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# Robust Multivariable Control of Ill-Conditioned Plants – A Case Study for High-Purity Distillation

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## 1. Introduction

Distillation is one of the most important unit operations in the chemical industry. Among various distillation operations, control of high-purity column poses difficult control due to a number of characteristics of these systems, including strong directionality, ill-conditioning and strongly nonlinear behavior. At the same time, the potential benefits that can be obtained through tight and economic control of the product compositions are very large. This is due to different reasons including the large energy consumption required by the columns and the market requirements which are becoming stricter and stricter. Because of these obvious features of high-purity distillation, this type of column has been studied extensively.

Control systems for chemical processes are typically designed using an approximate, linear, time-invariant model of the plant. The actual plant may differ from the nominal model due to many sources of uncertainty, such as nonlinearity, the selection of low-order models to represent a plant with inherently high-order dynamics, inaccurate identification of model parameters due to poor measurements or incomplete knowledge, and uncertainty in the manipulative variables. Considering the differences between the actual plant and nominal model, it is necessary to insure that the control system will be stable and meet some predetermined performance criteria when applied to the actual plant. The identification and control of distillation columns have been subjects of frequent study due to the ill-conditioned nature of the distillation process. An ill-conditioned plant is very close to singular, and unless care is taken, very small errors can make the model useless. In distillation, this means that a model may have features that are in conflict with physical knowledge (Luyben, 1987; Jacobsen & Skogestad, 1994; Böling & Häggblom, 1996). In addition, ill-conditioned dynamics of high-purity distillation columns leads to high sensitivity to uncertainties in the manipulated variables (Skogestad & Morari, 1988). This effect causes even small errors in the manipulated variables show significant deterioration of the product quality, a fact which explains why open-loop control of high-purity distillation columns is hardly ever satisfactory. The model of a high-purity distillation process has a steady-state gain matrix with a high condition number. The gain matrix is almost singular and its determinant may be affected by quite small model errors, and if

determinant of the gain matrix of the model and that of the plant have different signs, no controller with integral action exists that can stabilize both the model and plant (Grosdidier et al., 1985).

Many control design techniques have been applied to the high-purity distillation columns (e.g. Georgiou et al., 1988; Skogestad and Lundström, 1990; Srinivas et al., 1995; Christen et al., 1997; Shin et al., 2000; Razzaghi & Shahraki, 2005, 2007; Biswas et al., 2009). Some possible improvements for linear multivariable predictive control of high-purity distillation columns are proposed by Trentacapilli et al. (1997) and a simple way of inserting a local model that contains part of the process nonlinearity into the controller is described also. In addition, a reliable model of the column is generally considered as a prerequisite for the design of efficient two-product control by multivariable methods. Another important aspect of distillation control design is the choice of a good configuration. In fact, poor control performance can result from the improper choice of manipulated/controlled variable pairing (Hurowitz et al., 2003). Some authors have been considered control configuration selection (Shinskey, 1984; Skogestad and Morari, 1987a; Finco et al., 1989; Stichlmair, 1995; Heath et al., 2000; Hurowitz et al., 2003; Luyben, 2005; Hori & Skogestad, 2007; Razzaghi & Shahraki, 2009), but there is no general agreement among these authors in choosing the best control configuration, however, a complete review in this field is performed by Skogestad et al. (1990). The main works for selection of manipulated/controlled variable pairings have focused upon using controllability measures, such as relative gain array (Bristol, 1966) and structured singular value  $\mu$  (Doyle, 1982). The relative gain array (RGA) provides a steady-state measure of coupling in multivariable systems and can be used to evaluate the steady-state coupling of configurations. RGA is still the most commonly used tool for control structure selection for single-loop controllers. Shinskey (1984) used the relative gain array to choose configuration which is applied widely in industry. Several authors such as Skogestad et al. (1990) and Kariwala et al. (2006) have demonstrated practical applications of the RGA that it depends on the plant model only, that it is scaling independent and that all possible configurations can be evaluated base on the a single matrix. The structured singular value (SSV) approach provides necessary and sufficient conditions for robust stability and performance for the situation in which uncertainty occurs simultaneously and independently in various parts of the overall control system (e.g. input and output uncertainty) but the perturbation matrix is still norm-bounded. One of the most difficult steps in analysing the robust stability and performance of any control system is the specification of an estimate of the uncertainty associated with the nominal process model. This is a critical step because an overestimation of the model inaccuracy will lead to extensively poor control performance and an underestimation may lead to instability (McDonald et al., 1988). Several papers discuss ways in which model inaccuracy can be described and methods that can be used for assessing robust stability. The most common multivariable approaches that use singular values (Doyle and Stein, 1981; Arkun et al., 1984) and structured singular values assume that the actual plant can be described by a norm-bounded perturbation matrix in the frequency domain. In chemical process control, nonlinearity is one of the most significant sources of model inaccuracy. We usually have some knowledge about the structure of model inaccuracy due to nonlinearity, however, and this knowledge should be exploited in our robustness studies. In formulating the SSV problem, use of physically-based uncertainty description is important. Simplified models that predict gain and time constant changes as the process is perturbed over the expected operating regime can be used to characterise the uncertainty (McDonald et al., 1988).

The objective in this chapter is to show that acceptable closed-loop performance can be achieved for an ill-conditioned high-purity distillation column by use of the structured singular value  $\mu$ . The distillation column model used in this case study is a high-purity column, referred to as “column at operating point A” by Skogestad and Morari (1988). Table 1 summarizes the steady-state data of the model in detail. The following simplifying assumptions are also made for the column: (1) binary separation, (2) constant relative volatility, and (3) constant molar flows. To include the effect of neglected flow dynamics, we will add uncertainty when designing and analysing controller.

Column data	
Relative volatility	$\alpha = 1.5$
Number of theoretical trays	$N_T = 40$
Feed tray (1 = reboiler)	$N_F = 21$
Feed composition	$z_F = 0.50$
Operating data	
Distillate composition	$y_D = 0.99$
Bottom composition	$x_B = 0.01$
Distillate to feed ratio	$D/F = 0.500$
Reflux to feed ratio	$L/F = 2.706$

Table 1. Steady-state data for distillation column.

2. Process description

A simple two time-constant dynamic model presented by Skogestad and Morari (1988) is chosen as the basis for the controller design. The model is derived assuming the flow and composition dynamics to be decoupled, and then the two separate models for the composition and flow dynamics are simply combined. The nominal model of the column is given by

$$\begin{aligned} dy_D &= \frac{87.8}{1 + 194s} dL + \left( \frac{1.4}{1 + 15s} - \frac{87.8}{1 + 194s} \right) dV, \\ dx_B &= \frac{108.2}{1 + 194s} g_L(s) dL + \left( -\frac{1.4}{1 + 15s} - \frac{108.2}{1 + 194s} \right) dV. \end{aligned} \tag{1}$$

$g_L(s)$  expresses the liquid flow dynamics:

$$g_L(s) = \frac{1}{[1 + (2.46 / n)s]^n} \tag{2}$$

where  $n$  is the number of trays in the column ( $N_T - 1$ ). Fig. 1 shows a schematic of a binary distillation column that uses reflux and vapor boilup as manipulated inputs for the control of top and bottom compositions, respectively. This is denoted as the *LV*-configuration (structure). This structure is commonly used in industry for one-point composition control.

However, severe interactions often make two-point control difficult with this configuration. Although the closed-loop system may be extremely sensitive to input uncertainty when the *LV*-configuration is used, while it is shown that it is possible to obtain good control behavior (i.e. good performance) with the *LV*-configuration when model uncertainty and possible changes in the operating point are included (Skogestad and Lundström, 1990). The simultaneous control of overhead and bottoms composition in a binary distillation column using reflux and steam flow as the manipulated variables often proves to be particularly difficult because of the coupling inherent in the process. The result of this coupling, which cause the two control loops to interact, leads to a deterioration in the control performance of both composition control loops compared to their performance if the objective were control of only one composition. Since high-purity distillation columns can be very sensitive to uncertainties in the manipulated variables, it is important for successful implementation that a controller guarantees its performance in the presence of uncertainties. This particular design task is frequently solved by modeling a multiplicative uncertainty for a nominal plant model and subsequently calculating the controller using  $\mu$ -synthesis (Doyle, 1982).

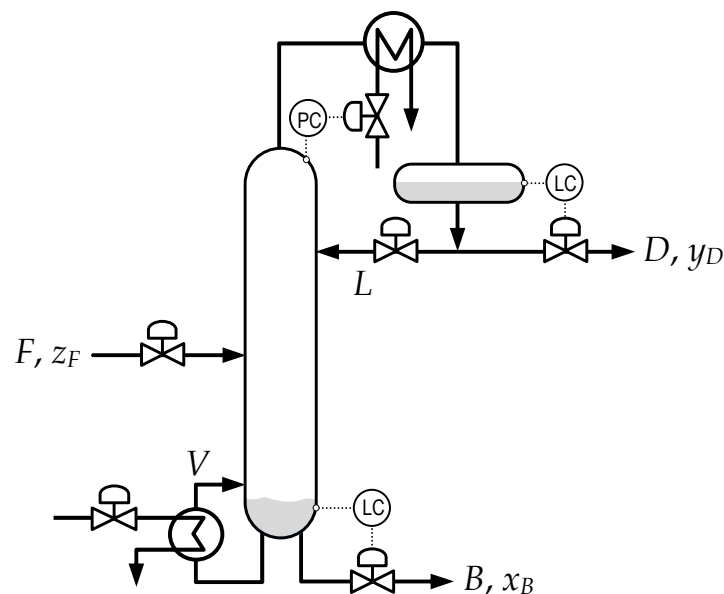


Fig. 1. Schematic of a binary distillation column using the *LV*-configuration.  $L$  and  $V$ : manipulated inputs;  $x_B$  and  $y_D$ : controlled outputs.

