6

Heat Conduction Problems of Thermosensitive Solids under Complex Heat Exchange

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1. Introduction

To provide efficient investigations for engineering problems related to heating/cooling process in solids, the effect of thermosensitivity (the material characteristics depend on the temperature) should be taken into consideration when solving the heat conductivity problems (Carslaw & Jaeger, 1959; Noda, 1986; Nowinski, 1962; Podstrihach & Kolyano, 1972). It is important to construct the solutions to the aforementioned heat conduction problems in analytical form. This requirement is motivated, for instance, by the need to solve the thermoelasticity problems for thermosensitive bodies, for which the determined temperature is a kind of input data, and thus, is desired in analytical form.

In general, the model of a thermosensitive body leads to a nonlinear heat conductivity problem. It is mentioned in (Carslaw & Jaeger, 1959) that the exact solutions of such problems can be determined when the temperature or heat flux is given on the surface by assuming the material to be "simply nonlinear" (thermal conductivity λ_t and volumetric volumetric heat capacity c_v depend on the temperature, but the relation, called thermal diffusivity $a = \lambda_t / c_v$, is assumed to be constant). For construction of the solution in this case, it is sufficient to use the Kirchhoff's transformation to obtain the corresponding linear problem for the Kirchhoff's variable. This problem can be solved (Ditkin & Prudnikov, 1975; Galitsyn & Zhukovskii, 1976; Sneddon, 1951) by application of classical methods (separation of variables, integral transformations, etc.). The solutions to the heat conductivity problems for crystal bodies, whose thermal characteristics are proportional to the third power of the absolute temperature, can be constructed in a similar manner for the case of radiation heat exchange with environment.

In the case of complex heat exchange, the Kirchhoff transform makes the heat conductivity problem to be linear only in part. In the heat conductivity problem for the Kirchhoff's variable, the heat conduction equation is nonlinear due to dependence of the thermal diffusivity on the Kirchhoff's variable. The boundary condition of the complex heat exchange is also nonlinear due to a nonlinear expression of the temperature on the surface. Herein we discuss several approaches, developed by the authors for determining temperature distribution in thermosensitive bodies of classical shape under complex (convective, radiation or convective-radiation) heat exchange on the surface (Kushnir & Popovych, 2006, 2007, 2009; Kushnir & Protsiuk, 2009; Kushnir et al., 2001, 2008; Popovych,

1993a, 1993b; Popovych & Harmatiy, 1996, 1998; Popovych & Sulym, 2004; Popovych et al. 2006). Note that the necessity of these investigations is emphasized in (Carslaw & Jaeger, 1959).

2. The step-by-step linearization method for solving the one-dimensional transient heat conductivity problems with simple thermal non-linearity

Let us consider the step-by-step method for determining one-dimensional transient temperature field $t(x,\tau)$, which can be found from the following non-linear heat conduction equation:

$$\frac{1}{x^m}\frac{\partial}{\partial x}\left(x^m\lambda_t(t)\frac{\partial t}{\partial x}\right)=c_v(t)\frac{\partial t}{\partial \tau}-W, \tag{1}$$

where $\lambda_t(t)$ is the thermal conductivity; $c_v(t)$ is the volumetric heat capacity; $m=0;1;2$ corresponds to Cartesian, cylindrical and spherical coordinate systems, respectively; $a\le x\le b,\ a\ge 0,\ a<b\le\infty$. The thermosensitive body of consideration is made of a material with simple nonlinearity. The density of heat sources W is a function of coordinate x and time τ. Let the surface $x=a$, for instance, is exposed to convective-radiation heat exchange

$$\left[\lambda_t(t)\frac{\partial t}{\partial x}-\alpha_a(t)(t-t_a)-\sigma\varepsilon_a(t)(t^4-t_a^4)\right]_{x=a}=0 \tag{2}$$

with the environment of constant temperature t_a, where $\alpha_a(t)$ is the temperature dependent coefficient of heat exchange between the surface and the environment; $\varepsilon_a(t)$ is the temperature dependent emittance; σ is the Stefan-Boltzmann constant. The surface $x=b$ is heated with constant temperature t_b or constant heat flux q_b:

$$t\big|_{x=b}=t_b \quad \text{or} \quad \lambda_t(t)\frac{\partial t}{\partial x}\bigg|_{x=b}=q_b. \tag{3}$$

At the initial moment of time, the temperature is uniformly distributed within the body:

$$t\big|_{\tau=0}=t_p\ . \tag{4}$$

The key point of the solution method for the formulated non-linear heat conductivity problem (1)–(4), which is presented below, consists in the step-by-step linearization involving the Kirchhoff transformation along with linearization of the nonlinear term in the boundary conditions by means of the spline approximation.

By introducing the dimensionless coordinates $\bar{x}=x/l_0$, temperature $T=t/t_0$, and time $\mathrm{Fo}=a\tau/l_0^2$ (the Fourier number), we can present the functional parameters $\lambda_t(t)$, $c_v(t)$, $\alpha_a(t)$, and $\varepsilon_a(t)$ in the form $\chi(t)=\chi_0\chi^*(T)$, where χ_0 is a reference value and $\chi^*(T)$ stands for the dimensionless function; t_0 is a reference temperature and l_0 is a characteristic dimension. The density of heat sources can be presented as $W=q_0\ q(\bar{x},\mathrm{Fo})$, where q_0 is the

dimensional constants, $q(\bar{x},\mathrm{Fo})$ is the dimensionless function describing the time variation of the heat sources. As a result, the problem (1)–(4) takes the form

$$\frac{1}{\bar{x}^m}\frac{\partial}{\partial\bar{x}}\left(\bar{x}^m\lambda_t^*(T)\frac{\partial T}{\partial\bar{x}}\right)=c_v^*(T)\frac{\partial T}{\partial\mathrm{Fo}}-\mathrm{Po}\,q(\bar{x},\mathrm{Fo}), \tag{5}$$

$$\left[\lambda_t^*(T)\frac{\partial T}{\partial\bar{x}}-\mathrm{Bi}_a\alpha_a^*(T)(T-T_a)-\mathrm{Sk}_a\varepsilon_a^*(T)(T^4-T_a^4)\right]_{\bar{x}=\bar{a}}=0, \tag{6}$$

$$T\big|_{\bar{x}=\bar{b}}=T_b \quad \text{or} \quad \lambda_t^*(T)\frac{\partial T}{\partial\bar{x}}\bigg|_{\bar{x}=\bar{b}}=\mathrm{Ki}_b, \tag{7}$$

$$T\big|_{\mathrm{Fo}=0}=T_p. \tag{8}$$

Here $P_0=q_0l_0^2/(t_0\lambda_{t0})$ (the Pomerantsev number), $\mathrm{Bi}_a=\alpha_a^{(0)}r_0/\lambda_{t0}$ (the Biot number), $\mathrm{Sk}_a=\sigma\varepsilon_a^{(0)}l_0t_0^3/\lambda_{t0}$ (the Starc number), $\mathrm{Ki}_b=q_bl_0/(t_0\lambda_{t0})$ (the Kirpichev number), $T_b=t_b/t_0$, $T_p=t_p/t_0$.

Let us apply the Kirchhoff's integral transformation (Carslaw & Jaeger, 1959; Noda, 1986; Podstrihach & Kolyano, 1972)

$$\theta=\int_{T_p}^{T}\lambda_t^*(t)dt \tag{9}$$

to the problem (5)–(8). By taking into account the feature of simple nonlinearity ($\lambda_t^*(T)\approx c_v^*(T)$) and expressions $\frac{\partial\theta}{\partial\bar{x}}=\lambda_t^*(T)\frac{\partial T}{\partial\bar{x}}$, $\frac{\partial\theta}{\partial\mathrm{Fo}}=\lambda_t^*(T)\frac{\partial T}{\partial\mathrm{Fo}}\approx c_v^*(T)\frac{\partial T}{\partial\mathrm{Fo}}$, the equation

$$\frac{1}{\bar{x}^m}\frac{\partial}{\partial\bar{x}}\left(\bar{x}^m\frac{\partial\theta}{\partial\bar{x}}\right)=\frac{\partial\theta}{\partial\mathrm{Fo}}-\mathrm{Po}\,q(\bar{x},\mathrm{Fo}) \tag{10}$$

follows from the nonlinear heat conductivity equation (5). The boundary condition of convective-radiation heat exchange (6) can be partially linearized and presented as

$$\left[\frac{\partial\theta}{\partial\bar{x}}-Q_a\big(T(\theta)\big)\right]_{\bar{x}=\bar{a}}=0, \tag{11}$$

where $Q_a\big(T(\theta)\big)=\mathrm{Bi}_a\alpha_a^*\big(T(\theta)\big)\big(T(\theta)-T_a\big)+\mathrm{Sk}_a\varepsilon_a^*\big(T(\theta)\big)\big(T(\theta)^4-T_a^4\big)$. The boundary conditions (7) and initial condition (8) yield

$$\theta\big|_{\bar{x}=\bar{b}}=\theta_b \quad \text{or} \quad \frac{\partial\theta}{\partial\bar{x}}\bigg|_{\bar{x}=\bar{b}}=\mathrm{Ki}_b, \tag{12}$$

$$\theta\big|_{\mathrm{Fo}=0}=0, \tag{13}$$

where $\theta_b = \int_{T_p}^{T_b} \lambda_t^*(T)dT$, $T(\theta)$ denotes the temperature expressed through the Kirchhoff's variable and determined for certain $\lambda_t^*(T)$ by means of the integral equation (9).

