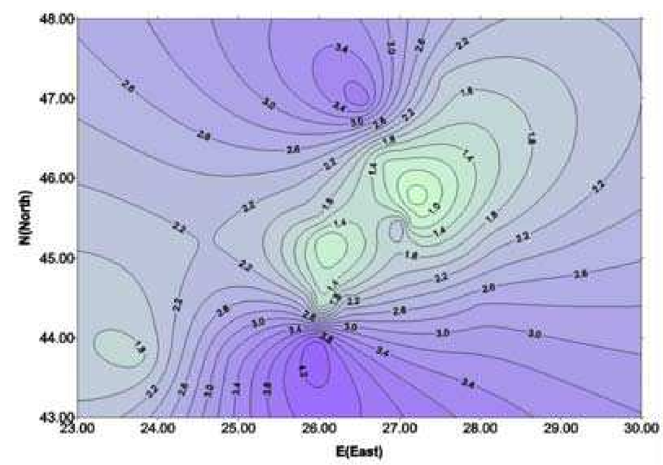


Fig. 12. The earthquake of 30th August 1986 - Normalized seismic field for the VLM azimuth

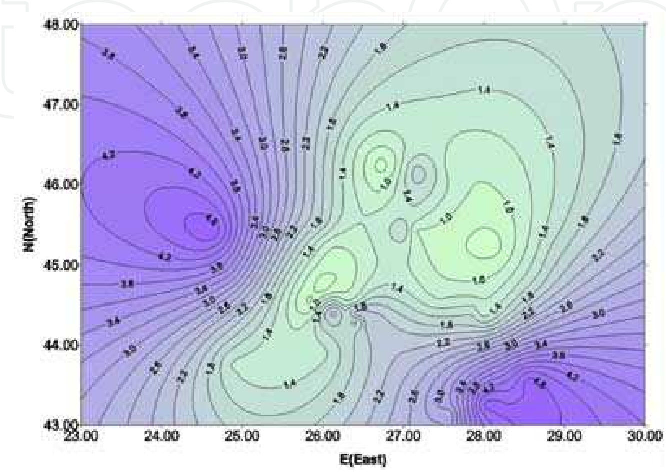


Fig. 13. The earthquake of 30th May 1990 – Normalized seismic field for the VLM azimuth

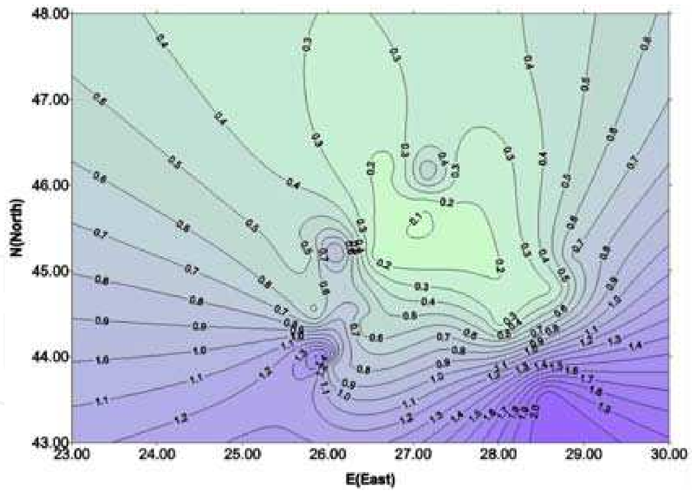


Fig. 14. The earthquake of 31st May 1990 – Normalized seismic field for the VLM azimuth

Mathematical Model: $\ln PGA = b + b_M M + b_R \ln Rh + \sigma_{\ln PGA}$

Azimuth	Regression coefficients			Standard deviation $\sigma_{\ln PGA}$
	b	b_M	b_R	
INC	-1.84230	1.50539	-0.79342	0.37103
BUC	-2.08125	1.61035	-0.87901	0.33432
CFR	0.52772	0.98049	-0.53216	0.40309
CVD	2.53490	0.77706	-0.67739	0.35774
IASI	1.19074	1.03637	-0.75200	0.32129
VLM	-4.33168	1.78635	-0.64281	0.40225
VRI	2.13673	0.82625	-0.63389	0.31867