## 2.1 General control problem formulation

Fig. 2 shows general control problem formulation, where  $G$  is the generalized plant and  $C$  is the generalized controller. The controller design problem is divided into the analysis and synthesis phases. The controller  $C$  is synthesized such that some measure, in fact a norm, of the transfer function from  $w$  to  $z$  is minimized, e.g. the  $H_\infty$ -norm. Then the controller design problem is to find a controller  $C$  (that generates a signal  $u$  considering the information from  $v$  to mitigate the effects of  $w$  on  $z$ ) minimizing the closed-loop norm from  $w$  to  $z$ . For the analysis phase, the scheme in Fig. 2 is to be modified to group the generalized plant  $G$  and the resulting synthesized controller  $C$  in order to test the closed-loop performance achieved with  $C$ . To get meaningful controller synthesis problems, weights on the exogenous inputs  $w$  and outputs  $z$  are incorporated. The weighting matrices are usually frequency dependent

and typically selected such that the weighted signals are of magnitude one, i.e. the norm from  $w$  to  $z$  should be less than one.

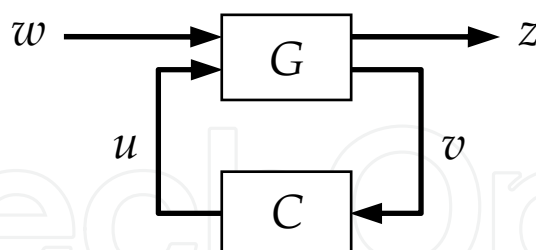


Fig. 2. General control problem formulation with no model uncertainty.

Once the stabilizing controller  $C$  is synthesized, it rests to analyze the closed-loop performance that it provides. In this phase, the controller for the configuration in Fig. 2 is incorporated into the generalized plant  $G$  to form the system  $N$ , as it is shown in Fig. 3. The expression for  $N$  is given by

$$N = G_{11} + G_{12}C(I - G_{22})^{-1}G_{21} \equiv F_l(G, C) \quad (3)$$

where  $F_l(G, C)$  denotes the lower Linear Fractional Transformation (LFT) of  $G$  and  $C$ . In order to obtain a good design for  $C$ , a precise knowledge of the plant is required. The dynamics of interest are modeled but this model may be inaccurate and may not reflect the changes suffered by the plant with time. To deal with this problem, the concept of model uncertainty comes out. The plant  $G$  is assumed to be unknown but belonging to a class of models,  $\mathbf{P}$ , built around a nominal model  $G_o$ . The set of models  $\mathbf{P}$  is characterized by a matrix  $\Delta$ , which can be either a full matrix or a block diagonal matrix that includes all possible perturbations representing uncertainty to the system. The general control configuration in Fig. 2 may be extended to include model uncertainty as it is shown in Fig. 4.

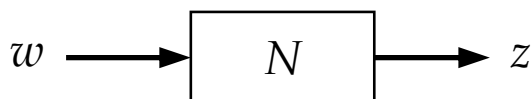


Fig. 3. General block diagram for analysis with no model uncertainty.

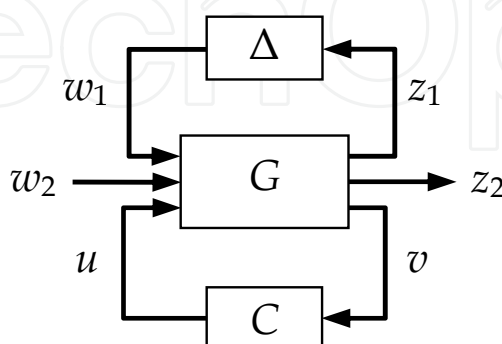


Fig. 4. General control problem formulation including model uncertainty.

The block diagram in Fig. 4 is used to synthesize the controller  $C$ . To transform it for analysis, the lower loop around  $G$  is closed by the controller  $C$  and it is incorporated into the



generalized plant  $G$  to form the system  $N$  as it is shown in Fig. 5. The same lower LFT is obtained as in Eq. (3) where no uncertainty was considered.

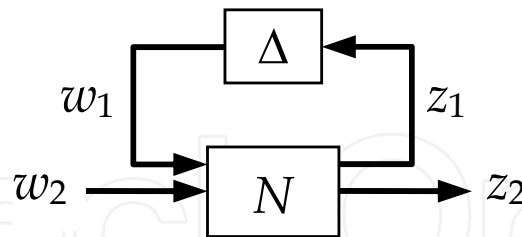


Fig. 5. General block diagram for analysis including model uncertainty.

To evaluate the relation between  $w = [w_1 \ w_2]^T$  and  $z = [z_1 \ z_2]^T$  for a given controller  $C$  in the uncertain system, the upper loop around  $N$  is closed with the perturbation matrix  $\Delta$ . This results in the following upper LFT:

$$F_u(N, \Delta) \equiv N_{22} + N_{21} \Delta (\mathbf{I} - N_{11})^{-1} N_{12} . \quad (4)$$

To represent any control problem with uncertainty by the general control configuration in Fig. 4, it is necessary to represent each source of uncertainty by a single perturbation block  $\Delta_i$ , normalized such that  $\bar{\sigma}(\Delta_i) \leq 1$ . The individual uncertainties  $\Delta_i$  are combined into one large block diagonal matrix  $\Delta$ ,

$$\Delta = \text{diag}\{\Delta_1, \Delta_2, \dots, \Delta_m\} , \quad (5)$$

satisfying

$$\bar{\sigma}(\Delta) \leq 1 . \quad (6)$$

Structured uncertainty representation considers the individual uncertainty present on each input channel and combines them into one large diagonal block. This representation avoids the norm-physical coupling at the input of the plant that appears with the full perturbation matrix  $\Delta$  in an unstructured uncertainty description. Consequently, the resulting set of plants is not so large as with an unstructured uncertainty description and the resulting robustness analysis is not so conservative (Balas et al., 1993).

## 2.2 Robust performance and robust stability

For obtaining good set point tracking, it is obvious that some performance specifications must be satisfied in spite of unmeasured disturbances and model-plant mismatch, i.e. uncertainty. The performance specification should be satisfied for the worst-case combination of disturbances and model-plant mismatch (robust performance). In order to achieve robust performance, some specifications have to be satisfied. The following terminologies are used:

1. *Nominal Stability*—The closed-loop system has Nominal Stability (NS) if the controller  $C$  internally stabilizes the nominal model  $G_o$ , i.e. the four transfer matrices  $N_{11}$ ,  $N_{12}$ ,  $N_{21}$  and  $N_{22}$  in the closed-loop transfer matrix  $N$  are stable.
2. *Nominal Performance*—The closed-loop system has Nominal Performance (NP) if the performance objectives are satisfied for the nominal model  $G_o$ , i.e.  $\|N_{22}\|_{\infty} < 1$ .

3. *Robust Stability*—The closed-loop system has Robust Stability (RS) if the controller  $C$  internally stabilizes every plant  $G \in \mathbf{P}$ , i.e. in Fig. 5,  $F_u(N, \Delta)$  is stable and  $\|\Delta\|_\infty \leq 1$ .
4. *Robust Performance*—The closed-loop feedback system has Robust Performance (RP) if the performance objectives are satisfied for  $G \in \mathbf{P}$ , i.e. in Fig. 5,  $\|F_u(N, \Delta)\|_\infty < 1$  and  $\|\Delta\|_\infty \leq 1$ .

The structured singular value is used as a robust performance index. To use this index one must define performance using the  $H_\infty$  framework. The  $H_\infty$ -norm of a transfer function  $G(s)$  is the peak value of the maximum singular value over all frequencies

$$\|G(s)\|_\infty \equiv \sup_\omega \bar{\sigma}(G(j\omega)). \quad (7)$$

Uncertainties are modeled by the perturbations and uncertainty weights included in  $G$ . These weights are chosen such that  $\|\Delta\|_\infty \leq 1$  generates the family of all possible plants to be considered (Fig. 4).  $\Delta$  may contain both real and complex perturbations, but in this case study only complex perturbations are used. The performance is specified by weights in  $G$  which normalized  $w_2$  and  $z_2$  such that a closed-loop  $H_\infty$ -norm from  $w_2$  to  $z_2$  of less than one (for worst-case  $\Delta$ ) means that the control objectives are achieved. Fig. 6 is used for robustness analysis where  $N$  is a function of  $G$  and  $C$ , and  $\Delta_p$  ( $\|\Delta_p\|_\infty \leq 1$ ) is a fictitious “performance perturbation” connecting  $z_2$  to  $w_2$ .

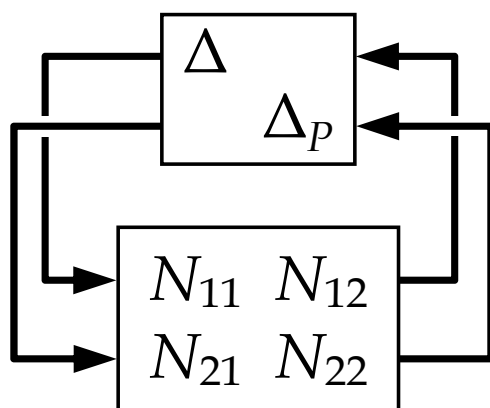


Fig. 6. General block diagram for robustness analysis.

