We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900
Open access books available

186,000
International authors and editors

200M
Downloads

154
Countries delivered to

Our authors are among the

TOP 1%
most cited scientists

12.2%
Contributors from top 500 universities

CLARIVATE ANALYTICS
BOOK CITATION INDEX
INDEXED

WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com

Modeling and Investigation of One-Dimensional Flexural Vibrating Mechatronic Systems with Piezoelectric Transducers

Andrzej Buchacz and Marek Płaczek
Silesian University of Technology,
Poland

1. Introduction

Piezoelectricity has found a lot of applications since it was discovered in 1880 by Pierre and Jacques Curie. There are many applications of the direct piezoelectric effect - the production of an electric potential when stress is applied to the piezoelectric material, as well as the reverse piezoelectric effect - the production of strain when an electric field is applied (Moheimani & Fleming, 2006). In this chapter analysis of mechatronic systems with both direct and reverse piezoelectric effects applications in mechatronic systems are presented. In considered systems piezoelectric transducers are used as actuators – the reverse piezoelectric effect application, or as vibration dampers with the external shunting electric circuit – the direct piezoelectric effect. In the first case piezoelectric transducers can be used as actuators glued on the surface of a mechanical subsystem in order to generate desired vibrations or also to control and damp vibrations in active damping applications (Kurnik et al., 1995; Gao & Liao, 2005). In this case electric voltage is generated by external control system and applied to the transducer. In the second case piezoelectric transducers are used as passive vibration dampers. A passive electric network is adjoined to transducer's clamps. The possibility of dissipating mechanical energy with piezoelectric transducers shunted with passive electric circuits was experimentally investigated and described in many publications (Buchacz & Płaczek, 2009a; Fein, 2008; Hagood & von Flotow, 1991; Kurnik, 2004). There are two basic applications of this idea. In the first method only a resistor is used as a shunting circuit and in the second method it is a passive electric circuit composed of a resistor and inductor. Many authors have worked to improve this idea. For example multimode piezoelectric shunt damping systems were described (Fleming et al., 2002). What is more there are many commercial applications of this idea (Yoshikawa et al., 1998).

Mechatronic systems with piezoelectric sensors or actuators are widely used because piezoelectric transducers can be applied in order to obtain required dynamic characteristic of designed system. It is very important to use very precise mathematical model and method of the system's analysis to design it correctly. It was proved that it is very important to take into consideration influence of all analyzed system's elements including a glue layer between piezoelectric transducer and mechanical subsystem (Pietrzakowski, 2001; Buchacz & Płaczek, 2010b). It is indispensable to take into account geometrical and material

parameters of all system's components because the omission of the influence of one of them results in inaccuracy in the analysis of the system.

This work presents the issues of modeling and testing of flexural vibrating mechatronic systems with piezoelectric transducers used as actuators or vibration dampers. Analysis method of the considered system will be presented, started from development of the mathematical model, by setting its characteristics, to determine the influence of the system's properties on these characteristics.

The discussed subject is important due to increasing number of applications, both simple and reverse piezoelectric phenomena in various modern technical devices. The process of modeling of technical devices with piezoelectric materials is complex and requires large amounts of time because of the complexity of the phenomena occurring in these systems. The correct description of the system by its mathematical model during the design phase is fundamental condition for proper operation of designed system. Therefore, in the work the processes of modeling, testing and verification of used mathematical models of one-dimensional vibrating mechatronic systems will be presented. A series of discrete – continuous mathematical models with different simplifying assumptions will be created. Using created models and corrected approximate Galerkin method dynamic characteristics of considered systems will be designated. An analysis of influence of some geometrical and material parameters of system's components on obtained characteristics will be conducted. Mathematical model that provides the most accurate analysis of the system and maximum simplification of used mathematical tools and minimize required amount of time will be indicated. Identification of the optimal mathematical model that meets the assumed criteria is the main purpose of this work, which is an introduction to the task of synthesis of one-dimensional vibrating continuous systems.

2. Considered system with piezoelectric transducer and assumptions

The main aim of this work is to designate dynamic characteristics of a mechatronic system with piezoelectric transducer used as an actuator or passive vibration damper. It is a cantilever beam which has a rectangular constant cross-section, length l , width b and thickness h_b. Young's modulus of the beam is denoted E_b. A piezoelectric transducer of length l_p is bonded to the beam's surface within the distance of x_1 from a clamped end of the beam. The transducer is bonded by a glue layer of thickness h_k and Kirchhoff's modulus G. The glue layer has homogeneous properties in overall length. The system under consideration in both cases (with piezoelectric actuator or vibration damper) is presented in Fig. 1.

In order to analyze vibration of the systems following assumptions were made:

- material of which the system is made is subjected to Hooke's law,
- the system has a continuous, linear mass distribution,
- the system's vibration is harmonic,
- planes of cross-sections that are perpendicular to the axis of the beam remain flat during deformation of the beam – an analysis is based on the Bernoulli's hypothesis of flat cross-sections,
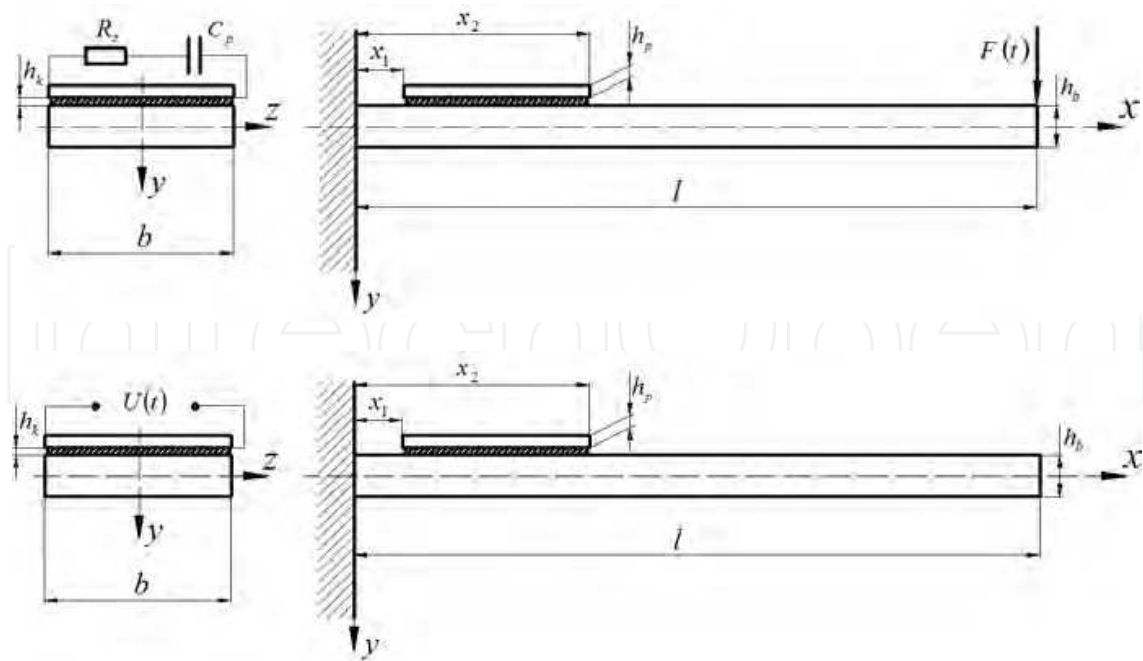- displacements are small compared with the dimensions of the system.

Fig. 1. Shape of the considered system: *a)* with piezoelectric passive vibration damper,
b) with piezoelectric actuator

Structural damping of the beam and glue layer was taken into account in mathematical
models of considered systems using Kelvin-Voigt model of material. It was introduced by
replacing Young's modulus of the beam and modulus of elasticity in shear of the glue layer
by equations:

$$E_b^* = E_b\left(1 + \eta_b \frac{\partial}{\partial t}\right),$$ (1)

$$G^* = G\left(1 + \eta_k \frac{\partial}{\partial t}\right),$$ (2)

where η_b and η_k denote structural damping coefficients of the beam and the glue layer that
have time unit (Pietrzakowski, 2001).

It was assumed that the beam is made of steel and piezoelectric transducer is a PZT transducer.
Geometric and material parameters of the system's elements: mechanical subsystem – the beam,
the glue layer and the piezoelectric transducer are presented in tables 1, 2 and 3.

Geometric parameters	Material parameters
$l = 0,24\,[m]$	$E_b = 210000\,[MPa]$
$b = 0,04\,[m]$	$\rho_b = 7850\left[\dfrac{kg}{m^3}\right]$
$h_b = 0,002\,[m]$	$\eta_b = 8 \cdot 10^{-5}\,[s]$

Table 1. Parameters of the mechanical subsystem

Geometric parameters	Material parameters
$x_1 = 0{,}01\,[m]$	$d_{31} = -240 \cdot 10^{-12} \left[\dfrac{m}{V}\right]$
$x_2 = 0{,}09\,[m]$	$e_{33}{}^T = 2900 \cdot \varepsilon_0 \left[\dfrac{F}{m}\right]$
$h_p = 0{,}001\,[m]$	$s_{11}{}^E = \dfrac{1}{c_{11}{}^E} = 17 \cdot 10^{-12} \left[\dfrac{m^2}{N}\right]$
$b_p = 0{,}04\,[m]$	$\rho_p = 7450 \left[\dfrac{kg}{m^3}\right]$

Table 2. Parameters of the piezoelectric transducer

Geometric parameters	Material parameters
$h_k = 0{,}0001\,[m]$	$G = 1000 \cdot 10^6\,[Pa]$
	$\eta_k = 10^{-3}\,[s]$

Table 3. Parameters of the glue layer

Symbols ρ_b and ρ_p denote density of the beam and transducer. d_{31} is a piezoelectric constant, $e_{33}{}^T$ is a permittivity at zero or constant stress, $s_{11}{}^E$ is flexibility and $c_{11}{}^E$ is a Young's modulus at zero or constant electric field.

Dynamic characteristics of considered systems are described by equations:

$$y(x,t) = \alpha_Y \cdot F(t), \tag{3}$$

$$y(x,t) = \alpha_V \cdot U(t), \tag{4}$$

where $y(x,t)$ is the linear displacement of the beam's sections in the direction perpendicular to the beam's axis. In case of the system with piezoelectric vibration damper it is dynamic flexibility – relation between the external force applied to the system and beam's deflection (equation 3). In case of the system with piezoelectric actuator it is relation between electric voltage that supplies the actuator and beam's deflection (equation 4) (Buchacz & Płaczek, 2011). Externally applied force in the first system and electric voltage in the second system are described as:

$$F(t) = F_0 \cdot \cos \omega t, \tag{5}$$

$$U(t) = U_0 \cdot \cos \omega t, \tag{6}$$

and they were assumed as harmonic functions of time.