Application of the Kirchhoff's variable allows us to linearize the nonlinear heat conductivity equation (5) and the second boundary condition (7) completely, whereas the convective-radiation heat exchange condition is linearized in a part. Due to the nonlinear expression $Q_a(T(\theta))$, it is impossible to apply any classical method to solve the boundary problem (10)–(13). Therefore, it is necessary to linearize the boundary condition (11). In (Nedoseka, 1988; Podstrihach & Kolyano, 1972), the convective heat exchange condition has been considered. Therefore, the nonlinear expression $T(\theta)$ is simply replaced by θ. As a result, the nonlinear convective heat exchange condition on θ becomes linear. However, it has been shown in (Kushnir & Popovych, 2009; Popovych, 1993b; Popovych & Harmatiy, 1996) that this unsubstantiated linearization leads to the numerically or physically incorrect results. In our case, when we take into account the radiation constituent (which is nonlinear even for a non-thermosensitive material) and dependence of the heat transfer coefficient and emittance on the temperature, the considered substitution does not provide the complete linearization of the condition (11). Instead, the boundary condition (11) can be linearized by means of interpolation of the nonlinear expression $Q_a(T(\theta))$ by special splines with order 0 or 1. For $\overline{x} = \overline{a}$, the expression $Q_a(T(\theta))$ is a function of Fo only. Let us select a finite set of points Fo_i $(i = \overline{1,n};\ 0 = \mathrm{Fo}_0 < \mathrm{Fo}_1 < \mathrm{Fo}_2 < \ldots < \mathrm{Fo}_n)$, which divides the region of time variation into $n+1$ intervals. Let us construct the spline $S_a^{(0)}(\mathrm{Fo})$ with order 0, whose values coincide with the values of expression $Q_a(\mathrm{Fo}) = Q_a(T(\theta))\big|_{\overline{x}=\overline{a}}$ at $\mathrm{Fo} = \mathrm{Fo}_i$ and

$$S_a^{(0)}(\mathrm{Fo}) = Q_1^{(a)} + \sum_{i=1}^{n-1} (Q_{i+1}^{(a)} - Q_i^{(a)}) S_+(\mathrm{Fo} - \mathrm{Fo}_i)\,; \tag{14}$$

$$Q_i^{(a)} = \mathrm{Bi}_a \alpha_a^*(T_i^{(a)})(T_i^{(a)} - T_a) + \mathrm{Sk}_a \varepsilon_a^*(T_i^{(a)})\left((T_i^{(a)})^4 - T_a^4\right) \tag{15}$$

on the every interval of interpolation. Here $T_i^{(a)}$ $(i \equiv \overline{1,n})$ are the values of temperature $T(\overline{x}, \mathrm{Fo})$, which are to be found on the surface $\overline{x} = \overline{a}$ at the moments of time Fo_i (the unknown parameters of spline approximation), $S_+(\cdot)$ denotes the asymmetric unit Heaviside function (H. Korn & T. Korn, 1977).

Having presented the nonlinear expression $Q_a(T(\theta))\big|_{\overline{x}=\overline{a}}$ by spline (14), the boundary condition (11) becomes linear

$$\left.\frac{\partial\theta}{\partial\overline{x}}\right|_{\overline{x}=\overline{a}} - S_a^{(0)}(\mathrm{Fo}) = 0\ . \tag{16}$$

Similarly, the first-order spline $S_a^{(0)}(\mathrm{Fo})$, whose values coincide with values of expression $Q_a(\mathrm{Fo})$ at the points $\mathrm{Fo_i}$ and on every segment of decomposition approximates $Q_a(\mathrm{Fo})$ by

the linear polynom $P_i^{(a)}(\mathrm{Fo}) = k_i^{(a)}\mathrm{Fo} + b_i^{(a)}$, can be constructed by the abovementioned decomposition. This spline can be written as

$$S_a^{(1)}(\mathrm{Fo}) = P_1^{(a)}(\mathrm{Fo}) + \sum_{i=1}^{n-1}\left(P_{i+1}^{(a)}(\mathrm{Fo}) - P_i^{(a)}(\mathrm{Fo})\right)S_+(\mathrm{Fo} - \mathrm{Fo}_i)\,. \tag{17}$$

Here the coefficients $k_i^{(a)}, b_i^{(a)}$ of polynom $P_i^{(a)}(\mathrm{Fo})$ are calculated by formulae

$$k_i^{(a)} = \frac{Q_i^{(a)} - Q_{i-1}^{(a)}}{\mathrm{Fo}_i - \mathrm{Fo}_{i-1}}, \quad b_i^{(a)} = Q_{i-1}^{(a)} - k_i^{(a)}\mathrm{Fo}_{i-1}\,, \tag{18}$$

where $Q_i^{(a)}$ is expressed through $T_i^{(a)}$ by means of formula (15).

If $Q_a\left(T(\theta)\right)\Big|_{\overline{x}=\overline{a}}$ is expressed as the first-order spline (17), then boundary condition (11) becomes linear

$$\left.\frac{\partial\theta}{\partial\overline{x}}\right|_{\overline{x}=\overline{a}} - S_a^{(1)}(\mathrm{Fo}) = 0\,. \tag{19}$$

Having solved the obtained linear problem (10), (12), (13), (16) or (10), (12), (13), (19) by means of the classical methods, the Kirchhoff's variable is found as a function of $\overline{x}$ and Fo. Besides the input data of the problem, this variable contains $\mathrm{Fo_i}$ and unknown values $T_i^{(a)} = T(\overline{a}, \mathrm{Fo}_i)$:

$$\theta = \theta(\overline{x}, \mathrm{Fo}, \mathrm{Fo}_1, \ldots, \mathrm{Fo}_n, T_1^{(a)}, \ldots, T_n^{(a)})\,. \tag{20}$$

By substitution θ into the expression for $T(\theta)$ (for specific dependence $\lambda_t^*(T)$), the formula for determination of the temperature

$$T = f(\overline{x}, \mathrm{Fo}, \mathrm{Fo}_1, \ldots, \mathrm{Fo}_n, T_1^{(a)}, \ldots, T_n^{(a)}) \tag{21}$$

can be obtained at arbitrary point $\overline{x}$ and arbitrary moment of time Fo. For determination of unknown values $T_i^{(a)}$ in the expressions for temperature (21), the collocation method is used. Assuming $\mathrm{Fo} = \mathrm{Fo}_i$ $(i = \overline{1,n})$ in (21), the system of equation for determination $T_i^{(a)}$

$$\begin{cases} T_1 = f(\overline{a}, \mathrm{Fo}_1, T_1^{(a)}), \\ T_2 = f(\overline{a}, \mathrm{Fo}_1, \mathrm{Fo}_2, T_1^{(a)}, T_2^{(a)}), \\ \cdots \\ T_n = f(\overline{a}, \mathrm{Fo}_1, \ldots, \mathrm{Fo}_n, T_1^{(a)}, \ldots, T_n^{(a)}) \end{cases} \tag{22}$$

is obtained. The structure of system (22) makes it possible to determine all unknown values $T_i^{(a)}$, starting from $T_1^{(a)}$. Substitution of values, determined from (22), into the formula (21) completes the solution procedure.

The temperature at given point $\overline{x}$ and moment of time can be calculated in accordance to the following scheme:

a. to divide the time axis by Fo_i and then to determine the approximation parameters $T_i^{(a)}$ from the system (22); as a result, the value of temperature (21) $T_n^{(a)}$ is obtained;

b. to divide every interval in two; to compute the values of parameters $T_i^{(a)}$ for this new time-segmentation and then to obtain the values of temperature $T_{n+1}^{(a)}$;

c. to calculate the difference $T_{n+1}^{(a)} - T_n^{(a)}$. If $|T_{n+1} - T_n| < \varepsilon$, where ε is the accuracy, then the calculation is over. Otherwise, we shall return to the stage *b*.

The temperature can be computed with any given accuracy ε for arbitrary segmentation of the time axis. However, the increasing of number of time-segments decreases the convergence of the proposed scheme. An appropriate choice of the initial moment of time can be done by means of the estimated 'a priory' time-dependence of the temperature on the surface $\overline{x} = \overline{a}$. We can also use the solution of corresponding boundary value problem for the body of the same shape with constant characteristics. Then the initial choice for values Fo_i can be used as the appropriate one for the thermosensitive body.

The method of step-by-step linearization is applicable for determination of the temperature fields in thermosensitive plates, half-space, solid and hollow cylinders or spheres, space with cylindrical or spherical cavities, on the surfaces of which, the conditions of convective, radiation or convective-radiation heat exchange may be given. This method has been efficiently used for solving the two-dimensional steady problem in thermosensitive body.