Provided that the closed-loop system is nominally stable, the condition for robust performance (RP) is

$$\text{RP} \Leftrightarrow \mu_{\text{RP}} = \sup_\omega \mu_\Delta(N(j\omega)) < 1, \quad (8)$$

where  $\Delta = \text{diag}\{\Delta, \Delta_p\}$ .  $\mu$  is computed frequency-by-frequency through upper and lower bounds. Here we only consider the upper bound which is derived by the computation of non-negative scaling matrices  $D_l$  and  $D_r$  defined within a set  $\mathbf{D}$  that commutes with the structure  $\Delta$ :

$$\mu_\Delta(N) \leq \inf_{D \in \mathbf{D}} \bar{\sigma}(D_l N D_r^{-1}), \quad (9)$$



where  $\mathbf{D} = \{D \mid D\Delta = \Delta D\}$ . A detailed discussion on the specification of such a set  $\mathbf{D}$  of scaling matrices can be found in Packard and Doyle (1993).

### 2.3 Design procedure

The design procedure of a control system usually involves a mathematical model of the dynamic process, the plant model or nominal model. Consequently, many aspects of the real plant behavior cannot be captured in an accurate way with the plant model leading to uncertainties. Such plant-model mismatching should be characterized by means of disturbances signals and/or plant parameter variations, often characterized by probabilistic models, or unmodelled dynamics, commonly characterized in the frequency domain.

The modern approach to characterizing closed-loop performance objectives is to measure the size of certain closed-loop transfer function matrices using various matrix norms. Matrix norms provide a measure of how large output signals can get for certain classes of input signals. Optimizing these types of performance objectives, over the set of stabilizing controllers is the main thrust of recent optimal control theory, such as  $L_1$ ,  $H_2$ ,  $H_\infty$  and optimal control (Balas et al., 1993). Usually, high performance specifications are given in terms of the plant model. For this reason, model uncertainties characterization should be incorporated to the design procedure in order to provide a reliable control system capable to deal with the real process and to assure the fulfillment of the performance requirements. The term *robustness* is used to denote the ability of a control system to cope with the uncertain scheme. It is well known that there is an intrinsic conflict between performance and robustness in the standard feedback framework (Doyle and Stein, 1981; Chen, 1995). The system response to commands is an open-loop property while robustness properties are associated with the feedback. Therefore, one must make a trade-off between achievable performance and robustness. In this way, a high performance controller designed for a nominal model may have very little robustness against the model uncertainties and the external disturbances. For this reason, worst-case robust control design techniques such as  $\mu$ -synthesis, have gained popularity in the last thirty years.

### 3. Modeling of the uncertain system

Analyzing the effect of uncertain models on achievable closed-loop performance and designing controller to provide optimal worst-case performance in the face of the plant uncertainty are the main features that must be considered in robust control of an uncertain system. Skogestad et al. (1988) recommended a general guideline for modeling of uncertain systems. According to this, three types of uncertainty can be identified:

1. Uncertainty of the manipulated variables which is referred to input uncertainty.
2. Uncertainty because of the process nonlinearity, and
3. Unmodelled high-frequency dynamics and uncertainty of the measured variables which is referred to output uncertainty.

Fig. 7(a) shows a block diagram of a distillation column with related inputs ( $u$ ,  $d$ ) and outputs ( $y$ ,  $y_m$ ). In Fig. 7(b), we have added two additional blocks to Fig. 7(a). One is the controller  $C$ , which computes the appropriate input  $u$  based on the information about the process  $y_m$ . The other block,  $\Delta$ , represents the model uncertainty.  $\hat{G}$  and  $G$  are models only, and the actual plant is different depending on  $\Delta$ . Based on the measurements  $y_m$ , the

objective of the controller  $C$  is to generate inputs  $u$  that keep the outputs  $y$  as close as possible to their set points in spite of disturbances  $d$  and model uncertainty  $\Delta$ . The controller  $C$  is often non-square, as there are usually more measurements than manipulated variables. For the design of the controller  $C$ , information about the expected model uncertainty should be taken into account. Usually, there are two main ways for adding uncertainty to a constructed model: additive and multiplicative uncertainty. Fig. 7(c) represents additive uncertainty. In this case, the perturbed plant gain  $G_p$  will be  $G + \Delta$  where  $\Delta$  is unstructured uncertainty. Fig. 7(d) represents multiplicative uncertainty where the perturbed plant is equal to  $G(\mathbf{I} + \Delta)$ .

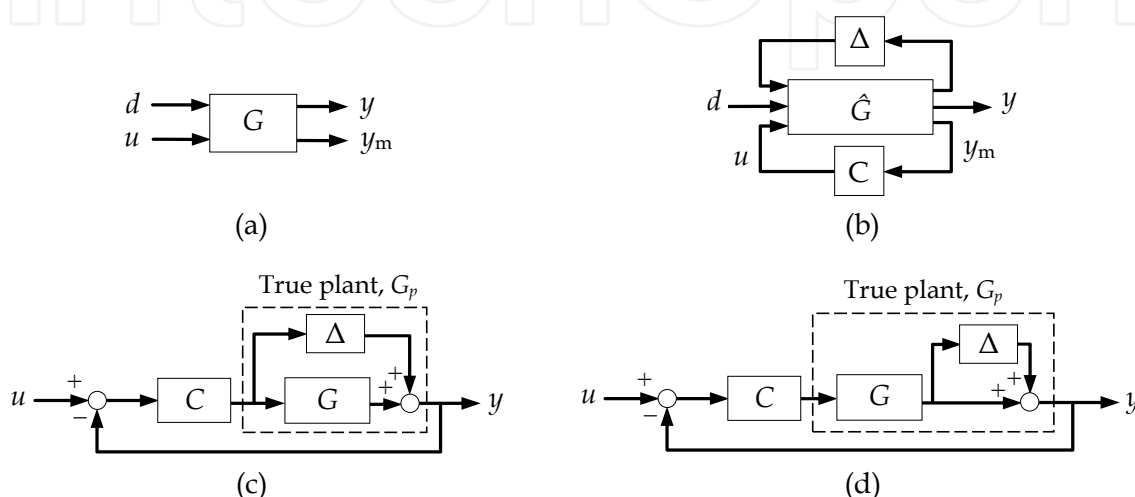


Fig. 7. (a) Schematic representation of distillation column; (b) general structure for studying any linear control problem; (c) additive unstructured uncertainty,  $G_p = G + \Delta$ ; (d) multiplicative unstructured uncertainty,  $G_p = G(\mathbf{I} + \Delta)$ .

Here we will consider only input and output uncertainties:

*Input uncertainty* – Input uncertainty always occurs in practice and generally limits the achievable closed-loop performance (Skogestad et al., 1988). Ill-conditioned plants can be very sensitive to errors in the manipulated variables. The bounds for the relative errors of the column inputs  $u$  are modeled in the frequency domain by a multiplicative uncertainty with two frequency-dependent error bounds  $w_u$ . These two bounds are combined in the diagonal matrix  $W_u = w_u \mathbf{I}$ . In this case

$$\tilde{u}(j\omega) = [\mathbf{I} + \Delta_u(j\omega)W_u(j\omega)]u(j\omega) \quad \text{with} \quad \|\Delta_u(j\omega)\|_\infty \leq 1. \quad (10)$$

The value of the bound  $W_u$  is almost very small for low frequencies (we know the model very well there) and increases substantially as we go to high frequencies where parasitic parameters come into play and unmodelled structural flexibility is common. If all flow measurements are carefully calibrated, an error bound of 10% for the low frequency range is reasonable (Christen et al., 1997). This error bound is not common among the researchers (e.g. Skogestad and Lundström, 1990, used an error bound of 20% at steady state). Higher errors must be assumed in the higher frequency range. Because of uncertain or neglected high-frequency dynamics or time delays, the input error exceeds 100%. The following weight is used as input uncertainty weight

$$w_u(s) = 0.1 \frac{1+10s}{1+s}. \quad (11)$$

The weight is shown graphically as a function of frequency in Fig. 8.

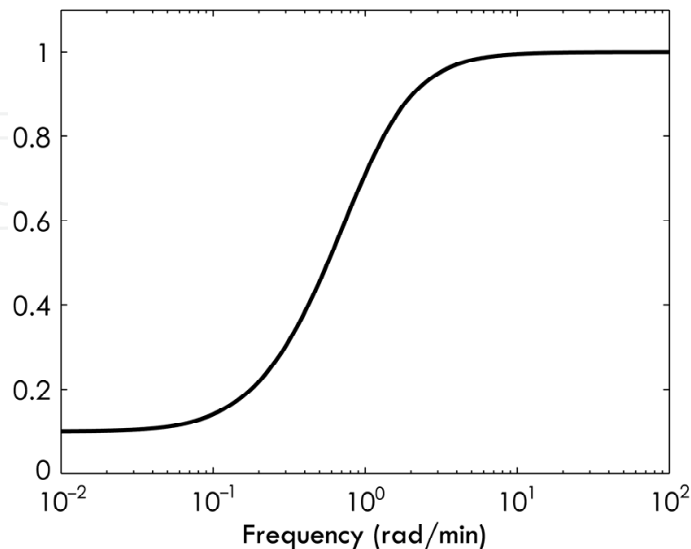


Fig. 8. Input uncertainty weight  $|w_u(j\omega)|$  as a function of frequency.