3. Approximate Galerkin method verification – Analysis of the mechanical subsystem

In order to designate dynamic characteristics of considered systems correctly it is important to use very precise mathematical model. Very precise method of the system's analysis is very important too. It is impossible to use exact Fourier method of separation of variables in analysis of mechatronic systems, this is why the approximate method must be used. To analyze considered systems approximate Galerkin method was chosen but verification of this method was the first step (Buchacz & Płaczek, 2010c). To check accuracy and verify if the Galerkin method can be used to analyze mechatronic systems the mechanical subsystem was analyzed twice. First, the exact method was used to designate dynamic flexibility of the mechanical subsystem. Then, the approximate method was used and obtained results were juxtaposed. The mechanical subsystem is presented in Fig.2.
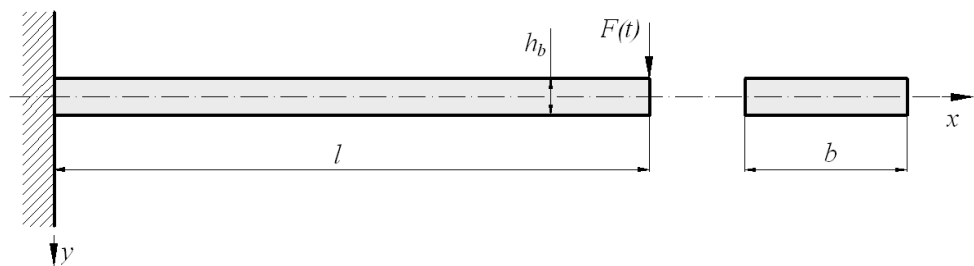

Fig. 2. Shape of the mechanical subsystem

The equation of free vibration of the mechanical subsystem was derived in agreement with d'Alembert's principle. The external force $F(t)$ was neglected. Taking into account equilibrium of forces and bending moments acting on the beam's element, after transformations a well known equation was obtained:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = -a^4 \frac{\partial^4 y(x,t)}{\partial x^4},$$ (7)

where:

$$a = \sqrt[4]{\frac{E_b J_b}{\rho_b A_b}}.$$ (8)

A_b and J_b are the area and moment of inertia of the beam's cross-section. In order to determine the solution of the differential equation of motion (7) Fourier method of separation of variables was used. Taking into account the system's boundary conditions, after transformations the characteristic equation of the mechanical subsystem was obtained:

$$\cos kl = -\frac{1}{\cosh kl}.$$ (9)

Graphic solution of the equation (9) is presented in Fig. 3. The solution of the system's characteristic equation approach to limit described by equation:

$$k_n = \frac{2n-1}{2l}\pi, \qquad n = 1,2,3... \tag{10}$$

This solution is precise for $n > 3$. For the lower values of n solutions should be readout from the graphic solution (Fig. 3) and they are presented in table 4.
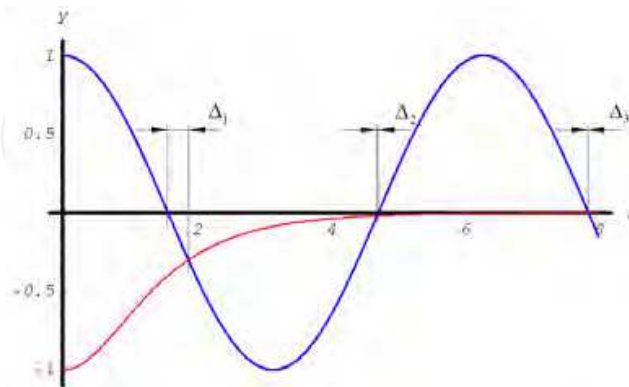

Fig. 3. The graphic solution of the characteristic equation of the system (equation 7)

Taking into account the system's boundary and initial conditions, after transformations the sequence of eigenfunctions is described by the equation:

$$X_n(x) = A_n \frac{\cos k_n l + \cosh k_n l}{\sin k_n l - \sinh k_n l}(\cos k_n x - \cosh k_n x) + \sin k_n x - \sinh k_n x. \tag{11}$$

Assuming zero initial conditions and taking into account that the deflection of the beam is a harmonic function with the same phase as the external force the final form of the solution of differential equation (7) can be described by the equation:

$$y_n(x,t) = \sum_{n=1}^{\infty} X_n(x) \cdot \cos \omega t, \tag{12}$$

and dynamic flexibility of the mechanical subsystem can be described as:

$$\alpha_Y = \frac{\left[\cosh \lambda^* l + \cos \lambda^* l\right]\left[-\sinh \lambda^* x + \sin \lambda^* x\right] + \left[\sinh \lambda^* l + \sin \lambda^* l\right]\left[\cosh \lambda^* x - \cos \lambda^* x\right]}{2E_b J_b \lambda^{*3}\left[1 + \cos \lambda^* l \cosh \lambda^* l\right]}, \tag{13}$$

where:

$$\lambda^* = \sqrt[4]{\omega^2 \cdot \frac{\rho_b A_b}{E_b J_b}}. \tag{14}$$

In the approximate method the solution of differential equation (7) was assumed as a simple equation (Buchacz & Płaczek, 2009b, 2010d):

$$y(x,t) = A \sum_{n=1}^{\infty} \sin k_n x \cos \omega t, \tag{15}$$

where A is an amplitude of vibration. It fulfils only two boundary conditions – deflection of the clamped and free ends of the beam:

$$y(x,t) = 0\big|_{x=0}, \tag{16}$$

$$y(x,t) = A\big|_{x=l}. \tag{17}$$

The equation of the mechanical subsystem's vibration forced by external applied force can be described as:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = -a^4 \frac{\partial^4 y(x,t)}{\partial x^4} + \frac{F(t)\delta(x-l)}{\rho_b A_b}. \tag{18}$$

Distribution of the external force was determined using Dirac delta function $\delta(x\text{-}l)$.

Corresponding derivatives of the assumed approximate solution of the differential equation of motion (15) were substituted in the equation of forced beam's vibration (18). Taking into account the definition of the dynamic flexibility (3), after transformations absolute value of the dynamic flexibility of the mechanical subsystem (denoted Y) was determined:

$$Y = \left| \sum_{n=1}^{\infty} \frac{\delta(x-l)}{\rho_b A_b \left(-\omega^2 + a^4 k_n^{\,4} \right)} \right|. \tag{19}$$

Taking into account geometrical and material parameters of the considered mechanical subsystem (see table 1), the dynamic flexibility for the first three natural frequencies are presented in Fig. 4. In this figure results obtained using the exact and the approximate
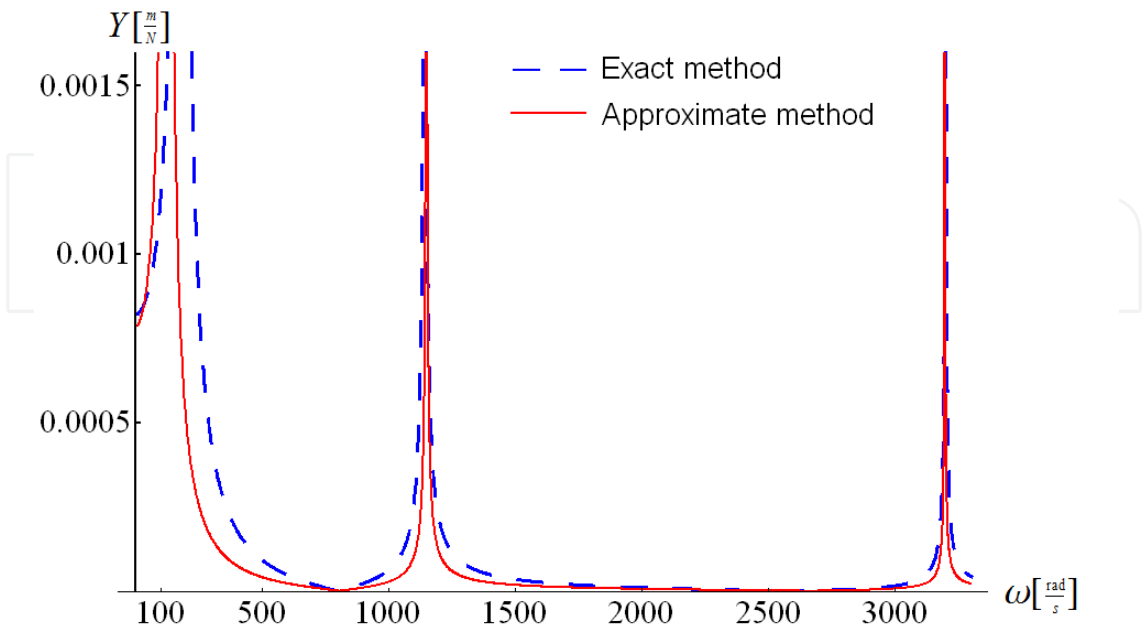

Fig. 4. The dynamic flexibility of the mechanical subsystem – exact and approximate method, for n=1,2,3

n	The exact method	The approximate method	$\Delta[\%]$
1	$k_1 = \dfrac{1,8751}{l}$	$k_1 = \dfrac{\pi}{2l}$	29,8
2	$k_2 = \dfrac{4,6941}{l}$	$k_2 = \dfrac{3\pi}{2l}$	−0,782
3	$k_3 = \dfrac{7,85477}{l}$	$k_3 = \dfrac{5\pi}{2l}$	0,023
>3	$k_n = (2n-1)\dfrac{\pi}{2l}$		0

Table 4. The first three roots of the characteristic equation and shifts of values of the system's natural frequencies

methods are juxtaposed. Inexactness of the approximate method is very meaningful for the first three natural frequencies. Shifts of values of the system's natural frequencies are results of the discrepancy between the assumed solution of the system's differential equation of motion in the approximate method and solution obtained on the basis of graphic solution of the system's characteristic equation in the exact method. These discrepancies are shown in table 4. So it is possible to identify discrepancies between the exact and approximate methods without knowing any geometrical and material parameters. Knowing the characteristic equation of the mechanical system with known boundary conditions and assumed solution of the differential equation of motion it is possible to determine whether the solution obtained using the approximate method differs from the exact solution.