Table 1. Regression coefficients and standard deviations based on two horizontal components

Mathematical Model: $\ln PGA = b + b_M M + b_R \ln Rh + \sigma_{\ln PGA}$

Azimuth	Regression coefficients			Standard deviation $\sigma_{\ln PGA}$
	b	b_M	b_R	
INC	-1.40590	1.49455	-0.84663	0.35791
BUC	- 1.60526	1.59385	-0.93390	0.32036
CFR	0.94361	0.96645	-0.57296	0.38277
CVD	2.95699	0.76408	-0.72328	0.33394
IASI	1.60496	1.02434	-0.79915	0.29758
VLM	-3.91229	1.76977	-0.68350	0.39286
VRI	2.58231	0.80355	-0.67176	0.29063

Table 2. Regression coefficients and standard deviations based on the larger component out of the two horizontal components

Azimuth	Magnitude M	Hypocentral Distance R_h (km)	Predicted PGA-L (cm/s^2)		Recorded PGA (two horizontal components) (cm/s^2)		Predicted PGA (cm/s^2)	
			50% non-exceedance	84% non-exceedance			50% non-exceedance	84% non-exceedance
INC	7.2	187.80	137.34	196.44	137.81	115.30	124.59	180.31
BUC	7.0	188.32	105.613	145.495	-95.77	-81.06	98.18	137.16
CFR	7.0	188.19	110.820	162.500	-70.04	-69.62	99.88	149.47
CVD	7.0	221.72	81.354	113.608	40.69	-51.13	74.85	107.04
IASI	7.0	241.85	80.589	108.520	51.27	76.36	75.05	103.48
VLM	7.0	139.56	164.125	243.104	-123.02	-146.71	148.15	221.52
VRI	7.0	137.87	134.006	179.202	-107.90	63.11	121.24	166.74
BUC	6.7	207.47	59.812	82.399	-63.34	-61.58	55.62	77.71
CFR	6.7	159.36	91.218	133.756	164.01	88.83	81.32	121.68
CVD	6.7	217.22	65.656	91.686	77.27	93.26	60.11	85.97
IASI	6.7	184.53	73.568	99.066	73.44	81.56	67.40	92.94
VLM	6.7	139.17	96.703	143.238	-118.19	91.52	86.85	129.86
VRI	6.7	99.87	130.764	174.867	91.66	-120.47	116.08	159.64
BUC	6.1	194.51	24.413	33.632	15.66	-16.56	22.40	31.29
CFR	6.1	152.41	52.401	76.838	-59.01	-46.55	46.24	69.19
IASI	6.1	181.26	40.364	54.354	38.02	-40.51	36.68	50.58
VLM	6.1	130.89	34.871	51.652	13.91	-13.85	30.93	46.25
VRI	6.1	89.95	86.623	115.839	-33.53	78.47	75.55	103.91

Table 3. Comparison between recorded and predicted values of PGA

Applying the regression coefficients and standard deviations from Tables 1 and 2, the PGA values have been computed with a non-exceedance of 50% and 84%, or as median and median + 1 standard deviation (Table 3).

The obtained results point to good fitting of the data from the mathematical model based on radius vectors, particularly in the case of use of the higher component from the two horizontal components. This is confirmed by the small values of the computed standard deviations ($\sigma_{\ln} Y \leq 0.4$) as well as the values of the median and median+1 standard deviation for the predicted PGA (PGA-L in Table 3).

The obtained PGA values depend on the instrument type, its transmission characteristics, maintenance, knowledge of the characteristics of the local profile of the instrument location, the procedures for processing of records, etc. The effect of the mathematical operations is reduced to minimum since only one multi linear regression analysis is performed.