*Output uncertainty* – Due to the nonlinear vapor/liquid equilibrium, the gains of the individual transfer functions between the two manipulated inputs and controlled outputs may change in opposite directions (gain directionality). This behavior can be described with independent multiplicative uncertainties for the two outputs of the model and a diagonal weighting matrix  $W_y = w_y \mathbf{I}$ . In mathematical form we can write

$$\tilde{y}(j\omega) = [\mathbf{I} + \Delta_y(j\omega)W_y(j\omega)] y(j\omega) \quad \text{with} \quad \|\Delta_y(j\omega)\|_\infty \leq 1. \quad (12)$$

For the low-frequency range, an uncertainty of 10% is assumed for the description of uncertainties in the measured outputs. The uncertainty weight is

$$w_y(s) = 0.1 \frac{1+180s}{1+2.5s}, \quad (13)$$

which has large gains in the high-frequency range that takes the effect of unmodelled dynamics into account.

*Performance* – The performance weight used in this study is the same in Skogestad and Morari (1988). The weight is defined as

$$w_p(s) = 0.5 \frac{1+10s}{10s}. \quad (14)$$

### 3.1 Controller

Skogestad and Lundström (1990) proposed two different approaches to tune controllers. The first approach is to fix the performance specification and minimize  $\mu_{RP}$  by adjusting the

controller tunings. The performance requirement is satisfied if  $\mu_{RP}$  is less than one, and lower  $\mu_{RP}$  values represent a better design. The second approach is to fix the uncertainty and find what performance can be achieved. In this approach, we adjust the time constant in the performance weight to make the optimal  $\mu_{RP}$  values equal to one. The latter approach has two disadvantages: (1) it introduces an outer loop in the  $\mu$  calculations, and (2) it may be impossible to achieve  $\mu_{RP}$  equal to one by adjusting the time constant in the performance weight. Here the first approach is used for tuning the controller because of the mentioned disadvantages of the second approach.

A diagonal PID controller based on internal model control (IMC) (Rivera et al., 1986) is used to investigate the process. Optimal setting for single-loop PID controller is found by minimizing  $\mu_{RP}$ . Furthermore, a  $\mu$ -optimal controller is designed since it gives a good indication of the best possible performance of a linear controller.

### 3.2 Analysis of controller

Comparison of controller is based mainly on computing  $\mu$  for robust performance. The main advantage of using the  $\mu$ -analysis is that it provides a well-defined basis for comparison.  $\mu$ -analysis is a worst-case analysis. It minimizes the  $H_\infty$ -norm with respect to the structured uncertainty matrix  $\Delta$ . A worst-case analysis is particularly useful for ill-conditioned systems in the cross-over frequency range (Gjøsæter and Foss, 1997). This is due to the fact that such systems may provide large difference between nominal and robust performance.

The value of  $\mu_{RP}$  is indicative of the worst-case response. If  $\mu_{RP} > 1$ , then the “worst-case” does not satisfy our performance objective, and if  $\mu_{RP} < 1$  then the “worst-case” is better than required by our performance objective. Similarly, if  $\mu_{NP} < 1$  then the performance objective is satisfied for the nominal case. However, this may not mean very much if the system is sensitive to uncertainty and  $\mu_{RP}$  is significantly larger than one. It is shown that this is the case, for example, if an inverse-based controller is used for the distillation column (Skogestad and Morari, 1988). Controller was obtained by minimizing  $\sup_{\omega} \mu_{RP}$  for the model using the input and output uncertainties and performance weight. The plots for RP for the  $\mu$ -optimal controller are of particular interest since they indicate the best achievable performance for the plant.  $\mu$  provides a much easier way of comparing and analyzing the effect of various combinations of controllers, uncertainty and disturbances than the traditional simulation approach. One of the main advantages with the  $\mu$ -analysis as opposed to simulations is that one does not have to search for the worst-case, i.e.  $\mu$  finds it automatically (Skogestad and Lundström, 1990).

### 3.3 Synthesis of controller

The structured singular value provides a systematic way to test for both robust stability and robust performance with a given controller  $C$ . In addition to this analysis tool, the structured singular value can be used to synthesize the controller  $C$ . The robust performance condition implies robust stability, since

$$\sup_{\omega} \mu_{\Delta}(N) \geq \sup_{\omega} \mu_{\Delta}(G). \quad (15)$$

Therefore, a controller designed to guarantee robust performance will also guarantee robust stability. Provided that the interconnection matrix  $N$  is a function of the controller  $C$ , the  $\mu$ -optimal controller can be found by

$$\text{minimize } \{\sup_{\omega} \mu_{\Delta}(N)\} \quad (16)$$

At the present time, there is no direct method to find the controller  $C$  by minimizing (16), however, combination of  $\mu$ -analysis and  $H_{\infty}$ -synthesis which is called  $\mu$ -synthesis or DK-iteration (Zhou et al., 1996) is a special method that attempts to minimize the upper bound of  $\mu$ . Thus, the objective function (16) is transformed into

$$\min_C \left( \inf_{D_l, D_r \in \mathbf{D}} \sup_{\omega} \bar{\sigma}(D_l N D_r^{-1}) \right) \quad (17)$$

The DK-iteration approach involves to alternatively minimize

$$\sup_{\omega} \bar{\sigma}(D_l N D_r^{-1}) \quad (18)$$

for either  $C$  or  $D_l$  and  $D_r$  while holding the other constant. For fixed  $D_l$  and  $D_r$ , the controller is solved via  $H_{\infty}$  optimization; for fixed  $C$ , a convex optimization problem is solved at each frequency. The magnitude of each element of  $D_l(j\omega)$  and  $D_r(j\omega)$  is fitted with a stable and minimum phase transfer function and wrapped back into the nominal interconnection structure. The procedure is carried out until  $\sup_{\omega} \bar{\sigma}(D_l N D_r^{-1}) < 1$ . Although convergence in each step is assured, joint convergence is not guaranteed. However, DK-iteration works well in most cases (Balas et al., 1993; Packard and Doyle, 1993). The optimal solutions in each step are of supreme importance to success with the DK-iteration. Moreover, when  $C$  is fixed, the fitting procedure plays an important role in the overall approach. Low order transfer function fits are preferable since the order of the  $H_{\infty}$  problem in the following step is reduced yielding controllers of low order dimension. Nevertheless, the method is characterized by giving controllers of very high order that must be reduced applying model reduction techniques (Glover, 1984).

### 3.4 Simulation

Simulations are carried out with the nonlinear model of the column and using single-loop controller, which generally is insensitive to steady-state input errors (Skogestad and Morari, 1988). In addition, input and output uncertainties are included to get a realistic evaluation of the controller. Simulations are for both cases with and without uncertainty.

## 4. Model analysis

### 4.1 RGA-analysis of the model

Let  $\times$  denote element-by-element multiplication. The RGA of the matrix  $G$  (Bristol, 1966) is defined as

$$\Lambda(G) = G \times (G^{-1})^T. \quad (19)$$

For 2×2 systems

$$\text{RGA} = \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix} = \begin{pmatrix} \lambda_{11} & 1 - \lambda_{11} \\ 1 - \lambda_{11} & \lambda_{11} \end{pmatrix} \quad \text{and} \quad \lambda_{11} = \frac{1}{1 - (g_{12}g_{21}/g_{11}g_{22})}, \quad (20)$$

where  $g_{ij}$ s are open-loop gain from the  $j$ th input to the  $i$ th output of the process. The RGA has been considered as an important MIMO system information for feedback control. Controllers with large RGA elements should generally be avoided, because otherwise the closed-loop system is very sensitive to input uncertainty (Skogestad and Morari, 1987b). Fig. 9 shows the magnitude of the diagonal element of the RGA ( $\lambda_{11}$ ). As seen in the figure, the plant is ill-conditioned at low frequencies, while at higher frequencies, the value of the RGA-element drops. This says that only based on the RGA plot, making a decision on the ill-conditionedness of the control problem may be misleading. On the other hand, the bandwidth area is located in a frequency range where the RGA elements are small or at lower frequencies where the RGA elements are large.

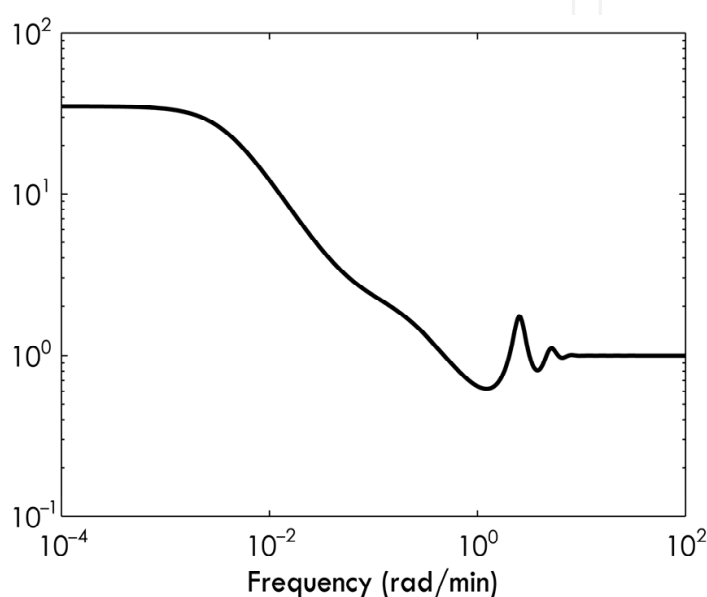


Fig. 9. Plot of  $|\lambda_{11}|$  as a function of frequency.