The approximate method was corrected for the first three natural frequencies of the considered system by introduction in equation (19) correction coefficients described by the equation:

$$\Delta\omega_n = \omega_n - \omega_n', \tag{20}$$

where ω_n and ω_n' are values obtained using the exact and approximate methods, respectively (Buchacz & Płaczek, 2010c). The dynamic flexibility of the mechanical subsystem before and after correction is presented in Fig. 5 separately for the first three natural frequencies.

Results of assumption of simplified eigenfunction of variable x (equation 15) are also inaccuracies of the system's vibration forms presented in Fig. 6.

The approximate Galerkin method with corrected coefficients gives a very high accuracy and obtained results can be treated as very precise (see Fig. 5). So it can be used to analyze mechatronic systems with piezoelectric transducers. The considered system – a cantilever beam was chosen purposely because inexactness of the approximate Galerkin method is the biggest in this way of the system fixing.
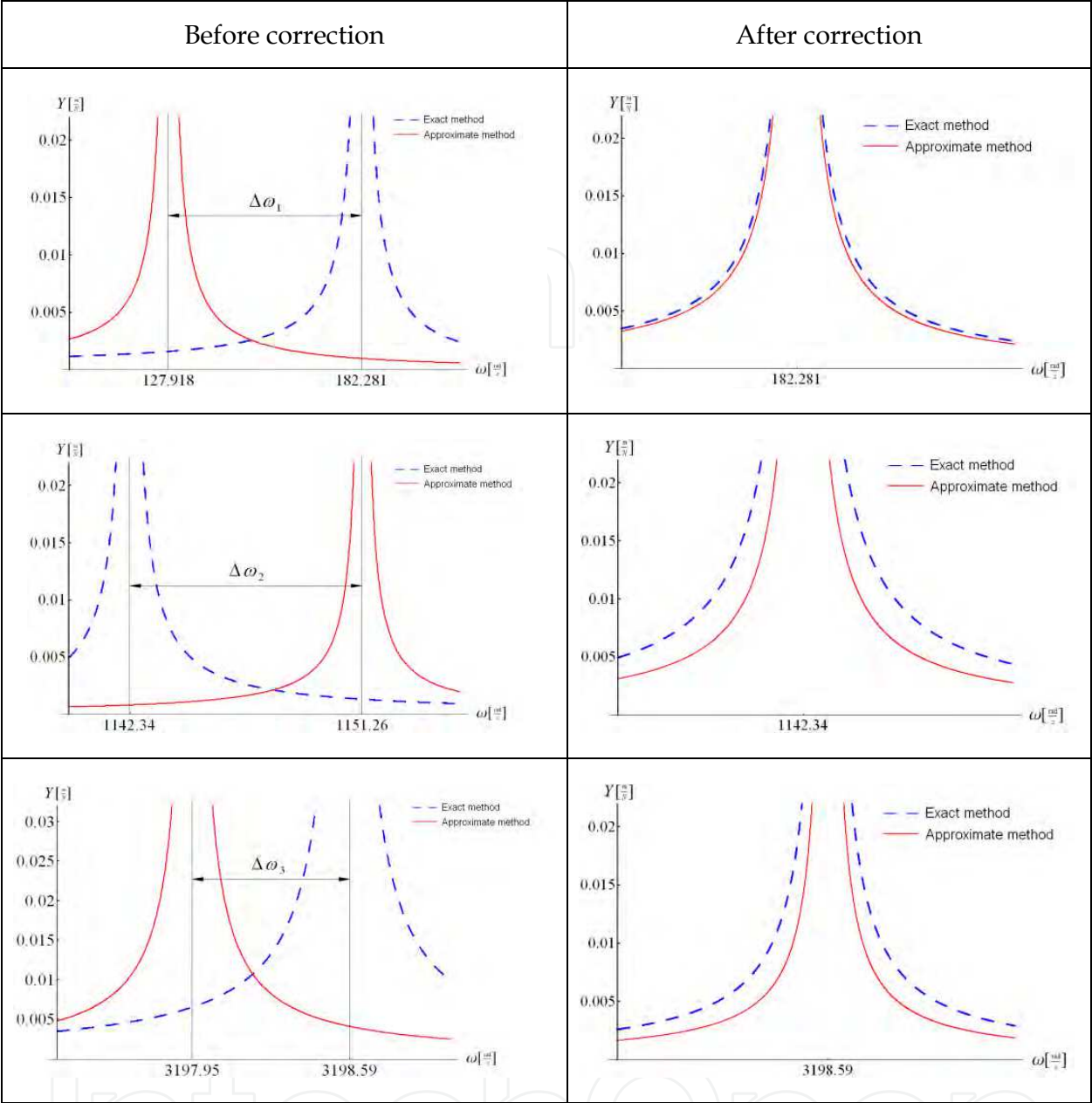
Fig. 5. The dynamic flexibility of the mechanical subsystem – exact and approximate method before and after correction.

4. Mechatronic system with broad-band piezoelectric vibration damper

The considered mechatronic system with broad-band, passive piezoelectric vibration damper was presented in Fig. 1. In this case, to the clamps of a piezoelectric transducer, an external shunt resistor with a resistance R_Z is attached. As a result of the impact of vibrating beam on the transducer and its strain the electric charge and additional stiffness of electromechanical nature, that depends on the capacitance of the piezoelectric transducer, are generated. Electricity is converted into heat and give up to the environment. Piezoelectric transducer with an external resistor is called a shunt broad-band damper (Buchacz & Płaczek, 2010c; Hagood & von Flotow, 1991;Kurnik, 1995).
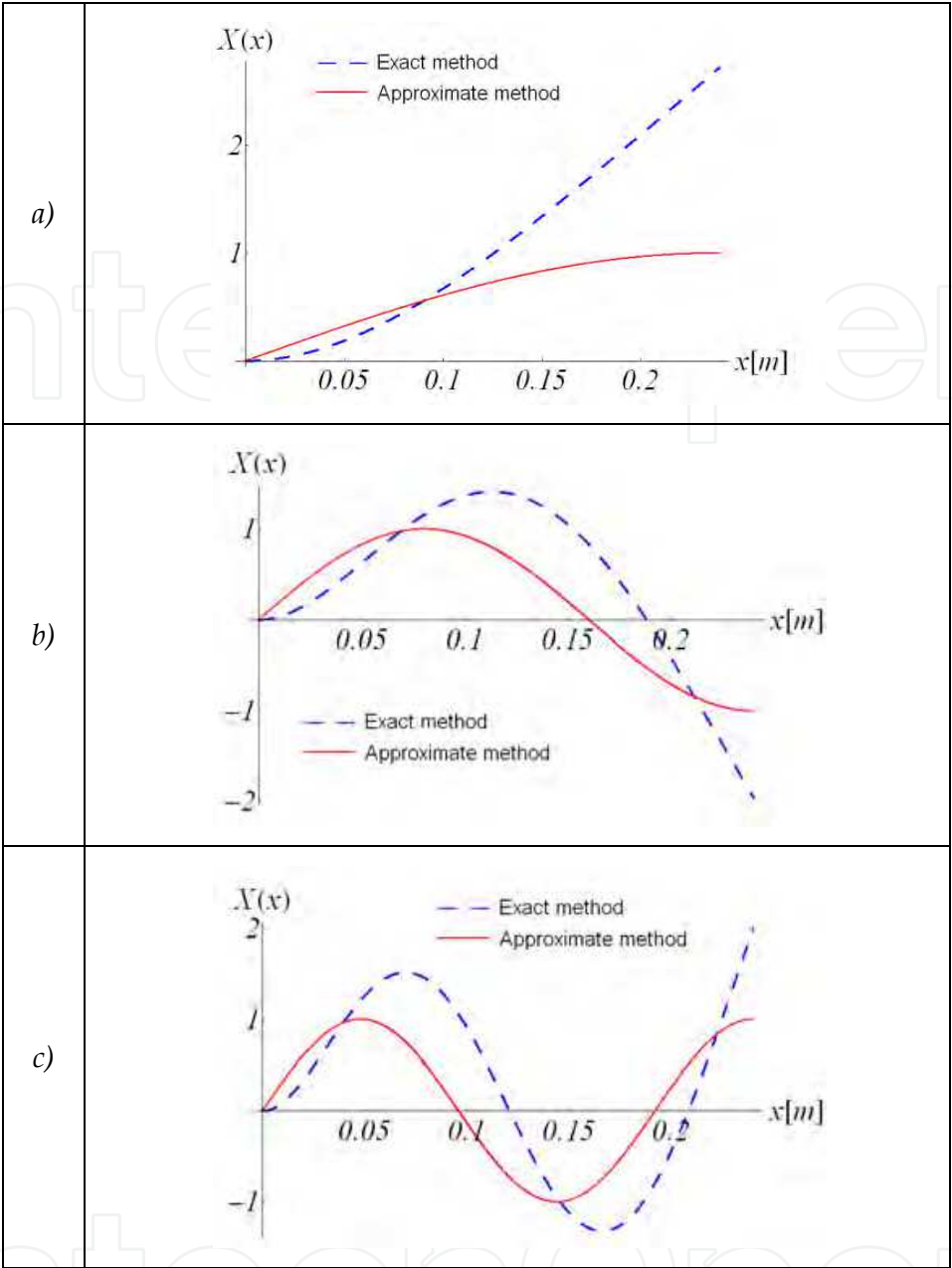
Fig. 6. Vibration forms of the mechanical subsystem – the exact and approximate methods,
a) the first natural frequency, b) the second natural frequency, c) the third natural frequency

Piezoelectric transducer can be described as a serial connection of a capacitor with capacitance C_P, internal resistance of the transducer R_P and strain-dependent voltage source U_P. However, it is permissible to assume a simplified model of the transducer where internal resistance is omitted. In this case internal resistance of the transducer, which usually is in the range 50 – 100 Ω (Behrens & Fleming, 2003) is negligibly small in comparison to the resistance of externally applied electric circuit (400 kΩ), so it was omitted. Taking into account an equivalent circuit of the piezoelectric transducer presented in Fig. 7, an electromotive force generated by the transducer and its electrical capacity are treated as a serial circuit. The considered mechatronic system can be represented in the form, as shown in Fig. 1. So, the piezoelectric transducer with an external shunt resistor is treated as a serial