3. Method of linearizing parameters

The method of step-by-step linearization makes it possible to determine the solutions to the two-dimensional heat conductivity problems in thermosensitive bodies with simple nonlinearity, when the nonlinear term in the condition of complex heat exchange for the Kirchhoff's variable depends on one (spatial or time) variable only. In this section, we consider an efficient method for solving the steady-state and transient heat conductivity problems of arbitrary dimension those describe the propagation of heat in thermosensitive bodies with simple nonlinearity under the convective heat exchange with environment.

Let the body occupies region D with surface S. The surface (whole or a part) is subjected to the convective heat exchange with the environment of temperature t_p. From the moment of time $\tau > 0$, the heat sources $W(x, y, z, \tau)$ are acting in the body. The temperature in the body shall be determined from the following heat conduction equation:

$$\operatorname{div}\left(\lambda_t(t)\operatorname{grad} t\right) = c_v(t)\frac{\partial t}{\partial \tau} - W \tag{23}$$

and the boundary

$$\left[\lambda_{\mathrm{t}}(t)\frac{\partial t}{\partial n} + \alpha(t - t_c)\right]_{\mathrm{s}} = 0 \tag{24}$$

and initial

$$t\,\big|_{\tau=0} = t_p \tag{25}$$

conditions, where α is the constant heat transfer coefficient; n is the external normal to surface S.

By making use of the above-introduced presentation for the material characteristics, heat sources, and dimensionless variables, the boundary value problem (23)–(25) can be reduced to the dimensionless form. After application of the Kirchhoff's transformation, the following boundary value problem for variable θ

$$\operatorname{div}\operatorname{grad}\theta = \frac{\partial\theta}{\partial \mathrm{Fo}} - \mathrm{Po}\, q(X,Y,Z,\mathrm{Fo}), \tag{26}$$

$$\left[\frac{\partial\theta}{\partial\overline{n}} + \mathrm{Bi}\left(T(\theta) - T_c\right)\right]_s = 0, \tag{27}$$

$$\theta\big|_{\mathrm{Fo}=0} = 0 \tag{28}$$

is obtained, where $X = x/l_0$, $Y = y/l_0$, $Z = z/l_0$ are dimensionless coordinates; $\overline{n} = n/l_0$, $q(X,Y,Z,\mathrm{Fo})$ is the dimensionless function of heat sources. As a result, the initial problem is partially linearized, meanwhile the condition (27) remains nonlinear. The latter conditions have been obtained from the conditions of convective heat exchange due to nonlinear expression $T(\theta)$ on the surface S. For solving the problem (26)–(28) by using an analytical method, it is necessary to linearize this condition. Let us prove the possibility of such linearization.

Consider the simplest case of linear dependence of heat conductivity coefficient on the temperature:

$$\lambda_t(t) = \lambda_{to} \quad \lambda_t^*(T) = \lambda_{to}\left[1 + k(T - T_p)\right], \tag{29}$$

where k is a constant. From the equation (9), the formula

$$\theta = (T - T_p) + \frac{k}{2}(T - T_p)^2 \tag{30}$$

follows, where

$$T(\theta) = k^{-1}(\sqrt{1 + 2k\theta} - 1) + T_p. \tag{31}$$

From the physical standpoint, the square root is chosen to be positive. After substitution of the equation (31) into the boundary condition (27), the last one takes the form

$$\left[\frac{\partial\theta}{\partial\overline{n}} + \mathrm{Bi}\left(\frac{\sqrt{1 + 2k\theta} - 1}{k} + T_p - T_c\right)\right]_s = 0. \tag{32}$$

Be decomposing the square root in (32) into the series and restricting this series with two terms, the boundary condition

$$\left[\frac{\partial\theta}{\partial\overline{n}} + \mathrm{Bi}\left(\theta - (T_c - T_p)\right)\right]_s = 0 \tag{33}$$

is obtained. The solution of equation (26) with boundary conditions (28), (33) is an approximate solution to the boundary value problem (26), (28), (32). To determine the exact solution, the equation (26) is to be solved under initial condition (28) and the following linear boundary condition

$$\left[\frac{\partial\theta}{\partial\overline{n}} + \mathrm{Bi}\Big((1+\kappa)\theta - (T_c - T_p)\Big)\right]_s = 0 \tag{34}$$

instead of the nonlinear condition (32), where κ is an unknown constant (linearized parameter). Note that the boundary condition (34) coincides at $\kappa = 0$ with the condition (33). Since the problem (26), (28), (34) is linear, the appropriate classical analytical method can be used for its solution. In addition to the original parameters of the problem ($\mathrm{Po}, \mathrm{Bi}, T_c, T_p$, dimensions of the body, coordinates and time), the solution involves the unknown linearized parameter κ:

$$\theta = \theta(X, Y, Z, \mathrm{Fo}, \kappa)\,. \tag{35}$$

For an arbitrary value of κ, the solution (35) meets the equation (26) and the initial condition (28). In order the solution (35) to satisfy the nonlinear conditions (32) and (34), the parameter κ is to be the solution of the equation

$$\left[\frac{\sqrt{1+2k\theta}-1}{k} - (1+\kappa)\theta\right]_s = 0\,.$$

After some transformations, this equation can be given as

$$\theta\big|_s = -\frac{2\kappa}{k(1+\kappa)^2}\,. \tag{36}$$

This equation holds for every moment of time Fo. After the paramenter κ is found, we substitute it into (35). In such manner, the expression for Kirchhoff's variable is obtained. The temperature in the body is then calculated by means of the relation (31).
Note that the boundary condition (34) can be represented as

$$\left[\frac{\partial\theta}{\partial\overline{n}} + \mathrm{Bi}^*(\theta - T_c^*)\right]_s = 0\,, \tag{37}$$

where $\mathrm{Bi}^* = \mathrm{Bi}(1+\kappa);\ T_c^* = (T_c - T_p)/(1+\kappa)$. This condition can be interpreted as a condition of convective heat exchange with certain parameters (the Biot number Bi^* and the temperature T_c^* of external environment) depending on the unknown parameter κ.
The equation (36) is nonlinear. It provides analytical solutions only for some cases of steady-state problems with substantional use of the numerical methods. Therefore, these solutions can be regarded as analytico-numerical solutions.
Let us consider the non-linear dependence of the heat conductivity coefficient on the temperature. For linearization of the boundary condition (27), we shall find the Kirhoff's variable for the case when the surface temperature of the thermosensitive body is equal to

the surface temperature of the body with constant characteristics. The latter temperature is to be found from the problem:

$$\operatorname{div}\operatorname{grad} T_H = \frac{\partial T_H}{\partial \mathrm{Fo}} - \mathrm{Po}\, q(X,Y,Z,\mathrm{Fo}), \tag{38}$$

$$\left[\frac{\partial T_H}{\partial \bar{n}} + \mathrm{Bi}\left((T_H - T_c\right)\right]_s = 0 , \tag{39}$$

$$T_H\big|_{\mathrm{Fo}=0} = T_p , \tag{40}$$

where $T_H = t_H / t_o$, t_H is temperature of the body with constant characteristics.
By subtraction equations of the problem (26)–(28) from corresponding equation of the problem (38)–(40) and taking into account that $T(\theta)\big|_s = T_H\big|_s$, we obtain:

$$\operatorname{div}\operatorname{grad}(T_H - \theta) = \frac{\partial (T_H - \theta)}{\partial \mathrm{Fo}}, \tag{41}$$

$$\left.\frac{\partial (T_H - \theta)}{\partial \bar{n}}\right|_s = 0 , \tag{42}$$

$$(T_H - \theta)\big|_{\mathrm{Fo}=0} = T_p . \tag{43}$$

The boundary value problem (41)-(43) is a problem of heat conductivity in the body with the surface S and uniform initial temperature T_p. The heat sources are absent and the boundary of the body is thermoinsulated. The evident solution of this problem is $T_H - \theta = T_p$. Consequently, if in the problem (26)–(28) for the Kirchhoff's variable the surface temperature for the thermosensitive body is replaced with the surface temperature for the body with constant characteristics (whose thermal diffusivity is equal to the thermal diffusivity of thermosensitive body and the heat conductivity coefficient is equal to the reference value of the heat conductivity coefficient λ_{t0}), then $\theta = T_H - T_p$.

Thus, if the surface temperature $T(\theta)\big|_s$ of the thermosensitive body in the condition (27) is equal to the corresponding temperature of the body with constant characteristics, then the boundary value problem for the Kirchhoff's variable θ should be solved with the condition (33). Then the solution of this problem presents the difference of the temperature in the same-shape body with constant characteristics and the initial temperature:

$$\theta = T_H - T_p . \tag{44}$$

As it was mentioned above, the substitution of $T(\theta)$ for $\theta + T_p$ in the case of linear dependence of the heat conductivity coefficient on the temperature is equivalent to keeping only two terms in the series, into which the square root in expression for the temperature through the Kirhoff's variable has been decomposed. This linearization does not guarantee a

sufficient solution approximation. To overcome this difficulty, we consider the boundary value problem for the variable θ with the linear condition (37) instead of the nonlinear condition (27), which involves an additional parameter κ. Having solved the obtained linear problem, the Kirhoff's variable θ is found as a function of the coordinates and parameter κ. The parameter κ should be chosen in the way to satisfy the nonlinear condition (27) with any given accuracy. Thus for determination of the temperature field in the body with simple nonlinearity for arbitrary temperature dependence of heat conductivity coefficient under convective heat exchange between the surface and environment, the corresponding solution of the nonlinear heat conductivity problem can be determined by following the proposed algorithm of the method of linearized parameters:

- to present the problem in dimensionless form;
- to linearize the problem in part by using integral Kihhoff transformation;
- to linearize the problem completely by linearizing the nonlinear condition on Kirchhoff's variable θ obtained from condition of convective heat exchange due to replacement of nonlinear expression $T(\theta)$ by $(1+\kappa)\theta + T_p$ with unknown parameter κ;
- to solve the obtained linear boundary value problem for variable θ by means of an appropriate classical method;
- to satisfy with given accuracy the nonlinear condition for variable θ by using the parameter κ;
- to determine the temperature using the obtained Kirchhoff's variable.