#### 4.2 Ill-conditionedness and process gain directionality

The common definition of an ill-conditioned plant is that it has a model with a large condition number ( $\gamma$ ). The condition number is defined as the ratio between the largest and smallest singular values ( $\bar{\sigma} / \underline{\sigma}$ ) of a process model. However, the condition number depends on the scaling of the process model. This problem arises from the scaling dependency of the Singular Value Decomposition (SVD). To eliminate the effect of scaling, the minimized condition number ( $\gamma_{\min}$ ) is defined as the smallest possible condition number that can be achieved by varying the scaling. Close relationship between  $\gamma_{\min}$  and RGA is proposed by Grosdidier et al. (1985). For  $2 \times 2$  systems

$$\gamma_{\min}(G) = \|\Lambda(G)\|_1 + \sqrt{\|\Lambda(G)\|_1^2 - 1}, \quad (21)$$

where the 1-norm of the RGA is defined as

$$\|\Lambda\|_1 = \max_j \sum_{i=1}^m |\lambda_{ij}|. \quad (22)$$



According to the above relationship, a  $2 \times 2$  system with small RGA elements always has a small  $\gamma_{\min}$ . In particular, if  $0 \leq \lambda_{11} \leq 1$  the minimized condition number is always equal to one. A process model with a large span in the possible gain of the model is said to show high directionality and a process model with the smallest singular value equal to the largest singular value is said to show no directionality. Waller et al. (1994) suggest redefined definition of process directionality. The definition divides the concept of process directionality into two parts. The minimized condition number is connected to stability aspects, whereas the condition number of a process model scaled according to the weight of the variables is connected to performance aspects. Fig. 10 shows the largest and smallest singular values and condition number of the process model as a function of frequency.

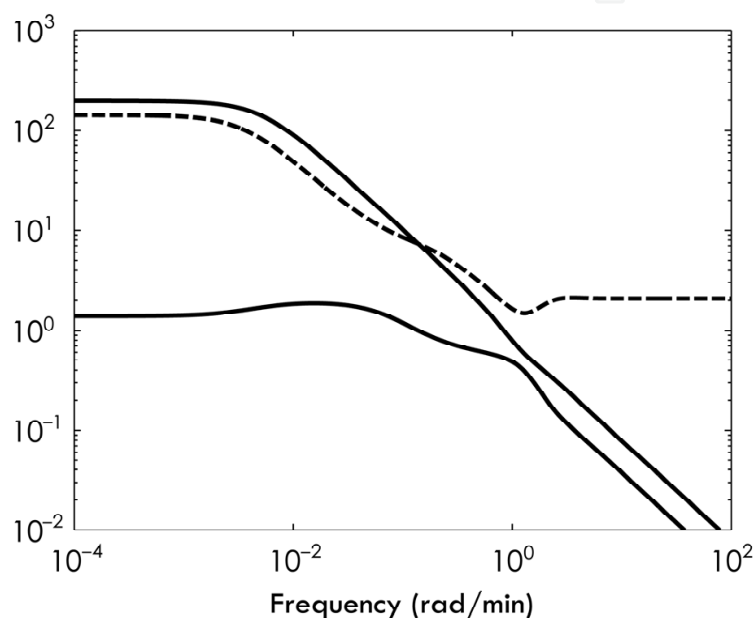


Fig. 10. Singular values (—) and condition number (----) of the distillation column.

The condition number of the process is about 10 times lower at high frequencies than at low frequencies (steady state). Fig. 11(a) represents the values of  $\gamma$  and  $\gamma_{\min}$  as a function of frequency. Values of  $\gamma$  and  $\gamma_{\min}$  match each other from low to intermediate frequencies, but  $\gamma_{\min}$  approaches one at high frequencies. For  $2 \times 2$  systems (Grosdidier et al., 1985):

$$\|\Lambda\|_1 - \frac{1}{\gamma_{\min}(G)} \leq \gamma_{\min}(G) \leq \|\Lambda\|_1. \quad (23)$$

Consequently, for  $2 \times 2$  systems the difference between these quantities is at most one and  $\|\Lambda\|_1$  approaches  $\gamma_{\min}$  as  $\gamma_{\min} \rightarrow \infty$ . Since  $\|\Lambda\|_1$  is much easier to compute than  $\gamma_{\min}$ , it is the preferred quantity to use. In Fig. 11(b),  $\gamma_{\min}$  and  $\|\Lambda\|_1$  are plotted as a function of frequency. The value of  $\gamma_{\min}$  at low frequencies is approximately twice  $\|\Lambda\|_1$ . At high frequencies, both  $\gamma_{\min}$  and  $\|\Lambda\|_1$  approach one (after  $\omega = 20$  rad/min). This is in agreement with the result obtained from  $\lambda_{11}$ -vs-frequency plot (Fig. 9). Since  $\gamma_{\min}$  is independent of scaling, therefore it is better to use  $\gamma_{\min}$  instead of  $\gamma$ , which is scale dependent.

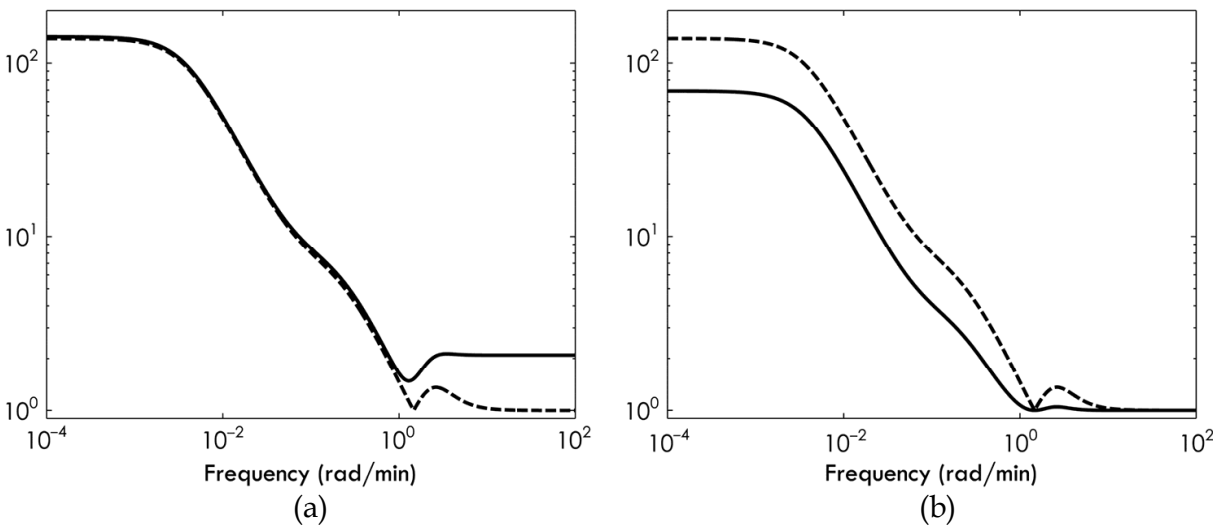


Fig. 11. (a) Plots of  $\gamma$  and  $\gamma_{\min}$  as a function of frequency (—  $\gamma$  and ----  $\gamma_{\min}$ ); (b) plots of  $\|\Lambda\|_1$  and  $\gamma_{\min}$  as a function of frequency (—  $\|\Lambda\|_1$  and ----  $\gamma_{\min}$ ).

4.3 Synthesis of the controller

The plots of the singular values of the sensitivity functions  $S = (\mathbf{I} + GC)^{-1}$  demonstrate good disturbance rejection properties, which indicate the closed-loop system is insensitive to uncertainties in inputs and outputs (Fig. 12(a)). The tracking properties of this controller are also adequate, which is illustrated by plots of the complementary sensitivity function,  $T = \mathbf{I} - S$  (Fig. 12(b)). Up to the mid-frequency range, the singular values are close to one and the maximum of the upper singular values is slightly greater than one.

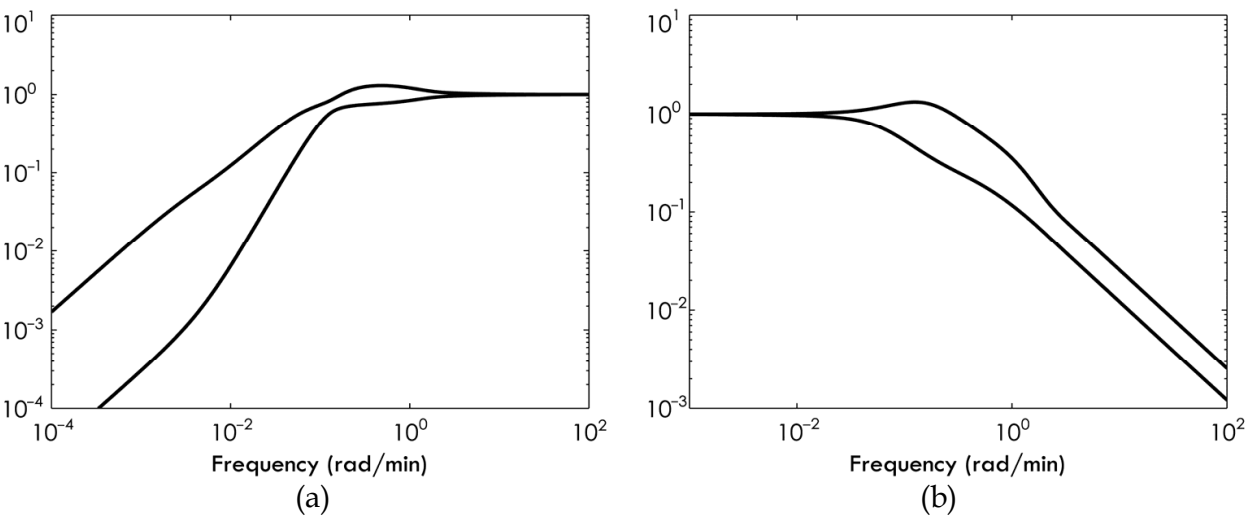


Fig. 12. Singular values of the closed-loop system. (a) Sensitivity function; (b) complementary sensitivity function.