RC circuit with a harmonic voltage source generated by the transducer (Behrens & Fleming, 2003; Moheimani & Fleming, 2006).
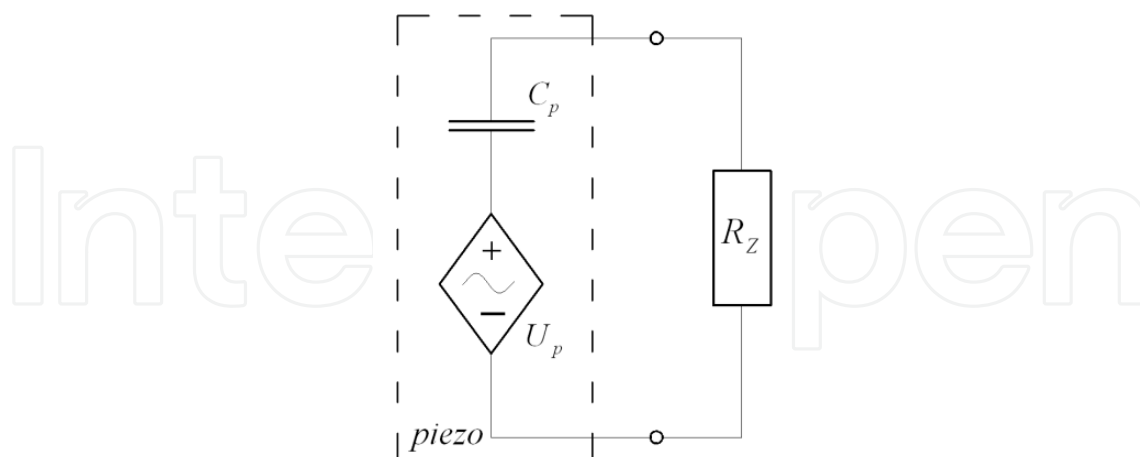

Fig. 7. The substitute scheme of the piezoelectric transducer with an external shunt resistor

4.1 A series of mathematical models of the mechatronic system with piezoelectric vibration damper

A series of mathematical models of the considered mechatronic system with broad-band, passive piezoelectric vibration damper was developed. Different type of the assumptions and simplifications were introduced so developed mathematical models have different degree of precision of real system representation. A series of discrete – continuous mathematical models was created. The aim of this study was to develop mathematical models of the system under consideration, their verification and indication of adequate model to accurately describe the phenomena occurring in the system and maximally simplify the mathematical calculations and minimize required time (Buchacz & Płaczek, 2009b, 2010b).

4.1.1 Discrete – continuous mathematical model with an assumption of perfectly bonded piezoelectric damper

In the first mathematical model of the considered mechatronic system there is an assumption of perfectly bonded piezoelectric transducer - strain of the transducer is exactly the same as the beam's surface strain. Taking into account arrangement of forces and bending moments acting in the system that are presented in Fig. 8, differential equation of motion can be described as:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = -a^4\left(1 + \eta_b \frac{\partial}{\partial t}\right)\frac{\partial^4 y(x,t)}{\partial x^4} + \frac{1}{\rho_b A_b}\frac{\partial^2 M_p(x,t)}{\partial x^2} + \frac{\delta(x-l)}{\rho_b A_b}F(t).\tag{21}$$

$T(x,t)$ denotes transverse force, $M(x,t)$ bending moment and $M_P(x,t)$ bending moment generated by the piezoelectric transducer that can be described as:

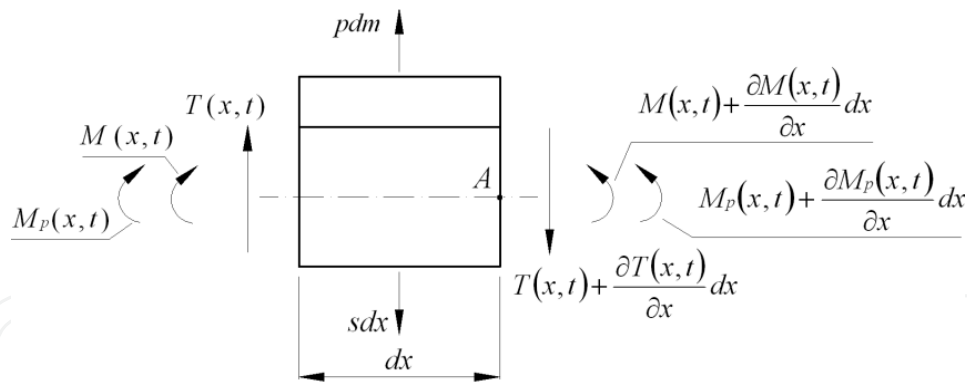$$M_p(x,t) = \frac{h_b + h_p}{2}\cdot F_p(t).\tag{22}$$

Fig. 8. Arrangement of forces and bending moments acting on the cut out part of the beam and the piezoelectric transducer with length dx

Piezoelectric materials can be described by a pair of constitutive equations witch includes the relationship between mechanical and electrical properties of transducers (Preumont, 2006; Moheimani & Fleming, 2006). In case of the system under consideration these equations can be written as:

$$D_3 = \varepsilon_{33}{}^{T}E_3 + d_{31}T_1,\tag{23}$$

$$S_1 = d_{31}E_3 + s_{11}{}^{E}T_1.\tag{24}$$

Symbols $\varepsilon_{33}{}^{T}$, d_{31}, $s_{11}{}^{E}$ are dielectric, piezoelectric and elasticity constants. Superscripts T and E denote value at zero/constant stress and zero/constant electric field, respectively. Symbols D_3, S_1, T_1 and E_3 denote electric displacement, strain, stress and the electric field in the directions of the axis described by the subscript. After transformation of equation (24), force generated by the transducer can be described as:

$$F_P(t) = c_{11}{}^{E}A_p\left[S_1(x,t) - \lambda_1(t)\right],\tag{25}$$

where:

$$\lambda_1(t) = d_{31} \cdot E_3 = d_{31}\frac{U_C(t)}{h_p}.\tag{26}$$

Symbol $c_{11}{}^{E}$ denotes Young's modulus of the transducer at zero/constant electric field (inverse of elasticity constant). $U_C(t)$ is an electric voltage on the capacitance C_p. Due to the fact that the piezoelectric transducer is attached to the surface of the beam on the section from x_1 to x_2 its impact was limited by introducing Heaviside function $H(x)$. Finally, equation (21) can be described as:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = -a^4\left(1 + \eta_b\frac{\partial}{\partial t}\right)\frac{\partial^4 y(x,t)}{\partial x^4} + c_1 \cdot \frac{\partial^2}{\partial x^2}\left[H \cdot S_1(x,t) - H \cdot \lambda_1(t)\right] + \alpha F(t),\tag{27}$$

where:

$$c_1 = \frac{(h_b + h_p) \cdot c_{11}{}^{E}A_p}{2\rho_b A_b},\tag{28}$$

$$\alpha = \frac{\delta(x-l)}{\rho_b A_b}, \tag{29}$$

$$H = H(x-x_1) - H(x-x_2). \tag{30}$$

Equation of the piezoelectric transducer with external electric circuit can be described as:

$$R_Z C_P \frac{\partial U_C(t)}{\partial t} + U_C(t) = U_p(t), \tag{31}$$

where: C_P is the transducer's capacitance, $U_P(t)$ denotes electric voltage generated by the transducer as a result of its strain. Voltage generated by the transducer is a quotient of generated electric charge and capacitance of the transducer. After transformation of the constitutive equations (23) and (24) electric charge generated by the transducer can be described as (Kurnik, 2004):

$$Q(t) = \frac{l_p b d_{31}}{s_{11}{}^E} S_1(x,t) + l_p b \varepsilon_{33}{}^T \frac{U_C(t)}{h_p}\left(1 - k_{31}{}^2\right), \tag{32}$$

where:

$$k_{31}{}^2 = \frac{d_{31}{}^2}{s_{11}{}^E \varepsilon_{33}{}^T}, \tag{33}$$

is an electromechanical coupling constant that determines the efficiency of conversion of mechanical energy into electrical energy and electrical energy into mechanical energy of the transducer, whose value usually is from 0,3 to 0,7 (Preumont, 2006). Equation (33) describes the electric charge accumulated on the surface of electrodes of the transducer with an assumption about uniaxial, homogeneous strain of the transducer. Assuming an ideal attachment of the transducer to the beam's surface its strain is equal to the beam's surface strain and can be described as:

$$S_1(x,t) = \frac{h_b}{2} \cdot \frac{\partial^2 y(x,t)}{\partial x^2}. \tag{34}$$

Finally, equation (31) can be described as:

$$R_Z C_P \frac{\partial U_C(t)}{\partial t} + U_C(t) = \frac{l_p b d_{31}}{C_p s_{11}{}^E} S_1(x,t) + l_p b \varepsilon_{33}{}^T \frac{U_C(t)}{C_p h_p}\left(1 - k_{31}{}^2\right). \tag{35}$$

Using the classical method of analysis of linear electric circuits and due to the low impact of the transient component on the course of electric voltage generated on the capacitance of the linear RC circuit the electric voltage $U_C(t)$ was assumed as:

$$U_C(t) = \frac{|U_p|}{\omega C_P |Z|} \cdot \sin(\omega t + \varphi), \tag{36}$$

where $|Z|$ and φ are absolute value and argument of the serial circuit impedance.

Equations (27) and (35) form a discrete-continuous mathematical model of the considered system.