3.4 Advantages and disadvantages

The advantages and disadvantages of the ground motion model based on radius vectors are:

- The advantage of the mathematical model based on radius vectors is that it uses a recorded seismic field. In this case, the uncertainties that are incorporated in the computation of the mathematical model of the earthquake ground motion result from the accuracy of the records.

- The disadvantage of this model is the case of use of a small number of records of occurred earthquakes and their non-uniform distribution in respect to the different azimuths. In such a case of a small number of records, the irregular closed polygon of the seismic field upon the surface will represent a polygonal figure with longer sides. This is not a deficiency of the method itself but a deficiency related to the available number of records and position of instruments. As such, it will be overcome by gradual increase of the number of instruments and records.

4. Conclusions and recommendations

The conclusions and recommendations referring to the presented ground motion models are the following:

- The azimuth dependent ground motion model defined by application of the double regression analysis contains all the specificities of the occurred individual earthquakes originating from a single seismic source;
- In an indirect way, by application of a parametric analysis, it includes in itself the characteristics of the seismic focus and the position of the location in respect to the projection of the fault plane upon the surface, or radiation pattern;
- The results obtained in the first step of the regression analysis can be controlled by the results computed by use of seismological data– seismograms. An example for this is the azimuth of the projection of the fault plane on the surface - β;
- It is possible to develop a method for computation of azimuth dependent ground motion model by use of results from seismological investigations, or taking the direction of the projection of the fault plane on the surface from the seismological investigations. This will extensively simplify the computation of the azimuth dependent ground motion model since the first step of the regression analysis will involve only parametric analysis of the relative ratio of the semi-axes of the ellipse of the seismic field $a:1=a$;
- Two models are applicable in seismic hazard analyses;
- The ground motion model based on radius vectors will yield even better results if the position of the instrument within an observation network is permanent, if it is regularly maintained and calibrated, if there are as many as possible instruments within the network and if the triggering thresholds are such that records of a number of occurred earthquakes are obtained from as many as possible instruments. So, the more exactly the recorded seismic field is defined, the more reduced will be the values of the standard deviations in the mathematical model of ground motion based on radius vectors.

The author believes that, in future, advantage will be given to the model based on radius vectors particularly due to the increasing number of recording instruments and number of records of occurred strong earthquakes generated from single seismic foci.