4.4 PID-tuning of the controller

Table 2 summarizes the PID controller setting that is used for the column. Fig. 13 shows  $\mu$ -plots of the controller. From a maximum peak-value point of view, it is seen that both robust

and nominal performance plots are less than one which satisfy the criterion. The plots approach 0.5 as frequency approaches infinity.

Type of controller	$k$	$\tau_I$ (min)	$\tau_D$ (min)
<i>PID Controller</i>			
Top composition control loop	0.37	5.16	0.58
Bottom composition control loop	0.20	3.70	1.18
<i><math>\mu</math>-Optimal Controller</i>			
Top composition control loop	0.26	3.43	1.33
Bottom composition control loop	0.31	4.71	0.67

Table 2. Tuning parameters for PID and  $\mu$ -optimal controllers.

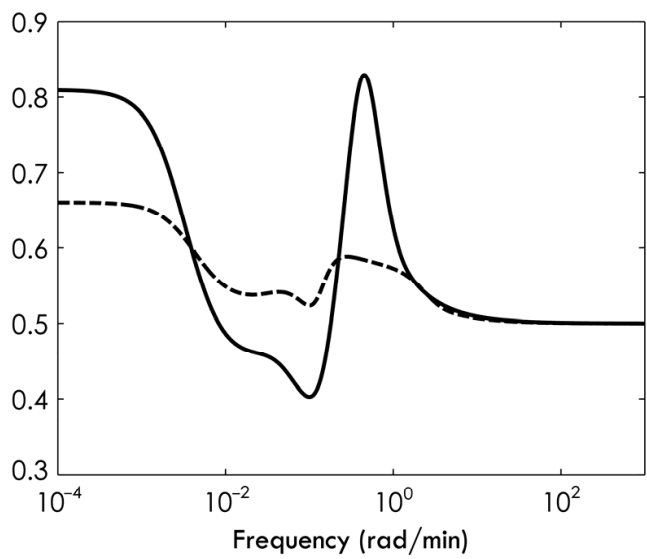


Fig. 13.  $\mu$  plots for PID controller: — robust performance; ---- nominal performance.

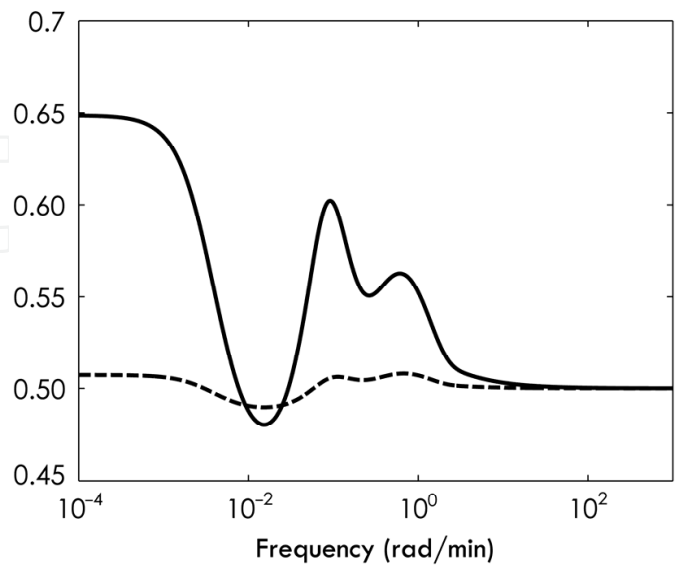


Fig. 14.  $\mu$ -plots for the  $\mu$ -optimal controller: — robust performance; ---- nominal performance.

4.5 Comparison with  $\mu$ -optimal controller

Nominal and robust performance plots of the  $\mu$ -optimal controller is shown in Fig. 14. Comparison of nominal performance of the controllers shows that for the  $\mu$ -optimal controller, the plot is nearly flat over a large frequency range which indicates that an optimal controller is achieved. Comparing robust performance of the controllers indicates that obtaining robust performance with the  $LV$ -configuration is also possible. This is also in agreement with the results presented by Skogestad and Lundström (1990).

5. Simulations

Simulations of a set-point change in  $y_D$  using the PID- and  $\mu$ -optimal controllers are shown in Figs. 15 and 16, respectively. As it is seen, the introduced uncertainties do not seriously affect the performance of the  $\mu$ -optimal controller, while for the PID-controller, the effect of uncertainties is more rather the  $\mu$ -optimal controller. It should be noted that the reference signal is filtered by a prefilter with a time constant of 5 min. Fig. 16 also shows that the PID controller has a slow return to steady state. This is due to the high  $\mu_{NP}$  value at lower frequencies compared with the  $\mu$ -optimal controller (Figs. 13 and 14). In Table 3, numerical values of  $\mu$  for nominal and robust performance are presented.

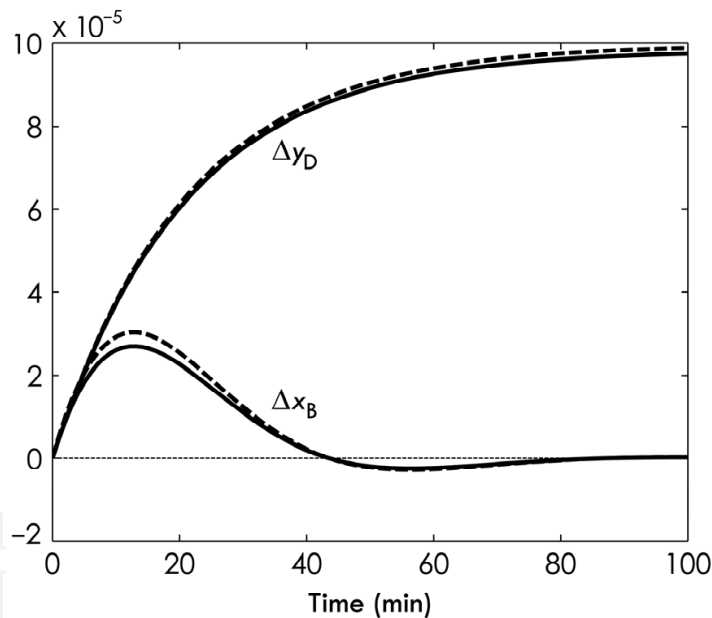


Fig. 15. Closed-loop response to small set-point change in  $y_D$  ( $\mu$ -optimal controller): — no uncertainty; ---- 10% uncertainty on input and output.

Controller	Nominal Performance	Robust Performance
PID	0.661	0.830
$\mu$ -optimal (both input and output uncertainties)	0.506	0.648
$\mu$ -optimal (only input uncertainty)	0.611	0.721

Table 3.  $\mu$  values of the controllers.

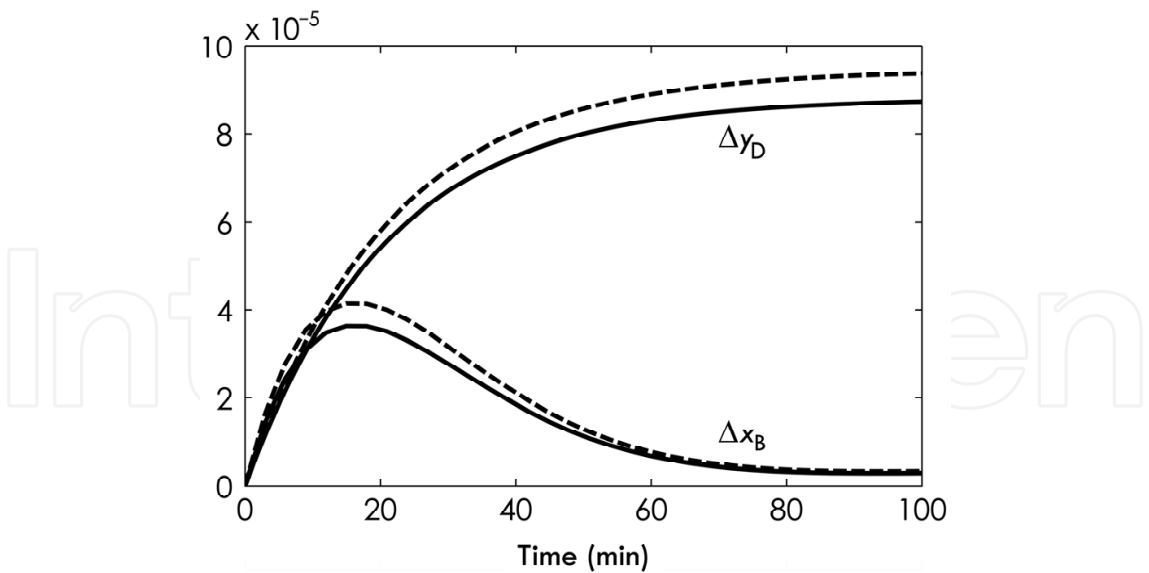


Fig. 16. Closed-loop response to small set-point change in  $y_D$  (PID controller): — no uncertainty; ---- 10% uncertainty on input and output.

Fig. 17 shows the closed-loop response of the  $\mu$ -optimal controller to a 20% increase in feed flow rate. In Fig. 18, the closed-loop response for both controllers is shown simultaneously. As the figure shows, the PID controller needs considerably more times to reach steady state than the  $\mu$ -optimal controller (see next page for the figures).