The main feature of the method of linearizing parameters consists in a possibility to obtain the solution of linearized boundary value problem for the Kirchhoff's variable in a thermosensitive body by solving the heat conductivity problem in the body with constant characteristics under convective heat exchange. This solution is obtained from (44) by setting $\mathrm{Bi}^* = \mathrm{Bi}(1+\kappa)$ and $T_c^* = (T_c - T_p)/(1+\kappa)$ instead of T_H Bi and T_c, respectively.

4. The method of linearizing parameters for the steady-state heat conduction problems in piecewise-homogeneous thermosensitive bodies

Determination of the temperature fields in piecewise-homogeneous bodies subjected to intensive thermal loadings is an initial stage that precedes the determination of steady-state or transient thermal stresses in the mentioned bodies. Let us assume that the elements of piecewise-homogeneous body are in the ideal thermal contact and the limiting surface is under the condition of complex heat exchange with environment. Mathematical model for determination of the temperature fields in such structures leads to the coupled problem for a set of nonlinear heat conduction equations with temperature-dependent material characteristics in the coupled elements. By making use of the Kirhoff's integral transformation for each element by assuming the thermal conductivity to be constants, the problem can be partially linearized. The nonlinearities remain due to the thermal contact conditions on the interfaces and the conditions of complex heat exchange on the surfaces. To obtain an analytical solution to the coupled problem for the Kirchhoff's variable, it is necessary to linearize this problem. The possible ways of such a linearization and, thus, determination of the general solution to the heat conduction problems in piecewise-homogeneous bodies are considered below in this section.

Let us adopt the method of linearizing parameters to solution of the steady-state heat conduction problems for coupled bodies of simple shape, for instance, n-layer thermosensitive cylindrical pipe. The pipe is of inner and outer radii $r = r_0$ and $r = r_n$, respectively, with constant temperatures t_b and t_H on the inner and outer surfaces. The layers of different temperature-dependent heat conduction coefficients are in the ideal thermal contact. The cylindrical coordinate system r, ϕ, z is chosen with z-axis coinciding with the axis of pipe. The temperature field in this pipe can be determined from the set of heat conduction equations

$$\frac{1}{r}\frac{d}{dr}\left[r\lambda_t^{(i)}(t_i)\frac{dt_i}{dr}\right] = 0,\ i = \overline{1,\ n}\ , \tag{45}$$

with the boundary conditions

$$t_1\big|_{r=r_0} = t_b,\quad t_n\big|_{r=r_n} = t_H\ , \tag{46}$$

$$t_i = t_{i+1},\ \lambda_t^{(i)}(t_i)\frac{dt_i}{dr} = \lambda_t^{(i+1)}(t_{i+1})\frac{dt_{i+1}}{dr}\ ,\ r{=}r_i, i = \overline{1,\ n-1}\ , \tag{47}$$

where $\lambda_t^{(i)}(t_i)$ denotes the heat conduction coefficient of the layers. We introduce the dimensionless values $T_i = t_i/t_0$, $\rho = r/r_0$ and $\lambda_t^{(i)}(t_i) = \lambda_{t0}^{(i)}\lambda_t^{*(i)}(T_i)$, where the constituents with the indices "0" are dimensional constants and the asterisked terms are dimensionless functions, t_0 is the reference temperature. In the dimensionless form, the problem (45)–(47) appears as

$$\frac{1}{\rho}\frac{d}{d\rho}\left[\rho\lambda_t^{*(i)}(T_i)\frac{dT_i}{d\rho}\right] = 0, i = \overline{1,\ n}\ , \tag{48}$$

$$T_1\big|_{\rho=1} = T_b,\quad T_n\big|_{\rho=\rho_n} = T_H\ , \tag{49}$$

$$T_i = T_{i+1},\ \lambda_{t0}^{(i)}\lambda_t^{*(i)}(T_i)\frac{dT_i}{d\rho} = \lambda_{t0}^{(i+1)}\lambda_t^{*(i+1)}(T_{i+1})\frac{dT_{i+1}}{d\rho}\ ,\ \rho = \rho_i,\ i = \overline{1,\ n-1}\,. \tag{50}$$

Consider the heat conduction coefficients in the form of linear dependence on the temperature $\lambda_t^{(i)}(t_i) = \lambda_{t0}^{(i)}(1 + k_i T_i)$, where k_i are constants. By introducing the Kirchhoff's variable

$$\theta_i = \int_0^{T_i}\lambda_t^{*(i)}(T)dT \tag{51}$$

in each layer, the following problem on Kirchhoff's variable

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d\theta_i}{d\rho}\right) = 0, i = \overline{1,\ n}\,, \tag{52}$$

$$\theta_1\big|_{\rho=1}=\theta_b,\ \theta_n\big|_{\rho=\rho_n}=\theta_{\text{н}}, \tag{53}$$

$$\left.\begin{aligned}&\left(\sqrt{1+2k_i\theta_i}-1\right)/k_i=\left(\sqrt{1+2k_{i+1}\theta_{i+1}}-1\right)/k_{i+1},\\&\lambda_{t0}^{(i)}\frac{d\theta_i}{d\rho}=\lambda_{t0}^{(i+1)}\frac{d\theta_{i+1}}{d\rho}\end{aligned}\right\}\ \text{at}\ \rho=\rho_i,\ i=\overline{1,n-1}, \tag{54}$$

is obtained from the problem (48)-(50). Here $\theta_b=\int_0^{T_b}\lambda_t^{*(1)}(T)dT$; $\theta_{\text{н}}=\int_0^{T_{\text{н}}}\lambda_t^{*(n)}(T)dT$.

The initially nonlinear heat conduction problem is partially linearized due to application of the Kirchhoff's variables. However, the conditions for temperature, that reflects the temperature equalities of the neighbouring layers, remain nonlinear (the first group of conditions (54)). By integrating the set of equations (52) with boundary conditions (53) and contact conditions (54), the set of transcendent equation can be obtained for determination of constant of integration. This set can be solved numerically. The efficiency of numerical methods depends on the appropriate initial approximation. Unfortunately, it is very complicated to determine the definition domain for the solution of this set of equations and thus to present a constructive algorithm for determination of the initial approximation.

The possible way around this problem is to decompose the square root in the first conditions (54) into series by holding only two terms. Then, instead of mentioned conditions, the following approximated conditions are obtained:

$$\theta_i=\theta_{i+1}\ \ \text{at}\ \ \rho=\rho_i,\ i=\overline{1,n-1}. \tag{55}$$

Application of the conditions (55), instead of exact ones, separates the interfacial conditions. This fact allows us to consider the boundary problem (52)–(54) replacing the conditions (54) by the following ones:

$$(1+\kappa_i)\theta_i=(1+\kappa_{i+1})\theta_{i+1}\ \ \text{at}\ \ \rho=\rho_i,\ i=\overline{1,\ n-1}, \tag{56}$$

where κ_i are unknown constants (linearizing parameters). By substitution

$$\theta_i^*=(1+\kappa_i)\theta_i, \tag{57}$$

we obtain

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d\theta_i^*}{d\rho}\right)=0, \tag{58}$$

$$\theta_1^*\big|_{\rho=1}=\theta_b^*,\quad \theta_n^*\big|_{\rho=\rho_n}=\theta_n^*, \tag{59}$$

$$\theta_i^*=\theta_{i+1}^*,\ \gamma_i\frac{d\theta_i^*}{d\rho}=\gamma_{i+1}\frac{d\theta_{i+1}^*}{d\rho}\ \ \text{at}\ \ \rho=\rho_i,\ i=\overline{1,\ n-1}, \tag{60}$$

where $\theta_b^*=(1+\kappa_i)\theta_b$; $\theta_{\text{н}}^*=(1+\kappa_n)\theta_{\text{н}}$; $\gamma_i=\lambda_{t0}^{(i)}/(1+\kappa_i)$, $i=\overline{1,n-1}$.