4.1.2 Discrete – continuous mathematical model with an assumption of pure shear of a glue layer between the piezoelectric damper and beam's surface

Concerning the impact of the glue layer between the transducer and the beam's surface, the mathematical model of the system under consideration was developed. It will allow more detailed representation of the real system. First, a pure shear of the glue layer was assumed. Arrangement of forces and bending moments acting in the system modeled with this assumption is presented in Fig. 9.
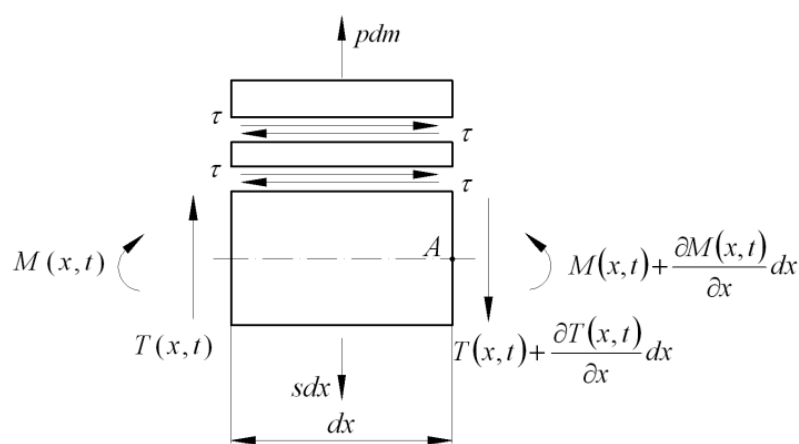
Fig. 9. Forces and bending moments in case of the pure shear of the glue layer

Shear stress was determined according to the Hook's law, assuming small values of pure non-dilatational strain:

$$\tau = \frac{\Delta l}{h_k} \cdot G. \tag{37}$$

Δl is a displacement of the lower and upper surfaces of the glue layer. Movements of the beam, the glue layer and the transducer are shown in Fig. 10.
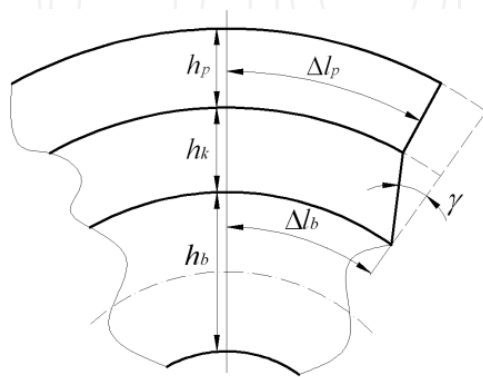

Fig. 10. Movements of the beam, the glue layer and the piezoelectric transducer in the case of pure shear of the glue layer

Uniform distribution of shear stress along the glue layer was assumed. The real strain of the transducer is a difference of the glue layer's upper surface strain and the free transducer's strain that is a result of electric field on the transducer's electrodes, so Δl can be described as:

$$\Delta l = l_p \left[\varepsilon_b(x,t) - \varepsilon_k(x,t) + \lambda_1(t) \right], \tag{38}$$

where: ε_b and ε_k are the beam's and the glue layer's upper surfaces strains.

Finally, obtained system of equations:

$$\begin{cases} \dfrac{\partial^2 y(x,t)}{\partial t^2} = -a^4 \left(1 + \eta_b \dfrac{\partial}{\partial t} \right) \dfrac{\partial^4 y(x,t)}{\partial x^4} + c_2 \left(1 + \eta_k \dfrac{\partial}{\partial t} \right) \dfrac{\partial}{\partial x} H \left[\varepsilon_b(x,t) - \varepsilon_k(x,t) + \lambda_1(t) \right] + \alpha F(t) \\ R_Z C_P \dfrac{\partial U_C(t)}{\partial t} + U_C(t) = \dfrac{l_p b d_{31}}{C_p s_{11}^E} S_1(x,t) + \dfrac{l_p b \varepsilon_{33}^T}{C_p h_p} \left(1 - k_{31}^2 \right) \cdot U_C(t) \end{cases}, \tag{39}$$

where:

$$c_2 = \frac{G l_p}{2 \rho_b h_k}. \tag{40}$$

is the discrete-continuous mathematical model of the system under consideration with the assumption about pure shear of the glue layer.

4.1.3 Discrete – continuous mathematical model taking into account a shear stress and eccentric tension of a glue layer between the piezoelectric damper and beam's surface

In the next mathematical model the system under consideration was modeled as a combined beam in order to unify parameters of all components (Buchacz & Płaczek, 2009c). Shear stress and eccentric tension of the glue layer were assumed. The substitute cross-section of considered system presented in Fig. 11 was introduced by multiplying the width of the piezoelectric transducer and the glue layer by factors:

$$m_p = \frac{c_{11}^E}{E_b}, \tag{41}$$

$$m_k = \frac{2G(1+v)}{E_b}. \tag{42}$$

Symbol υ denotes the Poisson's ratio of the glue layer.

Taking into account the eccentric tension of the glue layer under the action of forces presented in Fig.12 the stress on the substitute cross-section's surfaces was assigned.

$F_P(t)$ denotes force generated by the piezoelectric transducer and $F_b(t)$ denotes forces generated by the bending beam as a result of the beam's elasticity. The area, location of the central axis and moment of inertia of the substitute cross-section were calculated:
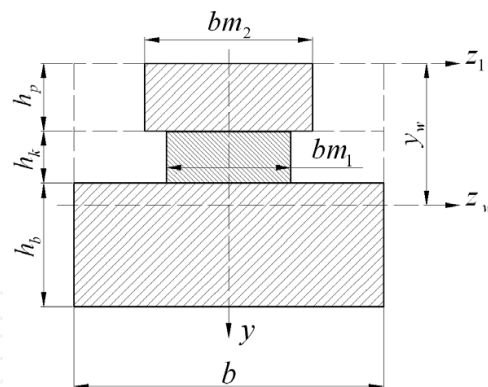
Fig. 11. Position of the center of gravity of substitute cross-section of the beam
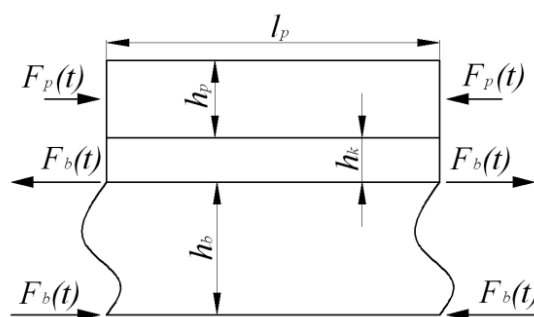

Fig. 12. Arrangement of forces in case of eccentric tension of the glue layer

$$A_w = bh_b + m_k bh_k + m_p bh_p, \tag{43}$$

$$y_w = A_w^{-1} b \left[h_b \left(h_k + h_p + \frac{h_b}{2} \right) + m_k h_k \left(h_p + \frac{h_k}{2} \right) + m_p \frac{h_p^{\,2}}{2} \right], \tag{44}$$

$$J_w = b \left\{ h_b \left[\frac{h_b^{\,2}}{12} + \left(h_p + h_k + \frac{h_b}{2} - y_w \right)^2 \right] + m_k h_k \left[\frac{h_k^{\,2}}{12} + \left(y_w - h_p - \frac{h_k}{2} \right)^2 \right] + m_p h_p \left[\frac{h_p^{\,2}}{12} + \left(y_w - \frac{h_p}{2} \right)^2 \right] \right\}, \tag{45}$$

were calculated and stress on the surfaces of the substitute cross-section was assigned:

$$\sigma_i = m_i \left[F_b(t) \left(-\frac{h_b y}{J_w} \right) - F_p(t) \left(\frac{1}{A_w} - \frac{(-y_w + 0.5h_p)y}{J_w} \right) \right], \tag{46}$$

where subscript i denotes element of the composite beam ($i=b,k,p$). In case of the beam, value of the symbol m_b is equal to one, while in case of the transducer and the glue layer m_p and m_k are described by equations (41) and (42). Using the basic laws and dependences from theory of strength of materials the real strain of the piezoelectric transducer was assigned:

$$S_1(x,t) = W_1 \cdot \varepsilon_b(x,t) - W_2 \cdot \lambda_1(t), \tag{47}$$

where:

$$W_1 = \frac{h_p - y_w}{\left(h_p + h_k - y_w\right)\left[1 - \dfrac{E_p A_p}{E_b A_w}\left(\dfrac{h_p - y_w}{h_p + h_k - y_w} - 1\right)\right]},$$
(48)

$$W_2 = \frac{E_p \dfrac{A_p}{A_w}\left(\dfrac{h_p - y_w}{h_p + h_k - y_w} - 1\right)}{E_b\left[1 - \dfrac{E_p A_p}{E_b A_w}\left(\dfrac{h_p - y_w}{h_p + h_k - y_w} - 1\right)\right]}.$$
(49)

To determine the value of shear stress on the plane of contact of the transducer and beam the following dependence was used:

$$\tau(x,y) = \frac{T(x,t) \cdot S_z(y)}{J_w \cdot b(y)},$$
(50)

where: $S_Z(y)$ is a static moment of cut off part of the composite beam's cross-section relative to the weighted neutral axis. Transverse force $T(x,t)$ can be calculated as a derivative of bending moment acting on the beam's cross-section:

$$T(x,t) = \frac{\partial\{H[W_3 \cdot \varepsilon_b(x,t) - W_4 \cdot \lambda_1(t)]\}}{\partial x},$$
(51)

where:

$$W_3 = c_{11}{}^E A_p W_1\left(y_w - \frac{h_p}{2}\right) + \frac{h_b E_b J_w}{h_b(y_w - h_p - h_k)} + \frac{h_b W_1 c_{11}{}^E A_p J_w}{h_b(y_w - h_p - h_k)}\left[A_w^{-1} + \frac{0{,}5h_p - y_w}{J_w}(-y_w + h_p + h_k)\right],$$
(52)

$$W_4 = c_{11}{}^E A_p(W_2 + 1) \cdot \left(y_w - \frac{h_p}{2}\right) + \frac{(W_2 + 1)c_{11}{}^E h_b A_p J_w}{h_b(y_w - h_p - h_k)}\left[A_w^{-1} + \frac{0{,}5h_p - y_w}{J_w}(-y_w + h_p + h_k)\right].$$
(53)

Finally, the discrete-continuous mathematical model of the system can be described as:

$$\begin{cases} \dfrac{\partial^2 y(x,t)}{\partial t^2} = -a^4\left(1 + \eta_b \dfrac{\partial}{\partial t}\right)\dfrac{\partial^4 y(x,t)}{\partial x^4} + \dfrac{S_z(y)}{2\rho_b J_w b}\left(1 + \eta_k \dfrac{\partial}{\partial t}\right)\dfrac{\partial^2}{\partial x^2}\left[HW_3\varepsilon_b(x,t) - HW_4\lambda_1(t)\right] + \alpha F(t) \\[4mm] R_Z C_P \dfrac{\partial U_C(t)}{\partial t} + U_C(t) = \dfrac{l_p b d_{31}}{C_p s_{11}{}^E}S_1(x,t) + \dfrac{l_p b \varepsilon_{33}{}^T}{C_p h_p}\left(1 - k_{31}{}^2\right)\cdot U_C(t) \end{cases}.$$
(54)

Obtained system of equations is a mathematical model with assumptions of shear stress and eccentric tension of the glue layer.