**5.1 Effect of output uncertainty**

Fig. 19 shows the effect of output uncertainty on closed-loop response of the  $\mu$ -optimal controller. For the case that both input and output uncertainties are considered, the response is faster than for the case that only input uncertainty is considered, however, this difference is not so large. The reason for this again returns to the  $\mu_{NP}$  values at low frequencies. The  $\mu$ -values of nominal performance for the case including both input and output uncertainties is close to the case where only input uncertainty included (Table 3)

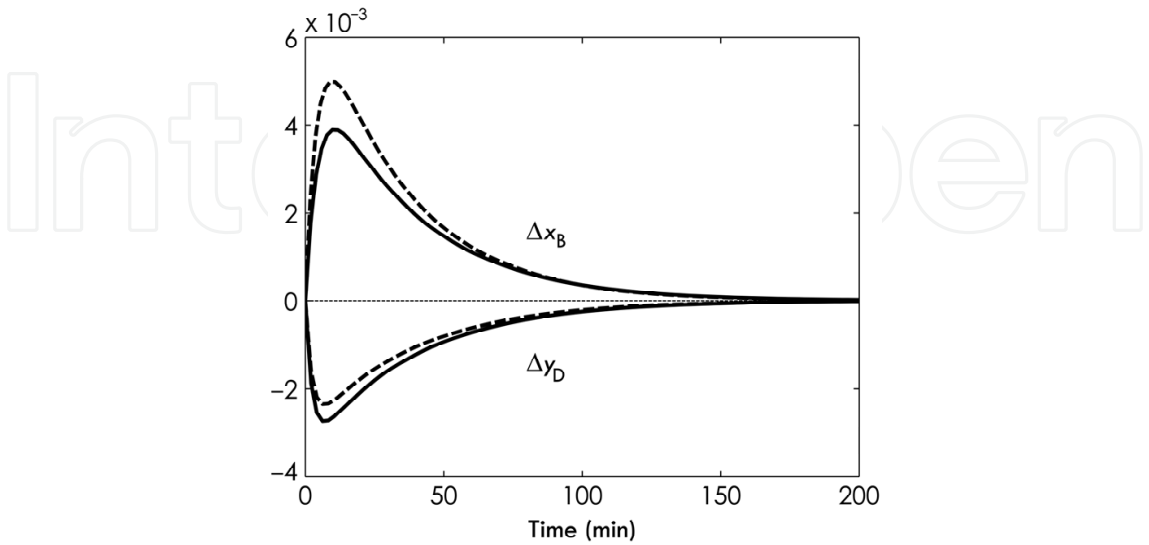


Fig. 17. Closed-loop response to a 20% increase in feed flow rate ( $\mu$ -optimal controller): — no uncertainty; ---- 10% uncertainty on inputs and outputs.

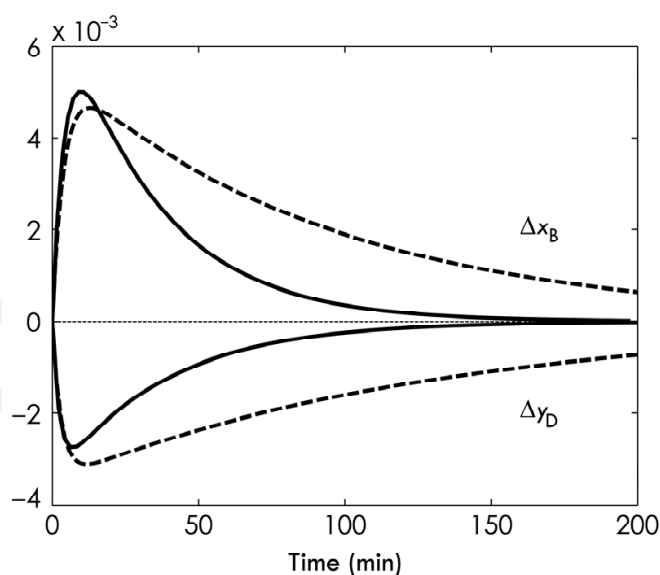


Fig. 18. Closed-loop response to a 20% increase in feed flow rate (including input and output uncertainties): —  $\mu$ -optimal controller; ---- PID controller.

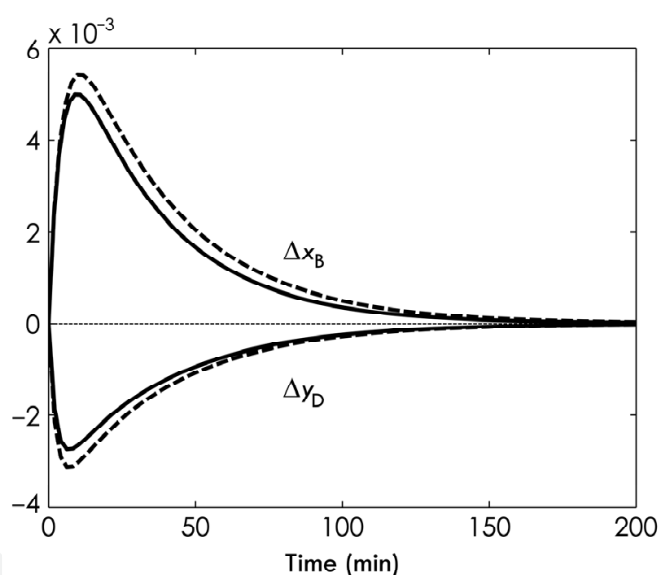


Fig. 19. Closed-loop response to a 20% increase in feed flow rate ( $\mu$ -optimal controller): — both input and output uncertainty; ---- input uncertainty only.

## 6. Discussion

The structured singular value ( $\mu$ ) is used to investigate the robust performance and robust stability of the PID controller. The control problem formulation used in this study is using weighted input and output uncertainties. Although other sources of uncertainty could be included, however, these two are the most severe uncertainties that may be considered. The inclusion of both input and output uncertainty prevents the control system from becoming sensitive to the uncertainties, as may happen with inverting controllers.

The solution of the problem leads to the inequality of Eq. (8). The numerical solution of this design task is difficult. At present, there is no direct method to synthesize a  $\mu$ -optimal



controller, however, combination of  $\mu$ -analysis and  $H_\infty$ -synthesis which called  $\mu$ -synthesis or DK-iteration, often yields good results. This algorithm has two drawbacks. Firstly, the algorithm cannot guarantee convergence, and secondly, the algorithm requires a scaling of the plant in each iteration step, which increases the order of the plant.

The  $\mu$  analysis advantageously avoids dealing explicitly with the bad condition of the plant. With the  $\mu$ -approach, the upper bound for the bandwidth of the control system is provided by the uncertainty model, whereas the lower bound is a matter of optimization.  $\mu$ -synthesis is ideally suited to deal with complex uncertainty models which takes into account such aspects as various operating points. A difficulty that one may encounter in synthesis of controller is high computation time, because the  $\mu$  approach requires scaling in each iteration. If, however, loop-shaping ideas are used to form the augmented plant,  $H_\infty$ -synthesis may be used to advantage. In this case, the results are as good as with the  $\mu$ -synthesis, but are obtained with less numerical efforts (Christen et al., 1997).

In this case study, the  $LV$ -configuration is used. The use of this configuration for columns with high condition number may be doubtful, but under special considerations, this configuration may yield acceptable performance. It is shown (Skogestad and Lundström, 1990) that it is possible to achieve good control behavior using the  $LV$ -configuration for two-point composition control provided measurement delays are not too large (typically less than 1–2 min). In addition, severe interactions and poor control often reported with the  $LV$ -configuration may be almost eliminated if the loops are tuned sufficiently tight. However, this does not imply that the  $LV$ -configuration is the best structure to use. Shinskey (1984) showed that the use of the  $(L/D)(V/B)$ -configuration is probably better in most cases, and in particular for columns with large reflux.

## 7. Concluding remarks

Based on a structured uncertainty model, which describes the column dynamics within the entire operating range, a decentralized PID controller is calculated using the  $\mu$ -synthesis technique. The controller was found to be robust with respect to model-plant mismatch, provided the RGA values of the column transfer function are not too large in the cross-over frequency range. The response of the system is improved by using a  $\mu$ -optimal controller.

In spite of high condition number of the process, nominal and robust performance is achieved by insertion of input and output uncertainties in the control system and using the structured singular value to synthesis the controller. Good set-point tracking and disturbance rejection of the controller is observed by simulations that carried out for the closed-loop system. It was also shown that good control performance can be obtained by using the  $LV$ -configuration which is difficult to implement for two-point control. The obtained results also verify the findings of Skogestad and Lundström (1990).