It can be shown (Podsdrihach et al., 1984) that the boundary value problem (58)–(60) is equivalent to the problem

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\gamma(\rho)\frac{d\theta^*}{d\rho}\right)=0\,, \tag{61}$$

$$\theta_1^*\Big|_{\rho=1}=\theta_b^*,\quad \theta_n^*\Big|_{\rho=\rho_n}=\theta_n^*\,, \tag{62}$$

where $\gamma(\rho)=\gamma_1+\sum_{j=1}^{n-1}(\gamma_{j+1})-\gamma_j)S_-(\rho-\rho_j)$. After integration of the equation (61), we obtain

$$\theta^*=C_1\left[\frac{\ln\rho}{\gamma(\rho)}-\sum_{j=1}^{n-1}\left(\frac{1}{\gamma_{j+1}}-\frac{1}{\gamma_j}\right)\ln\rho_j S_-(\rho-\rho_j)\right]+C_2\,. \tag{63}$$

Substitution of (63) into (62) yields

$$\theta^*=\frac{\theta_{\text{н}}^*-\theta_b^*}{\dfrac{\ln\rho_n}{\gamma_n}-\sum_{j=1}^{n-1}\left(\dfrac{1}{\gamma_{j+1}}-\dfrac{1}{\gamma_j}\right)\ln\rho_j}\left[\frac{\ln\rho}{\gamma(\rho)}-\sum_{j=1}^{n-1}\left(\frac{1}{\gamma_{j+1}}-\frac{1}{\gamma_j}\right)\ln\rho_j S_-(\rho-\rho_j)\right]+\theta_b^*\,, \tag{64}$$

or

$$\theta_i^*=A_i^*\ln\rho+B_i^*\,, \tag{65}$$

where

$$A_i^*=\left(\theta_{\text{н}}^*-\theta_b^*\right)\left(\gamma_j\left[\frac{\ln\rho_n}{\gamma_n}-\sum_{j=1}^{n-1}\left(\frac{1}{\gamma_{j+1}}-\frac{1}{\gamma_j}\right)\ln\rho_j\right]\right)^{-1};\ B_i^*=\theta_b^*-A_i^*\gamma_i\sum_{j=1}^{i-1}\left(\frac{1}{\gamma_{j+1}}-\frac{1}{\gamma_j}\right)\ln\rho_j\,.$$

For the Kirchhoff's variables, we have

$$\theta_i=A_i\ln\rho+B_i\,, \tag{66}$$

where

$$A_i=\left((1+\kappa_n)\theta_{\text{н}}-(1+\kappa_1)\theta_b\right)\left(\lambda_{t0}^{(i)}\left[\frac{1+\kappa_n}{\lambda_{t0}^{(n)}}\ln\rho_n-\sum_{j=1}^{n-1}\left(\frac{1+\kappa_{j+1}}{\lambda_{t0}^{(j+1)}}-\frac{1+\kappa_j}{\lambda_{t0}^{(j)}}\right)\ln\rho_j\right]\right)^{-1};$$

$$B_i=\frac{1}{1+\kappa_i}\left[(1+\kappa_1)\theta_b-A_i\lambda_{t0}^{(i)}\sum_{j=1}^{i-1}\left(\frac{1+\kappa_{j+1}}{\lambda_{t0}^{(j+1)}}-\frac{1+\kappa_j}{\lambda_{t0}^{(j)}}\right)\ln\rho_j\right].$$

Besides the initial data, the solution (66) contains n arbitrary constants κ_i and satisfies the equation (52), boundary conditions (53) and the second group of the contact conditions (54).

The linearized parameters κ_i will be selected to satisfy the first group of the conditions (54). By assuming that one of the linearizing parameters κ_i, for instance, is equal to zero, the following set of $n-1$ equations can be obtained

$$\left(\sqrt{1+2k_i\,\theta_i\big|_{\rho=\rho_i}}-1\right)/k_i=\left(\sqrt{1+2k_{i+1}\,\theta_{i+1}\big|_{\rho=\rho_i}}-1\right)/k_{i+1},\quad i=\overline{1,n-1} \tag{67}$$

for determination of the rest $n-1$ linearizing parameters. The solution should be found in a neighborhood of zero. From the set (67), we determine the values of linearization parameters and thus the Kirchhoff's variables. Then the temperature in layers is

$$T_i=k_i^{-1}(\sqrt{1+2k_i\theta_i}-1)\,. \tag{68}$$

For example, we consider the two-layer pipe $(n=2)$. The Kirchhoff's variables for this case are expressed as

$$\theta_1=K_\lambda\frac{(1+\kappa)\theta_{\text{н}}-\theta_b}{(1+\kappa)\ln\dfrac{\rho_2}{\rho_1}+K_\lambda\ln\rho_1}\ln\rho+\theta_b\,,\ \theta_2=\frac{(1+\kappa)\theta_{\text{н}}-\theta_b}{(1+\kappa)\ln\dfrac{\rho_2}{\rho_1}+K_\lambda\ln\rho_1}\ln\frac{\rho}{\rho_2}+\theta_{\text{н}}\,, \tag{69}$$

where $K_\lambda=\lambda_{t0}^{(2)}/\lambda_{t0}^{(1)}$; κ_1 is equal to zero, and κ_2 is denoted as κ. The value of κ shall be obtained from the equation

$$\frac{1}{k_1}\left[\sqrt{1+2k_1\left(K_\lambda\frac{(1+\kappa)\theta_{\text{н}}-\theta_b}{(1+\kappa)\ln\rho_2/\rho_1+K_\lambda\ln\rho_1}\ln\rho_1+\theta_b\right)}-1\right]=$$

$$=\frac{1}{k_2}\left[\sqrt{1+2k_2\left(\frac{(1+\kappa)\theta_{\text{н}}-\theta_b}{(1+\kappa)\ln\rho_2/\rho_1+K_\lambda\ln\rho_1}\ln\frac{\rho_1}{\rho_2}+\theta_{\text{н}}\right)}-1\right]. \tag{70}$$

If the heat conduction coefficients of the layers $\lambda_t^{(i)}$ $(i=1,2)$ are constants, then the temperature in each layer is determined by formula

$$T_1=NK_\lambda^*\ln\rho+T_b,\ T_2=N\ln\rho+T_{\text{н}}\,, \tag{71}$$

where $N=\left(T_{\text{н}}-T_b\right)\Big/\left((K_\lambda^*-1)\ \ln\rho_1+\ln\rho_2\right)$, $K_\lambda^*=\lambda_t^{(2)}/\lambda_t^{(1)}$.

Let the first layer of thickness $e-1\,(\rho_1=e)$ is made of steel C12 and the second layer of thickness $e^2-e(\rho_2=e^2)$ is made of steel C8 (Sorokin et al., 1989). Let $t_b=700^\circ\text{C}$, $t_{\text{н}}=0^\circ\text{C}$, and $t_0=t_b$. The heat conduction coefficients in the temperature range $0...700^\circ\text{C}$ are given in the form of linear relations: $\lambda_t^{(1)}=47.5(1-0.37T)$ $[W/(m\cdot K)]$, $\lambda_t^{(2)}=$ $64.5(1-0.49\ \text{T})$ $[W/(m\cdot K)]$. Then $k_1=-0.37$, $\lambda_{t0}^{(1)}=47.5$, $k_2=-0.49$, $\lambda_{t0}^{(2)}=64.5$, $K_\lambda=1.36$, $T_b=1$, $T_{\text{н}}=0$, $\theta_b=0.815$, $\theta_{\text{н}}=0$. At reference values, the linearized parameter κ (determined from equation (70)), is equal to 0.0249.

ρ	Thermosensitive layers				Layers with constant characteristics			
	$\kappa = 0{,}0249$		$\kappa = 0$		$\lambda_{tc}^{(1)}\ \lambda_{tc}^{(2)}$		$\lambda_{t0}^{(1)}\ \lambda_{t0}^{(2)}$	
	T	t°C	T	t°C	T	t°C	T	t°C
1	1	700	1	700	1	700	1	700
1,34	0,7945	556,1	0,7924	554,7	0,8369	585,9	0,8314	582,0
1,69	0,6500	455,0	0,6466	452,6	0,7077	495,4	0,6978	488,5
2,03	0,5395	377,7	0,5352	374,6	0,6055	423,9	0,5922	414,6
2,37	0,4506	315,4	0,4455	311,9	0,5193	363,5	0,5031	352,1
$e-0$	0,3764	263,5	0,3707	259,5	0,4429	310,0	0,4241	296,9
$e+0$	0,3765	263,6	0,3810	266,7	0,4429	310,0	0,4241	296,9
3,65	0,2570	179,9	0,2600	182,0	0,3124	218,6	0,2991	209,4
4,59	0,1701	119,1	0,1720	120,4	0,2109	147,6	0,2019	141,3
5,52	0,1023	71,6	0,1037	72,4	0,1292	90,4	0,1237	86,6
6,49	0,0468	32,8	0,0473	33,1	0,0602	42,1	0,0576	40,4
e^2	0	0	0	0	0	0	0	0