5. Acknowledgement

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Appendix A

No.	Data source code *)	Comp. **)	Instrument location			Mag. M	Depth h (km)	Corrected epicentral distance (km)	Hypocentral distance (km)	Peak ground acceleration PGA (cm/s²)	Normalized seismic field $\|\rho_{VLM}/\rho_i\|$
			Code	N (rad)	E (rad)						
1	1	2	FOC	0.798	0.474	7	131	36.897	136.097	227.7609	0.6441
2	1	2	VRI	0.801	0.466	7	131	58.413	143.433	-107.904	1.3596
3	1	1	DOC	0.819	0.463	7	131	580.25	594.854	-38.9911	3.7626
4	1	1	CFR	0.789	0.491	7	131	283.008	311.856	-70.039	2.0947
5	1	1	MLR	0.794	0.453	7	131	79.065	153.011	-79.122	1.8542
6	1	1	ISR	0.788	0.463	7	131	57.393	143.021	109.075	1.345
7	1	2	IAS	0.824	0.481	7	131	390.608	411.99	76.3557	1.9214
8	1	1	BAC	0.813	0.469	7	131	261.127	292.144	67.7456	2.1656
9	1	1	BUC	0.774	0.454	7	131	207.264	245.193	-95.7646	1.532
10	1	2	CVD	0.774	0.489	7	131	513.447	529.895	-51.1277	2.8694
11	2	2	BLV	0.776	0.451	7	131	277.749	307.092	67.2604	2.1812
12	2	1	BRN	0.777	0.46	7	131	221.264	257.136	-75.7762	1.9361
13	2	2	CVD	0.774	0.489	7	131	574.741	589.482	45.6613	3.213
14	2	2	EXP	0.776	0.456	7	131	158.18	205.382	113.8977	1.2881
15	2	1	FOC	0.798	0.474	7	131	38.055	136.416	220.8287	0.6644
16	2	1	GRG	0.767	0.453	7	131	798.046	808.727	33.5727	4.3699
17	2	1	INC	0.776	0.457	7	131	259.74	290.905	67.3488	2.1783
18	2	1	ONS	0.807	0.467	7	131	99.906	164.749	-119.651	1.2261
19	2	2	PRS	0.78	0.454	7	131	123.727	180.193	117.0445	1.2534
20	2	1	RMS	0.792	0.473	7	131	56.003	142.469	-126.626	1.1586
21	2	2	RMS	0.792	0.472	7	131	95.582	162.163	-70.9702	2.0672
22	2	1	TRM	0.764	0.434	7	131	743.82	755.267	46.128	3.1805
23	2	2	VLM	0.789	0.455	7	131	48.131	139.562	-146.708	1
24	3	2	KOZ	0.763	0.415	7	131	506.124	522.802	84.76	1.7309
25	1	1	ARR	0.792	0.43	6.7	99.1	881.87	887.421	-24.632	4.7984
26	1	1	BAC	0.813	0.469	6.7	99.1	90.24	134.03	-101.178	1.1682
27	1	2	BIR	0.807	0.482	6.7	99.1	74.799	124.16	113.7463	1.0391
28	1	1	BUC	0.774	0.454	6.7	99.1	340.121	354.264	-63.3387	1.8661
29	1	1	CFR	0.789	0.491	6.7	99.1	89.936	133.826	164.013	0.7206
30	1	1	CVD	0.774	0.489	6.7	99.1	257.233	275.662	-88.4745	1.3359
31	1	1	ARM	0.775	0.455	6.7	99.1	396.126	408.334	-52.2339	2.2628
32	1	1	MLR	0.794	0.453	6.7	99.1	151.993	181.446	-65.624	1.8011
33	1	2	SDR	0.794	0.46	6.7	99.1	70.538	121.641	-97.237	1.2155
34	1	2	VRI	0.801	0.466	6.7	99.1	12.149	99.842	-120.474	0.9811
35	1	2	IAS	0.824	0.481	6.7	99.1	225.596	246.403	81.5571	1.4492
36	2	1	ADJ	0.805	0.474	6.7	99.1	60.617	116.169	-66.3789	1.7806
37	2	2	BAA	0.781	0.5	6.7	99.1	319.905	334.903	-69.6289	1.6975
38	2	1	BIR	0.807	0.482	6.7	99.1	77.785	125.981	109.3795	1.0806
39	2	1	BLV	0.776	0.451	6.7	99.1	130.099	163.544	-159.892	0.7392
40	2	1	BRN	0.777	0.46	6.7	99.1	161.769	189.71	-115.588	1.0225
41	2	1	DRS	0.774	0.461	6.7	99.1	246.684	265.845	-82.9311	1.4252
42	2	2	FOC	0.798	0.474	6.7	99.1	43.077	108.057	83.2419	1.4199
43	2	2	FTS	0.775	0.486	6.7	99.1	276.548	293.768	76.9566	1.5359
44	2	2	GRG	0.767	0.453	6.7	99.1	309.034	324.535	-87.4576	1.3514
45	2	1	INC	0.776	0.457	6.7	99.1	279.72	296.756	69.8092	1.6931
46	2	1	MET	0.773	0.463	6.7	99.1	392.448	404.767	53.9582	2.1905
47	2	2	MLT	0.775	0.46	6.7	99.1	298.264	314.297	67.4054	1.7535
48	2	2	MTR	0.775	0.454	6.7	99.1	322.933	337.796	-65.0369	1.8173