### Symbols

$B$	Bottom product
$C$	Controller
$D$	Distillate, scaling matrix
$\mathbf{D}$	Set of scaling matrices
$F$	Feed flow rate, Linear Fractional Transformation (LFT)
$g_L$	Liquid flow dynamics
$G$	Plant transfer function

<b>I</b>	Identity matrix
<i>L</i>	Reflux
<i>n</i>	Number of trays in the column
<i>N</i>	Lower LFT
<i>N<sub>T</sub></i>	Number of theoretical trays in the column
<b>P</b>	Set of all possible plants
<i>S</i>	Sensitivity function
<i>T</i>	Complementary sensitivity function
<i>u</i>	Uncertain input
$\tilde{u}$	Weighted input
<i>V</i>	Vapor boilup
<i>w</i>	Scalar weight, input signal
<i>W</i>	Diagonal matrix weight
<i>x<sub>B</sub></i>	Bottom composition
<i>y</i>	Output
$\tilde{y}$	Weighted output
<i>y<sub>D</sub></i>	Distillate composition
<i>z</i>	Output signal
<i>z<sub>F</sub></i>	Feed composition
$\ \cdot\ _1$	1-norm
$\ \cdot\ _\infty$	$\infty$ -norm

*Greek letters*

$\alpha$	Relative volatility
$\gamma$	Condition number
$\Delta$	Perturbation matrix
$\Lambda$	Relative gain array
$\lambda_{ij}$	<i>i, j</i> element of the RGA
$\mu$	Structured singular value (SSV)
$\sigma$	Singular value
$\tau$	Time constant
$\omega$	Frequency (rad/min)

*Subscripts*

<i>D</i>	Derivative
<i>I</i>	Integral
<i>l</i>	Lower, left
min	Minimized
NP	Nominal performance
<i>o</i>	Nominal
<i>P</i>	Performance
<i>r</i>	Right
RP	Robust performance
<i>u</i>	Input, upper
<i>y</i>	Output

## 8. References

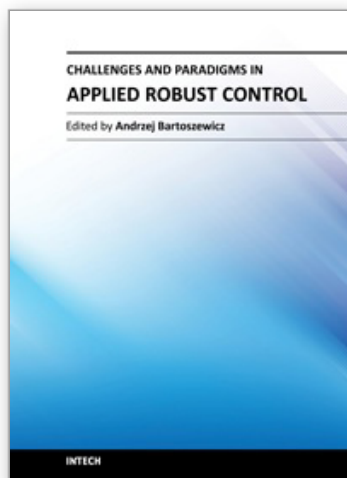
- Arkun, Y., Manousiouthakis, B. & Palazoglu, A. (1984). Robustness Analysis of Process Control Systems: A Case Study of Decoupling Control in Distillation. *Industrial and Engineering Chemistry Process Design and Development* 23(1), 93–101.
- Biswas, P.P., Ray, S. & Samanta, A.N. (2009). Nonlinear Control of High Purity Distillation Column under Input Saturation and Parametric Uncertainty. *Journal of Process Control* 19(1), 75–84.
- Böling, J.M. & Häggblom, K.E. (1996). Control-Relevant Identification of an Ill-Conditioned Distillation Column. *Proceedings of IEEE Conference on Control Applications*, Dearborn, MI, 570–575.
- Bristol, E.H. (1966). On A New Measure of Interactions for Multivariable Process Control. *IEEE Transactions on Automatic Control* 11(1), 133–134.
- Chen, J. (1995). Sensitivity Integral Relations and Design Trade-Offs in Linear Multivariable Feedback Systems. *IEEE Transactions on Automatic Control* 40(10), 1700–1716.
- Christen, U., Musch, H.E. & Steiner, M. (1997). Robust Control of Distillation Columns:  $\mu$ -vs.  $H_\infty$ -Synthesis. *Journal of Process Control* 7(1), 19–30.
- Doyle, J.C. (1982). Analysis of Feedback Systems with Structured Uncertainties. *IEEE Proceedings* 129(6), 242–250.
- Doyle, J.C. & Stein, G. (1981). Multivariable Feedback Design: Concepts for A Classical/Modern Synthesis. *IEEE Transactions on Automatic Control* 26(1), 4–16.
- Finco, M.V., Luyben, W.L. & Polleck, R.E. (1989). Control of Distillation Columns with Low Relative Volatilities. *Industrial and Engineering Chemistry Research* 28(1), 75–83.
- Georgiou, A., Georgakis, C. & Luyben, W.L. (1988). Nonlinear Dynamic Matrix Control for High-Purity Distillation Columns. *AIChE Journal* 34(8), 1287–1298.
- Glover, K. (1984). All Optimal Hankel-Norm Approximations of Linear Multivariable Systems and Their  $L^\infty$ -Error Bounds. *International Journal of Control* 39(6), 1115–1193.
- Grosdidier, P., Morari, M. & Holt, B.R. (1985). Closed-Loop Properties from Steady-State Gain Information. *Industrial and Engineering Chemistry Fundamentals* 24(2), 221–235.
- Heath, J.A., Kookos, I.K. & Perkins, J.D. (2000). Process Control Structure Selection Based on Economics. *AIChE Journal* 46(10), 1998–2016.
- Hori, E.S. & Skogestad, S. (2007). Selection of Control Structure and Temperature Location for Two-Product Distillation Columns. *Chemical Engineering Research and Design* 85(3), 293–306.
- Hurowitz, S., Anderson, J., Duvall, M. & Riggs, J.B. (2003). Distillation Control Configuration Selection. *Journal of Process Control* 13(4), 357–362.
- Jacobsen, E.W. & Skogestad, S. (1994). Inconsistencies in Dynamic Models for Ill-Conditioned Plants: Application to Low-Order Models of Distillation Columns. *Industrial and Engineering Chemistry Research* 33(3), 631–640.
- Kariwala, V., Skogestad, S. & Forbes, J.F. (2006). Relative Gain Array for Norm-Bounded Uncertain Systems. *Industrial and Engineering Chemistry Research* 45(5), 1751–1757.
- Luyben, W.L. (1987). Sensitivity of Distillation Relative Gain Arrays to Steady-State Gains. *Industrial and Engineering Chemistry Research* 26(10), 2076–2078.

- Luyben, W.L. (2005). Effect of Feed Composition on the Selection of Control Structures for High-Purity Binary Distillation. *Industrial and Engineering Chemistry Research* 44(20), 7800–7813.
- McDonald, K.A., Palazoglu, A. & Bequette, B.W. (1988). Impact of Model Uncertainty Descriptions for High-Purity Distillation Control. *AIChE Journal* 34(12), 1996–2004.
- Packard, A. & Doyle, J.C. (1993). The Complex Structured Singular Value. *Automatica* 29(1), 71–109.
- Razzaghi, K. & Shahraki, F. (2005). Robust Multivariable PID-Controller Design for a High-Purity Distillation Column Using  $\mu$ -Synthesis. *Proceedings of the 55th Canadian Chemical Engineering Conference*, Toronto, Ontario, Canada, October 16–19.
- Razzaghi, K. & Shahraki, F. (2007). Robust Control of an Ill-Conditioned Plant Using  $\mu$ -Synthesis: A Case Study for High-Purity Distillation. *Chemical Engineering Science* 62(5), 1543–1547.
- Razzaghi, K. & Shahraki, F. (2009). A Survey for the Selection of Control Structure for Distillation Columns Based on Steady State Controllability Indexes. *Iranian Journal of Chemical Engineering* 6(2), 29–36.
- Rivera, D.E., Morari, M. & Skogestad, S. (1986). Internal Model Control: 4. PID Controller Design. *Industrial and Engineering Process Design and Development* 25(1), 252–265.
- Shin, J., Seo, H., Han, M. & Park, S. (2000). A Nonlinear Profile Observer Using Tray Temperatures for High-Purity Binary Distillation Column Control. *Chemical Engineering Science* 55(4), 807–816.
- Shinskey, F.G. (1984). *Distillation Control*. 2nd ed., McGraw-Hill, New York.
- Skogestad, S. & Lundström, P. (1990). Mu-Optimal LV-Control of Distillation Columns. *Computers and Chemical Engineering* 14(4–5), 401–413.
- Skogestad, S., Lundström, P. & Jacobsen, E.W. (1990). Selecting the Best Distillation Control Configuration. *AIChE Journal* 36(5), 753–764.
- Skogestad, S. & Morari, M. (1987a). Control Configuration Selection for Distillation Columns. *AIChE Journal* 33(10), 1620–1635.
- Skogestad, S. & Morari, M. (1987b). Implications of Large RGA Elements on Control Performance. *Industrial and Engineering Chemistry Research* 26(11), 2323–2330.
- Skogestad, S. & Morari, M. (1988). LV-Control of a High-Purity Distillation Column. *Chemical Engineering Science* 43(1), 33–48.
- Skogestad, S., Morari, M. & Doyle, J.C. (1988). Robust Control of Ill-Conditioned Plants: High-Purity Distillation. *IEEE Transactions on Automatic Control* 33(12), 1092–1105.
- Sriniwas, G.R., Arkun, Y., Chien, I.-L. & Ogunnaike, B.A. (1995). Nonlinear Identification and Control of a High-Purity Distillation Column: A Case Study. *Journal of Process Control* 5(3), 149–162.
- Stichlmair, J. (1995). Conceptual Design of the Control Configurations of Distillation Columns. *Chemical Engineering and Processing* 34(2), 61–69.
- Trentacapilli, M., Semino, D. & Brambilla, A. (1997). High-Purity Distillation Control: Some Issues Regarding the Application of Multivariable Control. *Distillation and Absorption '97*, Vol. 1, IChemE, 313–322.

- Waller, J.B., Sågfors, M. & Waller, K.E. (1994). Ill-Conditionedness and Process Directionality – The Use of Condition Numbers in Process Control. *Proceedings of IFAC Symposium*, Kyoto, Japan, 465–470.
- Zhou, K., Doyle, J.C. & Glover, K. (1996). *Robust and Optimal Control*. Prentice-Hall, Inc., Upper Saddle River, New Jersey.

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The main objective of this book is to present important challenges and paradigms in the field of applied robust control design and implementation. Book contains a broad range of well worked out, recent application studies which include but are not limited to H-infinity, sliding mode, robust PID and fault tolerant based control systems. The contributions enrich the current state of the art, and encourage new applications of robust control techniques in various engineering and non-engineering systems.

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