Table 1. Distribution of temperature in a two layer pipe along its radius

Table 1 presents the temperature values in two-layer pipe versus its radius. In the first four columns, the values of dimensionless and real temperature T and t, respectively, are given; the first and second columns present the temperature values, obtained by method of linearizing parameters (formulae (68)-(70)); the third and fourth columns present the approximate values of the temperature, obtained by holding only two terms in the series into which the square roots in the first group of the conditions (54) were decomposed (formulae (68), (69) at $\kappa = 0$). The maximum difference between the exact and approximate values of temperature falls within 1.5%. But the approximate solution has a gap $7.2°\text{C}$ on the interface. This fact shows that the condition of the ideal thermal contact is not satisfied, which is physically improper result. In the last four columns, the values of dimensionless and real temperature in the pipe with constant thermal characteristics are presented. The values in the fifth and sixth columns describe the case when the heat conduction coefficients have the mean value in the temperature region $0...700°\text{C}$ i.e. $\lambda_{tc}^{(1)} = \frac{1}{700}\int_0^{700} \lambda_t^{(1)}(t)dt = 38.7$ $[W/(m\cdot K)]$, $\lambda_{tc}^{(2)} = \frac{1}{700}\int_0^{700} \lambda_t^{(2)}(t)dt = 48.7$ $[W/(m\cdot K)]$; the seventh and eighth columns present the maximum values of the heat conduction coefficients in the considered temperature range $\lambda_t^{(1)} = \lambda_{t0}^{(1)}$, $\lambda_t^{(2)} = \lambda_{t0}^{(2)}$. Thus, the maximum difference between the values of the temperature computed for the mean values of the heat conduction coefficients is about 15% $(\approx 48°\text{C})$. If the temperature is computed for the maximum values of the heat conduction coefficients, this difference is about 10% $(\approx 37°\text{C})$.

To simplify the explanation of the linearized parameters method for solving the heat conductivity problem in the coupling thermal sensitive bodies, the constant temperatures on bounded surfaces of piecewise-homogeneous bodies were considered. If the conditions of

convective heat exchange are given, then the final linearization of the obtained nonlinear conditions on Kirchhoff's variables may be fulfilled using the method of linearizing parameters.
The method of linearizing parameters can be successfully used for solution of the transient heat conduction problems.

5. Determination of the temperature fields by means of the step-by-step linearization method

To illustrate the step-by-step linearization method, consider the solution of the centro-symmetrical transient heat conduction problem. Let us consider the thermosensitive hollow sphere of inner radius r_1 and outer radius r_2. The sphere is subjected to the uniform temperature distribution t_p and, from the moment of time $\tau = 0$, to the convective-radiation heat exchange trough the surfaces $r = r_1$ and $r = r_2$ with environments of constant temperatures t_{c1} and t_{c2}, respectively. The transient temperature field in the sphere shall be determined from nonlinear heat conduction equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\lambda_t(t)\frac{\partial t}{\partial r}\right) = c_v(t)\frac{\partial t}{\partial \tau}, \tag{72}$$

with boundary and initial conditions

$$\left[\lambda_t(t)\frac{\partial t}{\partial r} + (-1)^j\left(\alpha_j(t)(t - t_{cj}) + \sigma\varepsilon_j(t)(t^4 - t_{cj}^4)\right)\right]_{r=r_j} = 0 \quad (j = 1,2), \tag{73}$$

$$t\big|_{\tau=0} = t_p \ . \tag{74}$$

Let us construct the solution to the problem (72)–(74) for the material with simple nonlinearity $(a = \lambda_t(t)/c_v(t) \approx \text{const})$. The temperature-dependent characteristics of the material are given as $\chi(t) = \chi_0\chi^*(T)$, where the values with indices zero are dimensional and the asterisked terms are dimensionless functions of the dimensionless temperature $T = t/t_0$ (t_0 denotes the reference temperature). Let the thickness of spherical wall $r_0 = r_2 - r_1$ be the characteristic dimension, and $\rho = r/r_0$, $\mathrm{Fo} = a\tau/r_0^2$, $\mathrm{Bi}_j = \alpha_a^{(j)}r_0/\lambda_{t0}$ (Biot number), and $\mathrm{Sk}_j = \sigma\varepsilon_a^{(j)}r_0t_0^3/\lambda_{t0}$ (Starc number). Then the problem (72)–(74) takes the dimensionless form

$$\frac{1}{\rho^2}\frac{\partial}{\partial \rho}\left(\rho^2\lambda_t^*(T)\frac{\partial T}{\partial \rho}\right) = c_v^*(T)\frac{\partial T}{\partial \mathrm{Fo}}, \tag{75}$$

$$\left[\lambda_t^*(T)\frac{\partial T}{\partial \rho} + (-1)^j\left(\mathrm{Bi}_j\alpha^*(T)(T - T_{cj}) + \mathrm{Sk}_j\varepsilon^*(T)(T^4 - T_{cj}^4)\right)\right]_{\rho=\rho_j} = 0\,(j = 1,2), \tag{76}$$

$$T\big|_{\mathrm{Fo}=0} = T_p \ , \tag{77}$$

where $T_{cj} = t_{cj}/t_0$. By application of the Kirchhoff transformation (9) to the nonlinear problem (75)–(77), the following problem for θ

$$\frac{\partial^2(\rho\theta)}{\partial\rho^2} = \frac{\partial(\rho\theta)}{\partial \mathrm{Fo}}, \tag{78}$$

$$\left[\frac{\partial\theta}{\partial\rho} + (-1)^j Q^{(j)}\left(T(\theta)\right)\right]_{\rho=\rho_j} = 0 \quad (j=1,2), \tag{79}$$

$$\theta\big|_{\mathrm{Fo}=0} = 0 \tag{80}$$

is obtained, where

$$Q^{(j)}\left(T(\theta)\right) = \mathrm{Bi}_j \alpha_j^*\left(T(\theta)\right)\left(T(\theta) - T_{cj}\right) + \mathrm{Sk}_j \varepsilon_j^*\left(T(\theta)\right)\left(\left(T(\theta)\right)^4 - T_{cj}^4\right). \tag{81}$$

The heat conduction equation for the Kirchhoff's variable θ is linear, meanwhile the conditions of convective-radiation heat exchange are partially linearized with the nonlinearities in the expressions $Q^{(j)}\left(T(\theta)\right)$. These expressions depend on the temperature which is to be determined on the surfaces $\rho = \rho_j$. The temperature of the sphere $T(\rho, \mathrm{Fo})$ on each surface $\rho = \rho_j$ is continuous and monotonic function of time. Because every continuous and monotonic function is an uniform limit of a linear combination of unit functions, these functions can be interpolated by means of the splines of order 0 as

$$Q^{(j)}(\mathrm{Fo}) = Q_i^{(j)} + \sum_{i=1}^{m_j - 1} (Q_{i+1}^{(j)} - Q_i^{(j)}) S_-(\mathrm{Fo} - \mathrm{Fo}_i^{(j)}), \tag{82}$$

$$Q_i^{(j)}\left(T(\theta)\right) = \mathrm{Bi}_j \alpha_j^*(T_i^{(j)})(T_i^{(j)} - T_{cj}) + \mathrm{Sk}_j \varepsilon_j^*(T_i^{(j)})((T_i^{(j)})^4 - T_{cj}^4), \tag{83}$$

where $T_1^{(j)} = T_p, T_i^{(j)}$ $(i = \overline{2, m_j})$ are unknown parameters of spline interpolation for the temperature which is to be determined on the surfaces $\rho = \rho_j$ at $\mathrm{Fo}_{i-1}^{(j)} \le \mathrm{Fo} < \mathrm{Fo}_i^{(j)}$ and $\mathrm{Fo}_{m_j}^{(j)} = \infty$, $S_-(\eta) = \begin{cases} 0, \eta < 0, \\ 1, \eta \ge 0 \end{cases}$ is asymmetric unit function (H. Korn & T. Korn, 1977; Podstrihach et al., 1984), $\mathrm{Fo}_i^{(j)}$ are the points of segmentation of the time axis $(0; \mathrm{Fo})$. After substitution of the expression (82) into the boundary conditions (79), the boundary value problem (78)–(80) becomes linear. For its solving, the Laplace integral transformation can be used (Ditkin & Prudnikov, 1975). As a result, the Laplace transforms of the Kirhoff's variables are determined as

$$\tilde{\theta} = -\frac{1}{\rho}\Big[\rho_1^2\left(Q_1^{(1)} + \sum_{i=1}^{m_1 - 1}(Q_{i+1}^{(1)} - Q_i^{(1)})e^{-s\mathrm{Fo}_i^{(1)}}\right)\frac{\Phi_2(s)}{\psi(s)} +$$

$$+\rho_2^2\left(Q_1^{(2)}+\sum_{i=1}^{m_2-1}(Q_{i+1}^{(2)}-Q_i^{(2)})e^{-s\mathrm{Fo}_i^{(2)}}\right)\frac{\Phi_1(s)}{\psi(s)}\Bigg], \tag{84}$$

where $\Phi_j(s)=\rho_j\, ch(\rho-\rho_j)\sqrt{s}+\dfrac{sh(\rho-\rho_j)\sqrt{s}}{\sqrt{s}}$; $\psi(s)=s\left[(\rho_1\rho_2 s-1)\dfrac{sh\sqrt{s}}{\sqrt{s}}+ch\sqrt{s}\right]$; s is the parameter of Laplace transformation; $\tilde{\theta}=\int_0^\infty \theta e^{-s\mathrm{Fo}}d\mathrm{Fo}$.