*) Source of data: 1 INFP – Romania; 2 INCERC – Romania; 3 Bulgaria; 4 Former Yugoslavia; 5 GEOTEC – Romania **) Components: 1 N-S; 2 E-W

Table A1. (continues on next page) Data used for definition of mathematical model based on radius vectors for the VLM azimuth

No.	Data source code *)	Comp. **)	Instrument location			Mag. M	Depth h (km)	Corrected epicentral distance (km)	Hypocentral distance (km)	Peak ground acceleration PGA (cm/s^2)	Normalized seismic field $\lvert\rho_{VLM}/\rho_i\rvert$
			Code	N (rad)	E (rad)						
49	2	1	ONS	0.807	0.467	6.7	99.1	26.624	102.614	177.9046	0.6644
50	2	2	PIT	0.783	0.434	6.7	99.1	651.248	658.745	-35.0827	3.369
51	2	2	PND	0.774	0.461	6.7	99.1	209.246	231.527	96.5762	1.2238
52	2	2	PRS	0.78	0.454	6.7	99.1	101.328	141.733	171.5427	0.689
53	2	1	RMS	0.792	0.473	6.7	99.1	55.295	113.483	121.5669	0.9723
54	2	2	RMS	0.792	0.472	6.7	99.1	89.55	133.566	73.699	1.6037
55	2	2	SLB	0.778	0.478	6.7	99.1	172.285	198.753	102.0212	1.1585
56	2	2	TIT	0.775	0.461	6.7	99.1	387.054	399.539	50.5505	2.3381
57	2	2	TLC	0.788	0.503	6.7	99.1	278.851	295.937	-71.7137	1.6481
58	2	2	TRM	0.764	0.434	6.7	99.1	389.337	401.751	86.0013	1.3743
59	2	1	VLM	0.789	0.455	6.7	99.1	97.704	139.165	-118.194	1
60	2	2	CVD	0.774	0.489	6.7	99.1	244.985	264.269	93.2554	1.2674
61	3	1	VRN	0.755	0.489	6.7	99.1	1442.841	1446.24	25.0339	4.7214
62	3	2	KVR	0.758	0.495	6.7	99.1	1280.872	1284.7	27.1648	4.351
63	3	1	SHB	0.76	0.498	6.7	99.1	1377.353	1380.913	24.9266	4.7417
64	3	2	RUS	0.766	0.454	6.7	99.1	314.193	329.452	87.8256	1.3458
65	3	1	BZV	0.752	0.48	6.7	99.1	807.732	813.788	45.5224	2.5964
66	3	2	PRV	0.753	0.479	6.7	99.1	942.059	947.257	38.6623	3.0571
67	1	2	ARM	0.775	0.455	6.1	89.1	145.281	170.427	-16.5572	0.8402
68	1	1	BIR	0.807	0.482	6.1	89.1	15.557	90.448	-65.7703	0.2115
69	1	1	CFR	0.789	0.491	6.1	89.1	29.153	93.748	-59.009	0.2358
70	1	1	CVD	0.774	0.489	6.1	89.1	48.303	101.351	-54.929	0.2533
71	1	1	ISR	0.788	0.463	6.1	89.1	12.582	89.984	92.414	0.1505
72	1	1	SDR	0.794	0.46	6.1	89.1	17.664	90.834	44.312	0.314
73	1	2	VRI	0.801	0.466	6.1	89.1	2.183	89.127	78.4674	0.1773
74	1	2	IAS	0.824	0.481	6.1	89.1	54.208	104.295	-40.5088	0.3434
75	2	1	ADJ	0.805	0.474	6.1	89.1	22.207	91.826	-22.4738	0.619
76	2	2	BAA	0.781	0.5	6.1	89.1	54.169	104.274	-48.0672	0.2894
77	2	2	BIR	0.807	0.482	6.1	89.1	14.95	90.345	-68.4408	0.2033
78	2	1	BLV	0.776	0.451	6.1	89.1	81.847	120.987	-29.5639	0.4706
79	2	2	BRN	0.777	0.46	6.1	89.1	114.489	145.075	18.9555	0.7339
80	2	1	CLS	0.772	0.477	6.1	89.1	131.325	158.698	-19.5534	0.7115
81	2	1	CMN	0.788	0.45	6.1	89.1	49.684	102.016	32.8062	0.4241
82	2	1	CMN	0.788	0.449	6.1	89.1	55.098	104.76	29.6712	0.4689
83	2	2	CVD	0.774	0.489	6.1	89.1	49.446	101.901	-53.6487	0.2593
84	2	1	DRS	0.774	0.461	6.1	89.1	90.552	127.037	26.2509	0.53
85	2	1	FOC	0.798	0.474	6.1	89.1	3.057	89.152	-132.605	0.1049
86	2	2	FTS	0.775	0.486	6.1	89.1	69.84	113.21	35.4889	0.392
87	2	1	GRG	0.767	0.453	6.1	89.1	392.623	402.606	-8.0253	1.7335
88	2	1	MTR	0.768	0.454	6.1	89.1	276.282	290.294	-10.9373	1.272
89	2	1	ONS	0.807	0.467	6.1	89.1	7.08	89.381	82.9353	0.1677
90	2	2	PND	0.774	0.461	6.1	89.1	85.804	123.698	27.3609	0.5085
91	2	2	SLB	0.778	0.478	6.1	89.1	61.524	108.278	-33.1588	0.4196
92	2	2	TLC	0.788	0.503	6.1	89.1	116.48	146.65	-20.113	0.6917
93	2	1	VLM	0.789	0.455	6.1	89.1	95.886	130.893	13.912	1
94	3	1	SHB	0.76	0.498	6.1	89.1	599.346	605.933	6.6992	2.0767
95	3	2	RUS	0.766	0.454	6.1	89.1	191.412	211.133	16.8093	0.8276