4.1.4 Discrete – continuous mathematical model taking into account a bending moment generated by the transducer and eccentric tension of a glue layer between the piezoelectric damper and beam's surface

Taking into account parameters of the combined beam introduced in section 4.1.4 the discrete-continuous mathematical model with influence of the glue layer on the dynamic characteristic of the system was developed. However, in this model the impact of the piezoelectric transducer was described as a bending moment, similarly as in the mathematical model with the assumption of perfectly attachment of the transducer. Homogeneous, uniaxial tension of the transducer was assumed and its deformation was described by the equation (47). In this case the bending moment generated by the transducer can be described as:

$$M_p(x,t) = \left(\frac{h_p + h_b}{2} + h_k \right) c_{11}{}^E A_p \left[W_1 \varepsilon_b(x,t) - (W_2 + 1)\lambda_1(t) \right]. \tag{55}$$

Obtained system of equations:

$$\begin{cases} \dfrac{\partial^2 y(x,t)}{\partial t^2} = -a^4 \left(1 + \eta_b \dfrac{\partial}{\partial t}\right) \dfrac{\partial^4 y(x,t)}{\partial x^4} + c_4 \dfrac{\partial^2}{\partial x^2} \left[T_1 \varepsilon_b(x,t)H - (T_2 + 1)\lambda_1(t)H \right] + \alpha F(t) \\ R_Z C_P \dfrac{\partial U_C(t)}{\partial t} + U_C(t) = \dfrac{l_p b d_{31}}{C_p s_{11}{}^E} S_1(x,t) + \dfrac{l_p b \varepsilon_{33}{}^T}{C_p h_p}\left(1 - k_{31}{}^2\right) \cdot U_C(t) \end{cases}, \tag{56}$$

where:

$$c_4 = \left(\frac{h_p + h_b}{2} + h_k \right) \frac{c_{11}{}^E A_p}{\rho_b A_b}, \tag{57}$$

is the discrete-continuous mathematical model of the system under consideration.

4.2 Dynamic flexibility of the system with broad-band piezoelectric vibration damper

Dynamic flexibility of the considered system was assigned using corrected approximate Galerkin method. Solution of the differential equation of the beam's motion with piezoelectric damper was assumed as a product of the system's eigenfunctions in accordance with the equation (15). For all mathematical models analogous calculations were done, therefore, an algorithm used to determine the dynamic flexibility of the system using the first mathematical model is presented. Obtained results for all mathematical models are presented in graphical form.

In the mathematical model of the considered mechatronic system with the assumption of perfectly bonded piezoelectric damper - equations (27) and (35) the derivatives of the approximate equation (15) were substituted. Assuming that the dynamic flexibility will be assigned on the free end of the beam (*x=l*), after transformations and simplifications a system of equations was obtained:

$$\begin{cases} P_1 A \cos \omega t + P_2 A \sin \omega t = P_3 B \sin \omega t + P_4 B \cos \omega t + \alpha F_0 \cos \omega t, \\ -P_5 A \cos \omega t + P_6 B \sin \omega t + P_7 B \cos \omega t = 0 \end{cases}$$ (58)

where:

$$B = \frac{|U_p|}{\omega C_P |Z|},$$ (59)

$$P_1 = \sin k_n l \left(-\omega^2 + a^4 k_n^4 - c_1 r_1 k_n^2 H'' \right),$$ (60)

$$P_2 = -a^4 k_n^4 \eta_b \omega \sin k_n l,$$ (61)

$$P_3 = -\frac{c_1 d_{31} H''}{h_p} \cos \varphi,$$ (62)

$$P_4 = -\frac{c_1 d_{31} H''}{h_p} \sin \varphi,$$ (63)

$$P_5 = \frac{l_p b d_{31} r_1 k_n^2 \sin k_n l}{s_{11}^E R_Z C_P^2},$$ (64)

$$P_6 = \frac{1}{R_Z C_P} \cos \varphi \left(1 - \frac{Z}{C_P} \right) - \omega \sin \varphi,$$ (65)

$$P_7 = \frac{1}{R_Z C_P} \sin \varphi \left(1 - \frac{Z}{C_P} \right) + \omega \cos \varphi.$$ (66)

Using mathematical dependences:

$$e^{\pm i \omega t} = \cos \omega t \pm i \cdot \sin \omega t,$$ (67)

$$\sin \omega t = \cos \left(\omega t - \frac{\pi}{2} \right),$$ (68)

after transformations the system of equations (58) can be written in matrix form:

$$\begin{bmatrix} P_1 - iP_2 & iP_3 - P_4 \\ -P_5 & -iP_6 + P_7 \end{bmatrix} \cdot \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \alpha F_0 \\ 0 \end{bmatrix}.$$ (69)

Using Cramer's rule amplitude of the system's vibration can be calculated as:

$$A = \frac{W_A}{W},$$ (70)

where W is a main matrix determinant and W_A is a determinant of the matrix formed by replacing the first column in the main matrix by the column vector of free terms. Obtained equation can be substituted in the assumed solution of the derivative equation of the beam's motion (15). Finally, in agreement with definition (3), the dynamic flexibility of the system under consideration can be described as:

$$\alpha_Y = \sum_{n=1}^{\infty} \frac{(\alpha P_7 - i\alpha P_6)\sin k_n l}{P_1 P_7 - P_2 P_6 - P_4 P_5 + i(P_3 P_5 - P_2 P_7 - P_1 P_6)}. \tag{71}$$

In order to eliminate complex numbers in equation (71) its numerator and denominator were multiplied by the number conjugate with the denominator. Absolute value of the obtained complex number was calculated:

$$Y = \sum_{n=1}^{\infty} \frac{\sqrt{R_1^2 + R_2^2}}{\left(P_1 P_7 - P_2 P_6 - P_4 P_5\right)^2 + \left(P_3 P_5 - P_2 P_7 - P_1 P_6\right)^2}, \tag{72}$$

where:

$$R_1 = \alpha \sin k_n l \left(P_1 P_7^2 - P_4 P_5 P_7 - P_3 P_5 P_6 + P_1 P_6^2\right), \tag{73}$$

$$R_2 = \alpha \sin k_n l \left(P_2 P_7^2 - P_3 P_5 P_7 + P_4 P_5 P_6 + P_2 P_6^2\right). \tag{74}$$

Taking into account geometrical and material parameters of the considered system presented in tables 1, 2 and 3, graphical solution of the equation (72) is presented in Fig. 13.
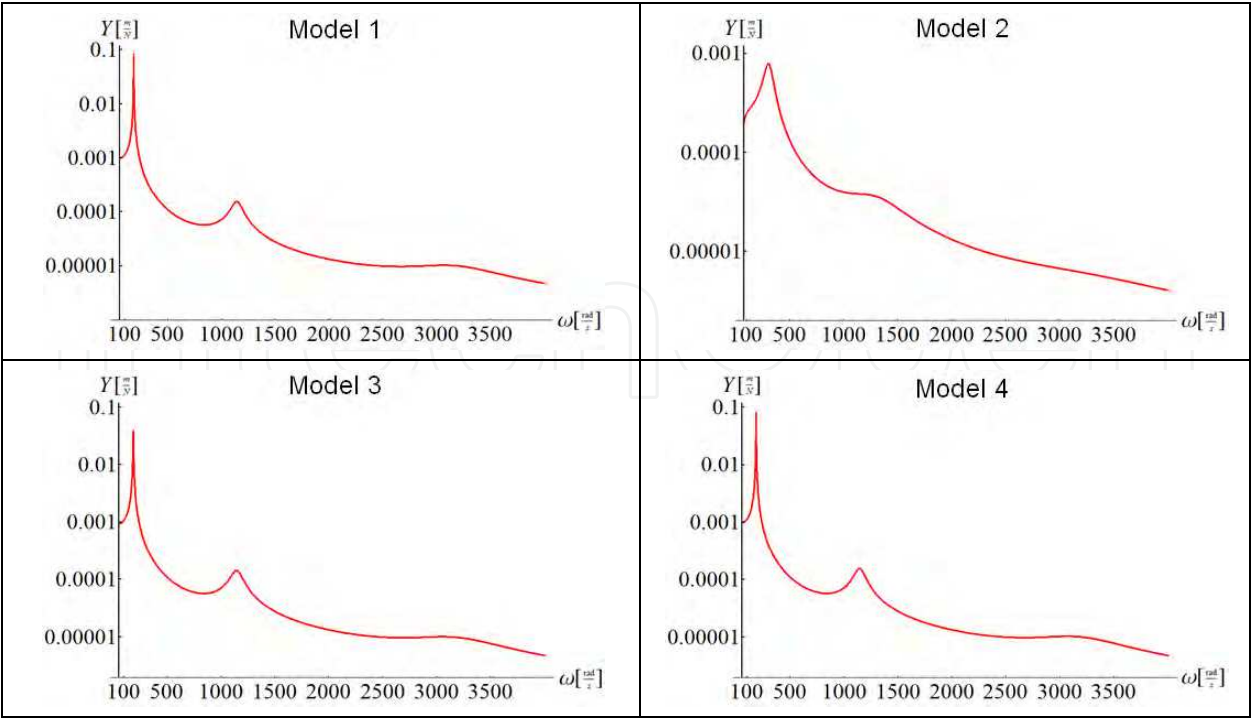
Fig. 13. Absolute value of the dynamic flexibility of mechatronic system with piezoelectric vibration damper, for the first three natural frequencies (in a half logarithmic scale)

Results obtained using the others mathematical models of the considered system are also presented in Fig. 13.

Using developed mathematical models and corrected approximate Galerkin method, very similar course of dynamic characteristics were obtained, except the second mathematical model with the assumption about pure shear of the glue layer. Shift of the natural frequencies in the direction of higher values of the mechatronic system in the direction of higher values can be observed. This shift is a result of increased stiffness of mechatronic system compared with the mechanical subsystem.

5. Mechatronic system with piezoelectric actuator

Developed mathematical models of the system with piezoelectric vibration damper were used to analyze the mechatronic system with piezoelectric actuator. In this case inverse piezoelectric effect is applied. Strain of the piezoelectric transducer is a result of externally applied electric voltage described by the equation (6). The considered system is presented in Fig. 1. Its parameters are presented in tables 1, 2 and 3. The aim of the system's analysis is to designate dynamic characteristic that is a relation between parameters of externally applied voltage and deflection of the free end of the beam (it was assumed that $x=l$), described by the equation (4).