The inverse Laplace transformation can be found by means of the Vashchenko-Zakharchenko expansion theorem of and shift theorem (Lykov, 1967). As a result, the following expression for Kirchhoff's variable

$$\theta=-\frac{1}{\rho}\Bigg[\rho_1^2\left(Q_1^{(1)}\Psi_2(\rho,\mathrm{Fo})+\sum_{i=1}^{m_1-1}(Q_{i+1}^{(1)}-Q_i^{(1)})\Psi_2(\rho,\mathrm{Fo}-\mathrm{Fo}_i^{(1)})S_-(\mathrm{Fo}-\mathrm{Fo}_i^{(1)})\right)+$$

$$+\rho_2^2\left(Q_1^{(2)}\Psi_1(\rho,\mathrm{Fo})+\sum_{i=1}^{m_2-1}(Q_{i+1}^{(2)}-Q_i^{(2)})\Psi_1(\rho,\mathrm{Fo}-\mathrm{Fo}_i^{(2)})S_-(\mathrm{Fo}-\mathrm{Fo}_i^{(2)})\right) \tag{85}$$

is obtained, where

$$\Psi_j(\rho,\mathrm{Fo})=\frac{1}{1+3\rho_1\rho_2}\left(3\rho\mathrm{Fo}+\frac{1}{2}(\rho-\rho_j)^2(\rho-2\rho_j)-\frac{3\rho(1+5\rho_1\rho_2)}{10(1+3\rho_1\rho_2)}\right)\sum_{n=1}^{\infty}A_n^{(j)}e^{-\mu_n^2\mathrm{Fo}};$$

$$A_n^{(j)}=\frac{2(1+\rho_1\rho_2\mu_n^2)}{\mu_n^2(1+3\rho_1\rho_2+\rho_1^2\rho_2^2\mu_n^2)\cos\mu_n}\left(\rho_j\cos(\rho-\rho_j)\mu_n+\frac{\sin(\rho-\rho_j)\mu_n}{\mu_n}\right); \tag{86}$$

μ_n are roots of characteristic equation

$$(1+\rho_1\rho_2\mu^2)\,\mathrm{tg}\,\mu=\mu\,. \tag{87}$$

For example, let the heat conduction coefficient be a linear function of the temperature $\lambda_t^*(T)=1-kT$. Then on the basis of formula (9),

$$T=k^{-1}\left(1-\sqrt{(1-kT_p)^2-2k\theta}\right). \tag{88}$$

The determined temperature is a function of coordinate ρ and time Fo; it contains $2(m_1+m_2)$ approximation parameters: m_1 values of the temperature $T_i^{(1)}$ on the surface $\rho=\rho_1$ (due to the expressions of $Q_i^{(1)}$) and $\mathrm{Fo}_i^{(1)}$ and m_2 values of the temperature $T_i^{(2)}$ on the surface $\rho=\rho_2$ (due to the expressions of $Q_i^{(2)}$) and $\mathrm{Fo}_i^{(2)}$. The collocation method has been used to determine the approximation parameters. If $\rho=\rho_j$ in (88), the expression of the temperature on the surface $\rho=\rho_j$ are determined as

$$T\big|_{\rho=\rho_j} = k^{-1}\left(1-\sqrt{(1-kT_p)^2-2k\theta\big|_{\rho=\rho_j}}\right) \quad (j=1,2). \tag{89}$$

If the values $\mathrm{Fo}_i^{(1)}$ and $\mathrm{Fo}_i^{(2)}$ are given ($\mathrm{Fo}=\mathrm{Fo}_1^{(j)}$, $\mathrm{Fo}=\mathrm{Fo}_2^{(j)}$, etc.) and $T\big|_{\substack{\rho=\rho_j \\ \mathrm{Fo}=\mathrm{Fo}_i^{(j)}}} = T_i^{(j)}$, then the set of m_1+m_2 algebraic equations will be obtained to determine m_1 values of $T_i^{(1)}$ and m_2 values of $T_i^{(2)}$:

$$\begin{cases} T_{i+1}^{(1)} = k^{-1}\left(1-\sqrt{(1-kT_p)^2-2k\theta\big|_{\substack{\rho=\rho_1 \\ \mathrm{Fo}=\mathrm{Fo}_i^{(1)}}}}\right) \quad (i=\overline{1,m_1-1}), \\ T_{i+1}^{(2)} = k^{-1}\left(1-\sqrt{(1-kT_p)^2-2k\theta\big|_{\substack{\rho=\rho_2 \\ \mathrm{Fo}=\mathrm{Fo}_i^{(2)}}}}\right) \quad (i=\overline{1,m_2-1}). \end{cases} \tag{90}$$

After solving this set of equations and substituting the values $T_i^{(j)}$ $(j=1,2)$ into (88), the expression for the temperature can be obtained.

For approximation of the nonlinear expressions $Q^{(j)}(T(\theta))$, we use the same segmentation of the time axis ($m_1=m_2=m$, $\mathrm{Fo}_i^{(1)}=\mathrm{Fo}_i^{(2)}=\mathrm{Fo}_i$) on the sphere surfaces $\rho=\rho_j$. In this case, the set of equations for determination of unknown values $T_i^{(1)}, T_i^{(2)}$ $(i=\overline{1,m})$ takes the following form: the first and second equations (obtained from (90) at $\mathrm{Fo}=\mathrm{Fo}_1$) contain only $T_2^{(1)}$ and $T_2^{(2)}$; the third and fourth equations (obtained from (90) at $\mathrm{Fo}=\mathrm{Fo}_2$) contain four values $T_i^{(1)}$ and $T_i^{(2)}$ $(i=2,3)$, etc.; in the last two equations (obtained from (90) at $\mathrm{Fo}=\mathrm{Fo}_{m-1}$), all $2(m-1)$ unknown values $T_i^{(1)}$ and $T_i^{(2)}$ $(i=\overline{2,m})$ are presented. After solving the first and second equations, the values $T_2^{(1)}$ and $T_2^{(2)}$ are determined. After substitution of these values into the third and fourth equations, the following two unknown values can be determined. The same procedure shall be repeated until all $T_i^{(j)}$ $(j=1,2)$ are determined.

Consider the transient temperature field in a solid thermosensitive sphere with simple nonlinearity under convective-radiation heat exchange between surface and environment of constant temperature t_c. The solution of such heat conduction problem can be obtained from solution of the problem for a hollow sphere. Putting $\rho_1=0$ and $\rho_2=1$ in (85) and denoting $\mathrm{Bi}_2=\mathrm{Bi}$, $Q_i^{(2)}=Q_i$, $T_{c2}=T_c$, the following expression for the Kirchhoff's variable

$$\theta = Q_1\Psi(\rho,\mathrm{Fo}) + \sum_{i=1}^{m-1}(Q_{i+1}-Q_i)\Psi(\rho_1,\mathrm{Fo}-\mathrm{Fo}_i)S_{-}(\mathrm{Fo}-\mathrm{Fo}_i) \tag{91}$$

can be obtained for the solid sphere, where $\Psi(\rho,\mathrm{Fo}) = \frac{3}{10} - \frac{\rho^2}{2} - 3\mathrm{Fo} - \frac{2}{\rho}\sum_{n=1}^{\infty} \frac{\sin\rho\,\mu_n}{\mu_n^3 \cos\mu_n} e^{-\mu_n^2 \mathrm{Fo}}$,

$Q_i = \mathrm{Bi}\alpha^*(T_i)(T_i - T_c) + \mathrm{Sk}\varepsilon^*(T_i)(T_i^4 - T_c^4)$, $T_1 = T_p$, and μ_n are roots of the equation

$$tg\,\mu = \mu\,. \tag{92}$$

The unknown parameters $T_i = T(\rho,\mathrm{Fo})\Big|_{\substack{\rho=1 \\ \mathrm{Fo}=\mathrm{Fo}_i}}$ are determined from the equations

$$T_{i+1} = k^{-1}\left(1 - \sqrt{(1-kT_p)^2 - 2k\theta\Big|_{\substack{\rho=1 \\ \mathrm{Fo}=\mathrm{Fo}_i}}}\right) \quad (i = \overline{1, m-1})\,. \tag{93}$$

If the Kirchhoff's variable is obtained, then the temperature in the sphere can be calculated by means of the formula (88).