*) Source of data: 1 INFP – Romania; 2 INCERC – Romania; 3 Bulgaria; 4 Former Yugoslavia; 5 GEOTEC – Romania **) Components: 1 N-S; 2 E-W

Table A1. (continued) Data used for definition of mathematical model based on radius vectors for the VLM azimuth

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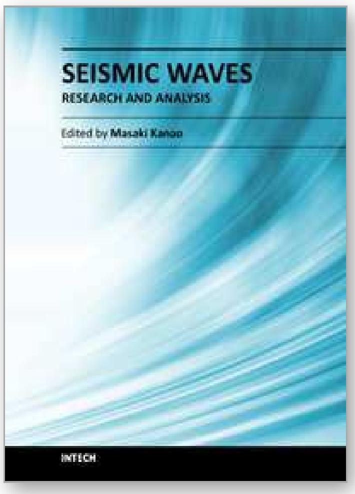

Seismic Waves - Research and Analysis
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The importance of seismic wave research lies not only in our ability to understand and predict earthquakes and tsunamis, it also reveals information on the Earth's composition and features in much the same way as it led to the discovery of Mohorovicic's discontinuity. As our theoretical understanding of the physics behind seismic waves has grown, physical and numerical modeling have greatly advanced and now augment applied seismology for better prediction and engineering practices. This has led to some novel applications such as using artificially-induced shocks for exploration of the Earth's subsurface and seismic stimulation for increasing the productivity of oil wells. This book demonstrates the latest techniques and advances in seismic wave analysis from theoretical approach, data acquisition and interpretation, to analyses and numerical simulations, as well as research applications. A review process was conducted in cooperation with sincere support by Drs. Hiroshi Takenaka, Yoshio Murai, Jun Matsushima, and Genti Toyokuni.

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