In this case the internal capacitance C_P and resistance R_P of the piezoelectric transducer were taken into account, so transducer supplied by the external harmonic voltage source can be treated as a serial RC circuit with harmonic voltage source and was described by the equation (Buchacz & Płaczek, 2011):

$$R_P C_P \frac{\partial U_C(t)}{\partial t} + U_C(t) = U(t). \tag{75}$$

Equations of motion of the beam with piezoelectric actuator for all developed mathematical models were designated in agreement with d'Alembert's principle similarly as in the case of mechatronic system with piezoelectric vibration damper. Obtained absolute value of dynamic characteristics for all mathematical models of the system are presented in Fig. 14.

Final results are very similar for all mathematical models, except the second model with the assumptions about pure shear of the glue layer, as it was in case of analysis of system with piezoelectric vibration damper.

6. Analysis of influence of parameters of considered systems on dynamic characteristics

Developed mathematical models of considered systems were used to analyze influence of geometric and material parameters of systems on obtained dynamic characteristics. This study was carried out in dimensionless form in order to generalize obtained results. Results are presented in the form of three-dimensional graphs that show the course of the dimensionless absolute value of dynamic characteristic in relation to dimensionless frequency of externally applied force or electric voltage and one of the system's parameters dimensionless value. Dimensionless values of dynamic characteristics were introduced as:

$$Y_W = YE_b\sqrt{A_b}\left[\frac{s^2}{kg}\cdot m\cdot\frac{kg}{m\cdot s^2}=1\right],\tag{76}$$

$$Y_{VW} = \frac{Y_V}{\sqrt{d_{31}}}\left[\frac{m}{V}\cdot\frac{V}{m}=1\right].\tag{77}$$
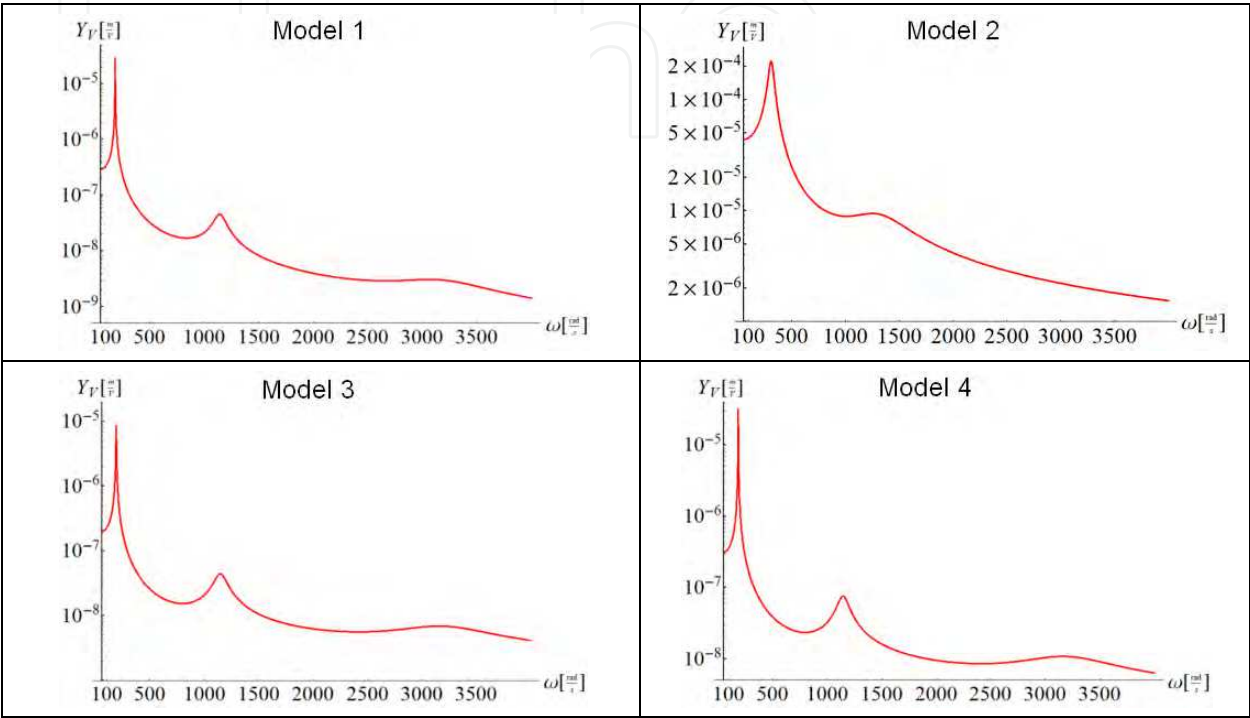

Fig. 14. Absolute value of the dynamic characteristic of mechatronic system with piezoelectric actuator, for the first three natural frequencies (a half logarithmic scale)

Dimensionless frequencyies of external force or electric voltage were introduced by dividing their values by the value of the first natural frequency of the mechanical subsystem. Dimensionless values of analyzed parameters were obtained by dividing them by their initial values. Obtained results for selected parameters are presented in Fig. 15 and Fig. 16.

Influence of the other parameters on characteristics of considered systems were analyzed in other publications (Buchacz & Płaczek, 2009b, 2010a).

7. Conclusions and selection of an optimal mathematical model

Realized studies have shown that the corrected approximate Galerkin method can be used to analyze mechatronic systems with piezoelectric transducers. Verification of the approximate method proved that obtained results can be treated as very precise. Precision of the mathematical model of considered system has no big influence on the final results. There are no significant differences between the values of natural vibration frequencies of considered systems and course of dynamic characteristics, except the second model. In case of the mathematical model with the assumption of pure shear of the glue layer a very significant shift of natural frequencies values and increase of piezoelectric damper or actuator efficiency were observed. These discrepancies are the results of the assumed
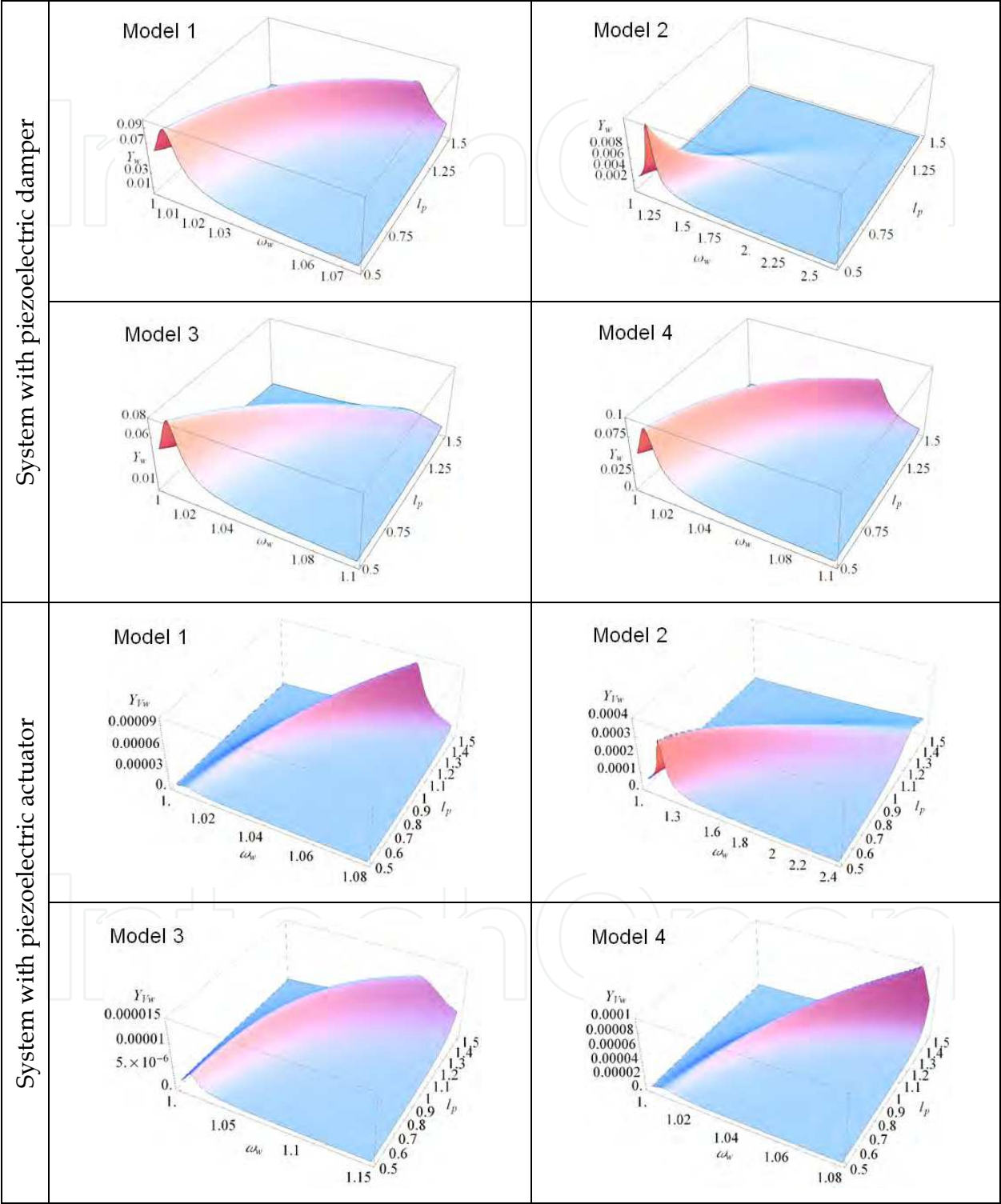
Fig. 15. Influence of length of the piezoelectric transducer on the absolute value of the dimensionless dynamic characteristics
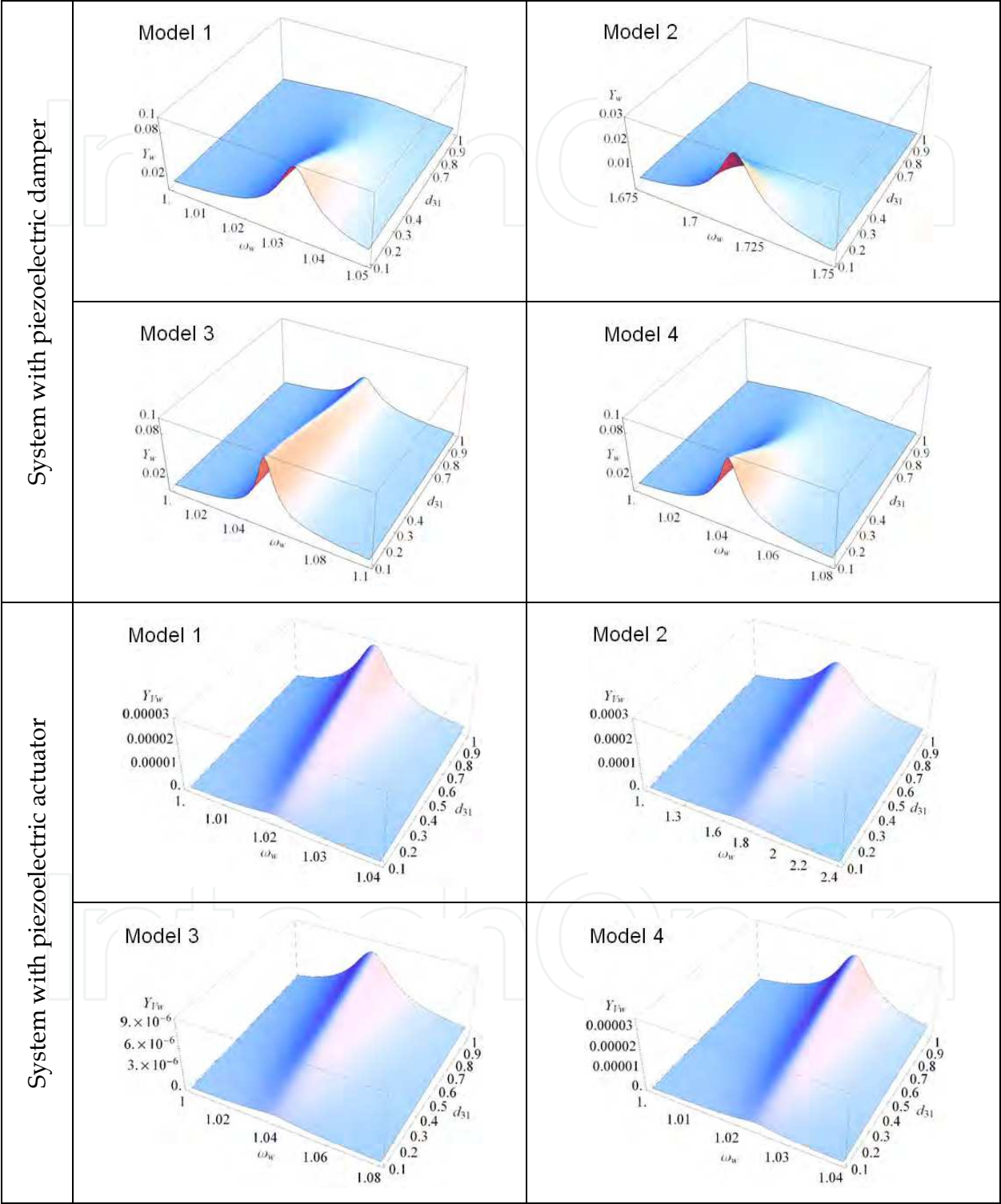
Fig. 16. Influence of piezoelectric constant of the piezoelectric transducer on the absolute value of the dimensionless dynamic characteristics