For the case when $\mathrm{Sk} = 0$ and the heat exchange coefficient is independent of the temperature $(\alpha^*(T) = 1)$, then formula (91) yields

$$\theta = \mathrm{Bi}\left[(T_p - T_c)\Psi(\rho,\mathrm{Fo}) + \sum_{i=1}^{m-1}\left(T_{i+1} - T_i\,\Psi(\rho,\mathrm{Fo} - \mathrm{Fo}_i)S_{-}(\mathrm{Fo} - \mathrm{Fo}_i)\right)\right]\,. \tag{94}$$

The unknown parameters of spline approximation T_i $(i = \overline{2, m})$ are determined from the set of equations (93) in the following manner. From the first equation of this set, T_2 can be found as

$$T_2 = \frac{1}{k}\left[L - \sqrt{L^2 - 2k\left(\theta_0 + \mathrm{Bi}\left[(T_p - T_c)\Psi(1,\mathrm{Fo}_1) - T_p\Psi(1,0)\right]\right)}\right], \tag{95}$$

where $L = 1 - \mathrm{Bi}\Psi(1,0)$; $\theta_0 = T_p - kT_p^2/2$. Then the solutions of second, third, and all the following equations can be written as

$$T_i = \frac{1}{k}[L - \{L^2 - 2k\left(\theta_0 + \mathrm{Bi}\left[(T_p - T_c)\Psi(1,\mathrm{Fo}_{i-1}) + \sum_{j=1}^{i-2}(T_{j+1} -\right.\right.$$

$$\left.\left. - T_j)\Psi(1,\mathrm{Fo}_{i-1} - \mathrm{Fo}_j) - T_{i-1}\Psi(1,0)\right]\right)\}^{\frac{1}{2}}]\ \ (i = \overline{3, m})\,. \tag{96}$$

To linearize the nonlinear boundary condition

$$\left[\frac{\partial\theta}{\partial\rho} + \mathrm{Bi}\left(T(\theta) - T_c\right)\right]_{\rho=1} = 0\,, \tag{97}$$

the substitution of the nonlinear expression $T(\theta)$ by θ (Nedoseka, 1988; Podstrihach & Kolyano, 1972) can be employed. Then the Kirchhoff's variable can be given as

$$\theta = T_c\left(1 - \frac{2}{\rho}\sum_{n=1}^{\infty}\frac{\sin\gamma_n - \gamma_n\cos\gamma_n}{\gamma_n(\gamma_n - \sin\gamma_n\cos\gamma_n)}\sin\rho\gamma_n\, e^{-\gamma_n^2\mathrm{Fo}}\right), \tag{98}$$

where γ_n are roots of the characteristic equation

$$(1+\mathrm{Bi})\,\mathrm{tg}\,\gamma = \gamma\,. \tag{99}$$

Let us provide the numerical implementation of the proposed solution method to determine the time-variation of the temperature on the surface $\rho = 1$ of solid sphere exposed to the condition of convective heat exchange. We assume t_c = 300°C (573 K) and this value is also chosen to be the reference temperature; the initial temperature is $t_p = 20$°C (293 K); the Biot number is $\mathrm{Bi} = 10$. In the expression $\lambda_t(t) = \lambda_{to}(1 - kT)$ we set $\lambda_{to} = 50{,}2$ W/(m°K) and $k = 0{,}018$. The results of computation are shown in Figure 1.

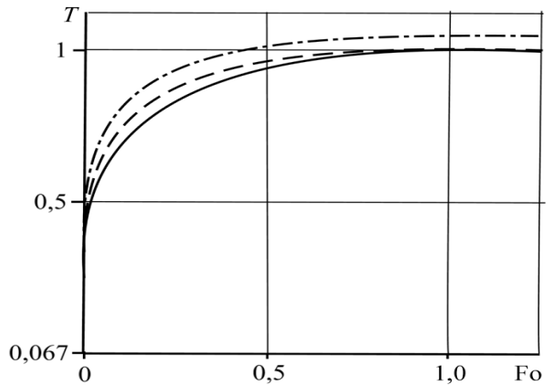

Fig. 1. Dependence of $T(\theta)$ on Fo

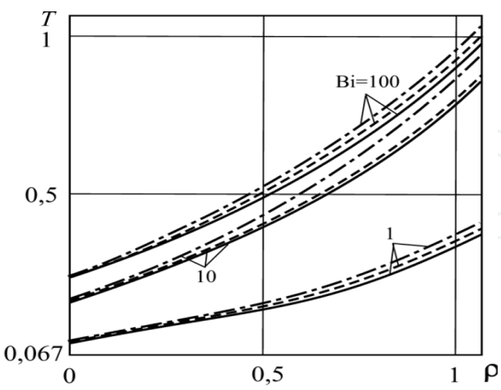

Fig. 2. Dependence of $T(\theta)$ on ρ

In Figure 2, the dependence of the temperature on the radial coordinate at the moment of time $\mathrm{Fo} = 0{,}1$ is shown for some values of the Biot number. The solid lines correspond to the solution of the heat conduction problem, obtained by using the step-by-step method, i.e., when the Kirchhoff's variable is computed by the formula (94). The dash-dot line

corresponds to the solution of the problem when the boundary condition is linearized by changing $T(\theta)$ for θ. In this case, the Kirchhoff's variable is calculated by formula (98). The dashed line presents the solution of corresponding linear problem when thermal characteristics are constant. In the considered case, neglecting the temperature dependence in thermal properties leads to the increasing of the temperature values. In the same time, the unsubstantiated linearization of boundary condition increases the temperature and leads to physically improper results. As it follows from the figures, at some moments of time, the temperature on surface of sphere is greater than the temperature of heating environment. The authors (Nedoseka, 1988; Podstrihach & Kolyano, 1972) did not give much attention to this matter because mainly they considered the temperature fields in thermosensitive bodies due to the internal heat sources. In this case increasing of the temperature is unbounded.

6. Conclusion

In this chapter, the formulations of non-linear heat conduction problems for the bodies with temperature-dependent characteristics (thermosensitive bodies) are given. The efficient analytico-numerical methods for solution of the formulated problems are developed. Particularly, the step-by-step linearization method is proposed for solution of one-dimensional transient problems of heat conduction, which describe the temperature fields in thermosensitive structure members of simple nonlinearity under complex (convective, radiation or convective-radiation) heat exchange boundary conditions. The coefficient of heat exchange and emissivity of the surface, that is under heat exchange with environment, are also dependent on the temperature. The method provides:

- reduction of the heat conduction problem to the corresponding dimensionless problem;
- partial linearization of the obtained problem by means of the Kirchhoff's transform;
- complete linearization of the nonlinear condition on the Kirchhoff's variable θ, that has been obtained from the condition of complex heat exchange due to approximation of the nonlinear term by specially constructed spline of zero or first order;
- construction of the solution to the linearized boundary value problem for θ by means of the appropriate analytical method;
- determination of the temperature in question by means of the inverse Kirchhoff's transform;
- determination of the unknown parameters of spline-approximation, those remain in the expression for the temperature, by means of the collocation method.

The method is verified by the solutions of transient heat conduction problems for thermosensitive solid and hollow spheres subjected to heating (cooling) due to the heat exchange over the limiting surface. This method can be efficiently used fro solution of two-dimensional steady-state heat conduction problems.

The efficient method of linearizing parameters is proposed for determination of the temperature fields in structure members with simple nonlinearity due to convective heat exchange through the limiting surfaces for an arbitrary dependence of the heat conduction coefficient on the temperature. The main feature of this method consists in the fact that the complete linearization of the nonlinear condition for the Kirchhoff's variable θ (obtained form the condition of convective heat exchange) is achieved by substitution of the nonlinear

term $T(\theta)$ by $(1+\kappa)\theta + T_p$ with unknown parameter κ. This parameter can be found by satisfaction of the nonlinear condition for θ with required accuracy.

The method of linearizing parameters is adopted to solution of the nonlinear steady-state and transient heat conduction problems for contacting thermosensitive bodies of simple geometrical shape under conditions of the ideal thermal contact at the interfaces and complex heat exchange on the limiting surfaces. Its approbation is provided for the *n*-layer cylindrical pipe under given temperatures on its inner and outer surfaces. It these surfaces are subjected to the convective heat exchange, then the complete linearization of the obtained nonlinear conditions for the Kirchhoff's variable θ can be done by means of the method of linearizing parameters.

7. Acknowledgment

This research is provided under particular support of the project within the joint program of scientific research between the Ukrainian National Academy of Sciences and Russian Foundation of Basic Research (2010-1011).

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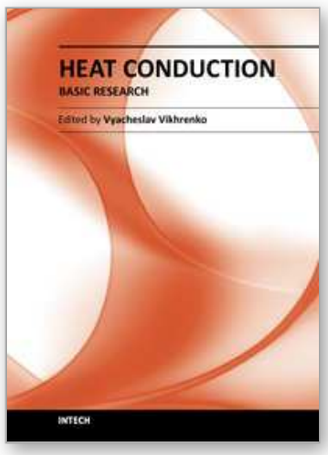

Heat Conduction - Basic Research

Edited by Prof. Vyacheslav Vikhrenko

ISBN 978-953-307-404-7
Hard cover, 350 pages
Publisher InTech
Published online 30, November, 2011
Published in print edition November, 2011

The content of this book covers several up-to-date approaches in the heat conduction theory such as inverse heat conduction problems, non-linear and non-classic heat conduction equations, coupled thermal and electromagnetic or mechanical effects and numerical methods for solving heat conduction equations as well. The book is comprised of 14 chapters divided into four sections. In the first section inverse heat conduction problems are discuss. The first two chapters of the second section are devoted to construction of analytical solutions of nonlinear heat conduction problems. In the last two chapters of this section wavelike solutions are attained.The third section is devoted to combined effects of heat conduction and electromagnetic interactions in plasmas or in pyroelectric material elastic deformations and hydrodynamics. Two chapters in the last section are dedicated to numerical methods for solving heat conduction problems.

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