simplifications of the real strain of the transducer and resulting generated shear stress in the glue layer. There was also an assumption about pure shear of the glue layer, while, in the real system, this layer is under the influence of forces that cause the eccentric tension of it.

The simplest is the mathematical model with the assumption about perfectly bonded piezoelectric transducer. But taking this assumption it is impossible to define influence of the glue layer on the dynamic characteristic of the system. Using this model it is not possible to meet requirements undertaken in this work. To take into account properties of the glue layer and its real loads to which it is subjected, mathematical models, where an eccentric tension of glue layer was considered, were developed. Interactions between elements of the system were being taken into consideration and real strain of the transducer was determined. The third mathematical model is much more complex then the last one, while obtained results are very similar. It is therefore concluded that the optima, in terms of assumed criteria, is the last mathematical model where a bending moment generated by the transducer and eccentric tension of a glue layer between the piezoelectric transducer and surface of the beam were taken into account. Using this model it is possible to analyze influence of all components of the system, including glue layer between the beam and transducer, while it is quite simple at the same time.

8. Acknowledgment

9. References

Behrens S., Fleming A. J., & Moheimani S. O. R. (2003). A broadband controller for shunt piezoelectric damping of structural vibration. *Smart Materials and Structures*, Vol. 12, (2003), pp. 18-28

Buchacz A., & Płaczek M. (2009a). Damping of Mechanical Vibrations Using Piezoelements, Including Influence of Connection Layer's Properties on the Dynamic Characteristic. *Solid State Phenomena*, Vols. 147-149, (2009), pp. 869-875, ISSN: 1012-0394

Buchacz A., & Płaczek M. (2009b). The discrete-continuous model of the one-dimension vibrating mechatronic system. *PAMM - Proc. Appl. Math. Mech.*, Vol. 9, No. 1, (2009), pp. 395-396, ISSN: 1617-7061

Buchacz A., & Płaczek M. (2009c). The vibrating mechatronic system modeled as the combined beam, *Proceedings of International Scientific and Technical Conference Reliability and Durability of Mechanic and Biomechanical Systems and Elements of their Constructions*, Sevastopol, September 2009

Buchacz A., & Płaczek M. (2010a). Selection of Parameters of External Electric Circuit for Control of Dynamic Flexibility of a Mechatronic System. *Solid State Phenomena*, Vol. 164, (2010), pp. 323-326, ISSN: 1012-0394

Buchacz A., & Płaczek M. (2010b). Development of Mathematical Model of a Mechatronic System. *Solid State Phenomena*, Vol. 164, (2010), pp. 319-322, ISSN: 1012-0394

Buchacz A., & Płaczek M. (2010c). The analysis of vibrating systems based on the exact end approximate method. *International Journal of Modern Manufacturing Technologies*, Vol. II, No. 1, (2010), pp. 19-24, ISSN: 2067-3604

Buchacz A., & Płaczek M. (2010d). The exact and approximate method in mechanical system's analysis, *PAMM - Proc. Appl. Math. Mech.*, Vol. 10, No. 1, (2010), pp. 379-380, ISSN: 1617-7061

Buchacz A., & Płaczek M. (2011). Characteristic of the mechatronic system with piezoelectric actuator modeled as the combined beam, *Proceedings of The 15th International Conference Modern Technologies, Quality and Innovation ModTech 2011*, ISSN: 2069-6736, Vadul lui Voda, Chisinau, Republic of Moldova, May 2011

Fein O. M. (2008). A model for piezo-resistive damping of two-dimensional structures. *Journal of Sound and Vibration*, Vol. 310, No. 4-5, (2008), pp. 865-880, ISSN: 0022-460X

Fleming A. J., Behrens S., & Reza Moheimani S. O. (2002). Optimization and Implementation of Multimode Piezoelectric Shunt Damping Systems. *IEEE/ASME Transactions on Mechatronics*, Vol. 7, No. 1, (March 2002), pp. 87-94.

Gao J. X., & Liao W. H. (2005). Vibration analysis of simply supported beams with enhanced self-sensing active constrained layer damping treatments. *Journal of Sound and Vibration*, Vol. 280, (2005), pp. 329-357

Hagood N. W., & von Flotow A. (1991). Damping of structural vibrations with piezoelectric materials and passive electric networks. *Journal of Sound and Vibration*, Vol. 146, No. 2, (1991), pp. 243-268

Kurnik W., Przybyłowicz P.M., & Tylikowski A. (1995). Torsional Vibrations Actively Attenuated by Piezoelectric System, *Proceedings of the 4th German-Polish Workshop on Dynamical Problems in Mechanical Systems*, Berlin, July 1995

Kurnik W. (2004). Damping of Mechanical Vibrations Utilising shunted Piezoelements. *Machine Dynamics Problems*, Vol. 28, No. 4, (2004), pp. 15-26

Moheimani S.O.R., & Fleming A.J. (2006). *Piezoelectric Transducers for Vibration control and Damping*, Springer, ISBN: 1-84628-331-0, London

Pietrzakowski M. (2001). Active damping of beams by piezoelectric system: effects of bonding layer properties. *International Journal of Solids and Structures*, Vol. 38, (2001), pp. 7885-7897, ISSN: 0020-7683

Preumont A. (2006). *Mechatronics: Dynamics of Electromechanical and Piezoelectric Systems*, Springer, ISBN: 1402046952, Dordrecht, The Netherlands

Yoshikawa S., Bogue A., & Degon B. (1998). Commercial Application of Passive and Active Piezoelectric Vibration Control, *Proceedings of the Eleventh IEEE International Symposium on Applications of Ferroelectrics*, ISBN: 0-7803-4959-8, Montreux, Switzerland, August 1998
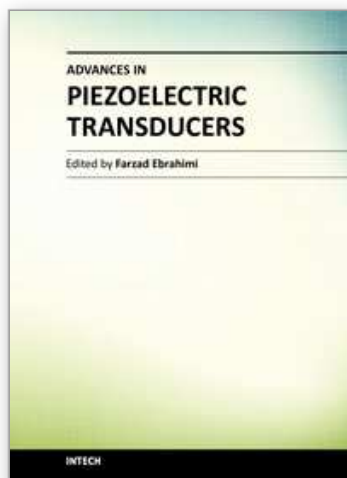
Advances in Piezoelectric Transducers

Edited by Dr. Farzad Ebrahimi

The piezoelectric transducer converts electric signals into mechanical vibrations or vice versa by utilizing the morphological change of a crystal which occurs on voltage application, or conversely by monitoring the voltage generated by a pressure applied on a crystal. This book reports on the state of the art research and development findings on this very broad matter through original and innovative research studies exhibiting various investigation directions. The present book is a result of contributions of experts from international scientific community working in different aspects of piezoelectric transducers. The text is addressed not only to researchers, but also to professional engineers, students and other experts in a variety of disciplines, both academic and industrial seeking to gain a better understanding of what has been done in the field recently, and what kind of open problems are in this area.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

INTECH
open science | open minds